The critical endpoint of the finite temperature phase transition in three flavor QCD

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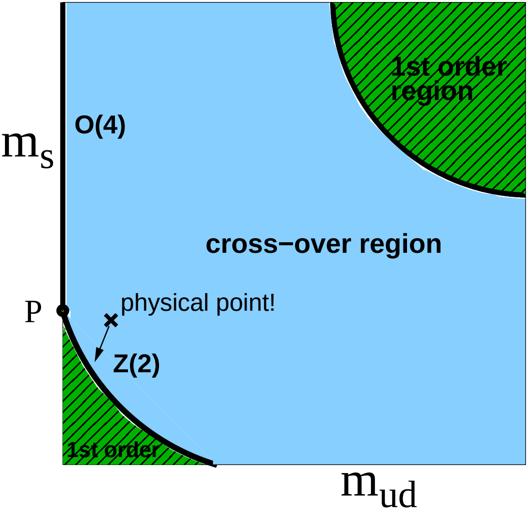
22 Jan. 2014, KEK

Columbia plot

- QCD phase transition for different quark masses at zero chemical potential



Critical endpoint(line) of $N_f = 3$ QCD

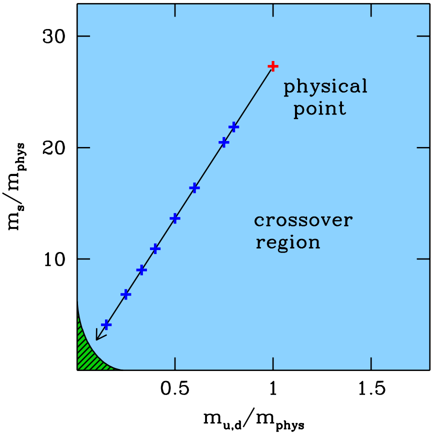
- staggered type: [de Forcrand, Philipsen ’07, Karsch, et. al. ’03, Endrődi, et. al. ’07, Ding, et. al. ’11]

N_t	action	m_π^E
4	unimproved	260 MeV
6	unimproved	150 MeV
4	p4-improved	70 MeV
6	stout-improved	$\lesssim$ 50 MeV
6	HISQ	$\lesssim$ 45 MeV

- m_π^E decreases with decreasing lattice spacing

Critical endpoint(line) of $N_f = 3$ QCD

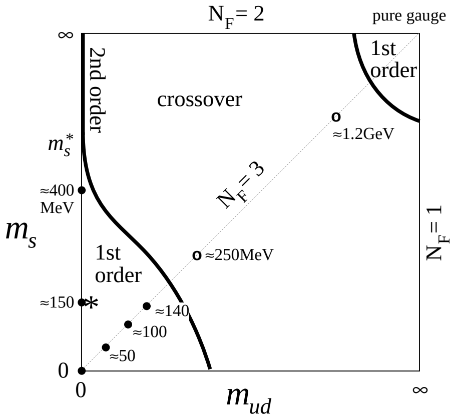
- the crossover may persist down to $\sim \mathbf{0.1}m^{phy}$



[Endrődi, et. at. '07]

Critical endpoint(line) of $N_f = 3$ QCD

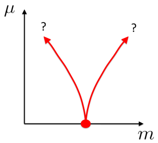
- Wilson type: [Iwasaki, et. al. '96], unimproved, $N_t = 4$



- 1st order at rather heavy m_q

Motivation

- Critical endpoint obtained with staggered and Wilson type fermios is inconsistent
- Results in the continuum limit is necessary and $N_f = 3$ study is a stepping stone
  - the order of phase transition around the physical point
  - curvature of critical surface
    → Takeda-san's talk



We determine the critical endpoint on $m_l = m_s$ line with clover fermions

Simulation parameters

- action: Iwasaki gluon + $N_f = 3$ clover
  (non perturbative c_{SW}, degenerate)
- temporal lattice size $N_t = 4, 6, 8$ for continuum extrapolation
- 3 spatial lattice sizes and a couple of β for each N_t to determine the critical endpoint by using intersection points of the Binder cumulants (kurtosis)
  - at $N_t = 4$, $N_l = 6, 8, 10$, $\beta = 1.60 - 1.70$
  - at $N_t = 6$, $N_l = 10, 12, 16$, $\beta = 1.73 - 1.77$
  - at $N_t = 8$, $N_l = 12, 16, 20, 24$, $\beta = 1.73 - 1.78$
- statistics: O(10,000) - O(200,000) traj.

