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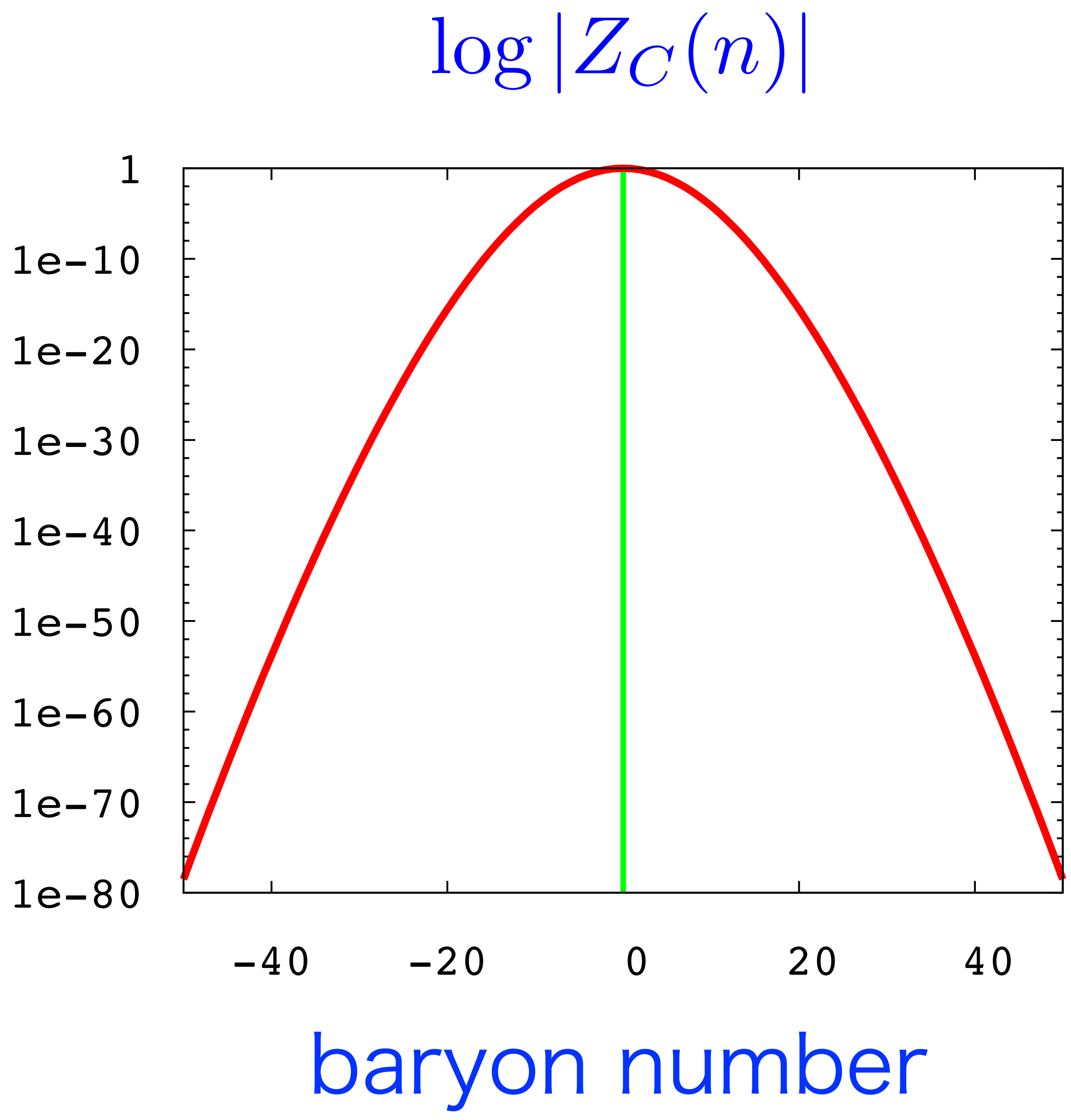
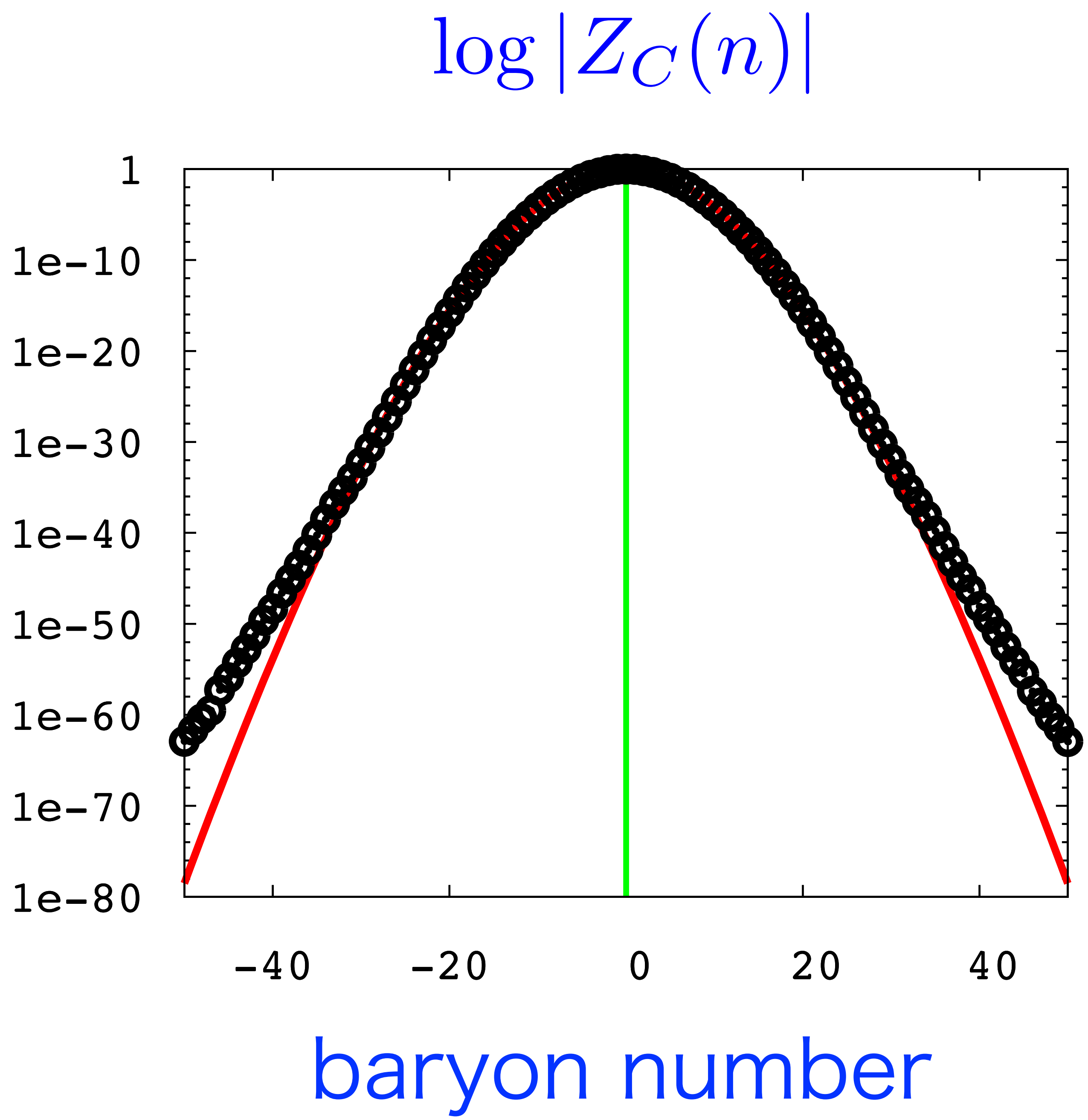
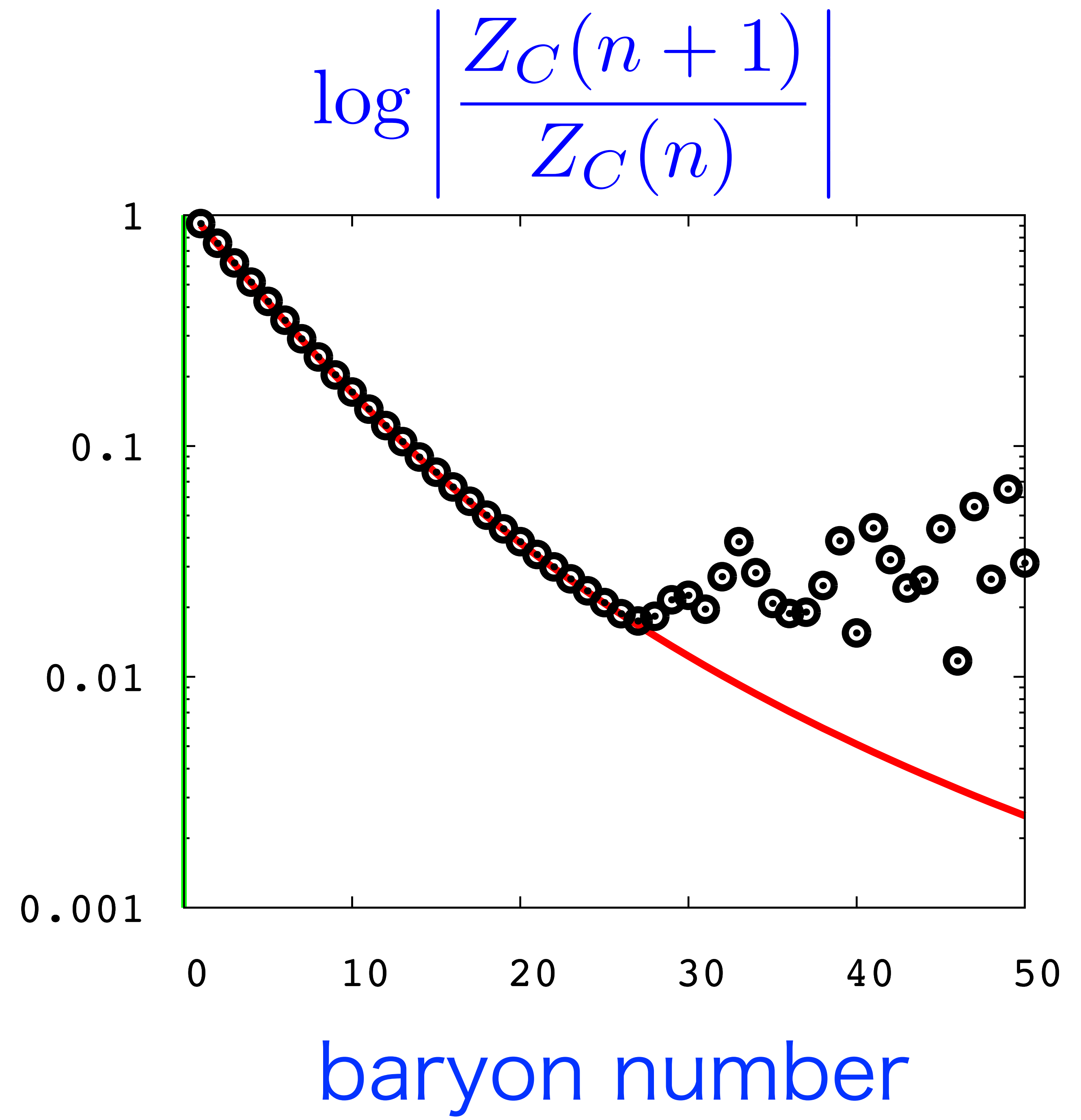
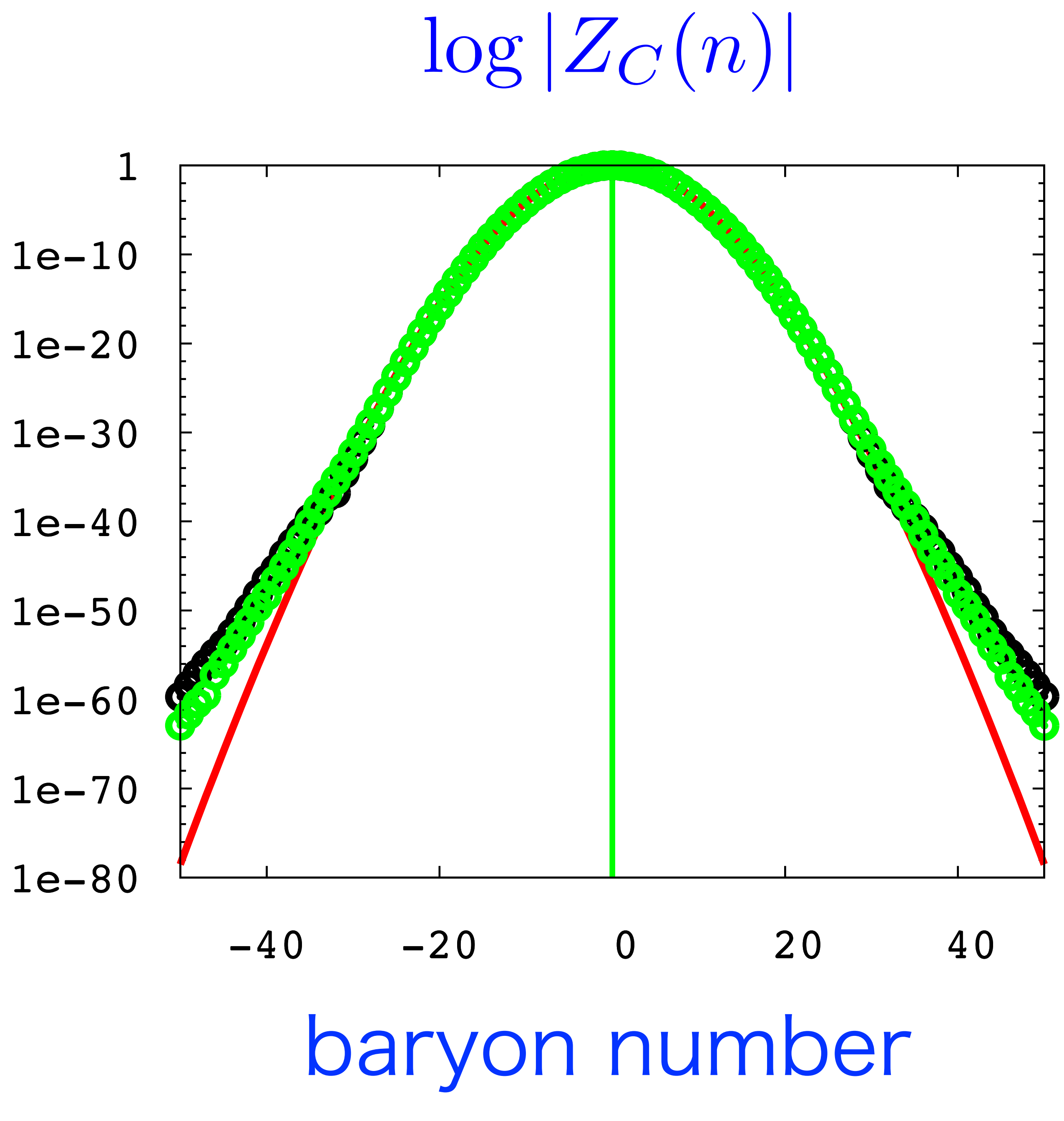


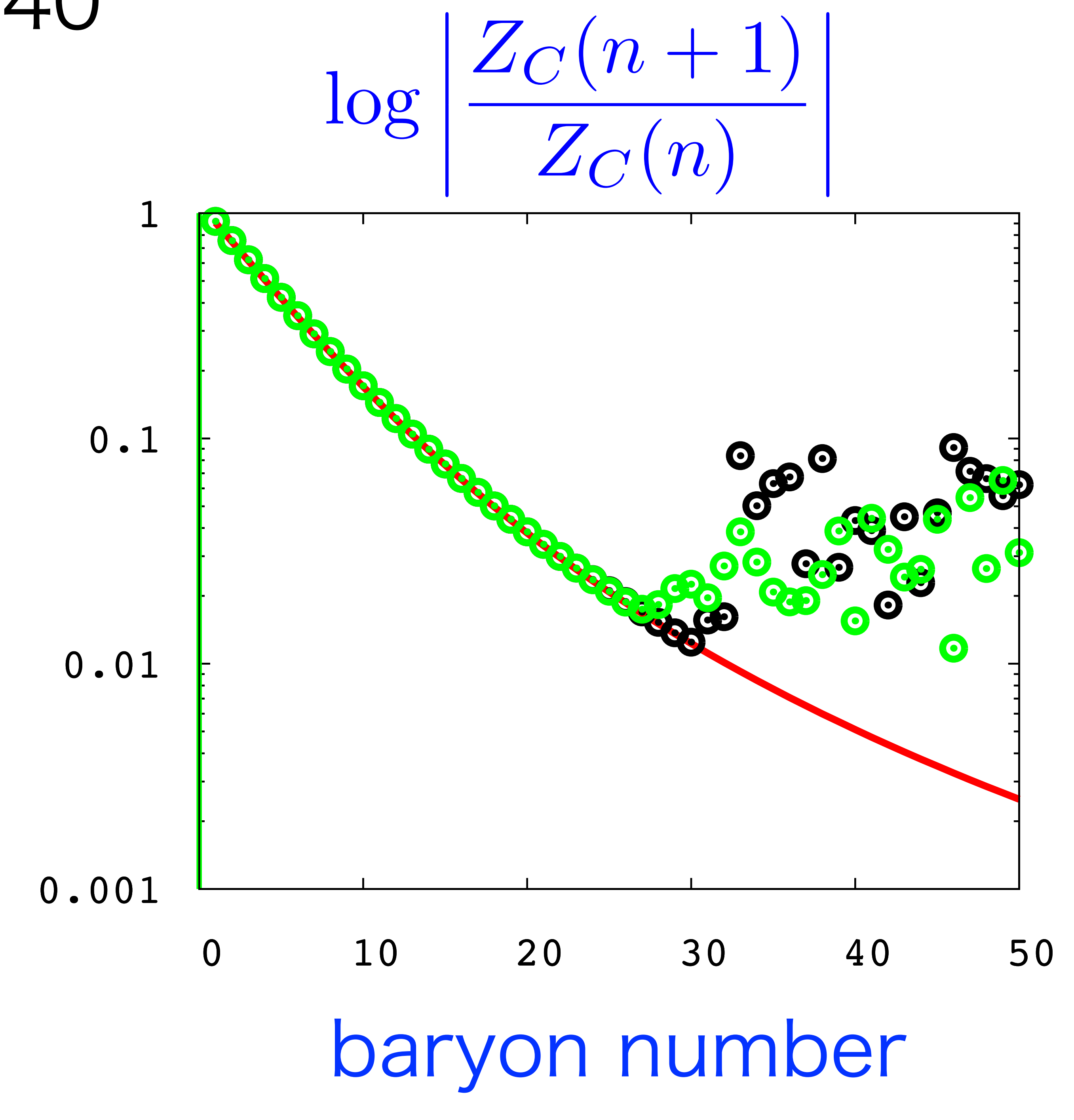
winding number=120 (number of hop =480)





winding number=120 vs 240





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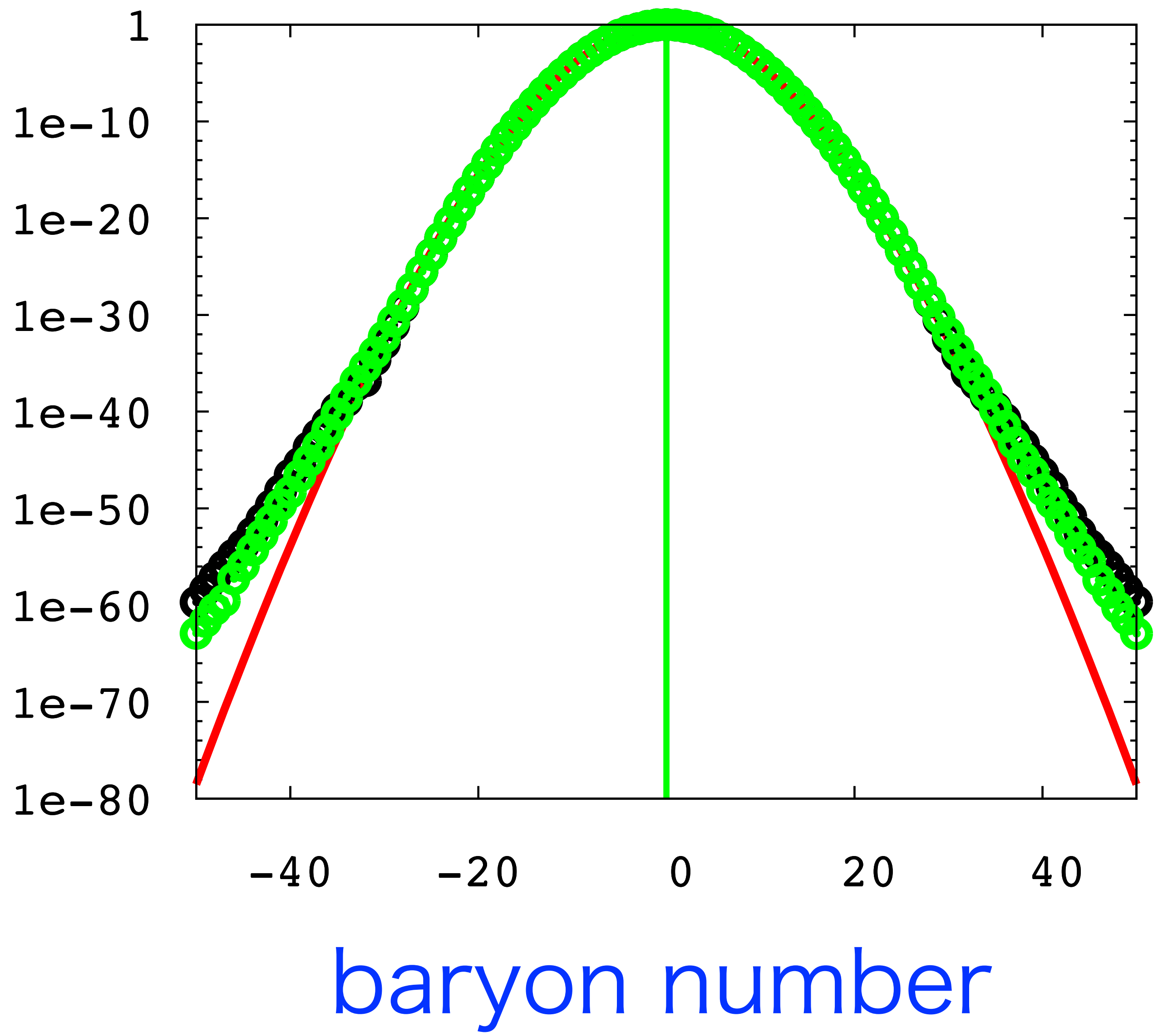
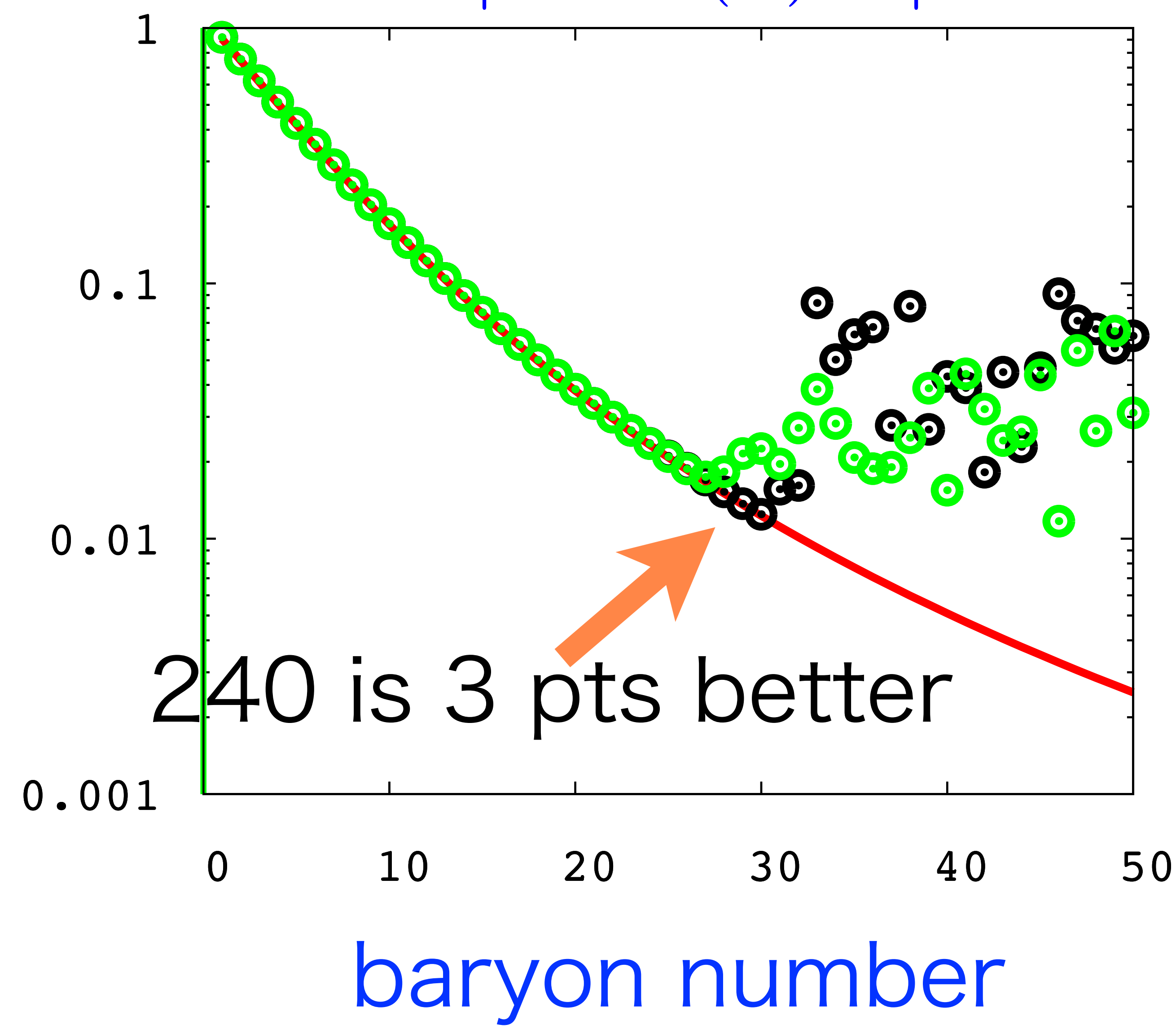
$$\log |Z_C(n)|$$



$$\log \left| \frac{Z_C(n+1)}{Z_C(n)} \right|$$



Use of Zc(n)

Get the grand partition function $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

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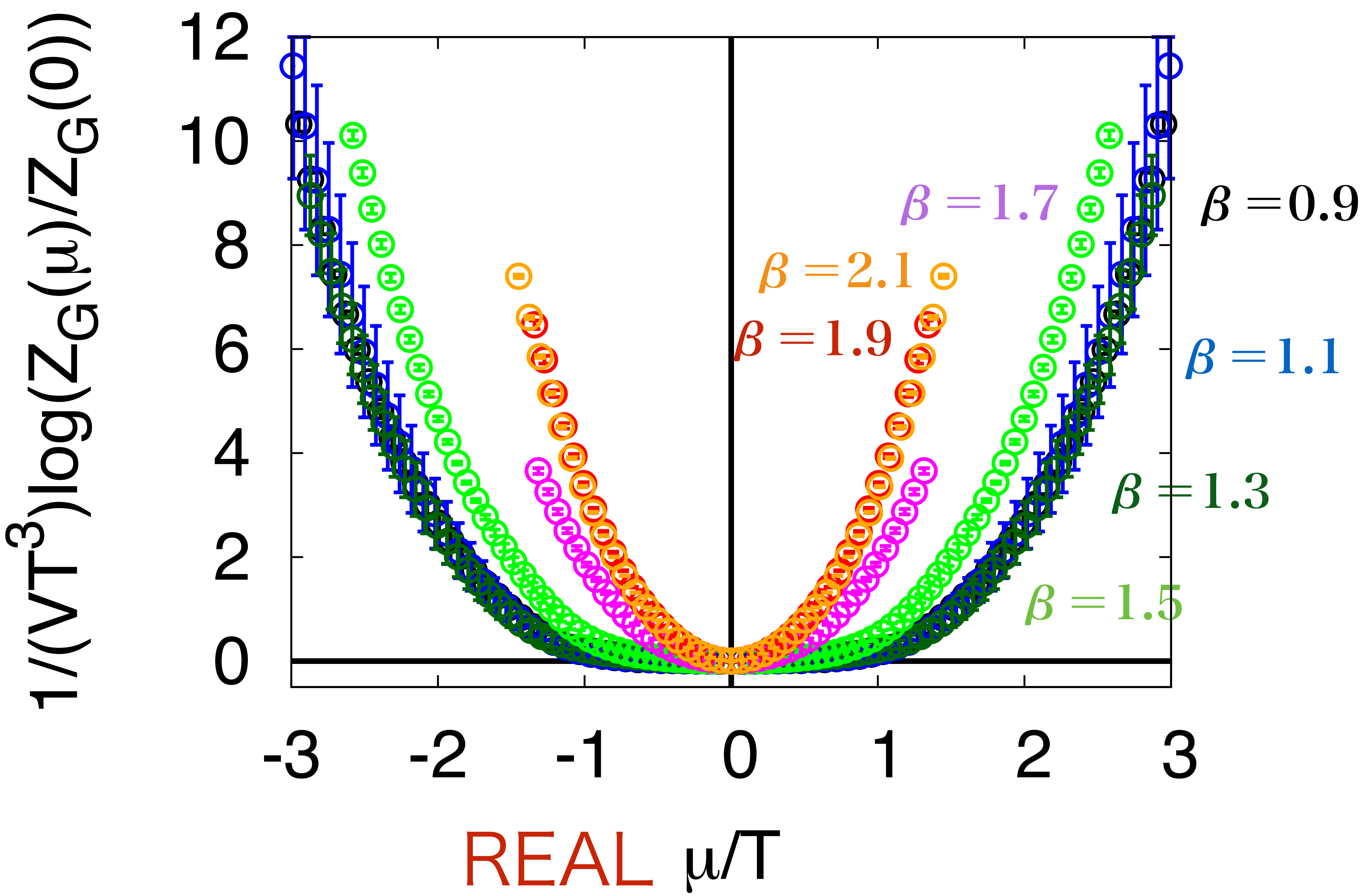

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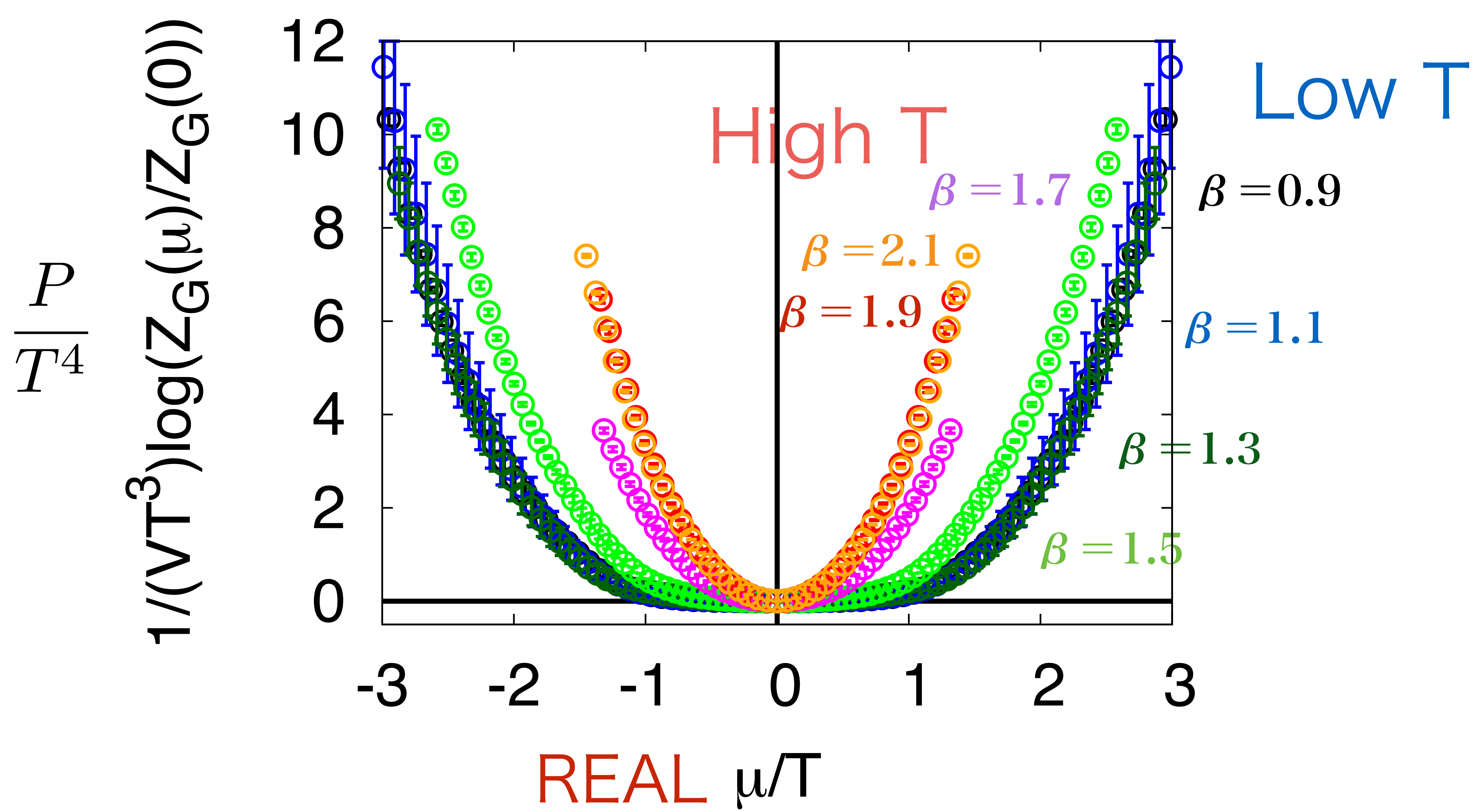

