Conformal window on the lattice

伊藤　悦子 (KEK)

PTEP (2013) 083B01 and arXiv:1307.6645[hep-lat]

「離散的手法による場と時空のダイナミクス」@KEK 2013/9/27

Summary of our work

- Study SU(3) gauge theory coupled to 12 massless fundamental fermions
- Measured the running coupling constant in Twisted Polyakov loop scheme and found an IR fixed point
- Derived the universal quantities around the fixed point
    - critical exponent of the beta fn.
    - mass anomalous dimension

Introduction

Conformal fixed point is the most important object in QFT

Scalar field theory

2d Gaussian fixed point

unitary discrete series

minimal model (IR fixed point of Landau-Ginzburg model)

3d Gaussian fixed point

Wilson-Fisher fixed point

non-linear sigma model

4d Gaussian only...

scaling dim. for the primary field

central charge

``c function”

H.Kawai and Y.Kikukawa: Phys.Rev.D83:074502,2011

S.Kamata and H.Suzuki: Nucl.Phys.B854:552-574,2012

Gauge theory

4d Gaussian fixed point

nontrivial fixed point?

Introduction

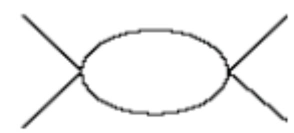
Higgs sector in the Standard Model Lagrangian

$$\mathcal{L}_H \sim \frac{1}{2} D_\mu \phi D^\mu \phi^\dagger + \frac{\lambda}{4}(\phi\phi^\dagger - v^2)^2$$

Problem with a fundamental Higgs boson

Hierarchy problem (need fine-tuning to cancel a quadratic divergence)

Triviality problem



$$\beta(\lambda) = \frac{3\lambda^2}{2\pi^2} > 0$$

No interaction at low energy

Running coupling constant diverges at a finite energy

Cuttoff theory?