Observables

plaquette

$$P = \frac{1}{3} \mathrm{Re}\,\mathrm{Tr}\, U_{\Box}$$

gauge action density

$$s_g = c_0(1 - P) + 2c_1(1 - R)$$

Polyakov loop

$$L = \frac{1}{N_l^3} \sum_{\vec{x}} L(\vec{x}), \; L(\vec{x}) = \frac{1}{3} \mathrm{Tr} \prod_{x_4=1}^{N_t} U_4(x)$$

Higher moments(1/2)

i-th derivative of weight with respect to control parameter c:

$$Q_i = \frac{\partial^i}{\partial c^i} e^{-S_g} (\det D)^{N_f}$$

- Variance

$$V = \frac{\partial^2 \ln Z}{\partial c^2} = \langle Q_2 \rangle - \langle Q_1 \rangle^2$$

Higher moments(2/2)

- Skewness

e.g. right-skewed → $S > 0$, left-skewed → $S < 0$

$$S = \frac{1}{V^{\frac{3}{2}}} \frac{\partial^3 \ln Z}{\partial c^3} = \frac{\langle Q_3 \rangle - 3\langle Q_2 \rangle \langle Q_1 \rangle + 2\langle Q_1 \rangle^3}{V^{\frac{3}{2}}}$$

- Kurtosis

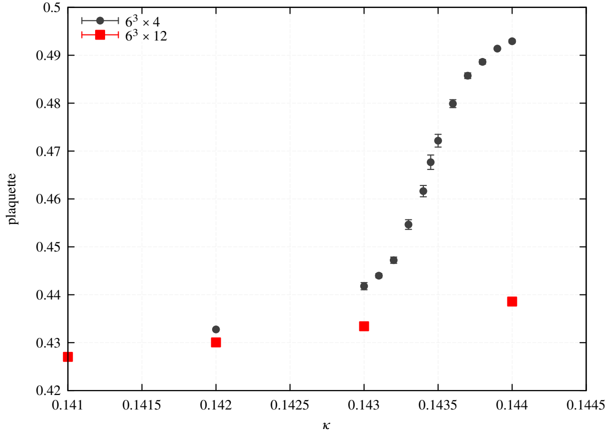
e.g. Gaussian → $K = 0$, uniform → $K = -1$, 2δ func. → $K = -2$

$$K = \frac{1}{V^2} \frac{\partial^4 \ln Z}{\partial c^4}$$

$$= \frac{\langle Q_4 \rangle - 4\langle Q_3 \rangle \langle Q_1 \rangle - 3\langle Q_2 \rangle^2 + 12\langle Q_2 \rangle \langle Q_1 \rangle^2 - 6\langle Q_1 \rangle^4}{V^2}$$

$$= B_4 - 3$$

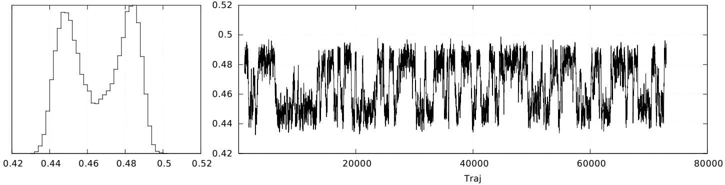
Finite temperature phase transition



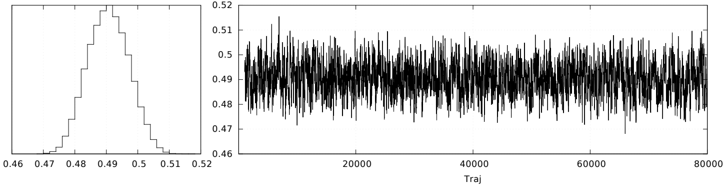
- Plaquette v.s. κ at lowest β $(= 1.60)$
- no bulk phase transition

1st order phase transition and crossover (like)

$\beta = 1.60$ and $\kappa = 0.14345$ on $10^3 \times 4$, clear two states, $K \sim -1.5$



$\beta = 1.70$ and $\kappa = 0.13860$ on $10^3 \times 4$, one state, $K \sim -0.5$



Distinguishing between 1st, 2nd and crossover

criterion	first order	second order	crossover
distribution	double peak	single peak	singe peak
χ_{peak}	$\propto N_l^d$	$\propto N_l^{\alpha/\nu}$	-
$\beta(\chi_{\text{peak}}) - \beta_c$	$\propto N_l^{-d}$	$\propto N_l^{-1/\nu}$	-
kurtosis at $N_l \to \infty$	K= -2	-2 < K < 0	K=0

problems....

