物理点ゲージ配位によるS=-1セクタのバリオン間力

根村英克 (RCNP)

arXiv:1702.00734[hep-lat]

Plan of research

QCD

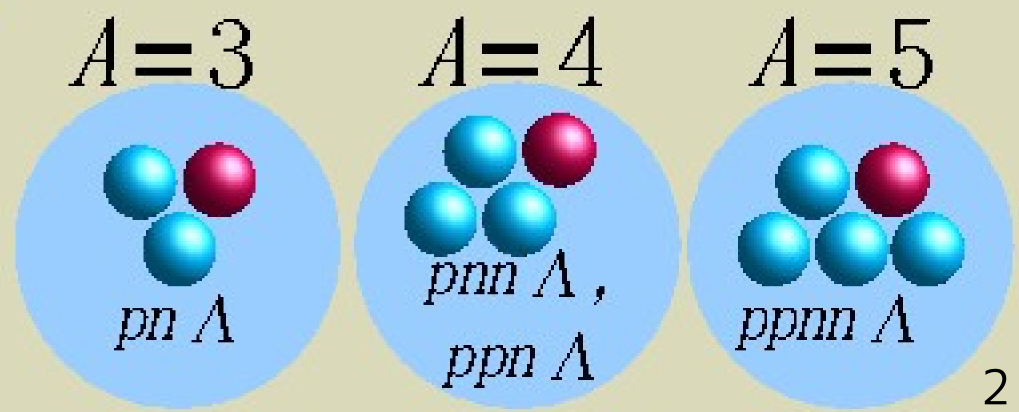
Baryon interaction

J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

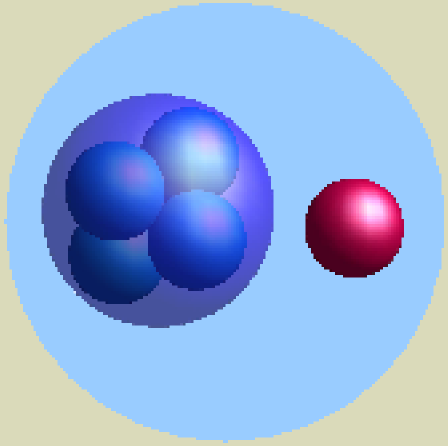
Neutron star and supernova



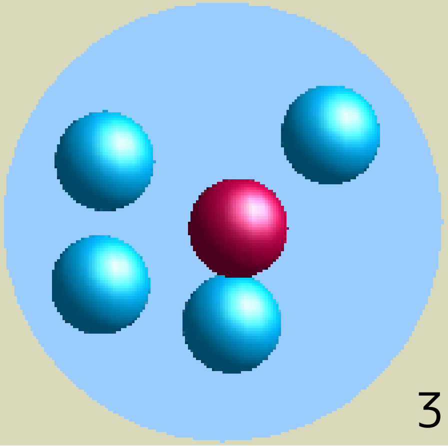
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^{4}\text{He}) + B_{\Lambda}(^{5}_{\Lambda}\text{He})$

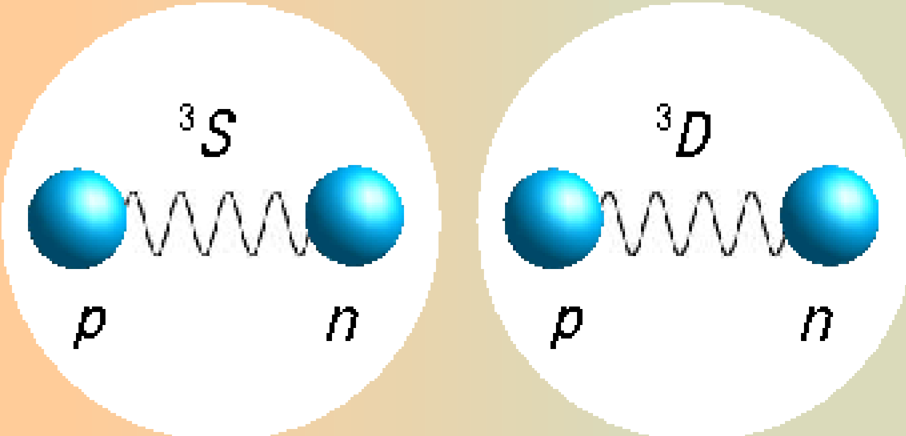
- A conventional picture:
  $B(\text{total})$
  $= B(^{4}\text{He}) + B_{\Lambda}(^{5}_{\Lambda}\text{He})$
  = 28+3 MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^{4}\text{He}) - \Delta E_c) + (B_{\Lambda}(^{5}_{\Lambda}\text{He}) + \Delta E_c)$
  = ??+?? MeV.

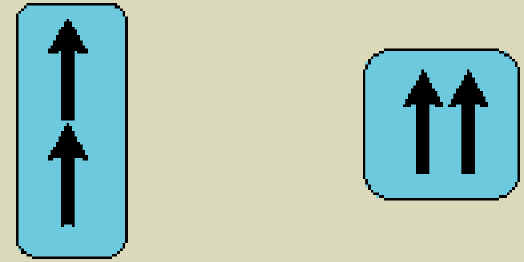


Comparison between d=p+n and core+Y



Tensor force makes bound state!

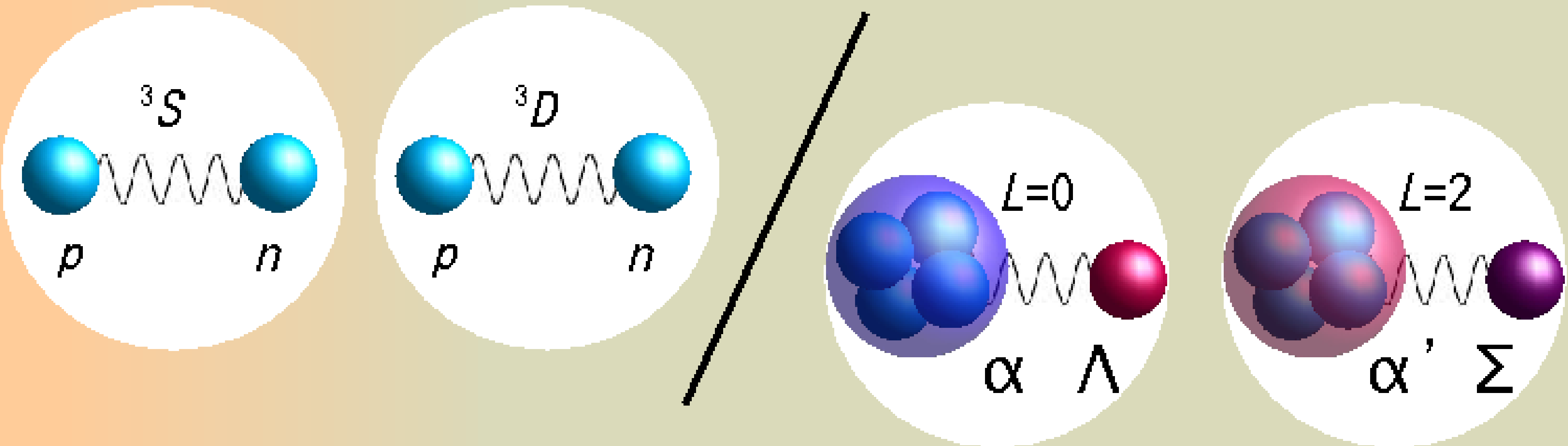
$$S_{12}=3(\boldsymbol{\sigma}_1\cdot\boldsymbol{r})(\boldsymbol{\sigma}_2\cdot\boldsymbol{r})-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_1$$



attractive repulsive

	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

Comparison between d=p+n and core+Y



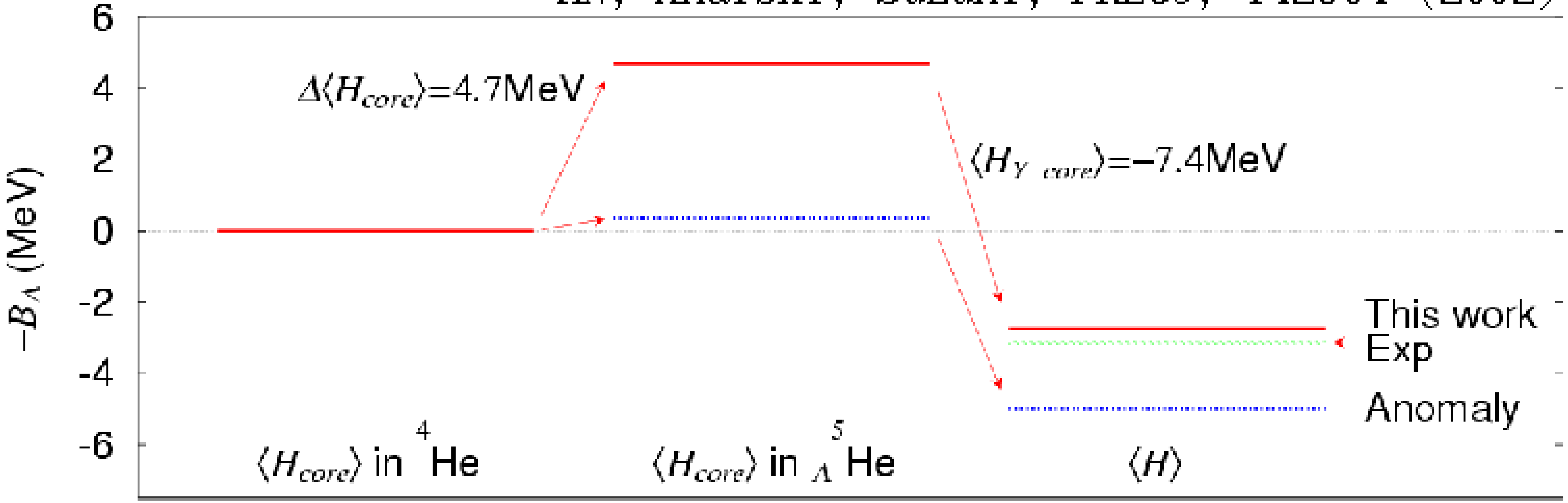
	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\mathrm{central}) \rangle$ (MeV)	$\langle V_{NN}(\mathrm{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\mathrm{LS}) \rangle$ (MeV)
AV8	8.57	11.31	−4.46	−16.64	−1.02
G3RS	10.84	5.64	−7.29	−11.46	0.00

	$\langle T_{Y\text{-}c} \rangle_\Lambda$	$\langle T_{Y\text{-}c} \rangle_\Sigma + \Delta \langle H_c \rangle$	$\langle V_{YN}$(のこり)$\rangle$	$2\langle V_{\Lambda N\text{-}\Sigma N}(\mathrm{tensor}) \rangle$
${}_\Lambda^5\mathrm{He}$	9.11	3.88+4.68	−0.86	−19.51
${}_\Lambda^4\mathrm{H}^*$	5.30	2.43+2.02	0.01	−10.67
${}_\Lambda^4\mathrm{H}$	7.12	2.94+2.16	−5.05	−9.22

HN, Akaishi, Suzuki, PRL89, 142504 (2002).

Rearrangement effect of $^{5}_{\Lambda}$He

HN, Akaishi, Suzuki, PRL89, 142504 (2002).