Candidates for the origin of Higgs sector

Supersymmetry

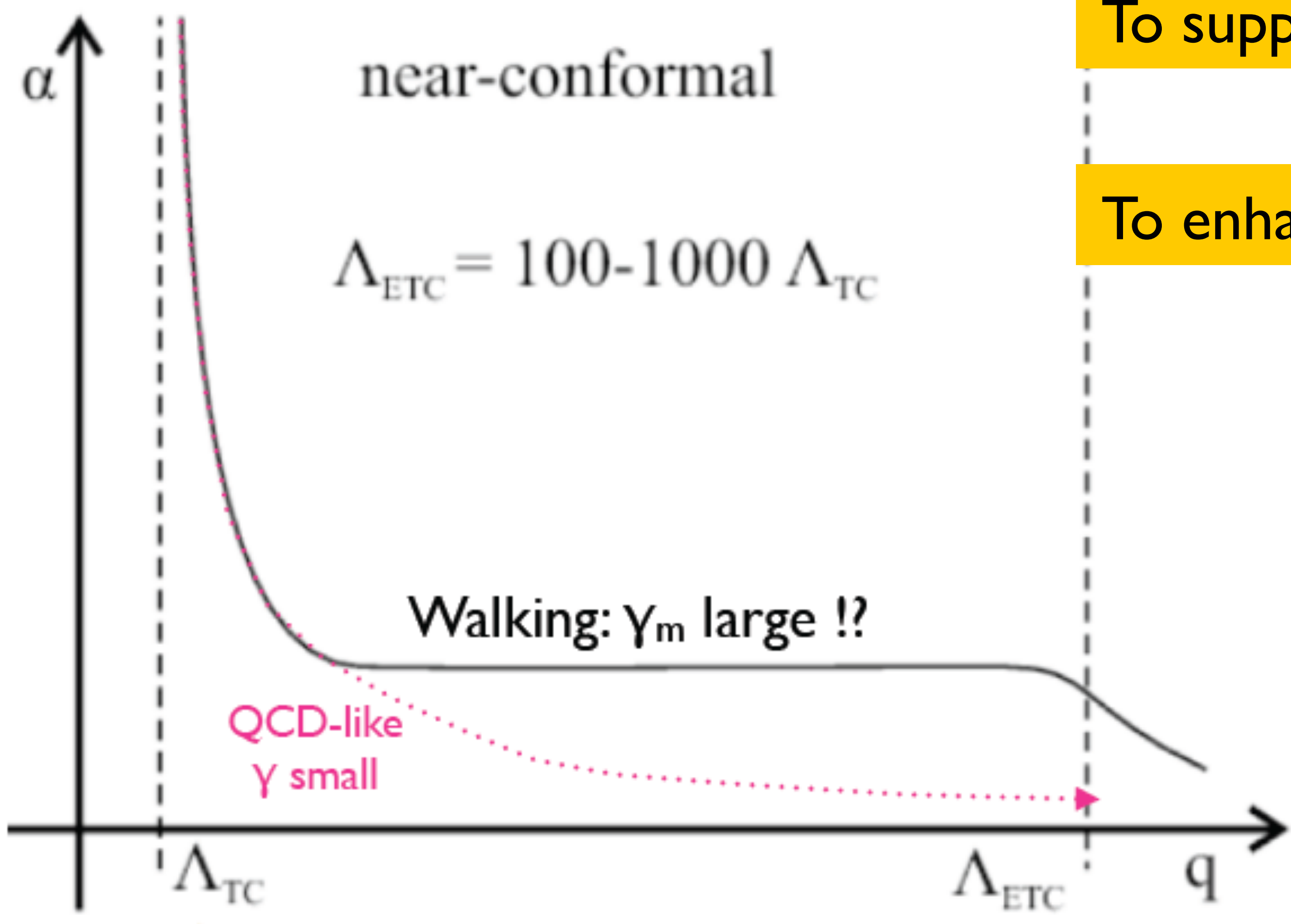
Extra dimension

Walking techni-color

.....