- finite size scaling and computing kurtosis require high statistics

problem on weak 1st order phase transition

- scaling might work with wrong exponents
- peaks in histgram might emerge only at large N_l

N_l dependence of K

$$M = N_l^{-\beta/\nu} f_M(tN_l^{1/\nu})$$

- K does not depend on volume at a second order phase transition point

$$K + 3 = B_4(M) = \frac{N_l^{-4\beta/\nu} f_{M^4}(tN_l^{1/\nu})}{[N_l^{-2\beta/\nu} f_{M^2}(tN_l^{1/\nu})]^2} = f_B(tN_l^{1/\nu})$$

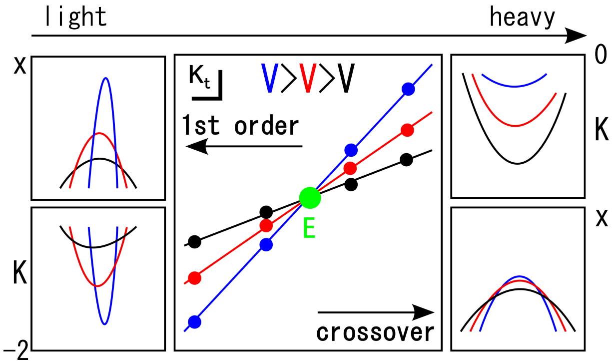
- At the first order phase transition point, for large volumes, K reaches the minimum [Billoire, et. at. '92]

$$K = -2 + \frac{c}{N_l^d} + O(1/N_l^{2d})$$

Our method

- determine the transition point by using a fit of around the peak of susceptibility (at $N_t = 8$ we also use β reweighting)
- determine kurtosis at transition point at each spatial lattice size
- find intersection point of kurtosis by fit(FSS inspired ansatz)

$$K_E + aN_l^{1/\nu}(\beta - \beta_E) + bN_l^{2/\nu}(\beta - \beta_E)^2$$



$$S(\beta, c_{\mathrm{SW}}) = \beta N_P(C - P) - N_f \ln \det D(c_{\mathrm{SW}})$$

where $C = c_0 + 2c_1$, $N_P = 6N_xN_yN_zN_t$,
$P = 1/3\mathrm{ReTr}(c_0W_{\mu\nu}^{1\times1} + 2c_1W_{\mu\nu}^{1\times2})$

