Towards Understanding QCD Phase Diagram Lattice and RHIC Experiments

Atsushi Nakamura
in Collaboration with K.Nagata

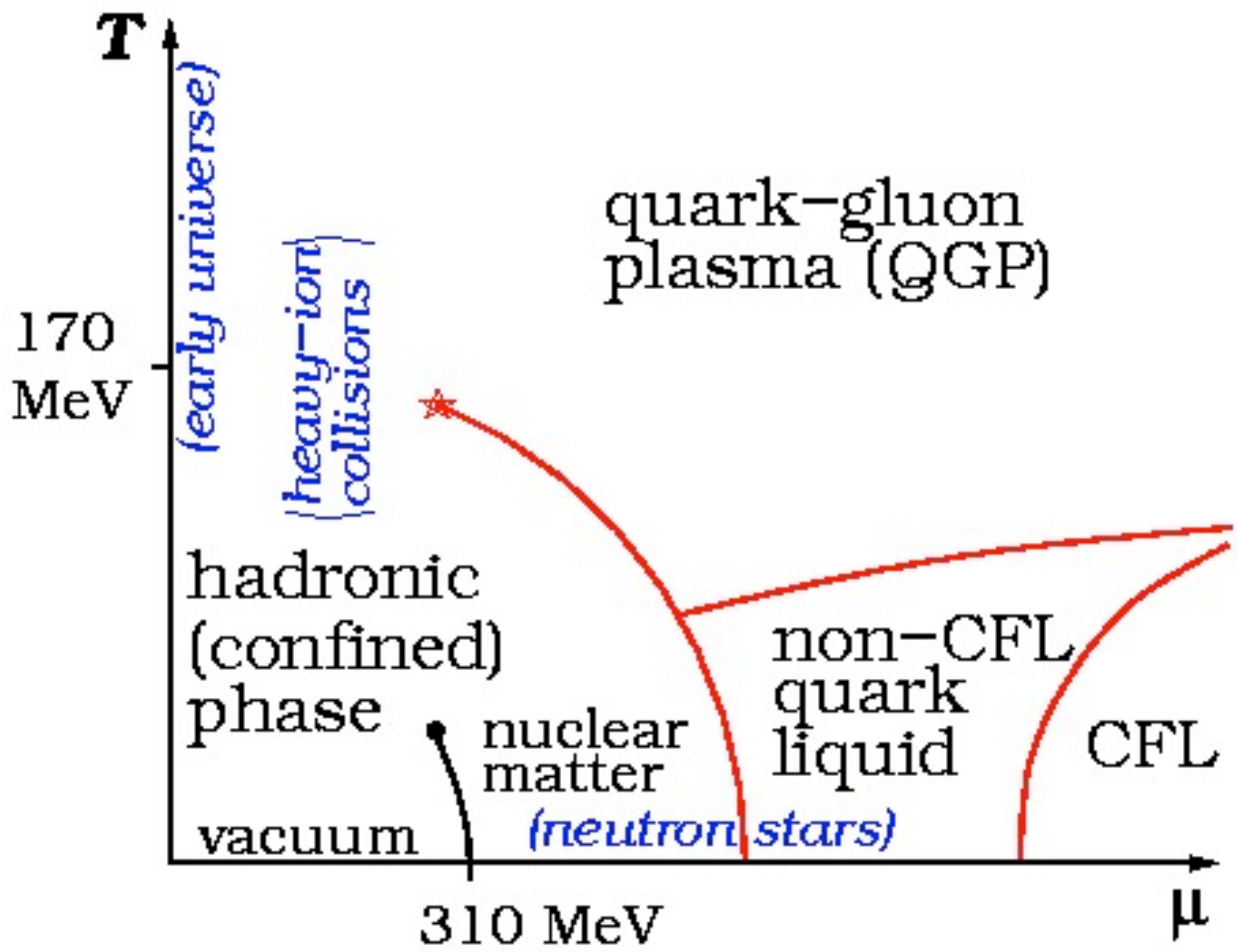
Lattice QCD at finite temperature and density
20 Jan. 2014 KEK

http://www.slideshare.net/atsushinakamura3/kek2014-v2

QCD Phase Diagram

Very Exciting !
Now it's time for
Lattice QCD.

But how do
you reveal it?
Please
No Sales-Talk !



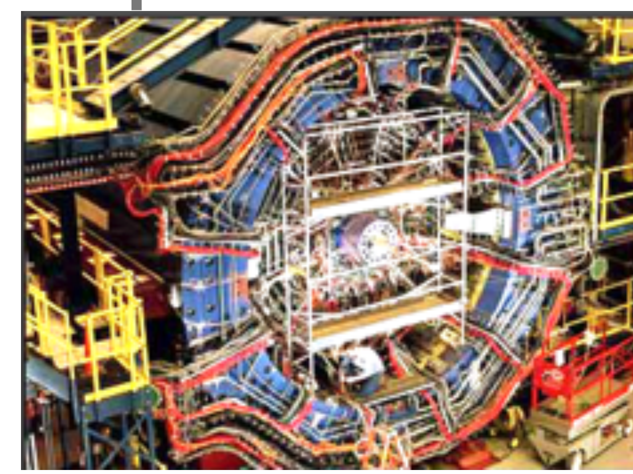
From Wiki-Pedia "QCD matter"

Canonical Partition Functions

Lattice QCD
Simulations

Z_N

Experiments



A few years ago,

$$Z(\mu) = \int DU \det \Delta(\mu) e^{-S_G}$$

- Nagata and I were looking for a Reduction formula for Wilson fermions in a finite density QCD project:

$$\det \boxed{\tilde{\Delta}} = \det \boxed{\Delta}$$

Reduction Matrix　　　　Original Fermion Matrix

$$\mathrm{Rank}(\det \tilde{\Delta}) < \mathrm{Rank}\,(\det \Delta)$$

Nagata and Nakamura

Phys. Rev. D82 094027 (arXiv:1009.2149)

- Obtained formula has the form of the fugacity expansion

$$Z(\mu) = \int DU \sum_n c_n \xi^n e^{-S_G}$$

$$= \sum_n Z_n \xi^n$$

$$\xi \equiv e^{\mu/T}$$ Fugacity

Z_n : Canonical Partition Function

$$Z(\mu, T) \Longleftrightarrow Z_n(T)$$

$$Z(\mu, T) = \mathrm{Tr}\, e^{-(H-\mu\hat{N})/T}$$

$$= \sum_n \langle n| e^{-(H-\mu\hat{N})/T} |n\rangle$$

If $[H, \hat{N}] = 0$

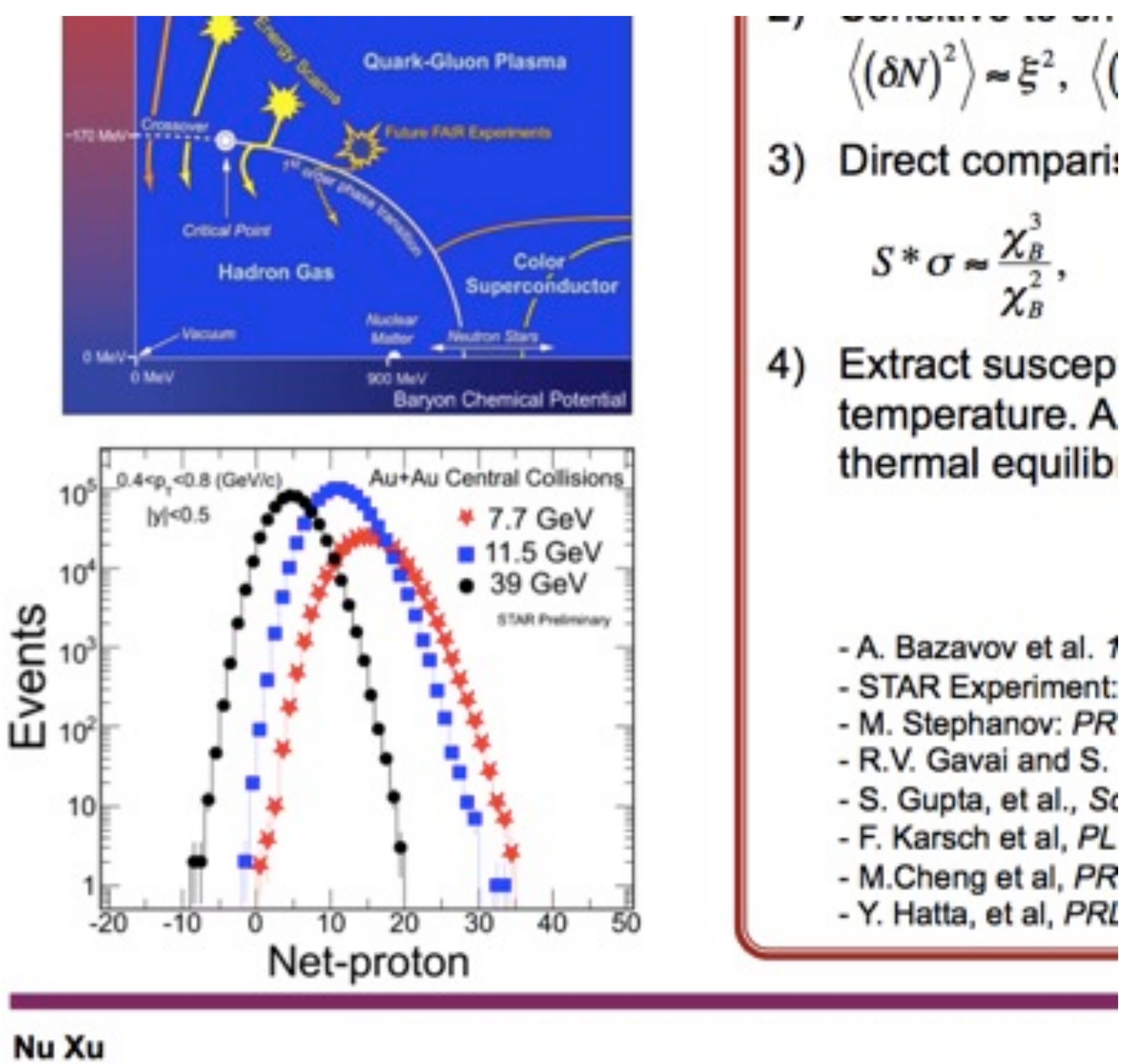
$$= \sum_n \boxed{\langle n| e^{-H/T} |n\rangle}\, e^{\mu n/T}$$

$$= \sum_n Z_n(T) \xi^n \qquad \left(\xi \equiv e^{\mu/T}\right)$$

Fugacity

RHIC
(Relativistic Heavy Ion Collider)

Multiplicity Distribution of RHIC



Wao,
Multiplicity !
Interesting !
It is almost Z_n

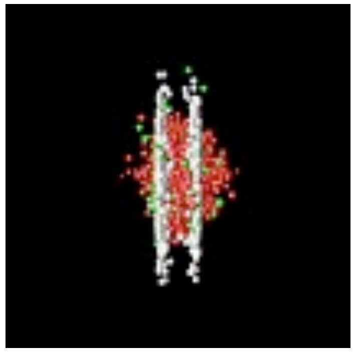
We assume

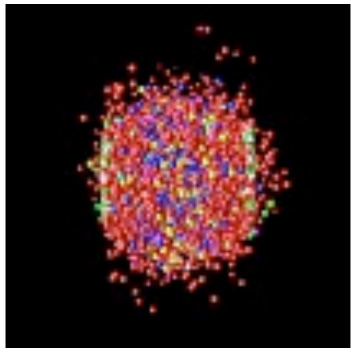
the Fireballs created in High Energy Nuclear Collisons are described as a Statistical System.

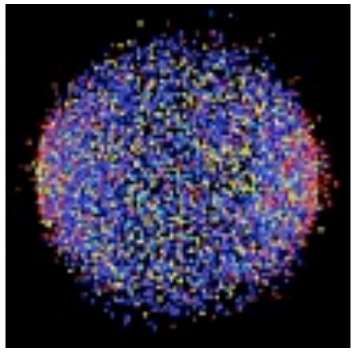
with μ (chemical Potential)
and T (Temperature)

$$Z(\mu,T)$$

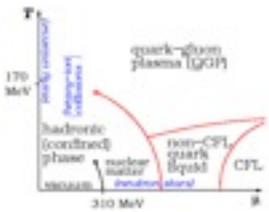
Grand Canonical partition function

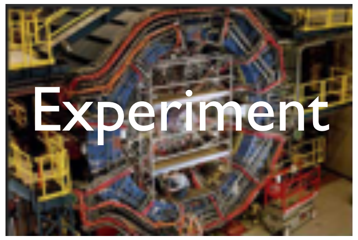






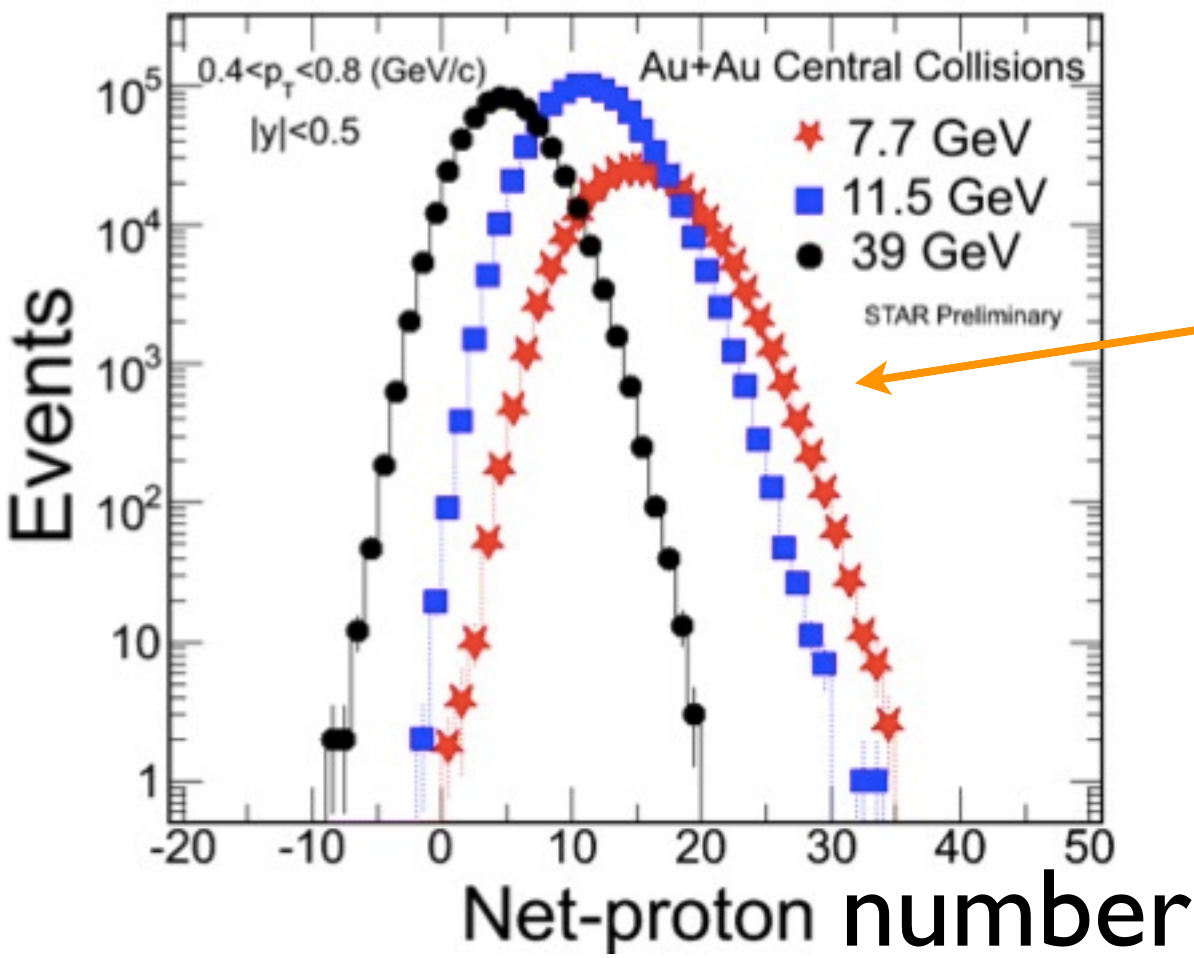
All QCD Phase Information is in $Z(\mu,T)$





$$Z(\xi, T) = \sum_n \boxed{Z_n(T)\,\xi^n}$$

Partition Function is
Sum of the Probabilities
with n ...
If I know ξ, then I have Zn.



