Lattice QCD at low temperature and finite density - early onset/Silver Blaze problem

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- ✓ KN, arXiv:1204.6480
- ✓ KN, Motoki, Nakagawa, Nakamura, Saito, PTEP01A103('12).
- ✓ KN, Nakamura, PRD82, 094027('10)

JICFuS Joint Institute for Computational Fundamental Science

NECOFuS NECO Fundamentalist Society

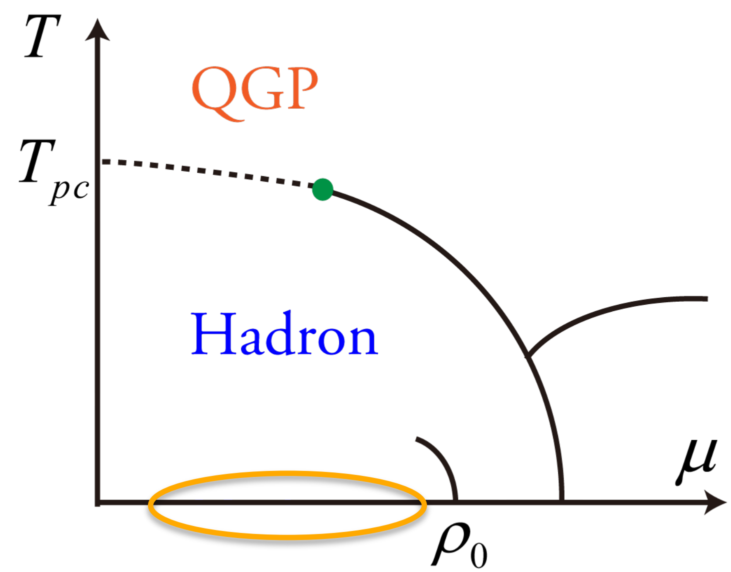
Outline

- Introduction
- Framework : reduction formula, reduced matrix
- Zero temperature limit of the fermion determinant
- $m_\pi/2$? or $M_N/3$?

Introduction

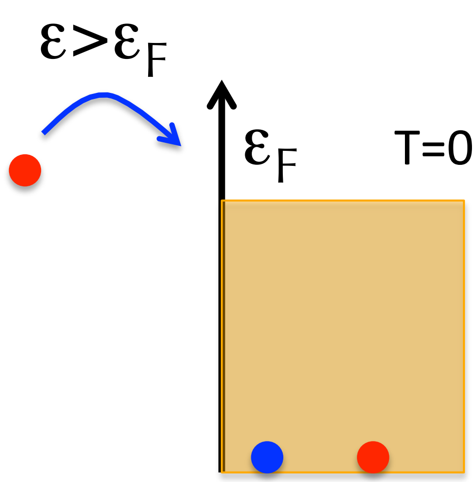
Introduction

- Hadronic phase at low-T
  - d.o.f is nucleons and pions



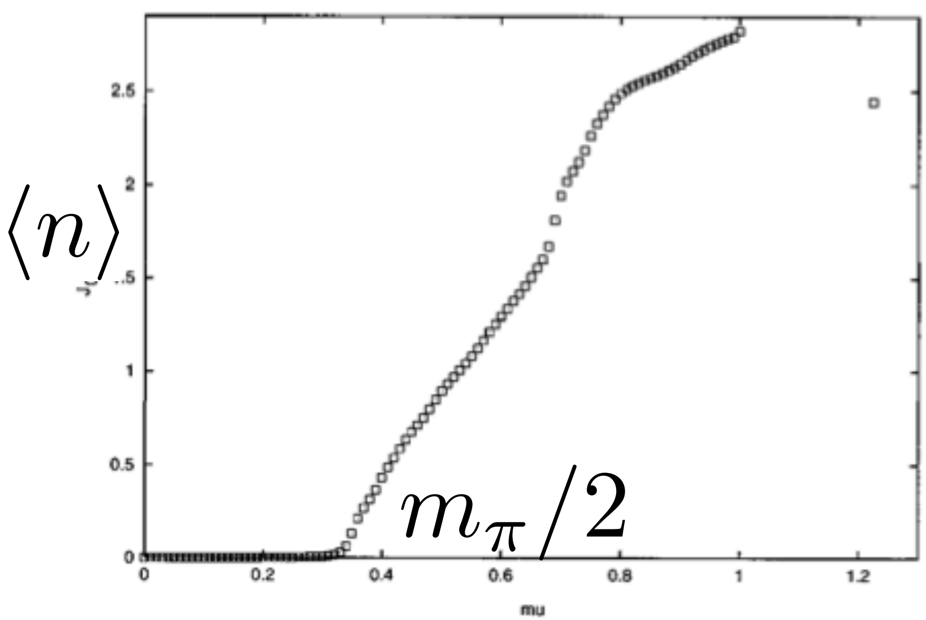
- In order to add one particle, energy larger than fermi energy is needed.