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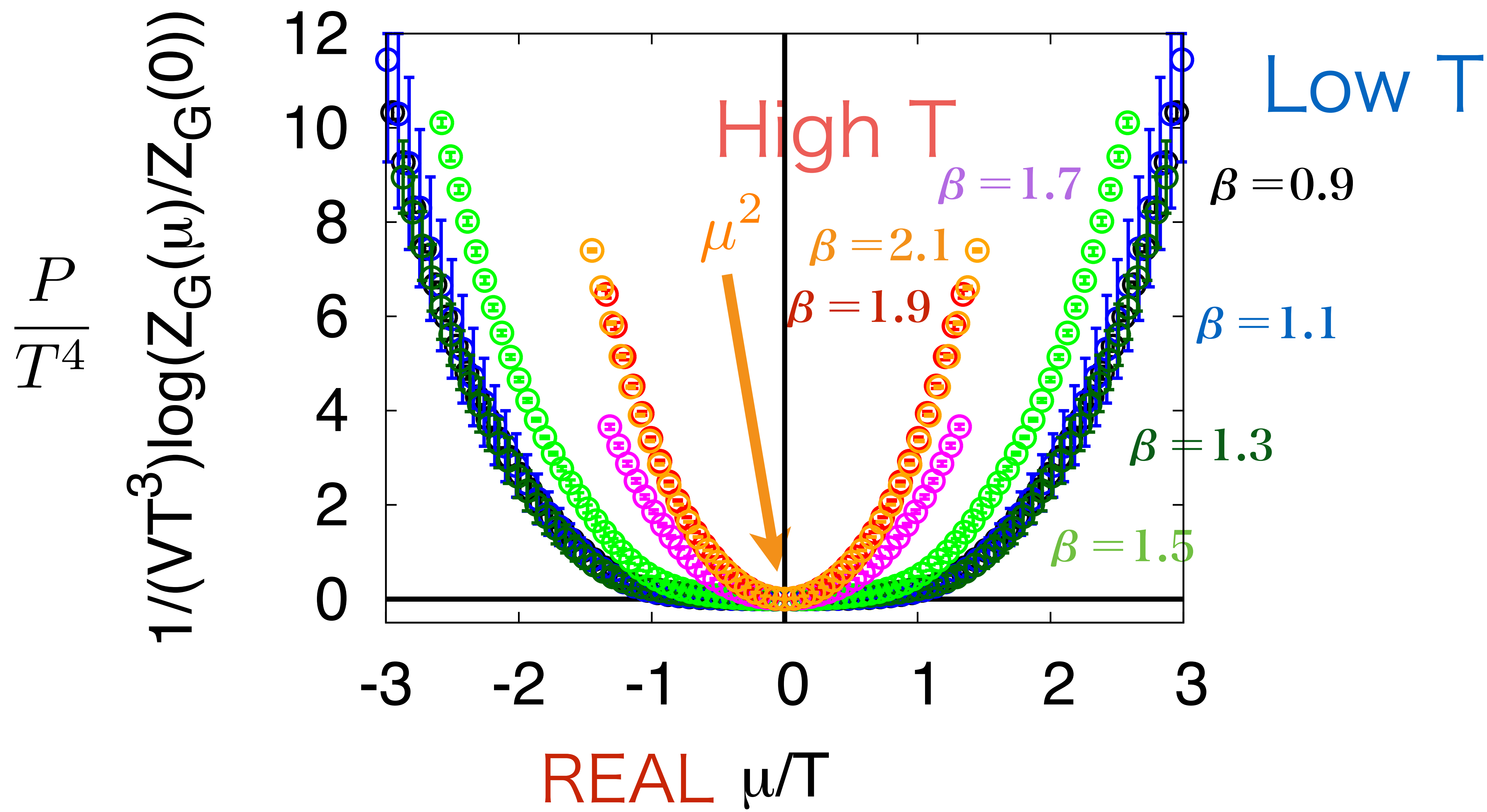

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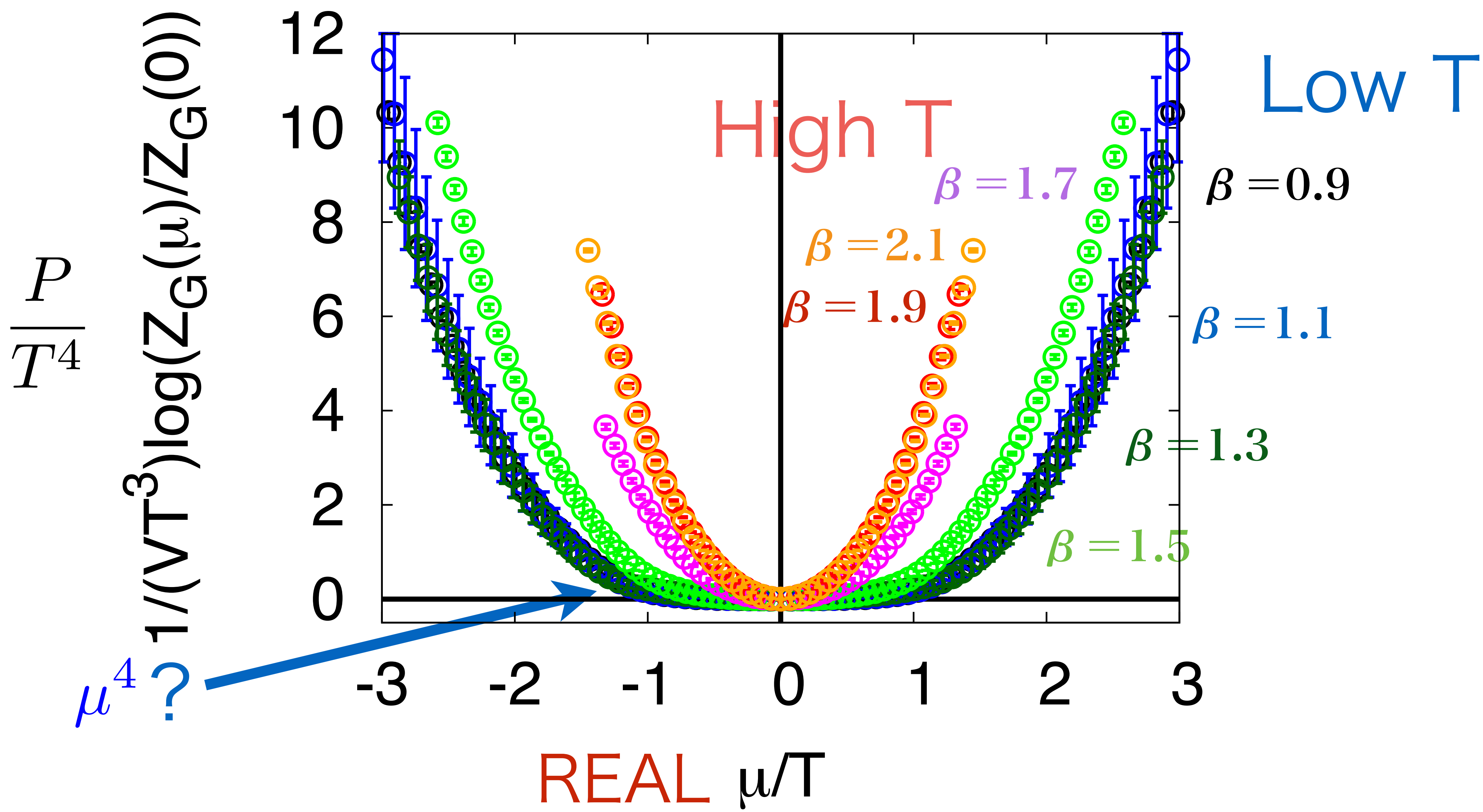

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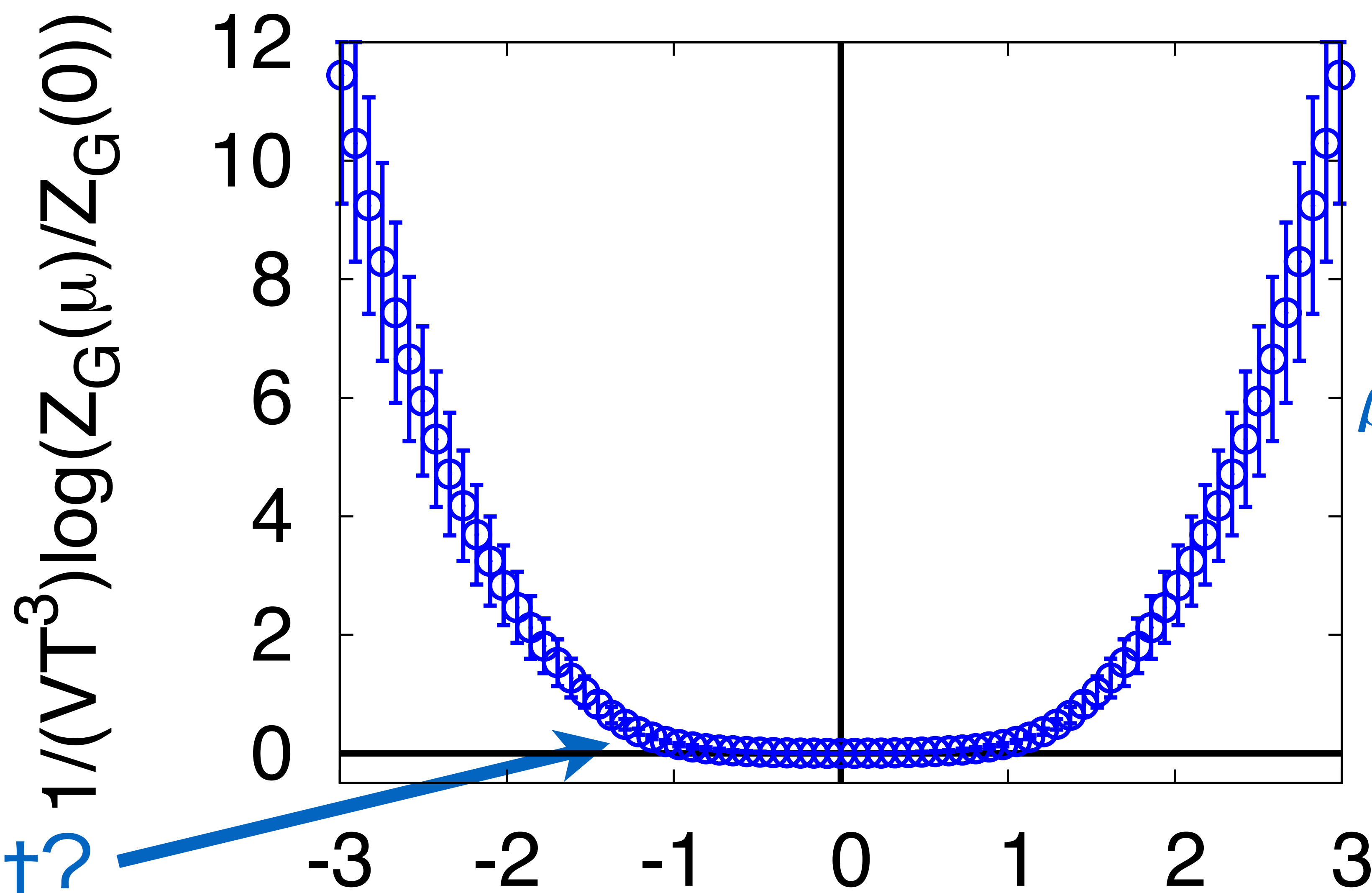

Low T

$\beta = 1.1$

flat?

REAL μ/T

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We have the canonical partition function Z_n

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2. quark number density

$$\langle N \rangle = \frac{1}{Z(\mu)} \sum_{n=-\infty}^{\infty} n|Z_n|\xi^n$$

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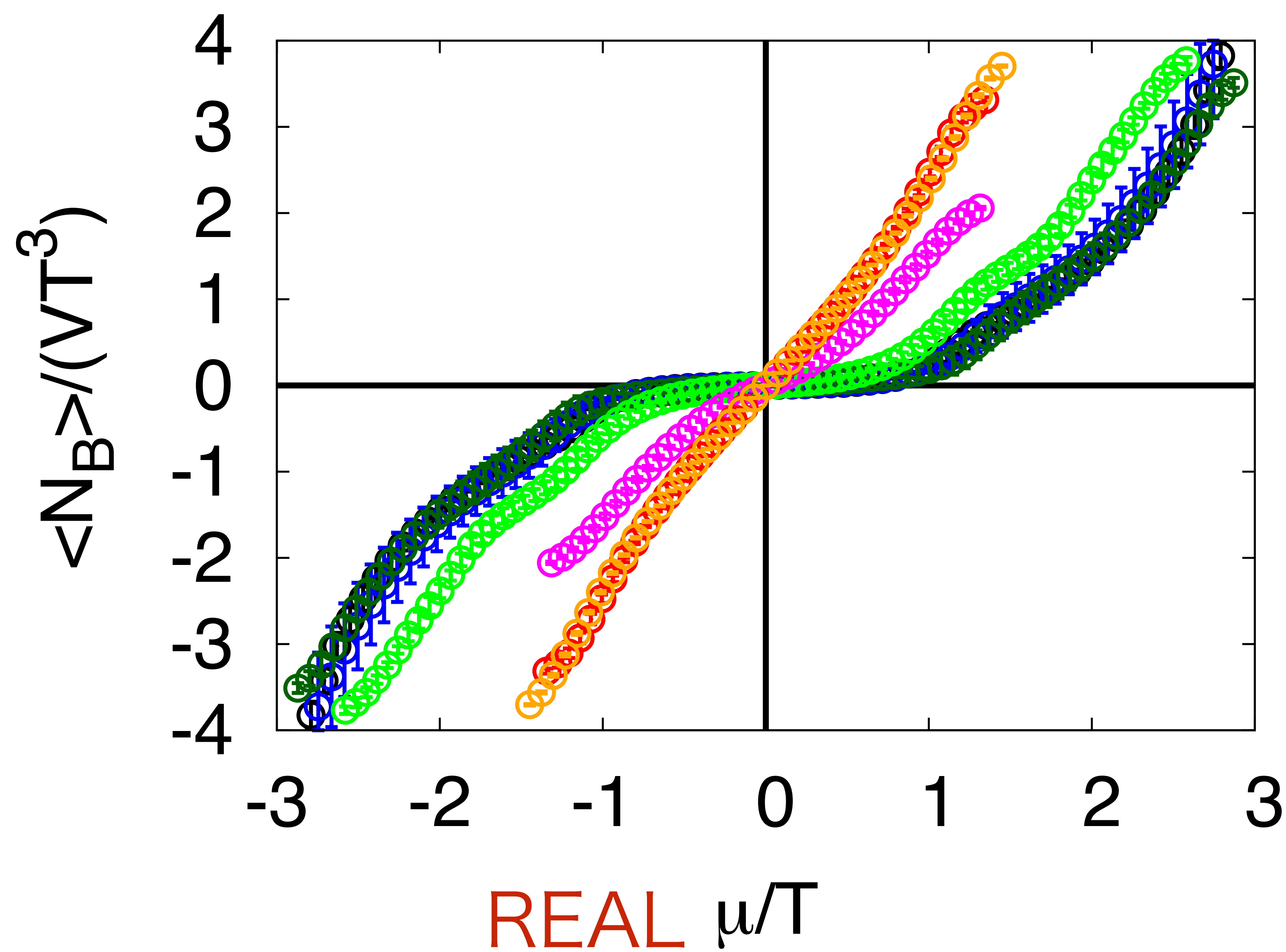

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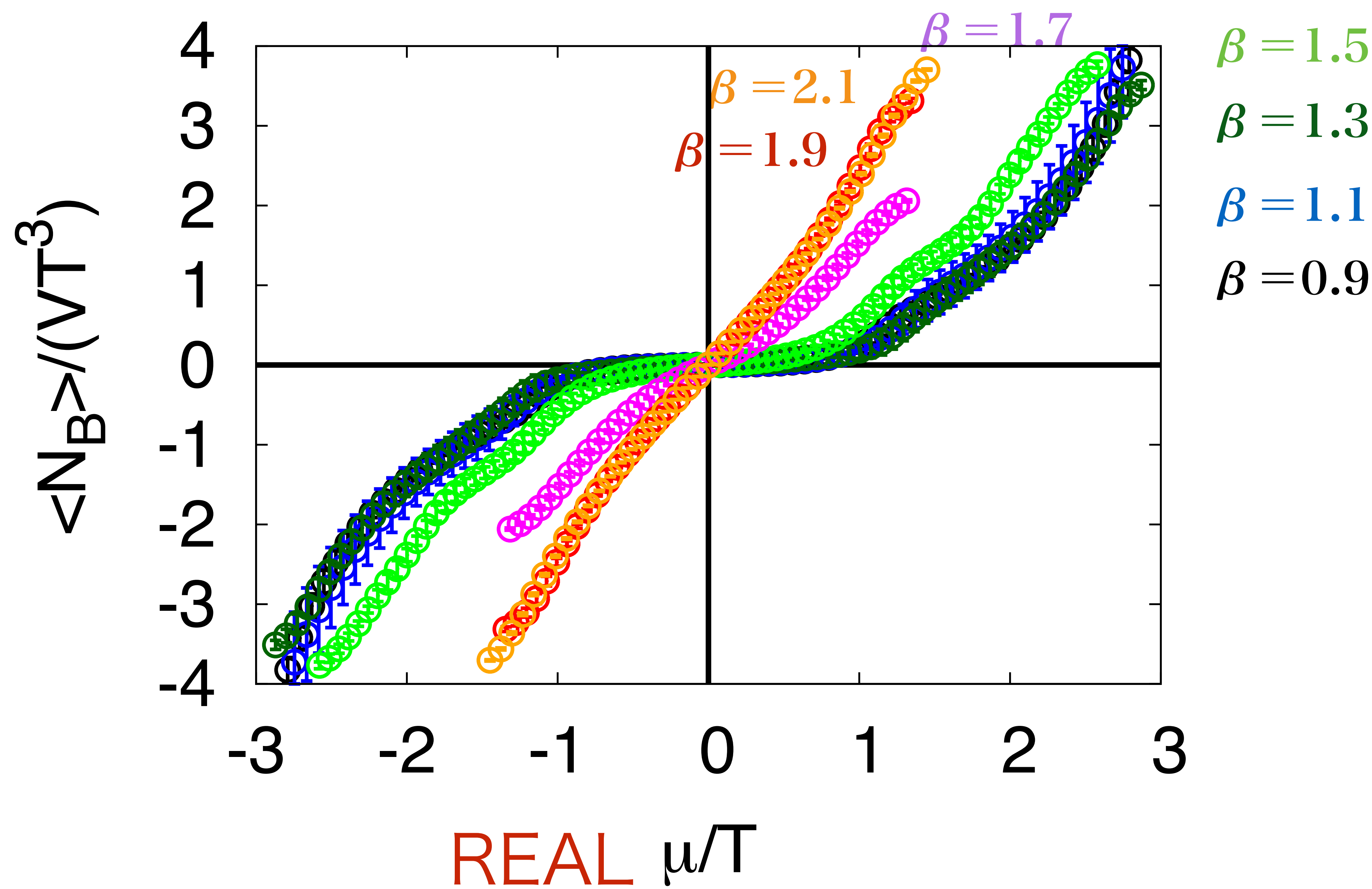

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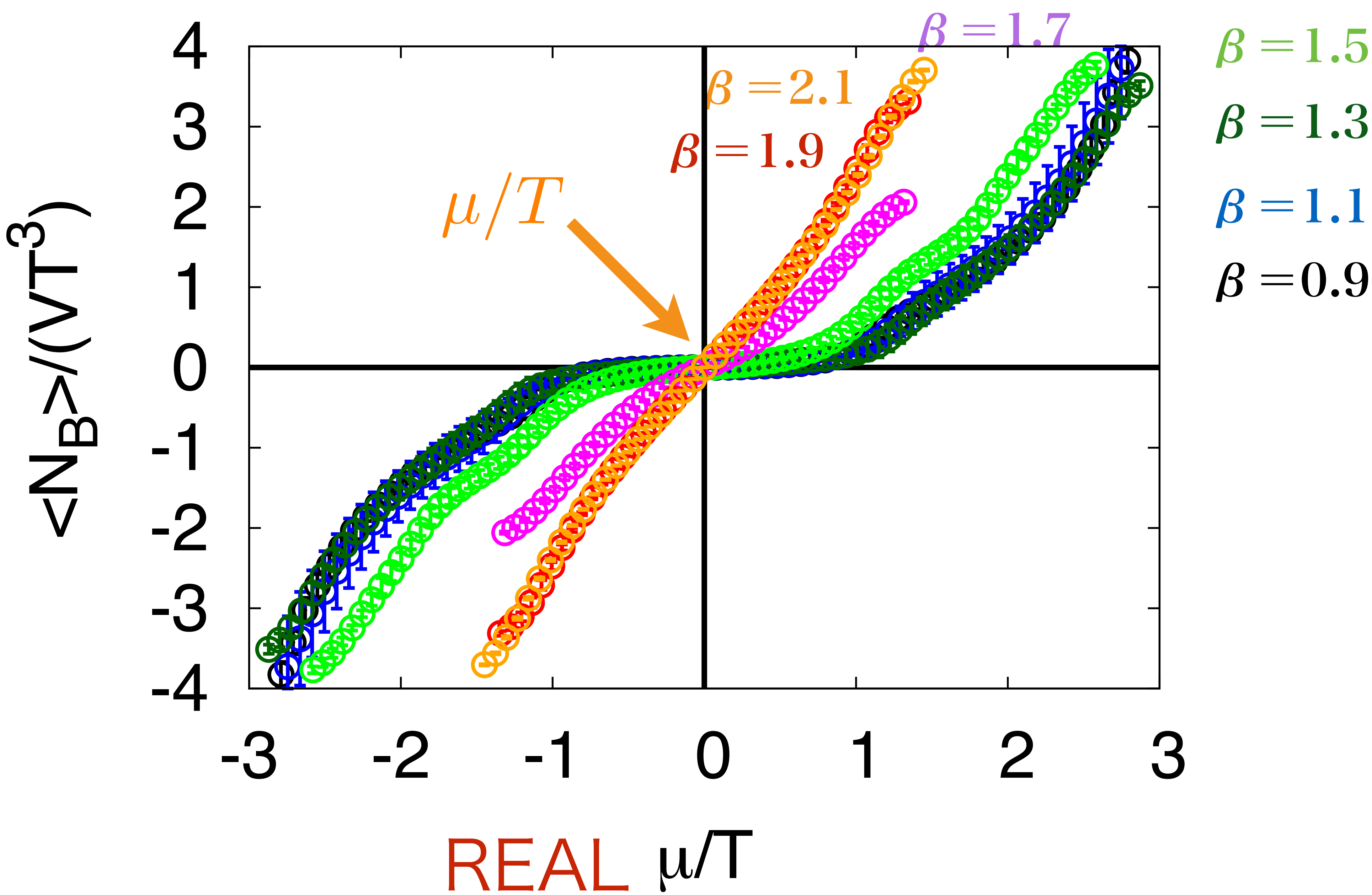

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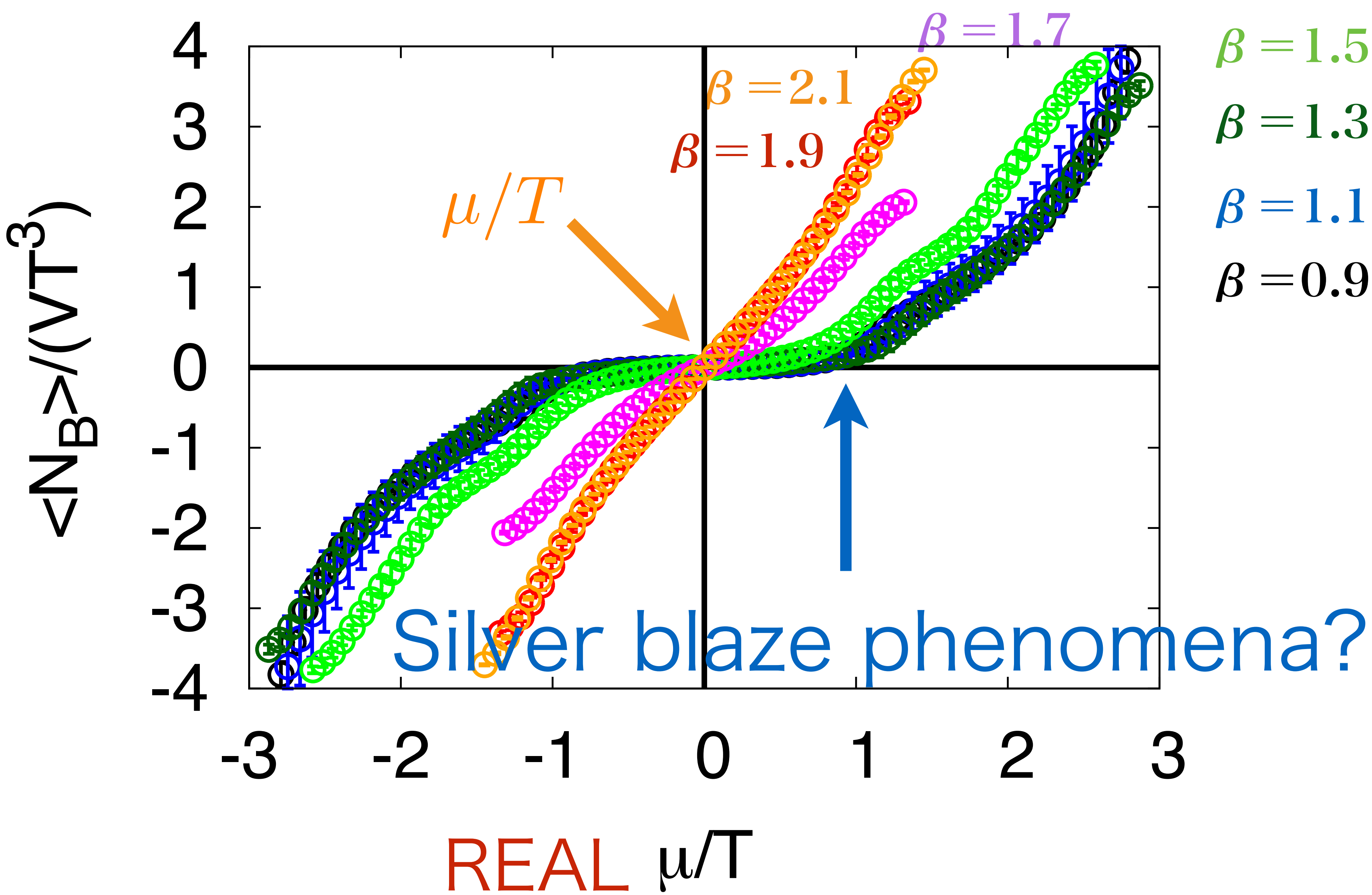

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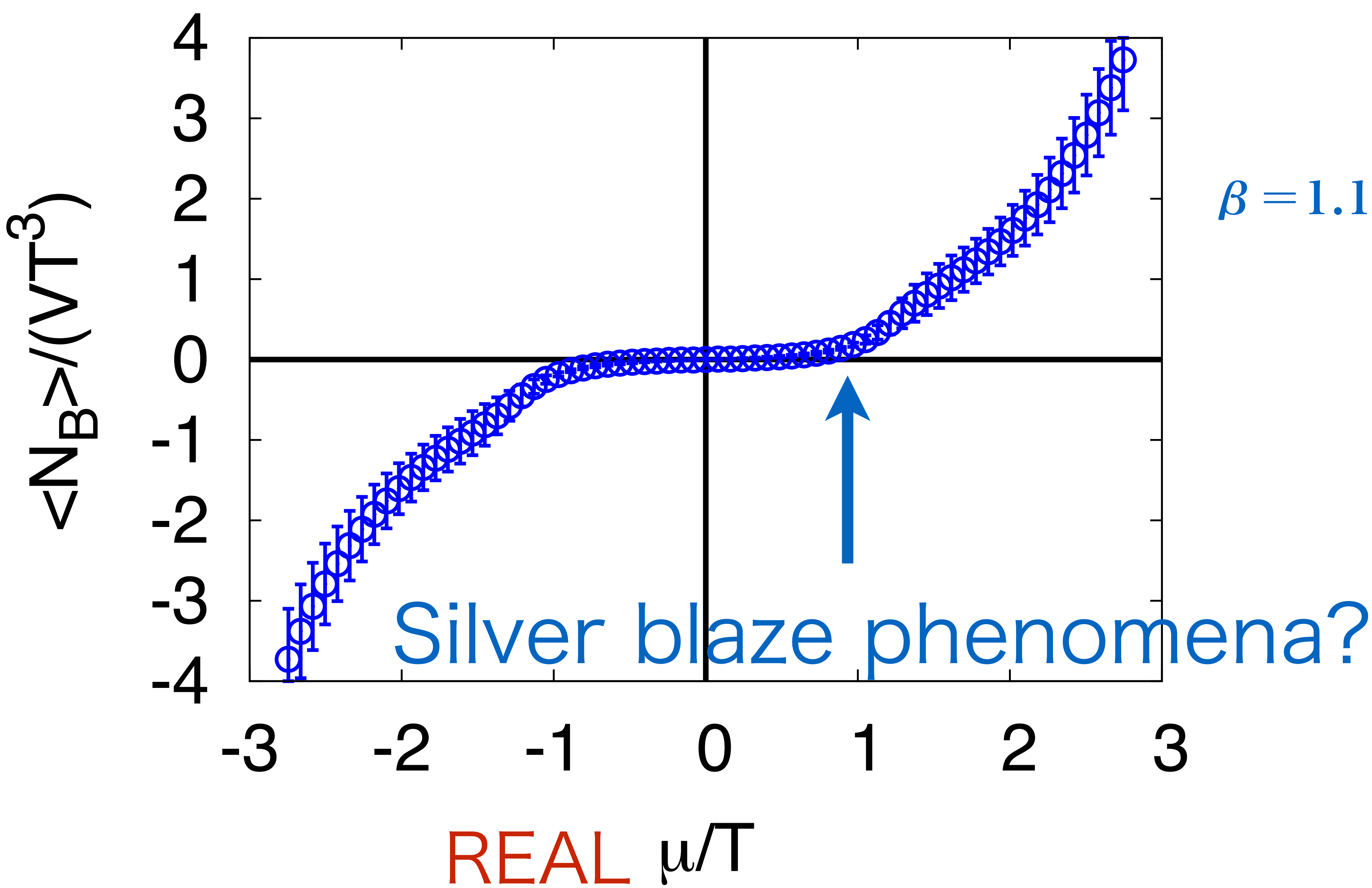

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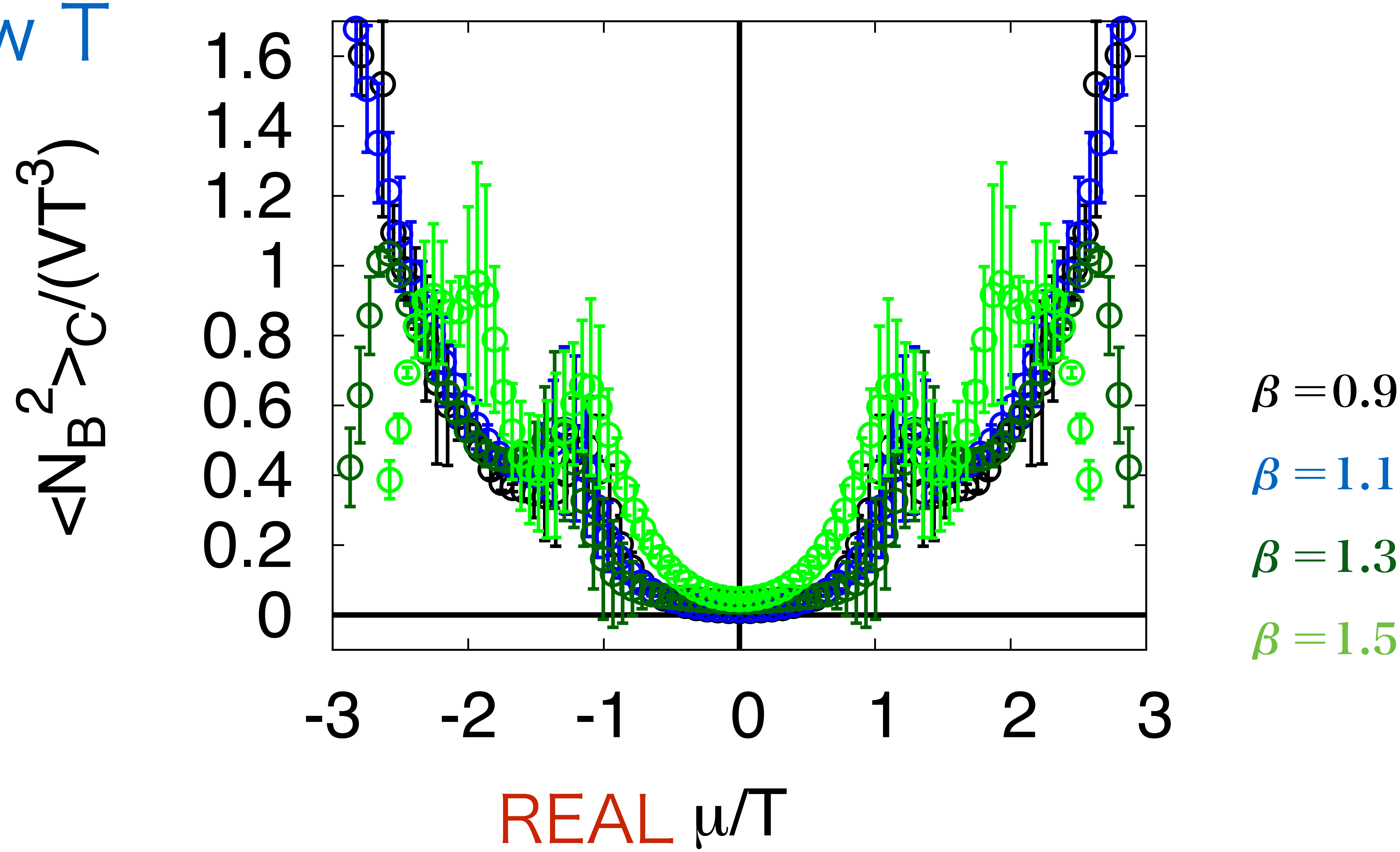
Low T



