Phase structure of finite density QCD

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in collaboration with
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Lattice QCD at finite temperature and density
2014/1/21 @KEK

Study of QCD thermodynamics so far

- Staggered fermions are mainly used in both zero and finite density
- Rooting trick?
- Universality check by solid ground formulation
- We use Wilson-type fermions HERE

QCD phase structure by Wilson-type fermions

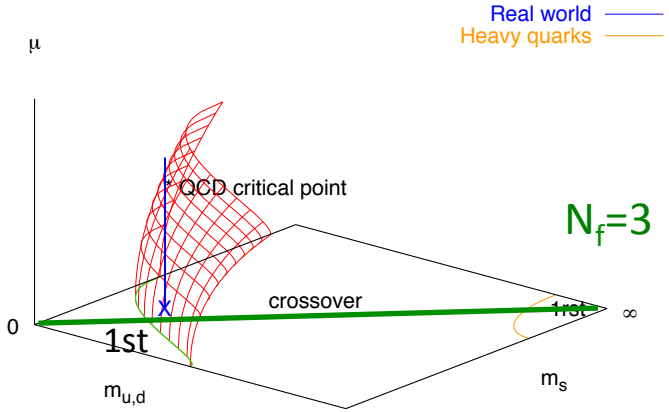
- Zero density
  - Fukugita et al. (‘84) Langevin algorithm
  - Iwasaki et al.(‘96), Bernard et al.(‘93)
    - Strong first order phase transition was observed
  - Aoki phase (‘96)
  - N_f=2: CP-PACS (‘00&’01), QCDSF(‘11)
  - We’ll explore N_f=3 ☞ Nakamura-san’s talk
- Finite density
  - WHOT-QCD : Heavy quark (‘12)
  - WHOT-QCD : N_f=2 by histogram method (‘12)
  - We’ll explore N_f=3

Goal

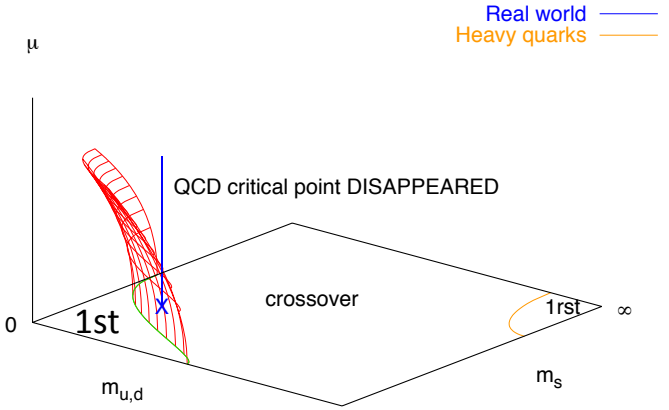
Slope/curvature of critical surface?

de Forcrand et al. 2006

Conventional scenario



Exotic scenario



$$\frac{m_c(\mu)}{m_c(0)} = 1 - 0.7(4)\left(\frac{\mu}{\pi T}\right)^2$$

Strategy

- Phase reweighting
- How to identify critical end point
- Physical scale
- Continuum limit of critical end point

Phase reweighting

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_{\rm f}\theta} \rangle_{||}}{\langle e^{iN_{\rm f}\theta} \rangle_{||}} \qquad \det D = |\det D| e^{i\theta}$$

$$\mathcal{Z}_{||}(T,\mu) = \int [dU] e^{-S_{\rm g}[U]} |\det D(\mu;U)|^{N_{\rm f}}$$

- Degenerate (N_f=3) mass & chemical potential
- No parameter β/μ-reweighting
- Evaluate phase of determinant exactly

How to compute det.

4-d 3-d

$$\det D = A_0 \det[1 - H_0 - e^{\mu/T} H_+ - e^{-\mu/T} H_-]$$

- Reduction of temporal direction Gattringer '10
- 3-d (dense) matrix H_0, $H_\pm$ are made by matrix product and inverse of 3-d (sparse) matrix in D
- Evaluate det. in 3-d matrix by LUD

☞ LAPACK & GPU

- Cost: 1.2h/conf. for 12^3x6 on 1node of HA-PACS

Strategy

- Phase reweighting
- How to identify critical end point
- Physical scale
- Continuum limit of critical end point

How to identify critical end point

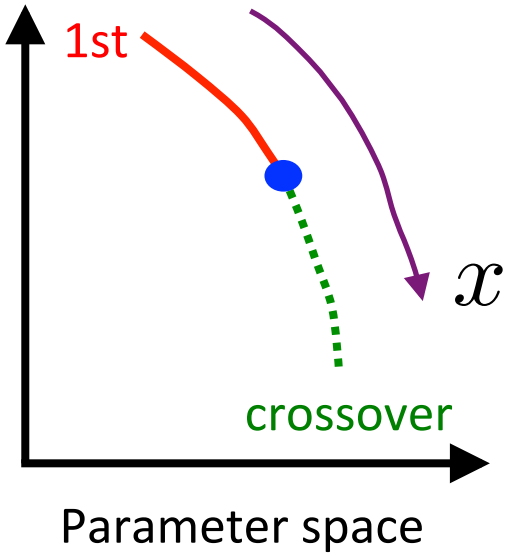
- Binder cumulant/Kurtosis intersection method
- Gap of transition point for two observables
- Lee-Yang zero analysis Fodor et al., ’02 & ‘04
- Histogram method Ejiri ‘07

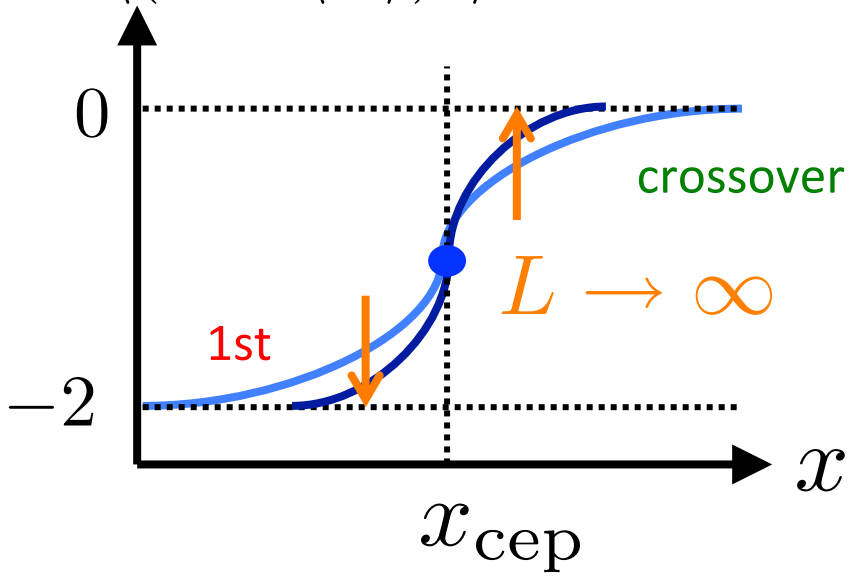
CEP by Kurtosis intersection method

Karsch et al. 2001

$$K = \frac{\langle (O - \langle O \rangle)^4 \rangle}{\langle (O - \langle O \rangle)^2 \rangle^2} - 3$$

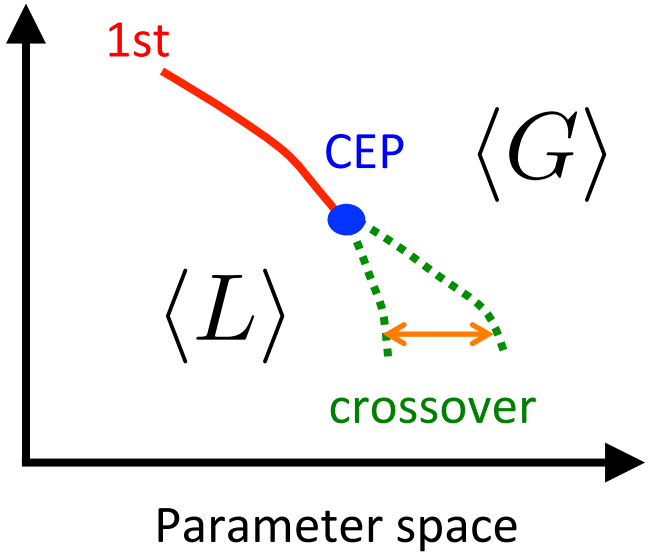
B_4





CEP by Gap of transition point

- Consider two independent observables (Gauge action density, Polyakov loop)
- Transition point at infinite volume limit
- In 1st order PT side, transition points are common for any observables
- While for crossover side, gap occurs
- CEP is defined such that gap vanishes

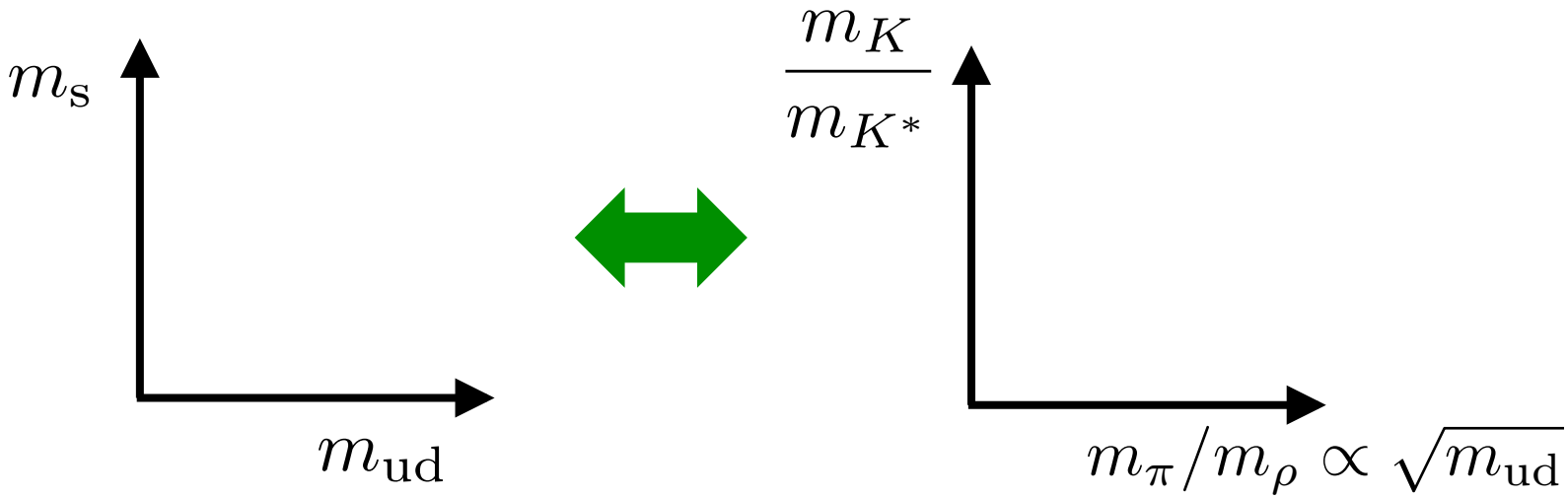


Strategy

- Phase reweighting
- How to identify critical end point
- Physical scale
- Continuum limit of critical end point