Introduce an additional gauge interaction and fermions

Walking Technicolor



To suppress FCNC process

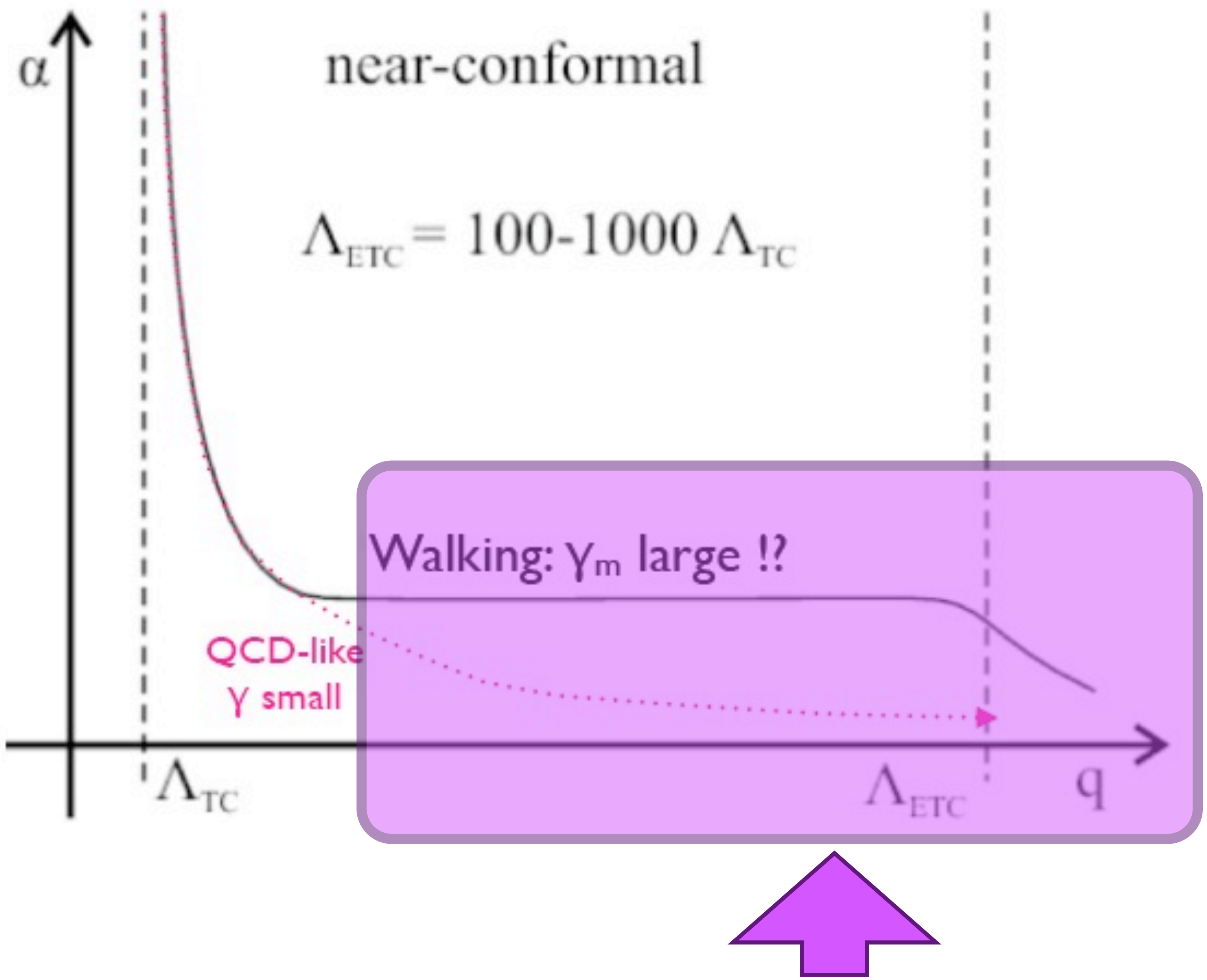
To enhance quark mass

Asymptotic free (UV complete)

IR fixed point (or walking behavior)

Large anomalous dimension for composite state

Is there a theory whose coupling constant show the behavior?