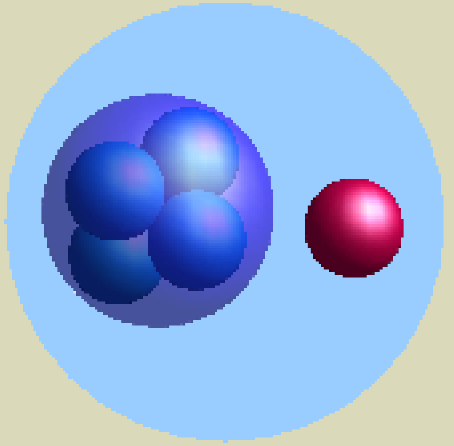
$$H = \sum_{i=1}^{A}\left(m_i c^2 + \frac{\boldsymbol{p}_i^2}{2\,\mathrm{m}_i}\right) - T_{CM} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} + \sum_{i=1}^{A-1} v_{iY}^{(NY)} = H_{core} + H_{Y-core} ,$$

$$H_{core} = \sum_{i=1}^{A-1}\frac{\boldsymbol{p}_i^2}{2\,\mathrm{m}_N} - \frac{\left(\sum_{i=1}^{A-1}\boldsymbol{p}_i\right)^2}{2(A-1)\,m_N} + \sum_{i<j}^{A-1} v_{ij}^{(NN)} = T_{core} + V_{NN} .$$

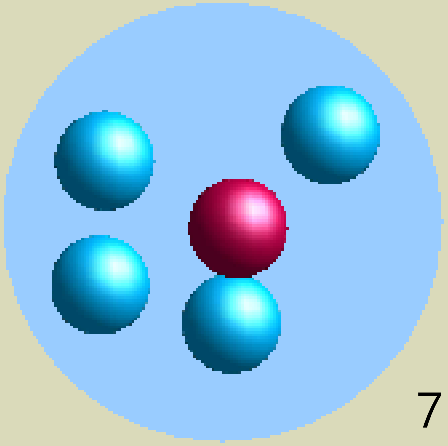
What is realistic picture of hypernuclei?

- $B(\text{total}) = B(^{4}\text{He}) + B_{\Lambda}(^{5}_{\Lambda}\text{He})$

- A conventional picture:
  $B(\text{total})$
  $= B(^{4}\text{He}) + B_{\Lambda}(^{5}_{\Lambda}\text{He})$
  $= 28 + 3$ MeV.



- A (probably realistic) picture:
  $B(\text{total})$
  $= (B(^{4}\text{He}) - \Delta E_c) + (B_{\Lambda}(^{5}_{\Lambda}\text{He}) + \Delta E_c)$
  $= 24 + 7$ MeV.



FY calculation with and w/o 3NF

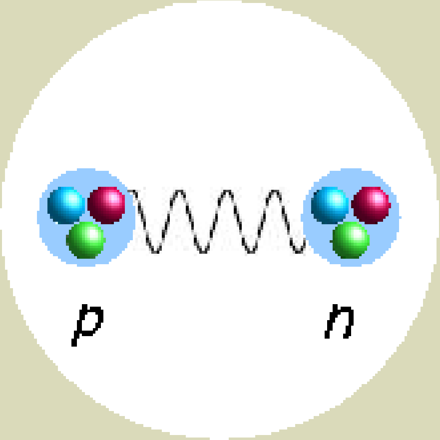
- Three nucleon force does not change the B_Λ so much.
  - A. Nogga, *et al.*, PRL **88**, 172501 (2002).

TABLE II. *NN* and 3*N* interaction dependence of the ${}^4_\Lambda$He SE's E^Λ_{sep} and the 0^+-1^+ splitting Δ. We show results for different combinations of *YN*, *NN*, and 3*N* forces (*YNF*, *NNF*, and 3*NF*). All energies are given in MeV.

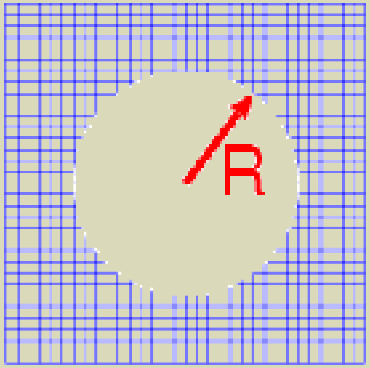
YNF	*NNF*	3*NF*	$E^\Lambda_{sep}(0^+)$	$E^\Lambda_{sep}(1^+)$	Δ
SC97*e*	Bonn *B*	···	1.66	0.80	0.84
SC97*e*	Nijm 93	···	1.54	0.72	0.79
SC97*e*	Nijm 93	TM	1.56	0.70	0.82
SC89	Bonn *B*	···	2.25	···	···
SC89	Nijm 93	···	2.14	0.02	2.06
SC89	Nijm 93	TM	2.19	···	···

172501-2

Lattice QCD calculation



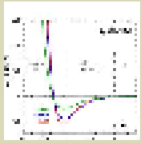
Multi-hadron on lattice

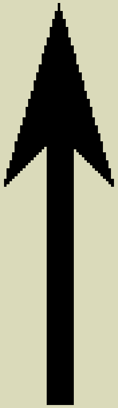


i) basic procedure:

asymptotic region

--> phase shift

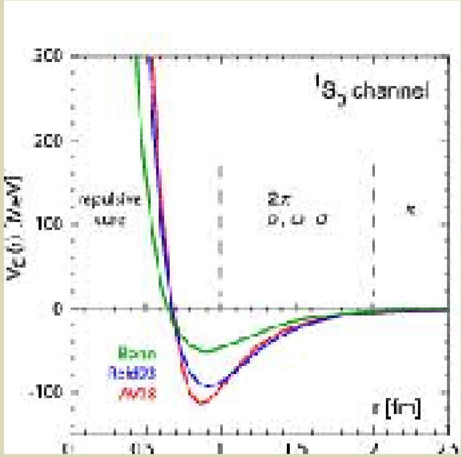




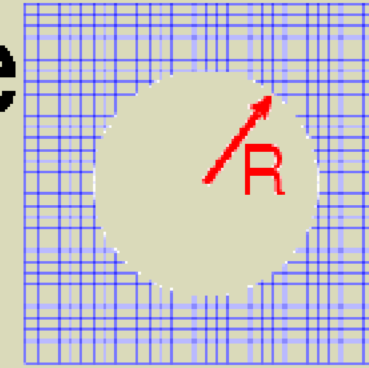
ii) HAL's procedure:

interacting region

--> potential



Multi-hadron on lattice



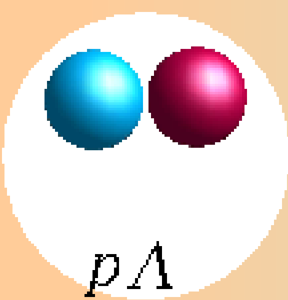
Lattice QCD simulation

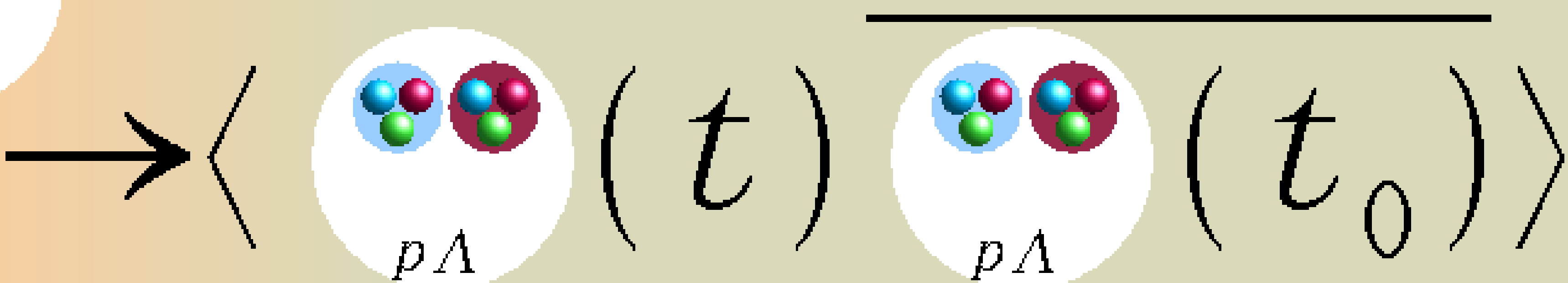
$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\partial_\mu - g t^a A^a_\mu) q - m\bar{q}q$$

$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U)$$

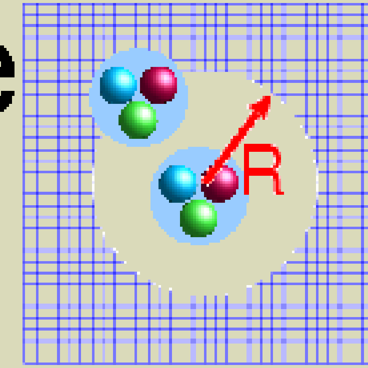
$$= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U))$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$



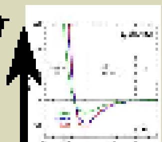


Multi-hadron on lattice



Lattice QCD simulation

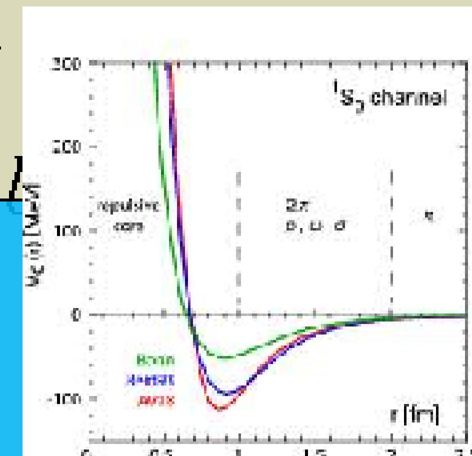
$$L = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^{\mu}(i\partial_{\mu} - g t^{a} A^{a}_{\mu}) q - m\bar{q}q$$



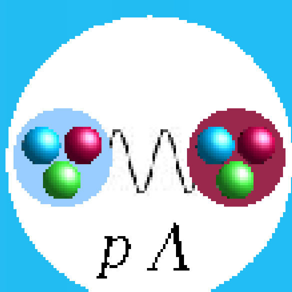
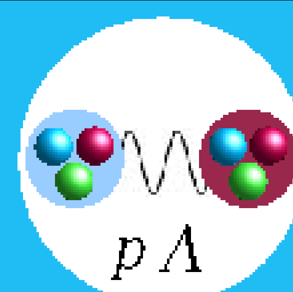
$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)} O(\bar{q}, q$$

$$= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$



$$F^{(JM)}_{\alpha\beta}(\vec{r}, t - t_0)$$

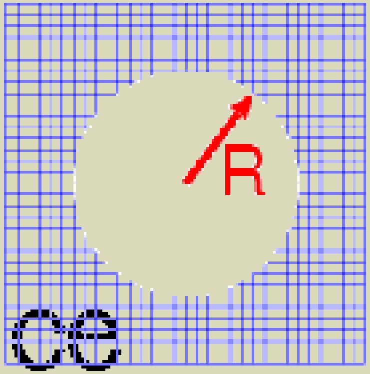
$\rightarrow \langle$  $(\vec{r}, t)$  $(t_0)\rangle$

Calculate the scattering state

Multi-hadron on lattice

ii) HAL's procedure:

make better use of the lattice output ! (wave function)



interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

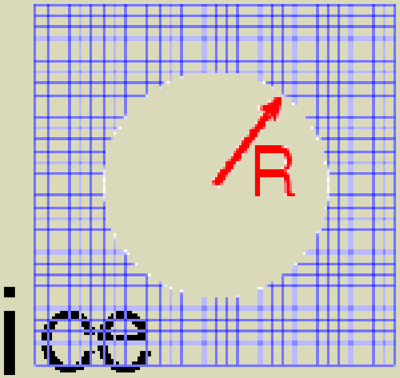
NOTE:
> Potential is not a direct experimental observable.
> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

Multi-hadron on lattice

ii) HAL's procedure:

make better use of the lattice



output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

=>

> Phase shift
> Nuclear many-body problems

In lattice QCD calculations, we compute the normalized four-point correlation function

$$R^{(J,M)}_{\alpha\beta}(\vec{r}, t \quad t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X} + \vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}^{(J,M)}_{B_3 B_4}(t_0)} \right| 0 \right\rangle / \exp\{ \quad (m_{B_1} + m_{B_2})(t \quad t_0)\},$$

$$p = \varepsilon_{abc}\left(u_a C \gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C \gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma \quad = \quad \varepsilon_{abc}\left(u_a C \gamma_5 s_b\right) u_c, \qquad \Sigma \quad = \quad \varepsilon_{abc}\left(d_a C \gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 - \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda - \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C \gamma_5 s_b\right) s_c, \qquad \Xi \quad = \quad \varepsilon_{abc}\left(d_a C \gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u - \varepsilon_{abc}\left(d_a C \gamma_5 s_b\right) u_c, \quad X_d - \varepsilon_{abc}\left(s_a C \gamma_5 u_b\right) d_c, \quad X_s - \varepsilon_{abc}\left(u_a C \gamma_5 d_b\right) s_c, \tag{6}$$