Physical scale

Use hadron mass ratio instead of quark mass to avoid renormalization issue



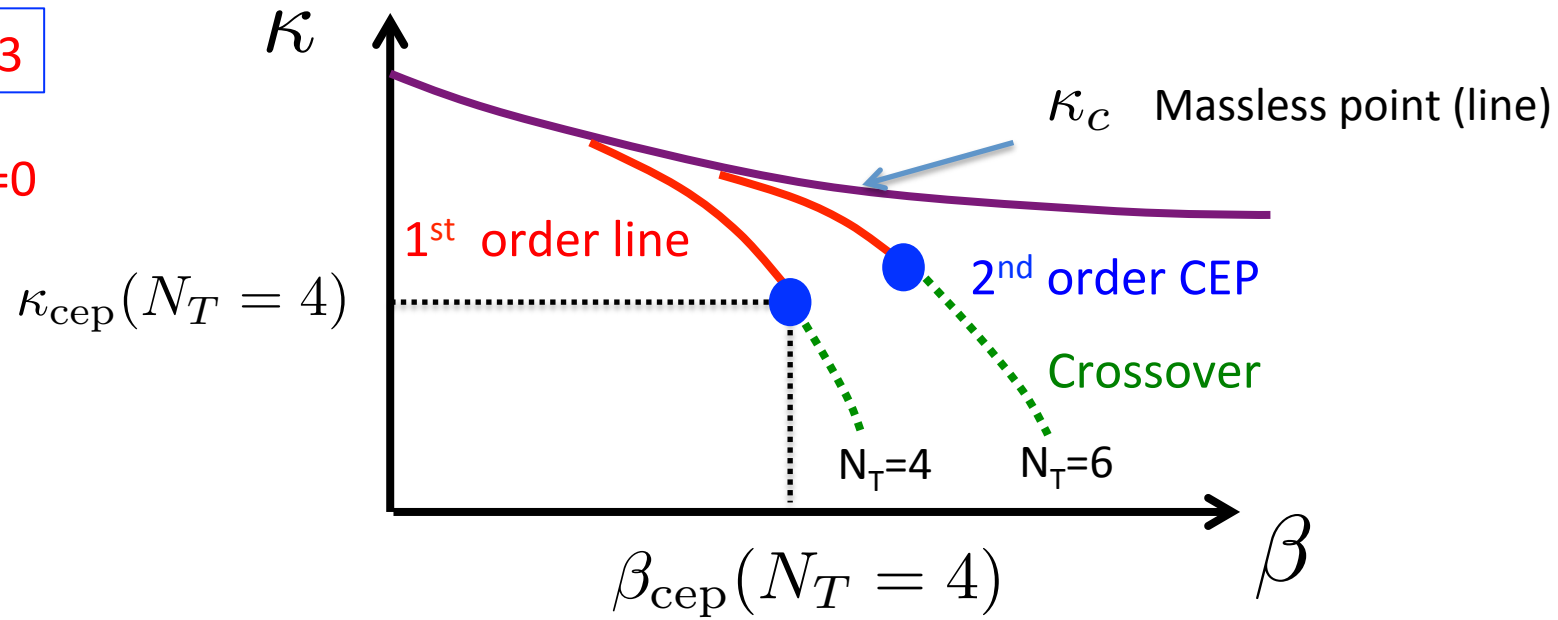
Strategy

- Phase reweighting
- How to identify critical end point
- Physical scale
- Continuum limit of critical end point

Continuum limit of CEP

N_f=3

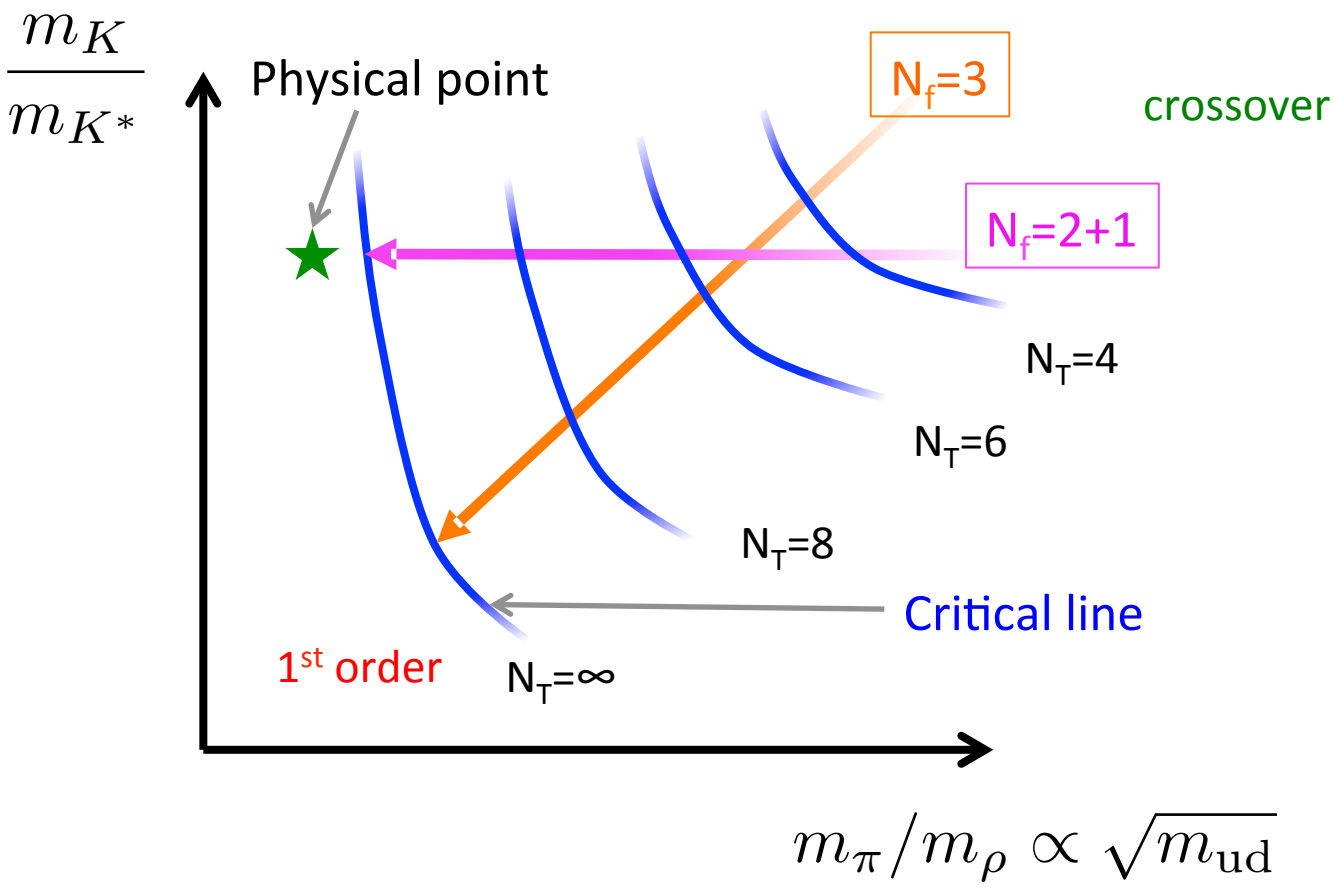
aμ=0



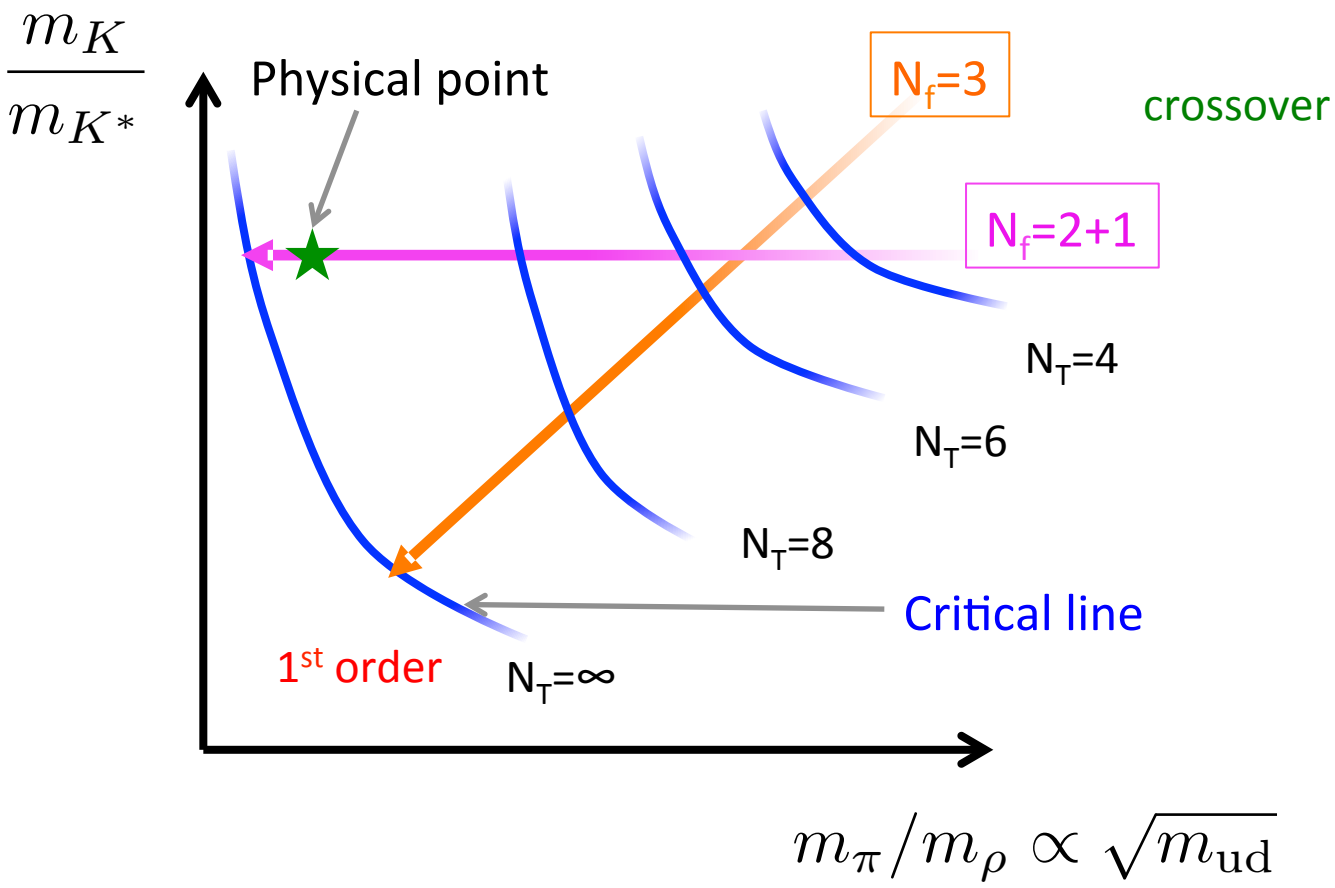
$1/N_{\rm T} = aT_{\rm cep}$

$$\lim_{1/N_{\rm T}\to 0} \left.\frac{m_\pi}{m_\rho}\right|_{\kappa_{\rm cep}(N_{\rm T}),\beta_{\rm cep}(N_{\rm T})}$$