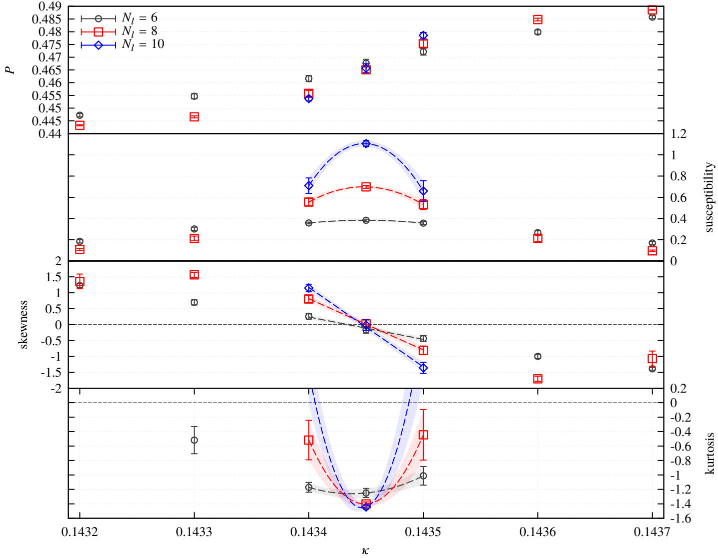
β reweighting

$$w(\beta') = \exp[(\beta - \beta')N_P(C - P)]$$

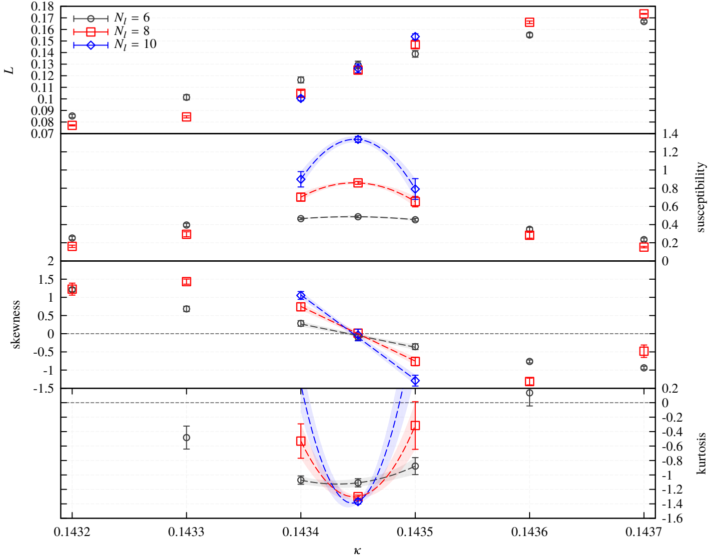
c_{SW} reweighting (Taylor expansion)

$$\ln \frac{\det D(c_{\mathrm{SW}})}{\det D(c_{\mathrm{SW0}})} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \mathrm{Tr}\left(D^{-1}\frac{\partial}{\partial c_{\mathrm{SW}}}D\right)^{n+1}(c_{\mathrm{SW}} - c_{\mathrm{SW0}})^{n+1}$$

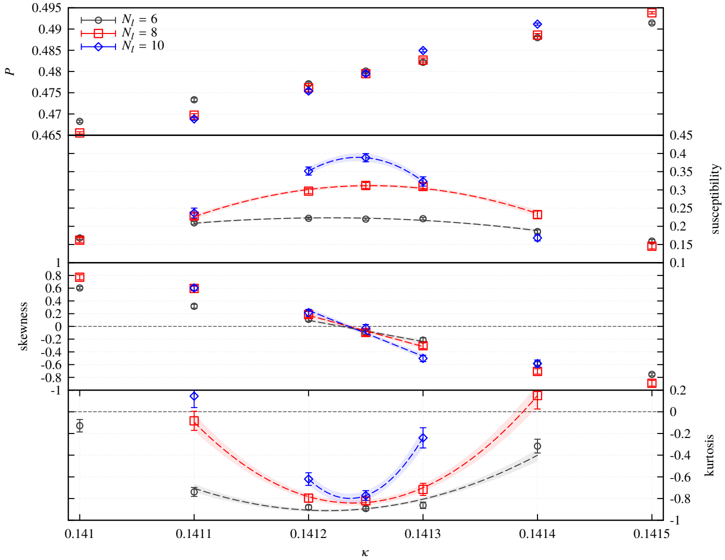
plaquette at $\beta = 1.60$, $N_t = 4$



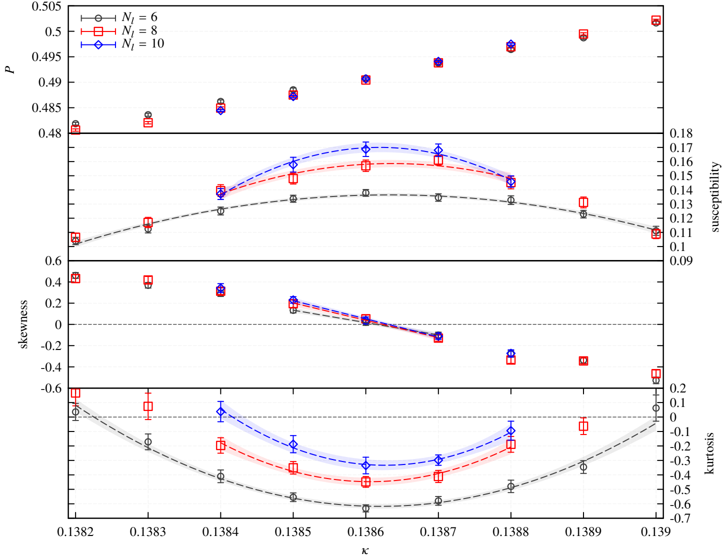
Polyakov loop at $\beta = 1.60$, $N_t = 4$