How can we extract Z_n from $\boxed{P_n} = Z_n \xi^n$?

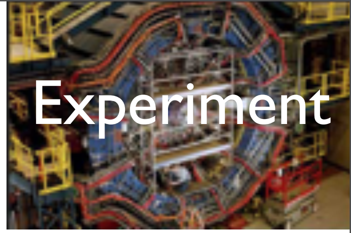
Observables in Experiments

$\boxed{\xi}$ unknown

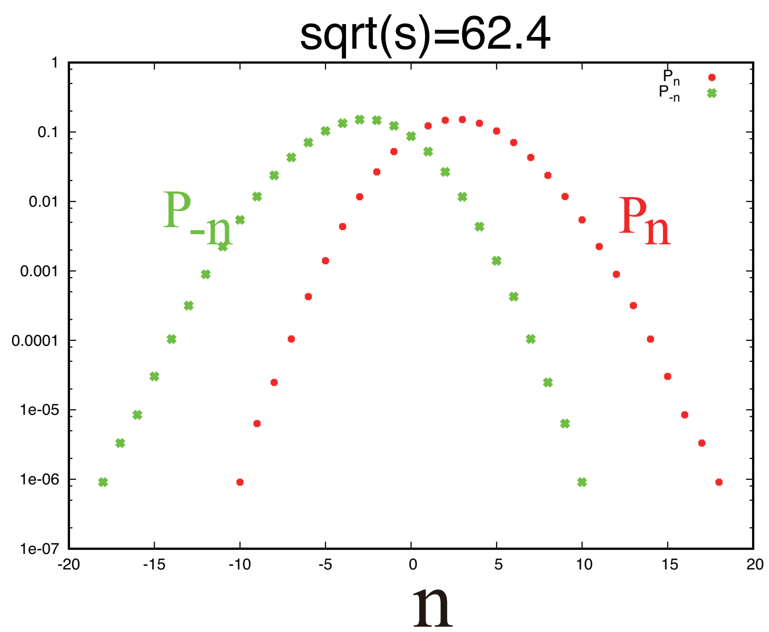
$$Z_n = P_n / \xi^n$$

We require (Particle-AntiParticle Symmetry)

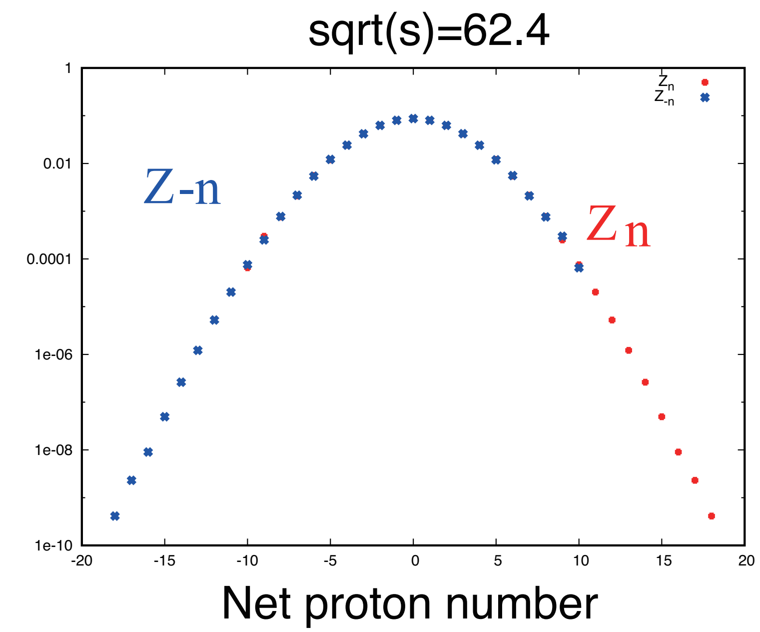
$$\boxed{Z_{+n} = Z_{-n}}$$



Demand $\boxed{Z_{+n} = Z_{-n}}$



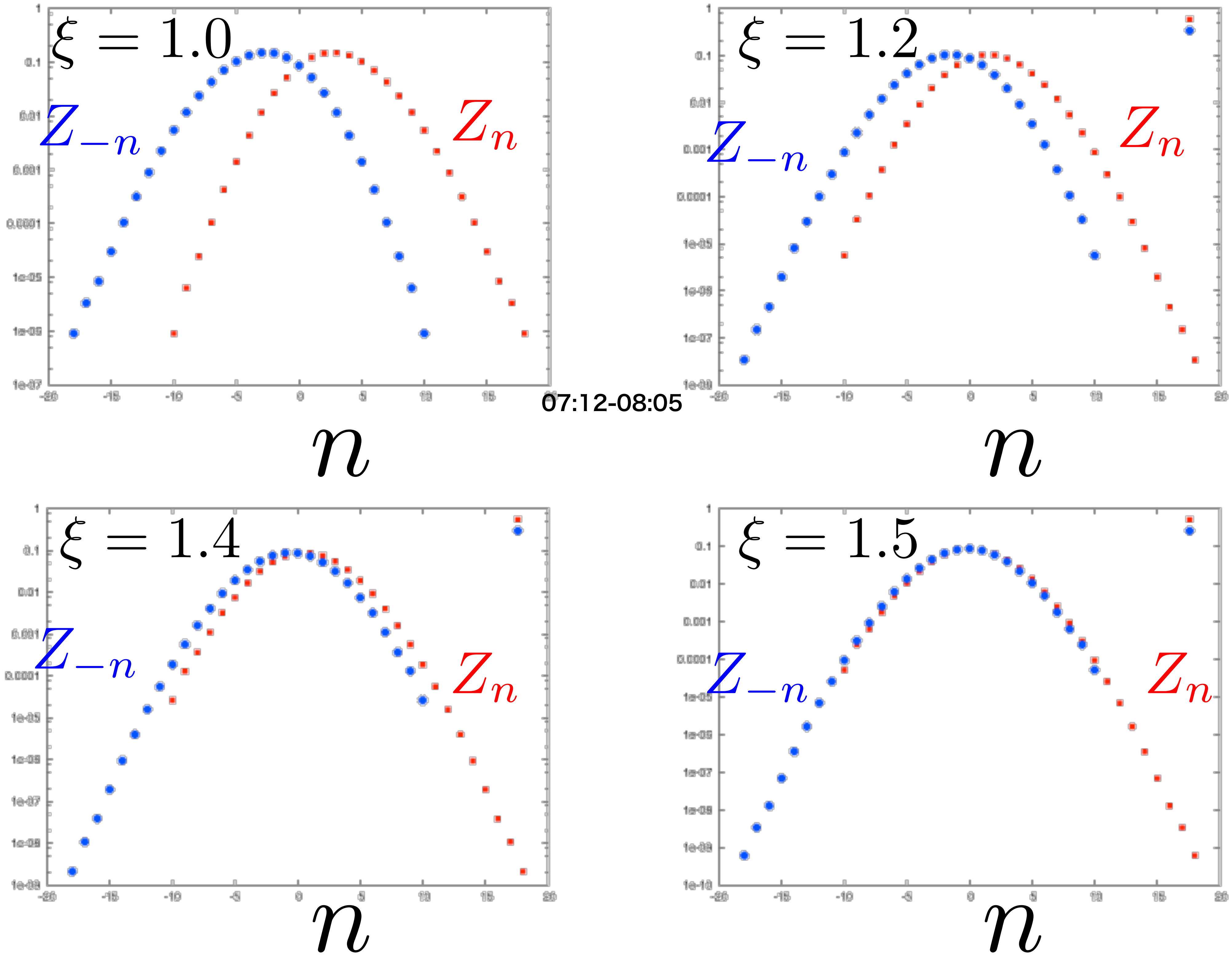
Fit ξ



$$Z_n = P_n/\xi^n$$

$$Z_n = \boxed{P_n} / \xi^n$$

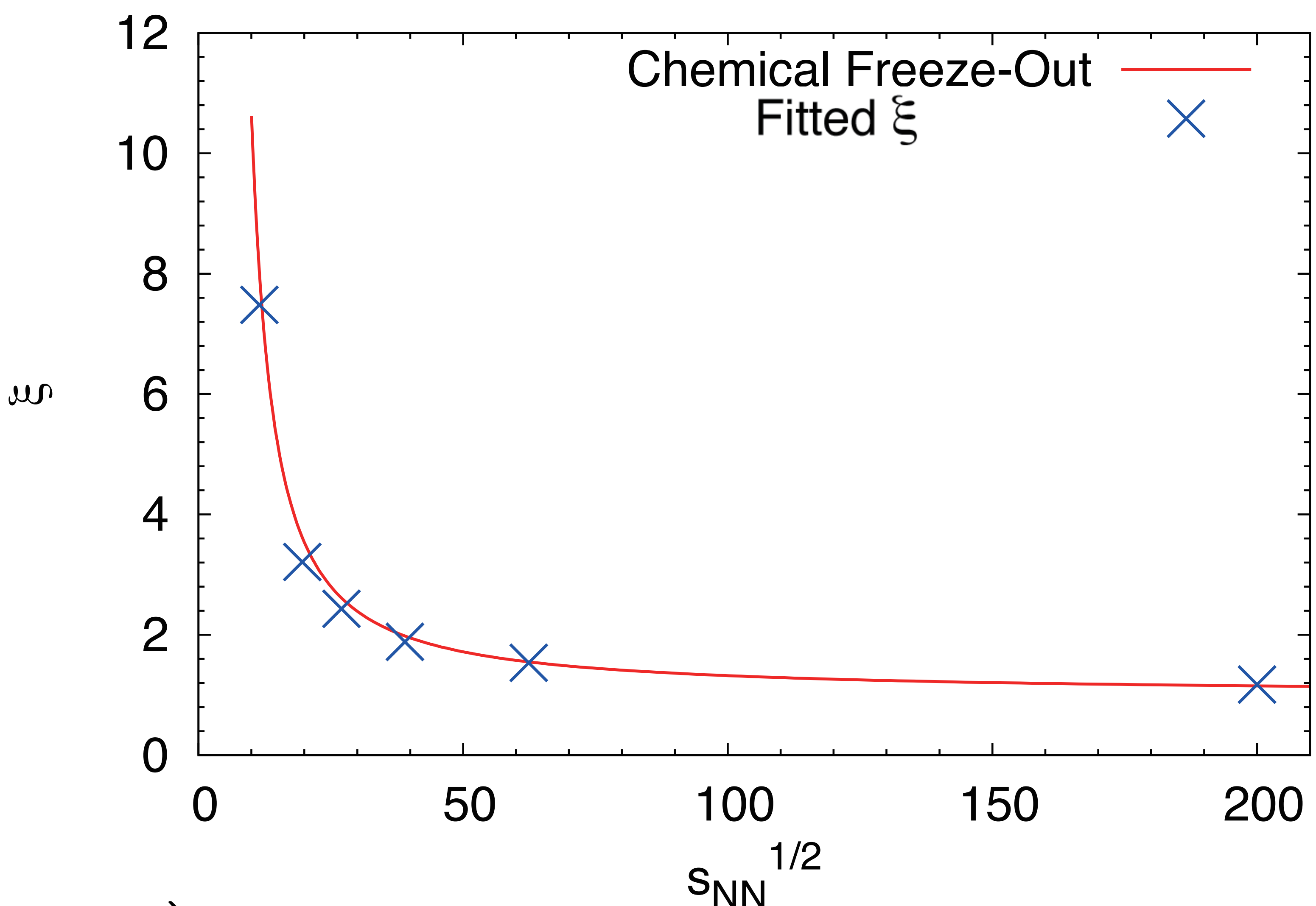
From Experiment



07:12-08:05

Final Value $\xi = 1.534$

Fitted ξ are very consistent with those by Freeze-out Analysis.