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R^{(J,M)}_{\alpha\beta}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle e^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)}$$
$$+O(e^{-(E_{th}-m_{B_1}-m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'} | 0 \rangle$. At moderately large $t-t_0$ where the inelastic contribution above the pion production $O(e^{-(E_{th}-2m_N)(t-t_0)}) = O(e^{-m_\pi(t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}')$. In lattice QCD calculations

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method [20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R^{(J,M)}_{\alpha\beta}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}^{(J,M)}_{B_3B_4}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \right| E_n \right\rangle e^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)}$$

$$+ O(e^{-(E_{th} - m_{B_1} - m_{B_2})(t-t_0)}), \quad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P^{(JM)}_{\alpha'\beta'} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'}(0) \rangle$. At moderately large $t-t_0$ where the inelastic contribution above the pion production $O(e^{-(E_{th}-2m_N)(t-t_0)}) = O(e^{-m_\pi(t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r' \, U(\vec{r}, \vec{r}') R(\vec{r}')$. In lattice QCD calculations

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method [20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

The potential is obtained at moderately large imaginary time; no single state saturation is required.

$$R_{\alpha\beta}^{(J,M)}(\vec{r}, t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, t) B_{2,\beta}(\vec{X}, t) \overline{\mathcal{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r}, 0) B_{2,\beta}(\vec{X}, 0) \right| E_n \right\rangle e^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)}$$

$$+O(e^{-(E_{th}-m_{B_1}-m_{B_2})(t-t_0)}), \qquad (4)$$

where E_n ($|E_n\rangle$) is the eigen-energy (eigen-state) of the six-quark system and $A_n = \sum_{\alpha'\beta'} P_{\alpha'\beta'}^{(JM)} \langle E_n | \overline{B}_{4,\beta'} \overline{B}_{3,\alpha'}, 0 \rangle$. At moderately large $t-t_0$ where the inelastic contribution above the pion production $O(e^{-(E_{th}-2m_N)(t-t_0)}) = O(e^{-m_\pi(t-t_0)})$ becomes exiguous, we can construct the non-local potential U through $\left(\frac{\nabla^2}{2\mu} - \frac{k^2}{2\mu}\right) R(\vec{r}) = \int d^3r'\, U(\vec{r}, \vec{r}') R(\vec{r}')$. In lattice QCD calculations

... The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm even for the physical pion mass. Furthermore, no single state saturation between the ground state and the first excited states, $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu L^2))^{-1}$, is required for the present HAL QCD method [20], which becomes $((2\pi)^2/(2\mu L^2))^{-1} \simeq 4.6$ fm if we consider $L \sim 6$ fm and $m_N \simeq 1$ GeV. In Ref. [14], the validity of the velocity expansion of the NN potential has been examined in quenched lattice QCD simulations at $m_\pi \simeq 530$ MeV and $L \simeq 4.4$ fm.

格子ＱＣＤによるポテンシャル導出の手順(超簡略版)

(1) 4点相関関数を計算する。

$$F_{\alpha\beta,JM}^{\langle B_1 B_2 \overline{B_3 B_4}\rangle}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}_{B_3 B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle, \qquad (2.3)$$

(2) 時間依存法を使うためにしきい値だけ時間相関をずらす

$$R_{\alpha\beta,JM}^{\langle B_1 B_2 B_3 B_4\rangle}(\vec{r},t-t_0) \equiv e^{(m_{B_1}+m_{B_2})(t-t_0)} F_{\alpha\beta,JM}^{\langle B_1 B_2 B_3 B_4\rangle}(\vec{r},t-t_0)$$
$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0) B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle e^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(e^{-(E_{th}-m_{B_1}-m_{B_2})(t-t_0)}) \qquad (2.4)$$

(3) チャネルごとにしきい値が異なるので、それを考慮した時間依存型Schroedinger方程式からポテンシャルを求める

$$\left(\frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t}\right) R_{\lambda\varepsilon}(\vec{r},t) \sim V_{\lambda\lambda'}^{(\mathrm{LO})}(\vec{r})\,\theta_{\lambda\lambda'} R_{\lambda'\varepsilon}(\vec{r},t), \text{ with } \theta_{\lambda\lambda'} = e^{-(m_{B_1}+m_{B_2}-m_{B'_1}-m_{B'_2})(t-t_0)}.$$

(※) “moderately large imaginary time” で計算を行う

(※※) 2種類の励起状態を区別している

[1]The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm. In addition, no single state saturation between the ground state and the excited states with respect to the relative motion, e.g., $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu(La)^2))^{-1} \simeq 8.0$ fm, is required for the HAL QCD method[13].

$$\begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{V_{\lambda\lambda'}^{(0)}(r) + V_{\lambda\lambda'}^{(\sigma)}(r) + V_{\lambda\lambda'}^{(T)}(r) S_{12}\right\} \theta_{\lambda\lambda'} R_{\lambda'\varepsilon}(\vec{r},t-t_0) = \begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{\frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t}\right\} R_{\lambda\varepsilon}(\vec{r},t-t_0), \qquad (2.7)$$

An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r})+\int d^3r' U(\vec{r},\vec{r}')R(t,\vec{r}')=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \frac{k^2}{2\mu}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

Determination of baryon-baryon potentials at nearly physical point

Effective block algorithm for various baryon-baryon correlators

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm

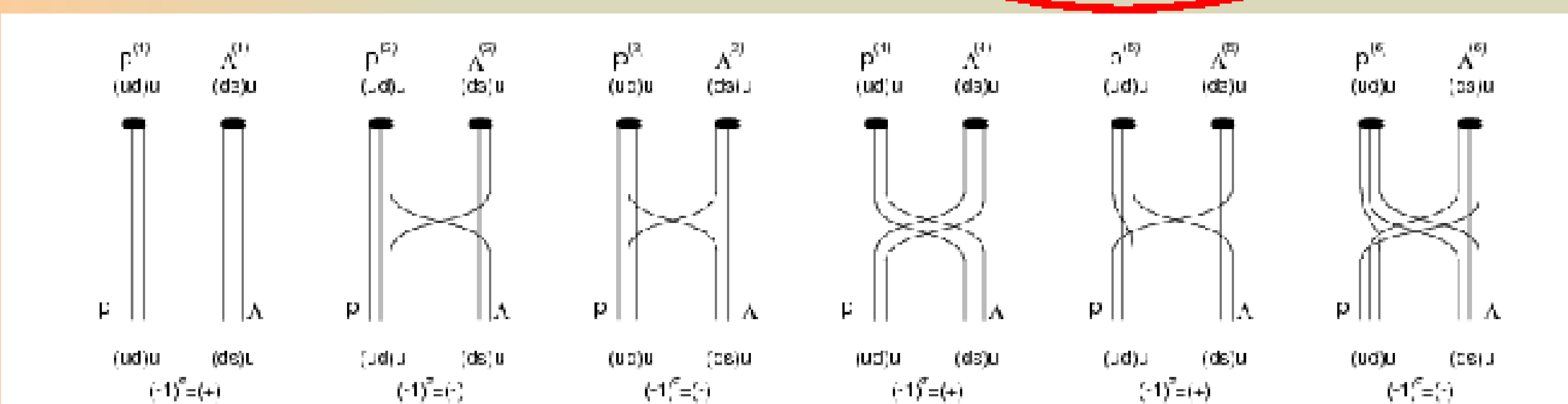
$$1+N_c^{2}+N_c^{2}N_\alpha^{2}+N_c^{2}N_\alpha^{2}+N_c^{2}N_\alpha+N_c^{2}N_\alpha=370$$

In an intermediate step:

$$(N_c!\,N_\alpha)^{B}\times N_u!\,N_d!\,N_s!\times 2^{N_\Lambda+N_{\Sigma^0}-B}=3456$$

In a naïve approach:

$$(N_c!\,N_\alpha)^{2B}\times N_u!\,N_d!\,N_s!=3,981,312$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle,\ \langle p\Lambda\overline{\Sigma^+ n}\rangle,\ \langle p\Lambda\Sigma^0 p\rangle,\\
&\langle \Sigma^+ np\Lambda\rangle,\ \langle \Sigma^+ n\Sigma^+ n\rangle,\ \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle,\ \langle \Sigma^0 p\overline{\Sigma^- n}\rangle,\ \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle, && \langle \Lambda\Lambda p\Xi^-\rangle, && \langle \Lambda\Lambda\overline{n\Xi^0}\rangle, && \langle \Lambda\Lambda\Sigma^+\Sigma^-\rangle, && \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle, && \\
&\langle p\Xi^-\overline{\Lambda\Lambda}\rangle, && \langle p\Xi^-\overline{p\Xi^-}\rangle, && \langle p\Xi^-\overline{n\Xi^0}\rangle, && \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle, && \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle, && \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle n\Xi^0\overline{\Lambda\Lambda}\rangle, && \langle n\Xi^0\overline{p\Xi^-}\rangle, && \langle n\Xi^0\overline{n\Xi^0}\rangle, && \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle, && \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle, && \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^+\Sigma^-\Lambda\Lambda\rangle, && \langle \Sigma^-\Sigma^-\overline{p\Xi^-}\rangle, && \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle, && \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle, && \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle, && \langle \Sigma^-\Sigma^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle, && \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle, && \langle \Sigma^0\Sigma^0 n\Xi^0\rangle, && \langle \Sigma^0\Sigma^0\overline{\Sigma^-\Sigma^-}\rangle, && \langle \Sigma^0\Sigma^0\Sigma^0\Sigma^0\rangle, && \\
& && \langle \Sigma^0\Lambda p\Xi^-\rangle, && \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle, && \langle \Sigma^0\Lambda\Sigma^+\Sigma^-\rangle, && && \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{aligned} \tag{4.3}$$