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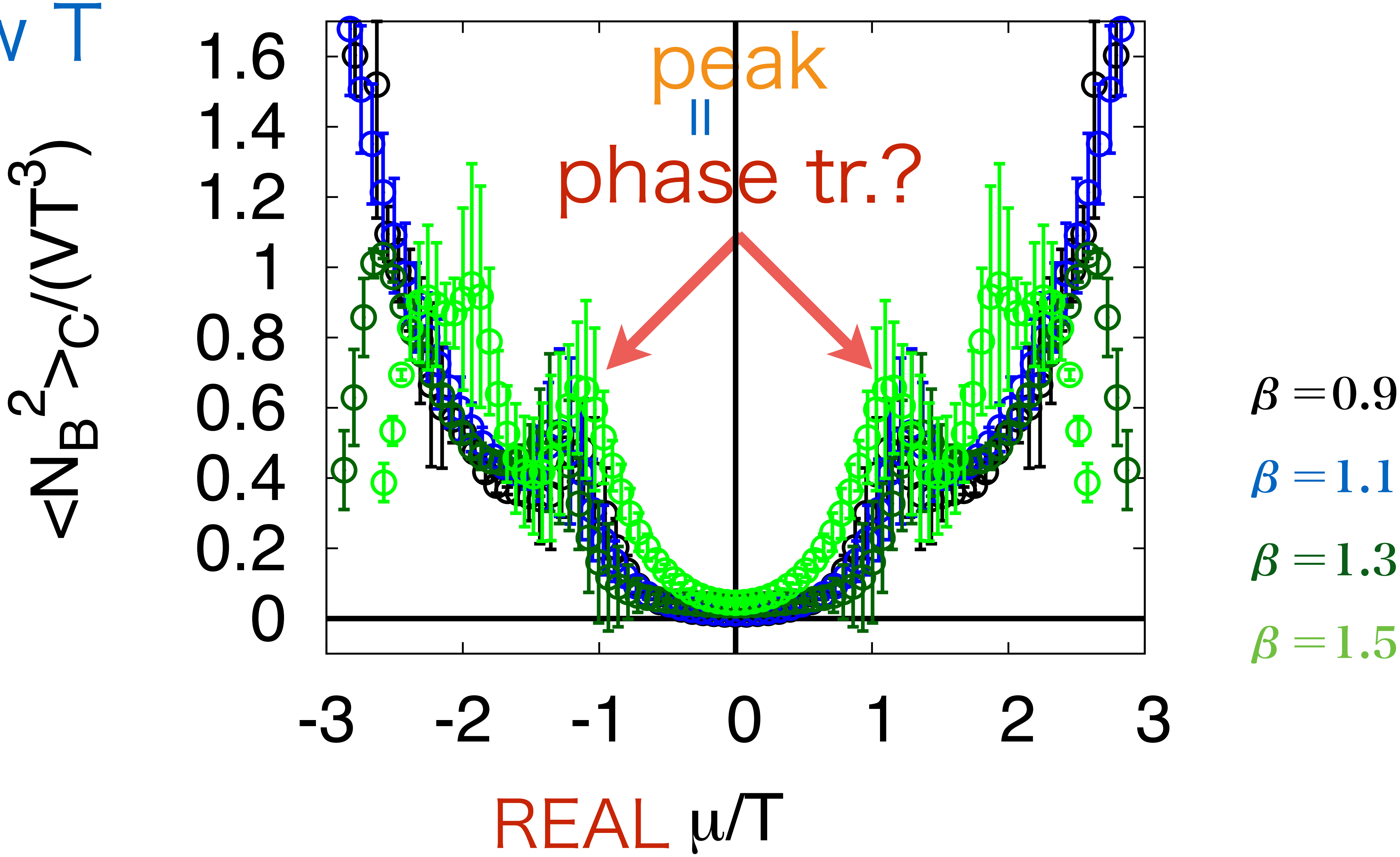

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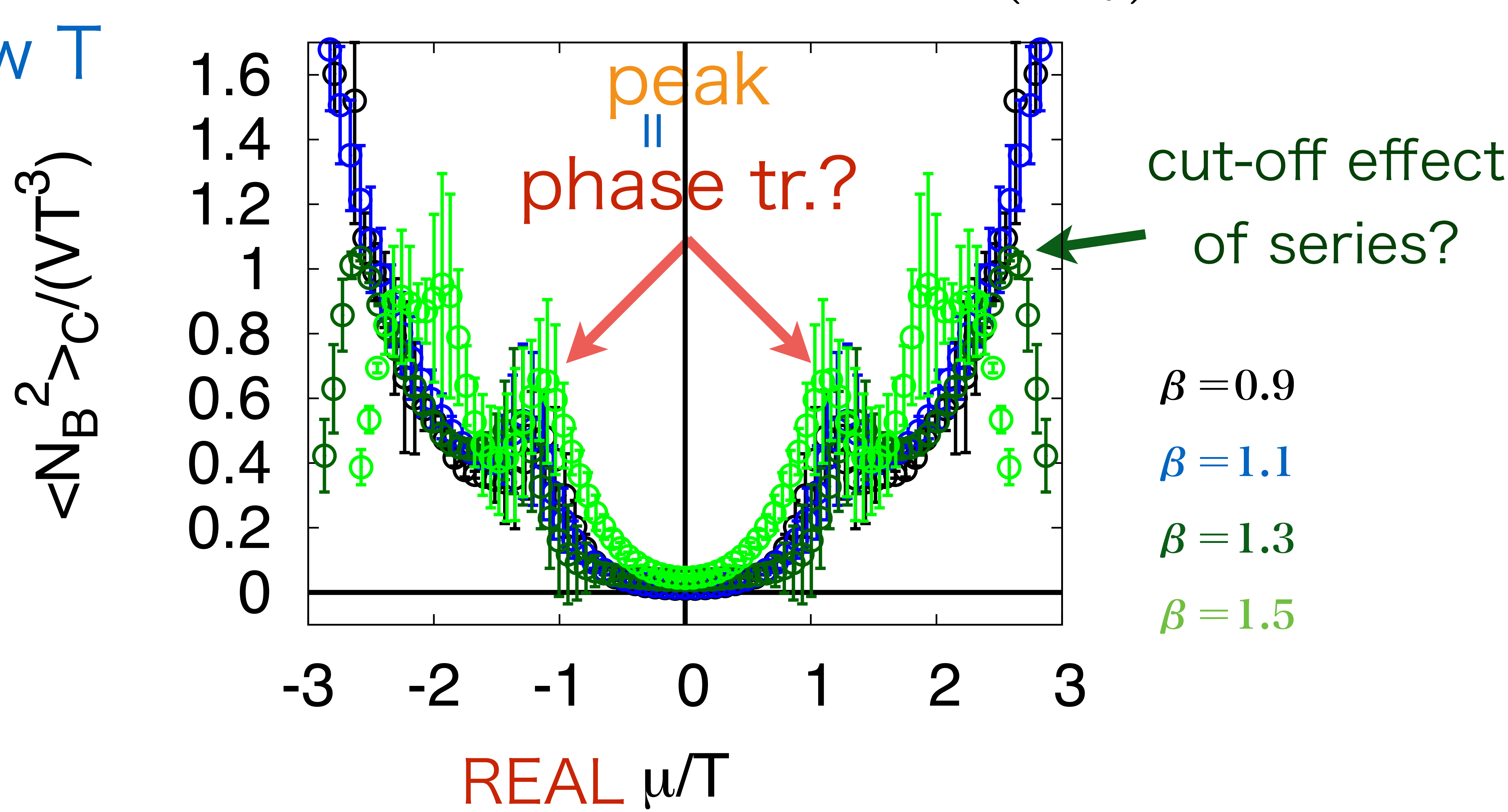

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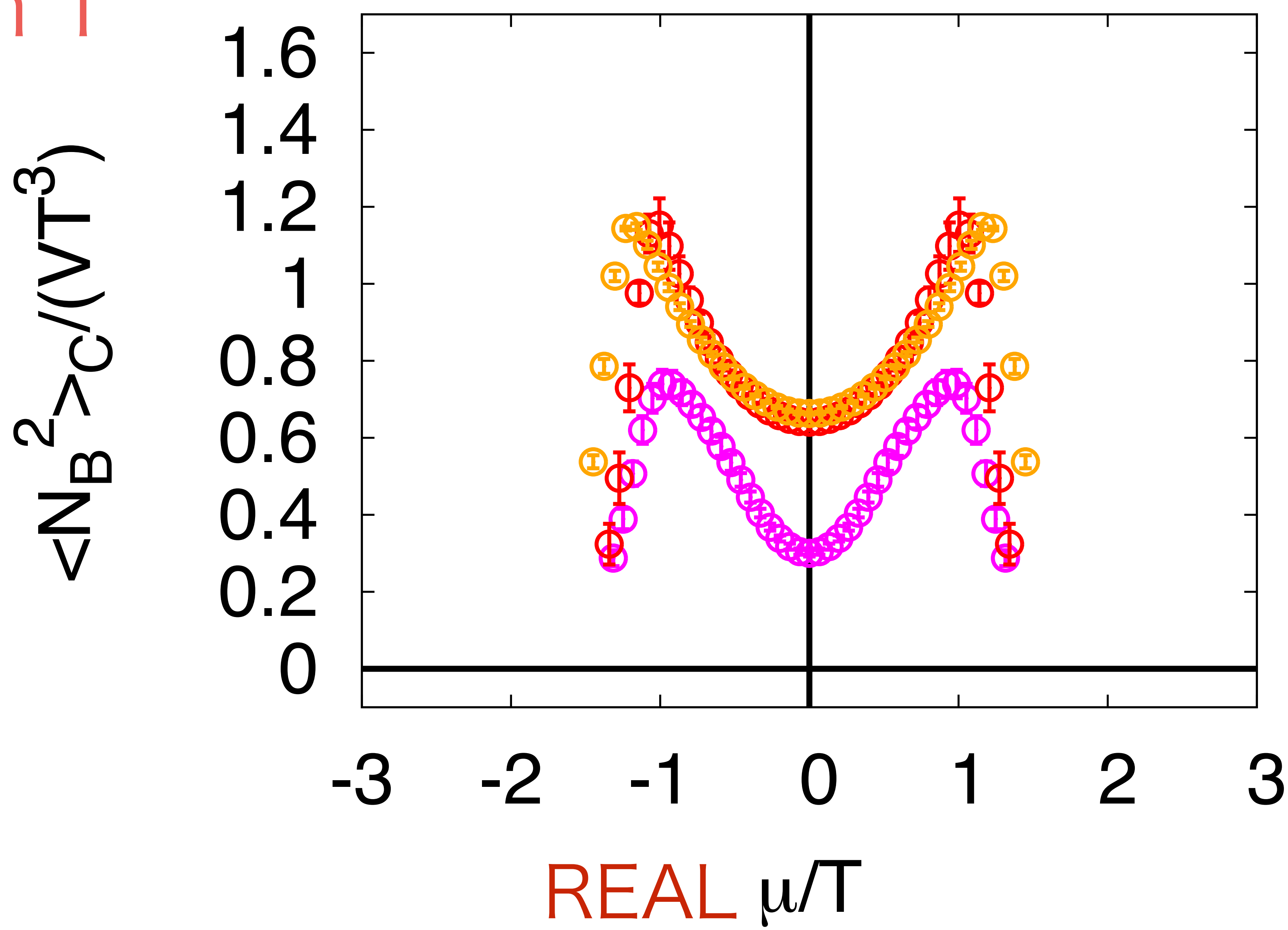

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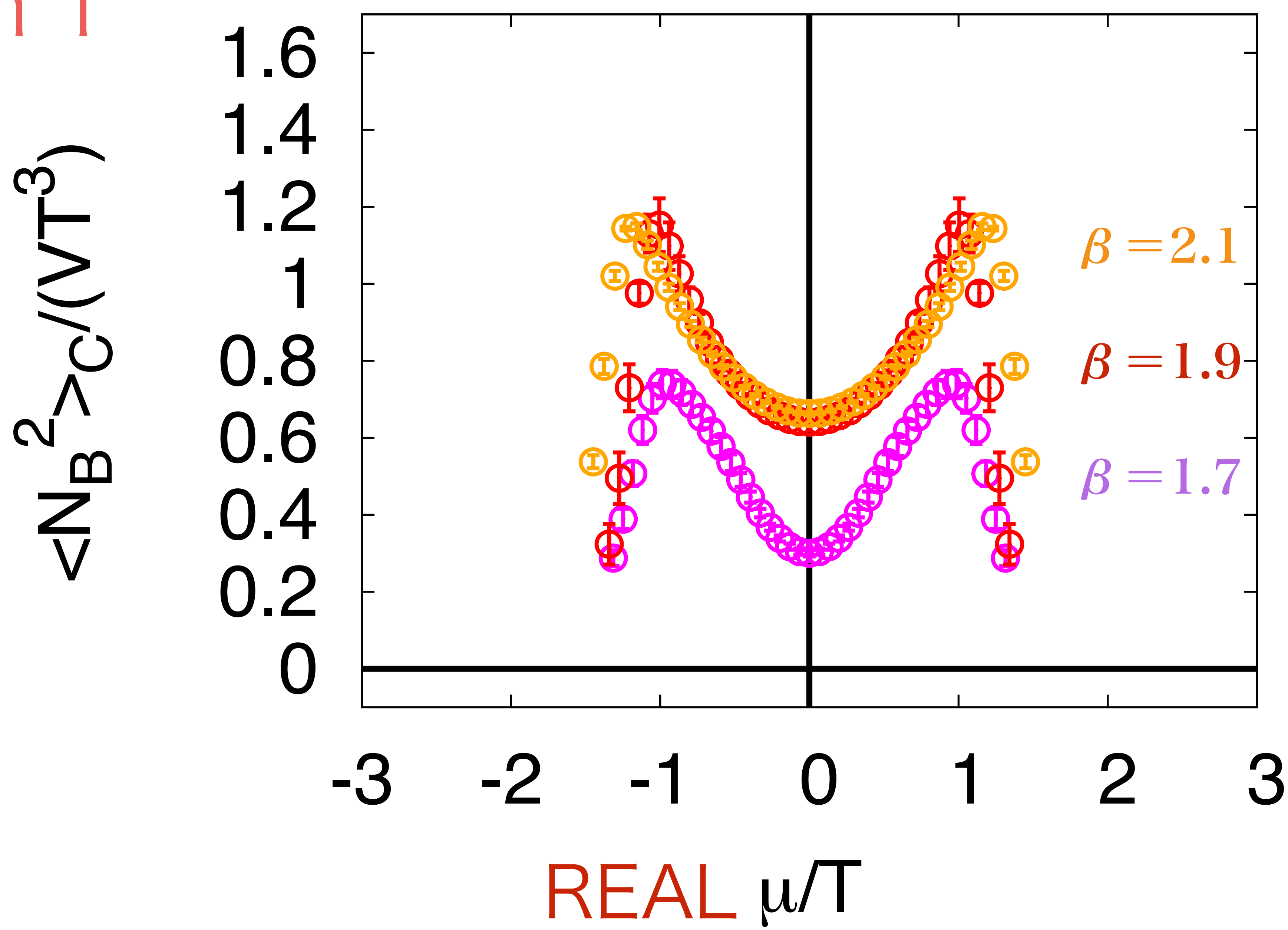

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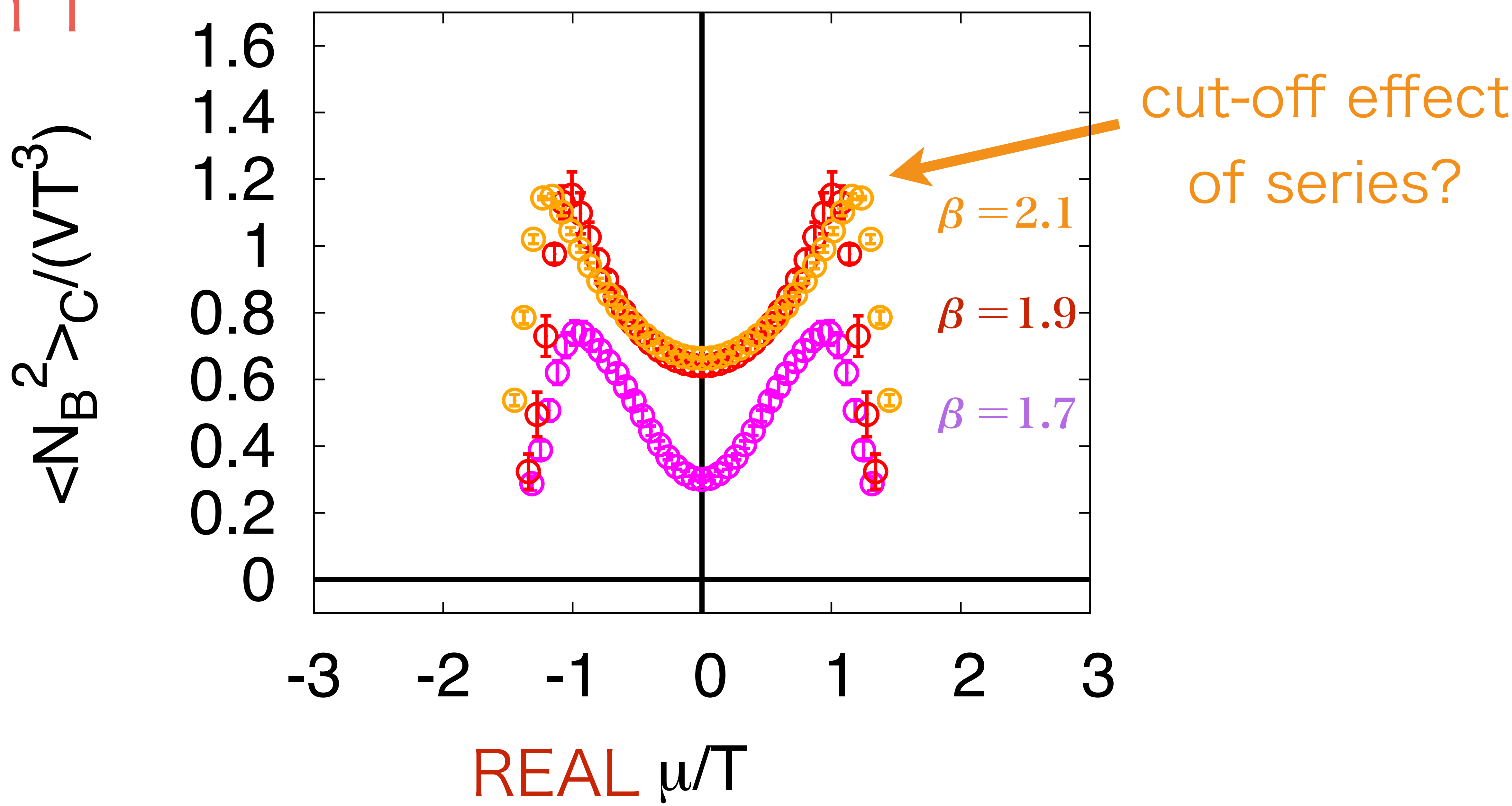

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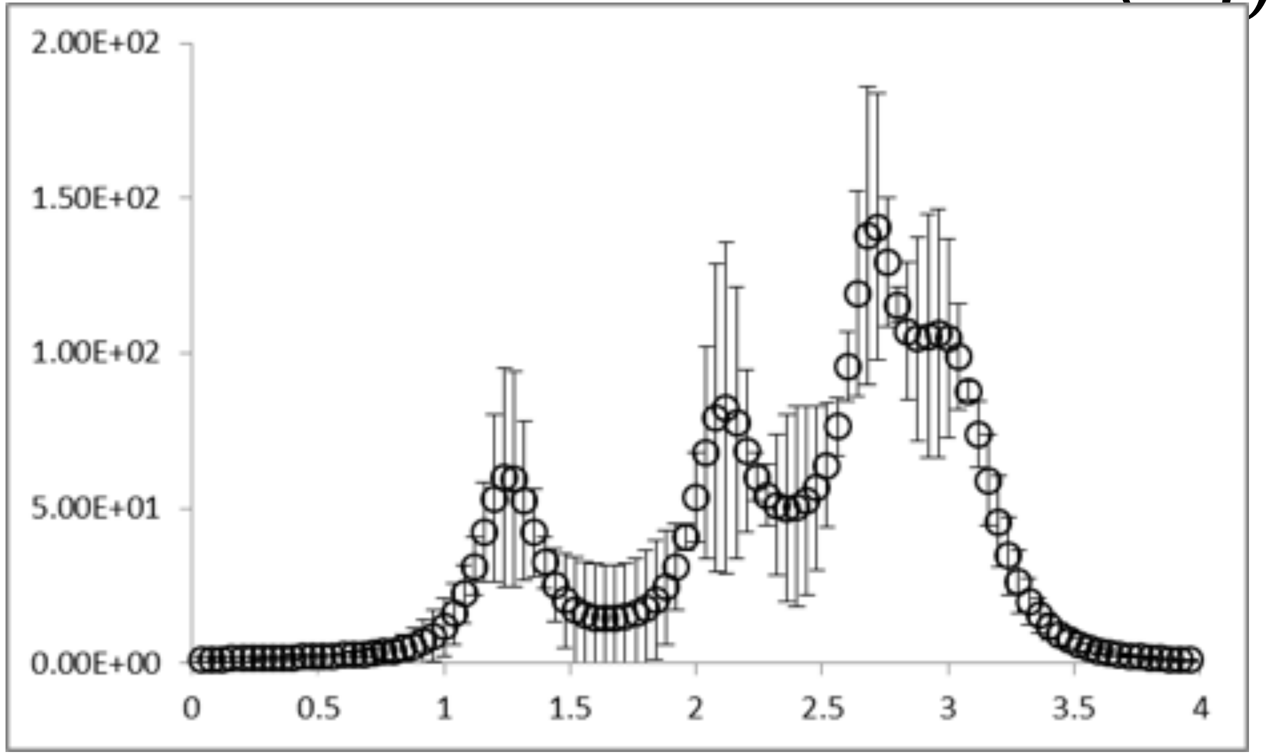
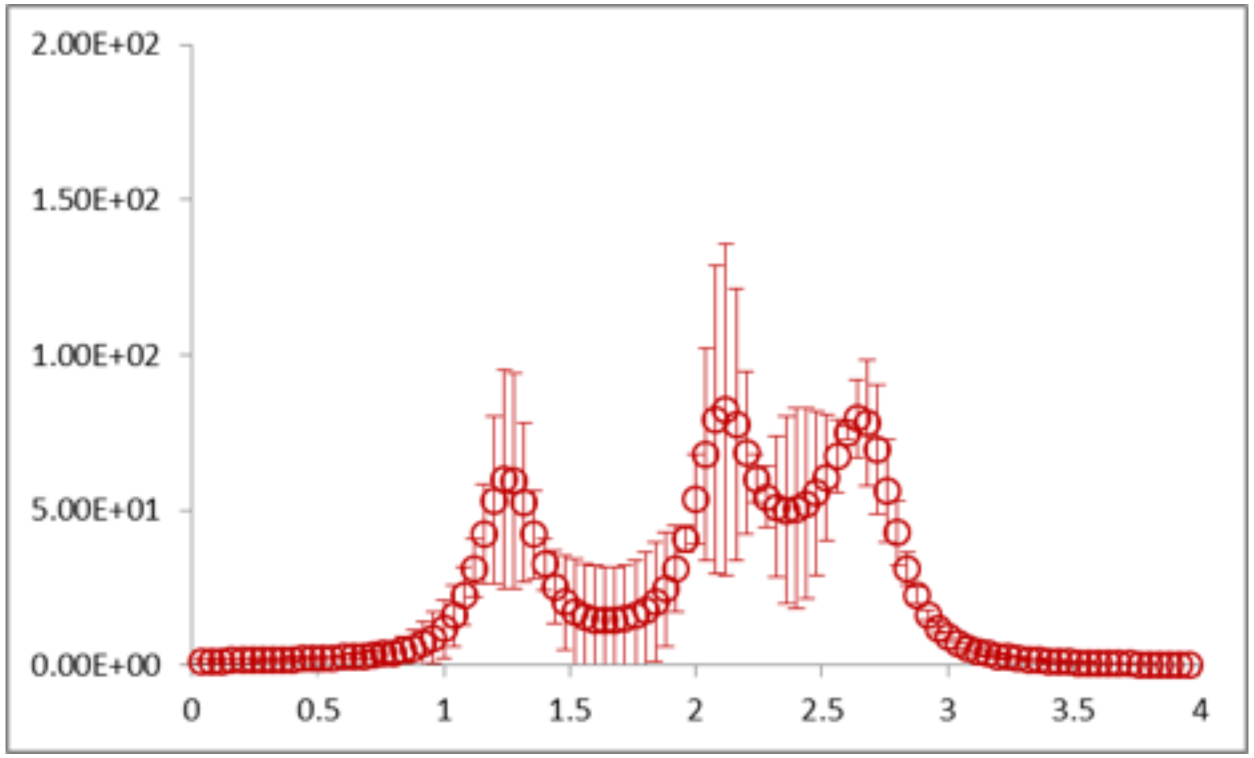
Cut-off effect of series?

β=1.5
κ=0.131



cut Zn 3 points earlier



Comparison with other methods

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canonical approach

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canonical approach ⟷ re-weighting
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Comparison with other methods

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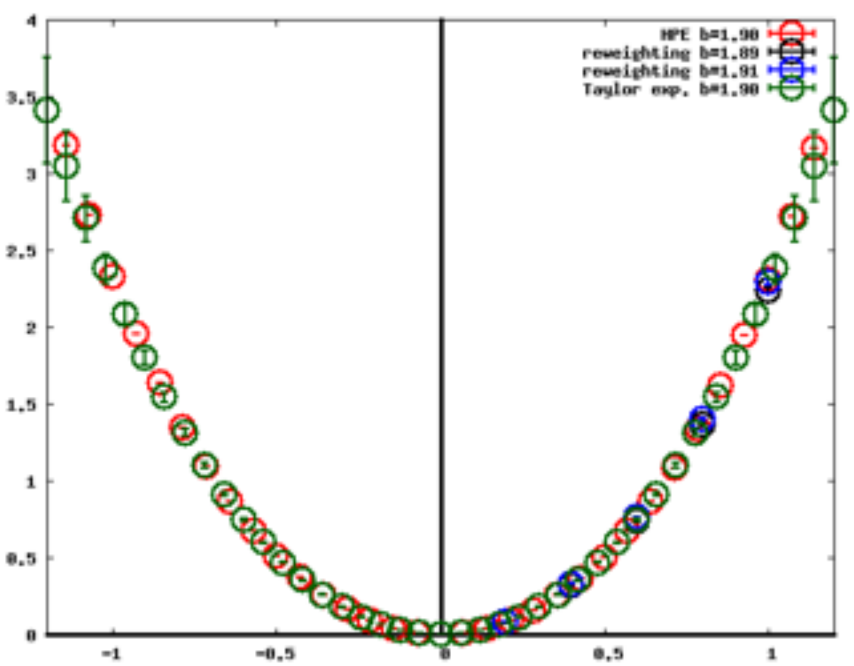
$\beta = 1.9$ T=1.09Tc (Nagata-Nakamura 2012)

pressure/T^4



μ_B/T

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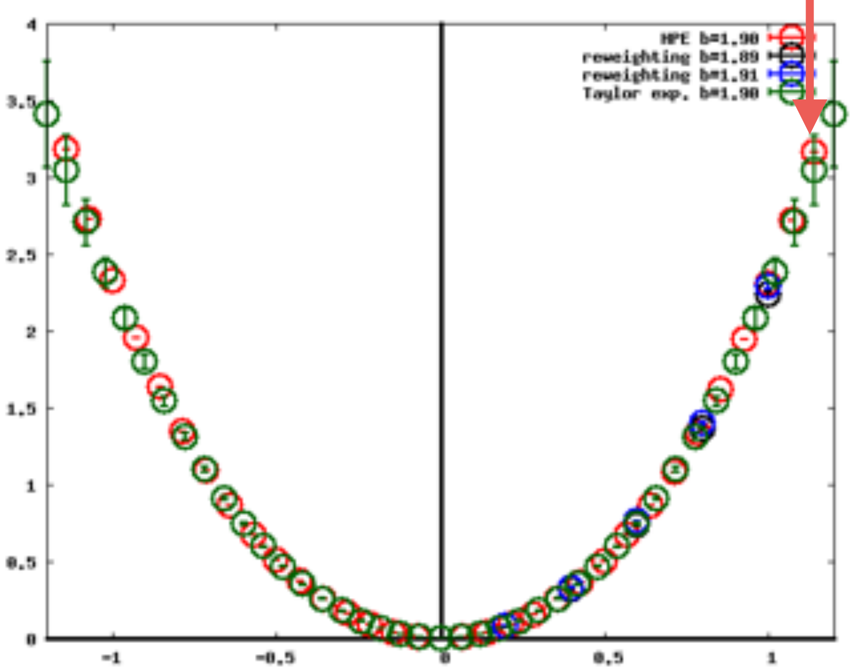

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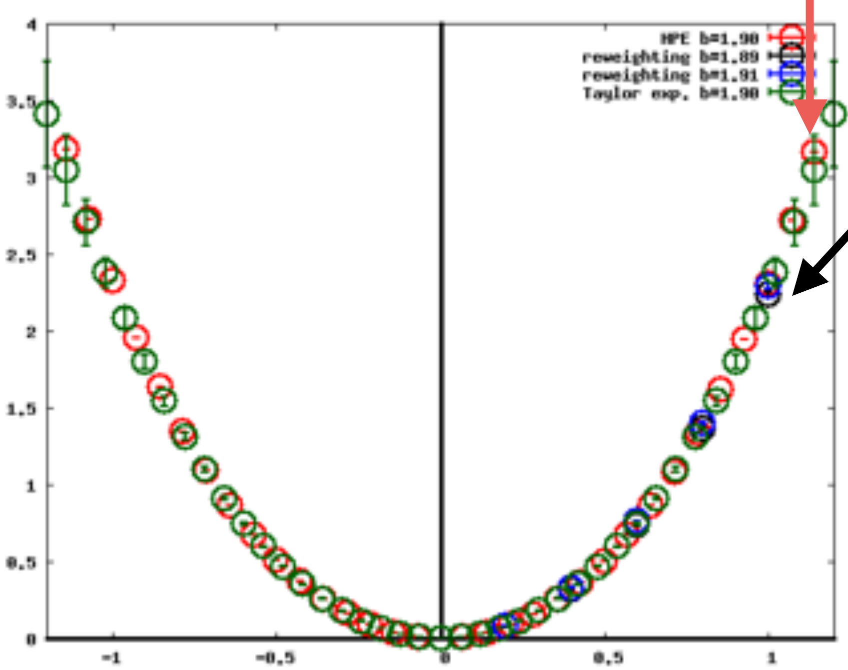

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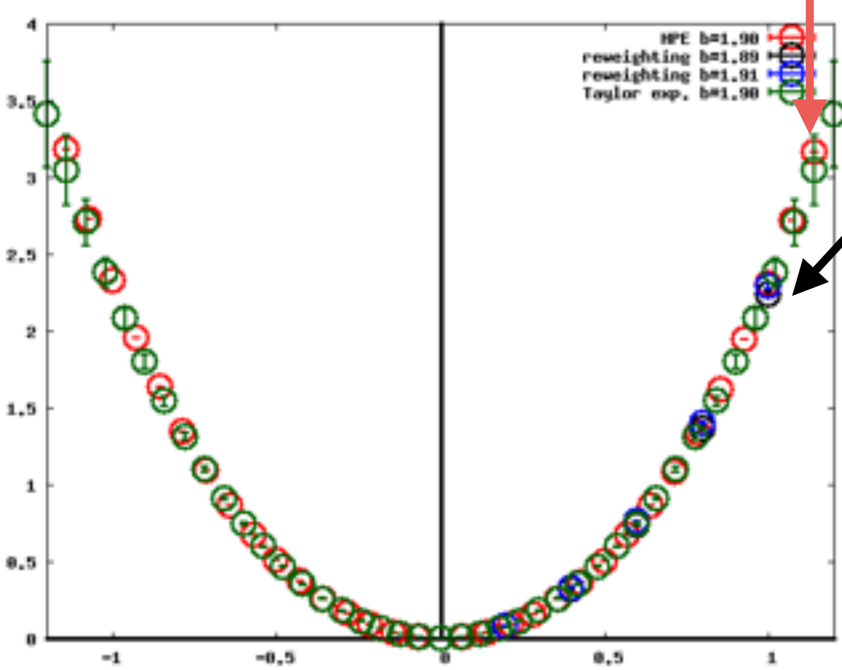