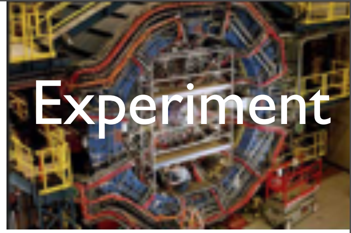


x This work

— Freeze-out

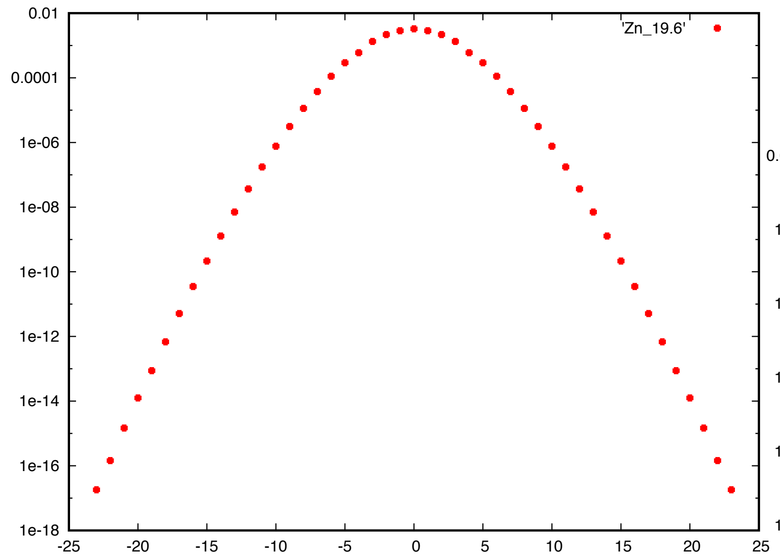
J.Cleymans, H.Oeschler, K.Redlich and S.Wheaton Phys. Rev. C73, 034905 (2006)

$$\left(\xi \equiv e^{\mu/T}\right)$$

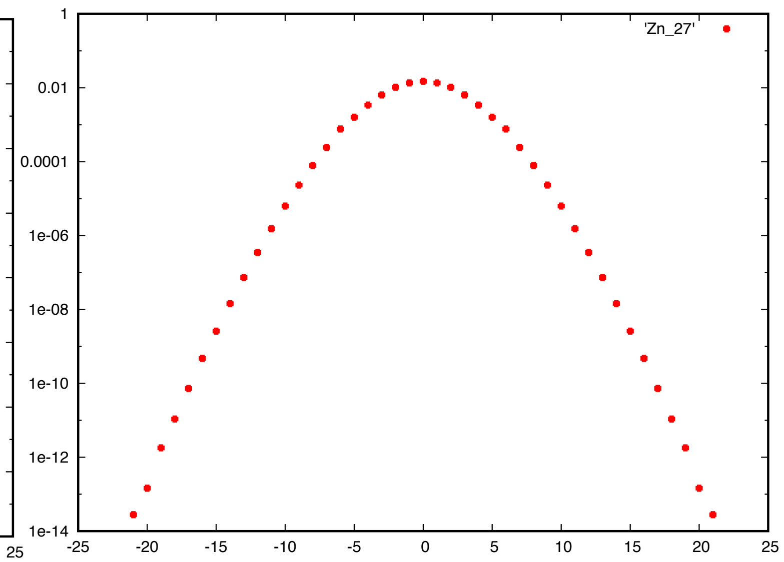


Z_n from RHIC data

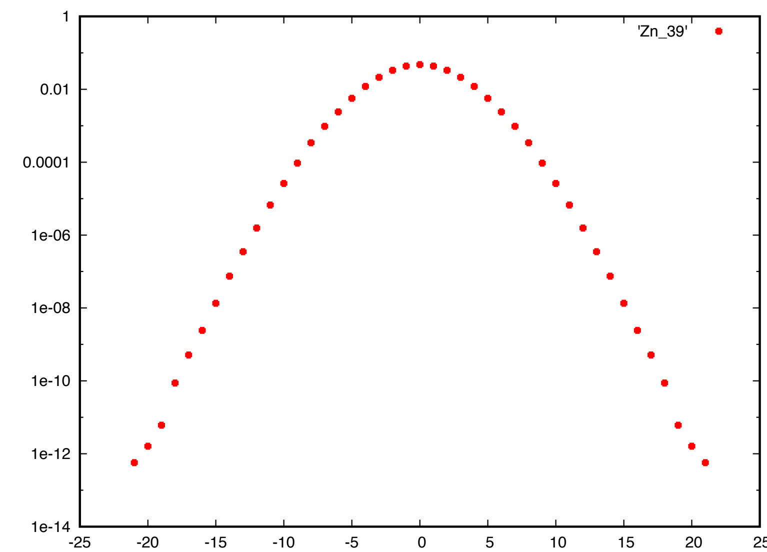
$\sqrt{s} = 19.6\mathrm{GeV}$



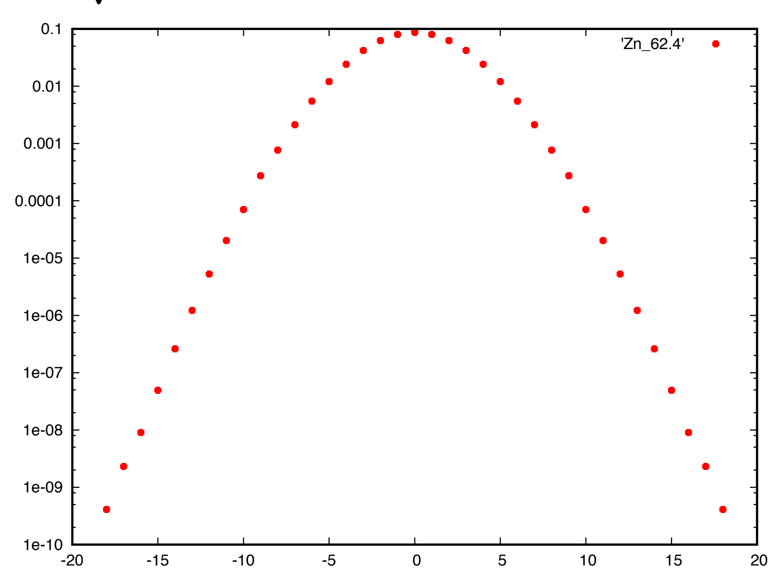
$\sqrt{s} = 27\mathrm{GeV}$



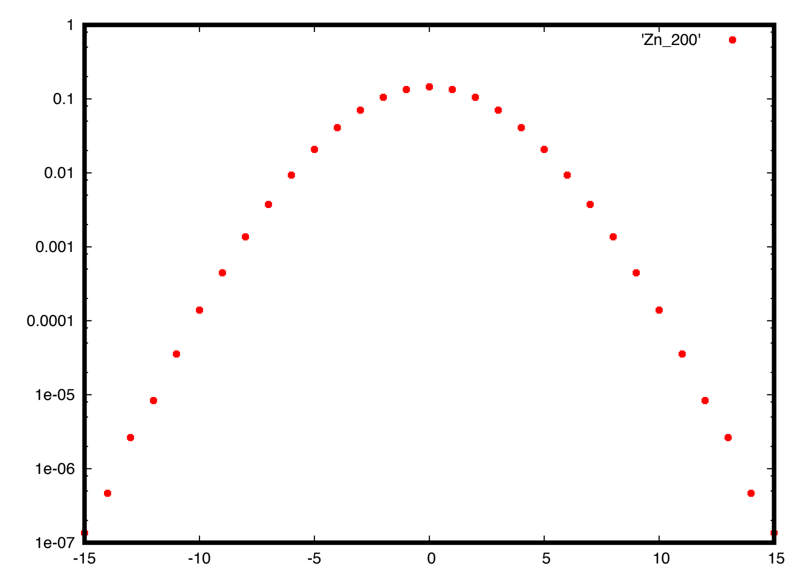
$\sqrt{s} = 39\mathrm{GeV}$



$\sqrt{s} = 62.4\mathrm{GeV}$

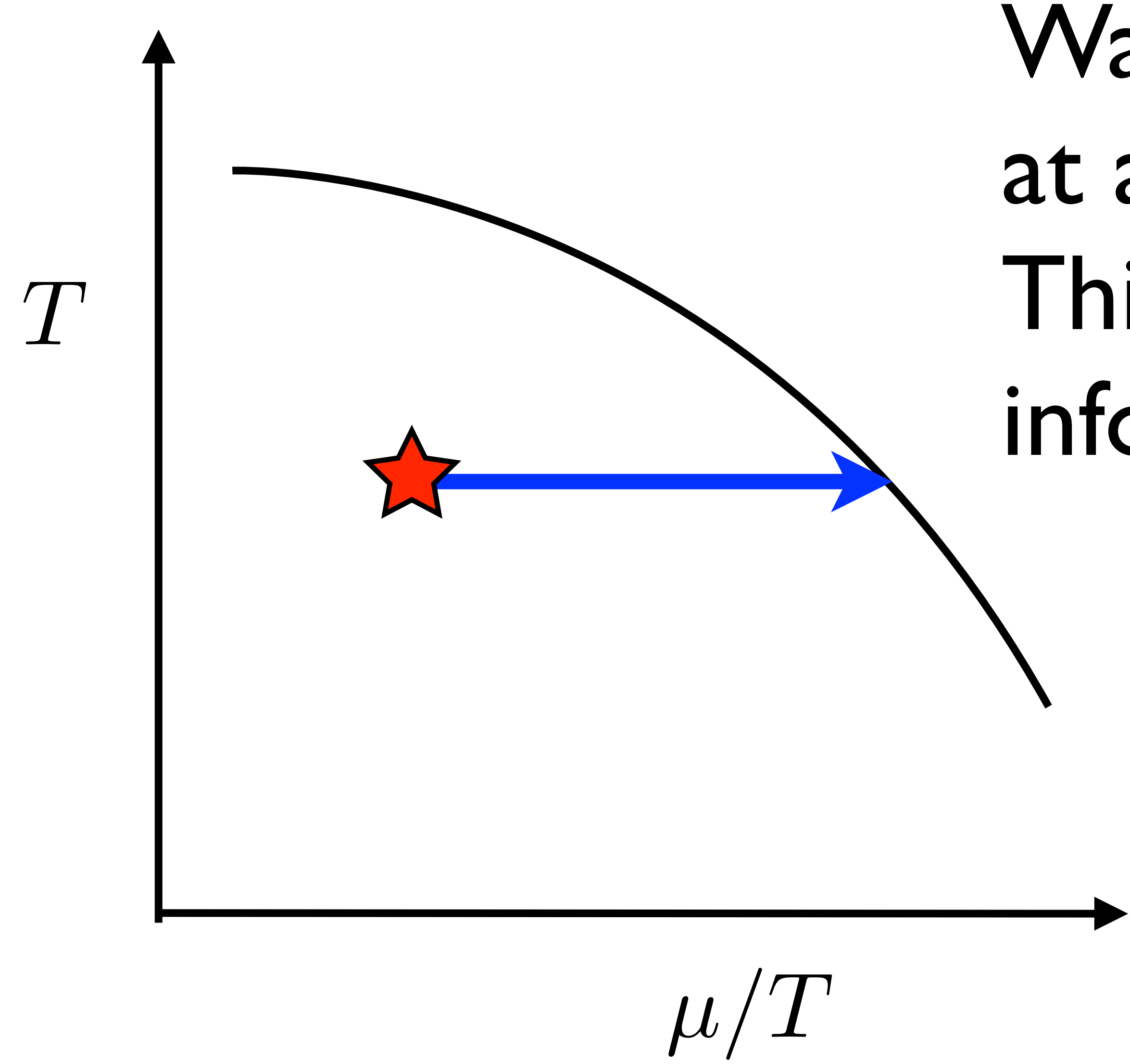


$\sqrt{s} = 200\mathrm{GeV}$



$$Z(\xi, T) = \sum_n Z_n(T)\, \xi^n$$

Now we have Zn of RHIC data
(sqrt(s)= 10.5,19.6, 27, 39, 62.4, 200 GeV)

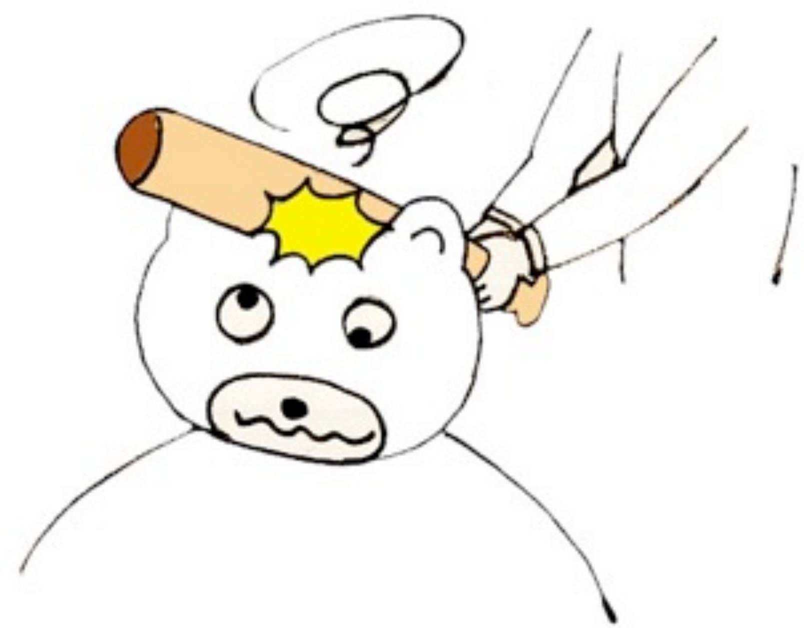


Wao ! We can calculate
at any density !
This includes all QCD Phase
information !

$(\xi \equiv e^{\mu/T})$

$$Z(\xi, T) = \sum_{n=\boxed{-N_{max}}}^{\boxed{+N_{max}}} Z_n(T)\, \xi^n$$

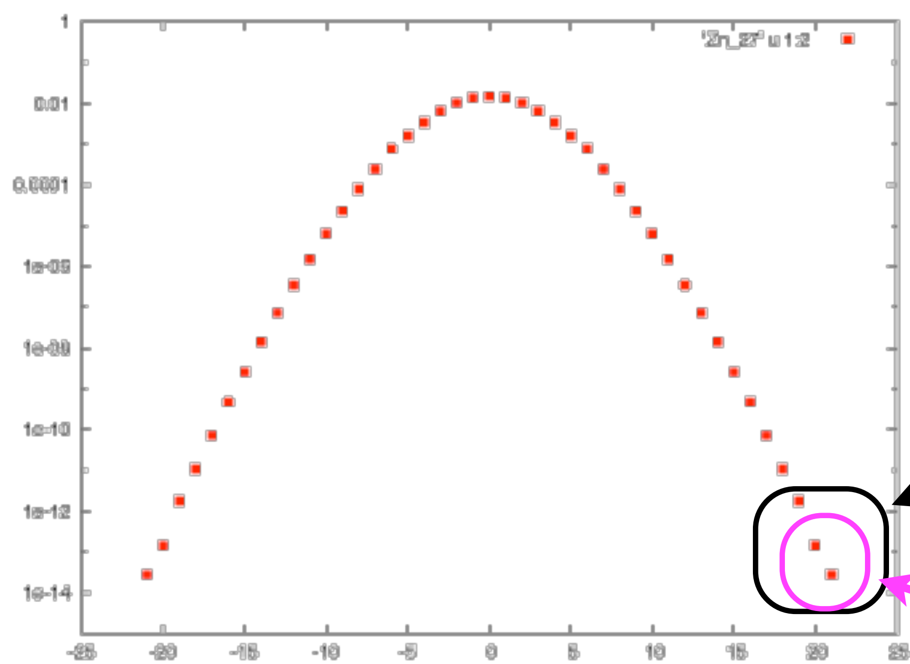
Do not forget that your *n* is finite !