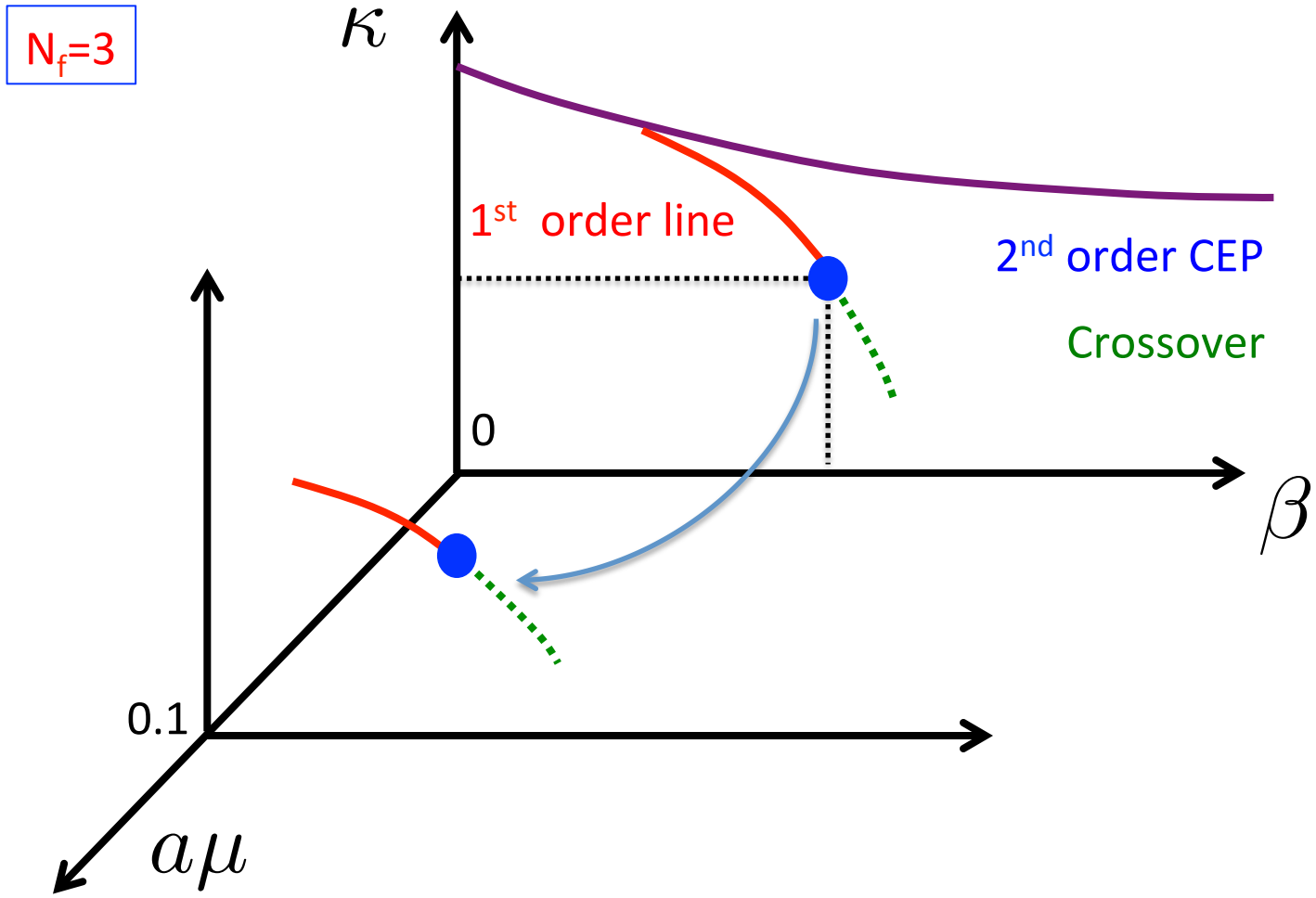
Continuum limit of CEP



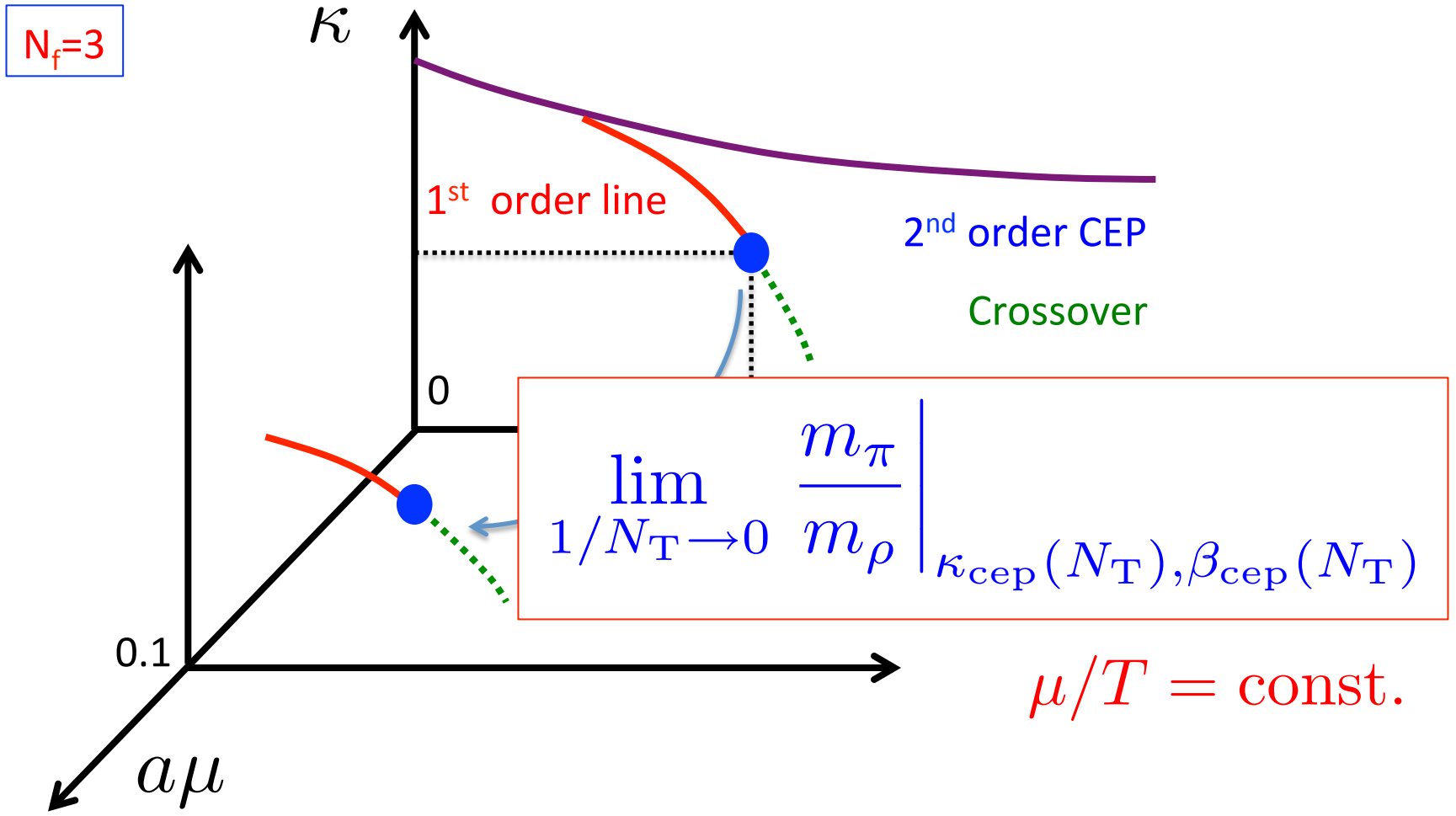
Continuum limit of CEP



For finite density



For finite density



Simulation results

Zero density

➔Y. Nakamura-san's talk tomorrow

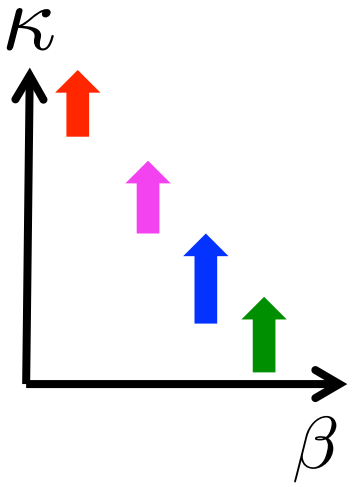
N_T=4,6,8

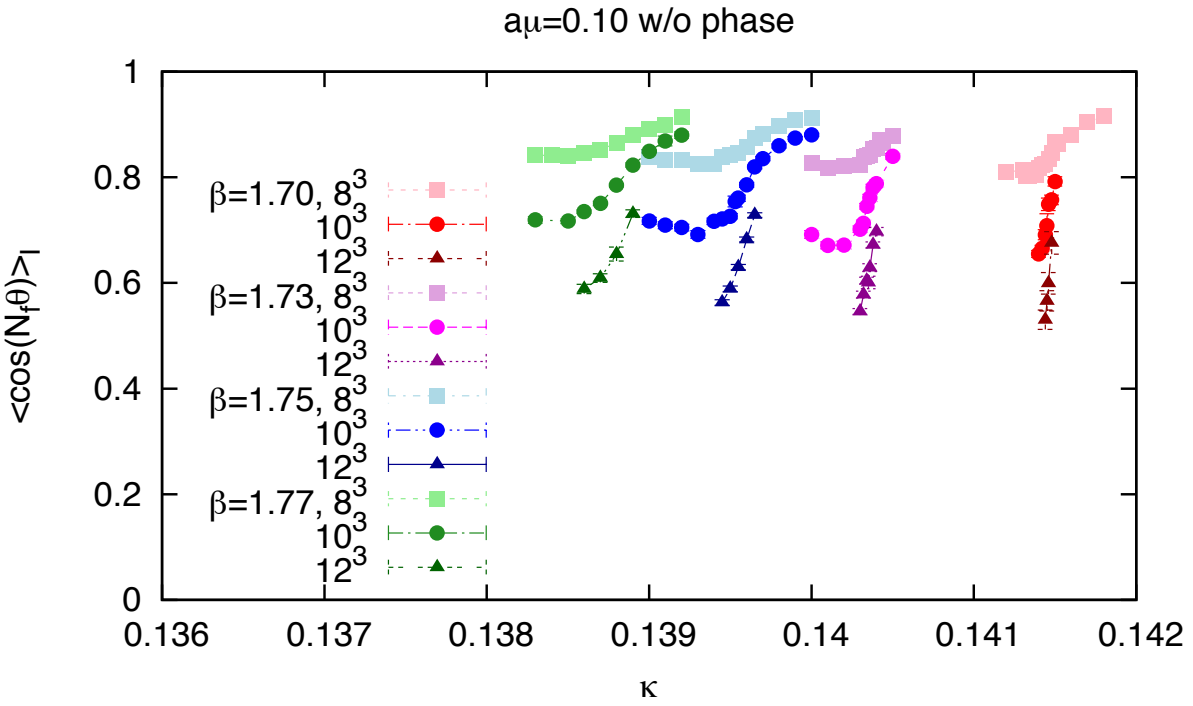
Finite density

➔Here N_T=6 & aμ=0.1 ⇒ μ/T=0.6