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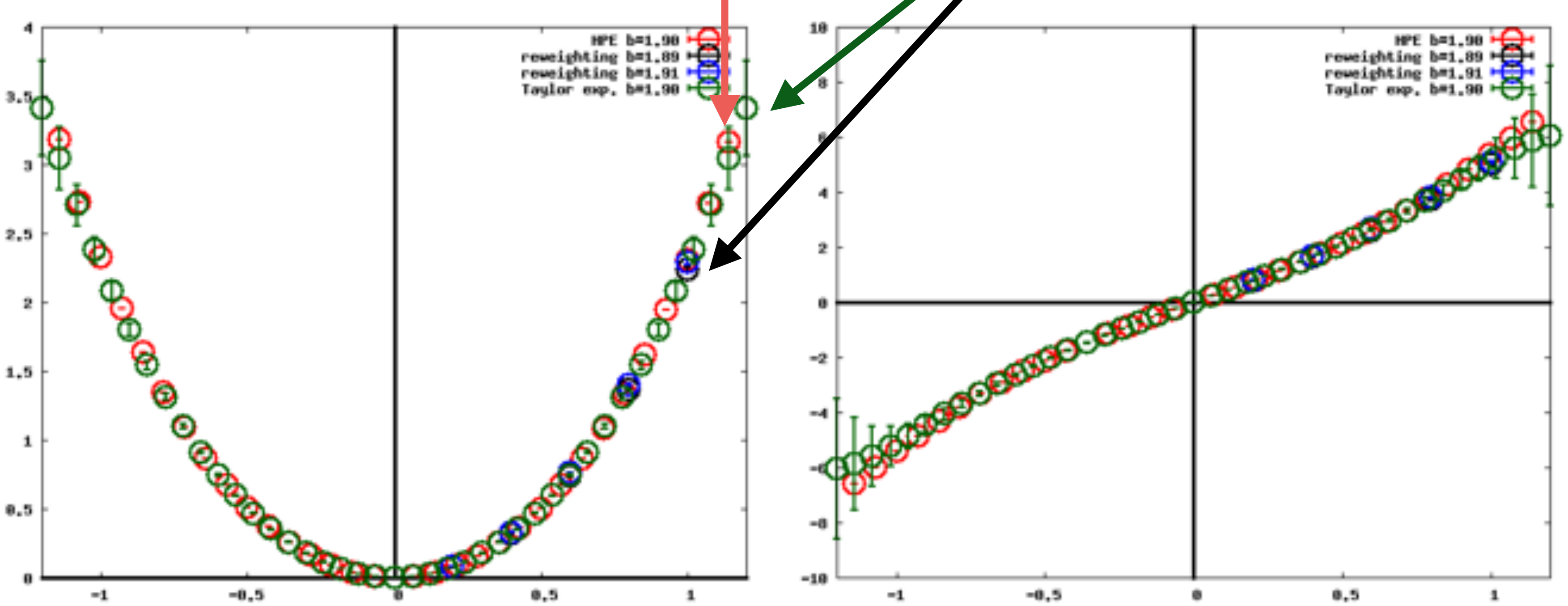
pressure/T^4 quark number



μ_B/T μ_B/T

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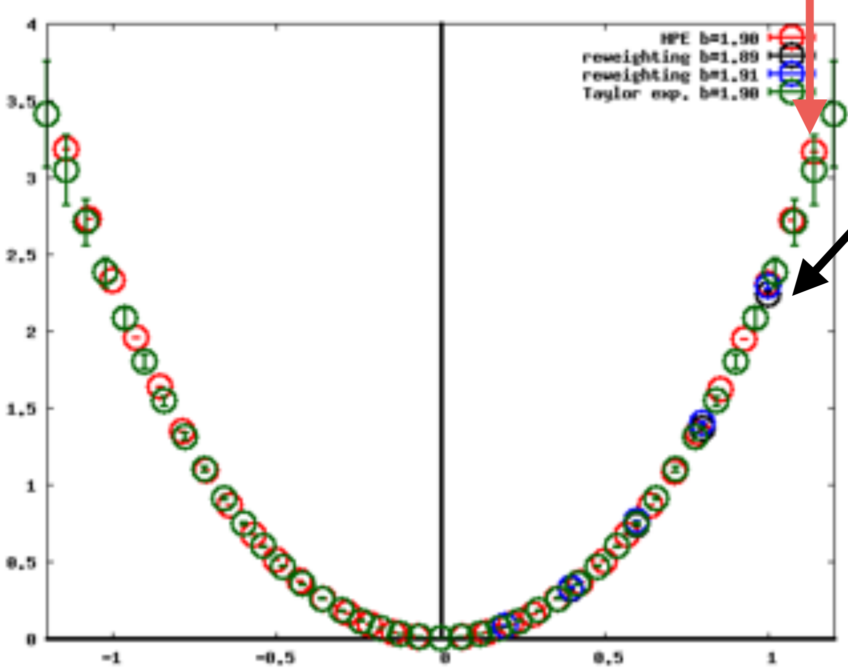

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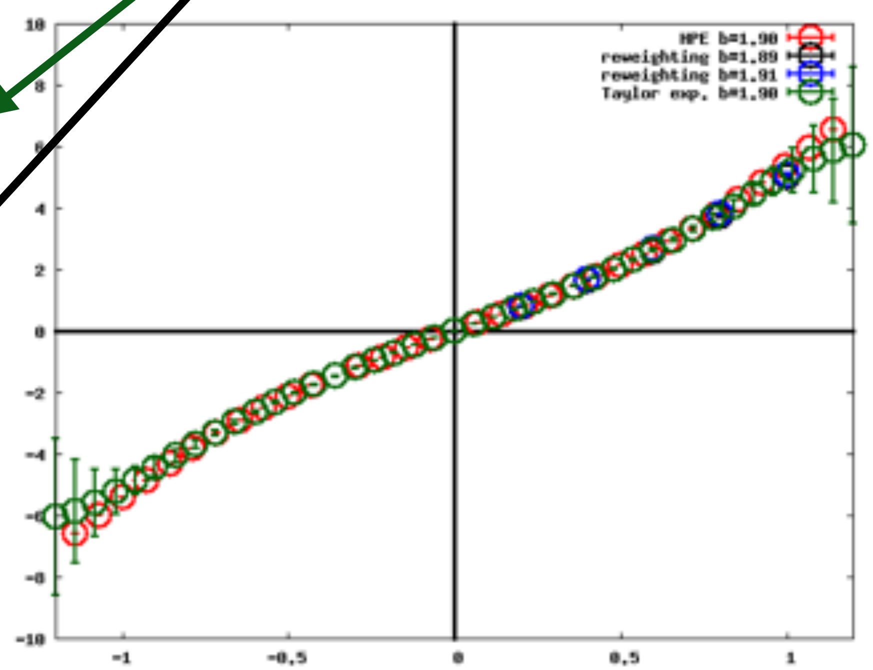
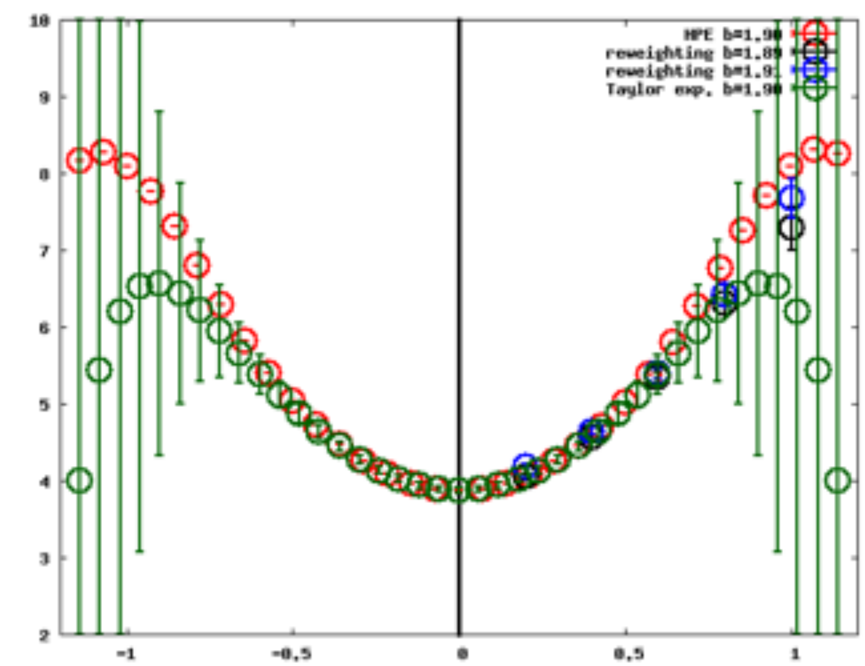


μ_B/T

susceptibility



μ_B/T

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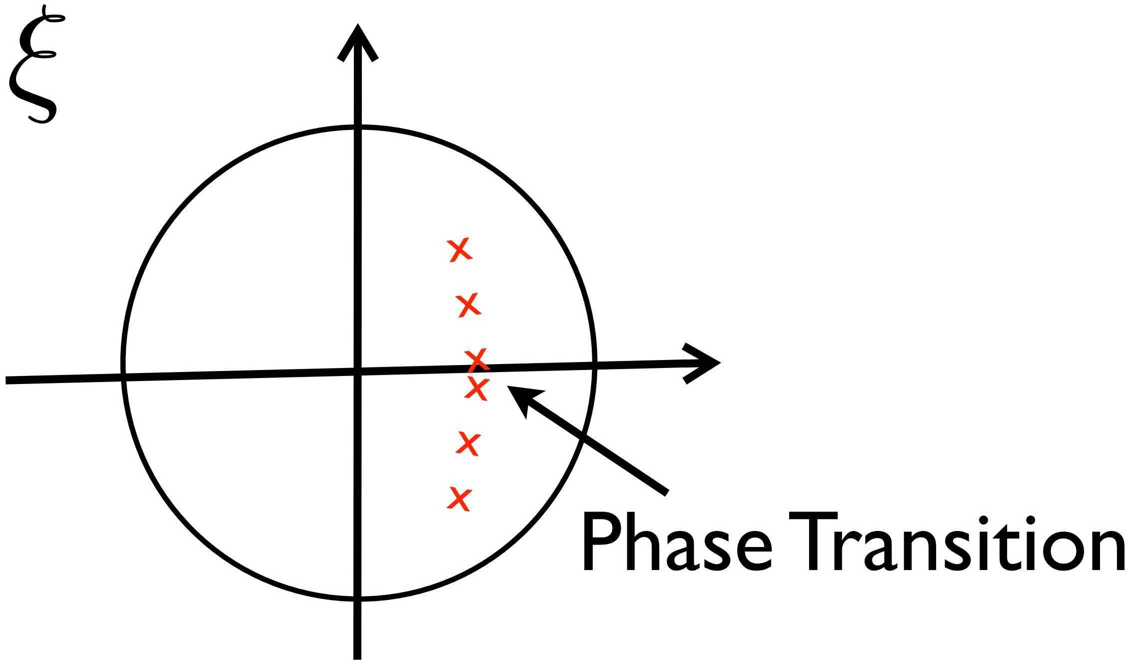

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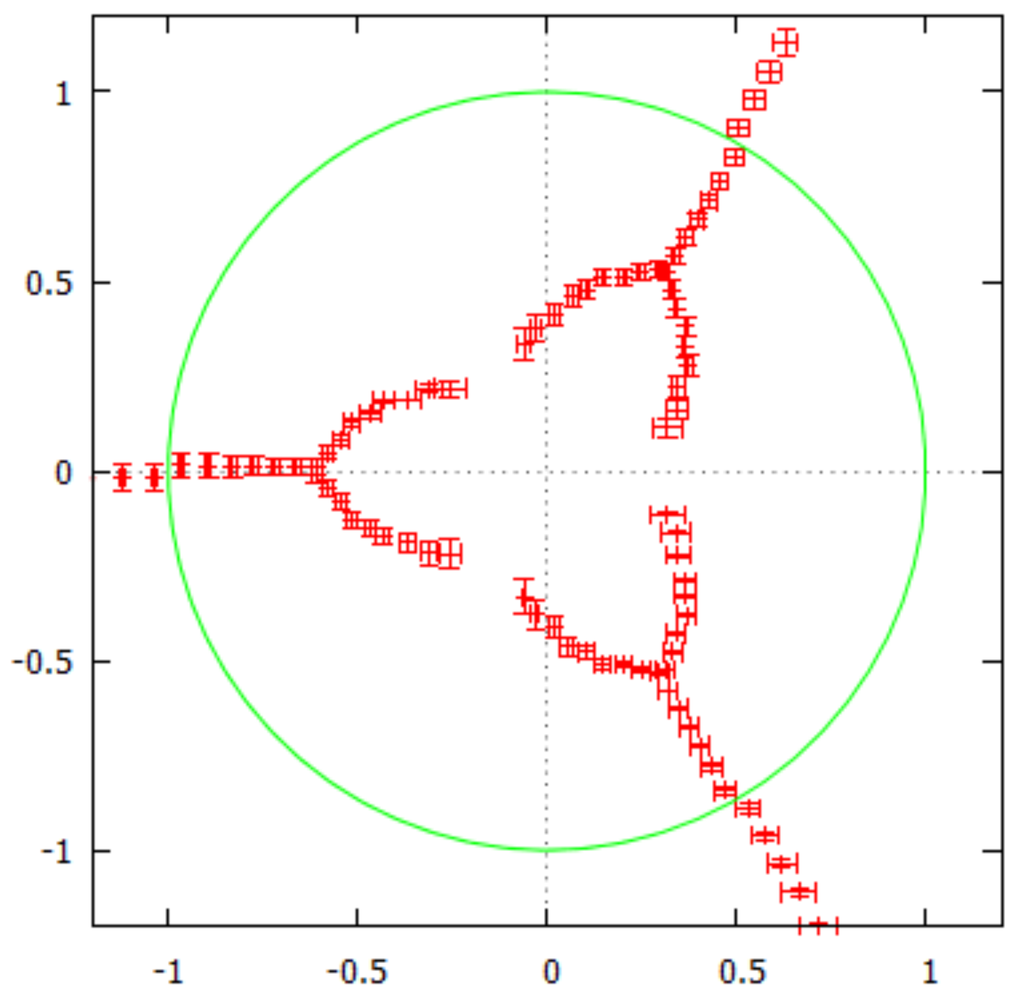
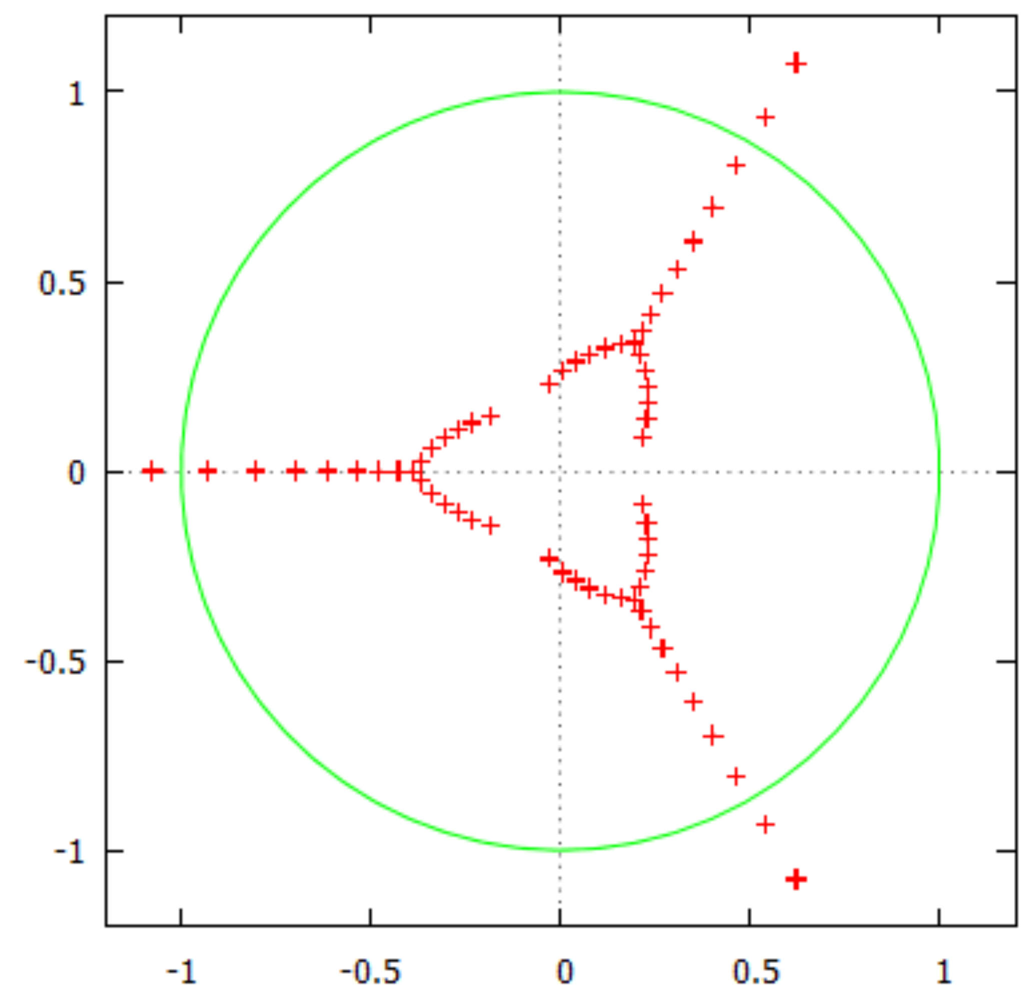
$\beta = 1.9$: T>Tc



$\beta = 1.7$: T≧Tc



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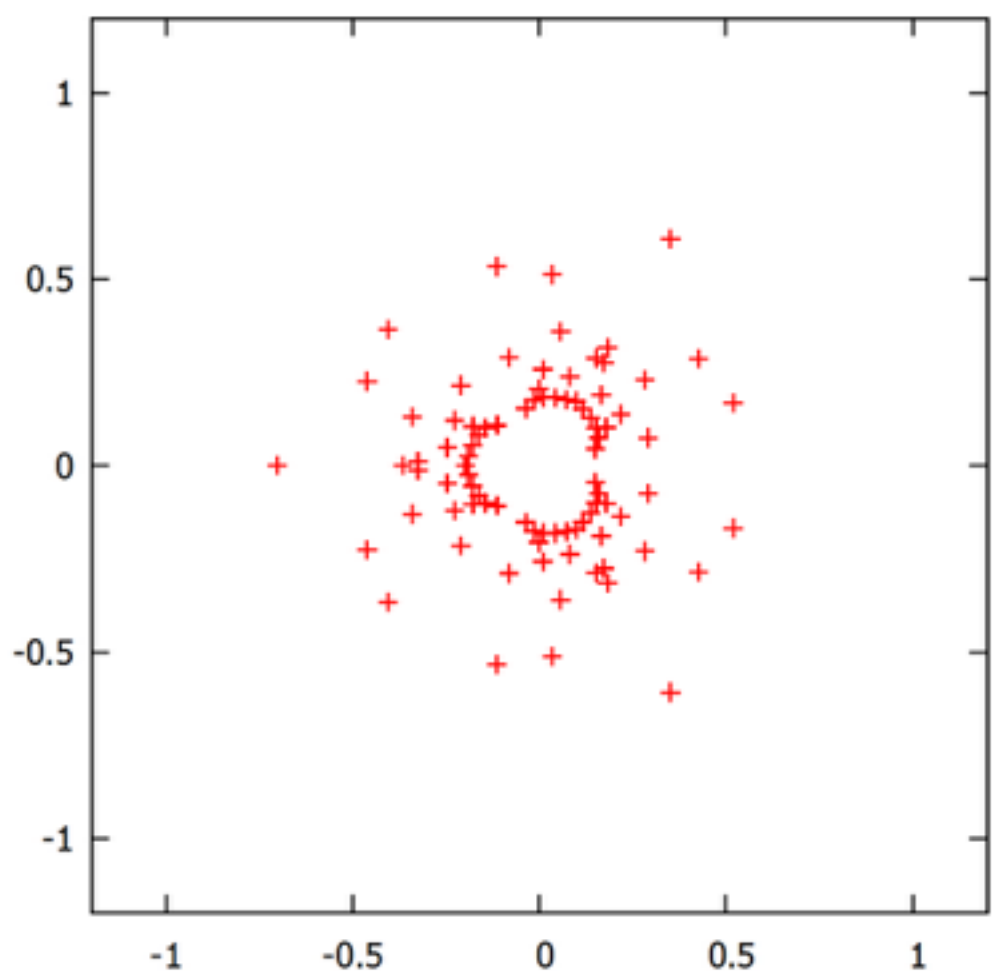
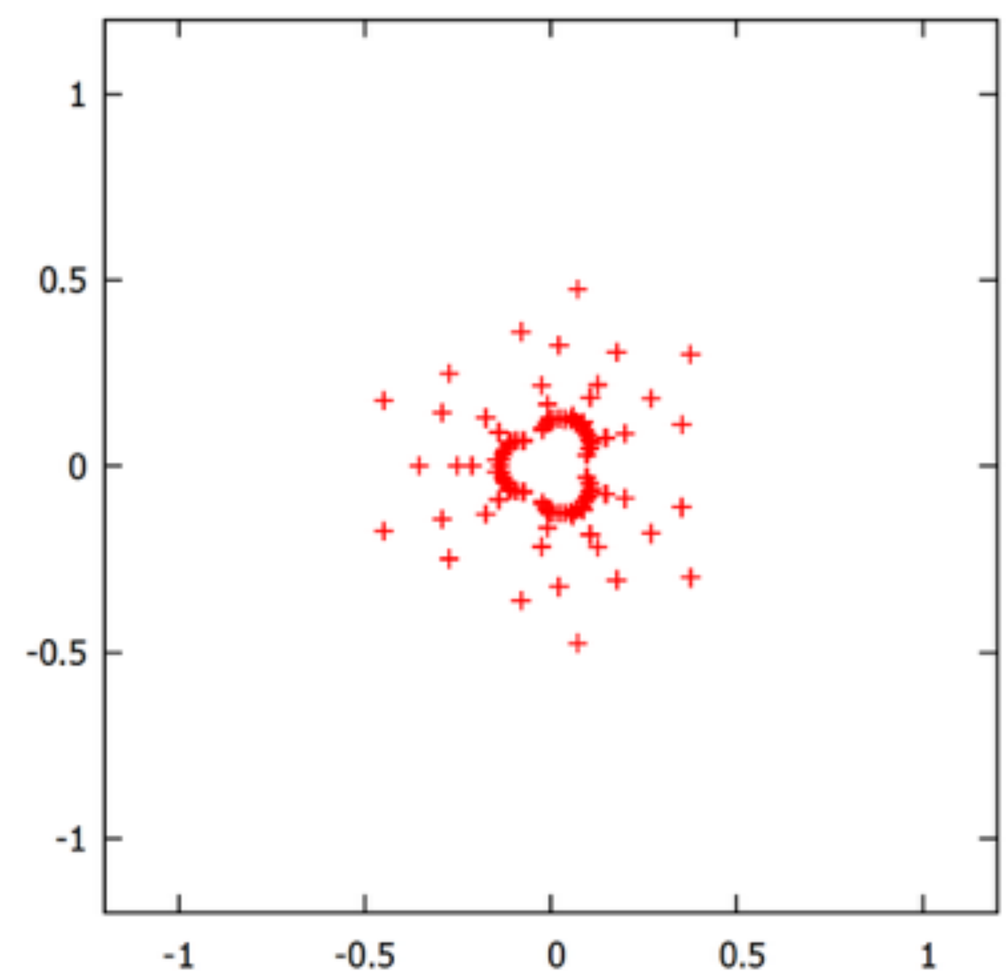


$\beta = 1.3$: T<Tc



Plan of the talk

✔ 1. Introduction

✔ 2. Hopping parameter expansion

✔ 3. Numerical setup

✔ 4. Canonical partition function Zn

5. Hadronic observables

6. Conclusion

Hadronic observables

Fugacity expansion of EV of GC observables

$$\langle\hat{O}\rangle_G(\beta,\mu,V) = \frac{\mathrm{Tr}\left[\hat{O}\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}{\mathrm{Tr}\left[\exp\left(-\beta\left(\hat{H}-\mu\hat{N}\right)\right)\right]}$$

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$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\mathrm{Det} D_W(\xi)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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function of ξ

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function of ξ HPE

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function of ξ HPE

$$\bar{\psi}\psi = -\mathrm{tr}\left(\frac{1}{D_W}\right) = -\mathrm{tr}\left(\frac{1}{1-\kappa Q}\right) = \sum_{m=0}^{\infty}\kappa^m \mathrm{tr} Q^m$$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\mathrm{Det} D_W(\xi)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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function of ξ ↓ HPE

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HPE of $\mathrm{Det} D_W$

Hadronic observables

Fugacity expansion of EV of GC observables

$$O_n = \oint \frac{d\xi}{2\pi i} \xi^{-n-1} \left\langle O(D_W(\xi)) \frac{\mathrm{Det} D_W(\xi)}{\mathrm{Det} D_W(\mu_0)} \right\rangle_0 Z_G(\mu_0)$$

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HPE of $\mathrm{Det} D_W$

re-sum → winding number expansion

Hadronic observables

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$$O_n = \oint \frac{d\xi}{2\pi i}\xi^{-n-1}\left\langle O(D_W(\xi))\frac{\mathrm{Det}D_W(\xi)}{\mathrm{Det}D_W(\mu_0)}\right\rangle_0 Z_G(\mu_0)$$

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HPE of $\mathrm{Det}D_W$

re-sum → winding number expansion

fugacity expansion of $O(D_W(\xi))\frac{\mathrm{Det}D_W(\xi)}{\mathrm{Det}D_W(\mu_0)}$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \middle| \hat{O} e^{-\beta \hat{H}} \middle| E, n \right\rangle$$

$$Z_n = \sum_E \left\langle E, n \middle| e^{-\beta \hat{H}} \middle| E, n \right\rangle$$

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VEV in canonical ensamble

Hadronic observables

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VEV in canonical ensamble

$$\langle \hat{O} \rangle_C(\beta, n, V) = \frac{O_n}{Z_n}$$

Hadronic observables

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VEV in canonical ensamble

$$\langle \hat{O} \rangle_C (\beta, n, V) = \frac{O_n}{Z_n}$$

VEV in the REAL μ!

Hadronic observables

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$$\langle \hat{O} \rangle_C (\beta, n, V) = \frac{O_n}{Z_n}$$

VEV in the REAL μ !

$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

Hadronic observables

$$O_n = \sum_E \left\langle E, n \middle| \hat{O} e^{-\beta \hat{H}} \middle| E, n \right\rangle$$

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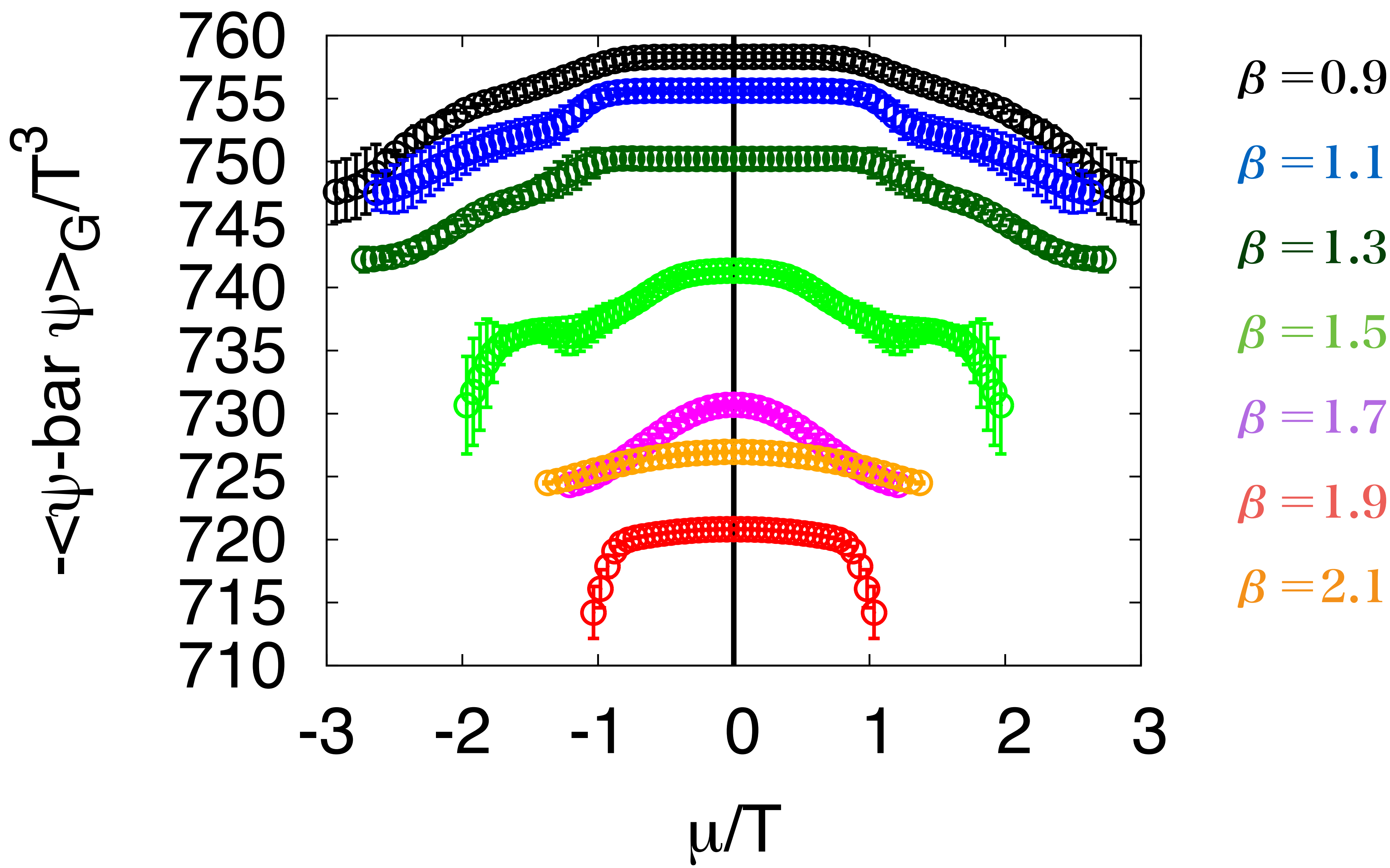
$$O(\mu) = \sum_{n=-\infty}^{\infty} O_n \xi^n$$

$$Z(\mu) = \sum_{n=-\infty}^{\infty} Z_n \xi^n$$

$$\longrightarrow \quad \langle \hat{O} \rangle_G (\beta, \mu, V) = \frac{O(\mu)}{Z(\mu)}$$

Chiral restoration?

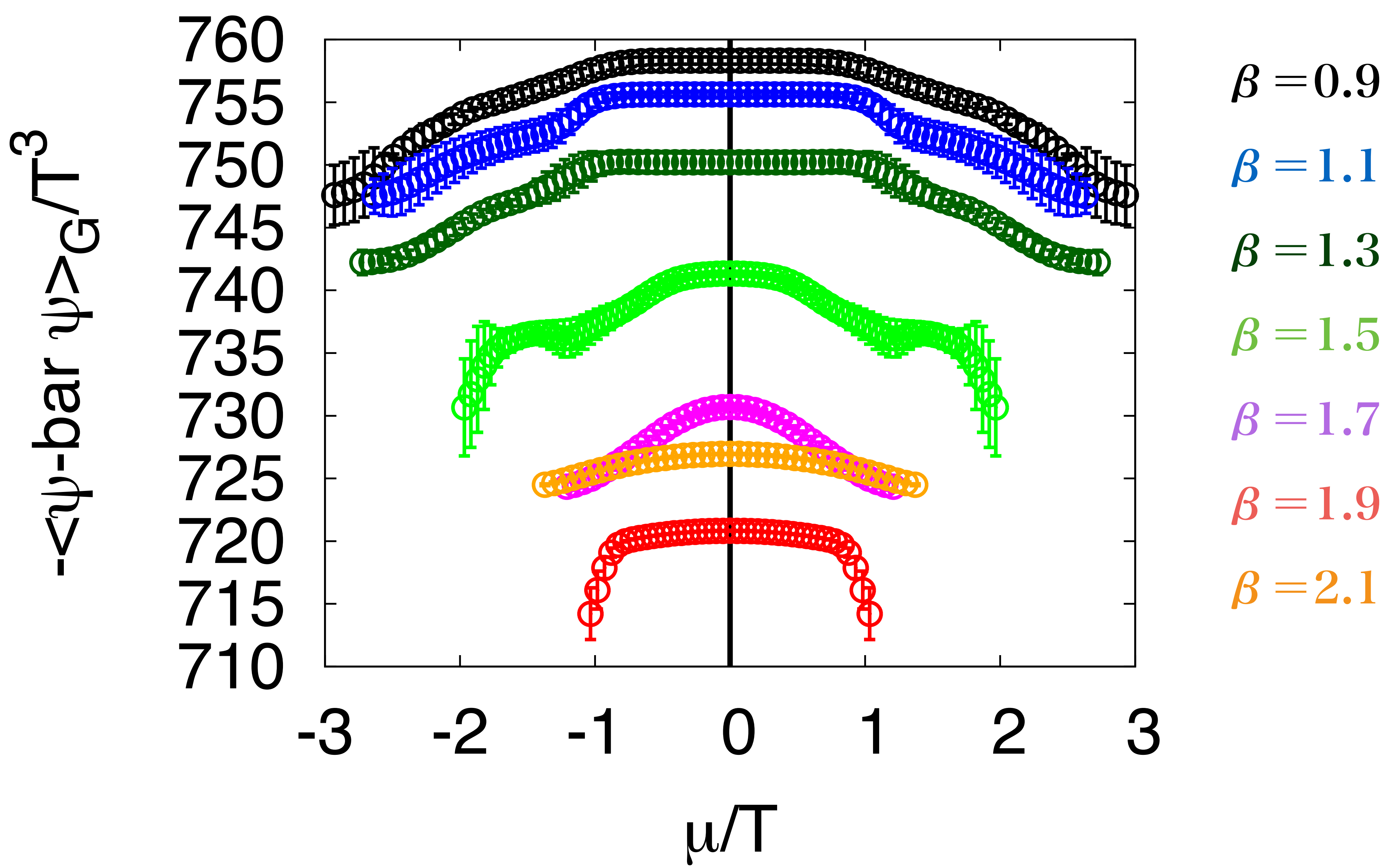
Grand canonical chiral condensate



Chiral restoration?