$$\mu > (1/3)M_N, (\mu_B > M_N)$$



Early onset problem

- Hadronic phase at low-T : d.o.f is nucleons and pions
- μ for the onset of quark number at T=0
  - expectation : $\mu = M_N/3$
  - lattice calculations : $\mu = m_\pi/2$



Glasgow method

- propagator matrix
- reweighting

[Barbour et. al. PRD56, 7063 (1997)] 6^4, staggered, mq=0.1

Silver Blaze

- The effect of μ does not appear, although the action explicitly depends on μ.
  - well-known property of fermion at T=0.
  - But it is unclear how it is realized in field theory).

- Cohen('04) considered isospin chemical potential case and showed.
  - fermion determinant of QCD is independent of μ at T=0 for μ less then half the pion mass.
  - (baryon Silver Blaze ($\mu < M_N/3$) has not been solved)

Motivation

- In previous studies by Glasgow method, it was unclear what is the origin of the early onset problem.
- Isospin Silver Blaze by Cohen provides an explanation of the early onset observed in Glasgow method.

- Motivation

1. lattice study is important to confirm it non-perturbatively.
2. It is still unclear why it is half the pion mass, not one-third of the nucleon mass.

- we mostly concentrate on
  - $\mu < m_\pi/2$
  - property of fermion determinant at configuration level