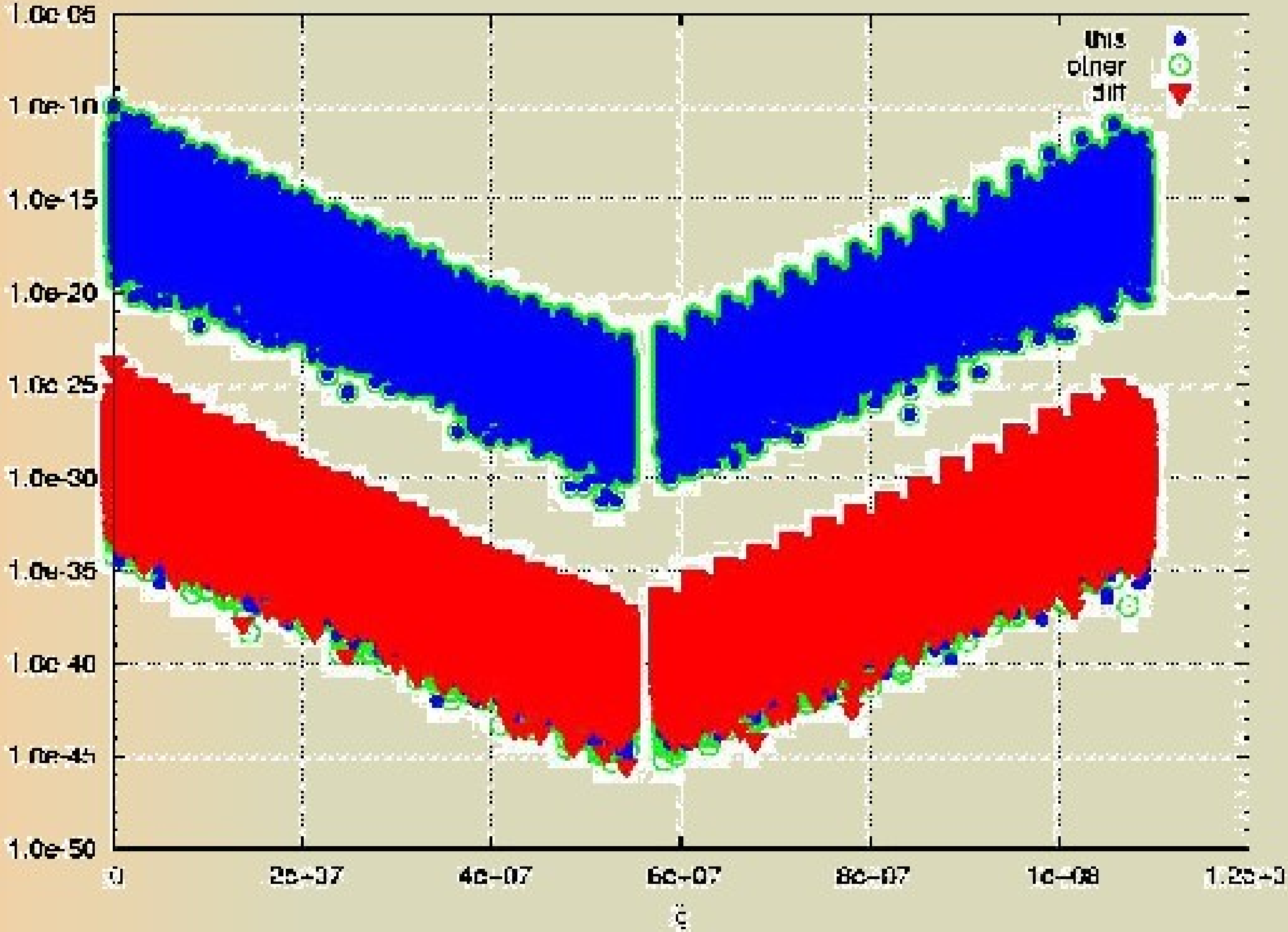
$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle,\ \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^-\Xi^0\Xi^-\Lambda\rangle,\ \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle,\ \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

Make better use of the computing resources!

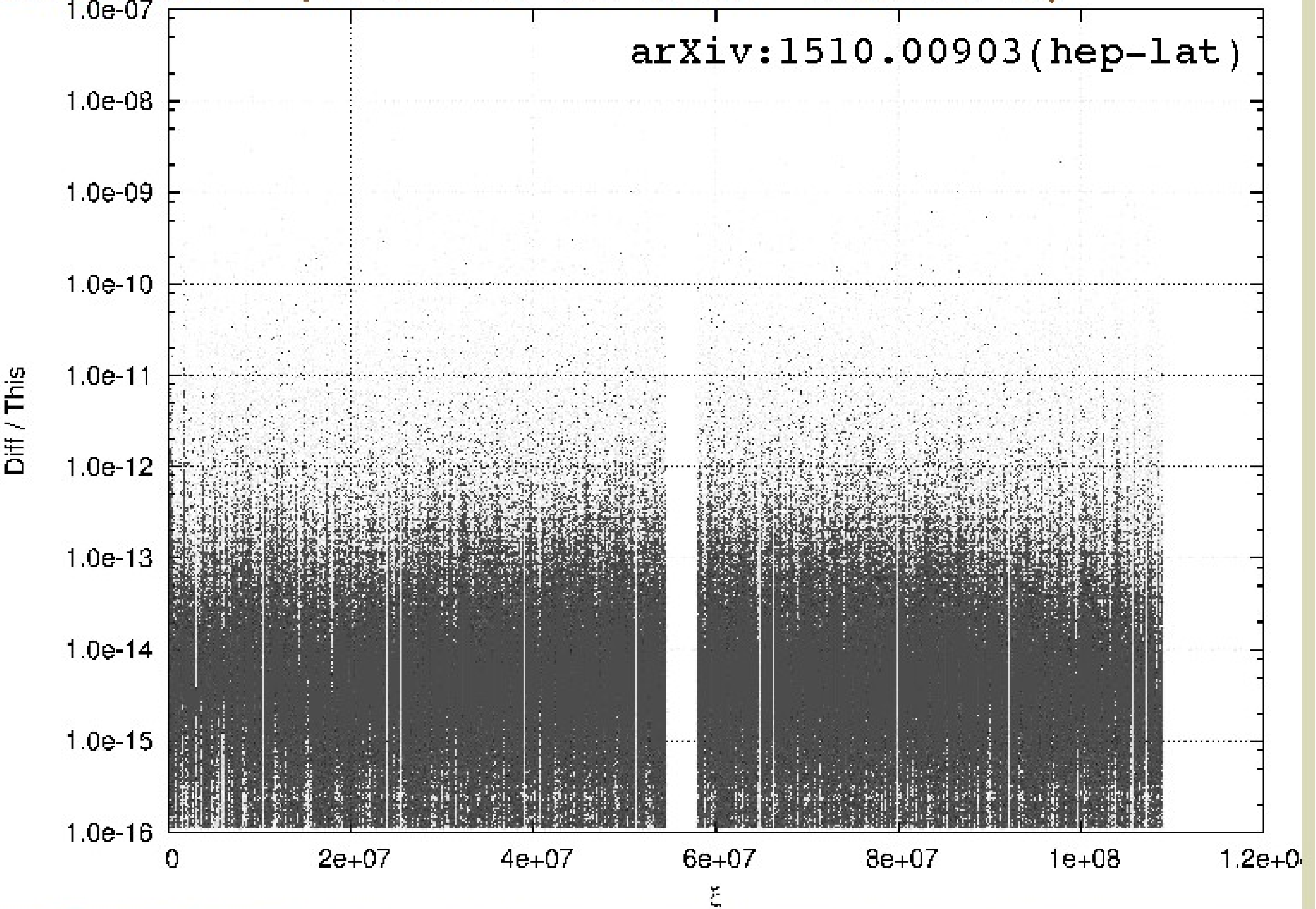
HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Benchmark (from NN to XiXi channels)



numerical results of the correlators of entire 52 channels from NN to $\Xi\Xi$ systems given in Eqs. (32)–(36), over 31 time-slices, 16^3 points for spatial, and 2^4 points for the spin degrees of freedom, obtained by using this effective block algorithm (dot) and by using the unified contraction algorithm (open circle) as a function of one-dimensionally aligned data point $\xi = \tilde{\alpha} + 2(\tilde{\beta} + 2(\tilde{\alpha}' + 2(\tilde{\beta}' + 2(x + 16(y + 16(z + 16(c + 52((t - t_0 + T) \bmod T)))))))$, where $c = 0, \cdots, 51$ selects one of the 52 channels. The absolute value of their difference is also shown (triangle).

Benchmark (from NN to XiXi channels)



$$\xi=\alpha+2(\beta+2(\alpha'+2(\beta'+2(x+16(y+16(z+16(c+52(t-t_0))))))))$$

Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (ρ=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \to (8\text{fm})^4$
- m_π=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc
  (Nsrc=4 → 20 → 52 → 96 (2015FY+))

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on:
Aoki, et al., Proc.Japan Acad. B87 (2011) 509.
Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015+) < 0.05 (==>0.2) ⇒ 0.54

$\Lambda N-\Sigma N\ (I=1/2)$

$V_C(^1S_0)$ $V_C(^3S_1-{}^3D_1)$ $V_T(^3S_1-{}^3D_1)$

$\Sigma N\ (I=3/2)$

$V_C(^1S_0)$ $V_C(^3S_1-{}^3D_1)$ $V_T(^3S_1-{}^3D_1)$

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on:
Aoki, et al., Proc.Japan Acad. B87 (2011) 509.
Sasaki, et al., PTEP 2015 (2015) no.11, 113B01.
Ishii, et al., JPS meeting, March (2016).

#stat: (this/scheduled in FY2015+) < 0.05 (==>0.2) ⇨ 0.54

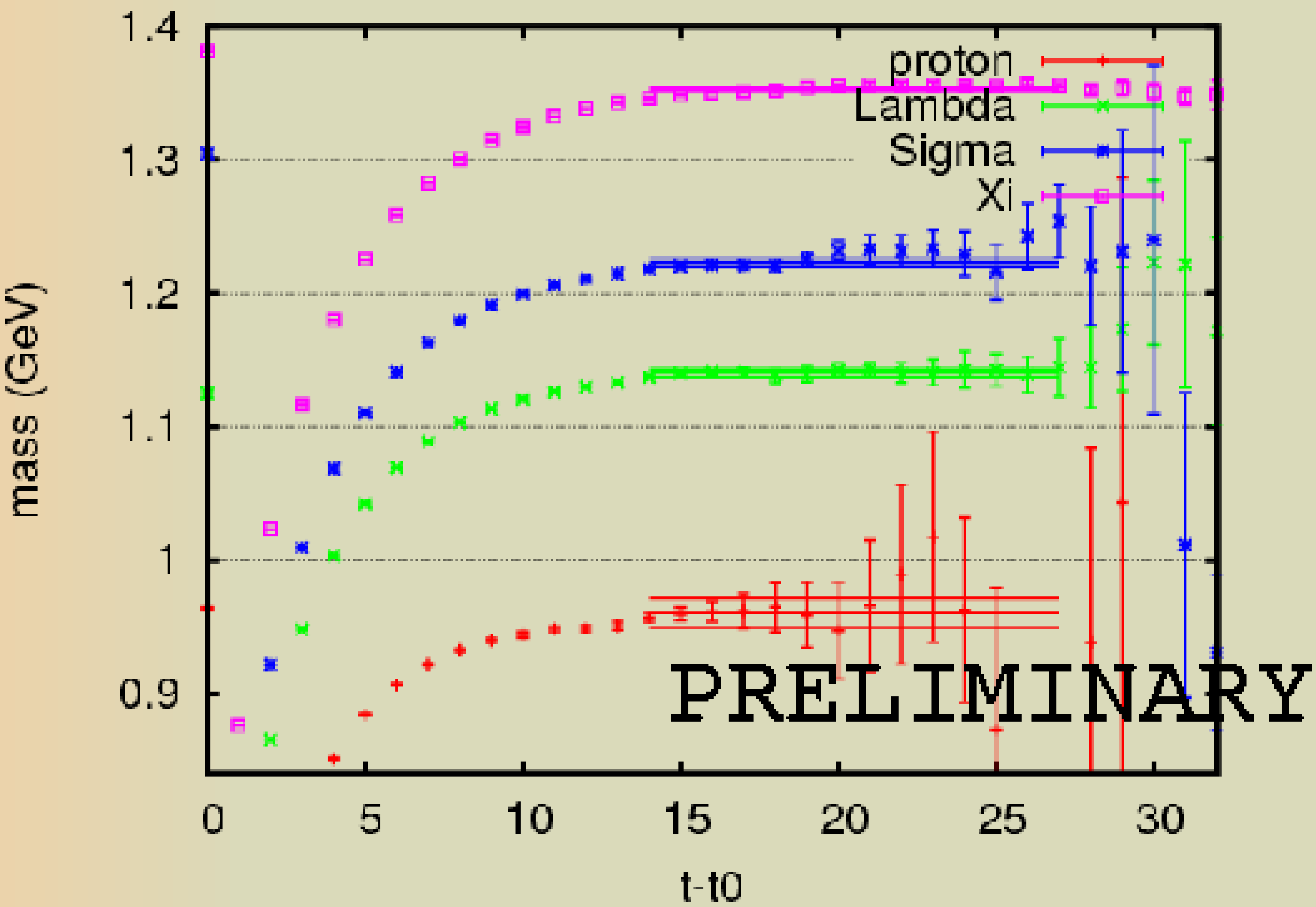
$\Lambda N - \Sigma N \; (I = 1/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - ^3D_1)$ $V_T(^3S_1 - ^3D_1)$

$\Sigma N \; (I = 3/2)$

$V_C(^1S_0)$ $V_C(^3S_1 - ^3D_1)$ $V_T(^3S_1 - ^3D_1)$

Effective mass plot of the single baryon's correlation function



Potentials obtained at t-t0 = 5 to 12 will be shown.