Grand canonical chiral condensate

No renormalization! No subtraction! Sorry...



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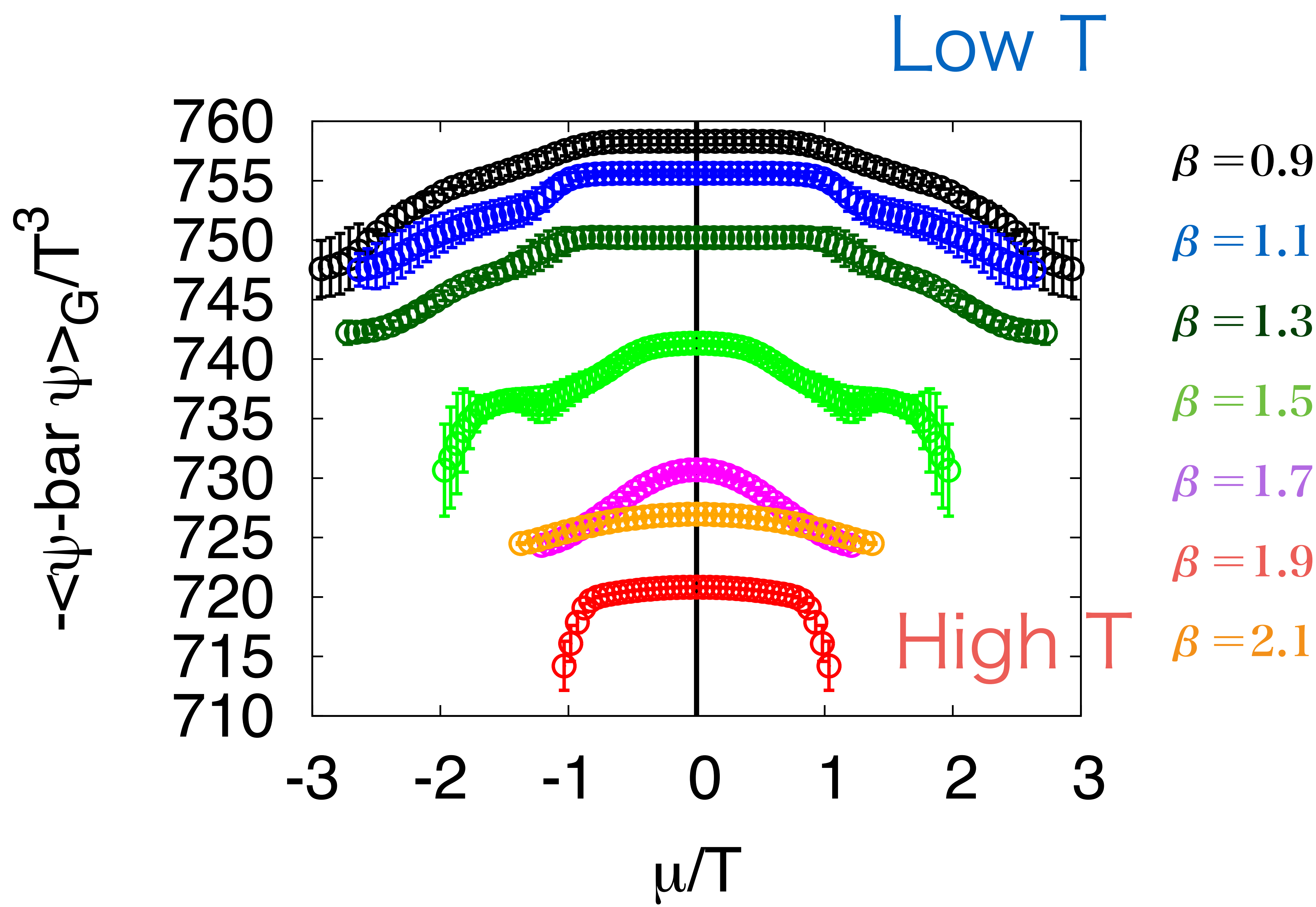

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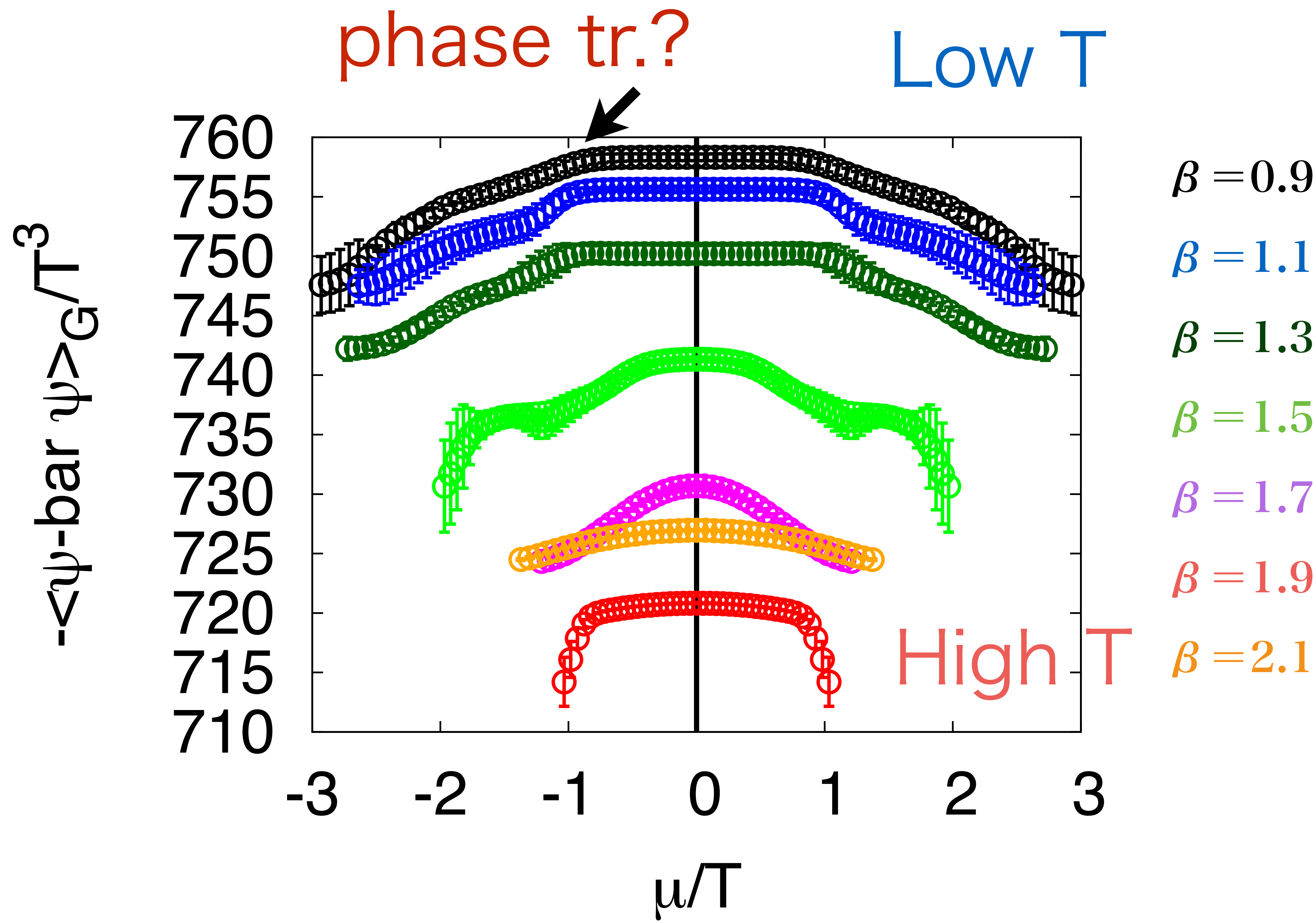

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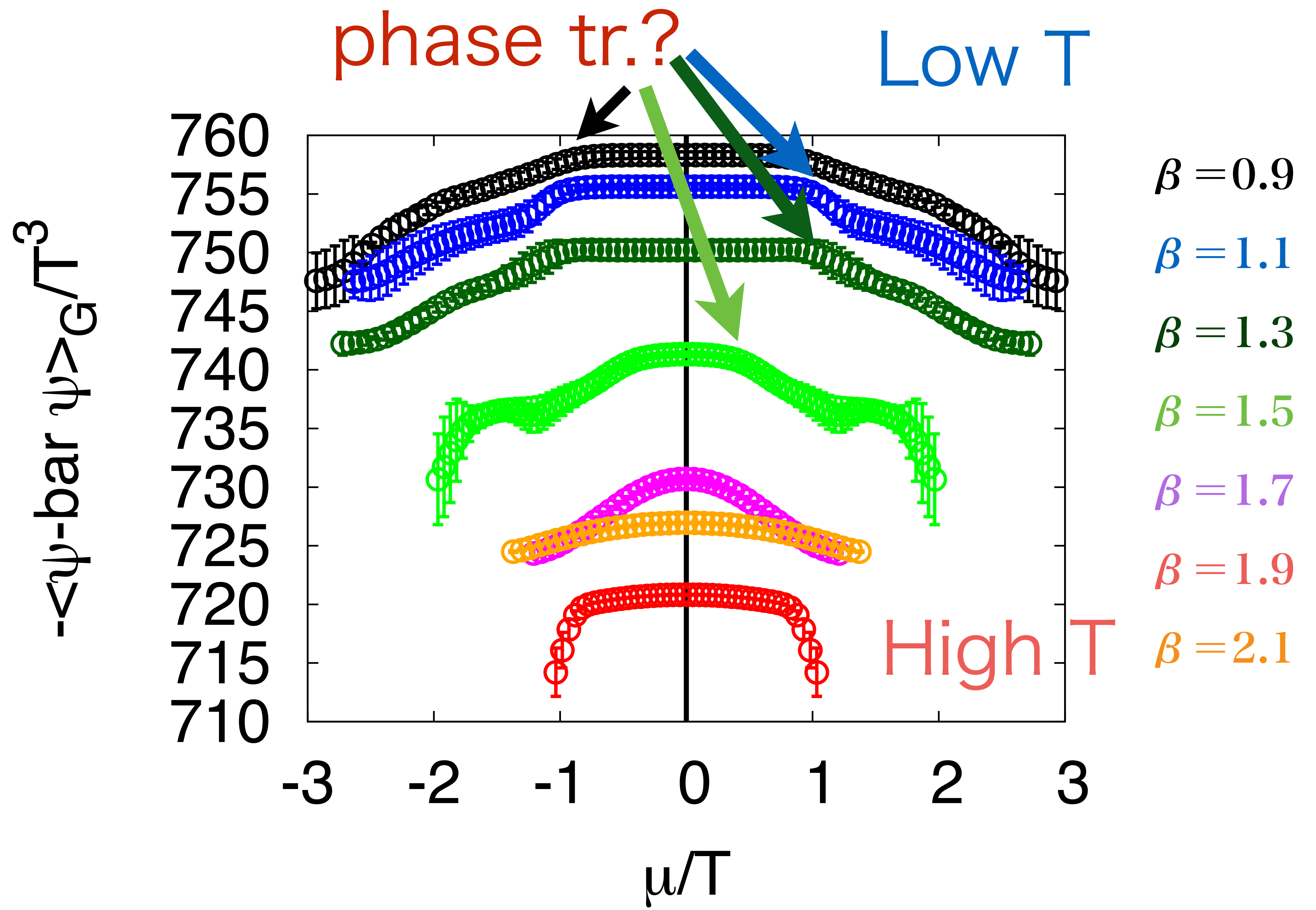

Chiral restoration?

Grand canonical chiral condensate

No renormalization! No subtraction! Sorry...

phase tr.?

Low T

$\beta = 0.9$

$\beta = 1.1$

$\beta = 1.3$

Chiral restoration?

2nd cumulant of chiral condensate

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Preliminary!

phase tr.?

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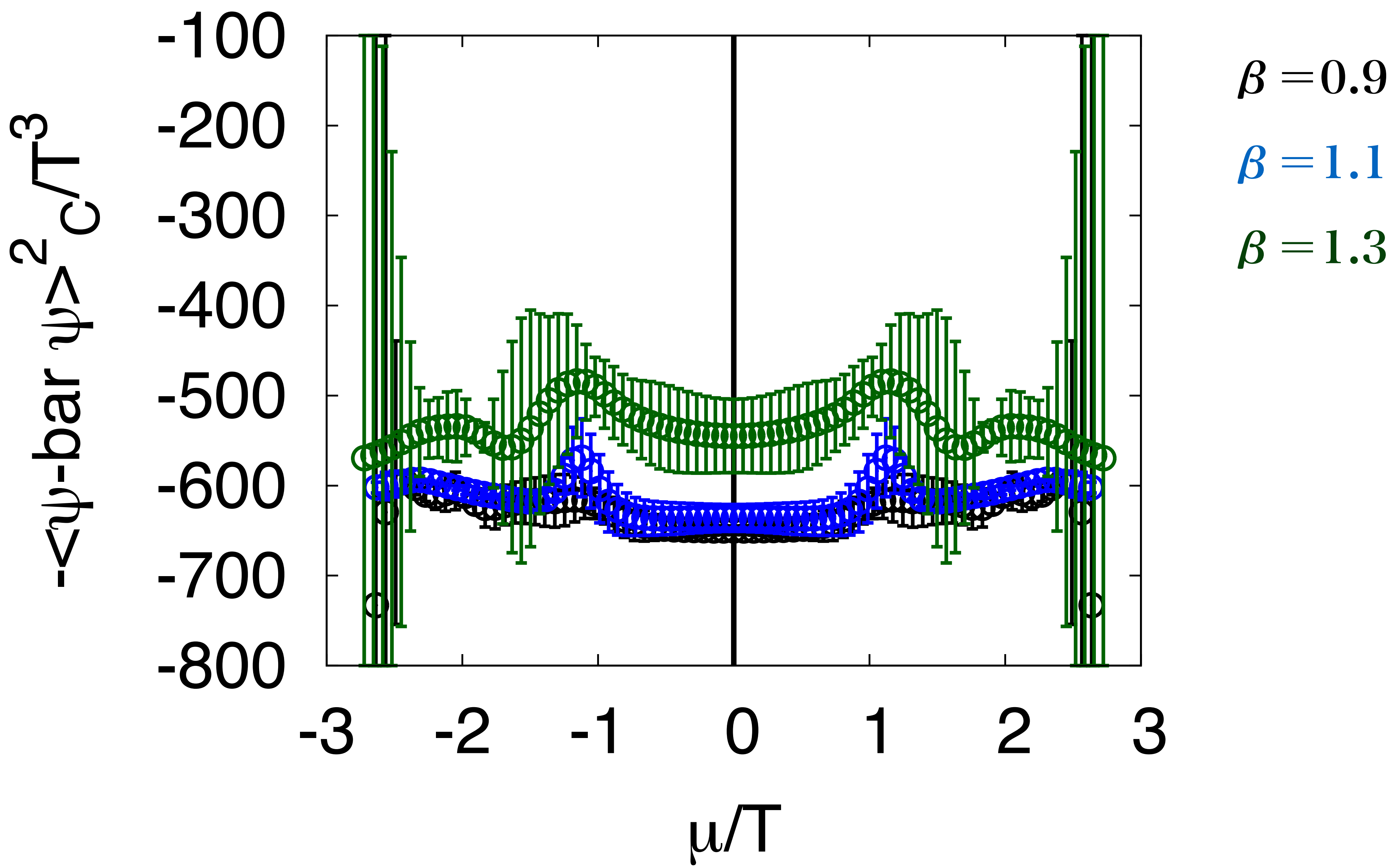

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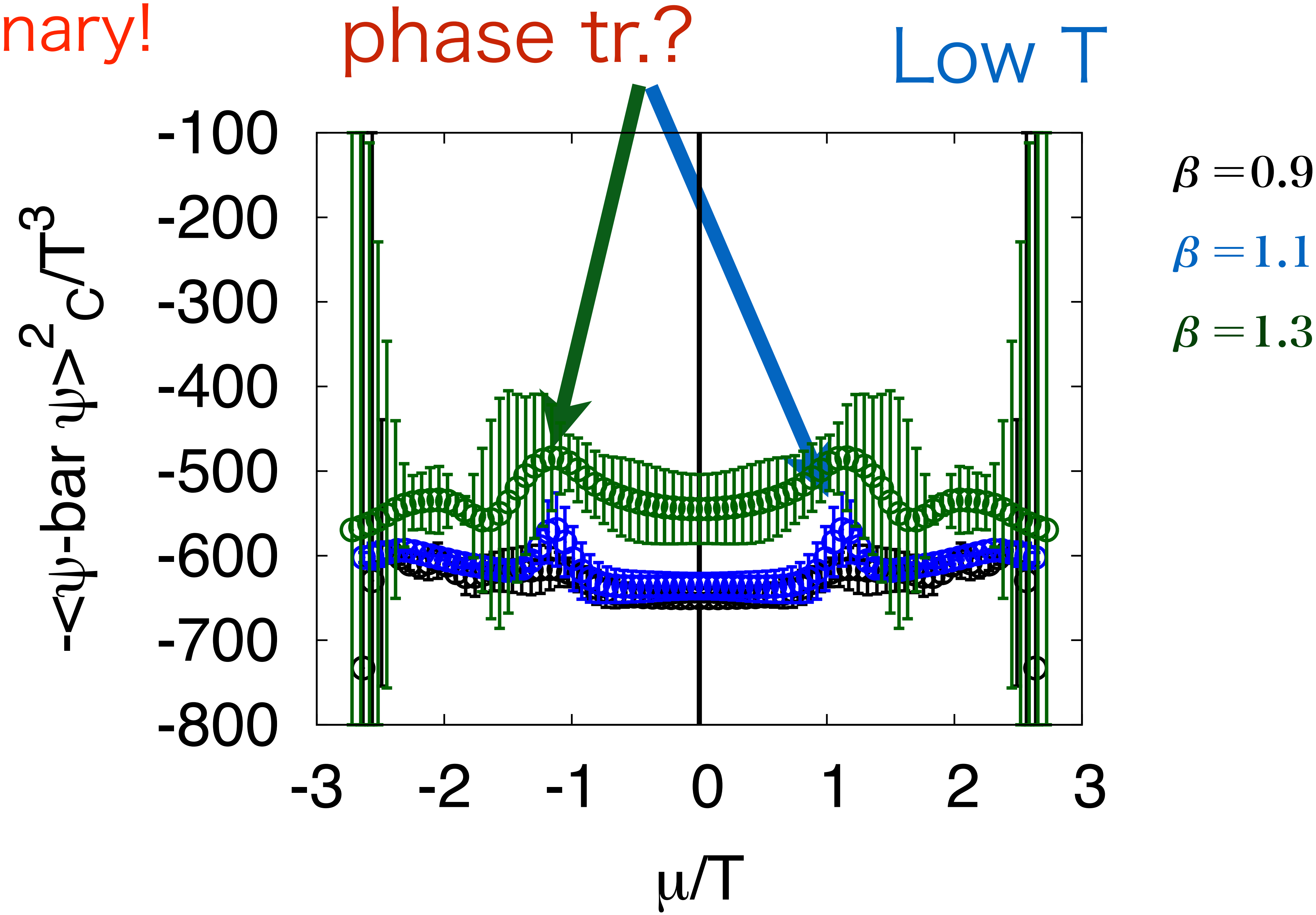
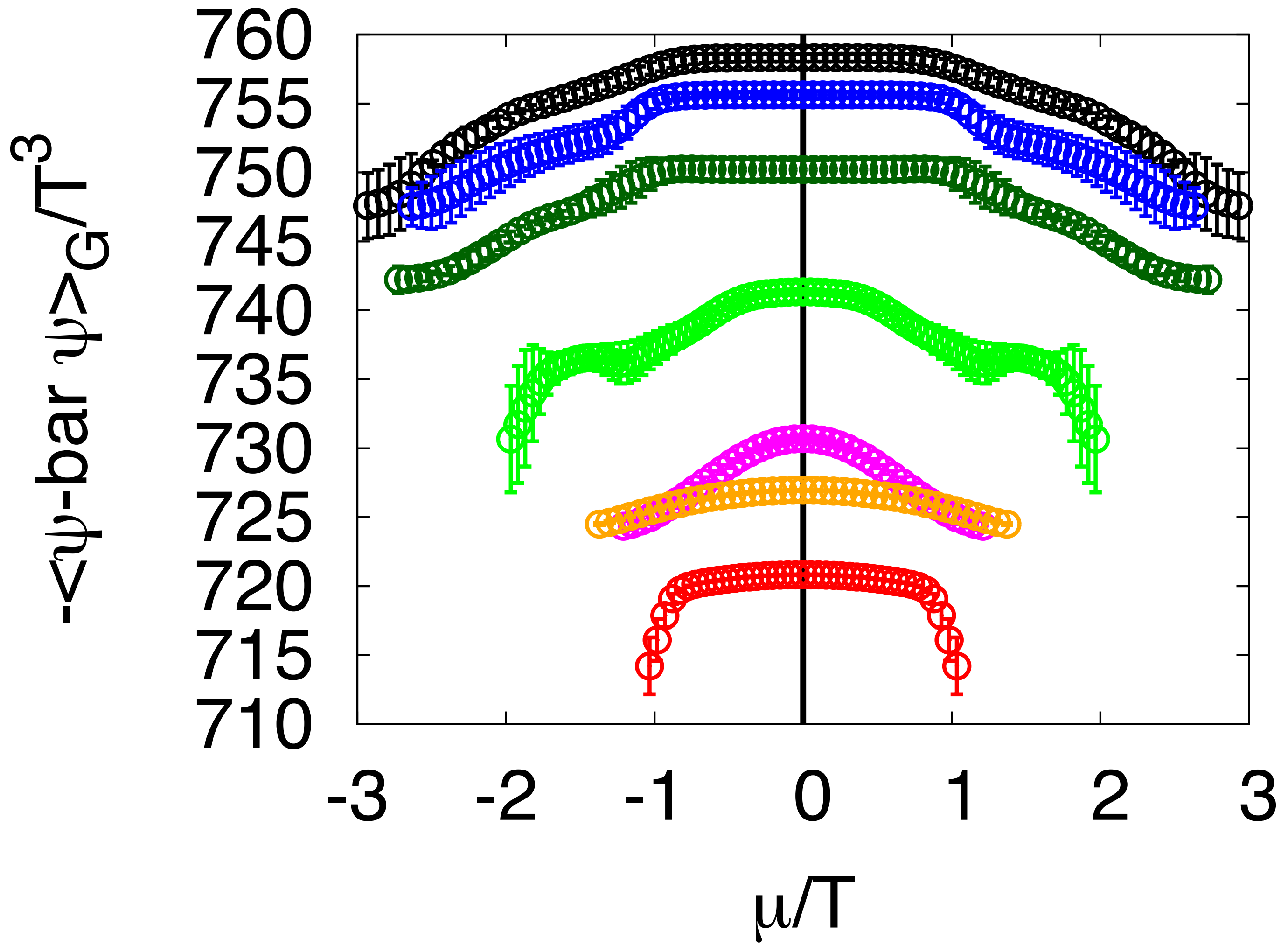


Chiral restoration?

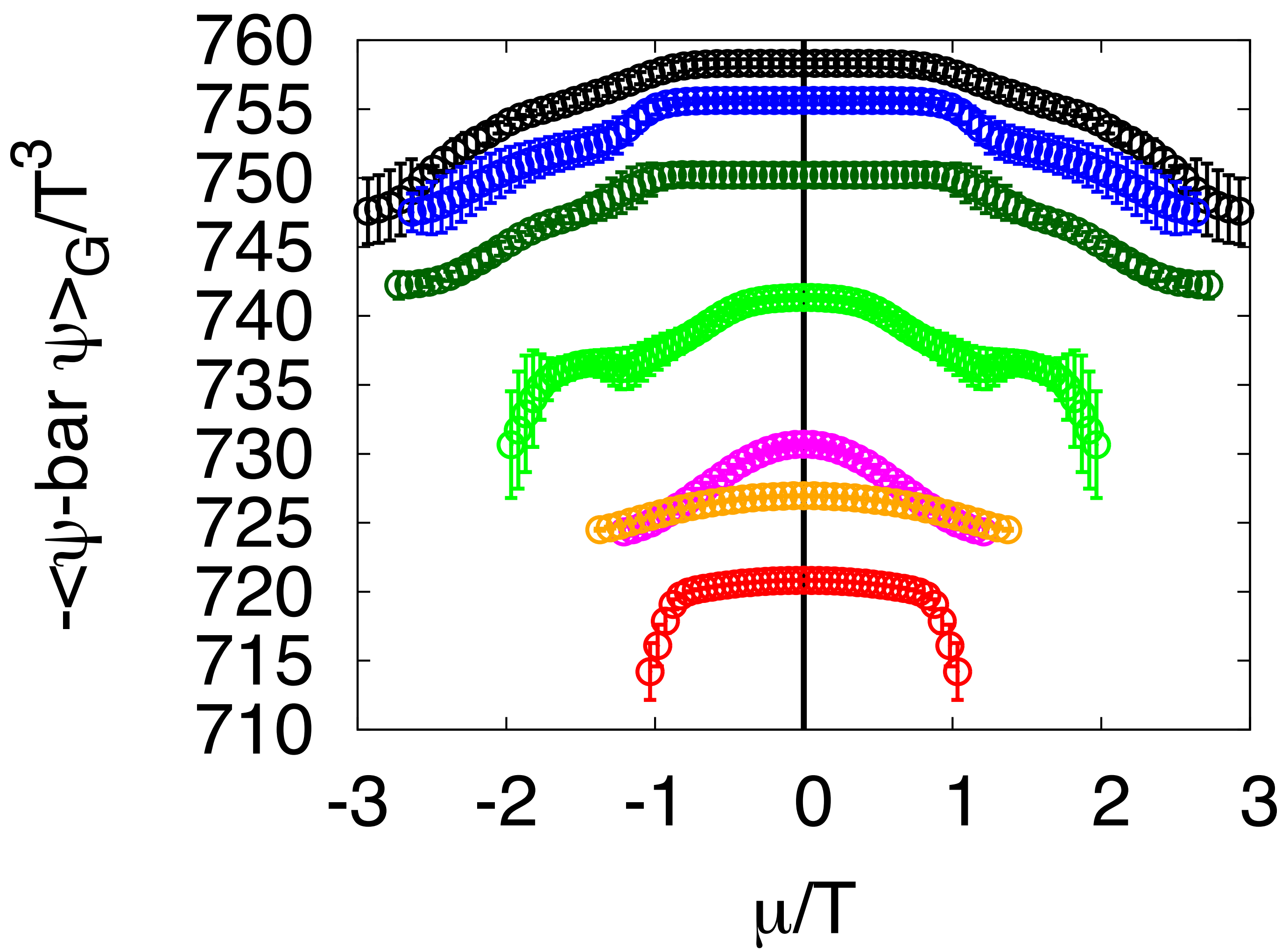
Grand canonical chiral condensate

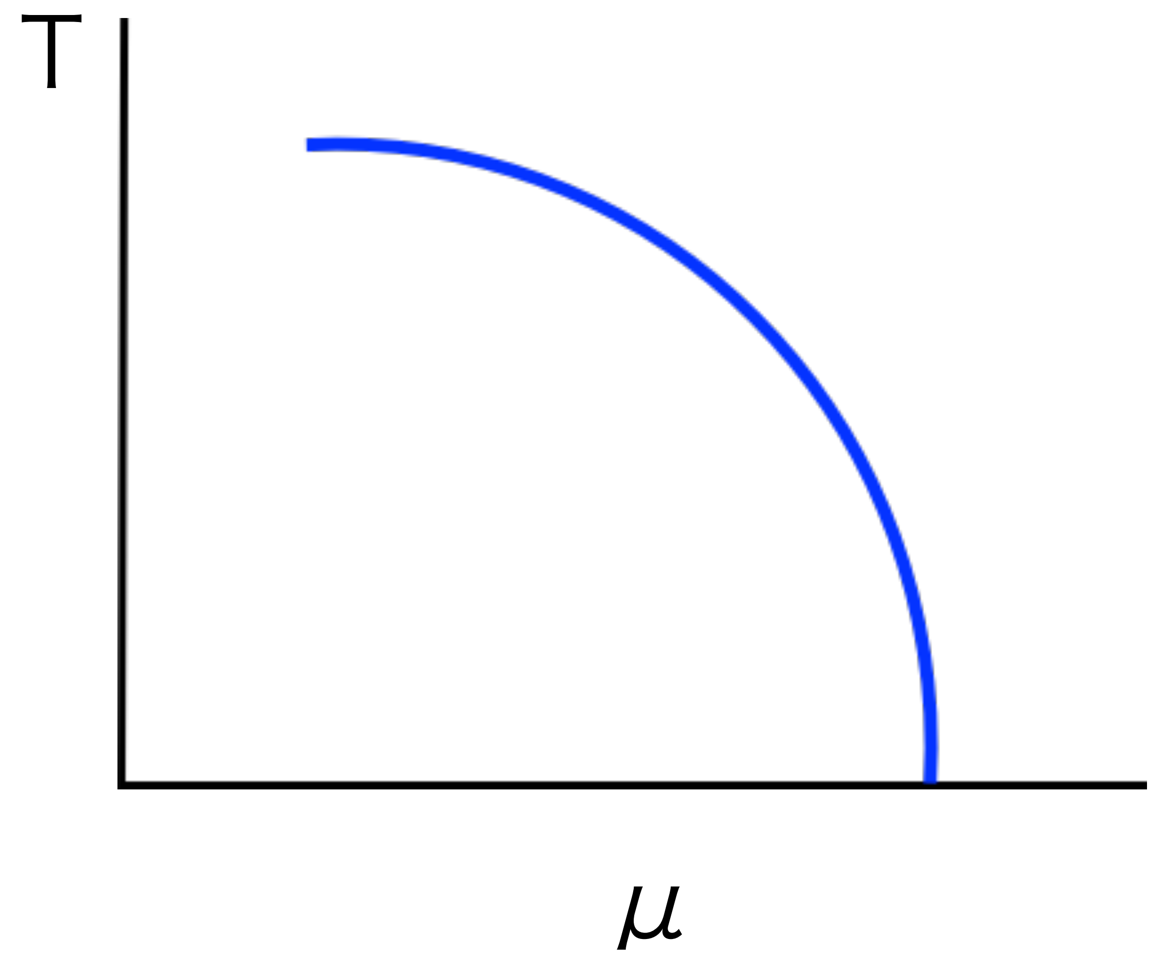


Chiral restoration?

Grand canonical chiral condensate

Correspondence with QCD phase diagram?