$V=8^3, 10^3, 12^3$

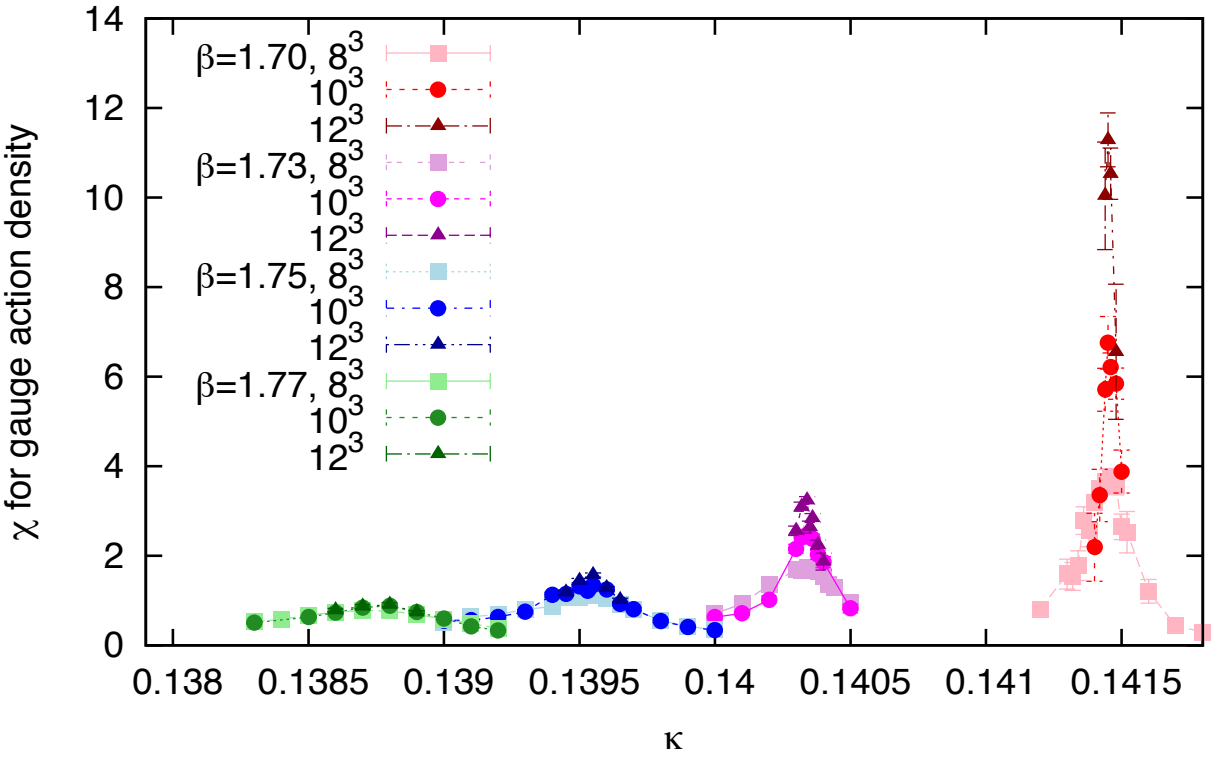
Phase re-weighting factor



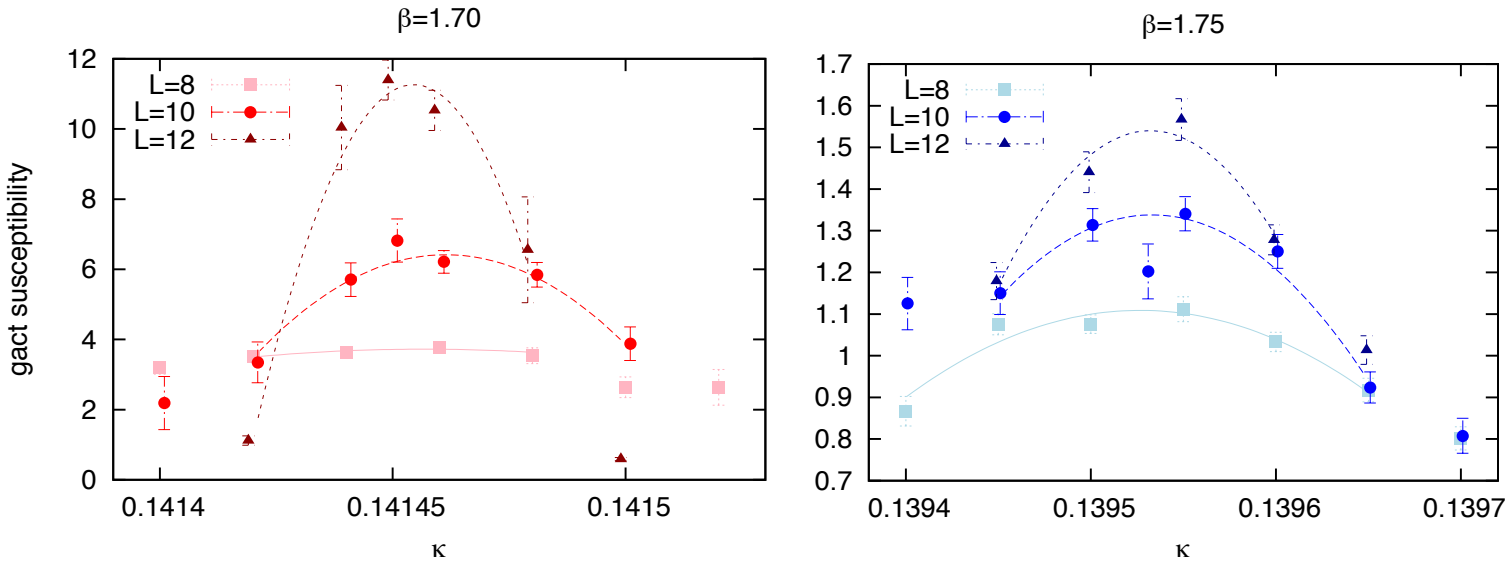


Sign problem is under controlled

Susceptibility of gauge action density

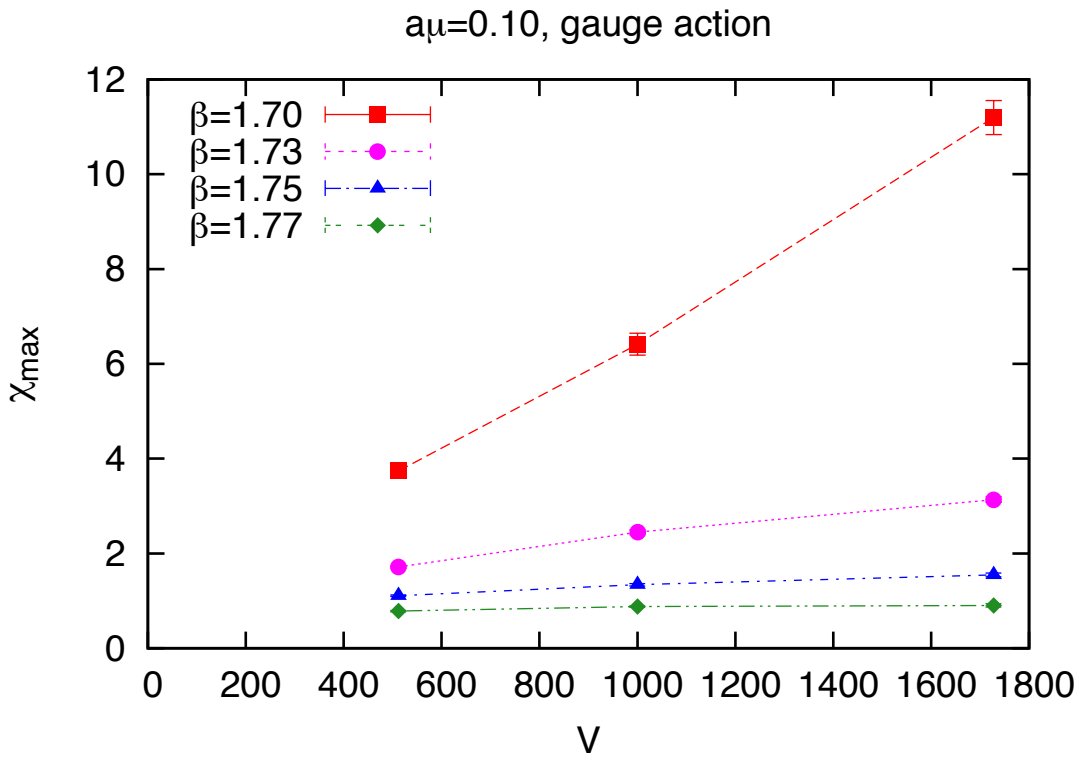


Determination of peak height



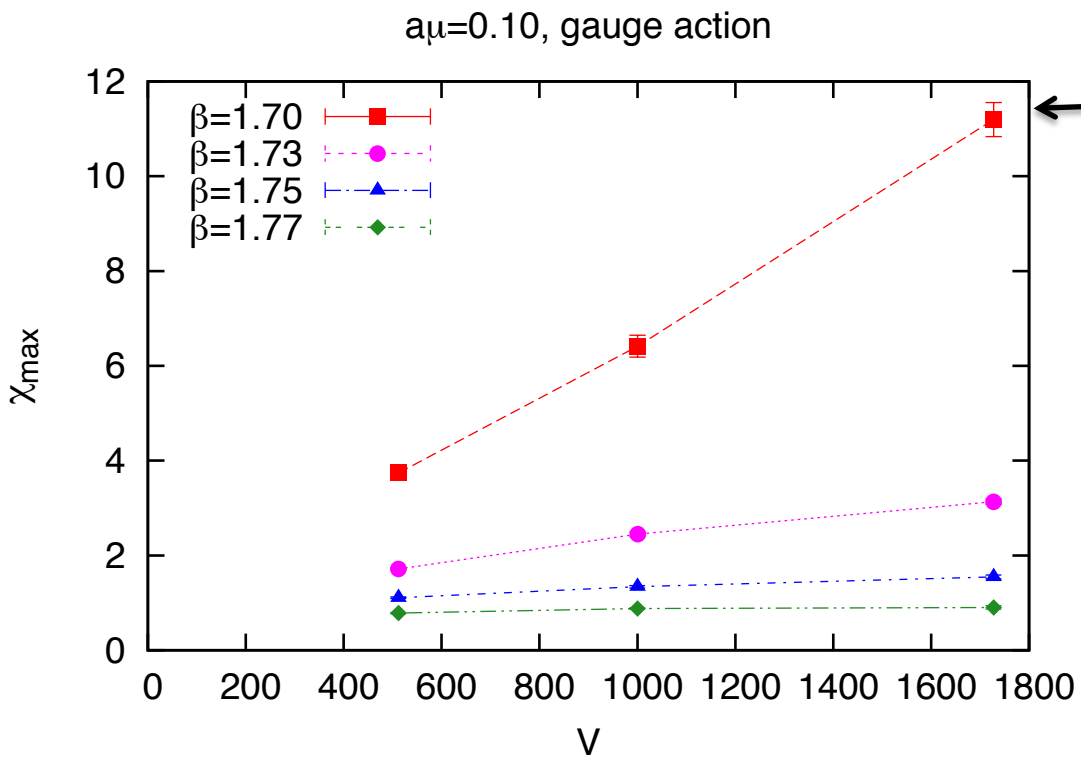
V-scaling of peak height of sus.