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for $S=-1$ two-baryon (BB) system

$S=-1$

I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
$\frac{1}{2}$	0	$N\Lambda$	1	0 $\frac{10}{9}$
		$N\Sigma$	$\frac{1}{9}$	
$\frac{1}{2}$	1	$N\Lambda$	1	$\frac{8}{9}$ $\frac{10}{9}$
		$N\Sigma$	1	
$\frac{3}{2}$	0	$N\Sigma$	$\frac{10}{9}$	
$\frac{3}{2}$	1	$N\Sigma$	$\frac{2}{9}$	

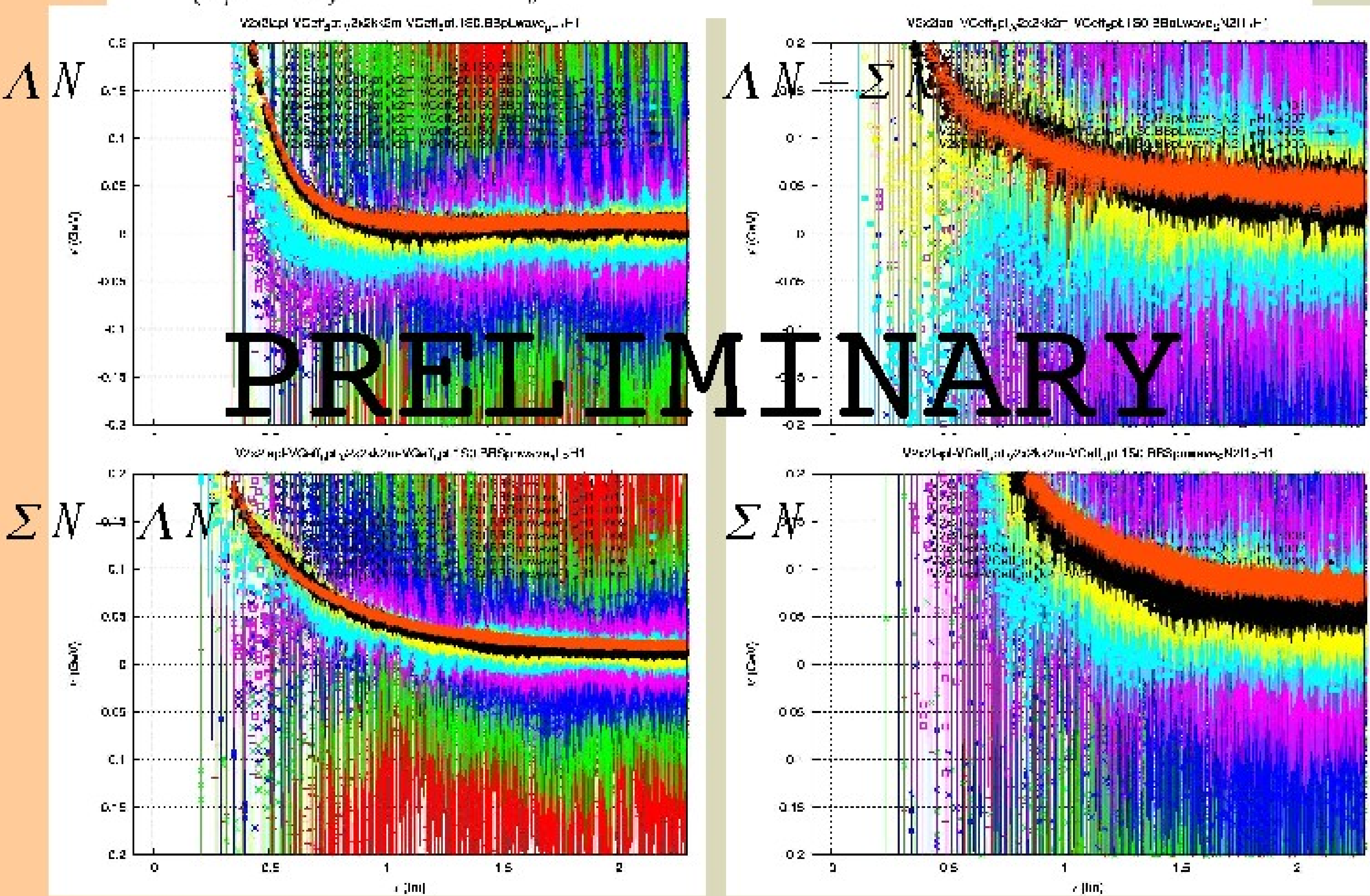
Eigenvalues of single and coupled channels are given.

Oka, Shimizu and Yazaki (1987)

Very preliminary result of LN potential at the physical point

$V_C(^1S_0)$

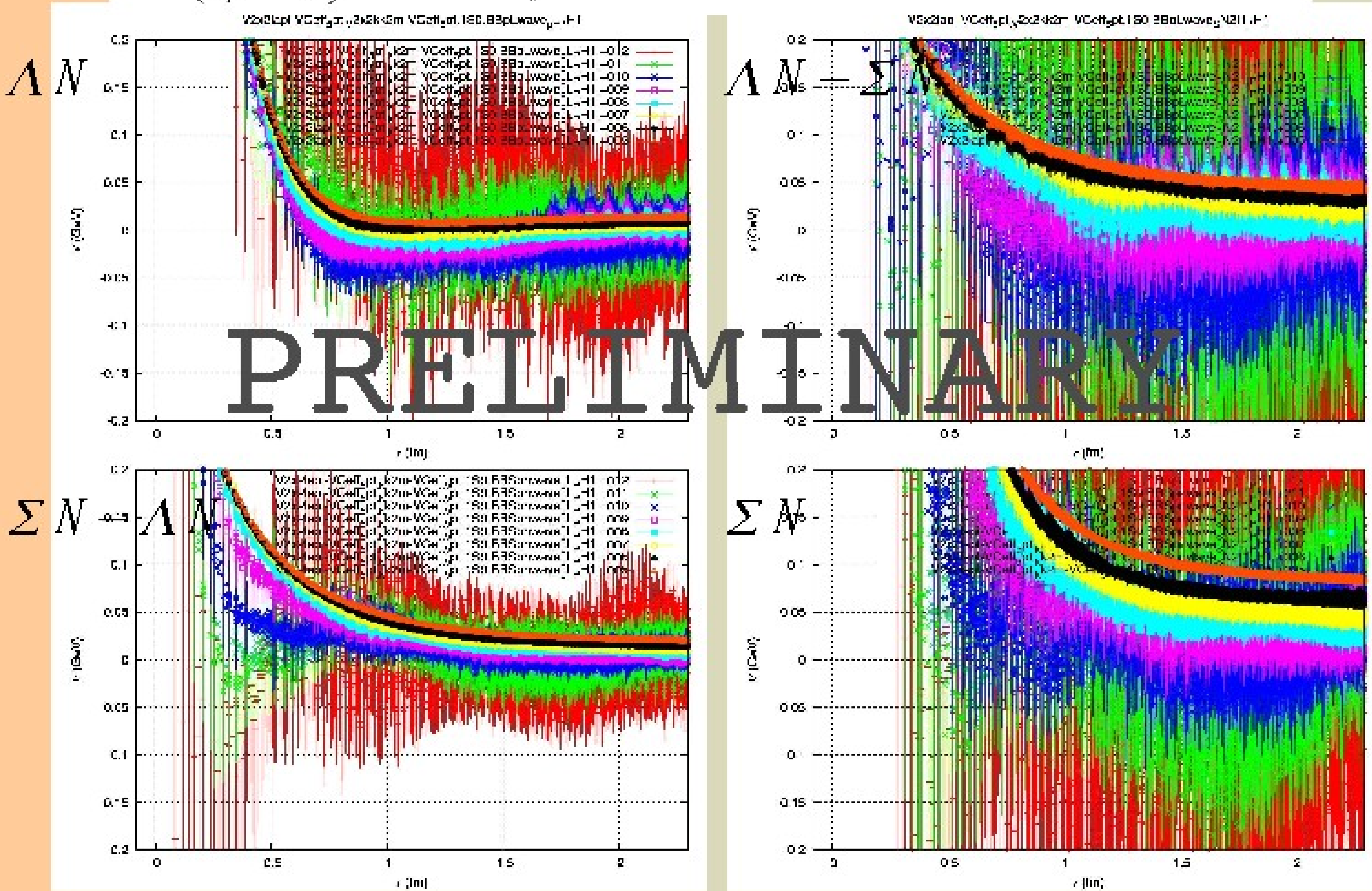
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C(\,^1S_0)$

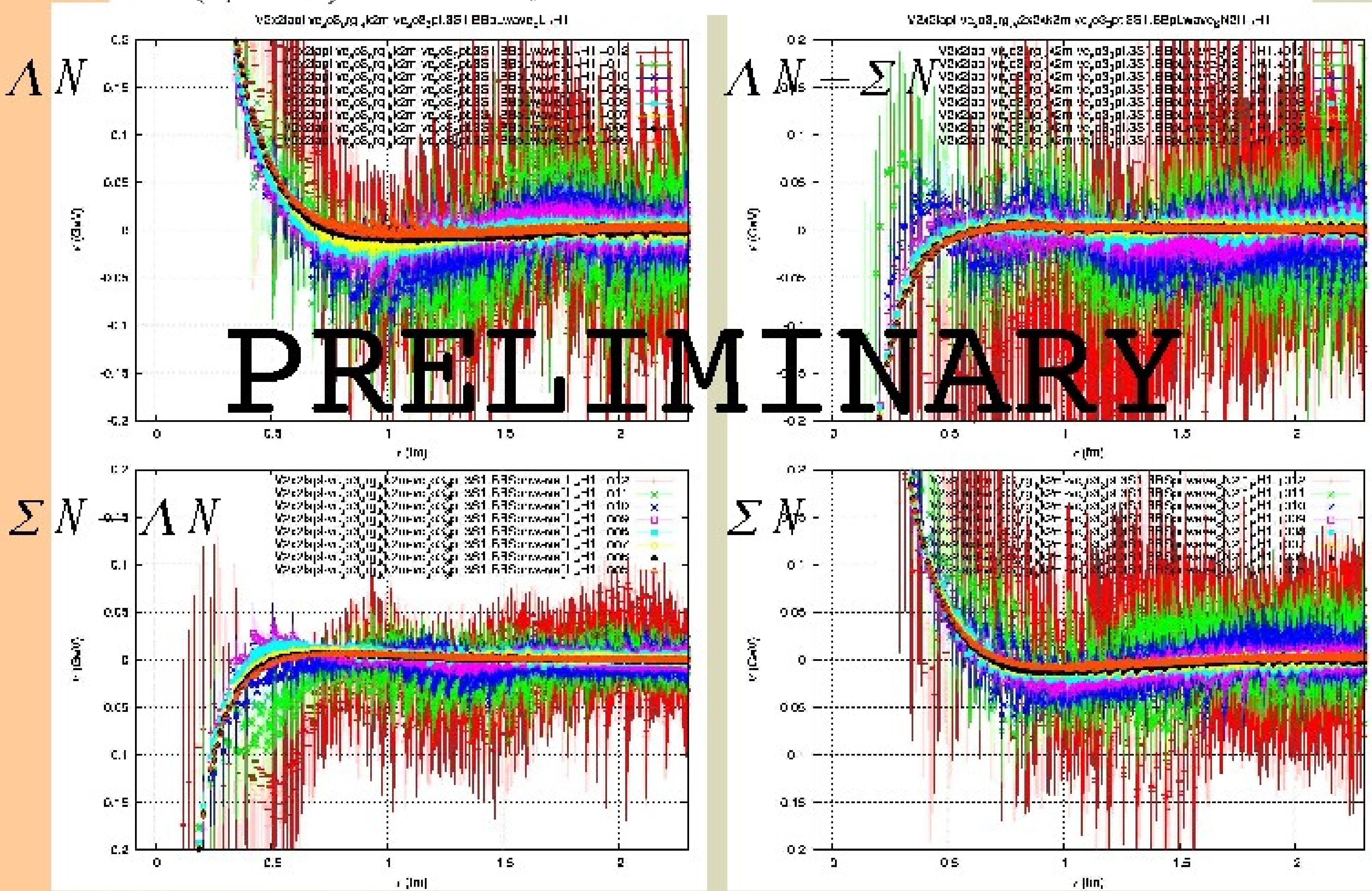
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r},t) = \cdot (8)$$