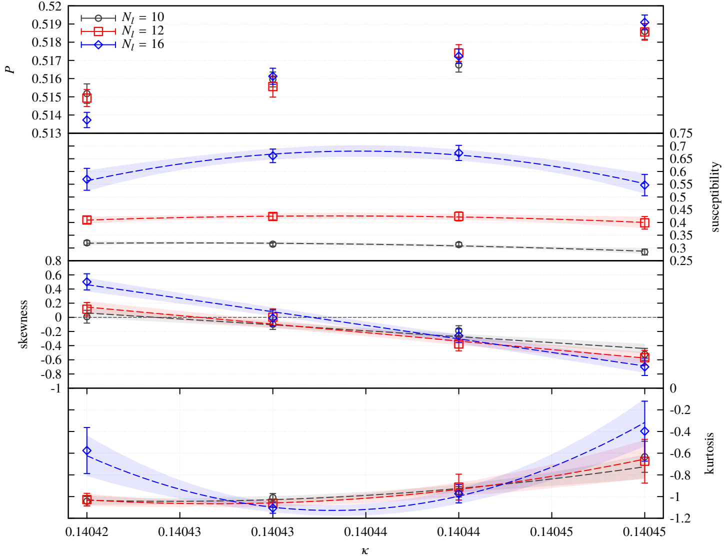
plaquette at $\beta = 1.65$, $N_t = 4$



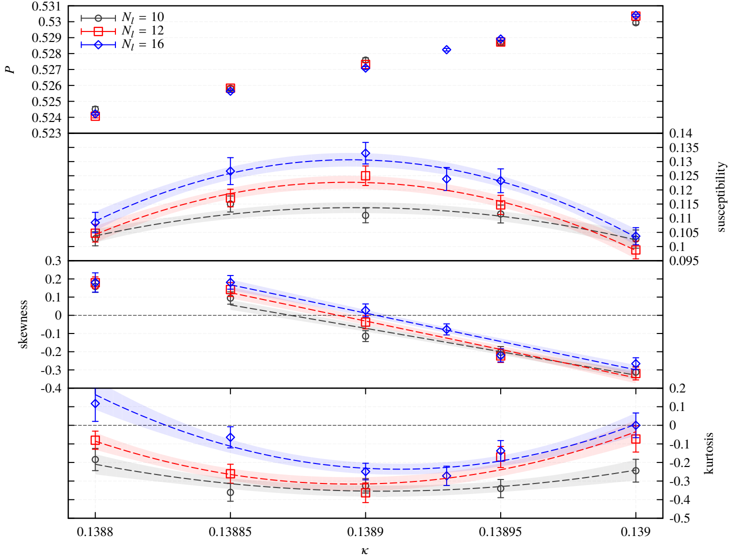
plaquette at $\beta = 1.70$, $N_t = 4$



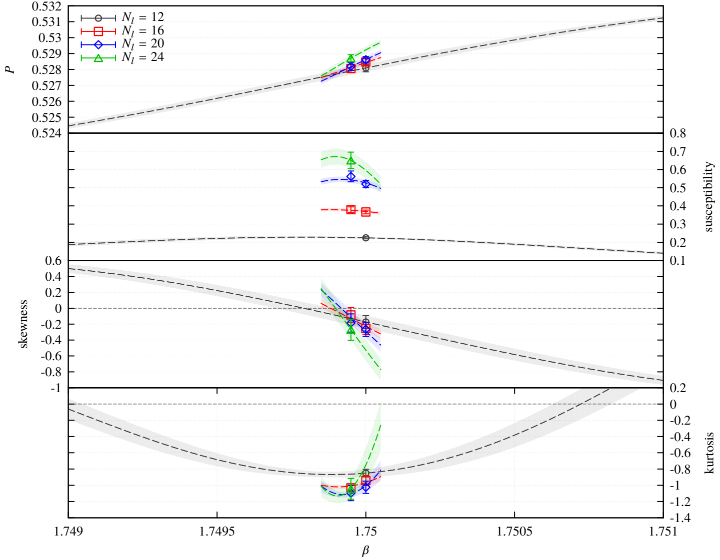