(μ/T=0.6)



V-scaling of peak height of sus.

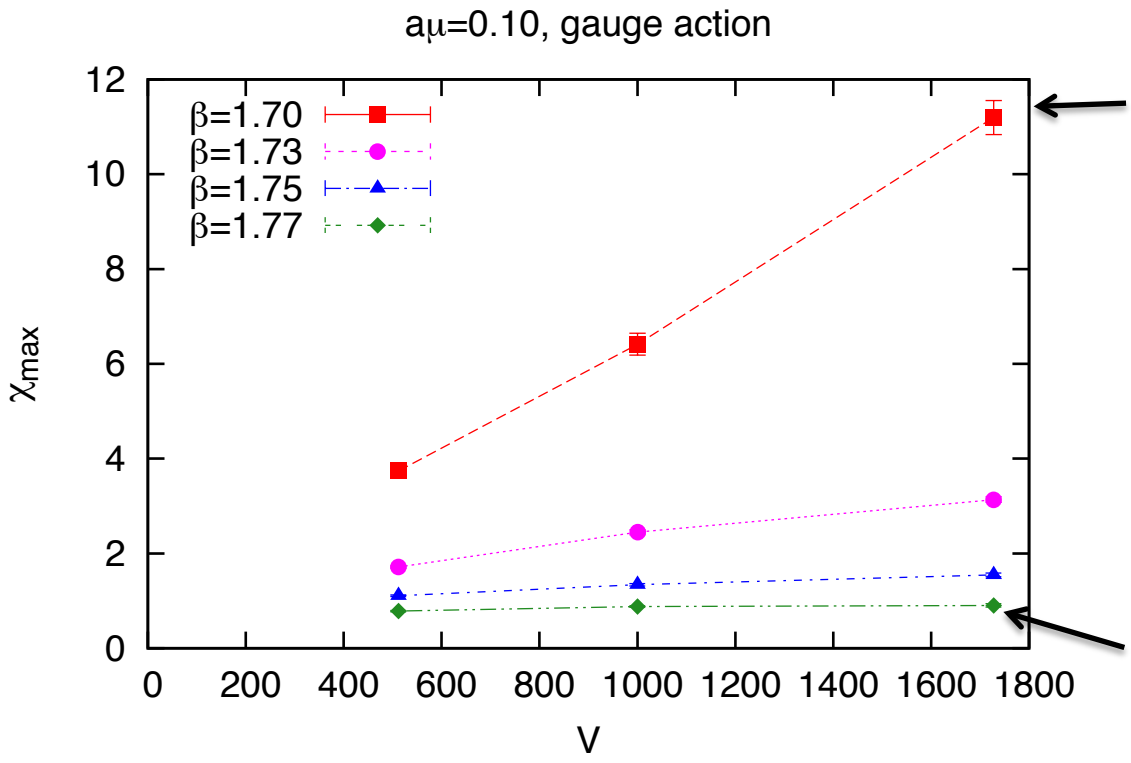
(μ/T=0.6)



1st order PT
∝ V

V-scaling of peak height of sus.

(μ/T=0.6)

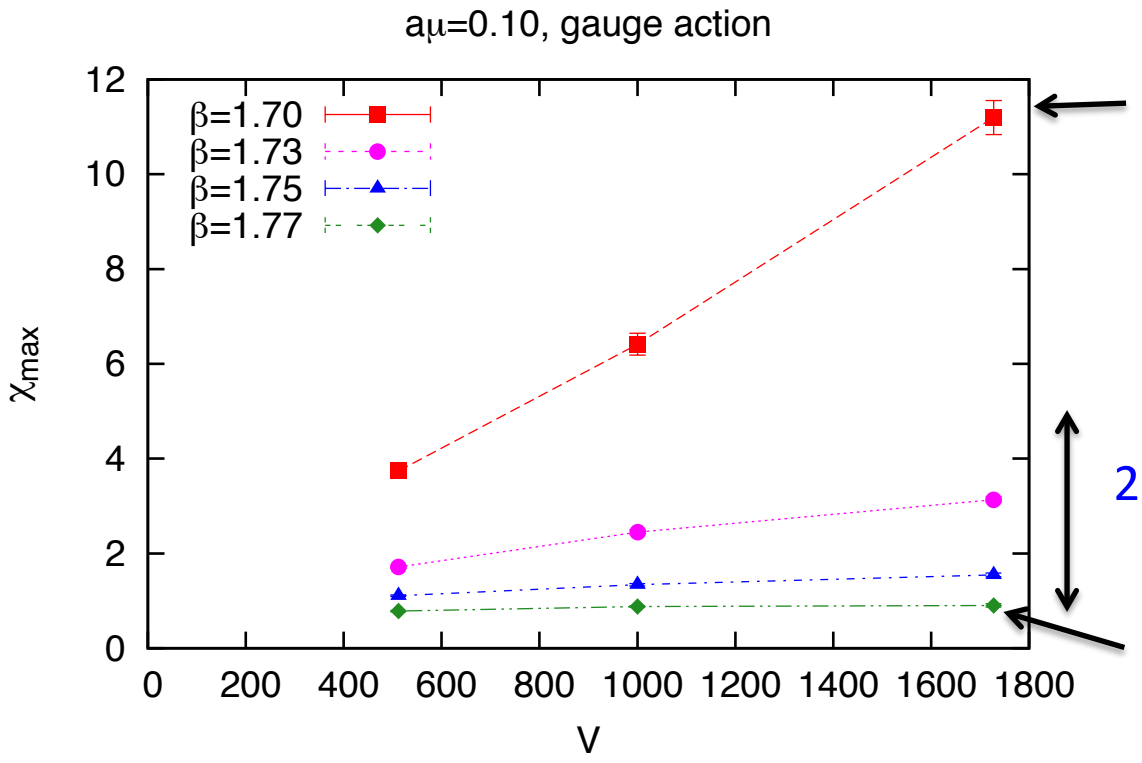


1st order PT
∝V

Crossover
Small V-dependence

V-scaling of peak height of sus.

(μ/T=0.6)



1st order PT
∝V

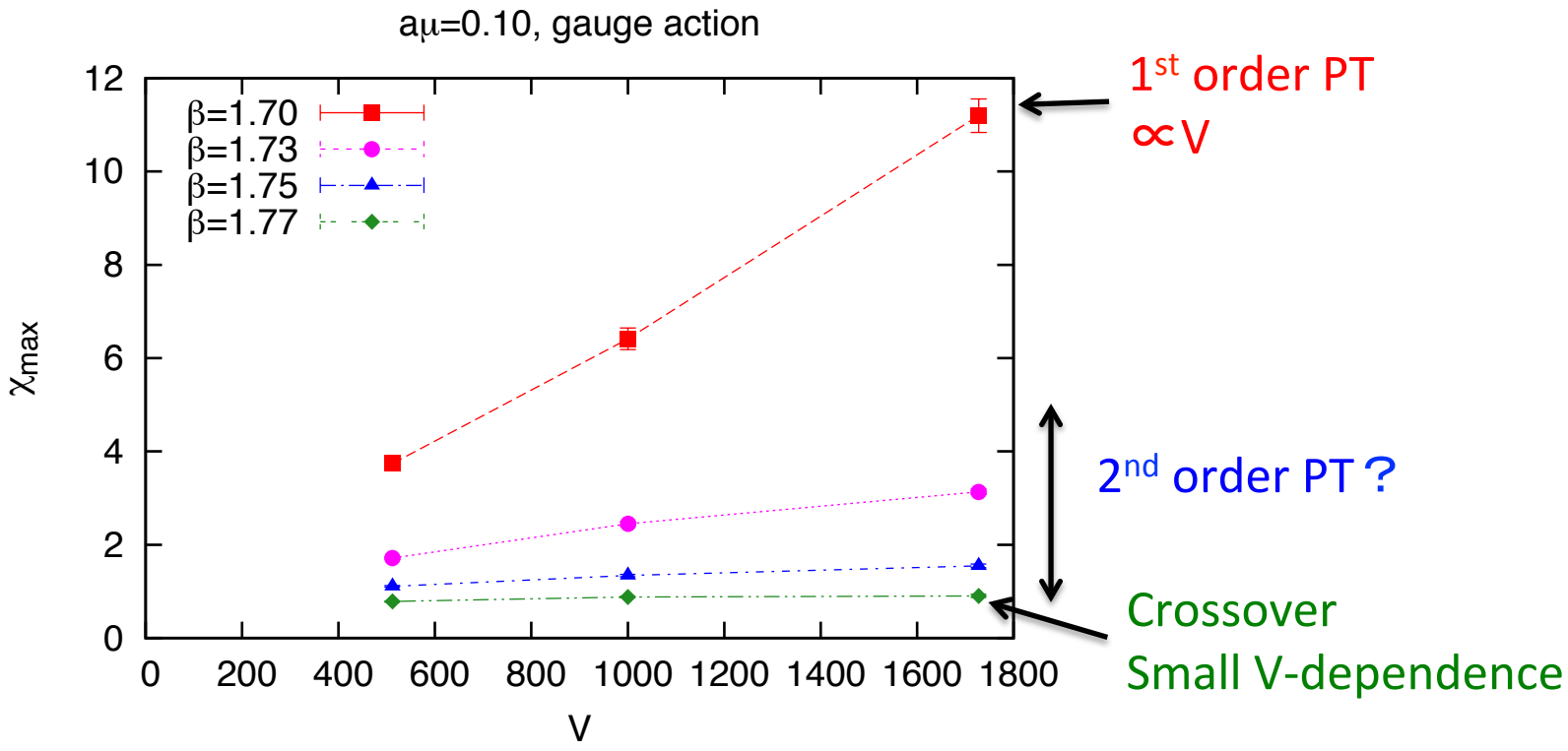
2nd order PT ?

