Calculation of nuclear transition matrix elements of neutrinoless double-beta decay using QRPA

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2. Neutrinoless double-beta decay
3. Nuclear matrix element
4. Status of calculation of nuclear matrix element
5. My application of QRPA
6. Summary

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What is the neutrino mass?

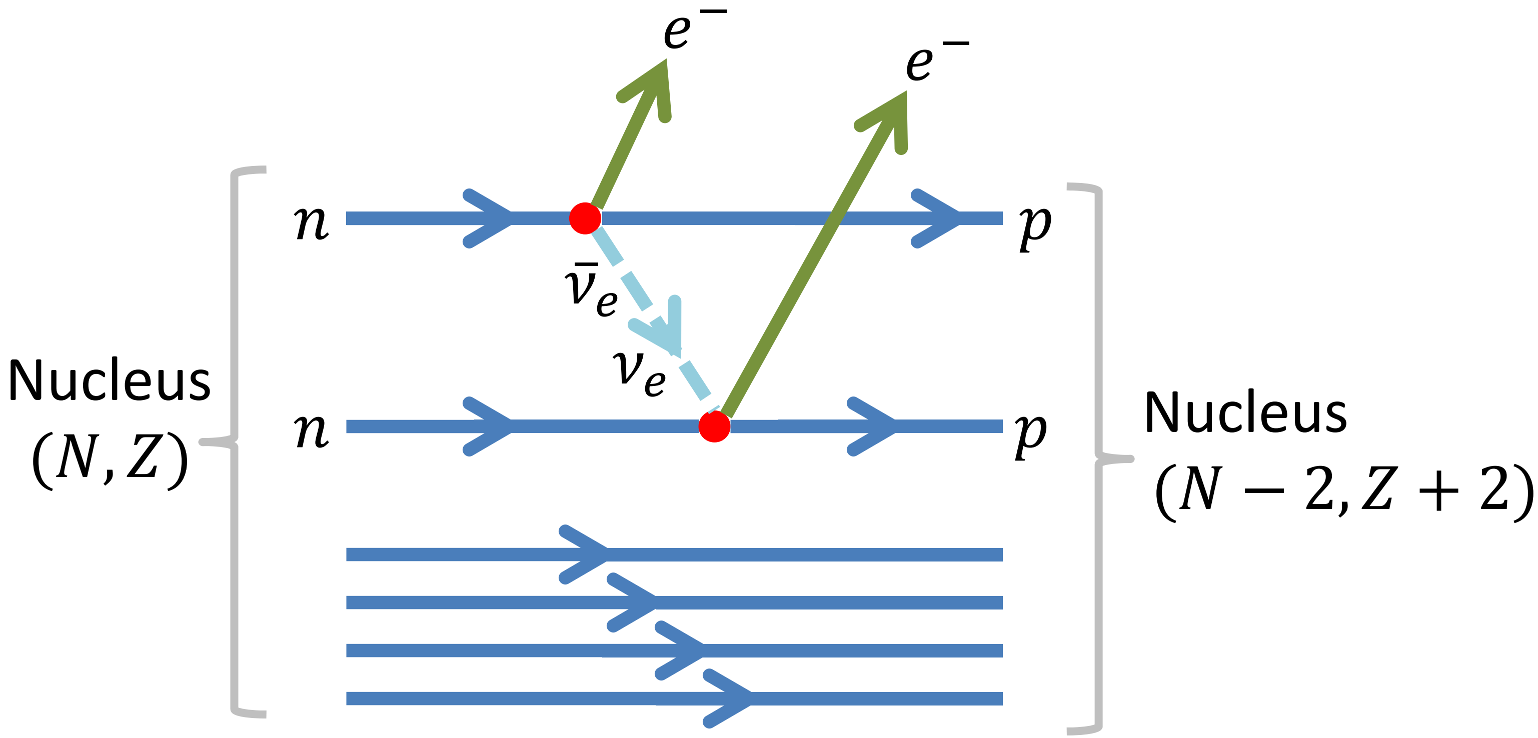
The neutrino is massless in the standard theory.