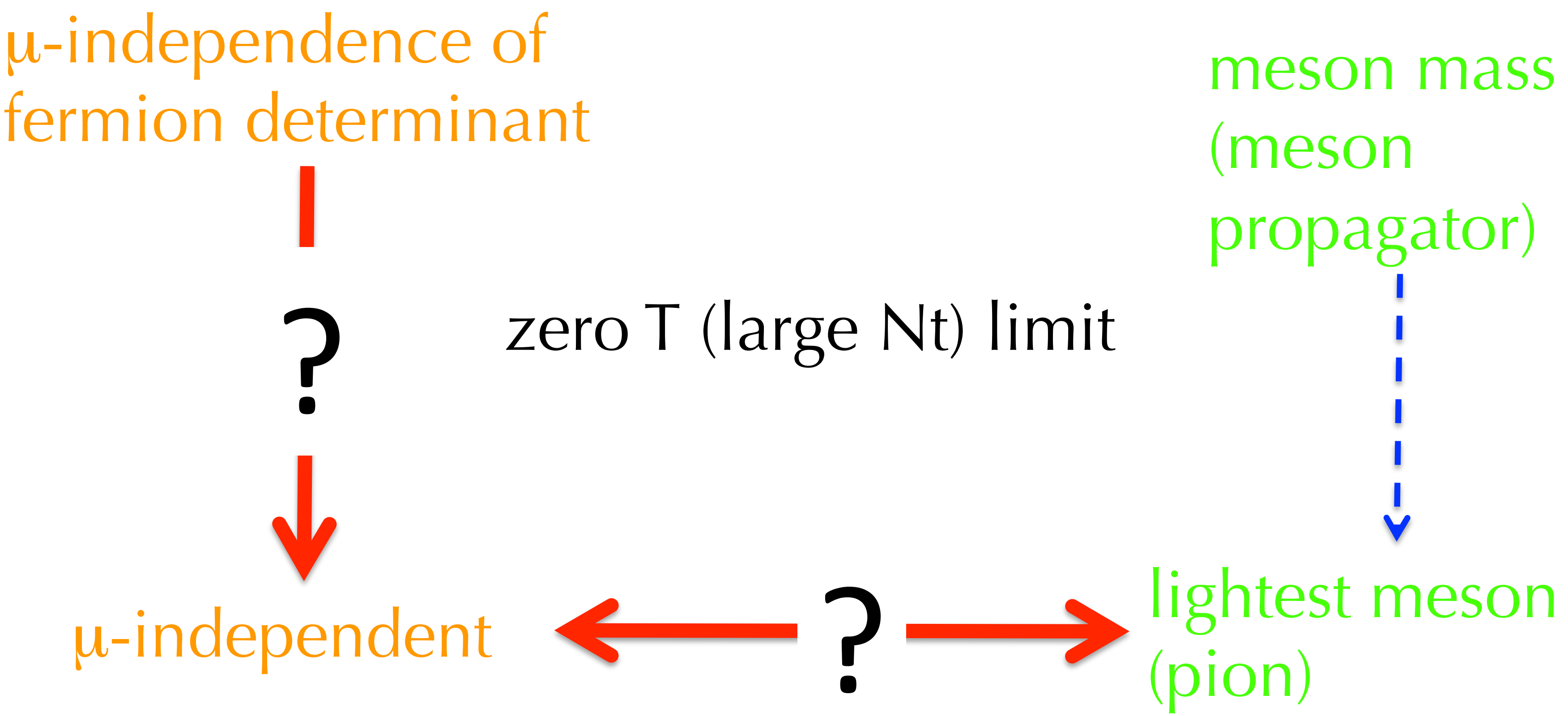
Motivation

- Question
  - Why does the quark number start at $\mu = m_\pi/2$ at $T=0$?

- We discuss the early onset problem in taking the zero temperature limit of the fermion determinant

  - relation between fermion determinant and pion mass

Idea and outline of talk



propagation of quarks

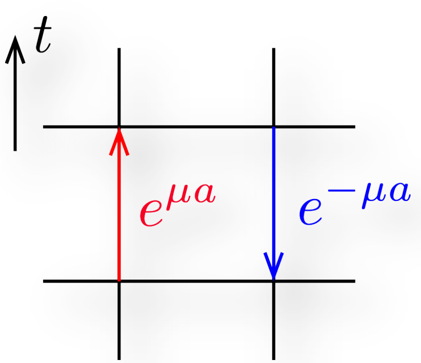
Framework

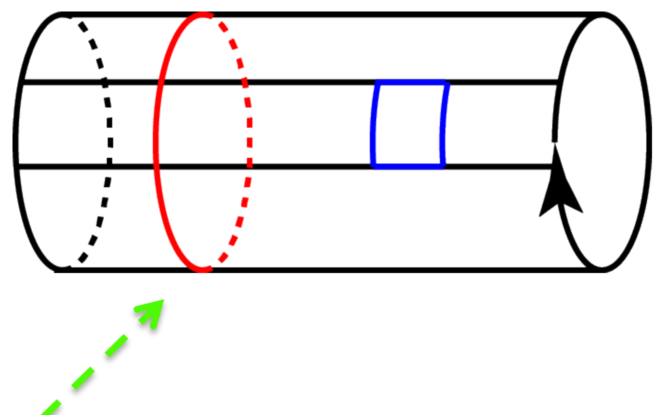
Fermion matrix

- Fermion path integral and fermion determinant

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-\sum \bar{\psi}\Delta\psi} = \det \Delta$$

  - Δ : matrix with the rank 4 Nc Nx Ny Nz Nt
  - it consists of fermion loops
  - chemical potential comes from temporal loops





$$e^{\mu a \times N_t} = e^{\mu/T}$$

Reduction formula

- A formula to perform t-part of the fermion determinant.
  - A fermion matrix in t-t matrix rep.

$$\Delta = B - e^{\mu a} V - e^{-\mu a} V^{\dagger}$$

$$\Delta = \begin{pmatrix} \square & \triangle & & & \triangle \\ \triangle & \square & \triangle & & \\ & \triangle & \ddots & & \\ & & & \ddots & \triangle \\ \triangle & & & \triangle & \square \end{pmatrix}$$

$\leftarrow$ det Δ can be calculcated analytically.

[Gibbs ('86). Hasenfratz, Toussaint('92). Adams('03, '04), Borici('04). KN&AN('10), Alexandru &Wenger('10)]

Reduction formula

- det Δ is reduced to

$$\det \Delta = \xi^{-N_{\mathrm{red}}/2} C_0 \det(\xi + Q)$$

$$\xi = e^{-\mu/T}$$

  - C_0 and Q are functions of gauge fields.
  - chemical potential and gauge fields are separated
  - Q is a matrix with rank $N_{\mathrm{red}}=12\, N_s^3$

Reduction formula

- det Δ is reduced to

$$\det\Delta = \xi^{-N_{\rm red}/2} C_0 \det(\xi + Q)$$

$$\xi = e^{-\mu/T}$$

- Using eigenvalues of Q, det Δ is given by

$$\det\Delta = \xi^{-N_{\rm red}/2} C_0 \det(\xi + Q) \qquad \xi = e^{-\mu/T}$$

$$= e^{\frac{N_{\rm red}}{2}\frac{\mu}{T}} C_0 \prod_n (e^{-\mu/T} + \lambda_n)$$