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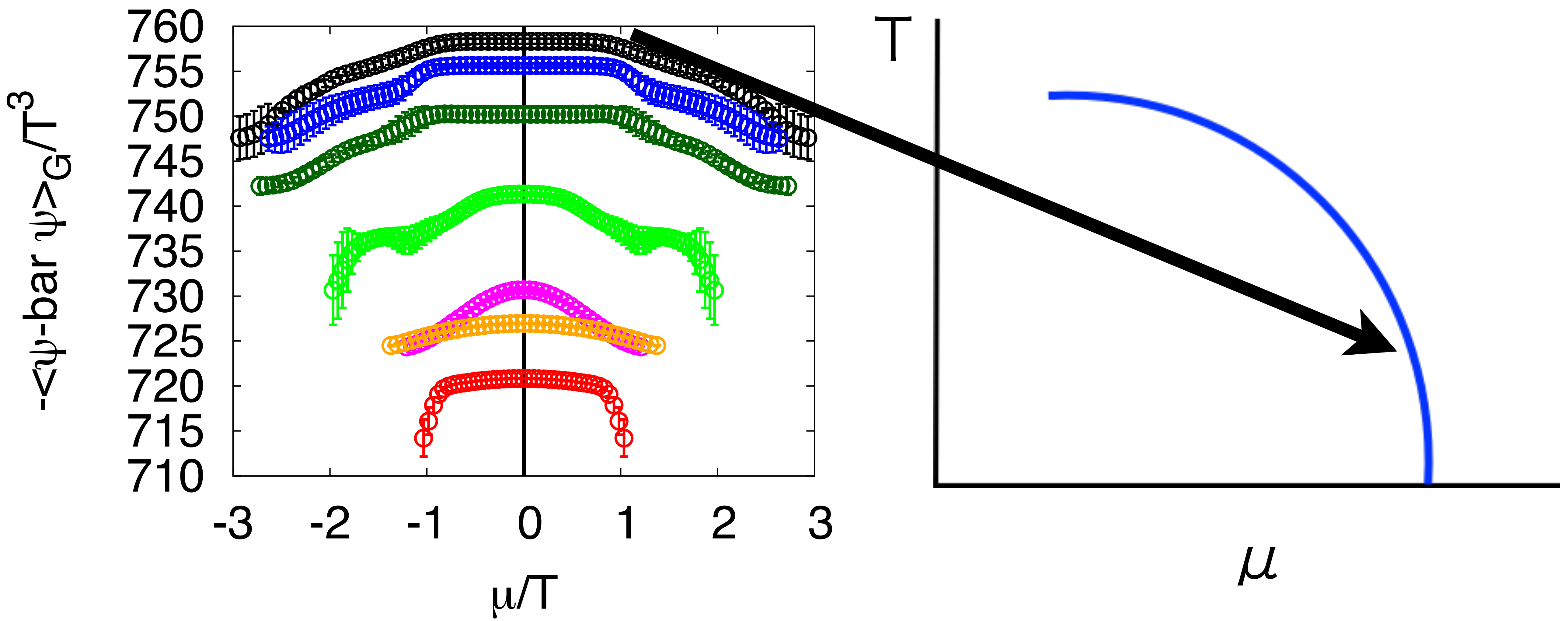

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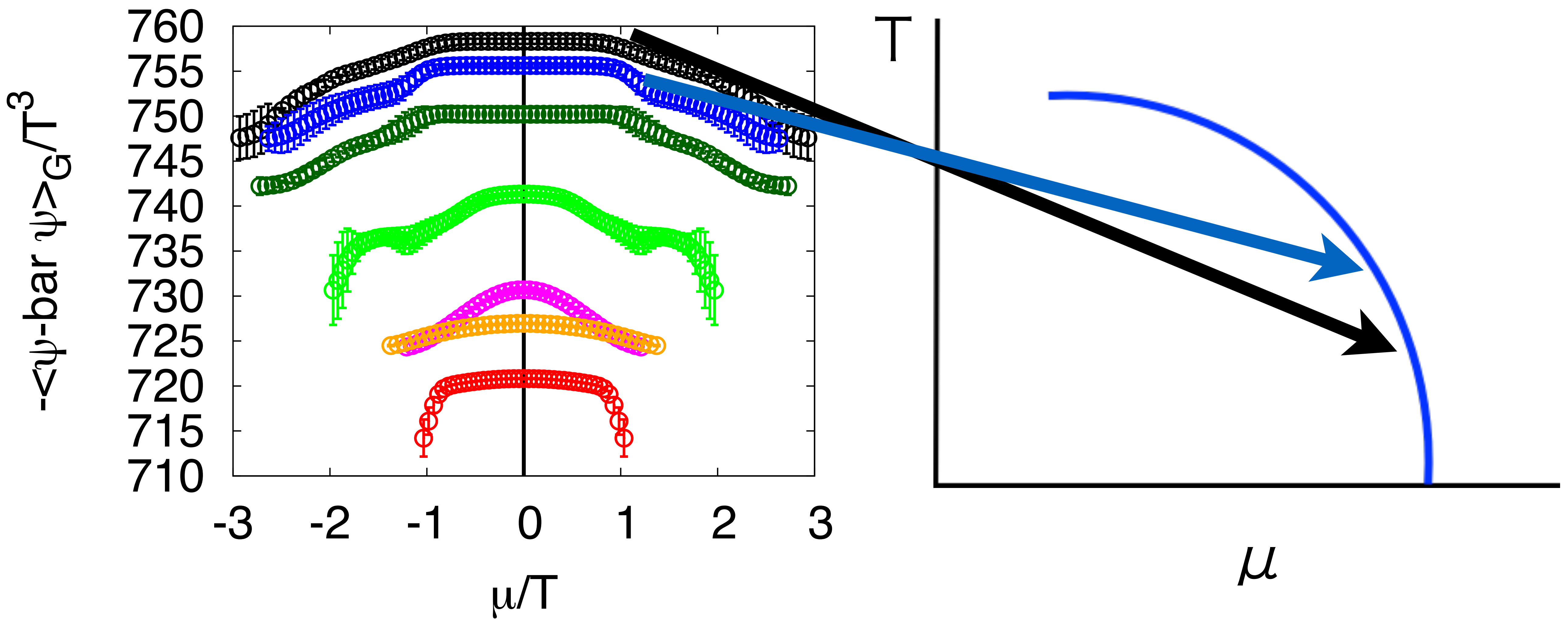

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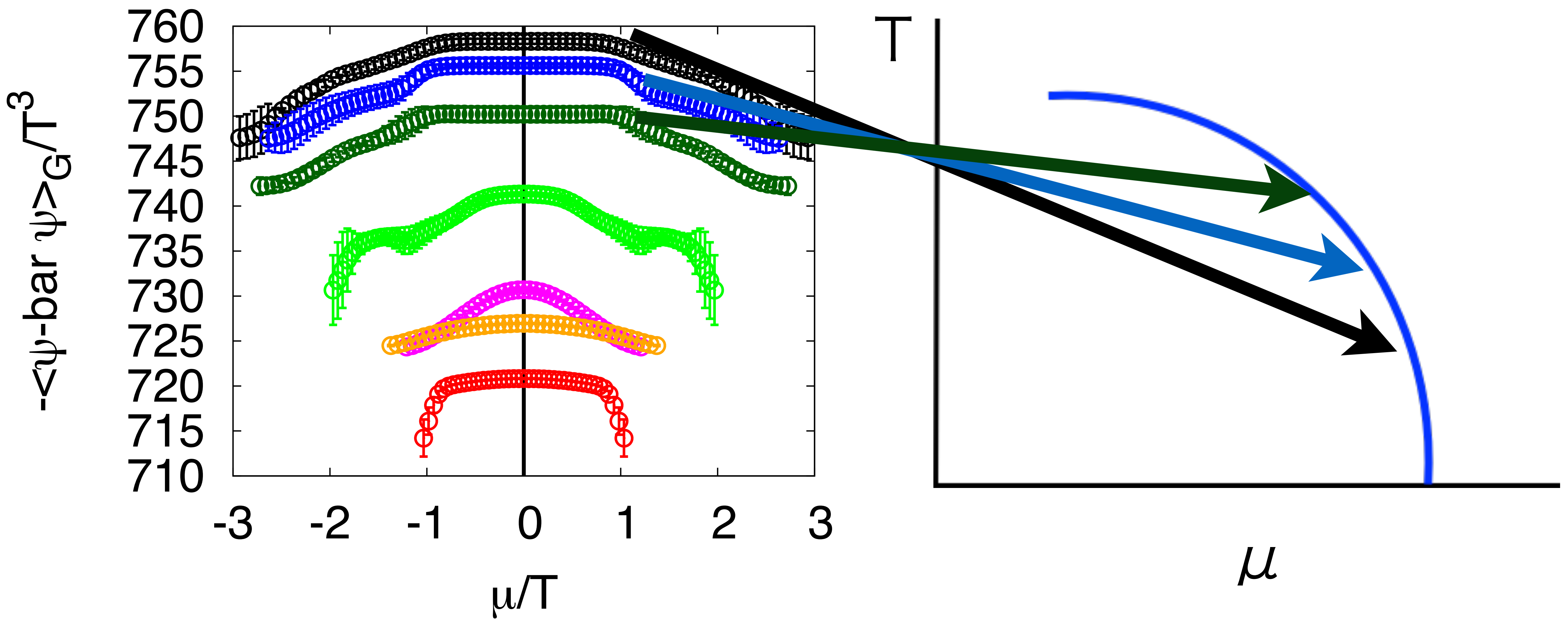

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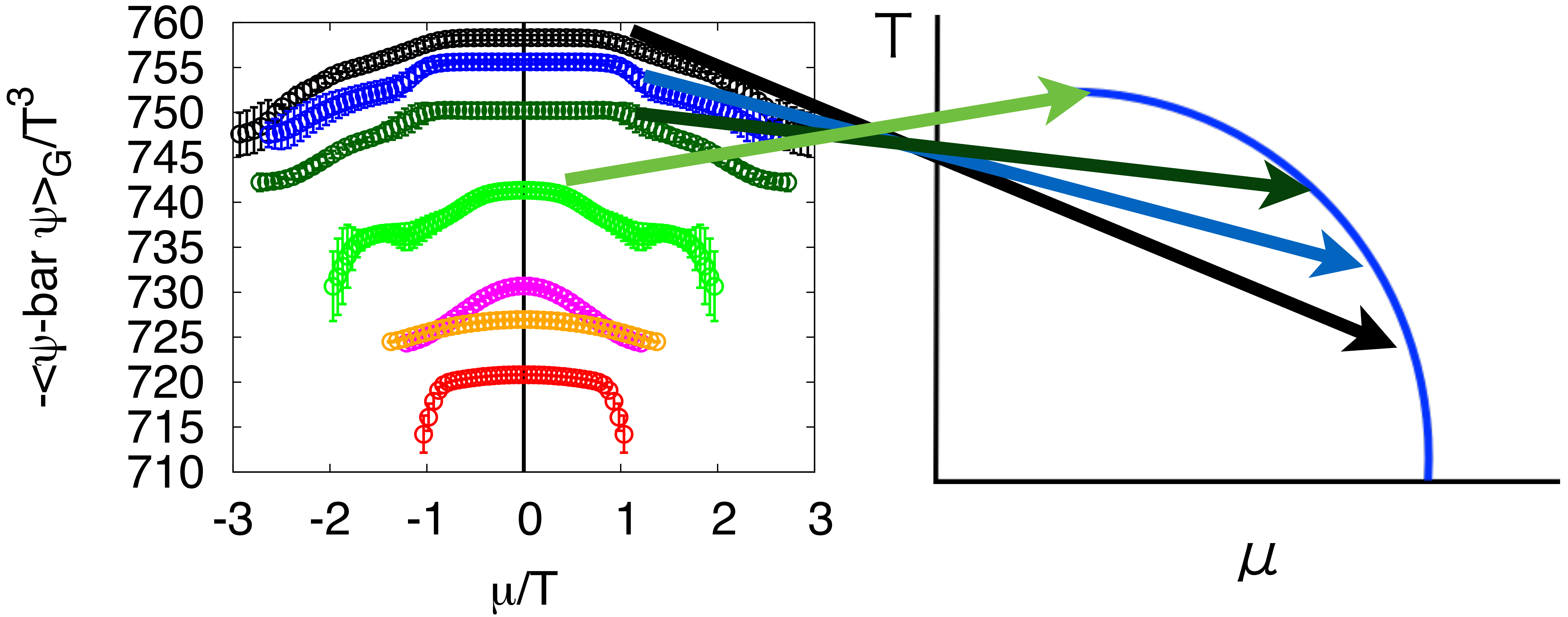

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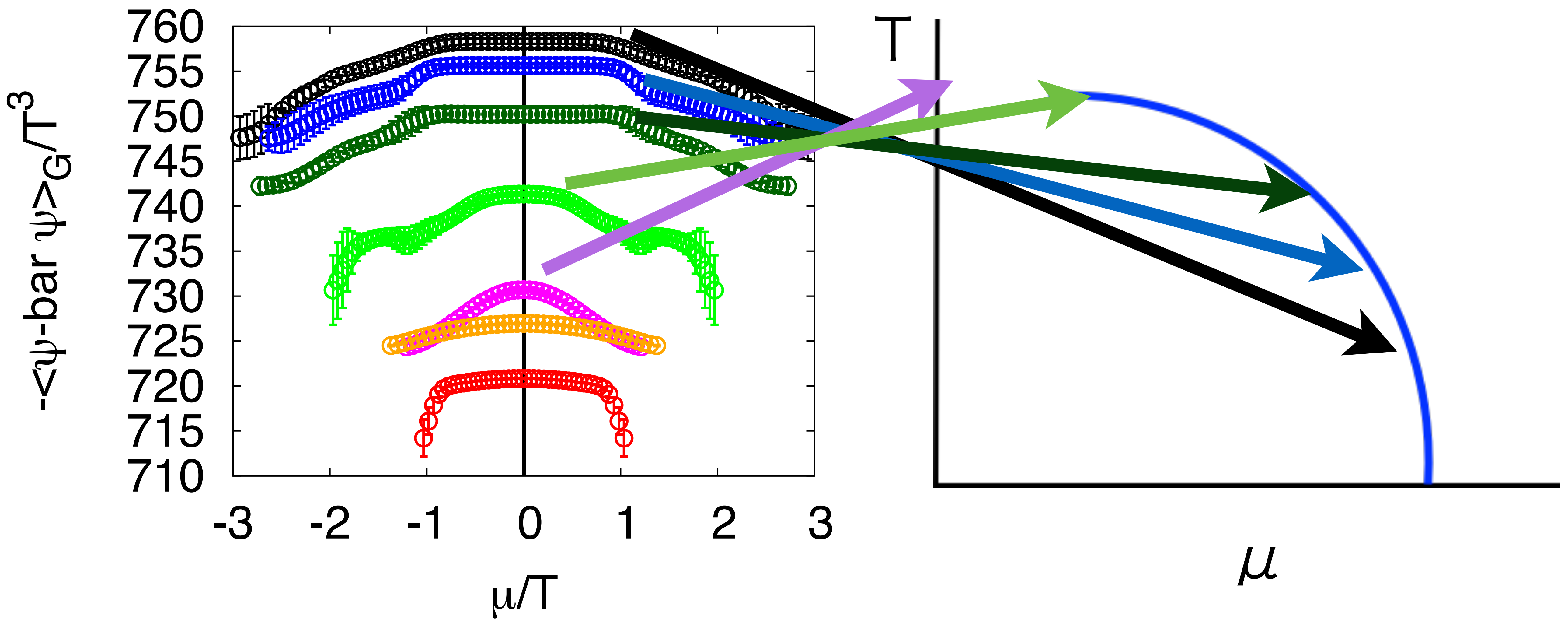

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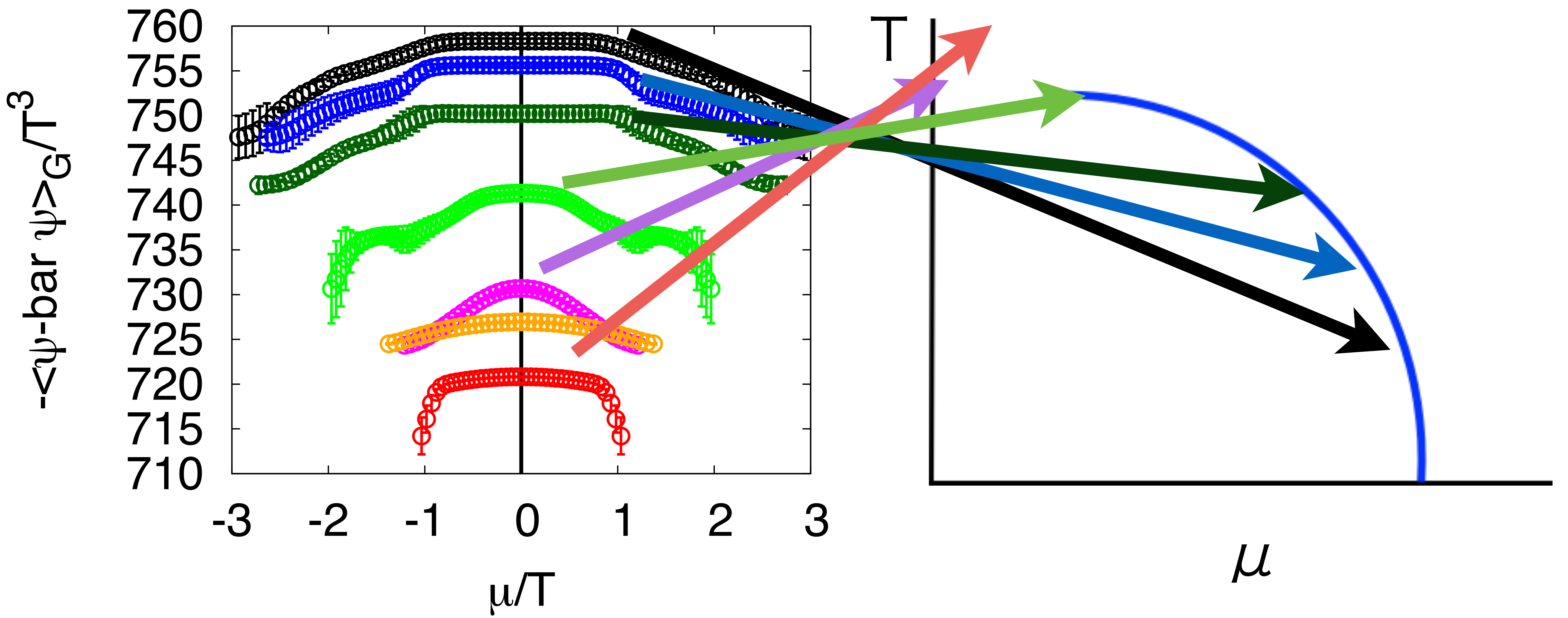

Chiral restoration?

Grand canonical chiral condensate

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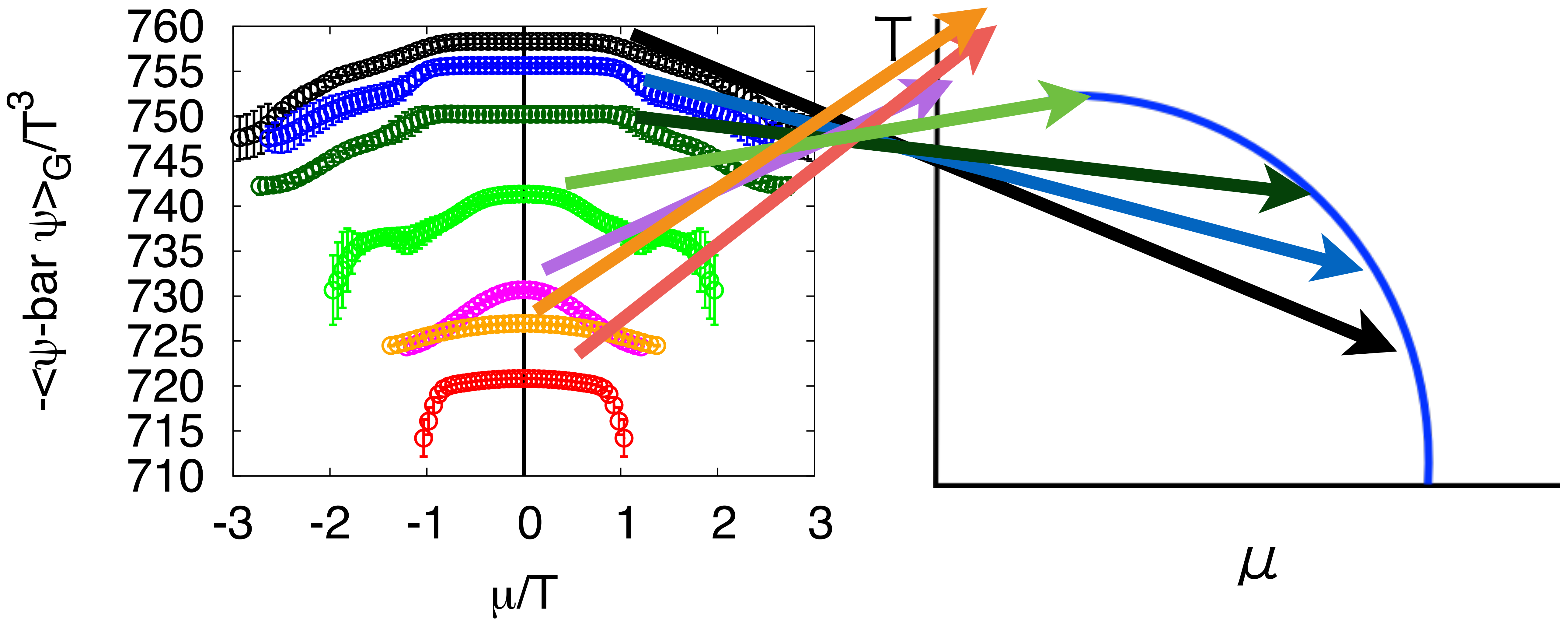

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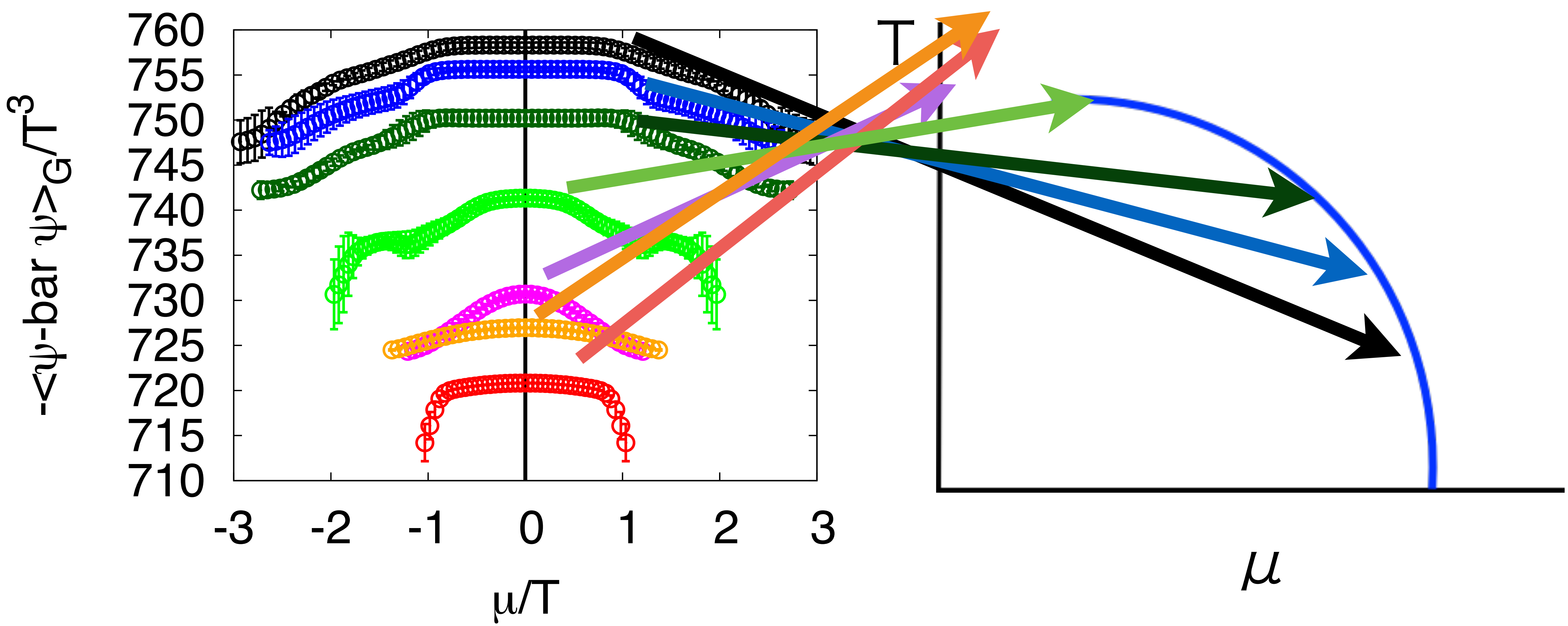
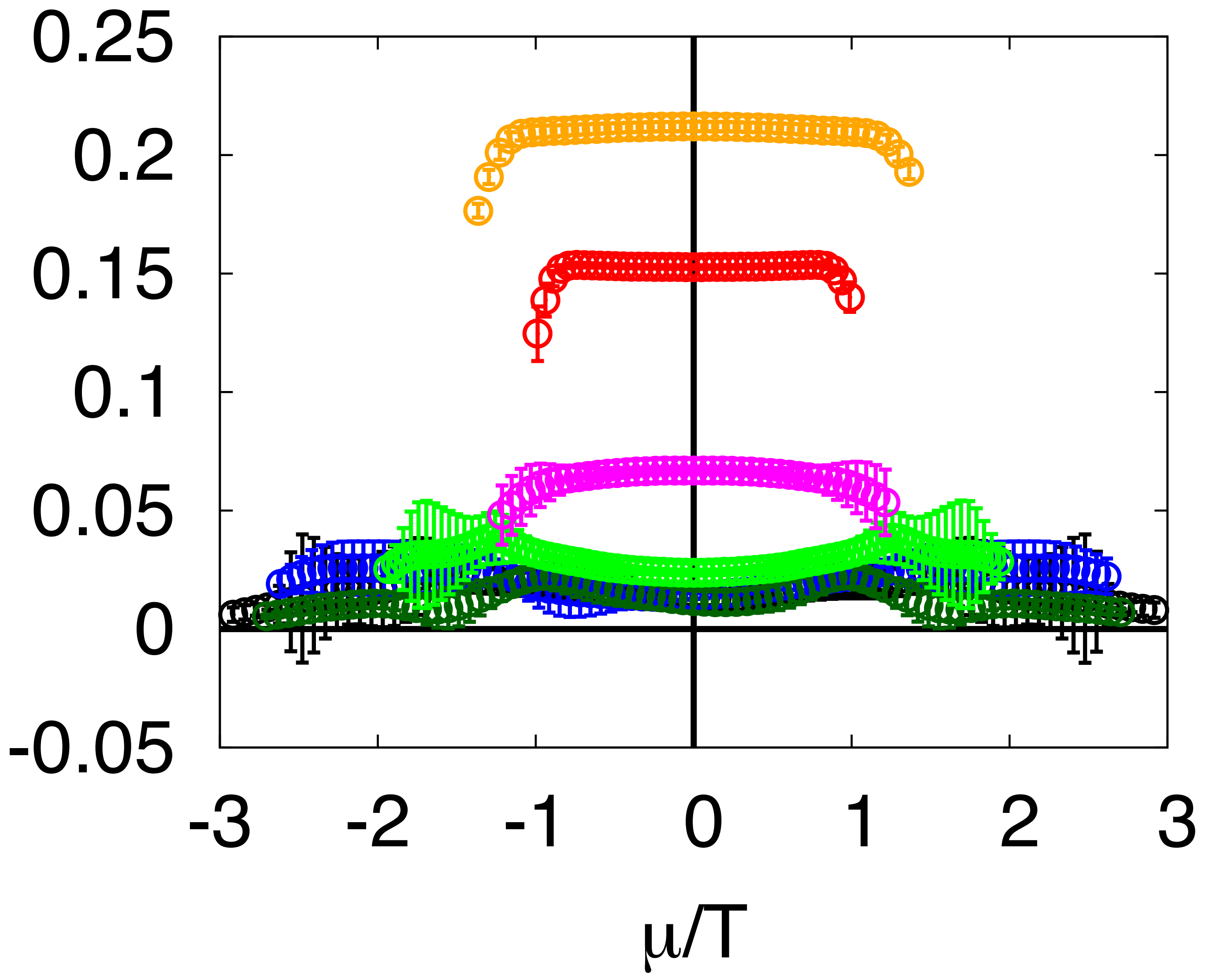


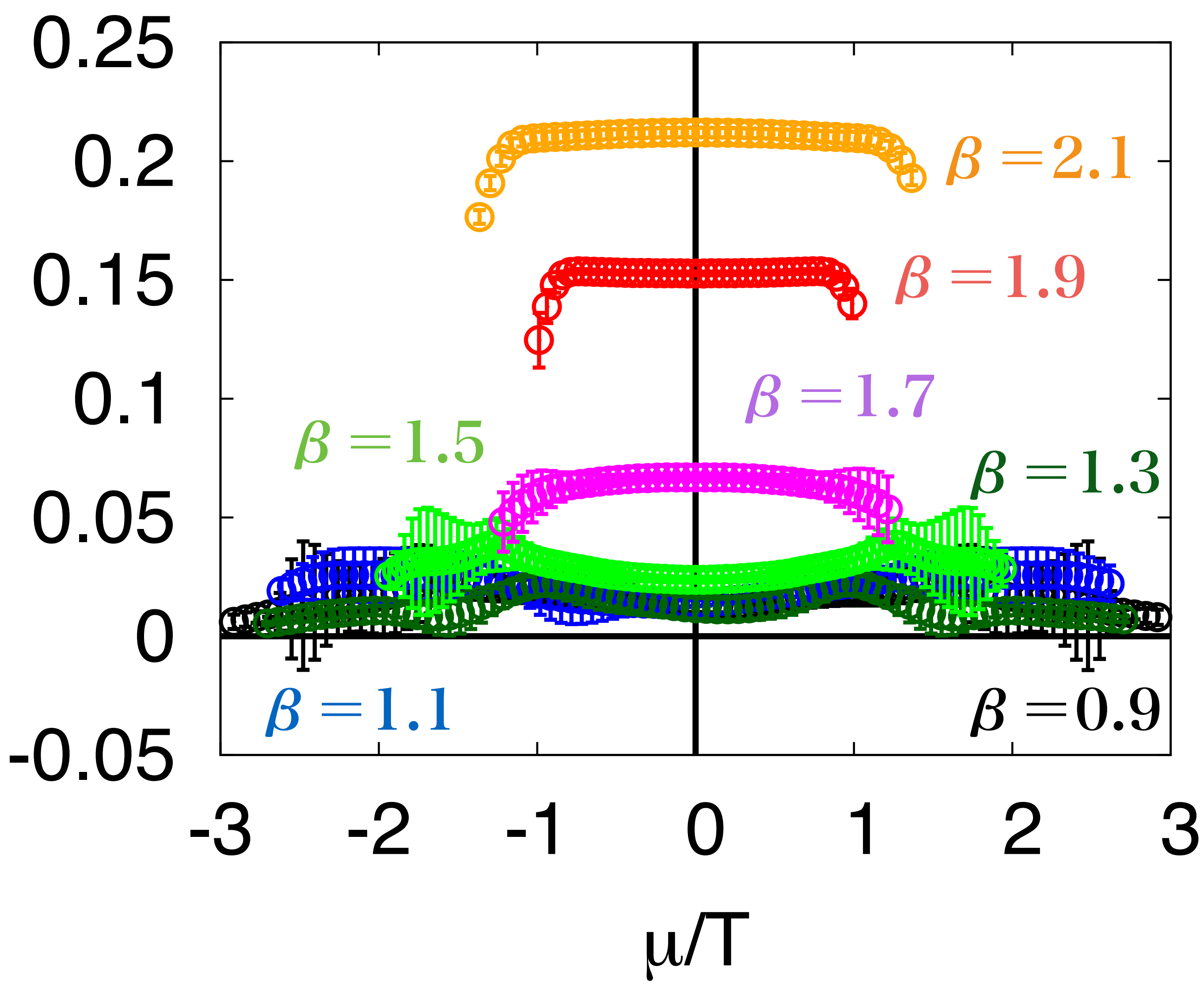
NOTE:

- μ/T vs μ
- mass is not the same.