Other physical issues of neutrino

- Dirac or Majorana particle?
- Breaking of the lepton number conservation?
- How is the right-handed neutrino experimentally observed? Does that neutrino have an interaction?

One of the intensively studied few methods to determine the neutrino mass:

Application of neutrinoless double-beta ($0\nu\beta\beta$) decay of nuclei



Neutrino assumed to be Majorana particle.

The principle to determine the effective neutrino mass using $0\nu\beta\beta$ decay

$$1/T_{0\nu}(0^+ \to 0^+) = \left|M^{(0\nu)}\right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e}\right)^2$$

Exp. → $T_{0\nu}$: half-life of the transition

Cal. →
- $M^{(0\nu)}$: transition matrix element of nucleus (nuclear matrix element)
- G_{01}: electron part of the transition amplitudes squared (phase space factor)

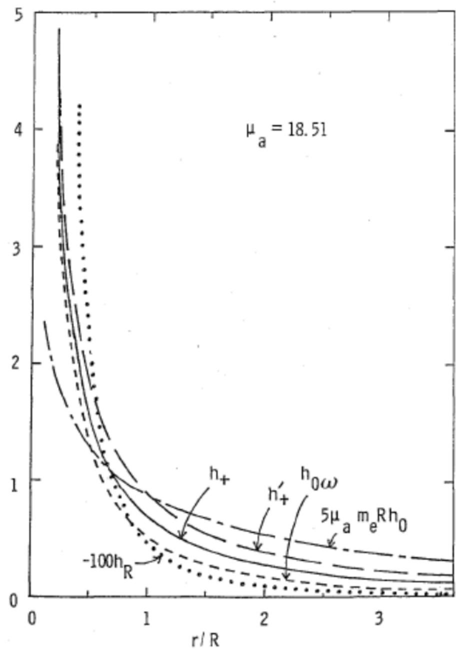
$\langle m_\nu \rangle$: effective neutrino mass

m_e : electron mass

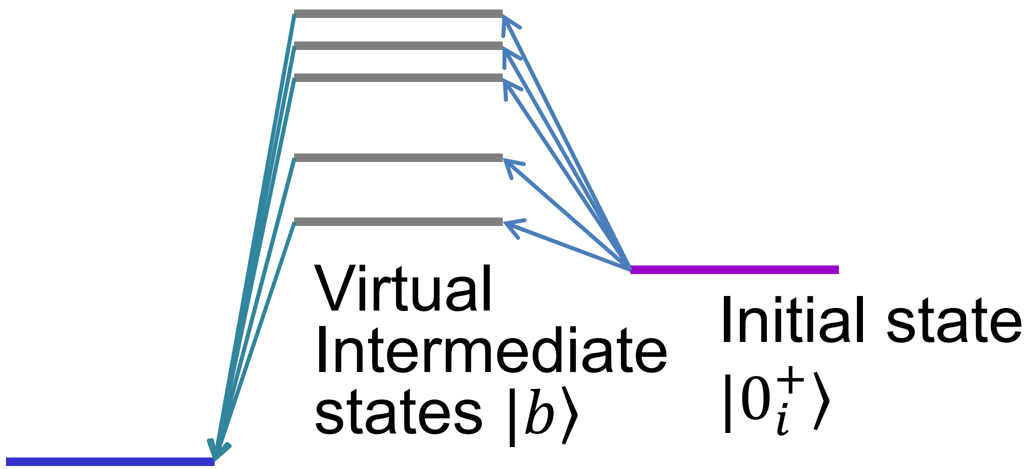
Nuclear matrix element

$$M^{(0\nu)} = \sum_{b}\sum_{pp'}\sum_{nn'}\langle pp'|V(r_{12},E_b)|nn'\rangle\langle 0_f^+|c_{p'}^\dagger c_{n'}|b\rangle\langle b|c_p^\dagger c_n\,|0_i^+\rangle$$

$$V(r_{12},E_b) = h_+(r_{12},E_b)\left\{-\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2)+\frac{g_V^2}{g_A^2}\right\}\tau^+(1)\tau^+(2)$$