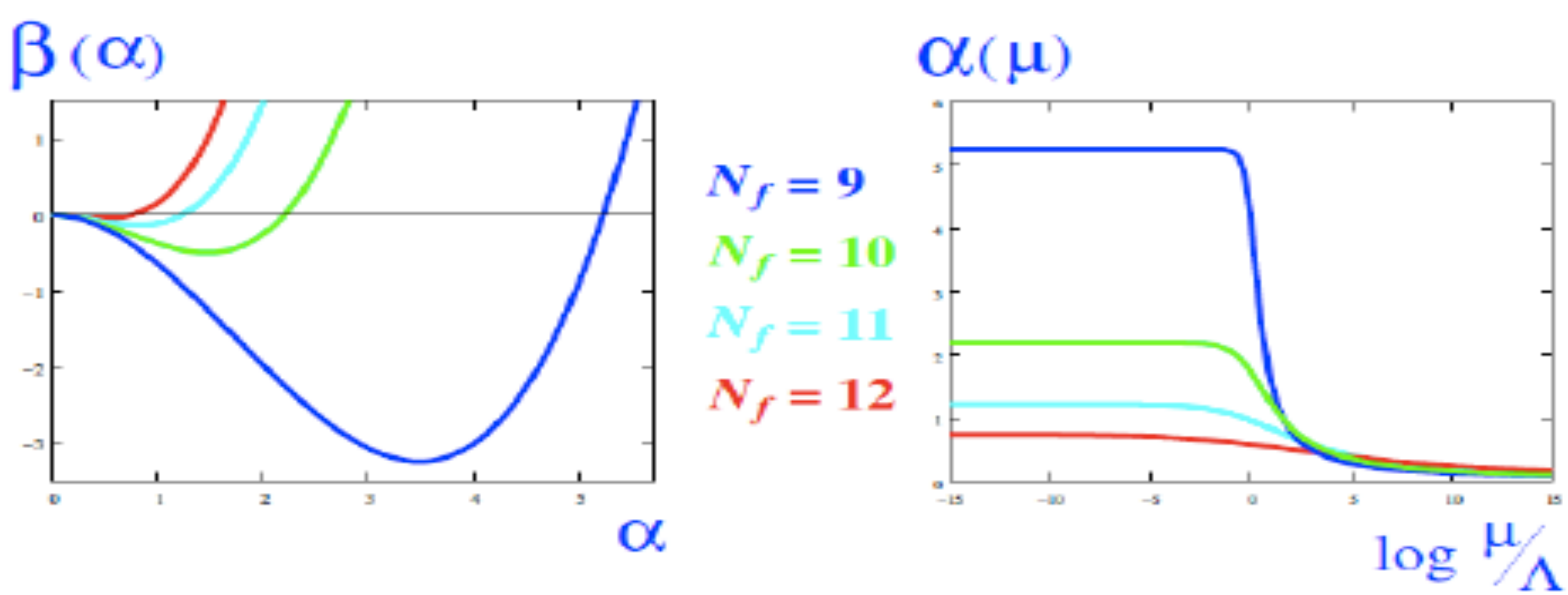


This part may be realized by

many flavor gauge theory.

SU(3) Nf gauge theory

Two loop analysis $\beta(\alpha) = -b\alpha^2 - c\alpha^3$



Perturbative (MS bar scheme)

	2-loop	3-loop	4-loop
(alpha)	0.75	0.44	0.47
(g^2)	9.4	5.5	5.9

T.A.Ryttov and R.Shrock,
Phys.Rev.D83,056011 (2011)
20th order in Wilson loop scheme is also done by Horsley et.al.
Phys.Rev. D86 (2012) 054502

S-D eq. with large Nc

$$N_f^{cr} = 11.9$$

Exact RG

$$N_f^{cr} = 10.0^{+1.6}_{-0.7}$$

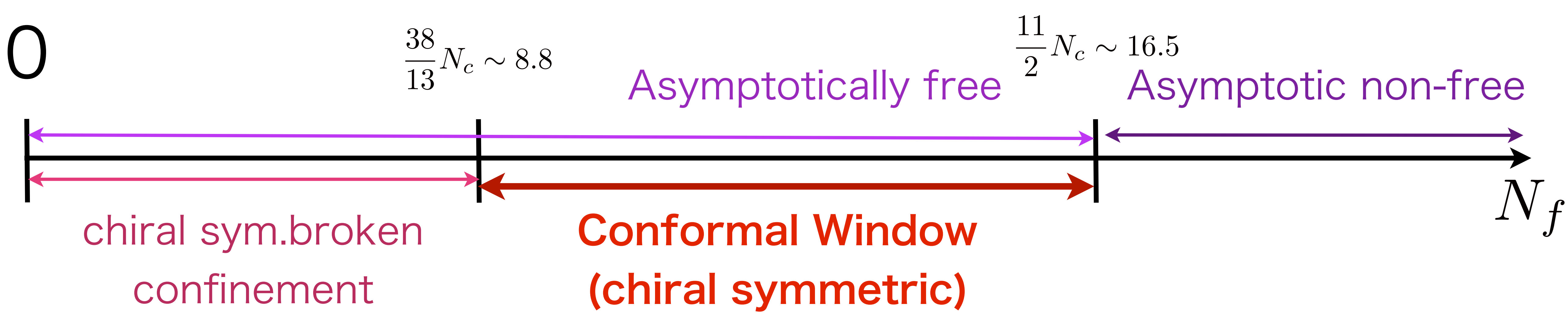
H.Gies and J.Jaeckel,
Eur.Phys.J. G46:433-438,2006

Exact RG (+ 4 fermi interaction)

$$N_f^{cr} = 11.58$$

Y.Kusafuka and H.Terao,
Phys.Rev. D84 (2011) 125006

Phase structure based on two loop



Is there an IR fixed point in SU(3) Nf=12 theory?

Ishikawa, Iwasaki, Nakayama, Yoshie (phase structure, correlation fn.)