Very preliminary result of LN potential at the physical point

$V_C(\,^3S_1 - \,^3D_1)$

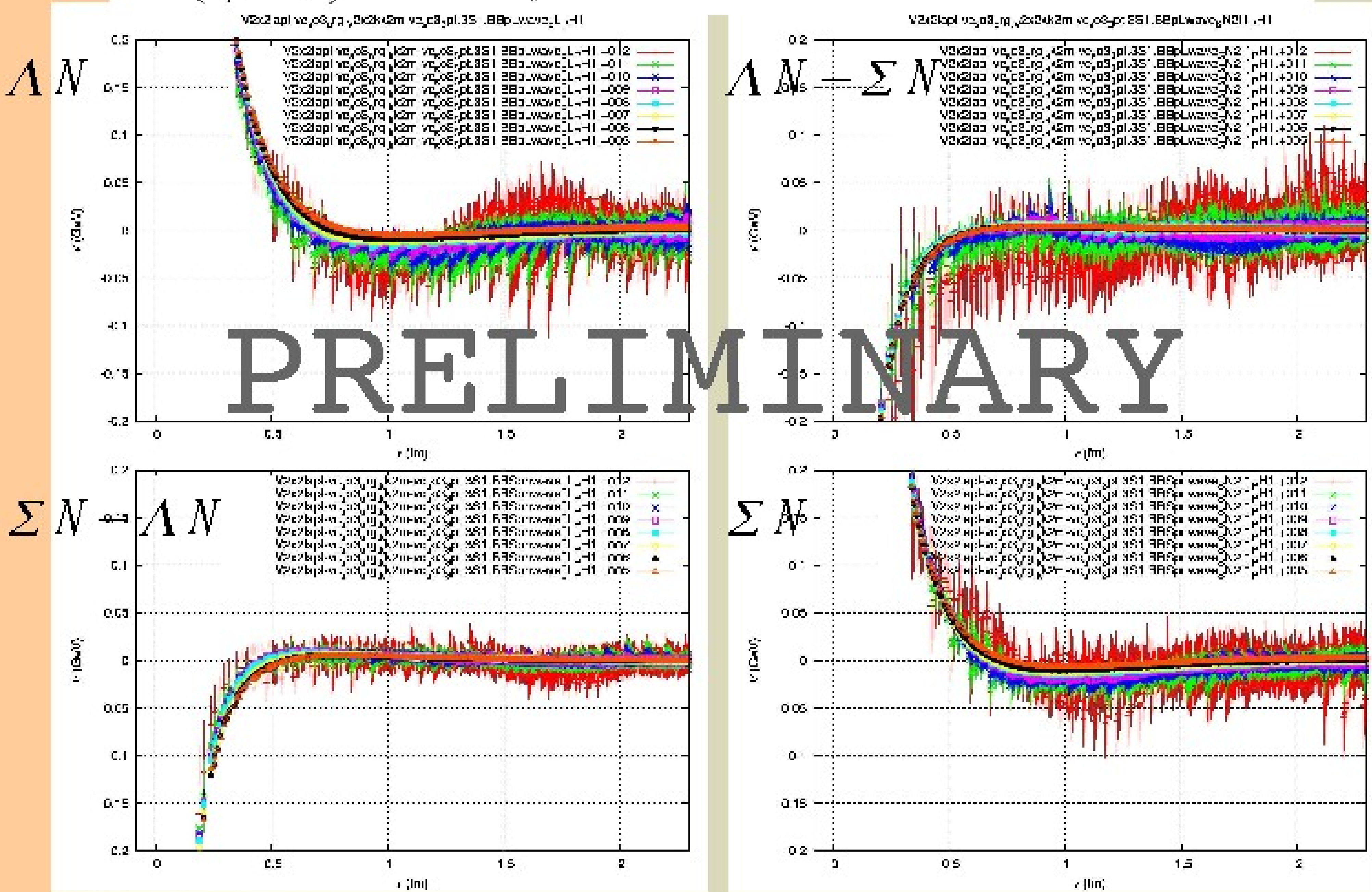
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' \, U(\vec{r}, \vec{r}\,') R(\vec{r}\,', t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r}, t) = \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C(\,^3S_1 - \,^3D_1)$

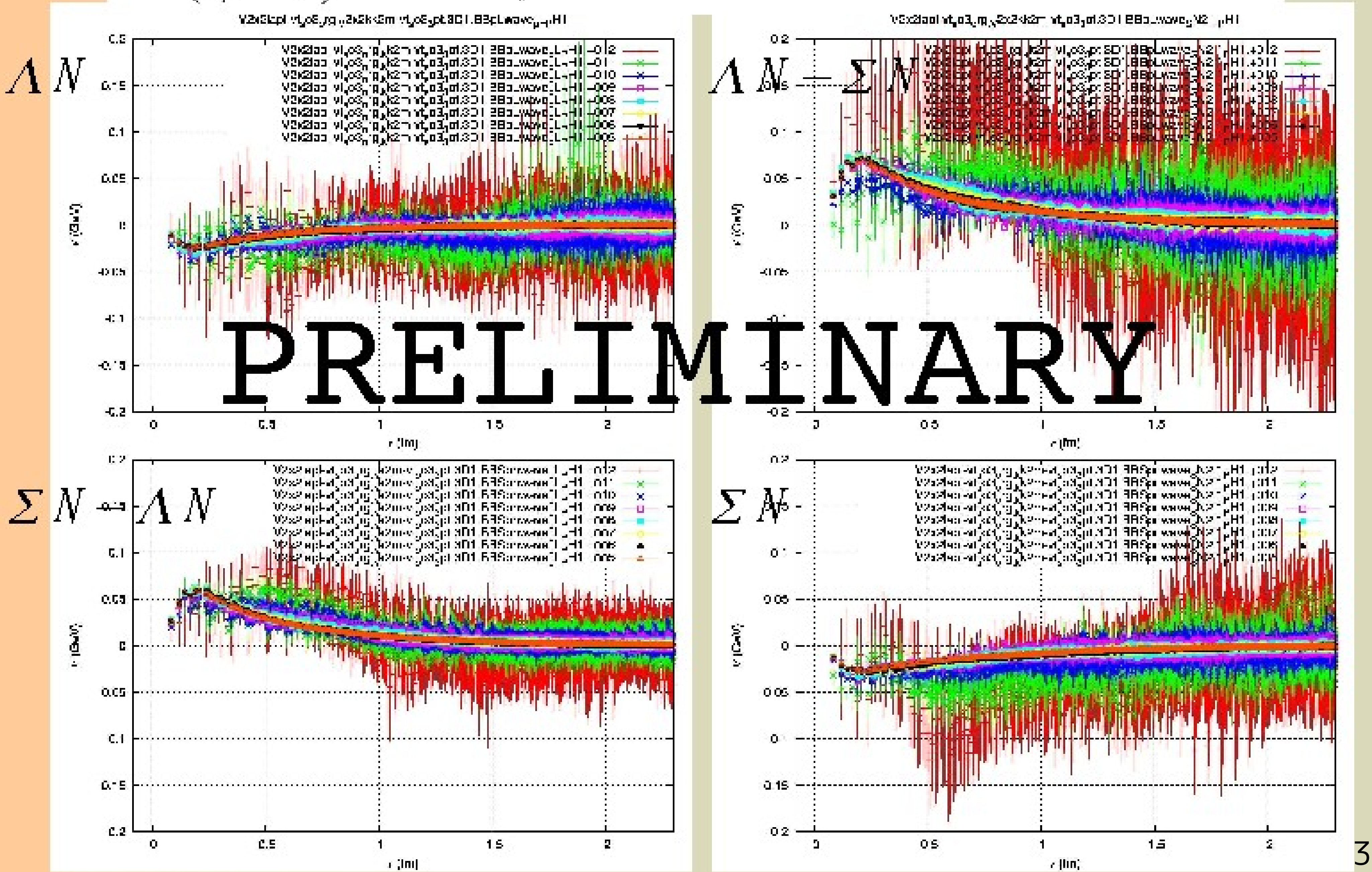
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}\,') R(\vec{r}\,',t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r},t) = \cdot (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

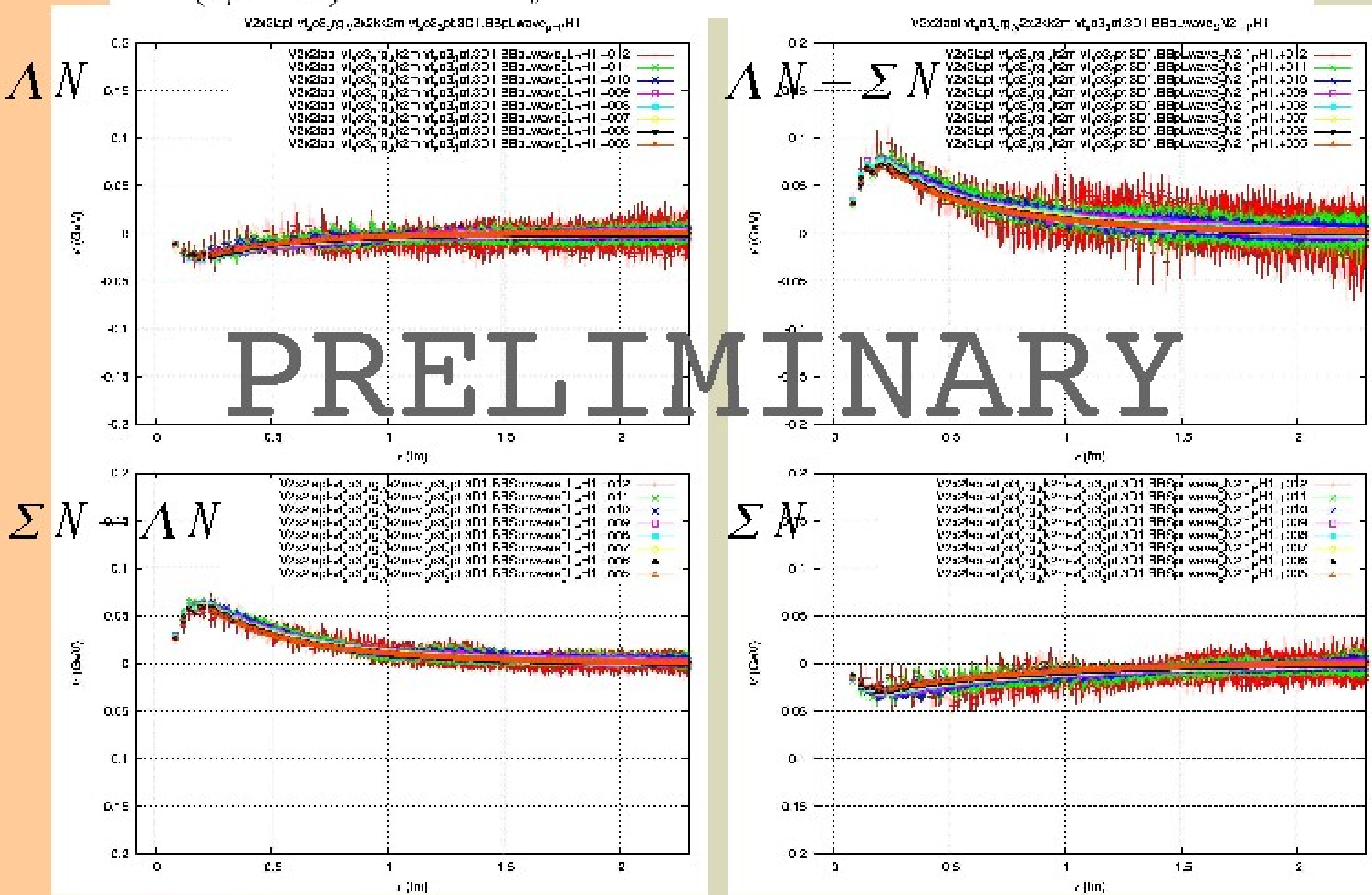
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}') R(\vec{r}',t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r},t) \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T({}^3S_1 - {}^3D_1)$