plaquette at $\beta = 1.73$, $N_t = 6$



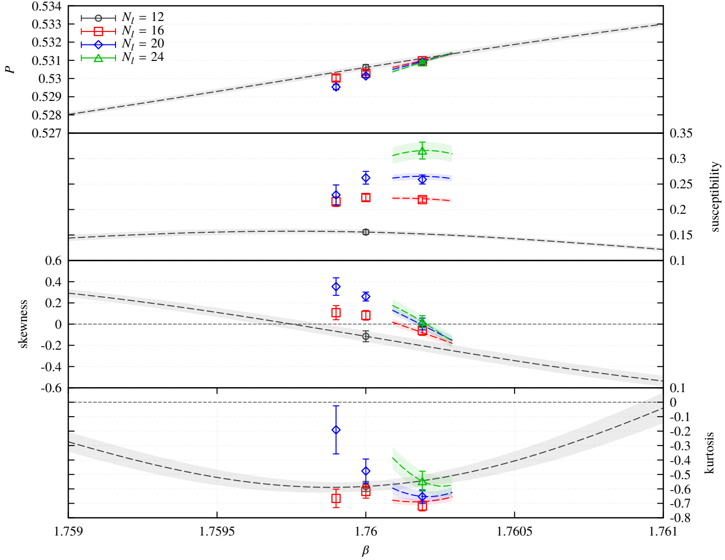
plaquette at $\beta = 1.77$, $N_t = 6$



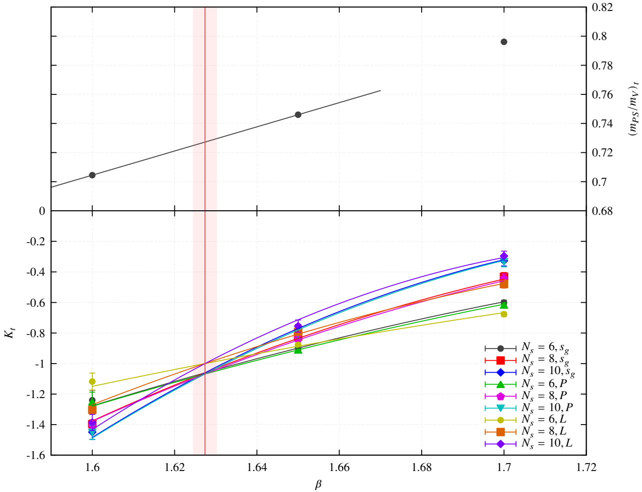
plaquette at $\kappa = 0.14024$, $N_t = 8$