Polyakov loop



Polyakov loop



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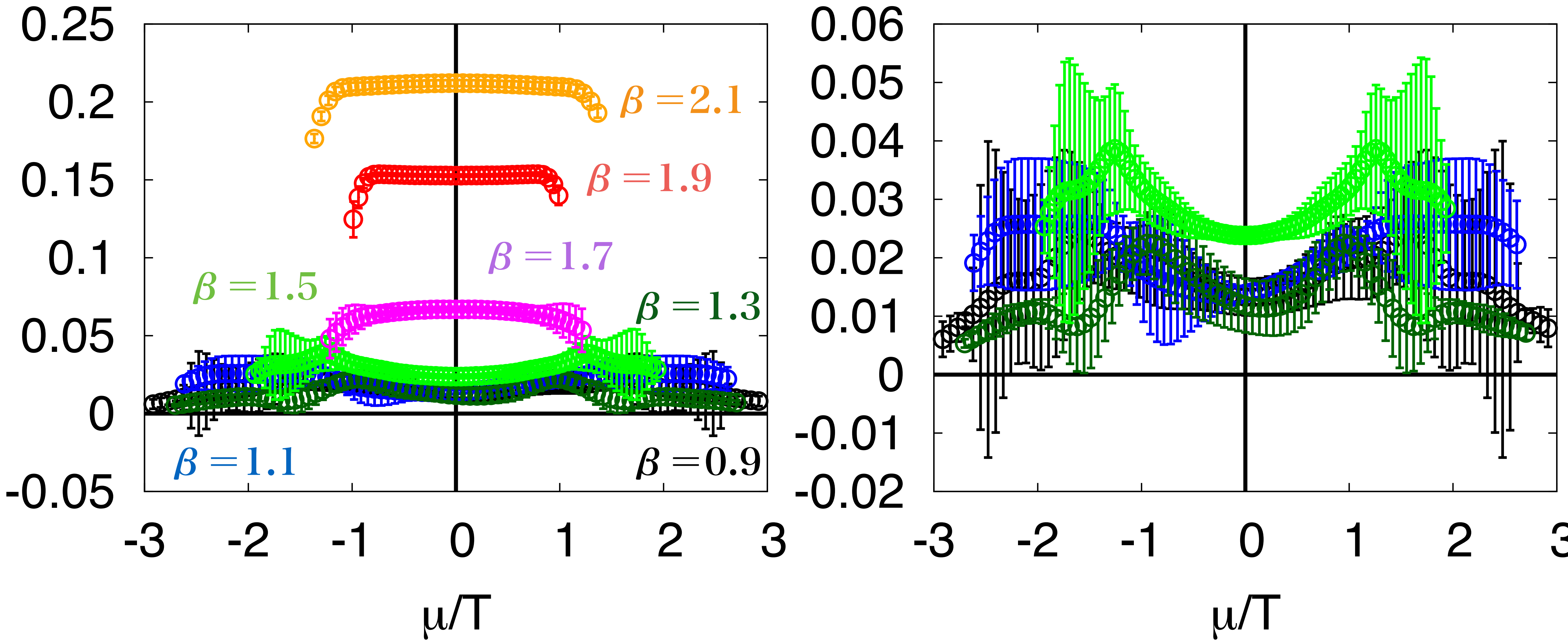
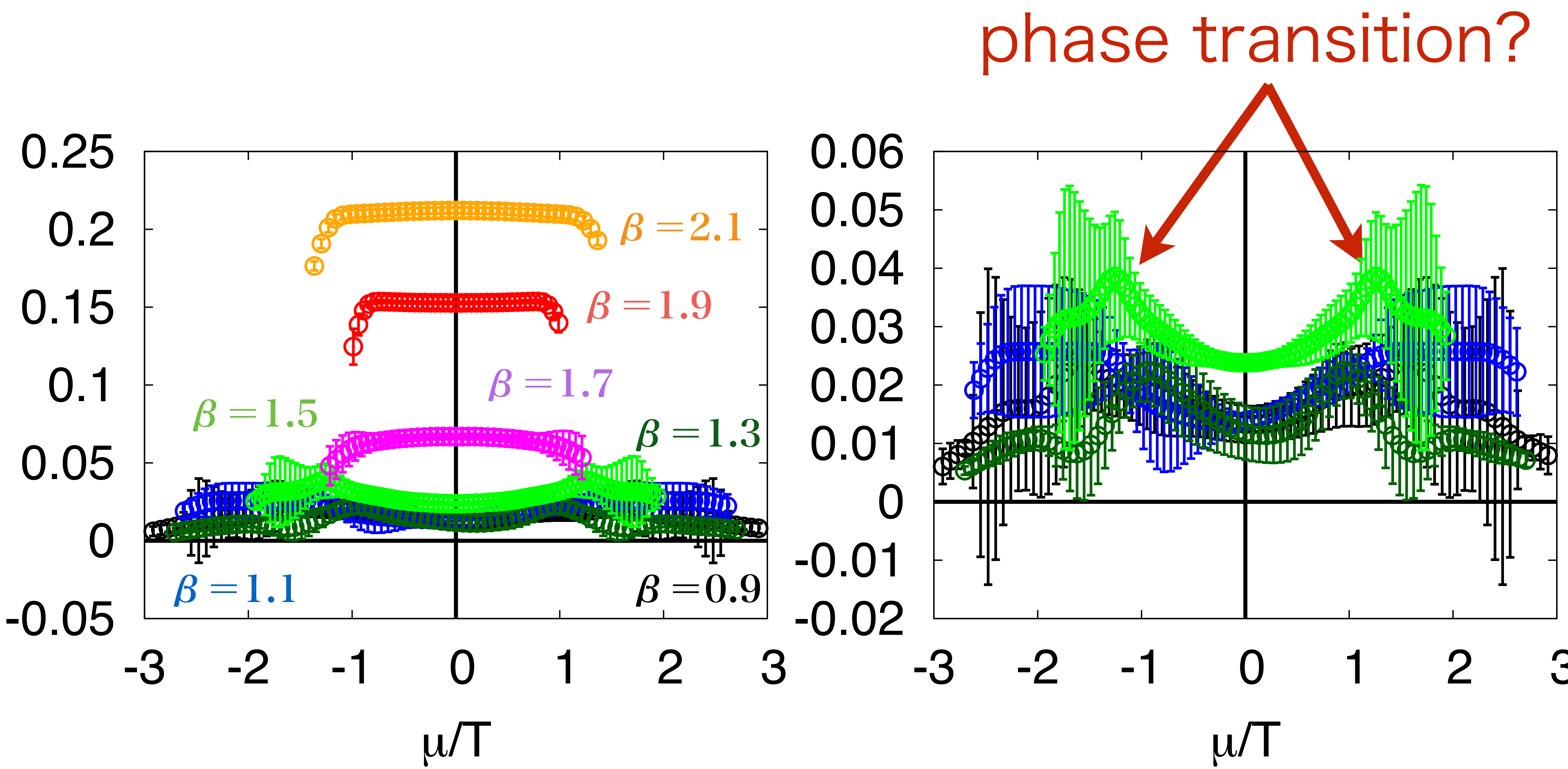


Polyakov loop



Where is the sign problem?

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I masked it! $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

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absolute value

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↑ absolute value

This is not a crazy idea.

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I masked it! $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

absolute value

This is not a crazy idea.

$$Z_C(T, n, V) = \sum_E \left\langle E, n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E, n \right\rangle$$

real, positive

Where is the sign problem?

I masked it! $$Z(\mu) = \sum_{n=-\infty}^{\infty} |Z_n| \xi^n$$

absolute value

This is not a crazy idea.

$$Z_C(T, n, V) = \sum_E \left\langle E, n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E, n \right\rangle$$ real, positive

If there is a well defined transfer matrix on the lattice.

Where is the sign problem?

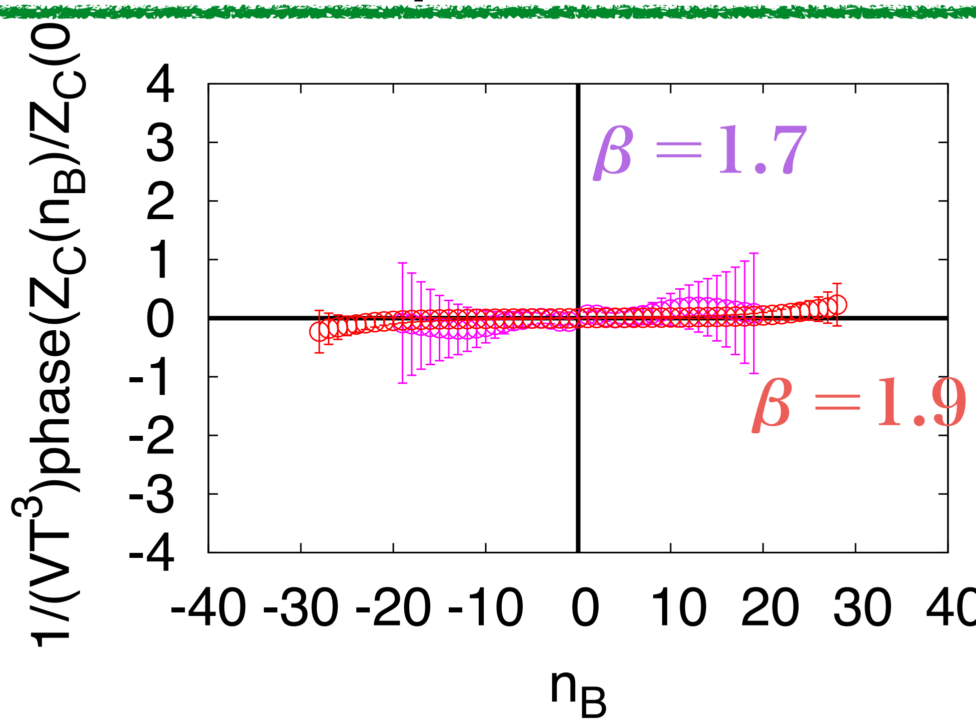
Where is the sign problem?

$\beta = 1.7$

$\beta = 1.9$

Where is the sign problem?

Zn is real and positive at high T.



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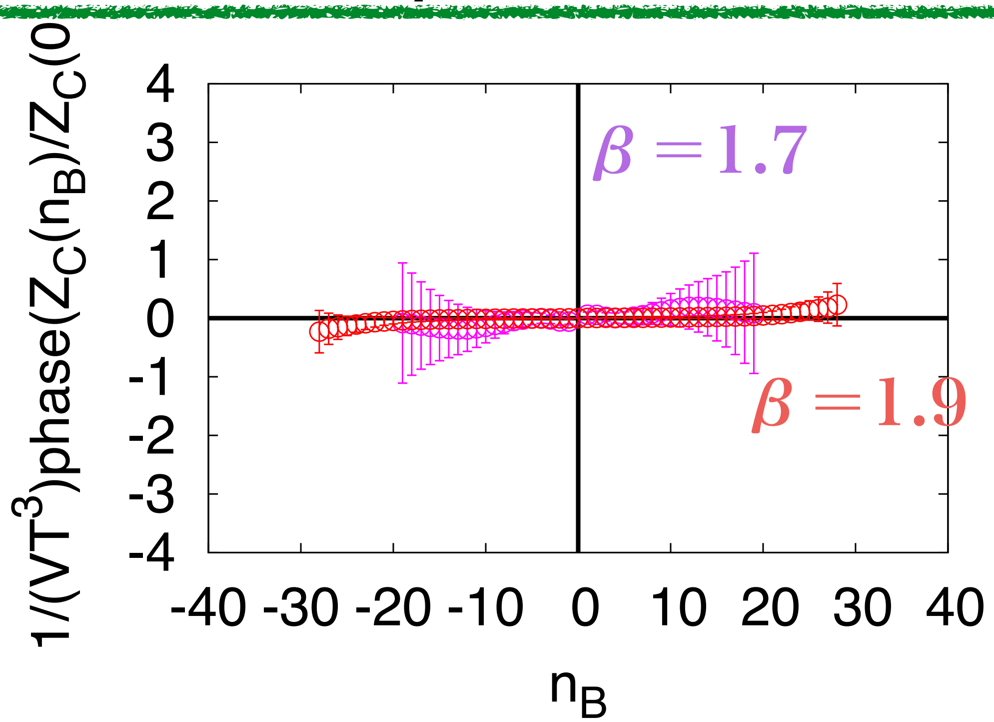
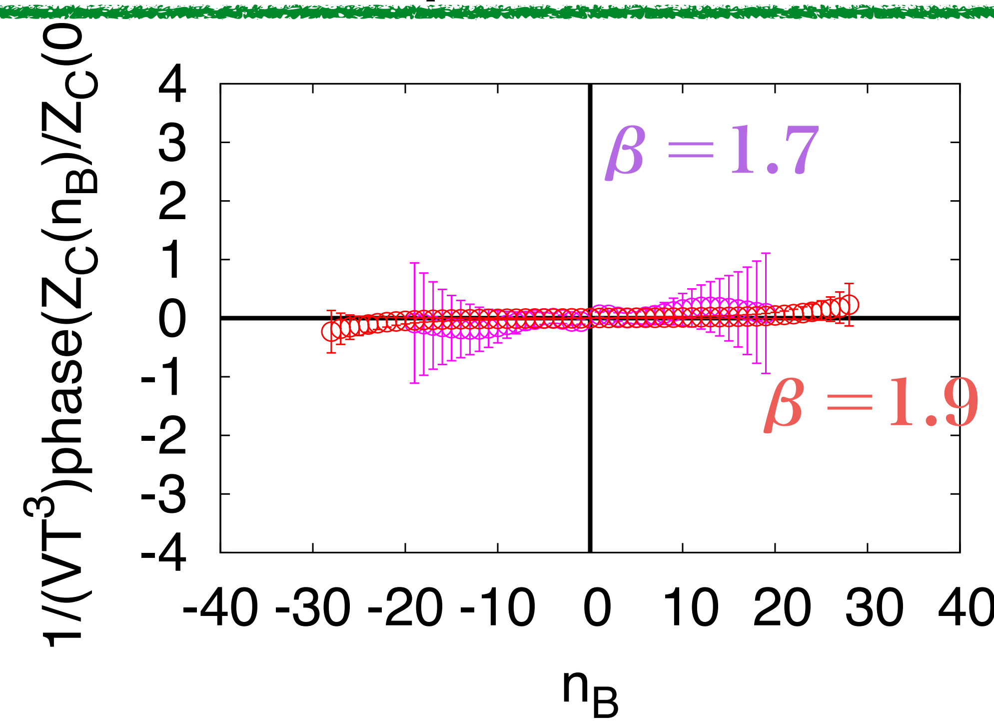


This is not the case at low T.

Where is the sign problem?

Zn is real and positive at high T.



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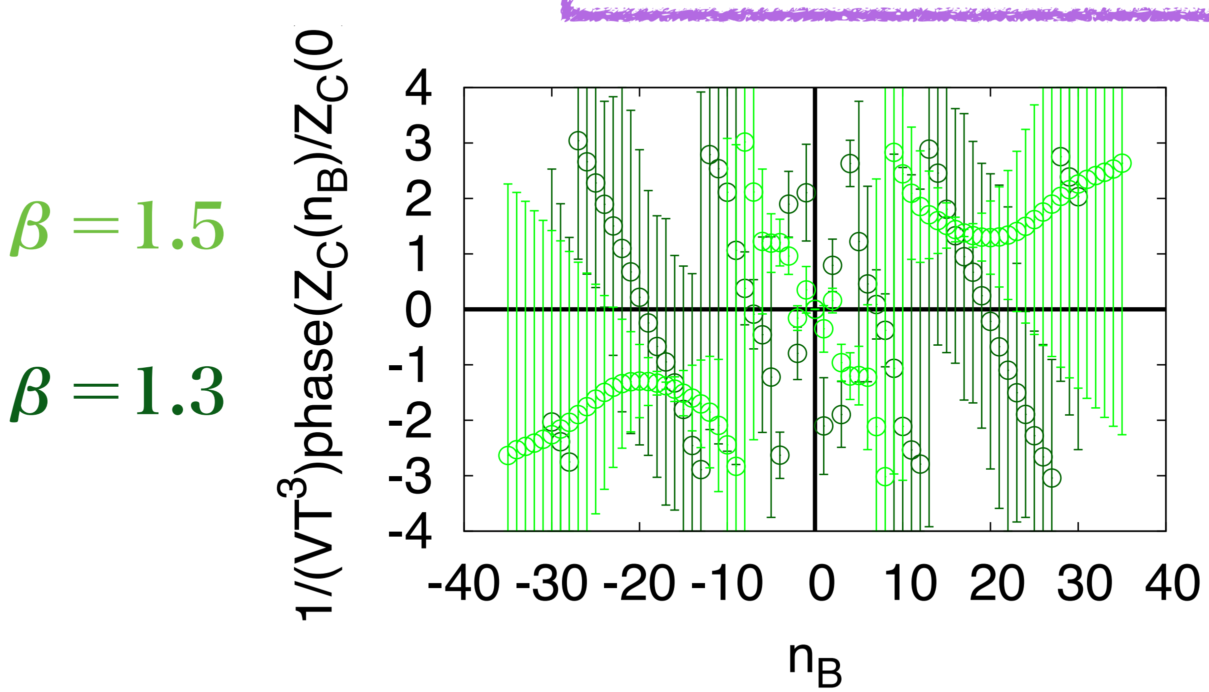

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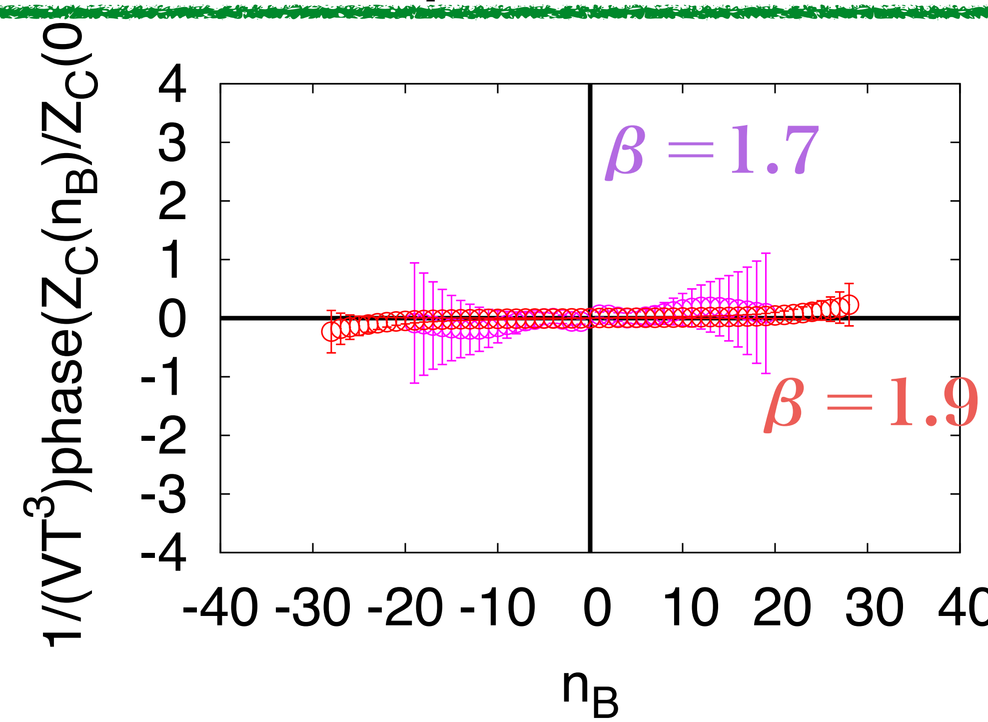
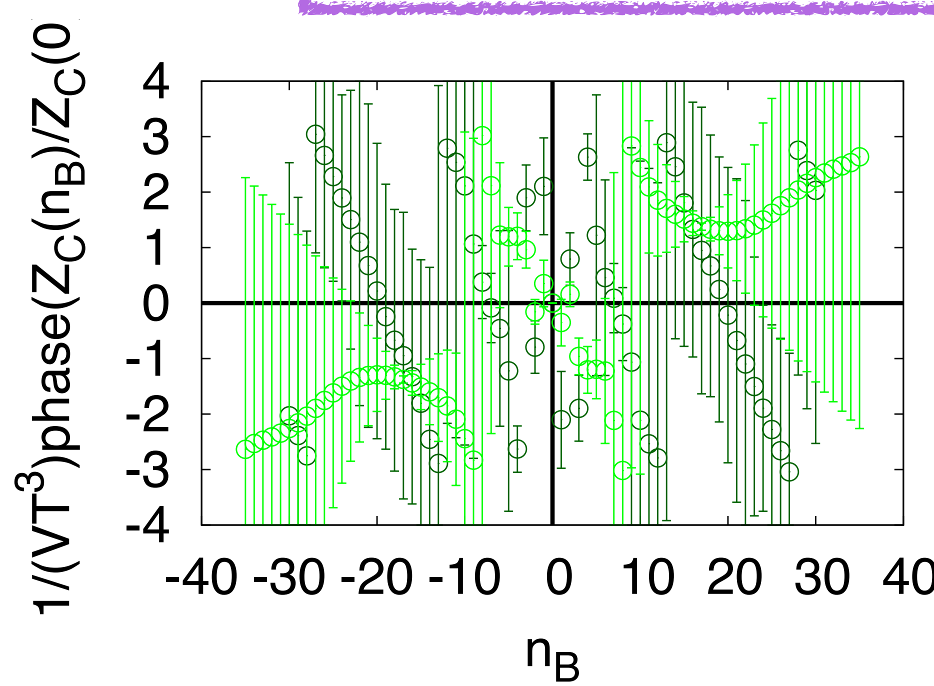
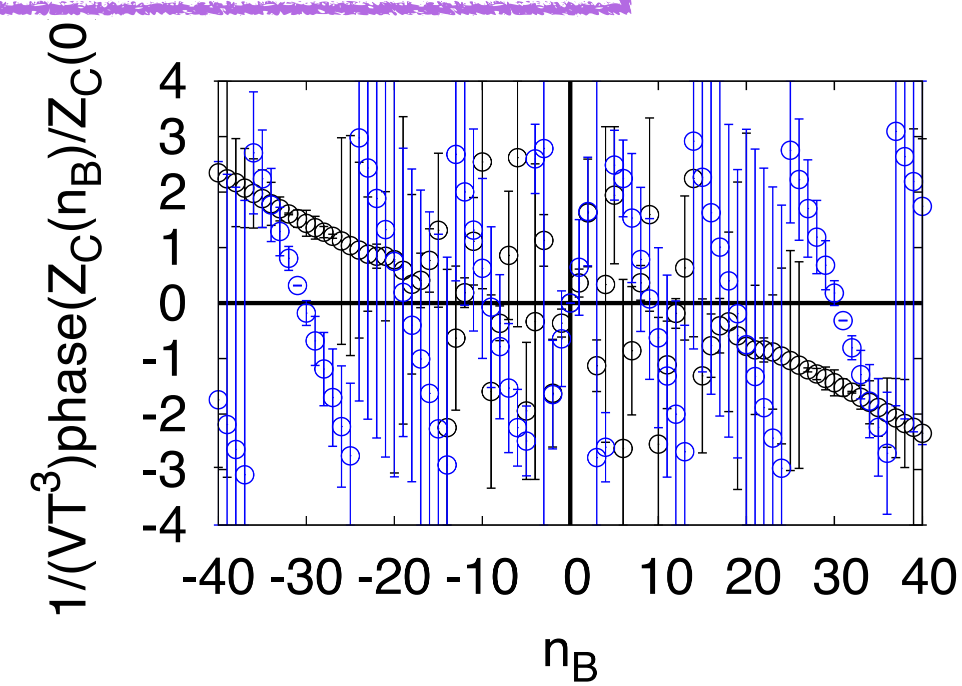


This is not the case at low T.

$\beta = 1.5$

$\beta = 1.3$





$\beta = 1.1$

$\beta = 0.9$

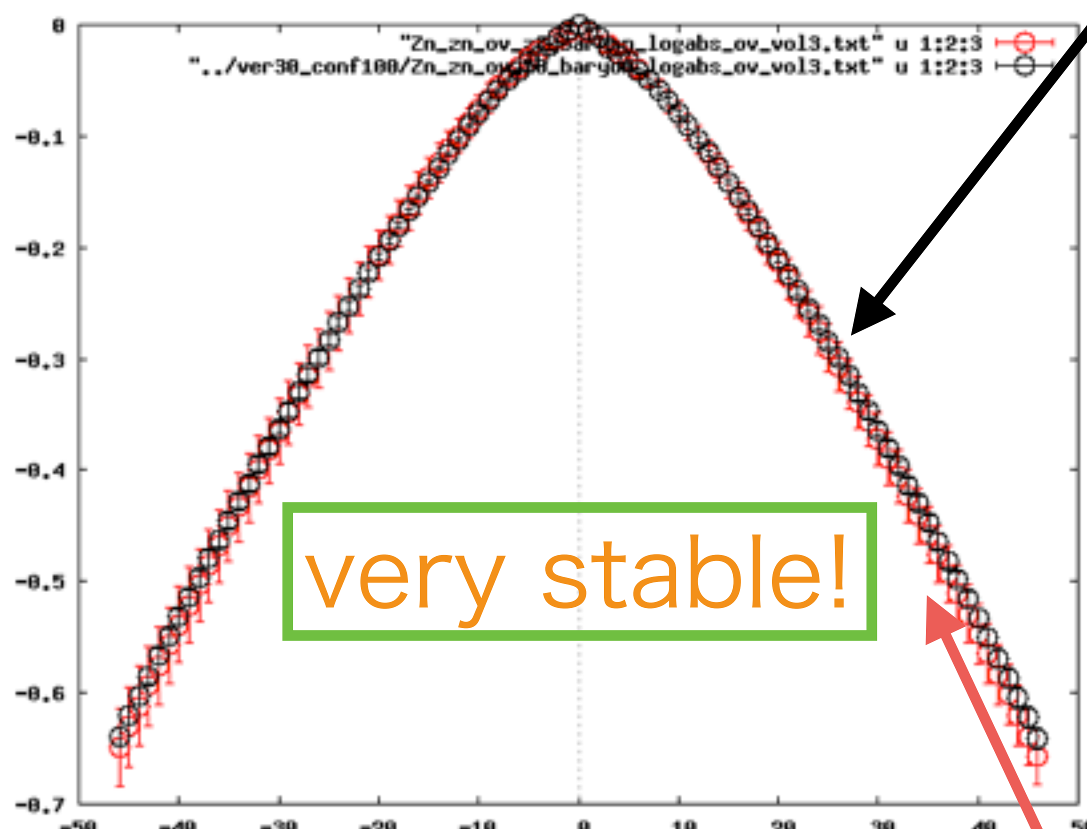
Where is the sign problem?

Fluctuation in phase does not wash out everything.

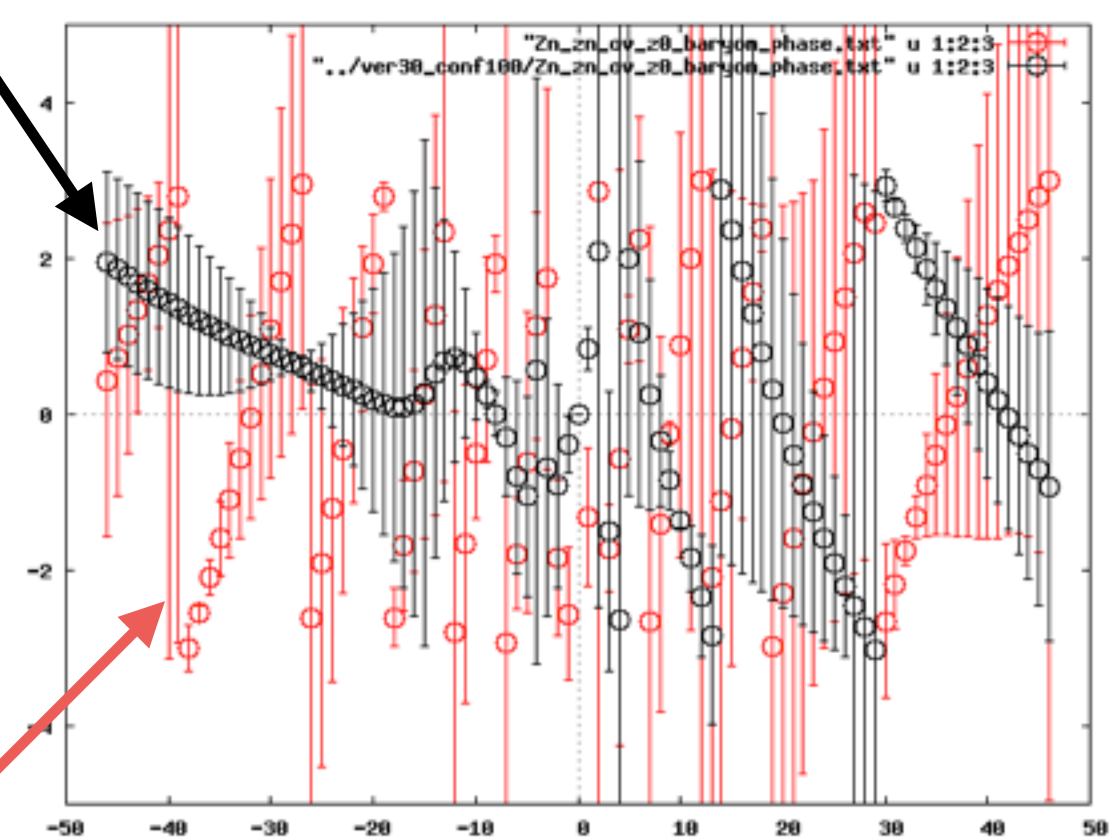
β=0.9, κ=0.137, 200 configurations

1st half 100confs

|Zn|



phase(Zn)

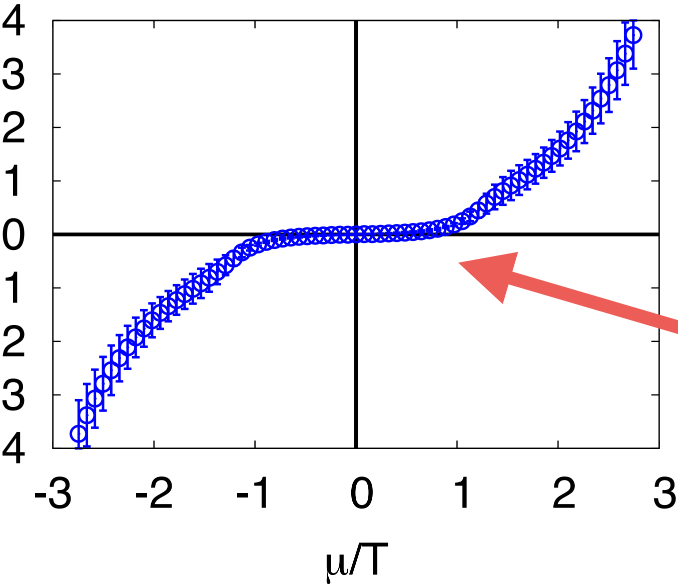


2nd half 100confs

Where is the sign problem?

Disaster of phase quenching

$$Z = \int DU |\mathrm{Det} D(U)| e^{-S_G(U)}$$



A phase transition is found!

But··· $\mu_c \propto m_\pi/2$ not $m_N/3$

DetD(U) has no reason to be real for each conf!

Zn has a good reason to be positive as a VEV!

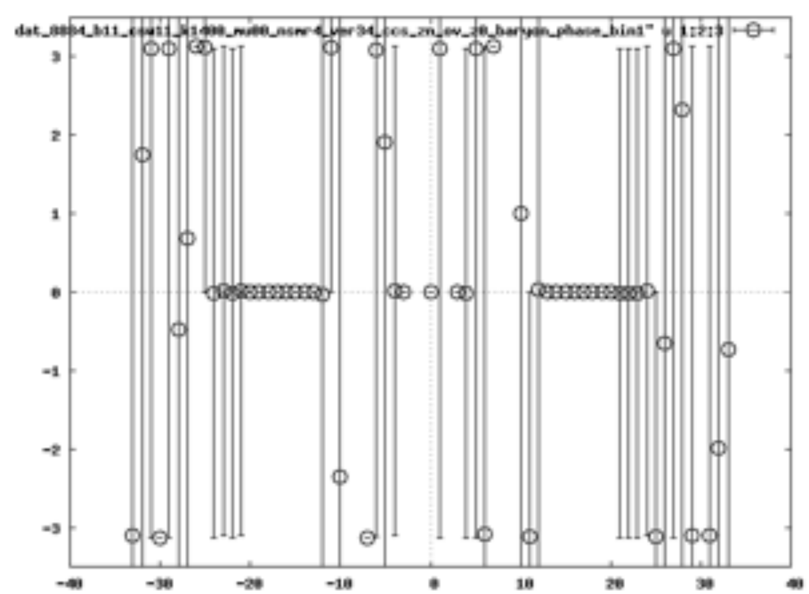
$$Z_C(T,n,V) = \sum_E \left\langle E, n \middle| \exp\left(-\frac{\hat{H}}{T}\right) \middle| E, n \right\rangle$$

Where is the sign problem?

Problems are two!

Zn is complex. → A_μ and charge conjugate A_μ^C does not appear with equal possibility

What if charge conjugation symmetry is enforced?



Zn is not real → transfer matrix is not well defined

Where is the sign problem?

β=0.9 is far from the continuum limit

Nt=4 is too small

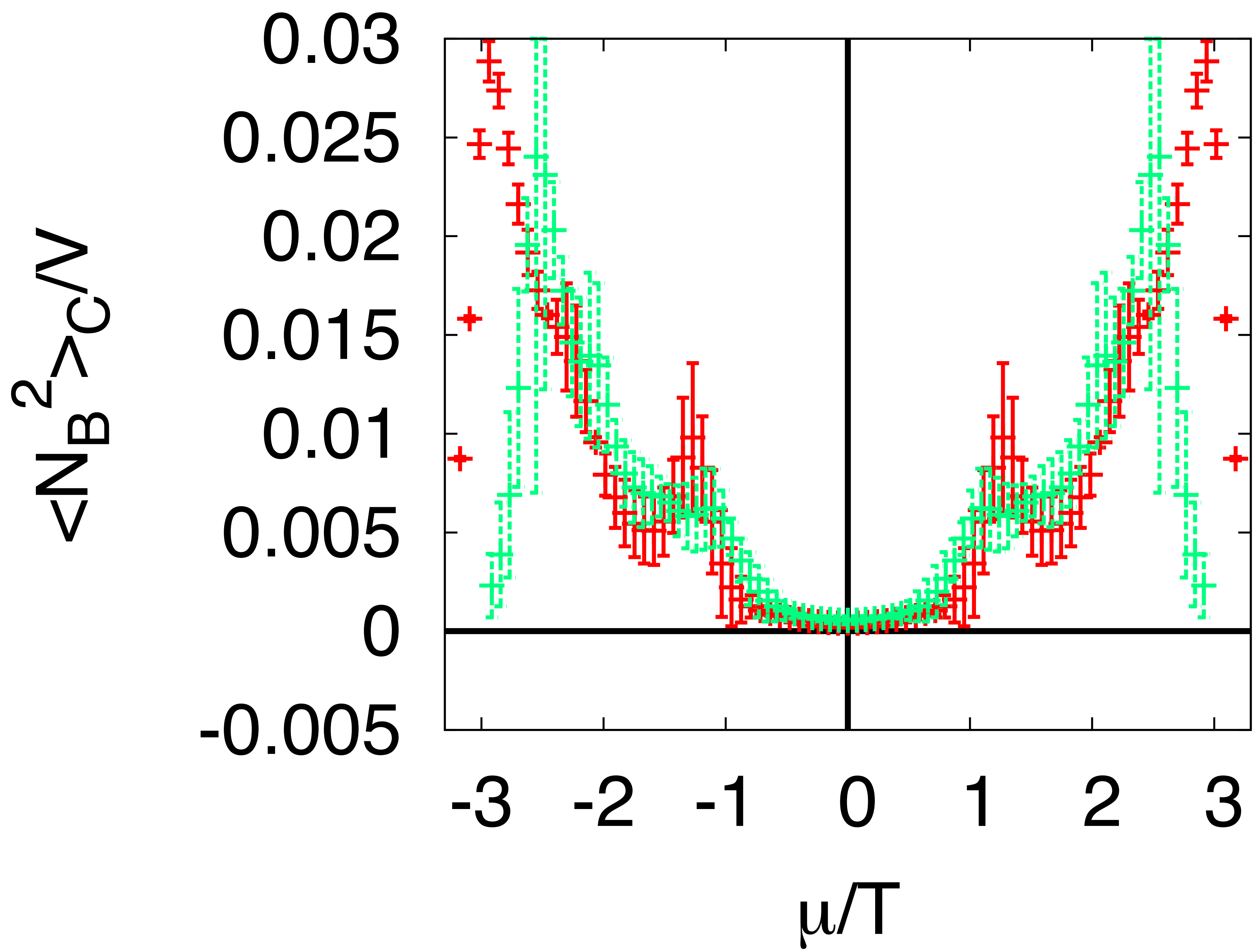
Zn should be real positive in the continuum!