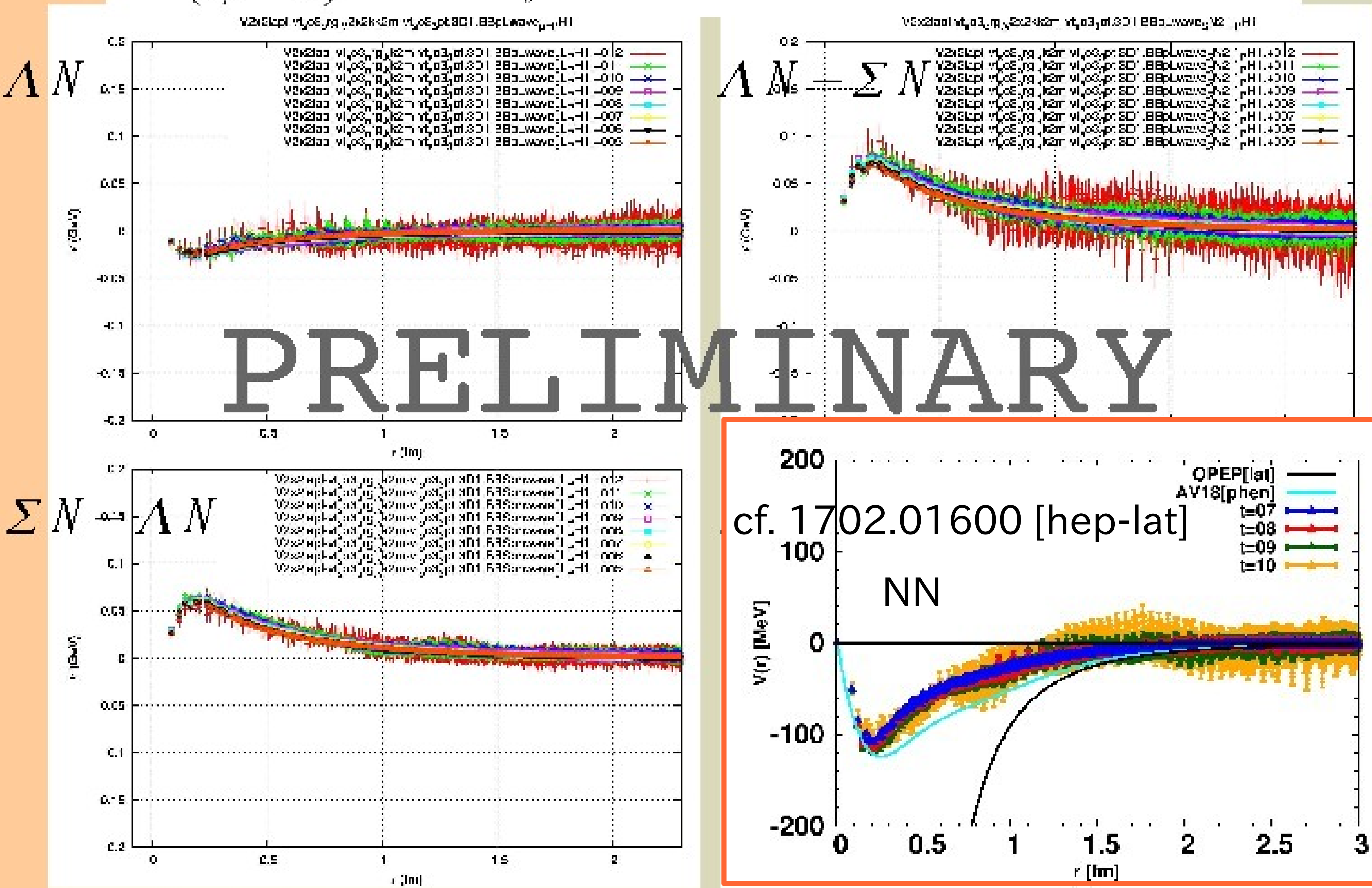
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')\, R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})\, R(\vec{r},t) \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_T(^3S_1 - ^3D_1)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{LO}(\vec{r}) R(\vec{r}, t) + \cdots (8)$$

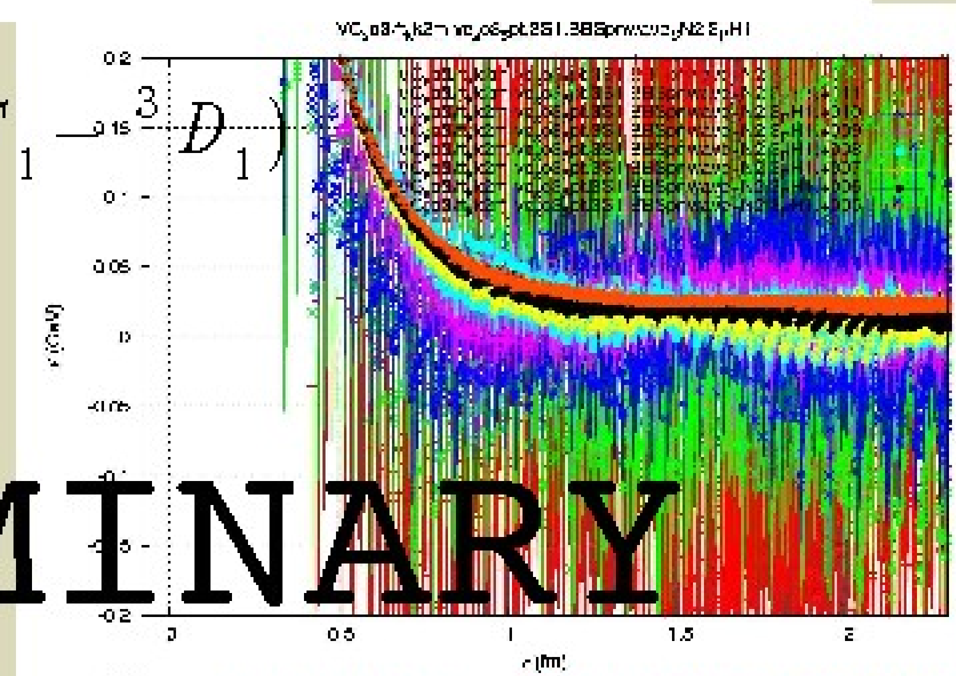


Very preliminary result of LN potential at the physical point

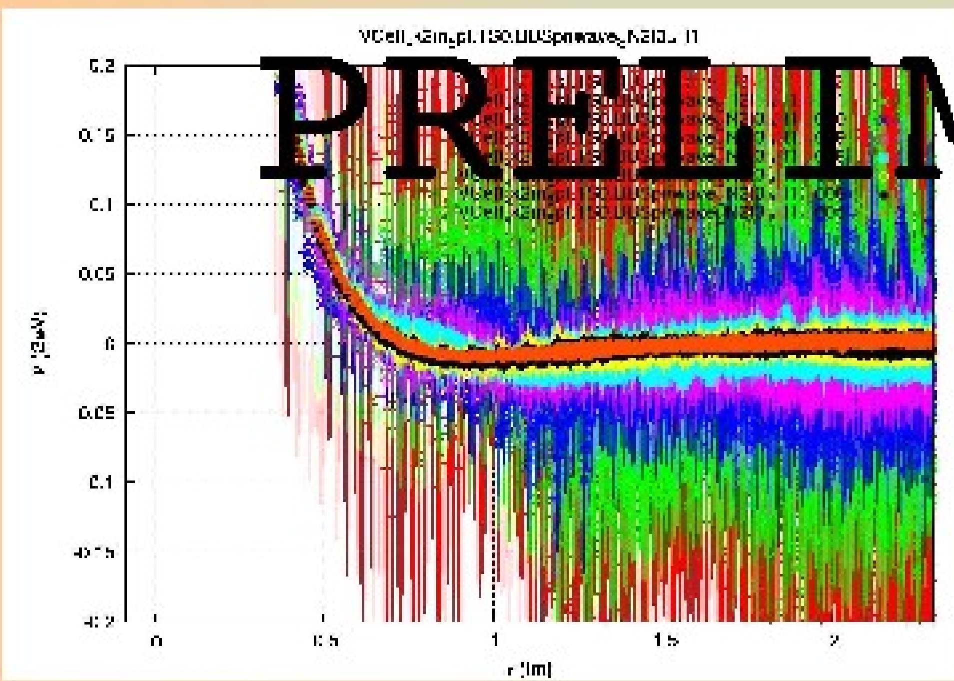
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{LO}(\vec{r})R(\vec{r},t) = \cdot(8)$$