Crossover
Small V-dependence

Where is critical end point?

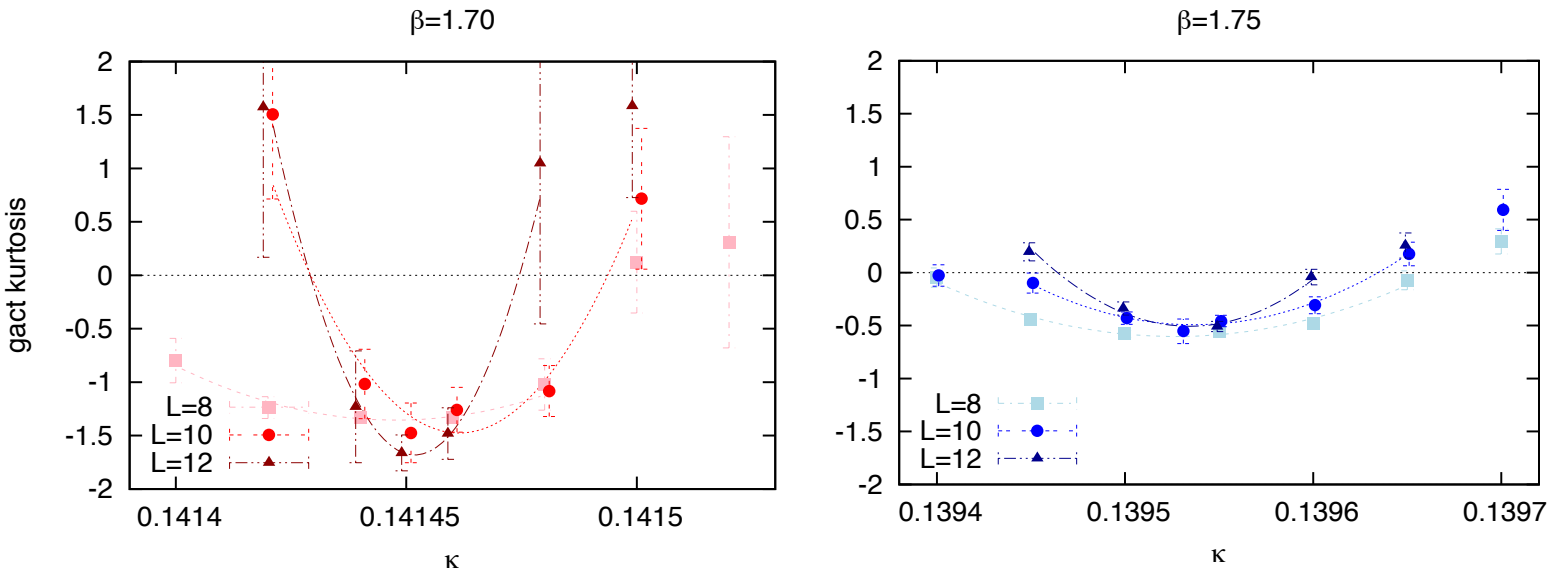
V-scaling of peak height of sus.

(μ/T=0.6)

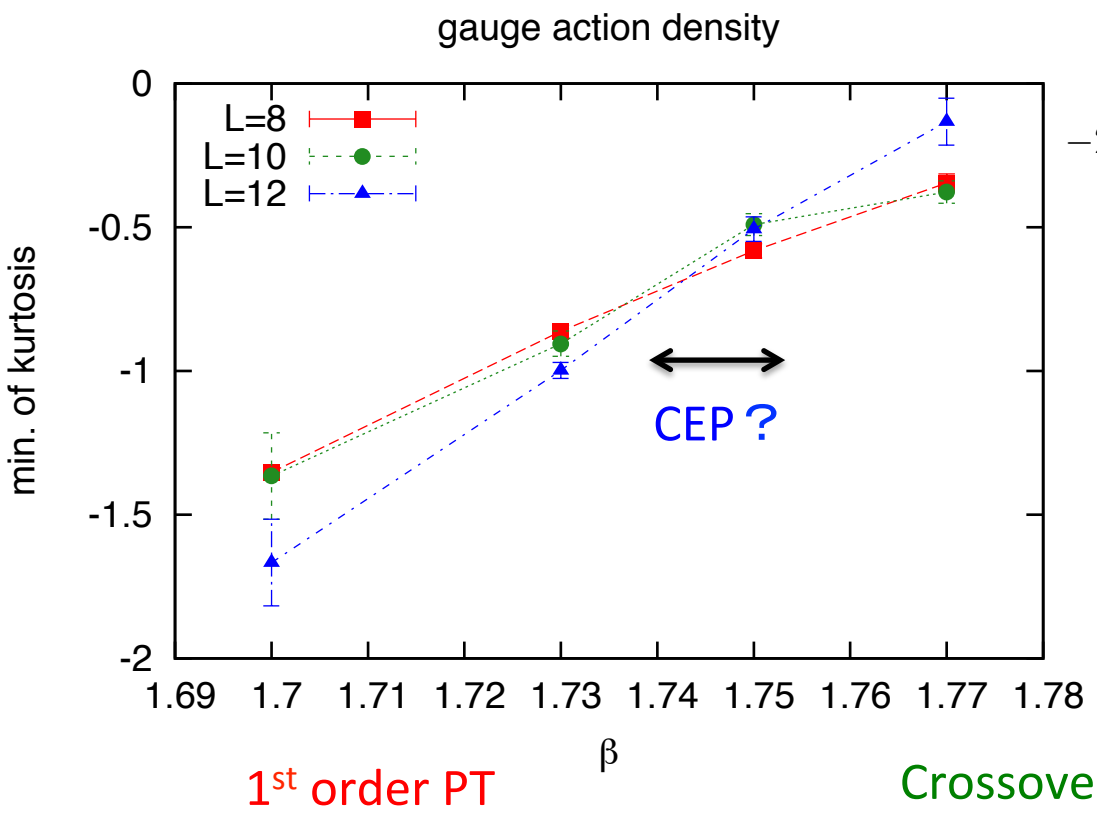


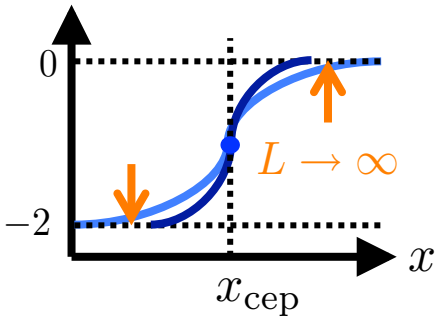
Where is critical end point? ☞ Kurtosis intersection

Kurtosis

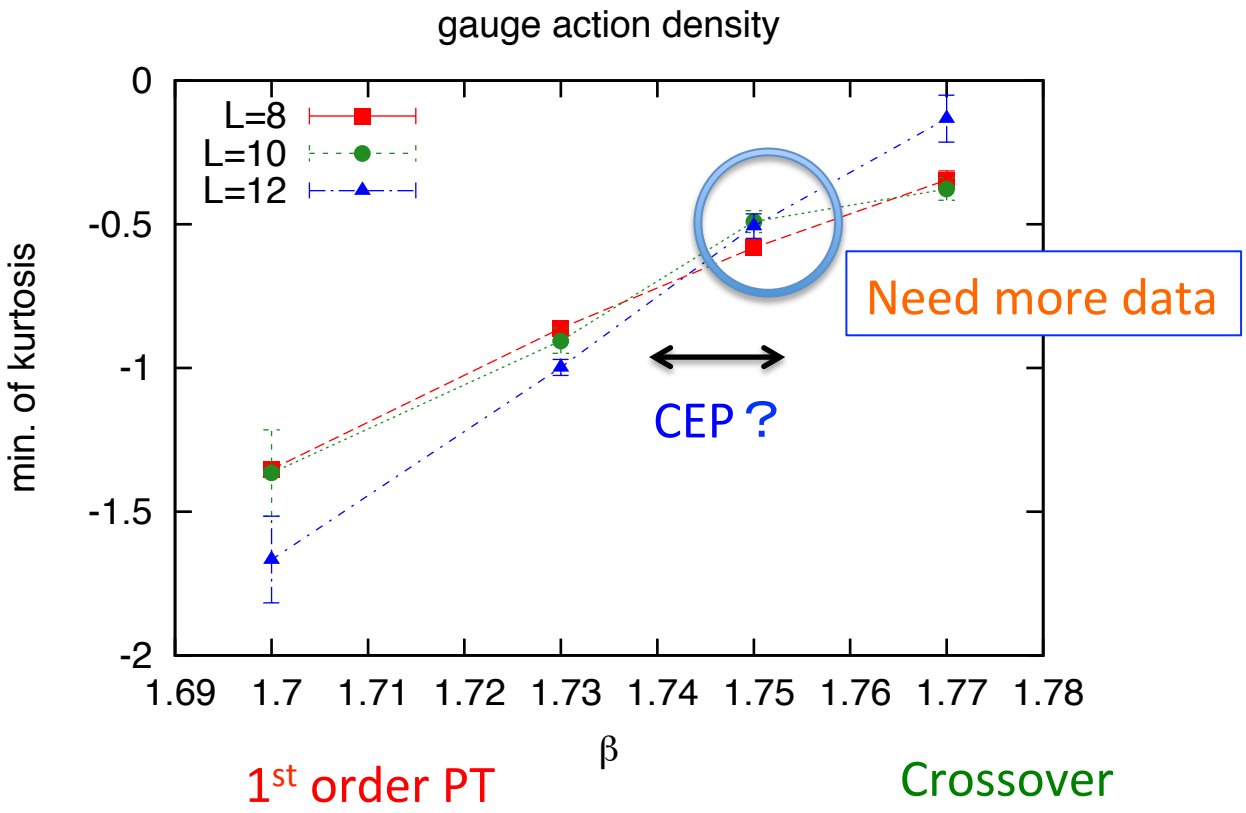

β=1.70
gact kurtosis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
L=8
L=10
L=12
0.1414
0.14145
0.1415
κ
β=1.75
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
L=8
L=10
L=12
0.1394
0.1395
0.1396
0.1397
κ