Moments λ_k

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\,\xi^n$$

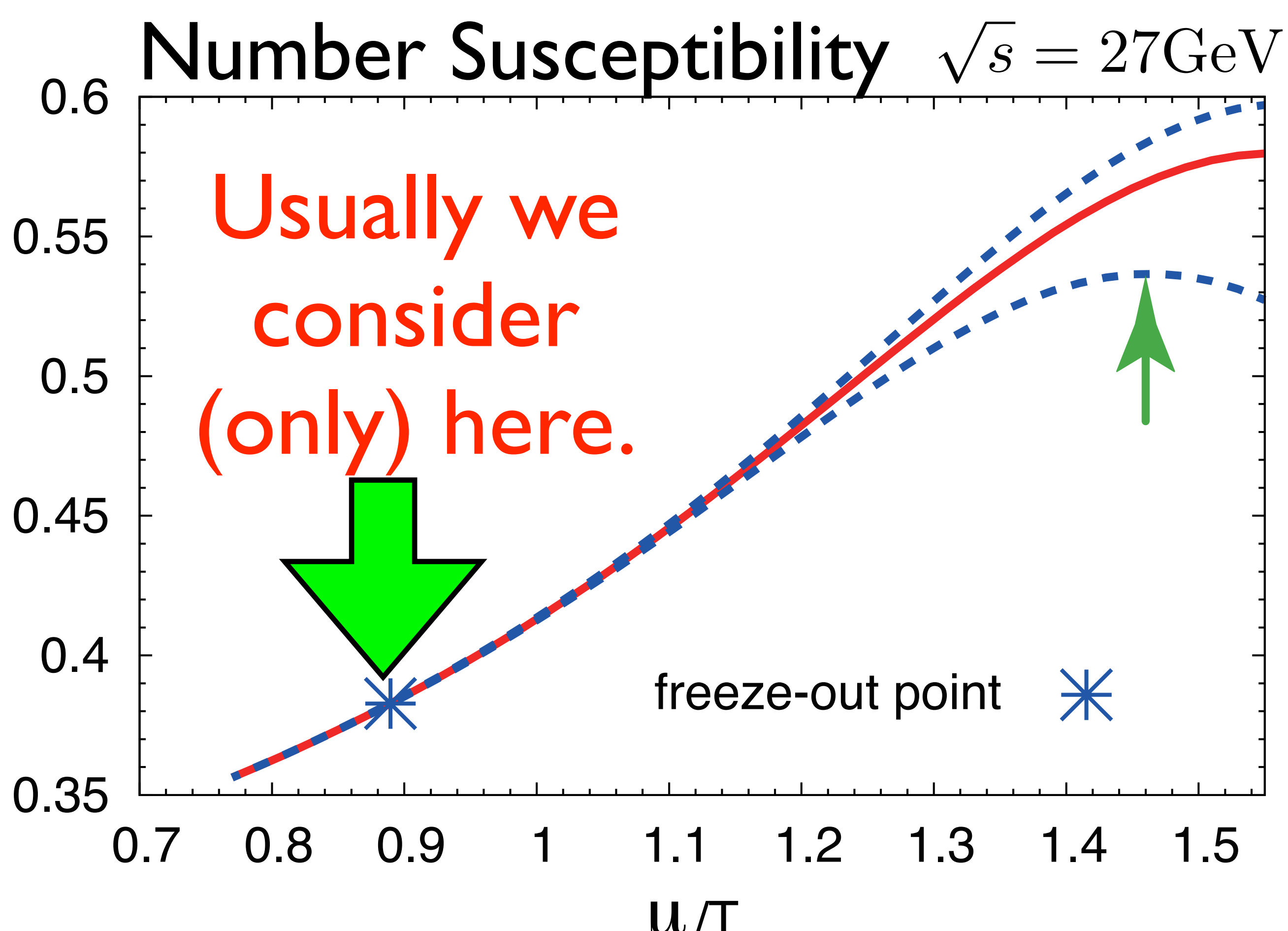
$$\lambda_k \equiv \left(T \frac{\partial}{\partial \mu} \right)^k \log Z$$

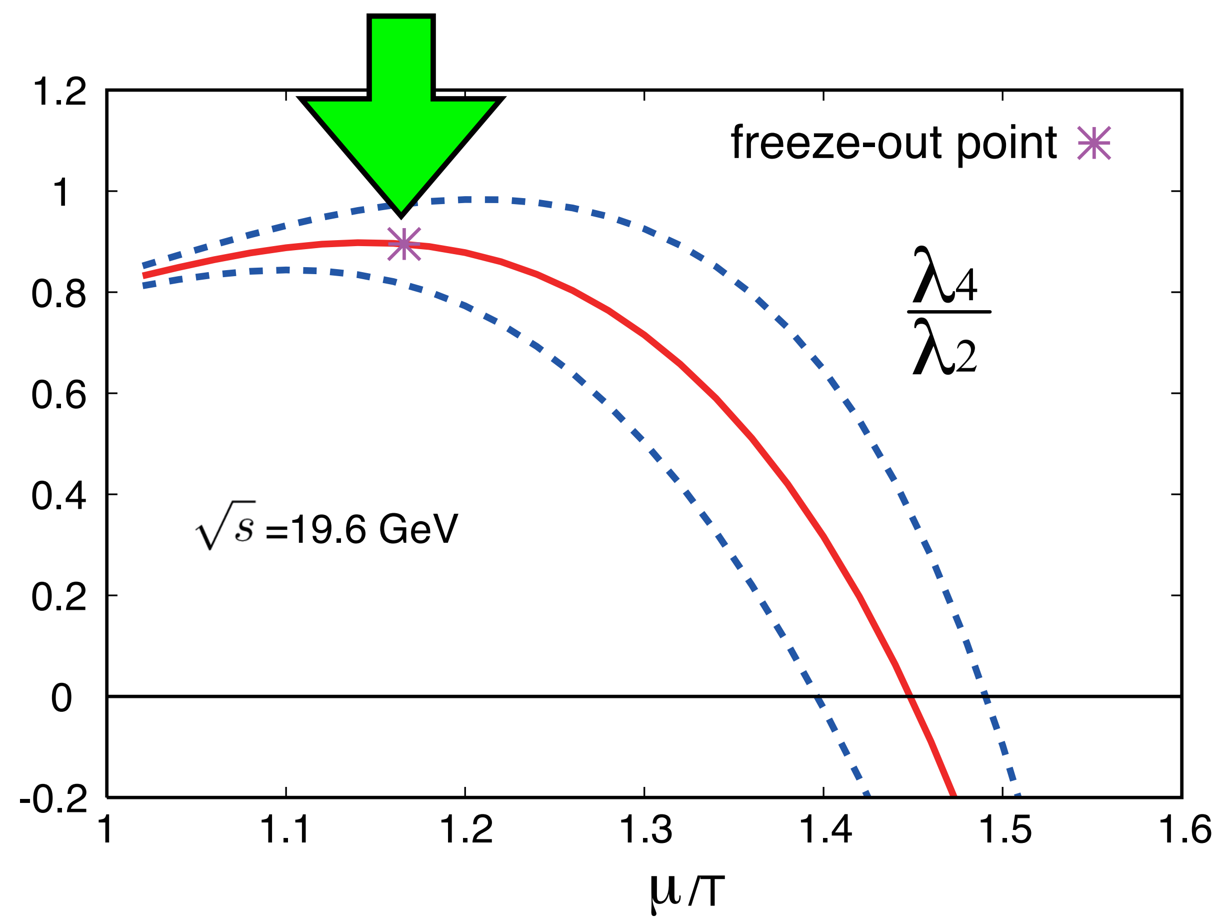


What happens ?