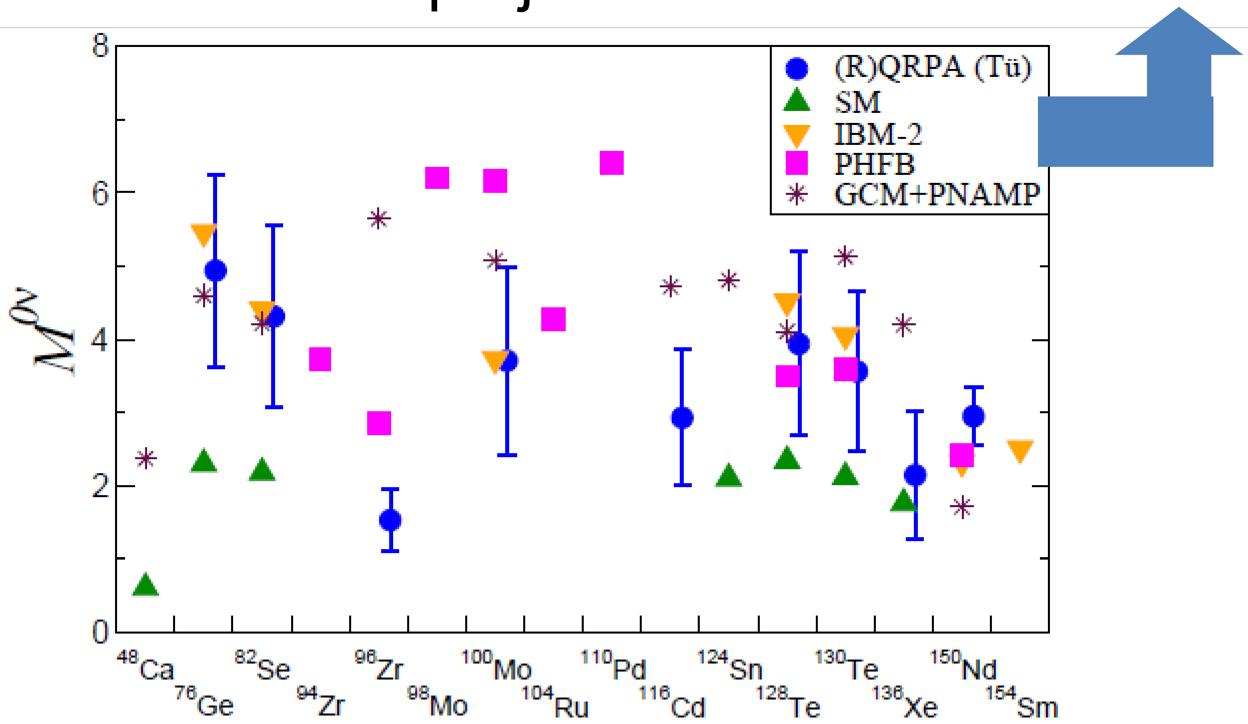
M. Doi et al. Prog. Theor. Phys. Suppl. No. 83 (1985) 1



Path of $0\nu\beta\beta$ decay in the calculation via virtual intermediate states

Status

(Relativistic) quasiparticle random-phase approximation
Shell model
Interacting boson model-2
Projected Hartree-Fock-Bogoliubov
Generator-coordinate method + Particle number and angular momentum projection



A. Feassler, arXiv:2103.3648 (2012)

Quasiparticle random-phase approximation (QRPA)

An approximation using only two-quasiparticle excitations $a_i^\dagger a_j^\dagger$ and $a_i a_j$ for the elementary mode of excitation