- eigenvalues of Q control the μ-dependence of det Δ

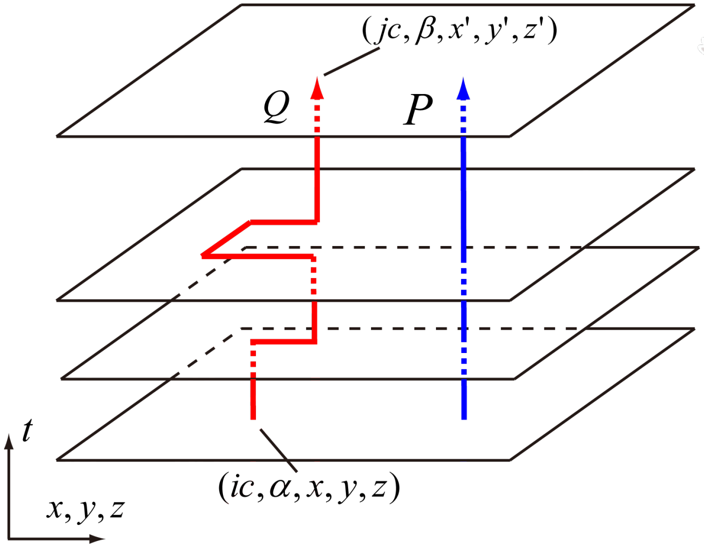
Reduced matrix (Wilson fermions)