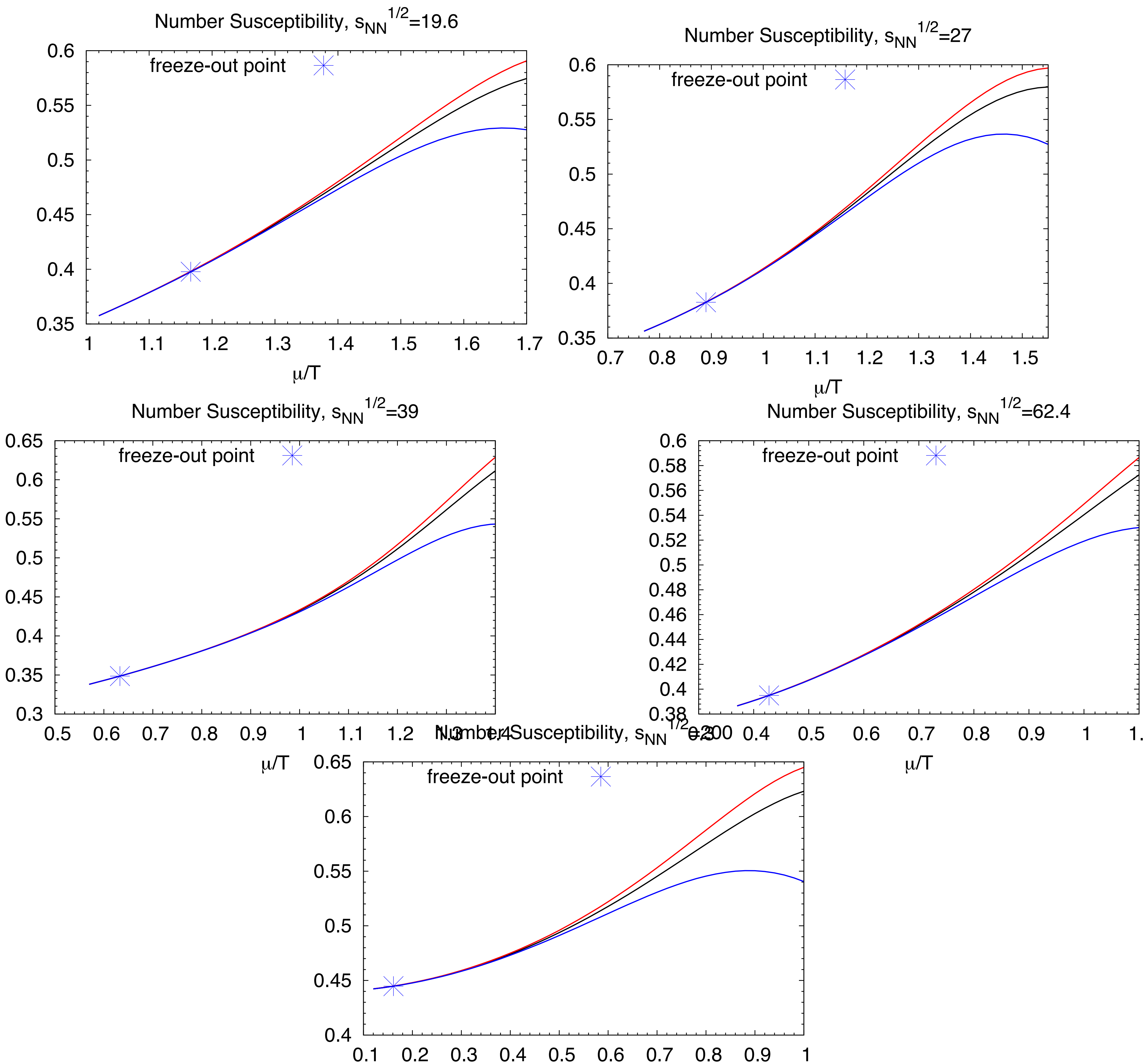
if we increase these points 15%

if we drop these points

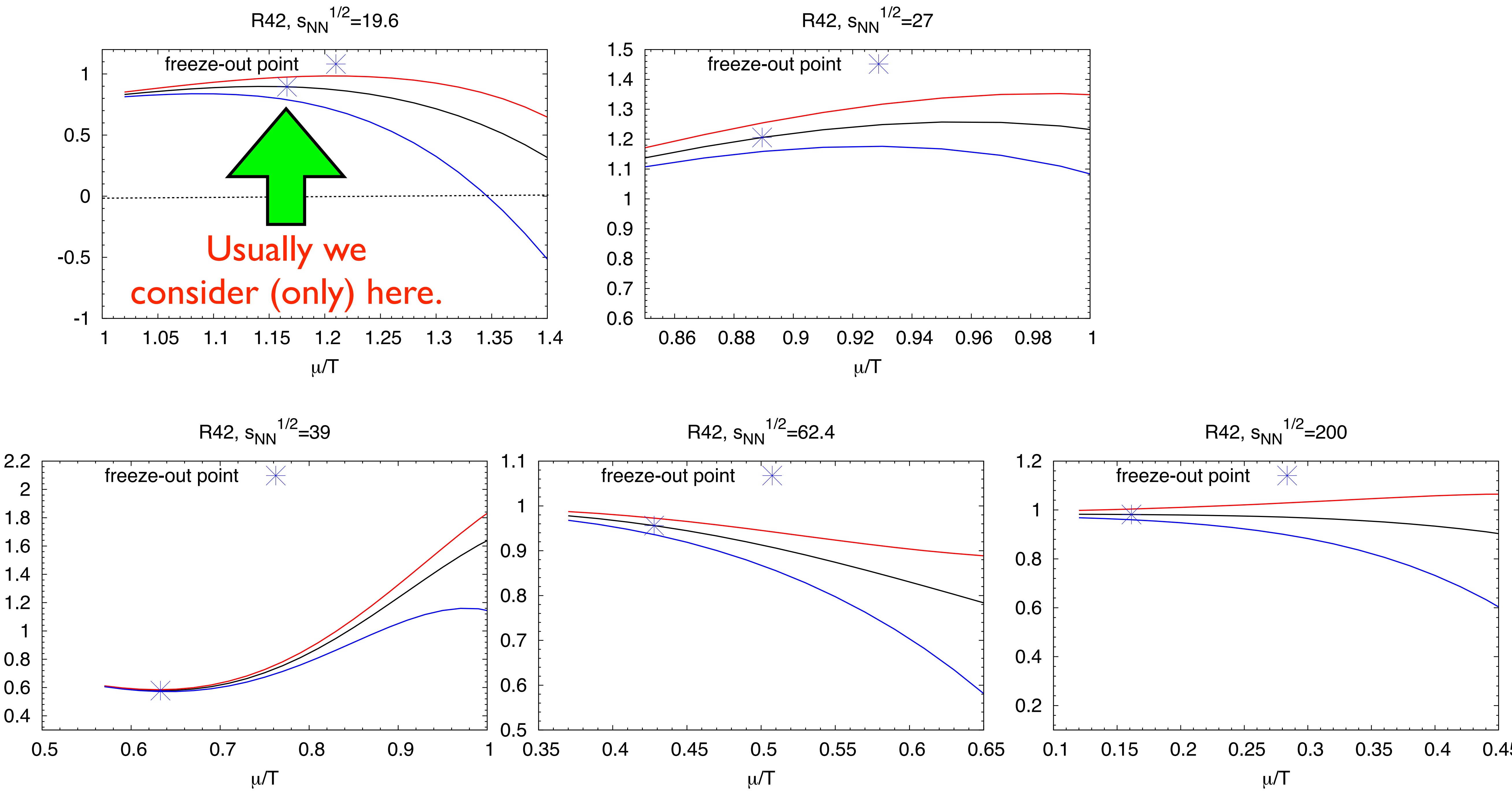




Susceptivility

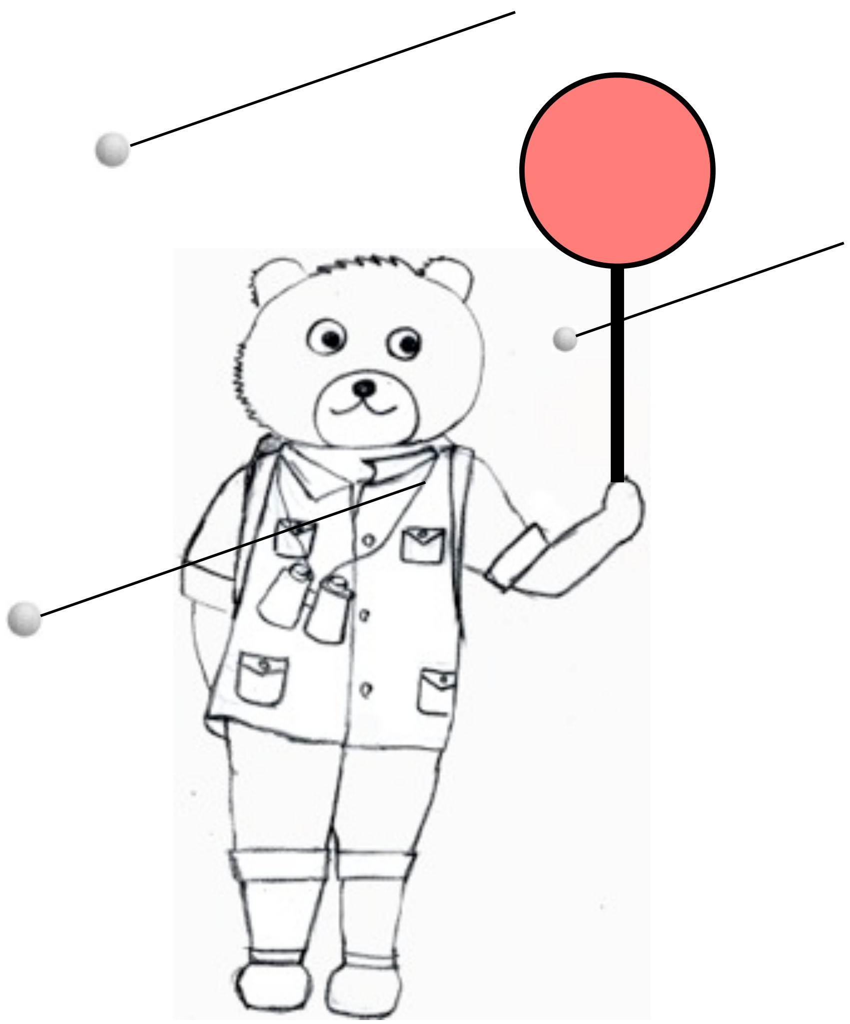


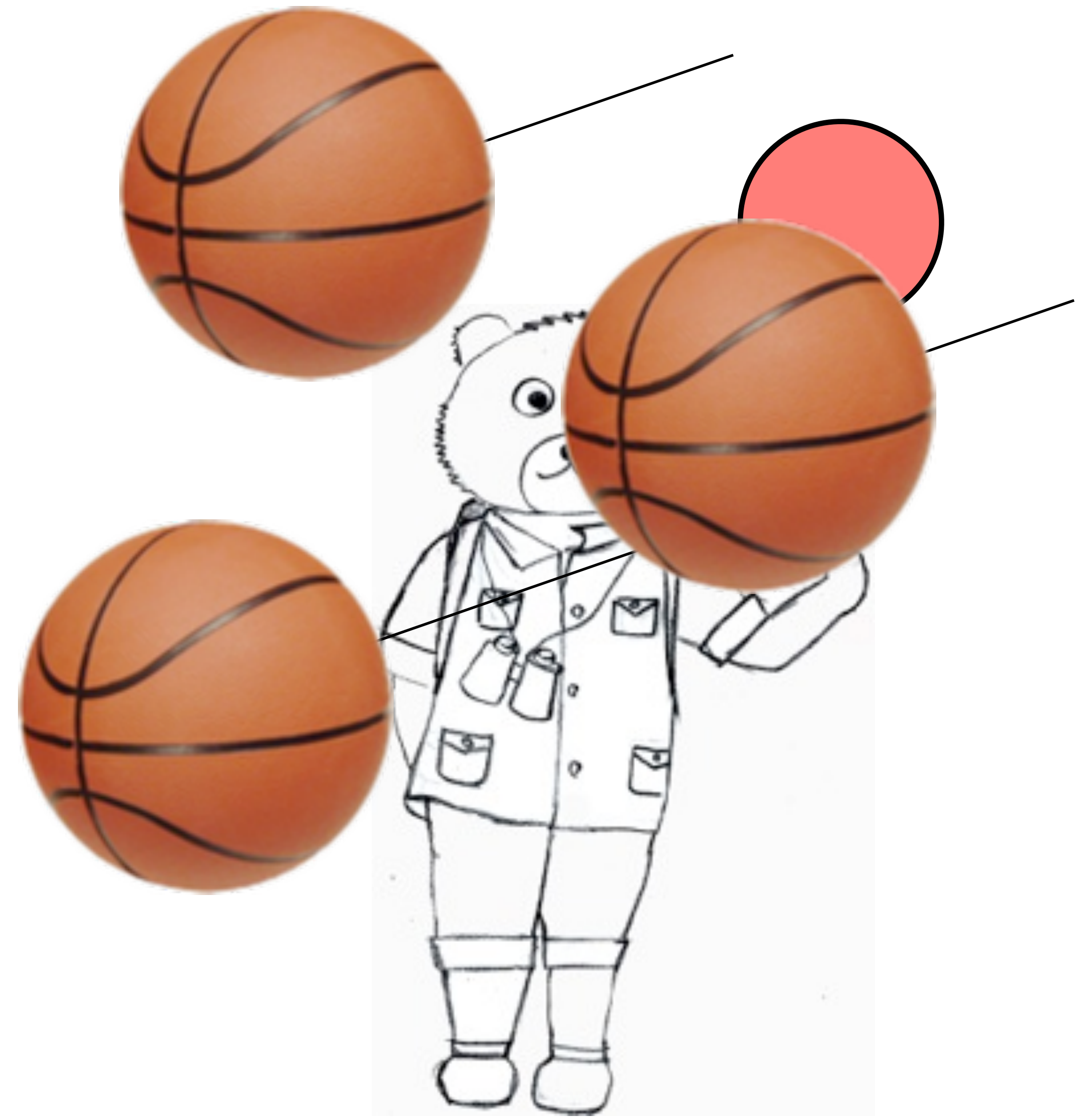
Kurtosis



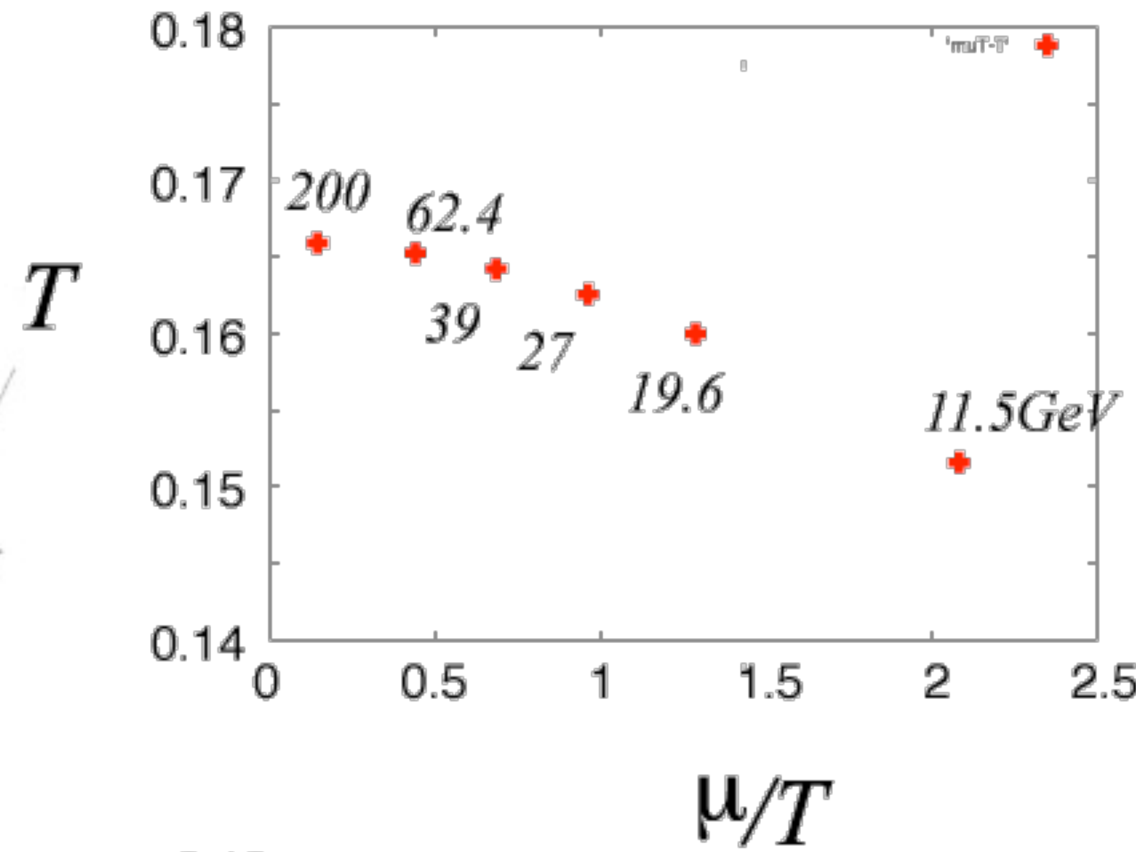
R42, $s_{NN}^{1/2}$=19.6
freeze-out point
Usually we
consider (only) here.
μ/T
R42, $s_{NN}^{1/2}$=27
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=39
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=62.4
freeze-out point
μ/T
R42, $s_{NN}^{1/2}$=200
freeze-out point
μ/T

BES(Beam Energy Scan)

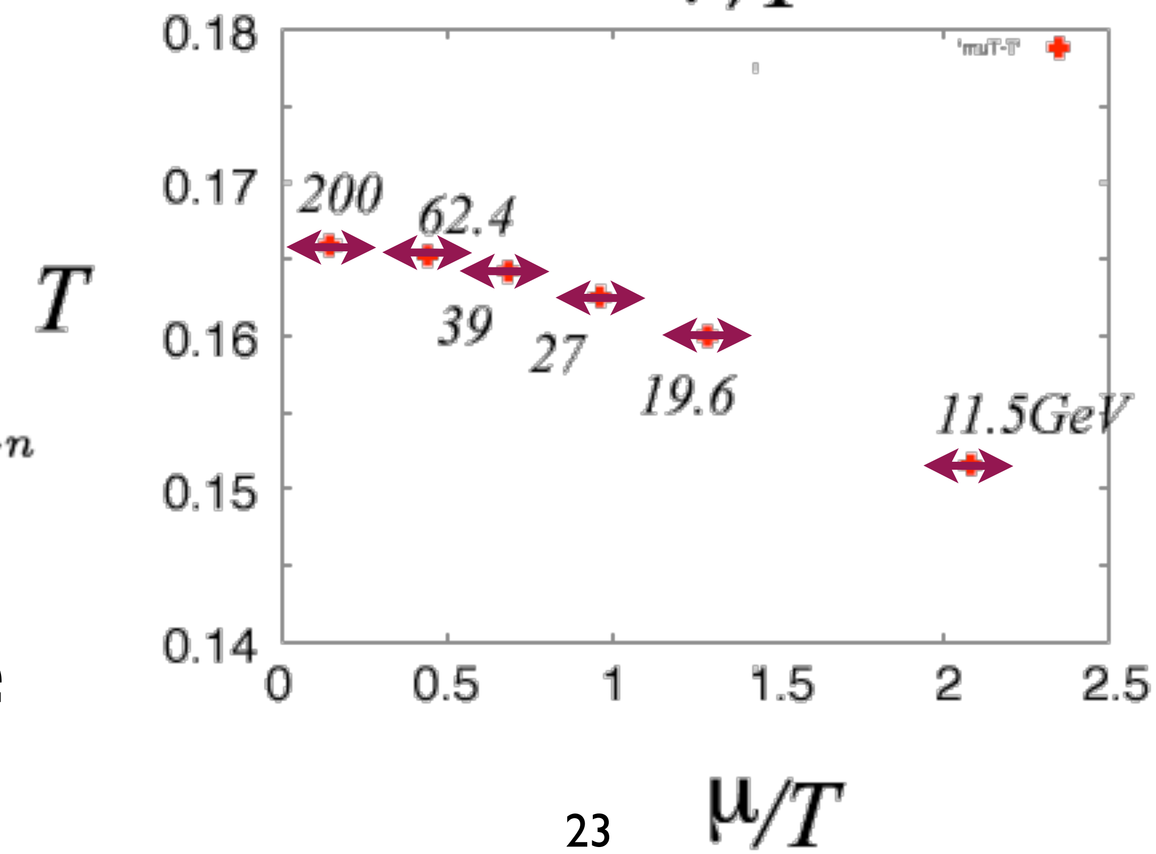




Multiplicity tells us



Not only Freeze -out points



Information of wider regions

$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Nmax →large

Wider

Lee-Yang Zeros

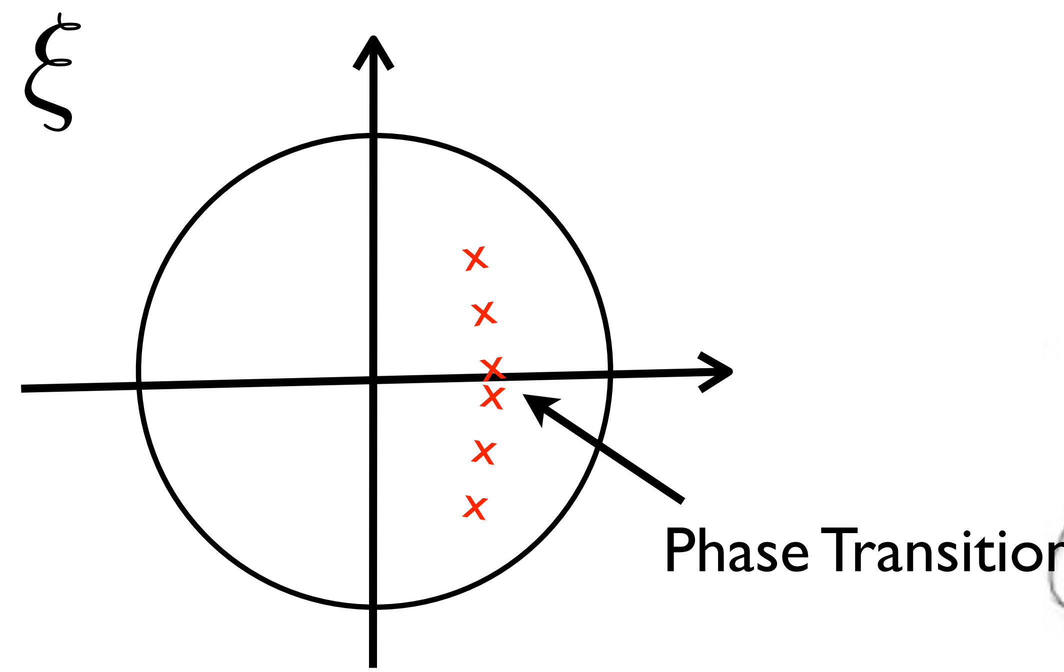
$$Z(\xi, T) = \sum_{n=-N_{max}}^{+N_{max}} Z_n(T)\, \xi^n$$