Kurtosis intersection



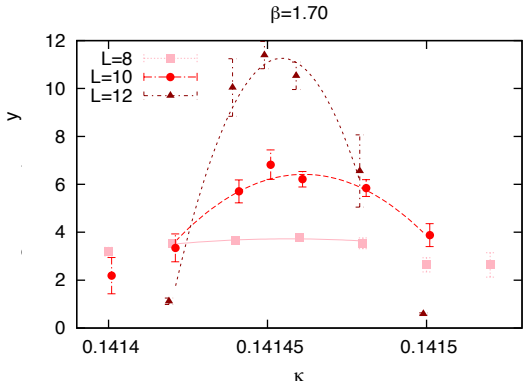


Kurtosis intersection

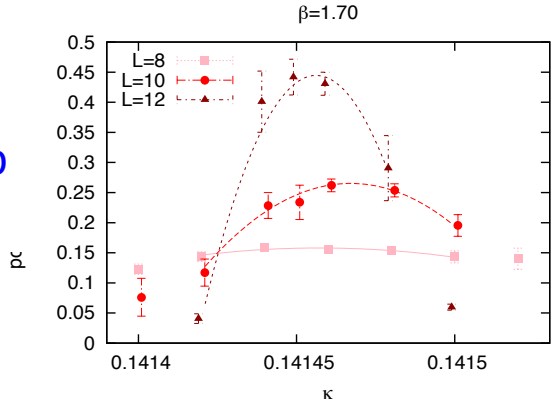


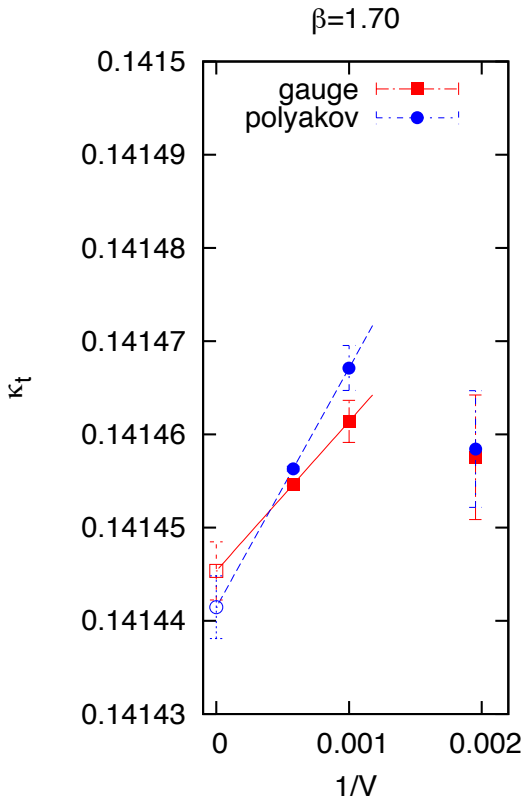
Gap of transition points

Gauge action susceptibility

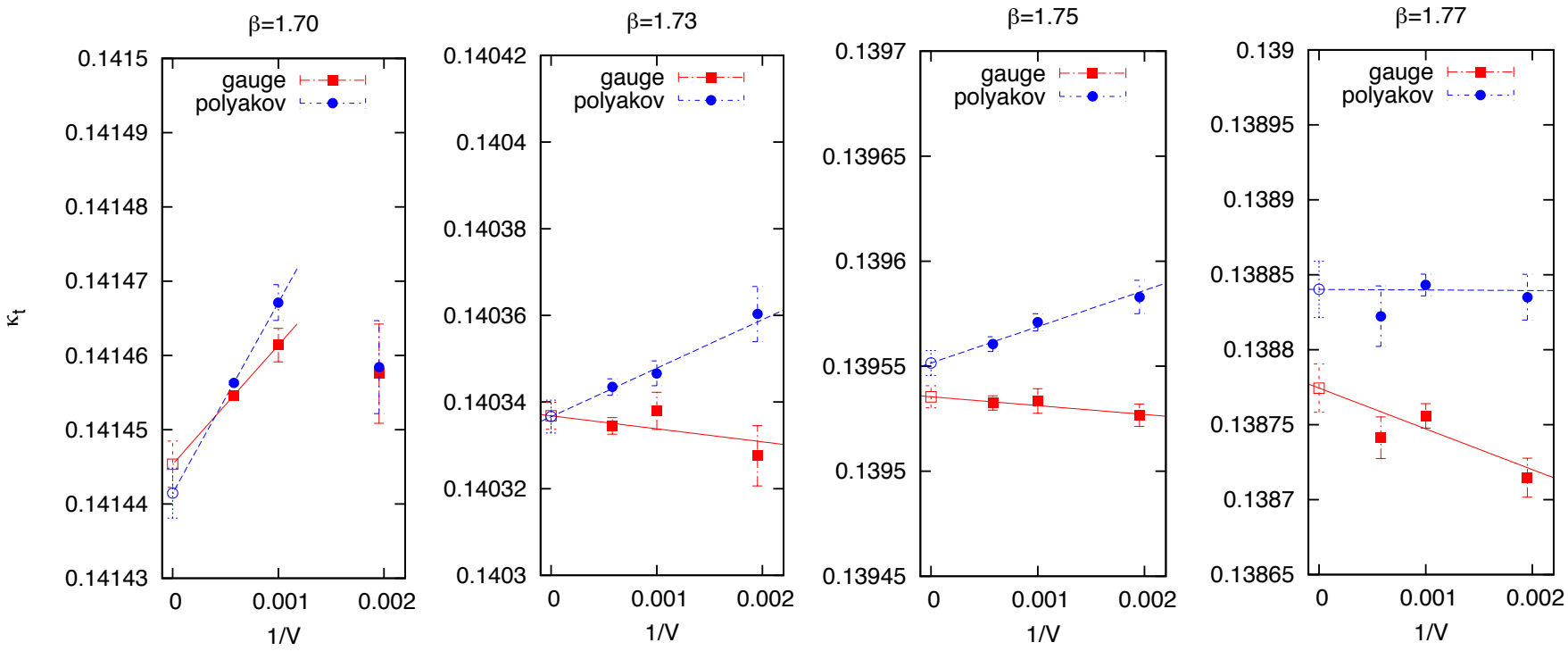


Polyakov loop susceptibility



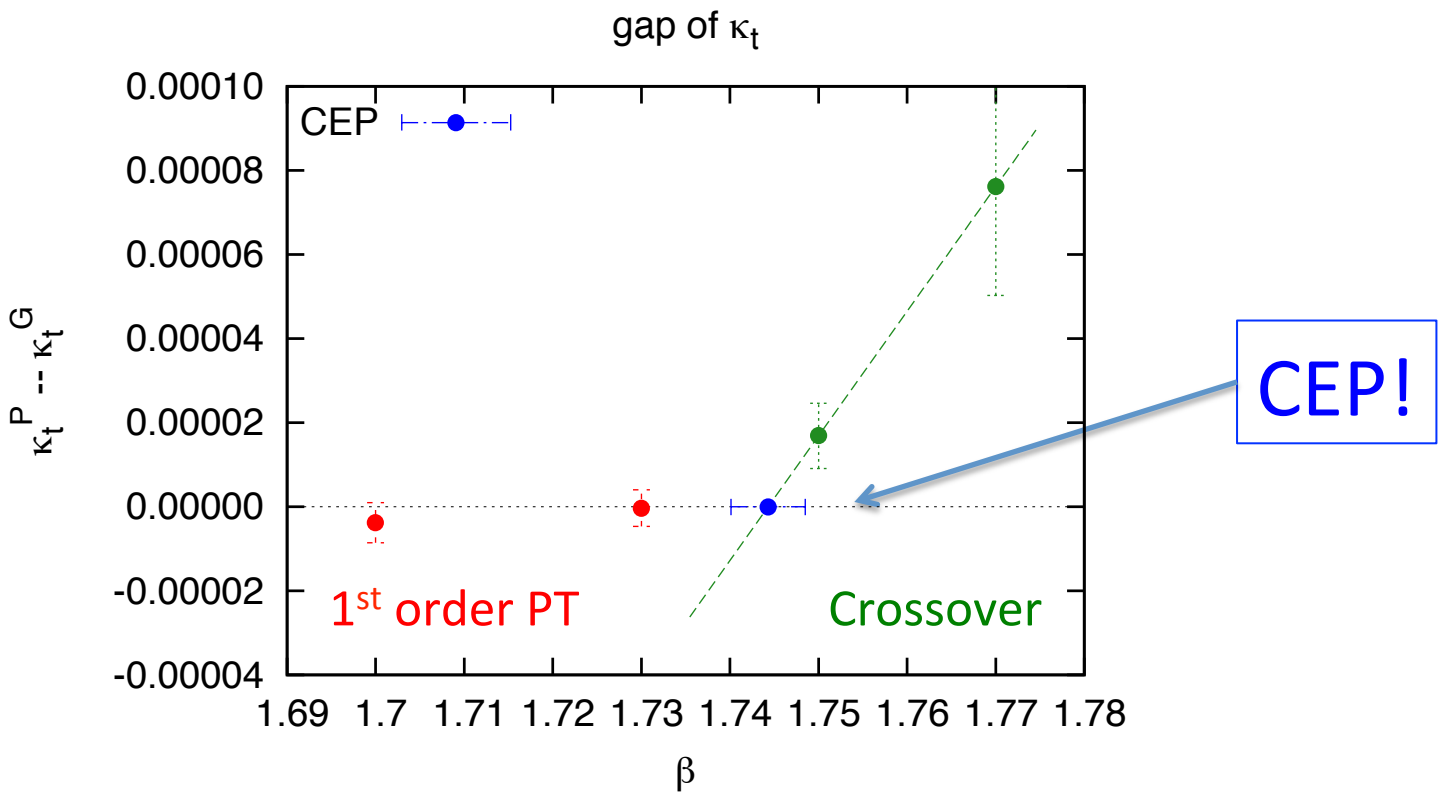


Gap of transition points



Gap is observed for larger β

Gap of transition point

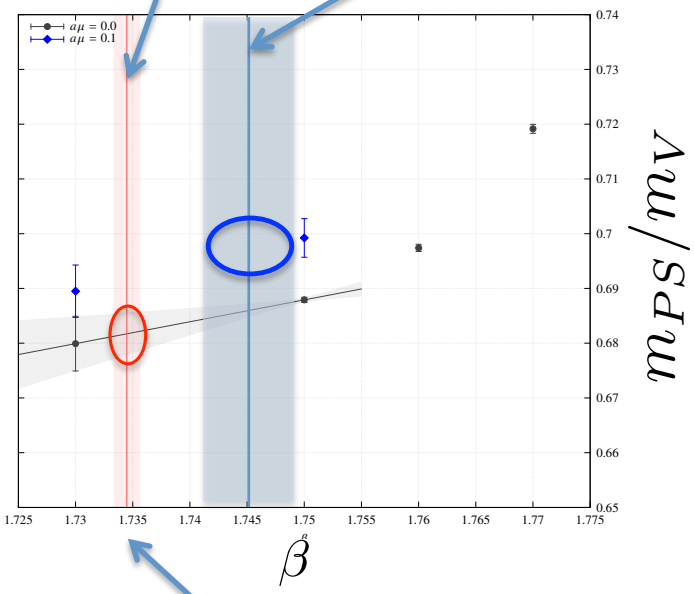


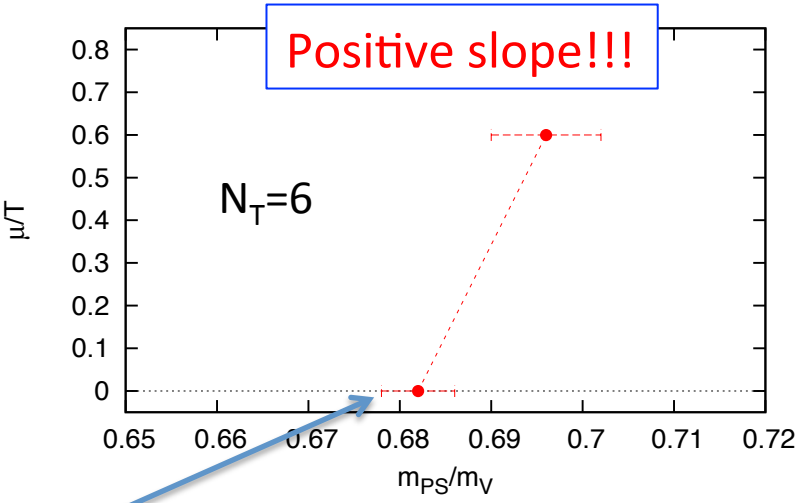
Need more statistics and β points in crossover side

Physical scale & Slope

CEP for aμ=0

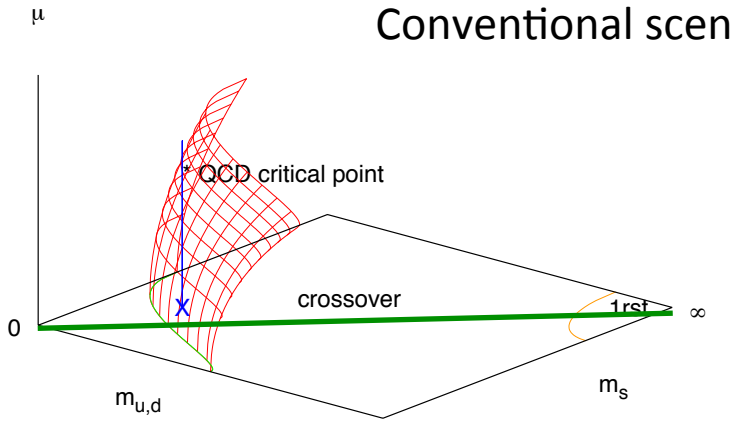
CEP for aμ=0.1





Nakamura-san's talk

Conventional scenario

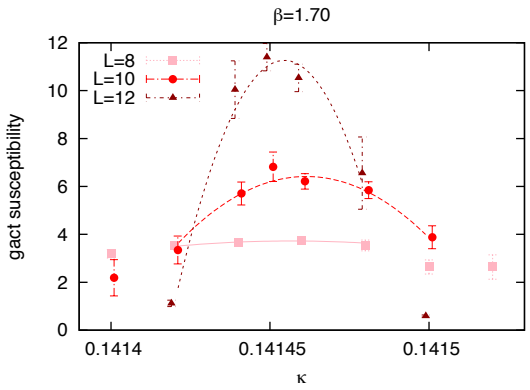