plaquette at $\kappa = 0.13995$, $N_t = 8$

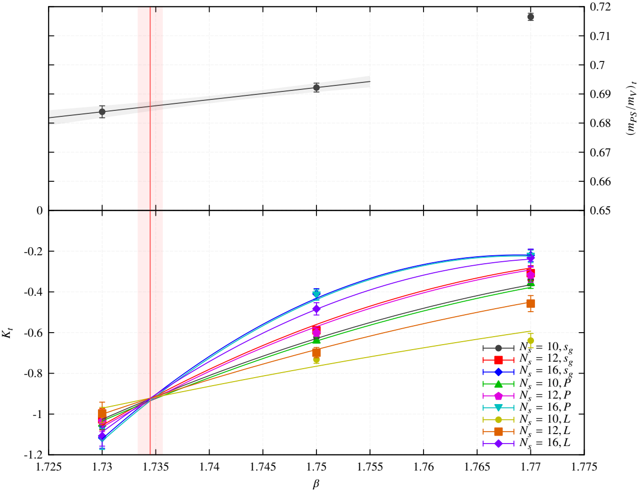


Critical endpoint at $N_t = 4$

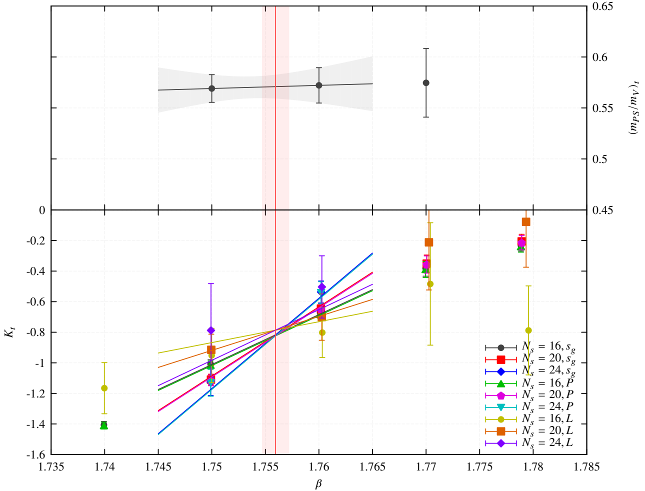


- $(m_{PS}/m_V)_t$ is the interpolated value to κ_t

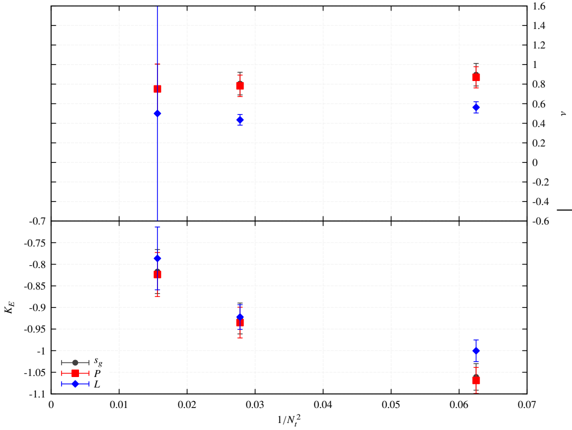
Critical endpoint at $N_t = 6$



Critical endpoint at $N_t = 8$



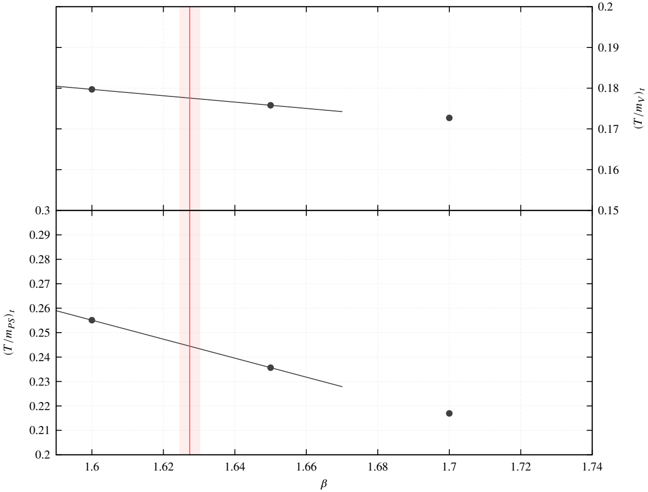
- need more statistics at $N_t = 8$



class	B_4	ν
Ising	1.604	0.63
O(2)	1.242	0.67
O(4)	1.092	0.75

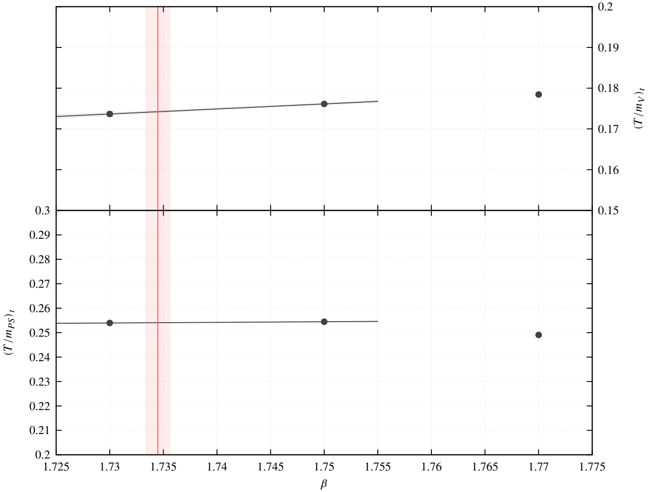
K_E and ν are not consistent with values of any classes