Appelquist, Fleming, Neil, M.Lin, Schaich (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura, da Silva (finite temperature)

Cheng, A. Hasenfratz, Petropoulos, Schaich (MCRG, phase structure, Dirac eigenmodes)

DeGrand (mass spectrum)

LatKMI (mass spectrum)

D.Lin, Ogawa, Ohki, Shintani (running coupling)

Fodor, Holland, Kuti, Nogradi, Schroeder, (running coupling, phase structure, spectrum)

Jin and Mawhinney (phase structure)

Why are these studies contradictory?

相互作用をする赤外固定点の見つけ方

(1) Step scaling 法によるrenormalized couplingの測定

Luescher, Weisz and Wolff, NPB 359 (1991) 221

(2) low betaでのchiral symmetry

(3) 質量変形した理論のhyperscaling

mass spectrum

Miransky, PRD59(1999)105003
Luty, JHEP 0904(2009)050
Del Debbio and Zwicky, PRD82(2010)014502

(4) Dirac固有モードのhyperscaling (Volume-scaling)

Patella,PRD86(2012)025006
Cheng, Hasenfratz, Petropulos and Schaich, JHEP1307(2013)061

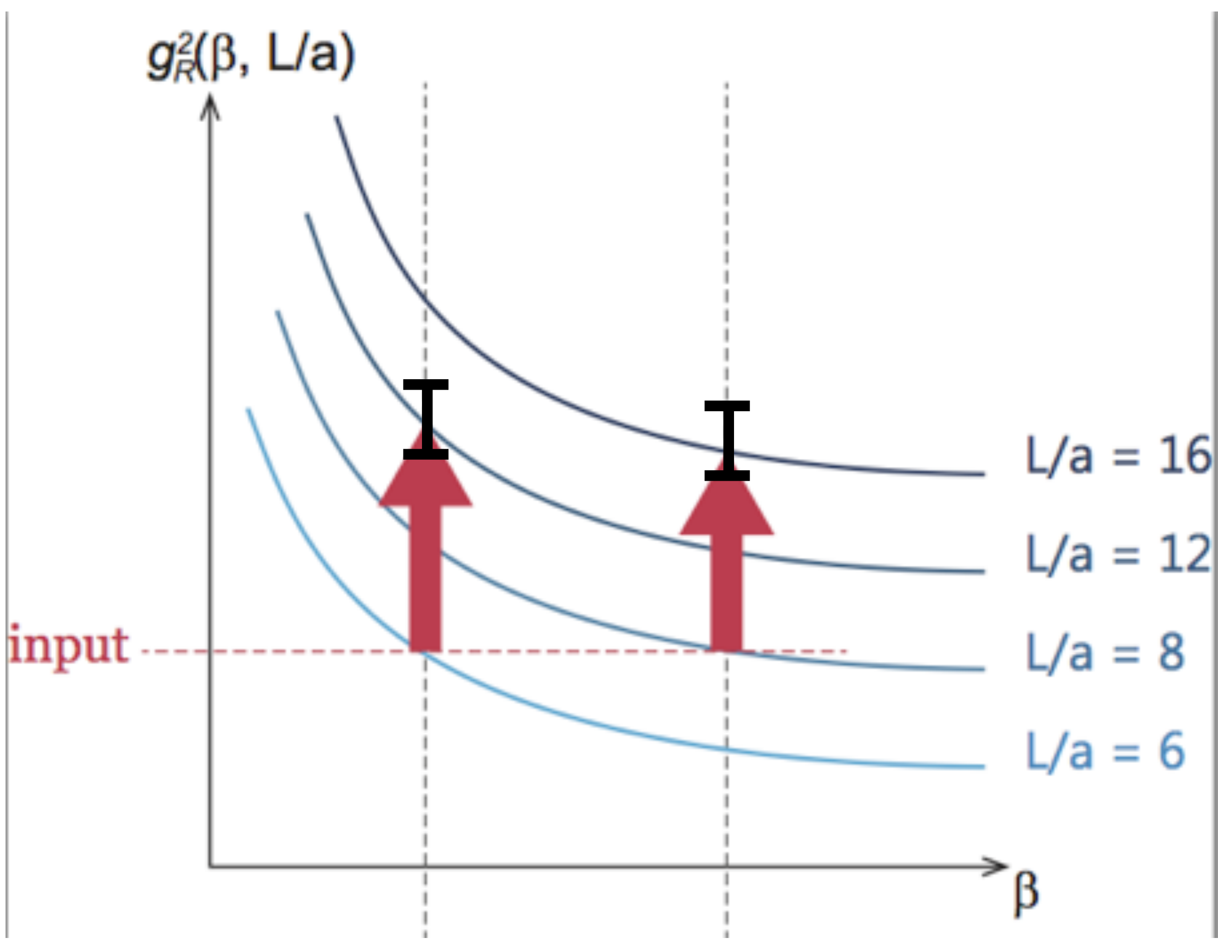
(5) メソン的演算子の相関関数の形

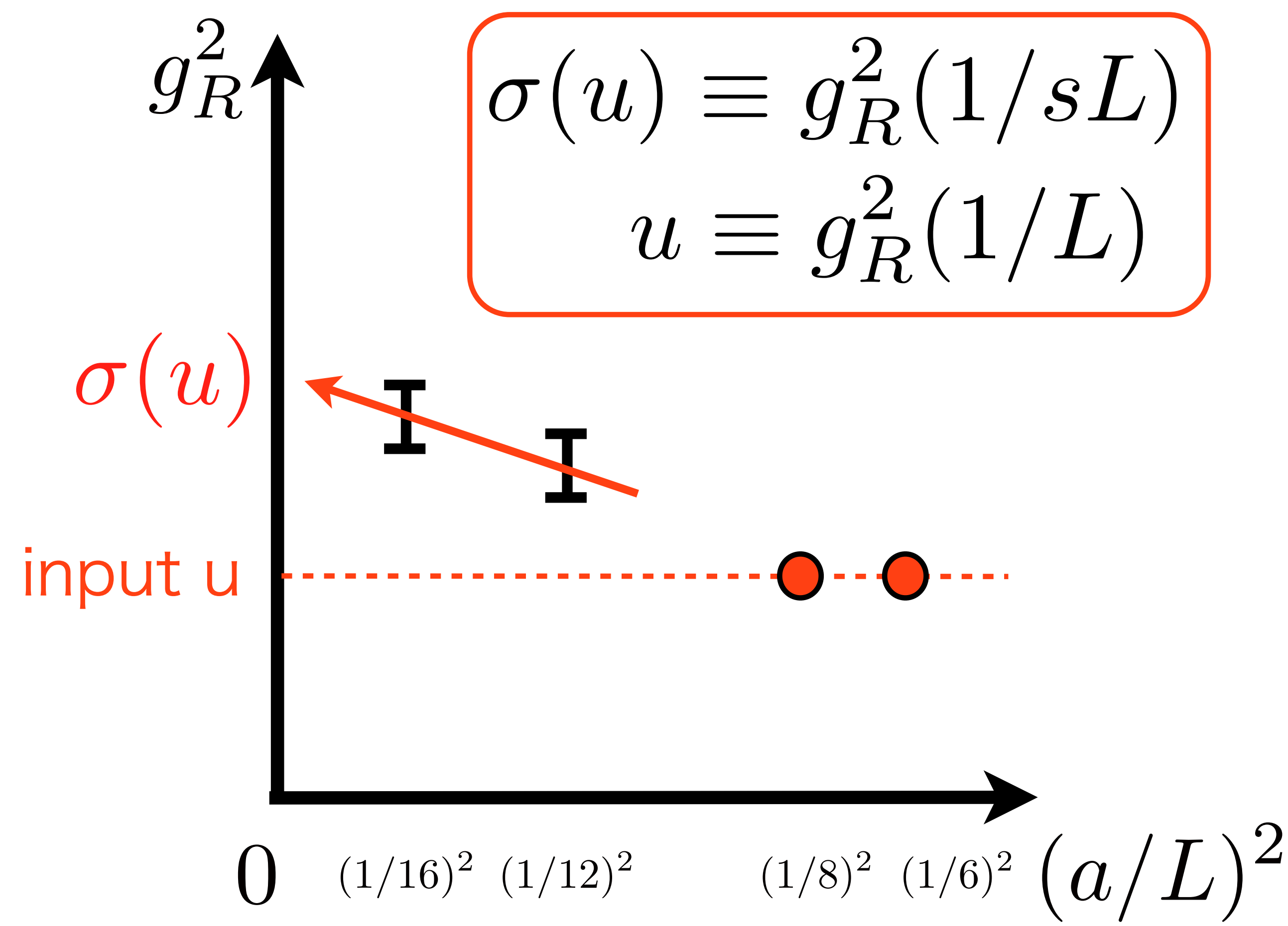
Ishikawa, Iwasaki, Nakayama and Yoshie, PRD87(2013)071503