$$Q = (\alpha_1^{-1}\beta_1)\cdots(\alpha_{N_t}^{-1}\beta_{N_t})$$

$$\alpha_i = B_i r_- - 2\kappa r_+,$$
$$\beta_i = (B_i r_+ - 2\kappa r_-) U_4$$

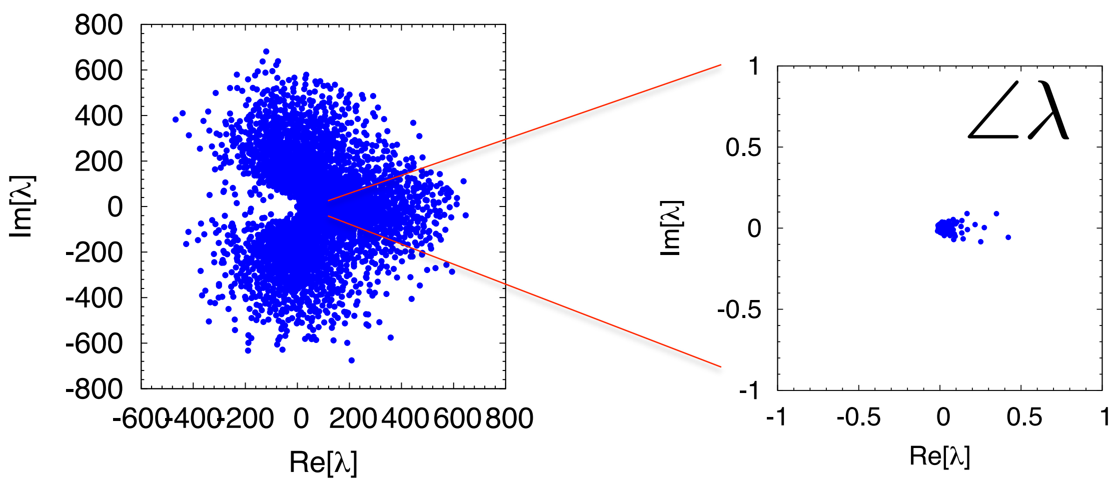
spatial temporal

r+-=(1+-gamma4)/2

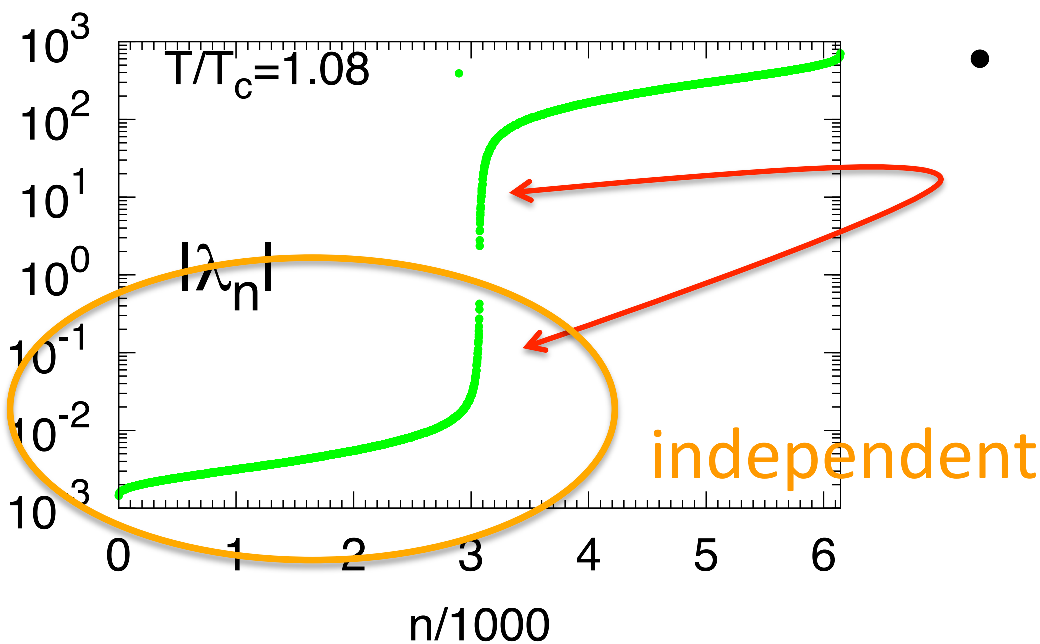


- It can be applied to e.g.
  - winding number expansions (Q, QQ, QQQ...)
  - hadron masses (QQ^+, QQQ ...) [Gibbs('86), Fodor, Szabo, Toth('07)]

Spectral properties of reduced matrix 1



Eigenvalue distribution
L: large scale
R: small scale

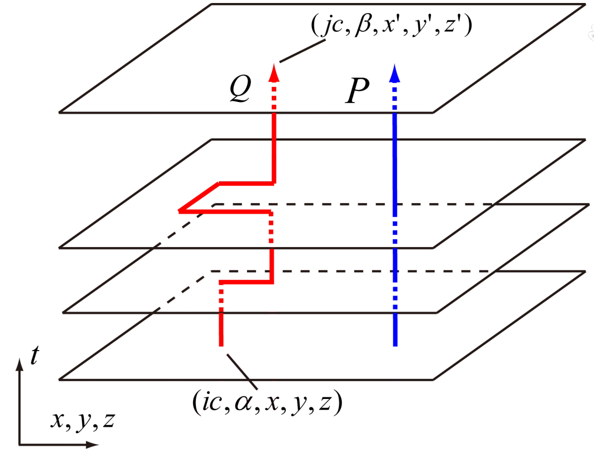


- Eigenvalues λ of Q -pair (γ_5-hermiticity)