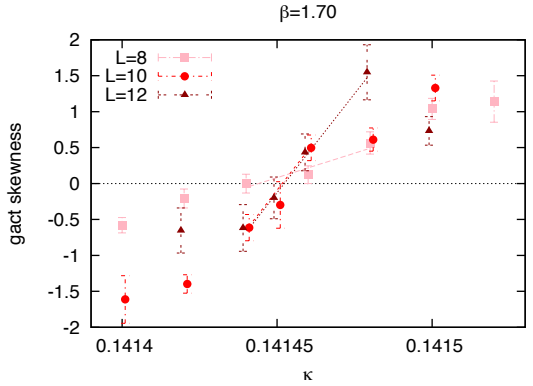
Summary & Outlook

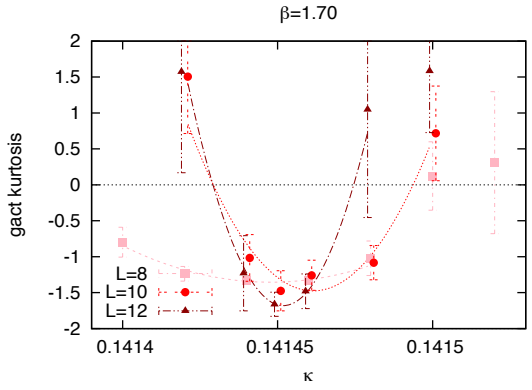
- Slope is consistent with conventional one
- Need more stat. and β points in X-over side
- Cross-check: Kurtosis intersection method
- Universality class of CEP?
- Continuum limit is not taken so far (N_T=6)
  - Challenging to increase N_T=8,10,…
- 2+1 flavor

BACKUP SLIDES

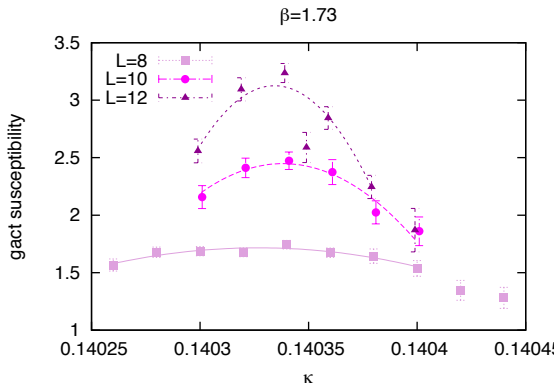
Moments of gauge action β=1.70

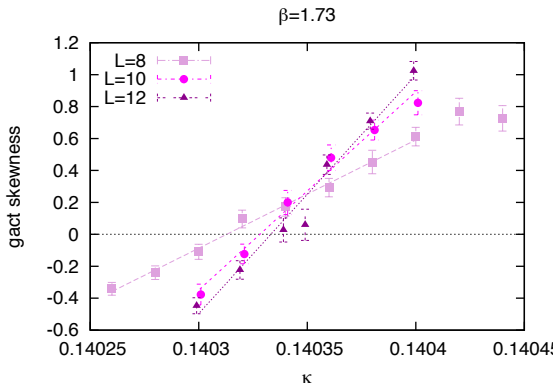


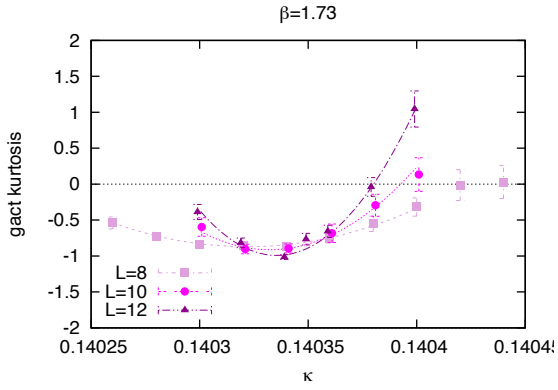




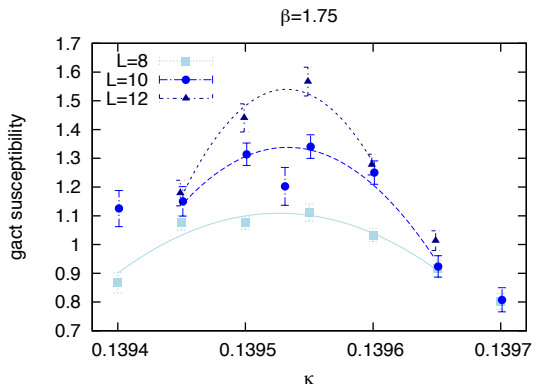
Moments of gauge action β=1.73

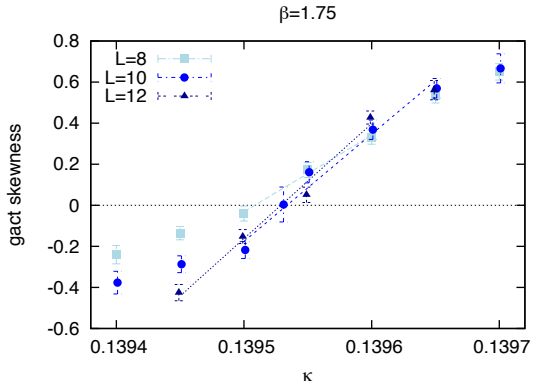


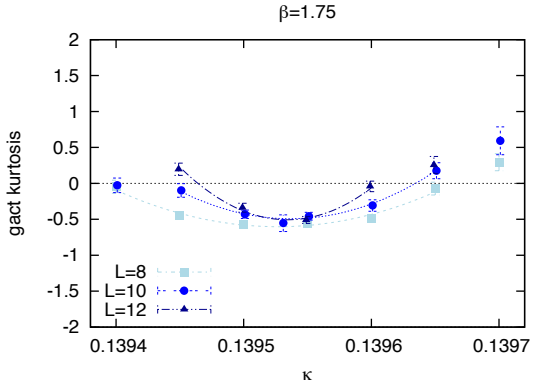




Moments of gauge action β=1.75







Moments of gauge action β=1.77