Application of QRPA

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'} c_p^\dagger c_n\ |0_i^+\rangle$$

$$\sum_{b_f:\mathrm{pnQRPA}} |b_f\rangle\langle b_f| \qquad \sum_{b_i:\mathrm{pnQRPA}} |b_i\rangle\langle b_i|$$

$$|b_i\rangle \sim (a_{\mathrm{proton}}^\dagger a_{\mathrm{neutron}}^\dagger + a_{\mathrm{neutron}} a_{\mathrm{proton}})|0_i^+\rangle$$

Application of QRPA

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'} c_p^\dagger c_n\, |0_i^+\rangle$$

$$\sum_{b_f:\text{pnQRPA}} |b_f\rangle\langle b_f| \qquad \sum_{b_i:\text{pnQRPA}} |b_i\rangle\langle b_i|$$

$$M^{(0\nu)} \cong \sum_{pp'nn'} \langle pp'|V(\bar{E})|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'} c_p^\dagger c_n\, |0_i^+\rangle$$

$$-c_{p'}^\dagger c_p^\dagger c_{n'} c_n$$

$$\sum_{b_f:\text{likeQRPA}} |b_f\rangle\langle b_f| \qquad \sum_{b_i:\text{likeQRPA}} |b_i\rangle\langle b_i|$$

Like-particle QRPA$\sim a_{\text{proton}}^\dagger a_{\text{proton}}^\dagger + a_{\text{neutron}}^\dagger a_{\text{neutron}}^\dagger +$ annihilator

QRPA states

The QRPA ground state is defined to be the vacuum to the QRPA quasiboson :

$$O_b|0_i^+\rangle \cong 0$$

O_b : annihilation operator of QRPA state b

$$|b_i\rangle = O_{bi}^\dagger|0_i^+\rangle = O_{bi}^\dagger \frac{1}{\mathcal{N}} e^{v}|0_{i\mathrm{HFB}}^+\rangle,$$

$v \sim a^\dagger a^\dagger a^\dagger a^\dagger$: product of quasiparticle creation operators

$\mathcal{N}$: normalization factor

The overlap $\langle b_f|b_i\rangle$ are calculated using that definition of the QRPA ground state.

J.T. PRC **86**, 021301(R) (2012); **87**, 024316 (2013)

Calculation of $M^{(0\nu)}$ of ^{150}Nd-^{150}Sm using like-particle QRPA. SkM*+volume pairing is used.
$\left|M^{(0\nu)}\right| \sim 0.02$

2.5 – 3.5 (QRPA , Tübingen),
1.8 – 3.5 including various approaches (the previous figure)

0.02 is too small, because the QRPA correlations are too large; it is known that the correlation energy diverges in the Skyrme QRPA.

The Skyrme and the volume pairing interaction $\propto \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$

My prescription

To pick up the QRPA solutions having the largest contributions so as to satisfy

$$E_{\mathrm{QRPA}}^{\mathrm{cor}} = E_{\mathrm{exp}} - E_{HFB}$$

and use only these states for calculating the QRPA ground states.

10 like-particle QRPA states for
$E_{\mathrm{QRPA}}^{\mathrm{cor}} = -1.72\mathrm{MeV}$ (^{150}Nd) and
18 like-particle QRPA states for
$E_{\mathrm{QRPA}}^{\mathrm{cor}} = -3.44\mathrm{MeV}$ (^{150}Sm)
were picked up. The pnQRPA correlations were small.

Revised calculations are in progress.

Summary

- Several physical issues of neutrino were shown.
- How the $0\nu\beta\beta$ decay is used for determining the effective neutrino mass was explained.
- Status was shown of the calculations of the nuclear matrix elements

  — the discrepancy problem of the results by methods
- "The QRPA ground state = QRPA quasiboson vacuum" is used.
- The QRPA correlations have the effect to reduce the nuclear matrix element through the normalization factor of the ground state.