Take continuum limit!

Mass dependence

$$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

Low T $\beta = 0.9$



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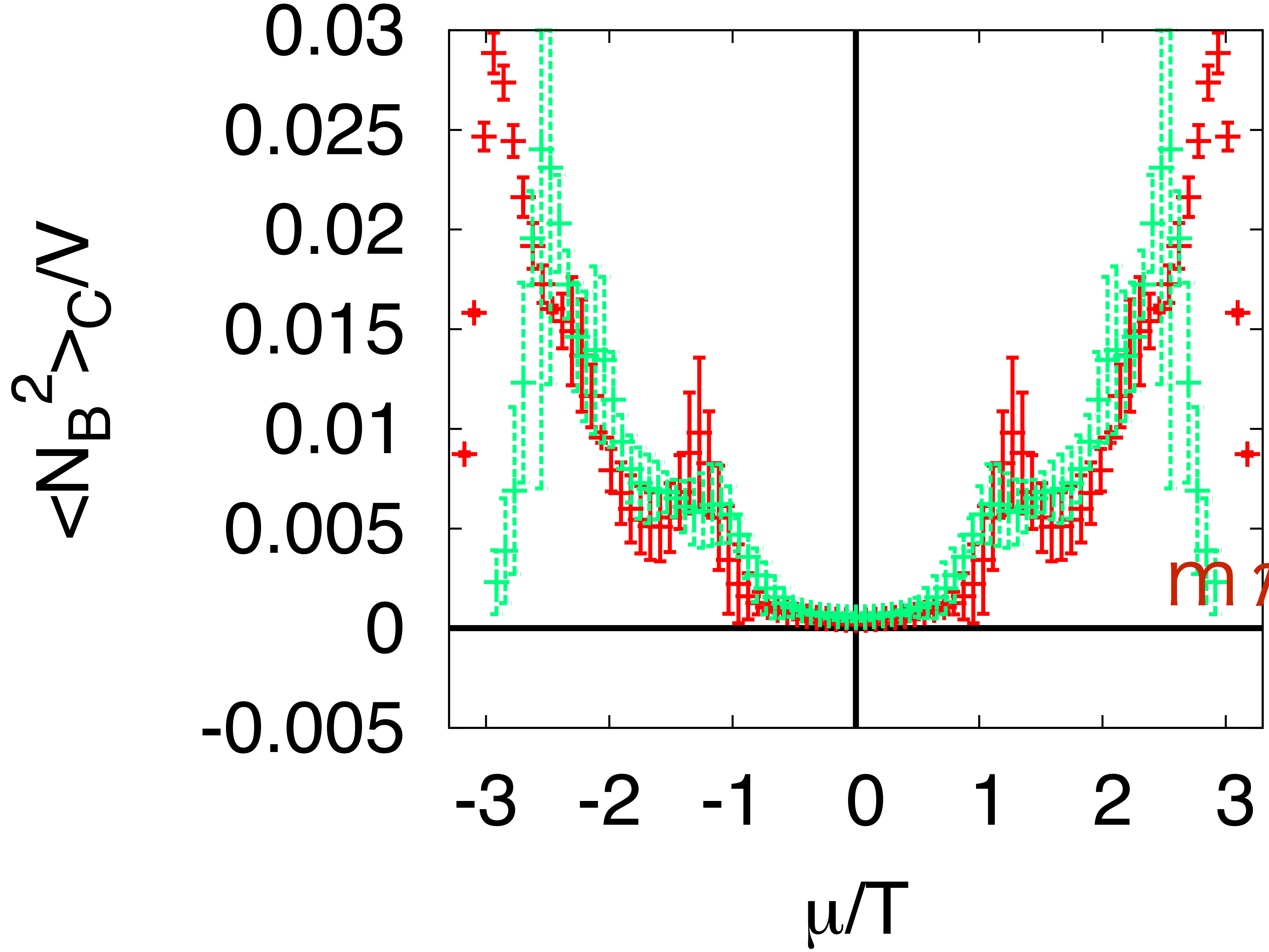
$$\left\langle N^2 \right\rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

Low T $\beta = 0.9$

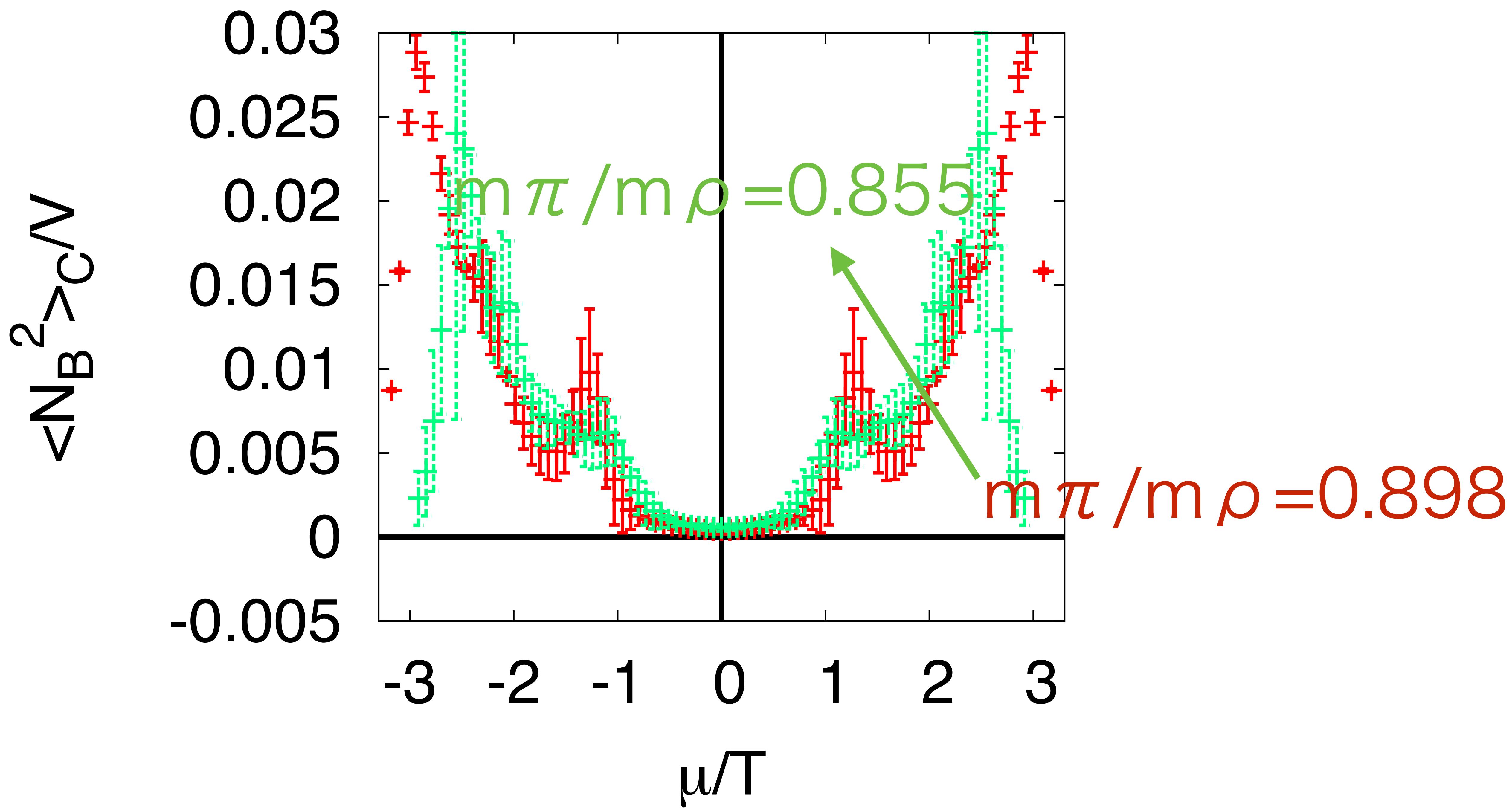


$m\pi / m\rho = 0.898$

Mass dependence

$$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

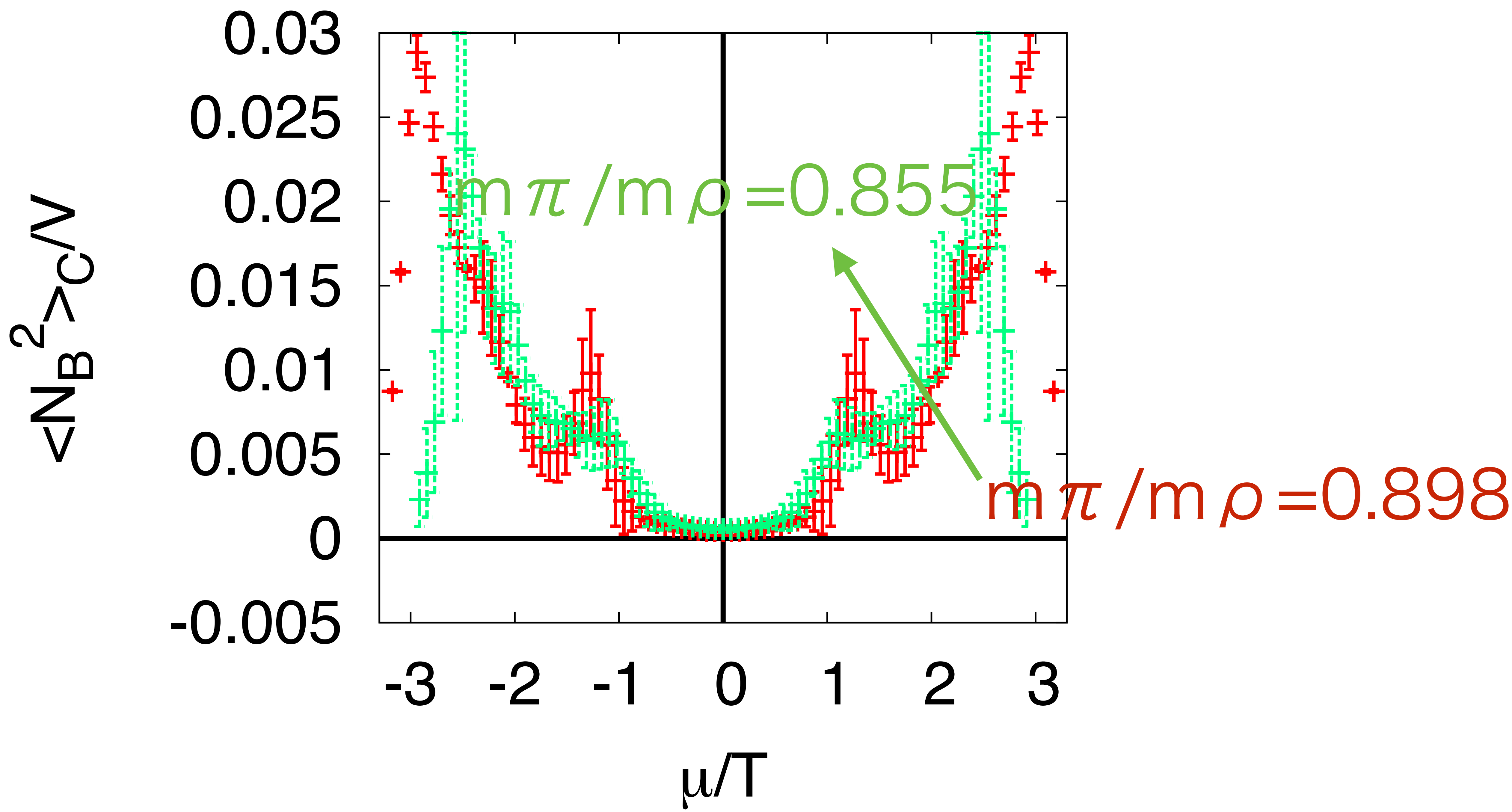
Low T $\beta = 0.9$



Mass dependence

3. cumulant of quark number density $\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$

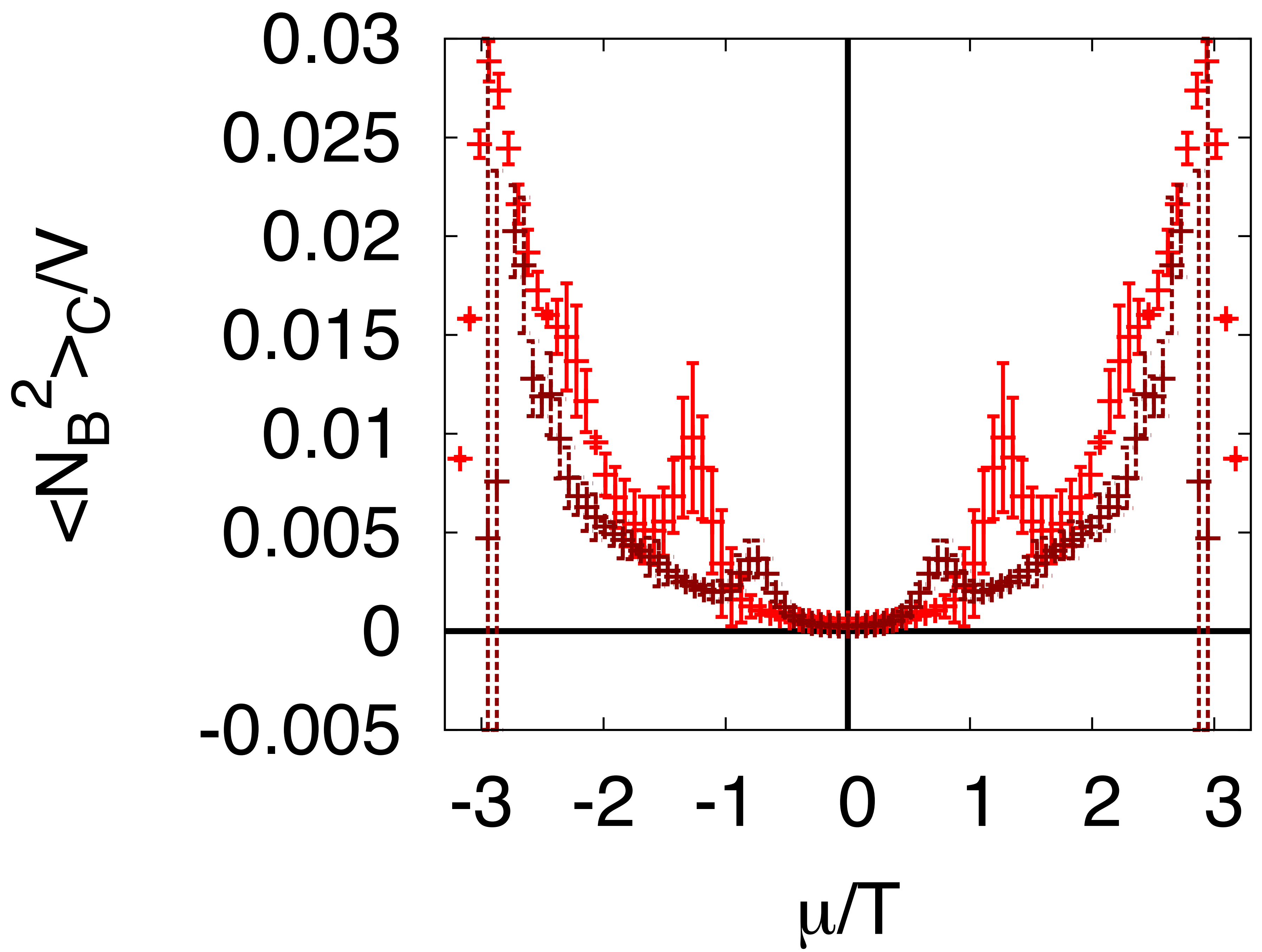
Low T $\beta = 0.9$



Volume dependence

$$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

Low T $\beta = 0.9$



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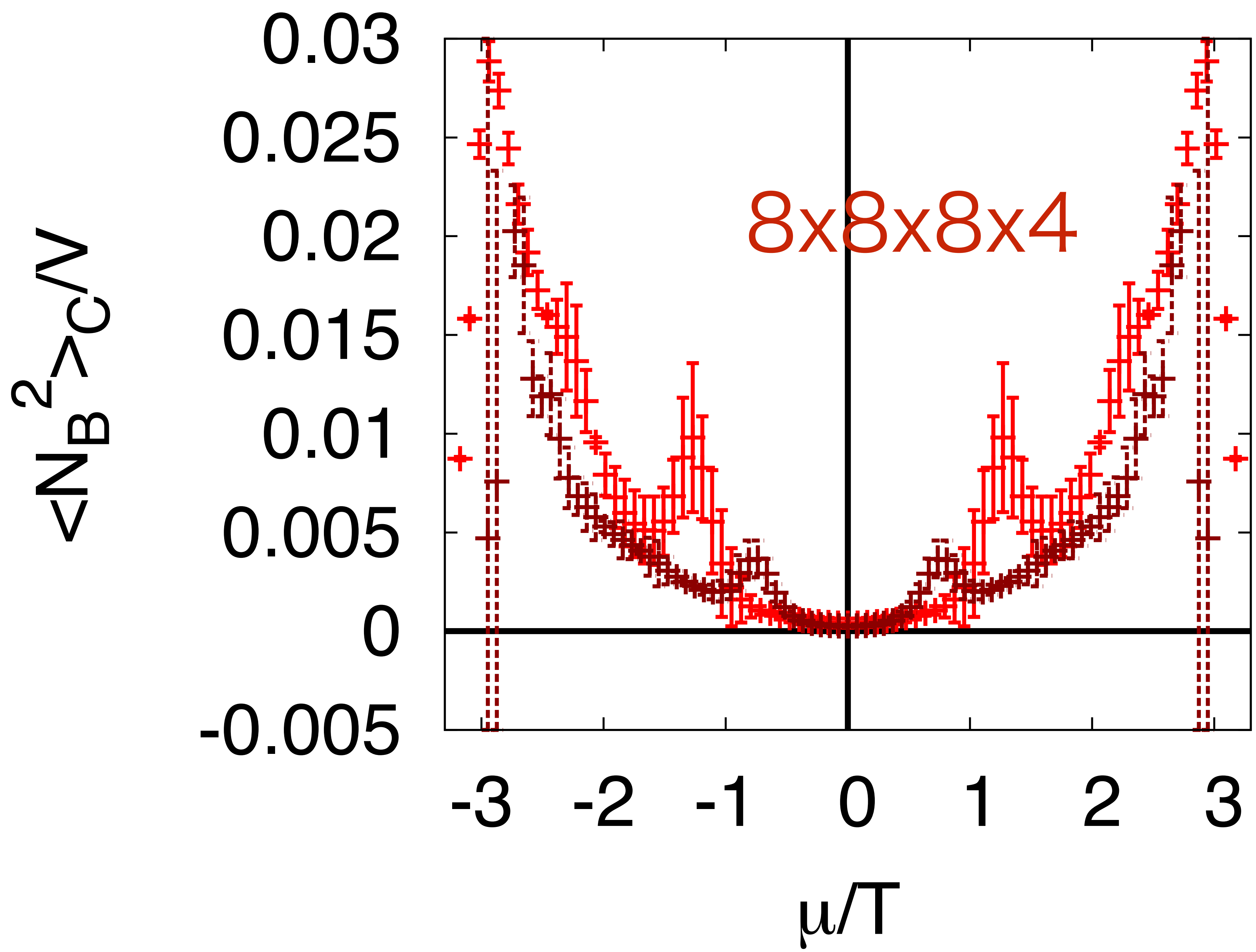

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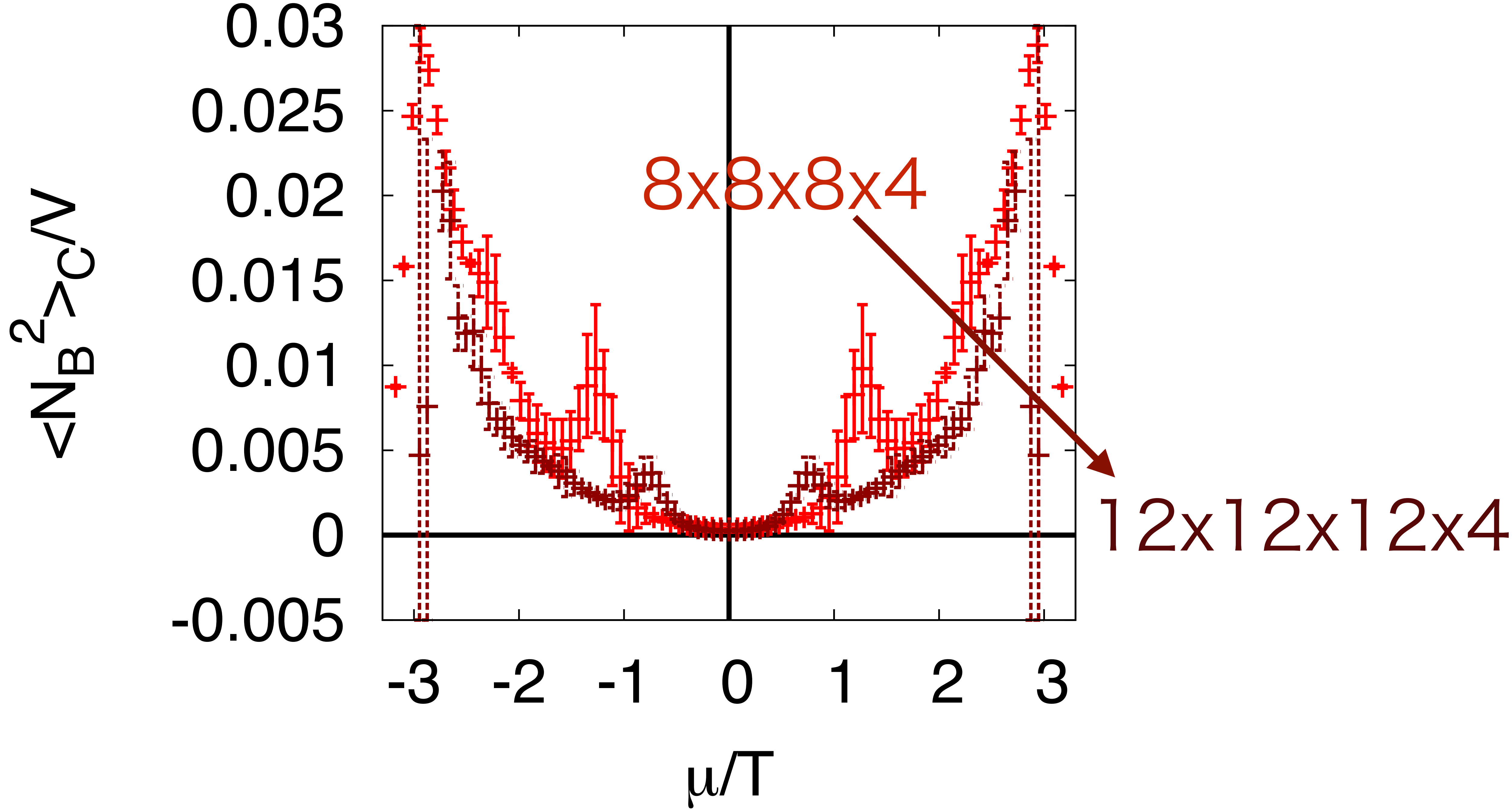
$$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

Low T $\beta = 0.9$

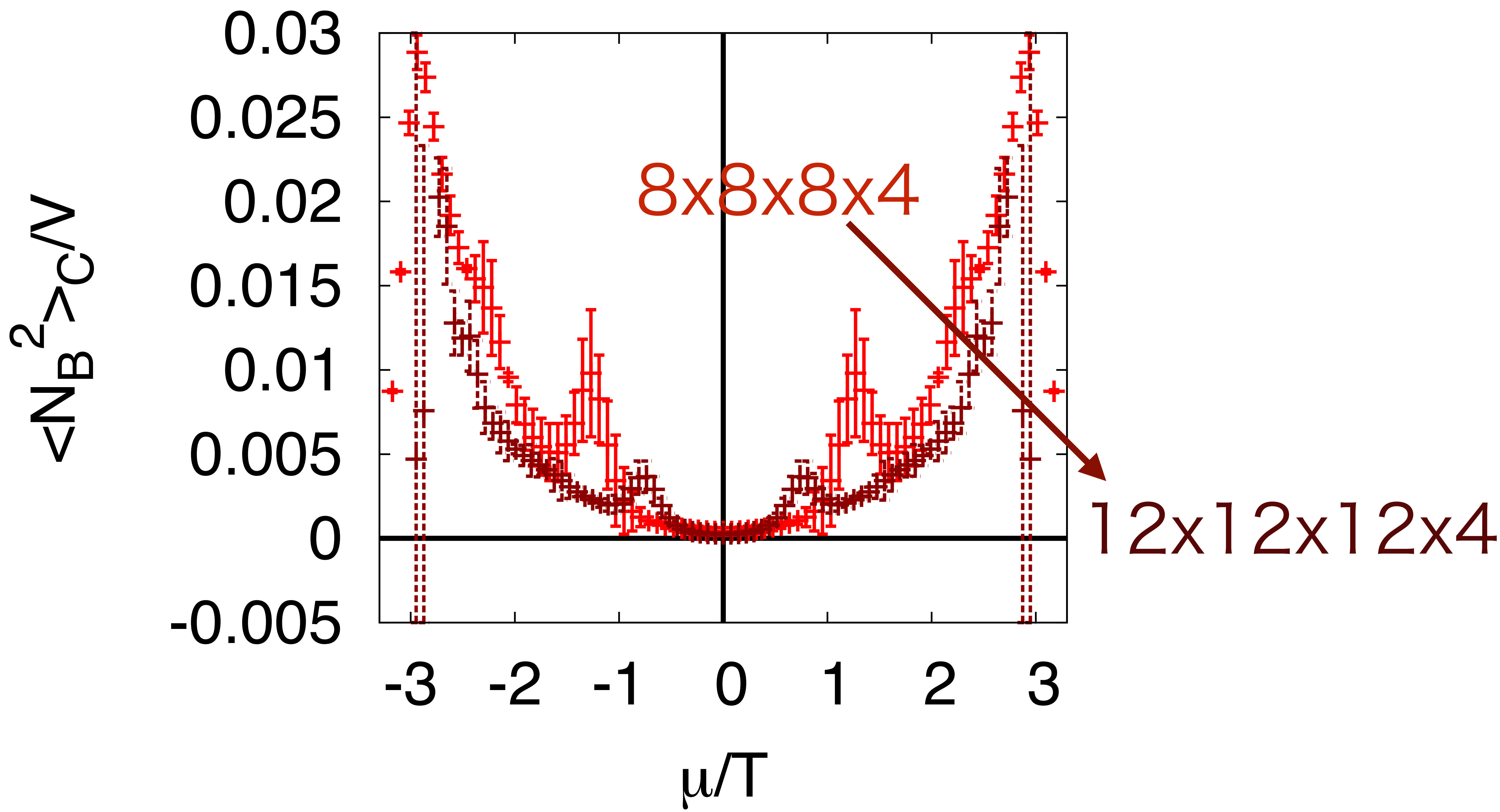


Volume dependence

3. cumulant of quark number density

$$\langle N^2 \rangle_c = \left(\xi \frac{\partial}{\partial \xi} \right)^2 \log Z(\xi)$$

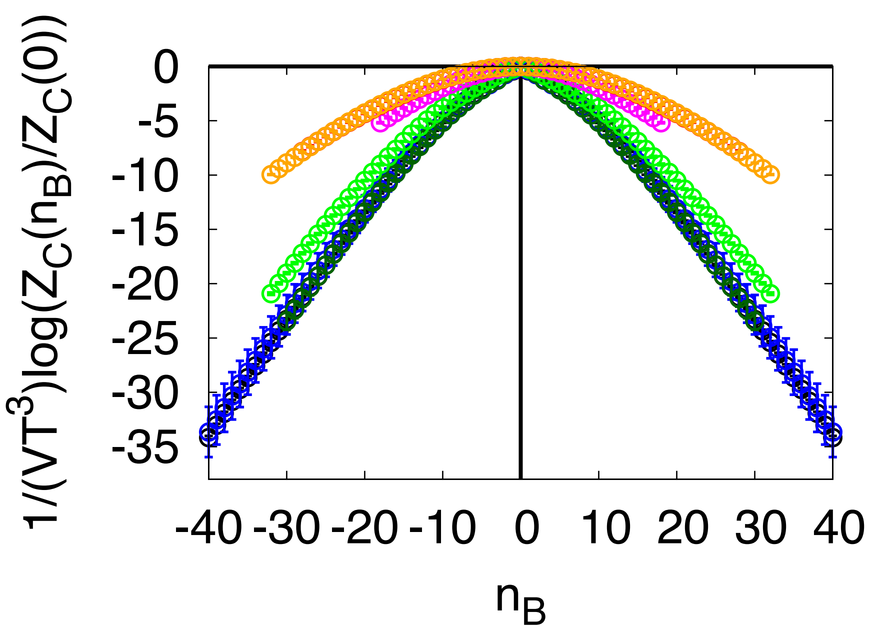
Low T $\beta = 0.9$



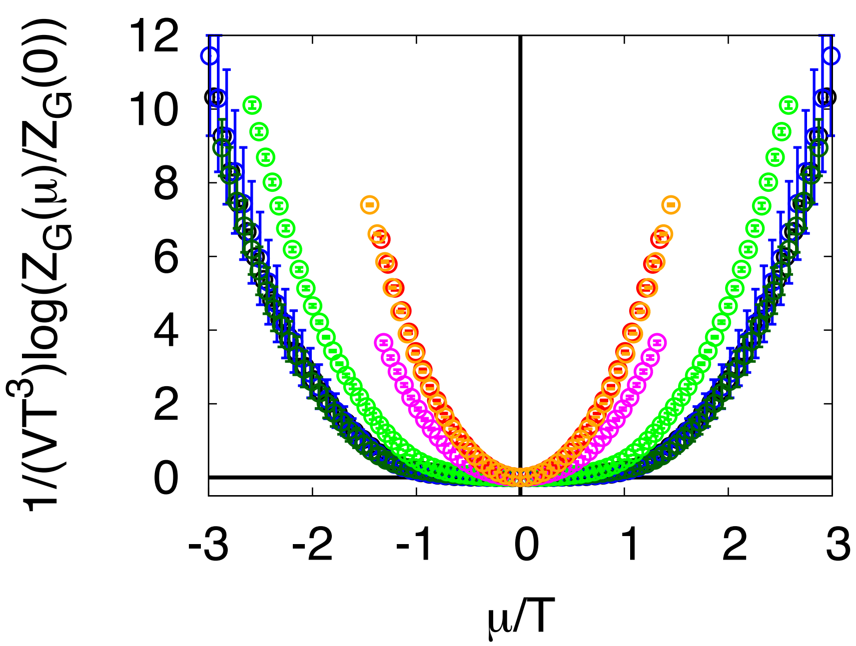
Conclusion

- Canonical approach is a good choice for finite density QCD.
- Hopping parameter expansion works more than we expected.
- We may observe the deconfinement phase transition.
- We may observe the chiral restoration.

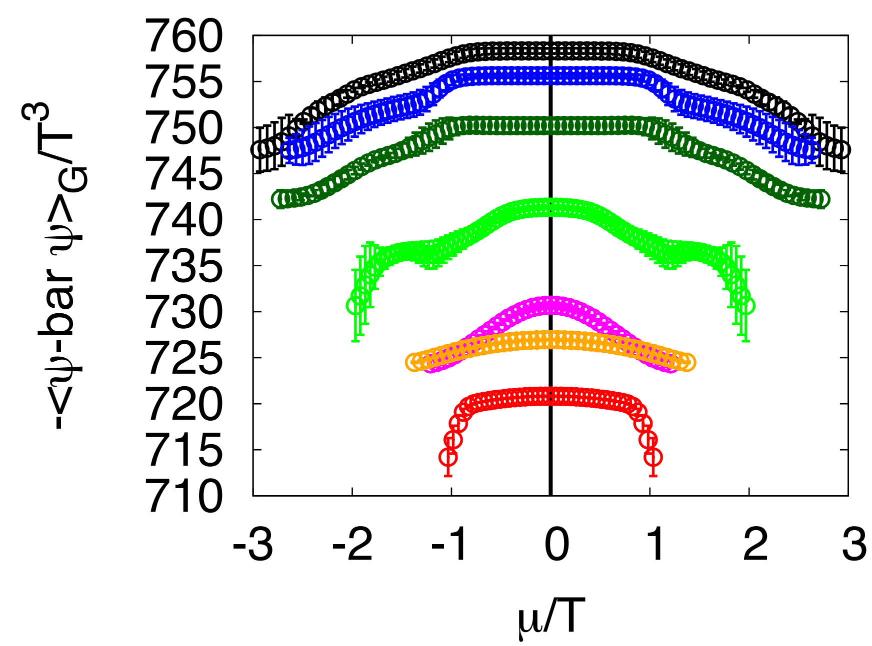
Zn



pressure



chiral condensate



$\beta = 0.9$ $\beta = 1.1$ $\beta = 1.3$ $\beta = 1.5$ $\beta = 1.7$ $\beta = 1.9$ $\beta = 2.1$

Future problems

- Why ZC(n) is complex?
  - Physics or artifact?
  - Fugacity expansion and transfer matrix?
- Overlap problem?
  - sea quarks at imaginary chemical potential.
- Study at various parameters.
  - Quark mass
  - Volume
  - Nt (continuum limit)
- Formulation beyond hopping parameter expansion.
  - Reduction formula
  - Numerical Fourier transformation