$$\lambda_n \leftrightarrow 1/\lambda_n^*$$

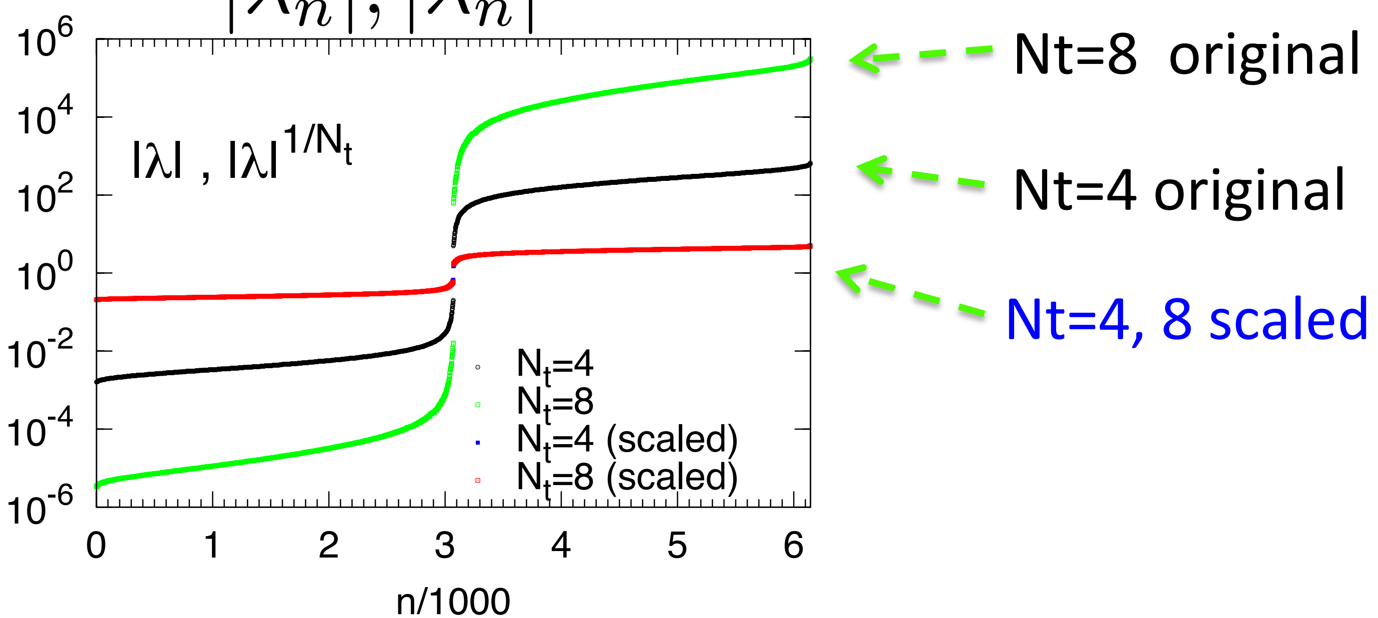
Spectral properties of reduced matrix 2

- Q follows Nt-scaling
  - Q is temporal quark line, analogous to the Polyakov loop



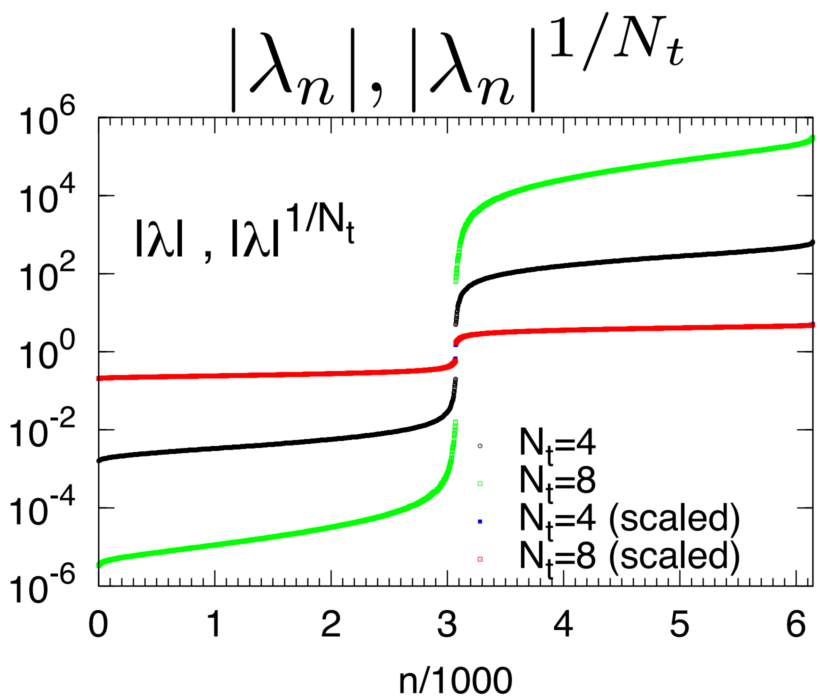
$$\lambda_n = l_n^{N_t} = e^{-\epsilon_n/T + i\theta_n}$$

$$|\lambda_n|, |\lambda_n|^{1/N_t}$$



Spectral properties of reduced matrix 3

- The gap gives the minimum value of epsilon



$$\lambda_n = l_n^{N_t} = e^{-\epsilon_n/T + i\theta_n}$$

- Lattice setup
  β=1.86(same value of a)
  8^3xNt, mps/mV=0.8,
  RG-improved gauge and
  clover-Wilson fermion with
  Nf=2

zero temperature limit

Fermion determinant

- Let us rewrite det Δ, using the properties of λ

$$\det \Delta(\mu) = C_0 \xi^{-N_r/2} \prod_{n=1}^{N_r} (\lambda_n + \xi)$$

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$$= C_0 \prod_{n=1}^{N_r/2} (\lambda_n^*)^{-1} \prod_{n=1}^{N_r/2} (1 + \lambda_n \xi^{-1})(1 + \lambda_n^* \xi)$$