Lee-Yang Zeros (1952)

Zeros of $Z(\xi)$ in Complex Fugacity Plane.

$$Z(\alpha_k) = 0$$

Great Idea to investigate a Statistical System

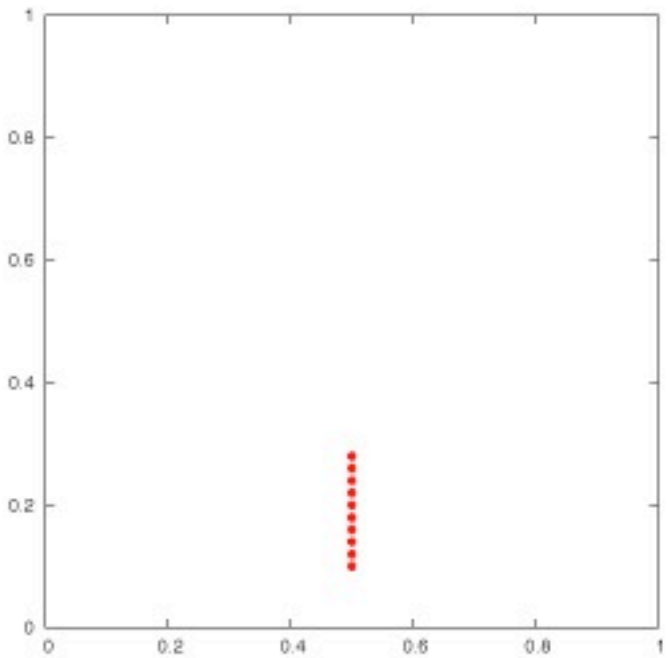


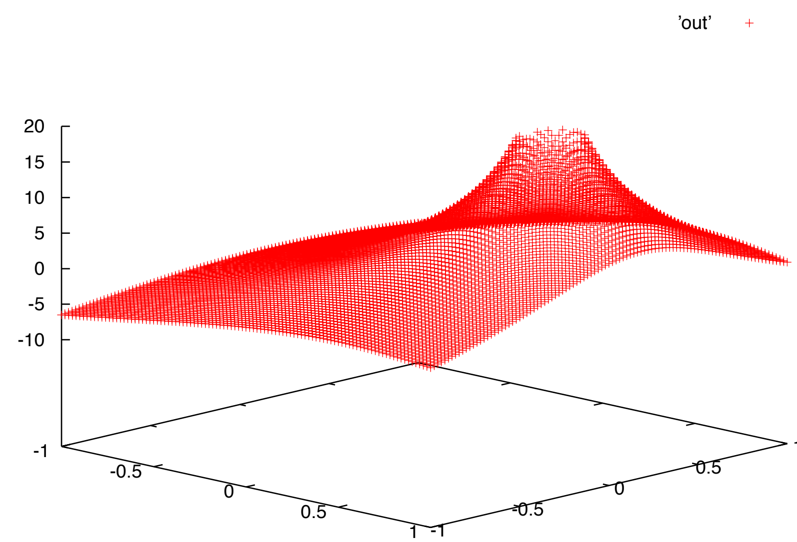
Lee-Yang Zeros

Non-trivial to obtain.
But once they are got, it is easy to figure out the Free-energy

$$Z(\xi, T) = e^{-F/T}$$

Lee-Yang zeros	2-d Electro-Magnetic
F: Free-energy	F: Potential
α_k: zeros	α_k:Point charge

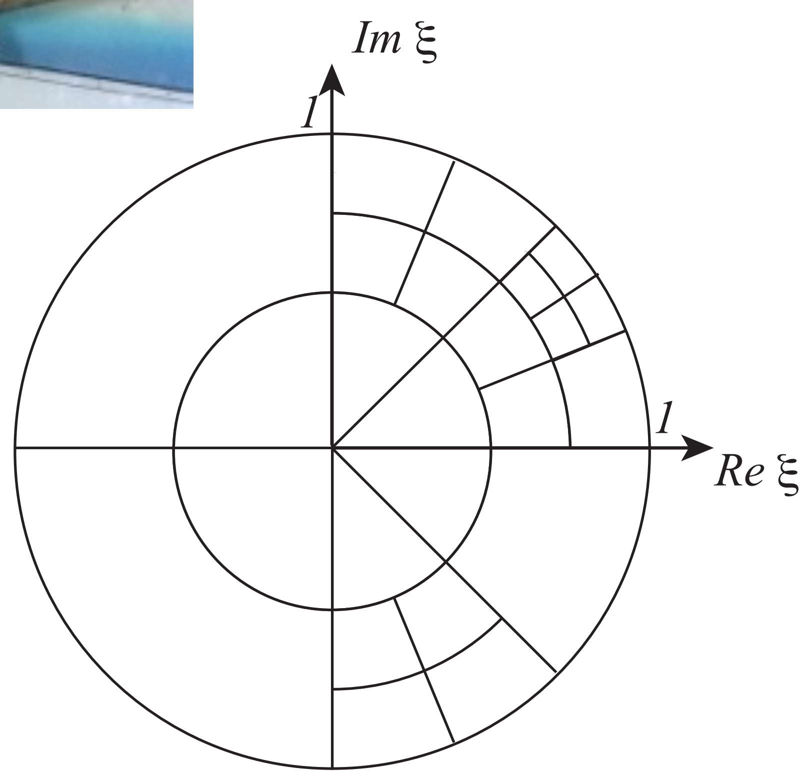




$$F(\xi) = -\sum_k \log(\xi - \alpha_k)$$

 and 

cut Baum-Kuchen (cBK) Algorithm



$$f(\xi) = \prod_k (\xi - \alpha_k)$$

$$\frac{f'}{f} = \sum_k \frac{1}{\xi - \alpha_k}$$

$$\frac{1}{2\pi i}\oint_C \frac{f'}{f} d\xi = \left(\begin{array}{c}\text{Number of} \\ \text{Zeros in} \\ \text{Contour } C\end{array}\right)$$

50 - 100 number
of significant digits

A Coutour is cut into
four pieces
if there are zeros inside.

Is this my Original ?

I donot think so.

Let us wait until someone claims.

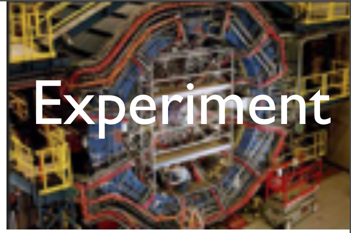
Is this my Original ?

I donot think so.

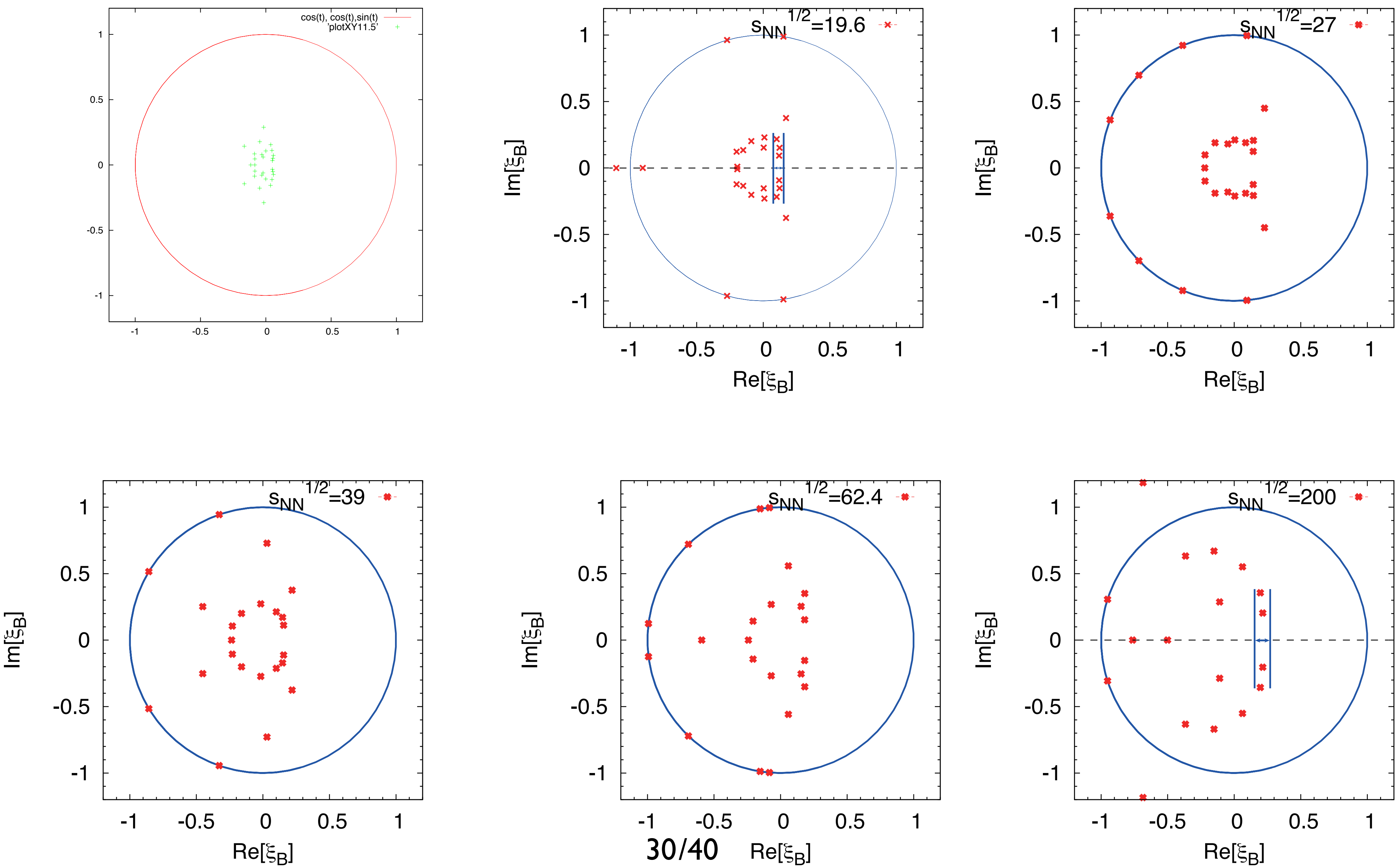
Let us wait until someone claims.

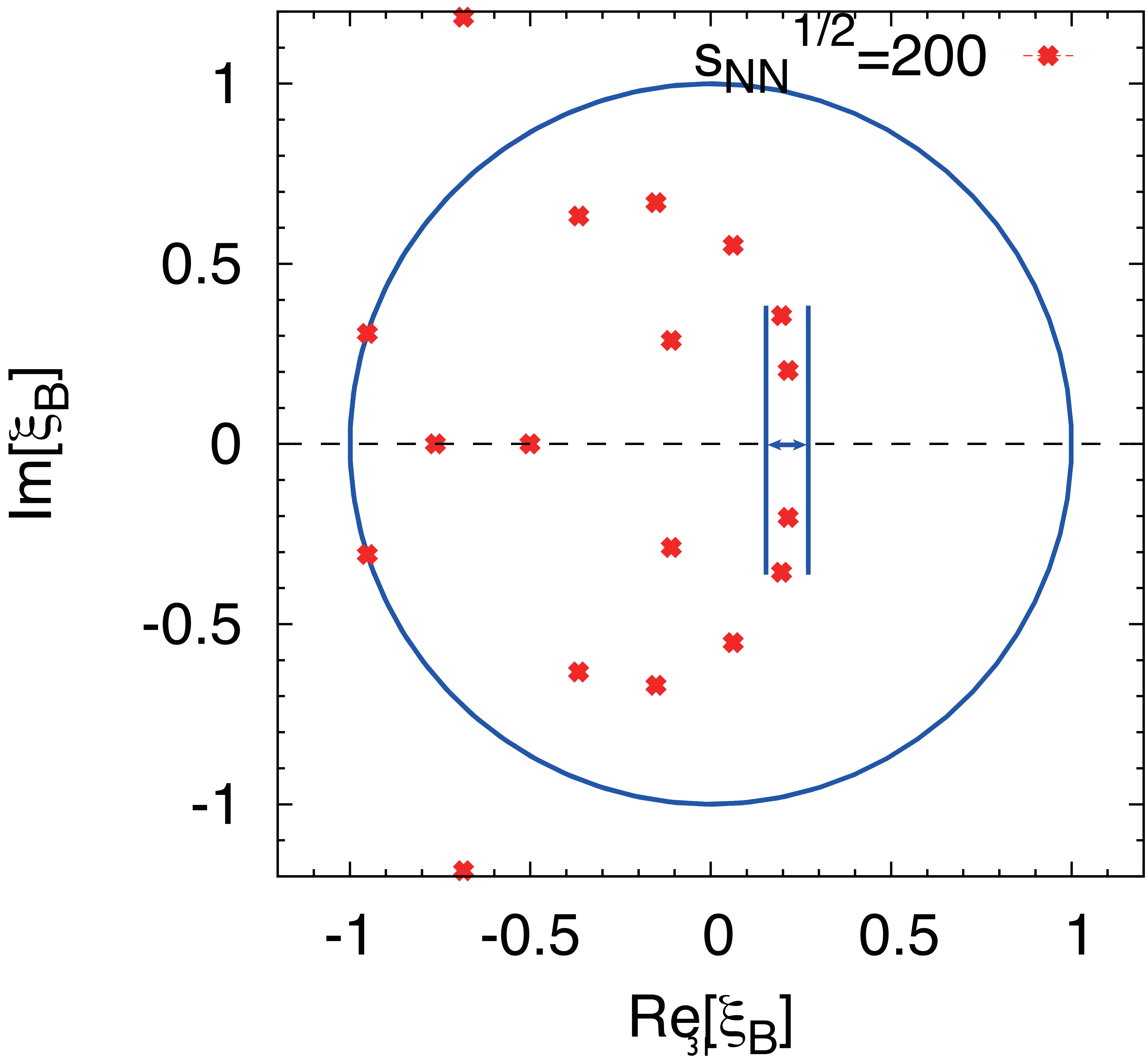
It’s me !