$\Sigma N(I=3/2)$

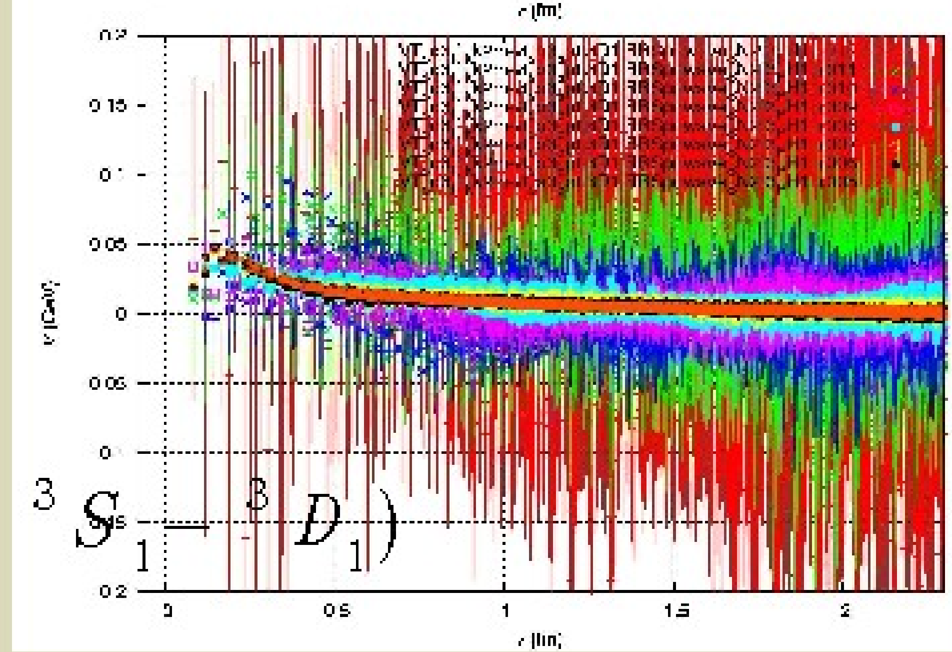
$V_C(^3S_1-^3D_1)$



PRELIMINARY



$V_C(^1S_0)$



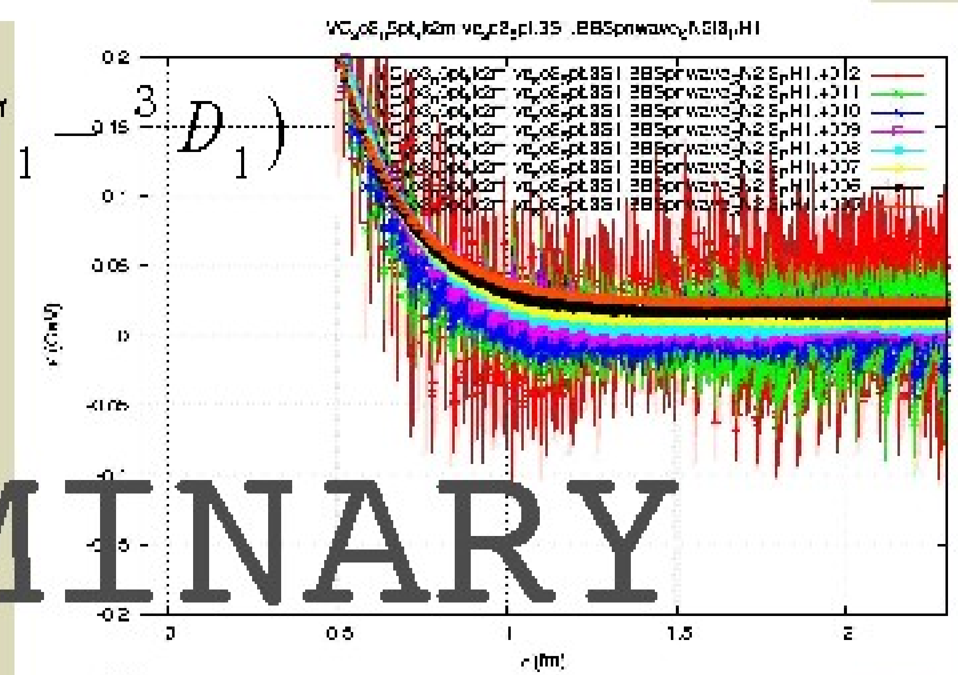
$V_T(^3S_1-^3D_1)$

Very preliminary result of LN potential at the physical point

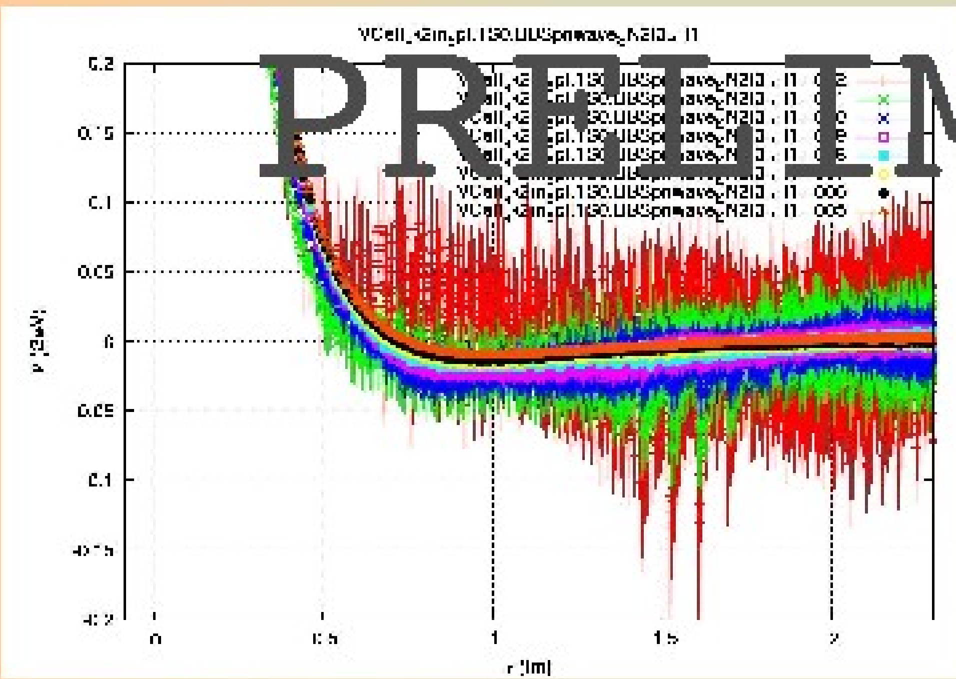
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r' \, U(\vec{r},\vec{r}\,') R(\vec{r}\,',t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r},t) \quad \cdots (8)$$

$\Sigma N(I=3/2)$

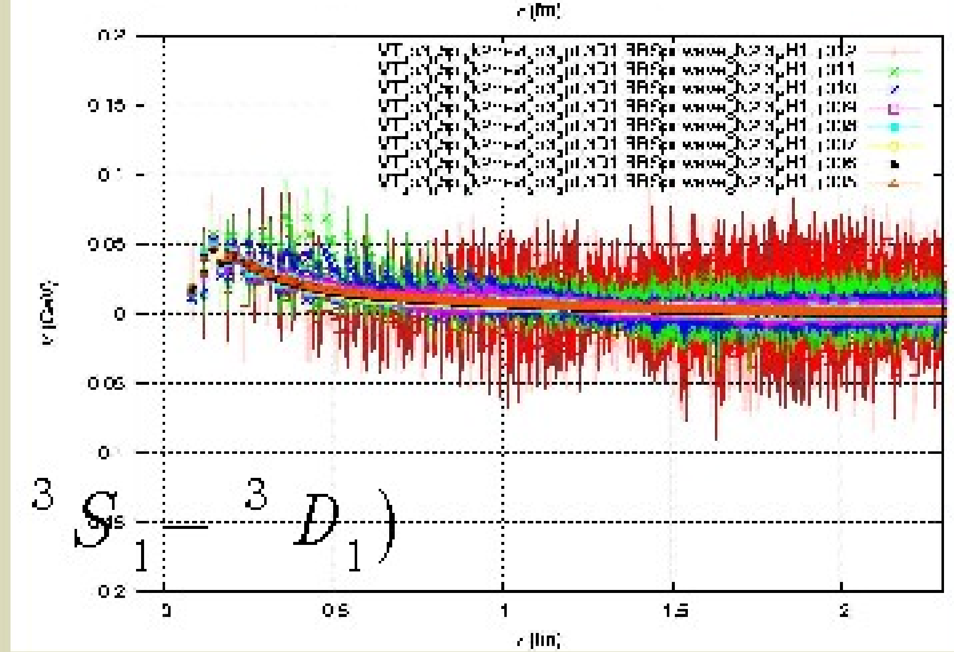
$V_C(^3S_1 - {}^3D_1)$



PRELIMINARY



$V_C(^1S_0)$



$V_T(^3S_1 - {}^3D_1)$

Summary

(I-1) Preliminary results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)
Statistics approaching to 0.54 (=present/scheduled)

Signals in spin-triplet are relatively going well smoothly.

We will have to increase still more statistics, particularly for spin-singlet channels

Several interesting features seem to be obtained with more high statistics.

(I-2) Effective hadron block algorithm for the various baron-baryon interaction

Paper published/available:

Comput.Phys.Commun.**207**,91(2016) [arXiv:1510.00903(hep-lat)]

Future work:
(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN potentials

PRL 94, 202502 (2005) PHYSICAL REVIEW LETTERS week ending 27 MAY 2005

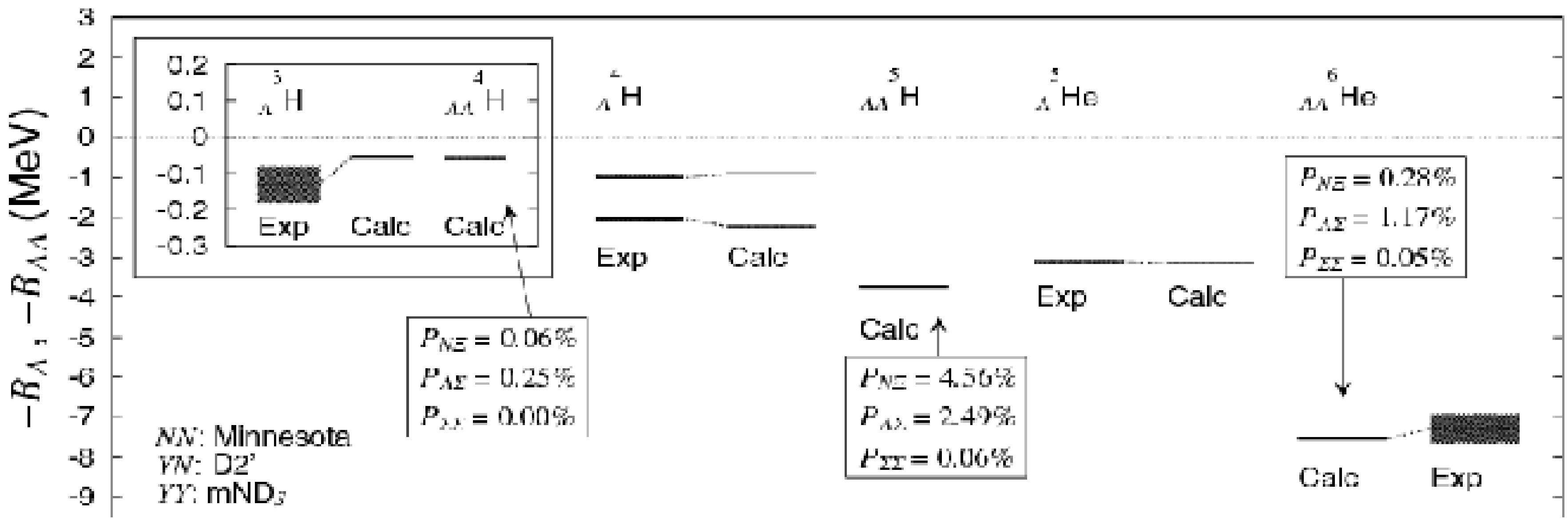


FIG. 1. Λ and ΛΛ separation energies of $A = 3 - 6$, $S = -1$ and -2 s-shell hypernuclei. The Minnesota NN, D2′ YN, and mND$_S$ YY potentials are used. The width of the line for the experimental B_Λ or $B_{\Lambda\Lambda}$ value indicates the experimental error bar. The probabilities of the $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$ components are also shown for the ΛΛ hypernuclei.

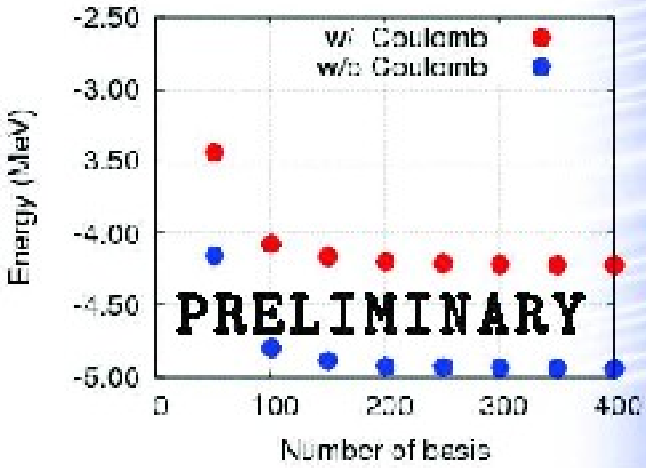
Benchmark test calculation of a four-nucleon bound state, Phys. Rev. C64, 044001 (2001).

TABLE I. The expectation values $\langle T \rangle$ and $\langle V \rangle$ of kinetic and potential energies, the binding energies E_b in MeV, and the radius in fm.

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

Results of few-body calculation

* Inputs:
  - m=1161.0 MeV,
  - hbar c = 197.3269602 MeV fm
  - hbar c/e^2 = 137.03599976
  - V_NN consists of AV8 type operators, determined from {1S0, 3S1, 3SD1, 1P1, 3P0, 3P1, 3PF2}.
    - V_0, V_σ, V_τ, $V_{\sigma\tau}$, V_T, $V_{T\tau}$, V_{LS}^{odd} are determined



* Preliminary results:
  - B(4He)=4.23 MeV (w/ Coulomb) (old: 4.37MeV)
    - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)
    - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
  - B(4He)=4.95 MeV (w/o Coulomb) (old: 5.09MeV)
    - Probabilities of (S, P, D) waves = (98.8%, 0.002%, 1.2%)