$$|\lambda| < 1$$

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$$|\lambda| < 1$$

$$\lambda = e^{-\epsilon/T + i\theta}$$

$$= C_0 \prod_{n=1}^{N_r/2} (\lambda_n^*)^{-1} \prod_{n=1}^{N_r/2} (1 + e^{-(\epsilon_n - \mu)/T + i\theta_n})(1 + e^{-(\epsilon_n + \mu)/T - i\theta_n})$$

analytic in μ and T
->possible to take T=0 limit

Fermion determinant

- Consider zero temperature limit

$$\det \Delta(\mu) = C_0 \prod_{n=1}^{N_r/2} (\lambda_n^*)^{-1} \qquad \epsilon(= -T \ln|\lambda|) > 0$$

$$\times \prod_{n=1}^{N_r/2} (1 + \underbrace{e^{-(\epsilon_n - \mu)/T + i\theta}}_{\to 0,\, \infty})(1 + \underbrace{e^{-(\epsilon_n + \mu)/T - i\theta}}_{\to 0})$$

Fermion determinant

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- Inequality $\quad e^{-(\epsilon_{\min} - \mu)/T} > e^{-(\epsilon_n - \mu)/T}$

  - If $\mu < \varepsilon_{min}$, then all the μ-dependence vanishes.

$$\det \Delta(\mu) = C_0 \prod_{n=1}^{N_r/2} (\lambda_n^*)^{-1}$$

Is ε_{min} $m_{\pi}/2$ or $M_N/3$?

Hadron masses in reduction formula

- Correlation function for charged meson

$$\langle tr(QQ^\dagger)\rangle = Ce^{-maN_t}$$

- Small eigenvalues contribute to hadron correlators.
- low temperature limit (large Nt)
  - m goes to the lightest mass.
  - Q is dominated by the eigenvalue near the gap.

$$\langle e^{-2\epsilon_{\min}aN_t}\rangle = C_1 e^{-m_\pi a N_t}$$

Phase vanishes

$$\epsilon_{\min} \sim m_\pi/2$$

Hadron masses in reduction formula

- Correlation function for baryon

$$\langle trQQQ \rangle = C_2 e^{-MaN_t}$$

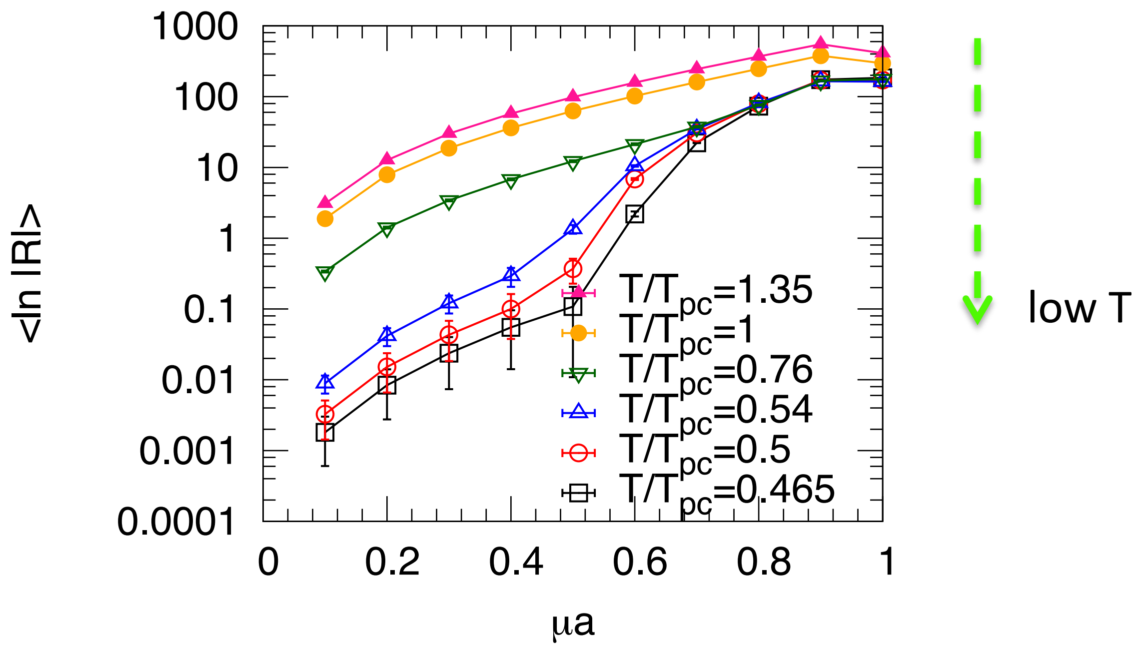
$$\langle e^{3(-\epsilon_{\min} aN_t + i\theta_n)} \rangle = C_2 e^{-M_N aN_t}$$