Riemann (1826 - 1866)



Lee-Yang Zeros: RHIC Experiments





$s_{NN}^{1/2}$=200
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1
Im[ξ_B]
Re[ξ_B]

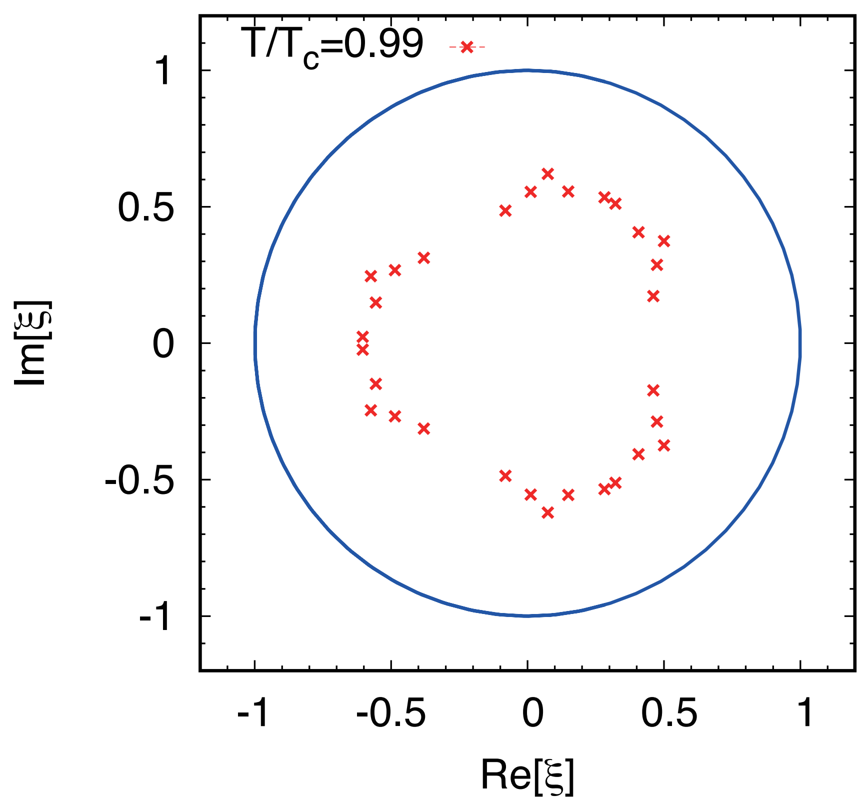
Lee-Yang Zeros
Lattice QCD

$$Z(\xi) = \sum_m Z_{3m} \times \xi^{3m}$$

$Z_n = 0$ **unless** $n \neq 3m$

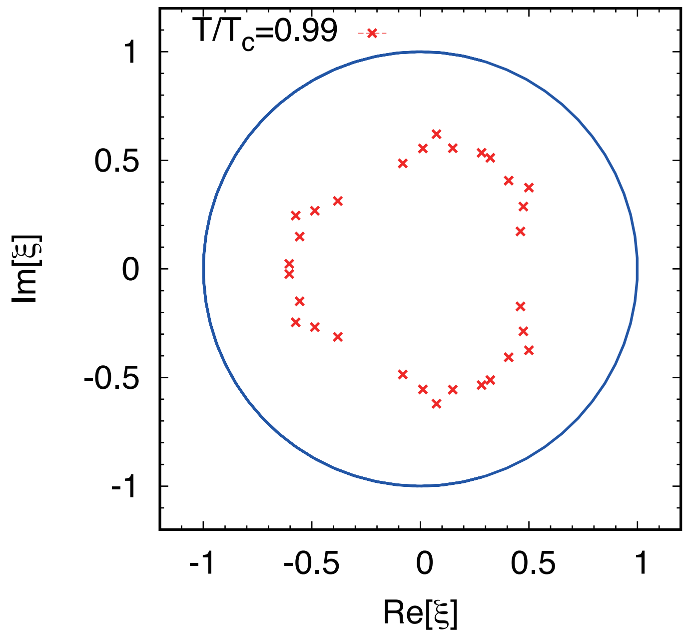
Periodicity $\theta = \dfrac{2\pi}{3}$

$(\xi = e^{i\theta})$

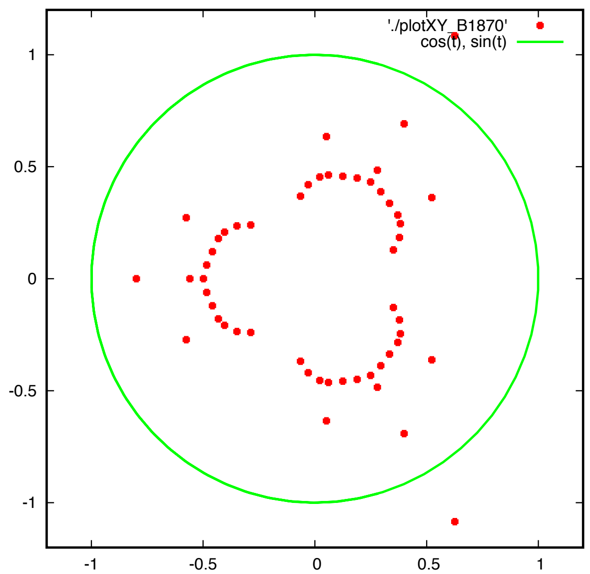


$$\beta = 1.85$$

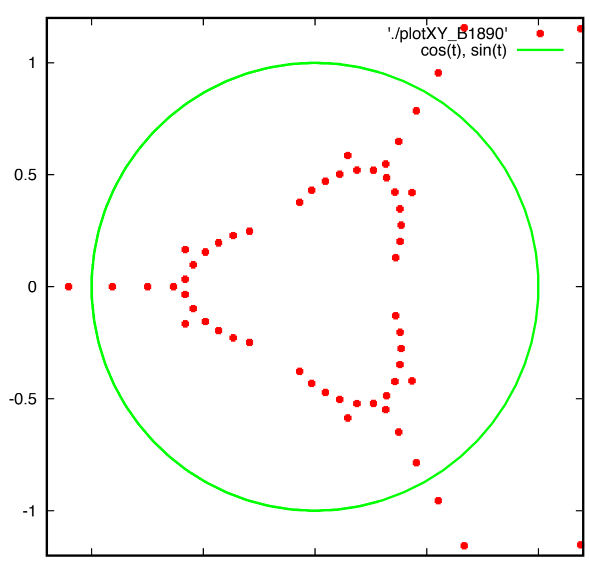
$$T/T_c \sim 0.99$$



$$\beta = 1.85 \qquad T/T_c \sim 0.99$$

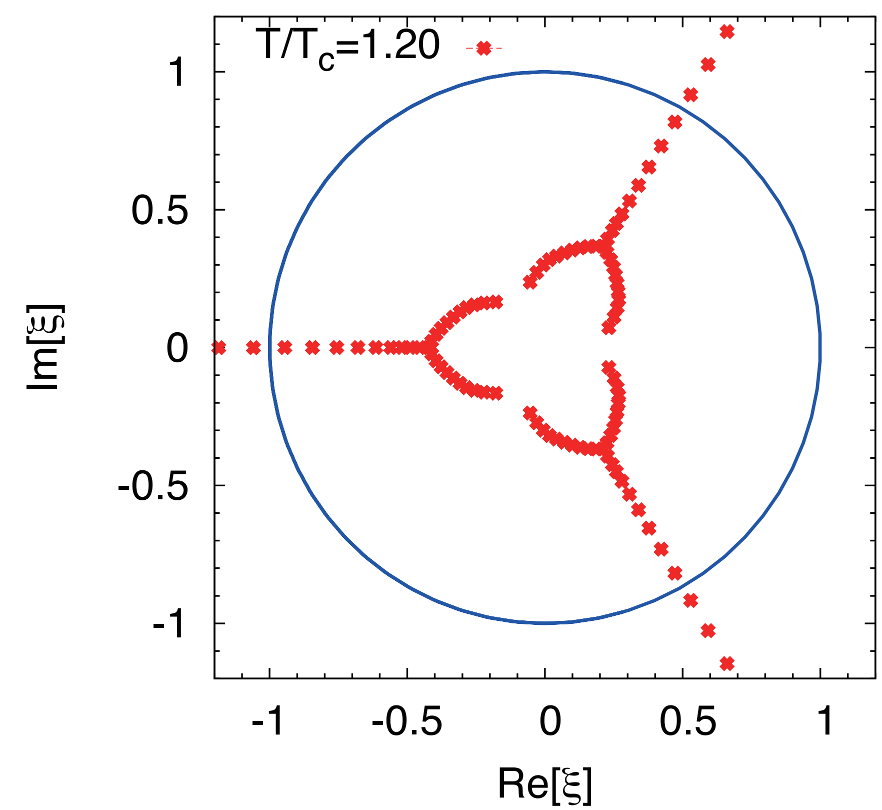


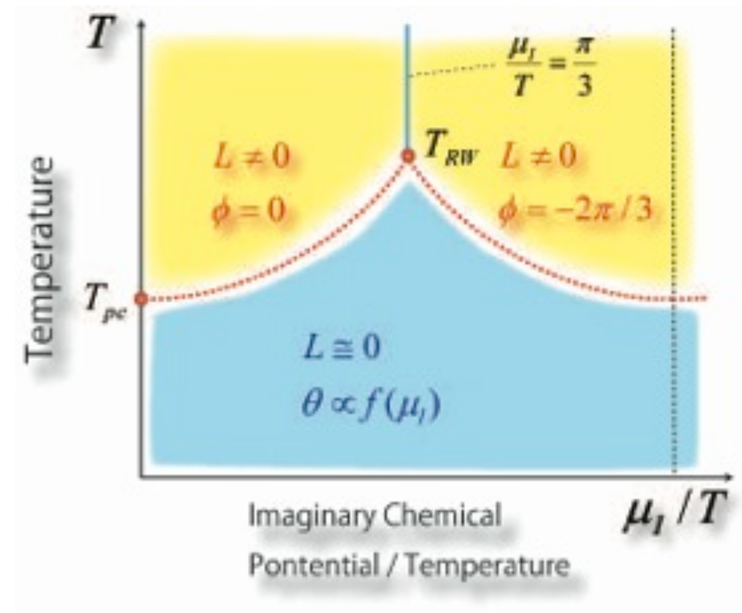
$$\beta = 1.87 \qquad T/T_c \sim 1.01$$



$$\beta = 1.89 \qquad T/T_c \sim 1.04$$

Lee-Yang Zeros
Lattice QCD





$$\xi = e^{\mu/T}$$

The Unit Cirle in ξ

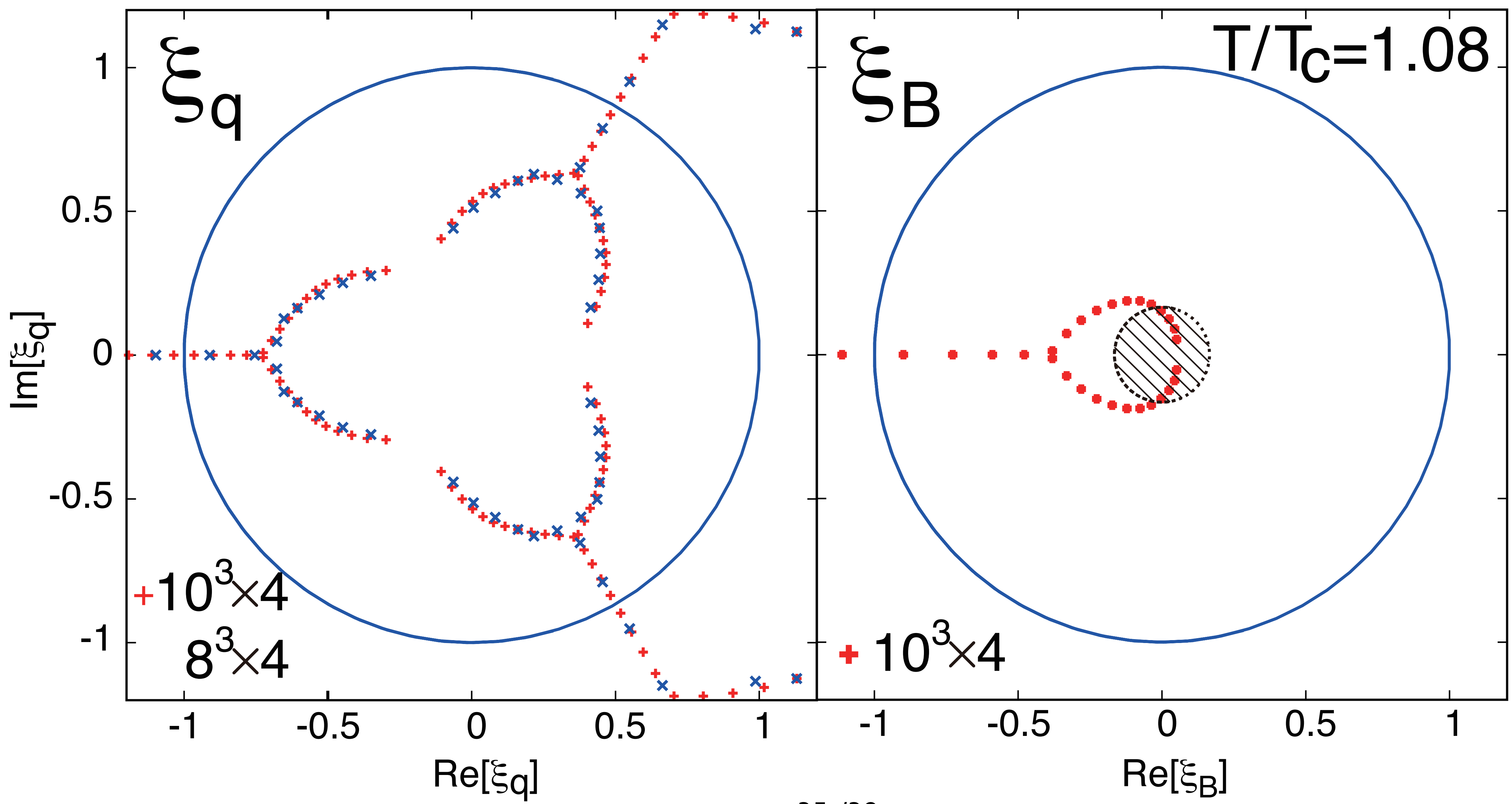
Imaginary μ

Roberge-Weise
Transition !