T_E at $N_t = 4$

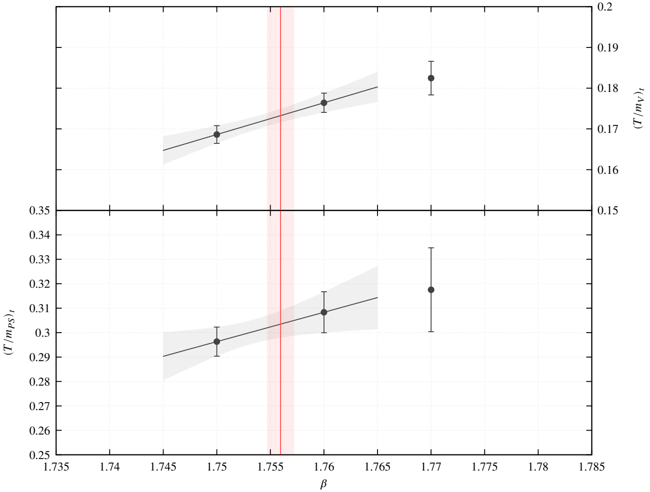


0.2
0.19
0.18
0.17
0.16
0.15
$(T/m_V)_t$
0.3
0.29
0.28
0.27
0.26
0.25
0.24
0.23
0.22
0.21
0.2
$(T/m_{PS})_t$
1.6
1.62
1.64
1.66
1.68
1.7
1.72
1.74
β

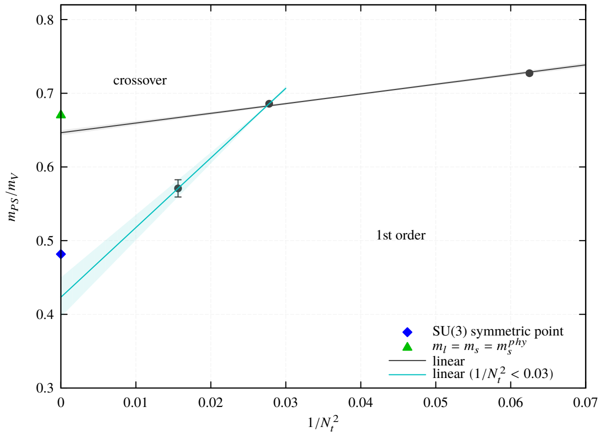
T_E at $N_t = 6$



T_{E} at $N_t = 8$



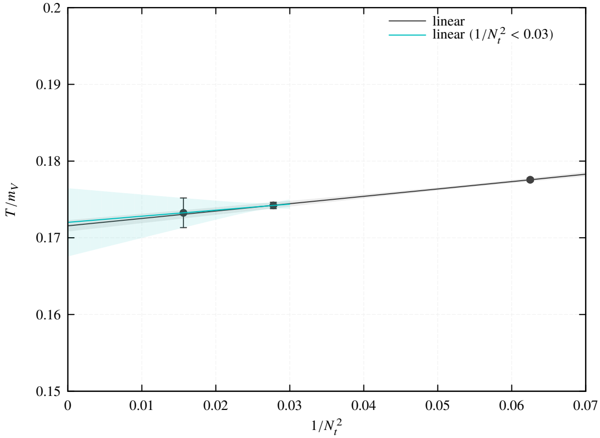
continuum extrapolation for $(m_{PS}/m_V)_E$



◆ : $m_{PS}^{phy;sym}/m_V^{phy;sym} = \sqrt{(m_\pi^2 + 2m_K^2)/3}/[(m_\rho + 2m_{K^*})/3] \sim 0.4817$

▲ : $m_{\eta_{ss}}/m_\phi \sim 0.6719$

continuum extrapolation for $(T/m_V)_E$



Summary

- We have investigated the critical endpoint of QCD with clover fermions and determined the critical endpoint by using the intersection points of the Binder cumulants at $N_t = 4, 6, 8$ and extrapolated to the continuum limit
- the critical endpoint at rather larger quark mass than results with staggered fermions
- continuum extrapolation : bad scaling, β is too small at $N_t = 4$
- future
  - more statistics
  - larger N_l/N_t
  - $N_t > 8$
  - chiral condensate
  - $m_l \neq m_s$