Phase survives

$$M_N > 3\epsilon_{\min}$$

Numerical result

$$R = \left(\frac{\det \Delta(\mu)}{\det \Delta(0)} \right)^2 = |R| e^{i\theta}$$



- μ-dependence appears at μa = 0.5(= about $m_\pi/2$)

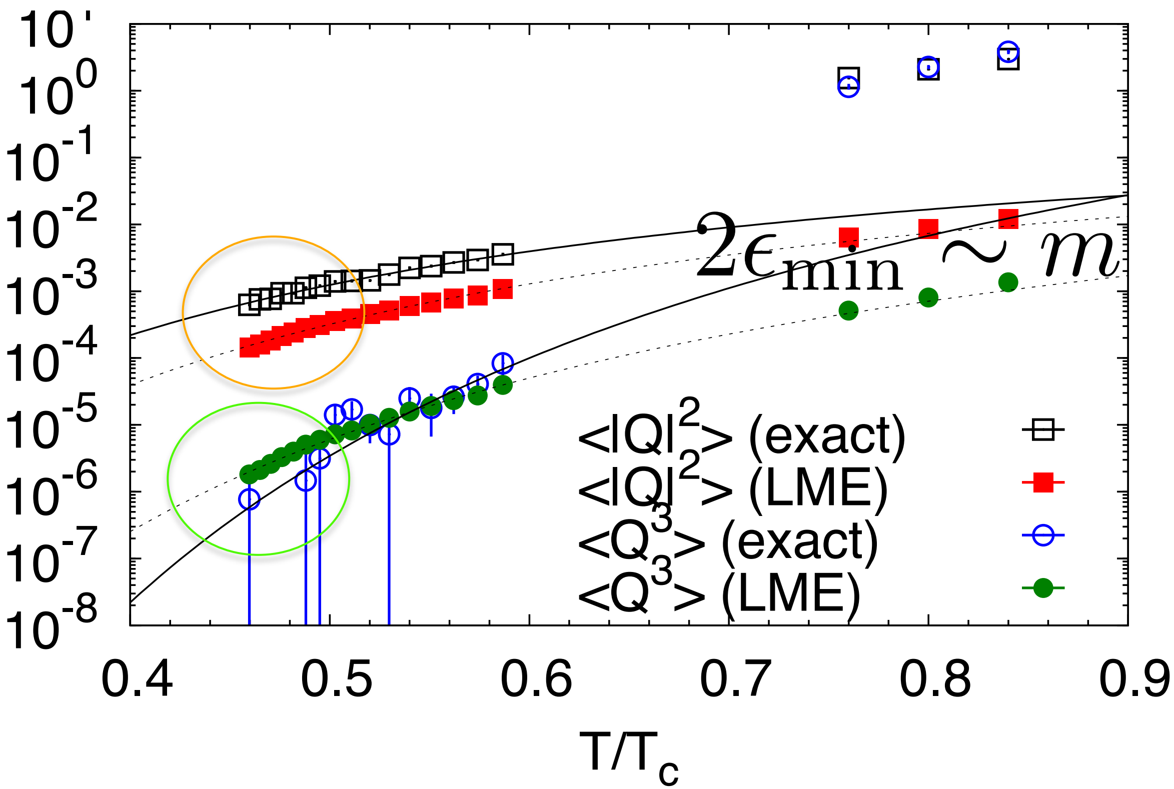
Numerical result

$$\langle tr(QQ^\dagger)\rangle = Ce^{-maN_t}$$

$$\langle trQQQ\rangle = C_2 e^{-MaN_t}$$

Low Mode approximation(LME)

$$Q \sim e^{-\epsilon_{\rm min}/T + i\theta_n}$$



$$m \sim 2\epsilon_{\rm min}$$

$$M > 3\epsilon_{\rm min}$$

Summary

- Using the reduction formula, we consider the property of the fermion determinant at T=0.
  - fermion determinant is μ-independent at T=0
  - threshold for μ-independence is given by $m_\pi/2$
- a lot of things to do
  - numerical confirmation
  - why does the quark number keep to be zero up to 1/3 M_N