$$T/T_c \sim 1.20$$

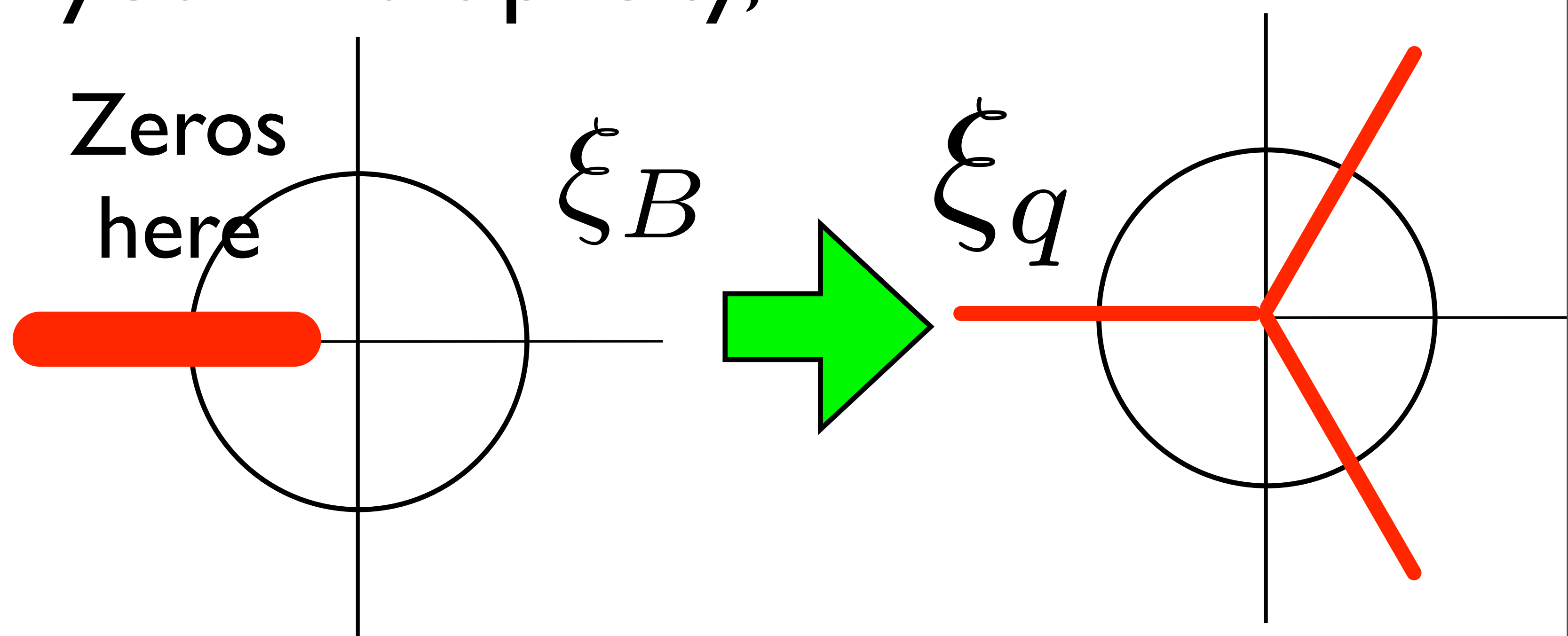
$$(\xi \equiv e^{\mu/T})$$

$$\xi_q \Rightarrow \xi_B = {\xi_q}^3$$



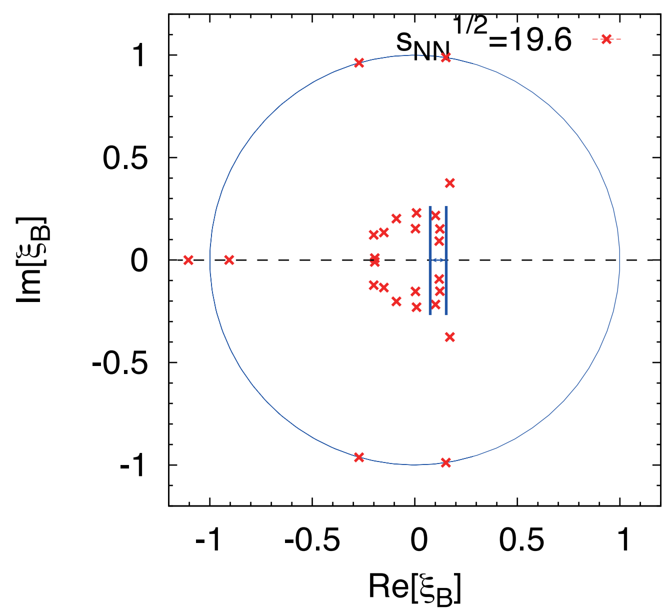
A Message to Experimentalists

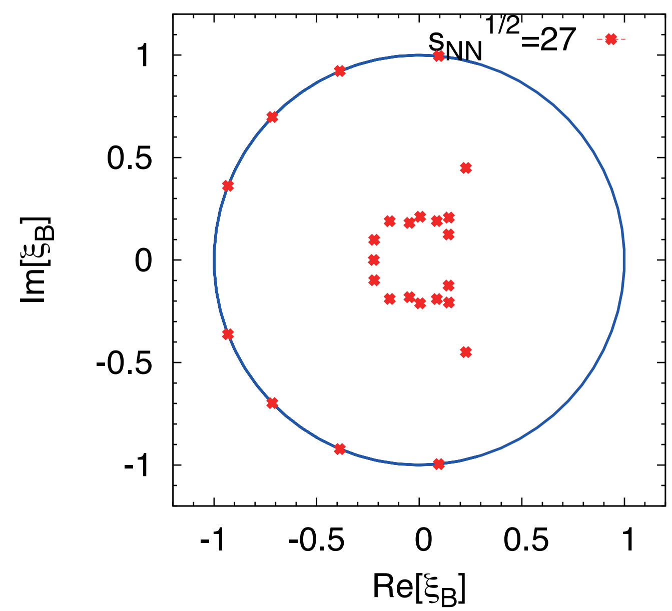
In the Lee-Yang Diagram constructed from your multiplicity,

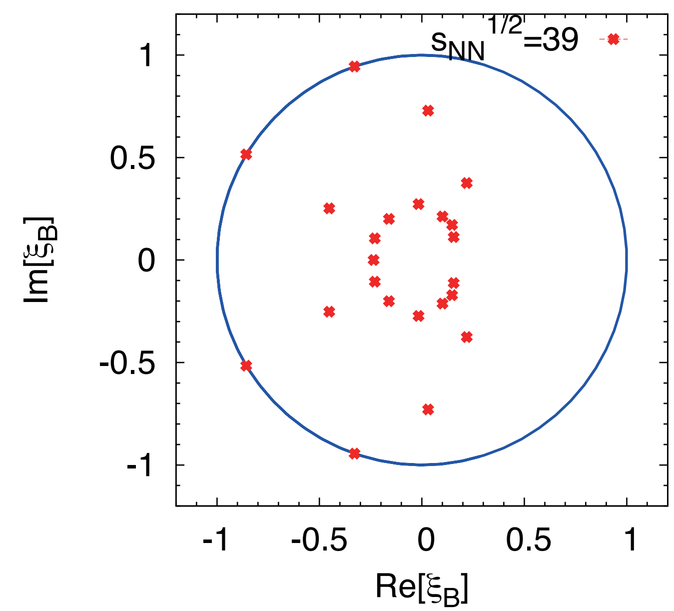


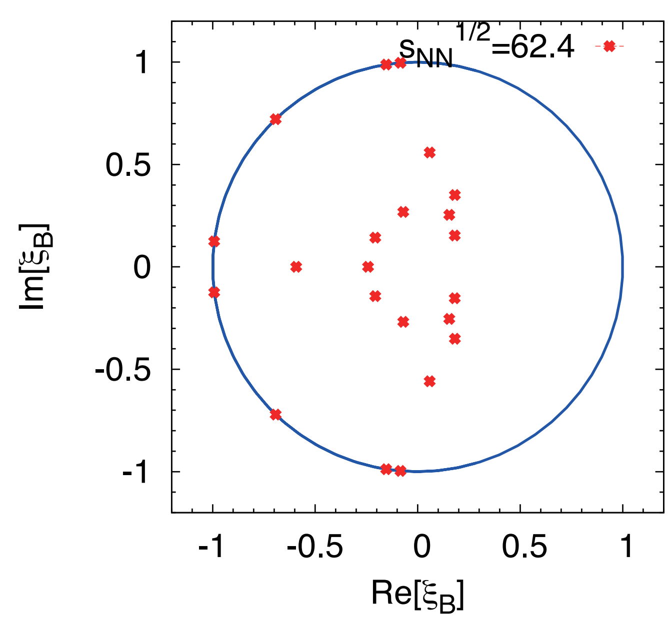
No Roberge-Weise Transition → Your Temperature $T \leq T_{RW} \sim 1.2T_c$

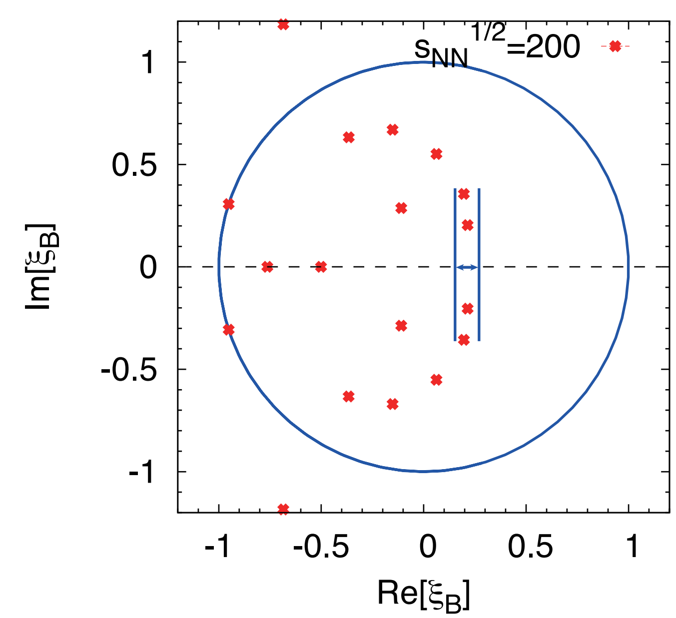
Lee-Yang Zeros: RHIC Experiments











Effects of Nmax
Kim's Model

In Confinement

$$Z(\mu_q) = I_0 + (\xi_q^3 + \xi_q^{-3})I_1 + (\xi_q^6 + \xi_q^{-6})I_2 + \cdots$$

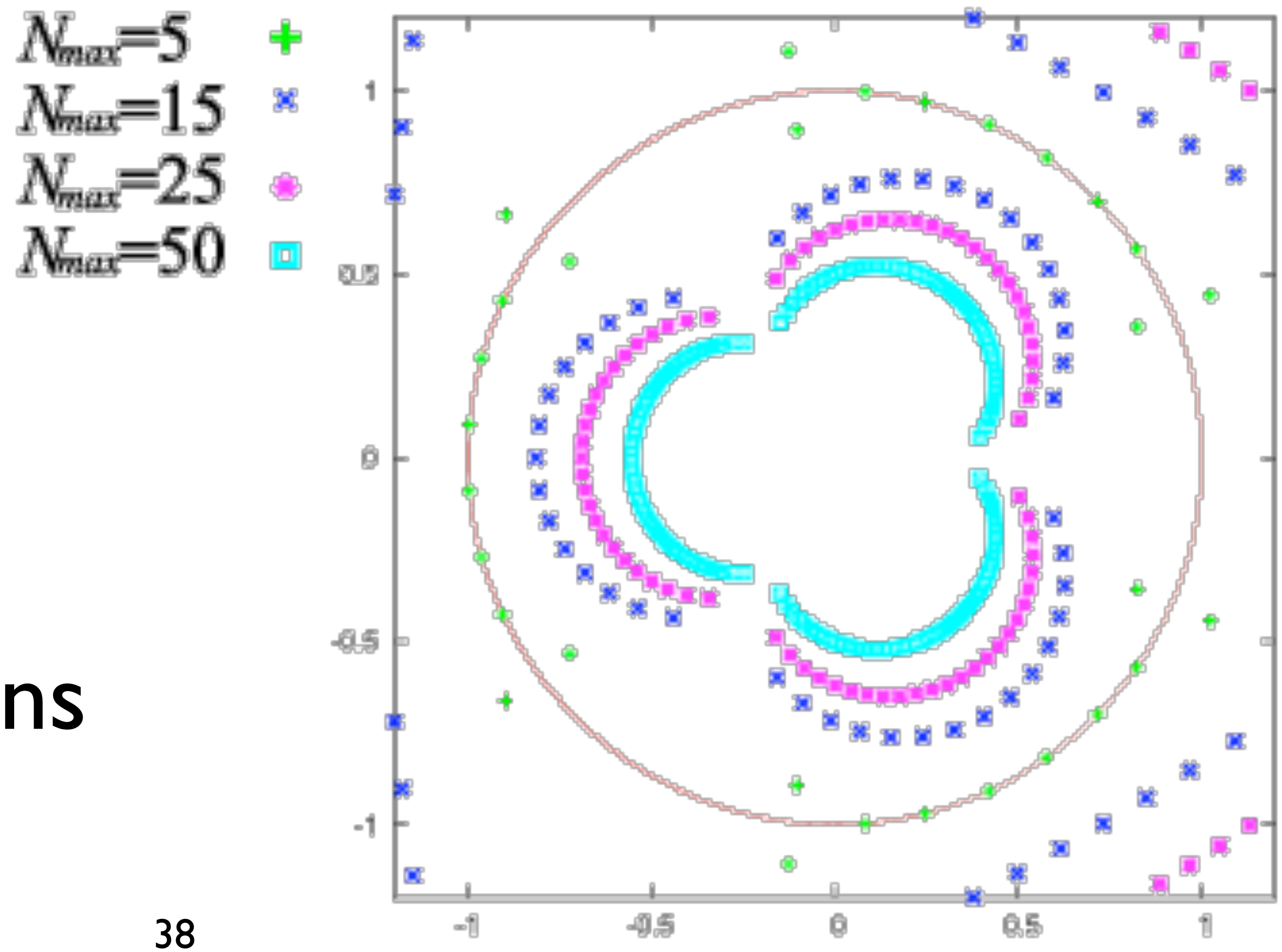
I_k :Modified Bessel

Lesson from the Model

Nmax ⇨ Large

Lee Yang Zeros ⇨ Large $|\mu|$ regions

It should be so!



Summary

- Grand-Partition functions, $Z(\mu, T)$, provide us the QCD phase information, which can be constructed from Z_n.
- Lattice QCD can calculate Z_n
  - But we need much more works to obtain reliable
- Experiments provide us the multiplicities
  - We can calculate Z_n from them.
  - Present data are those of net-proton, which are not conserved quantities.
    - Either correction, or ask experimentalists to measure net-baryon
    - Charge multiplicity is a conserved quantity, and another probe.
  - Large Nmax are wanted, but even finite Nmax data give us the lower bound.
- Lee-Yang zeros provide us a new tool of the QCD phase study.
  - They are sensitive to the data, i.e., they teach us which regions are important.