(1) Step scaling method
- measuring the running coupling constant -

Luescher, Weisz and Wolff, NPB 359 (1991) 221

- tune beta to reproduce the input renormalized coupling
- measure the g^2 at the larger lattice with the tuned beta
- take the continuum limit

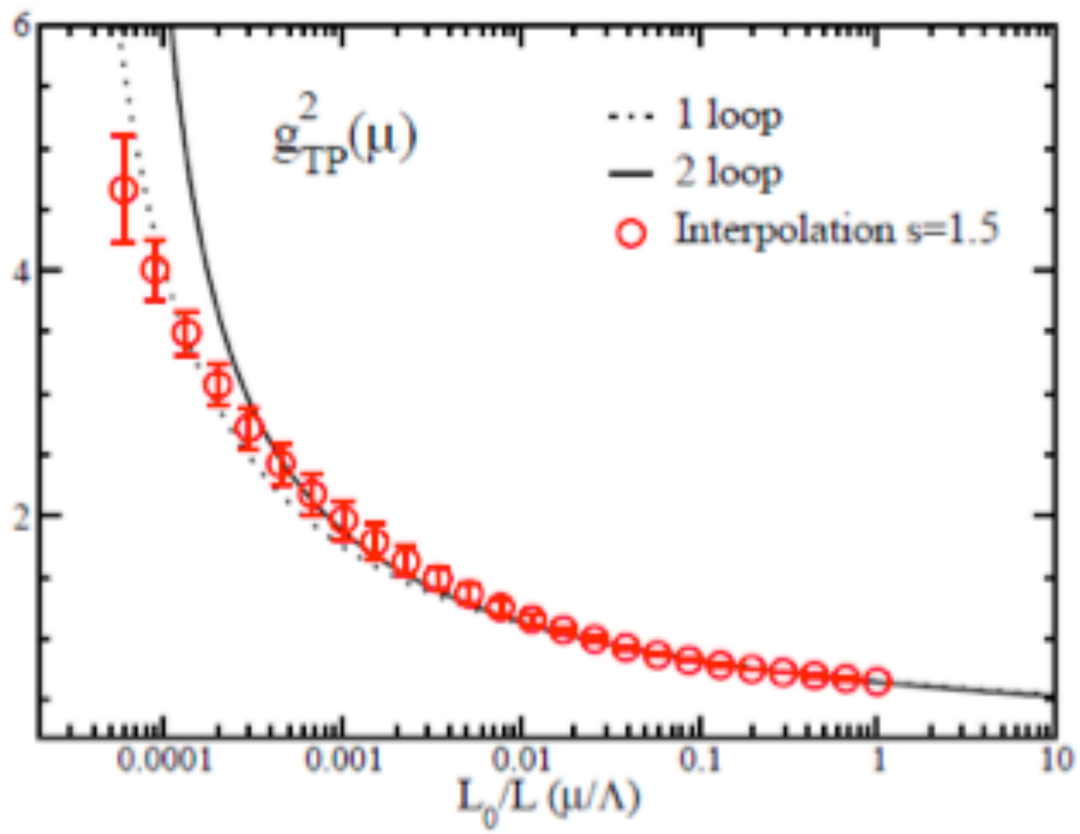




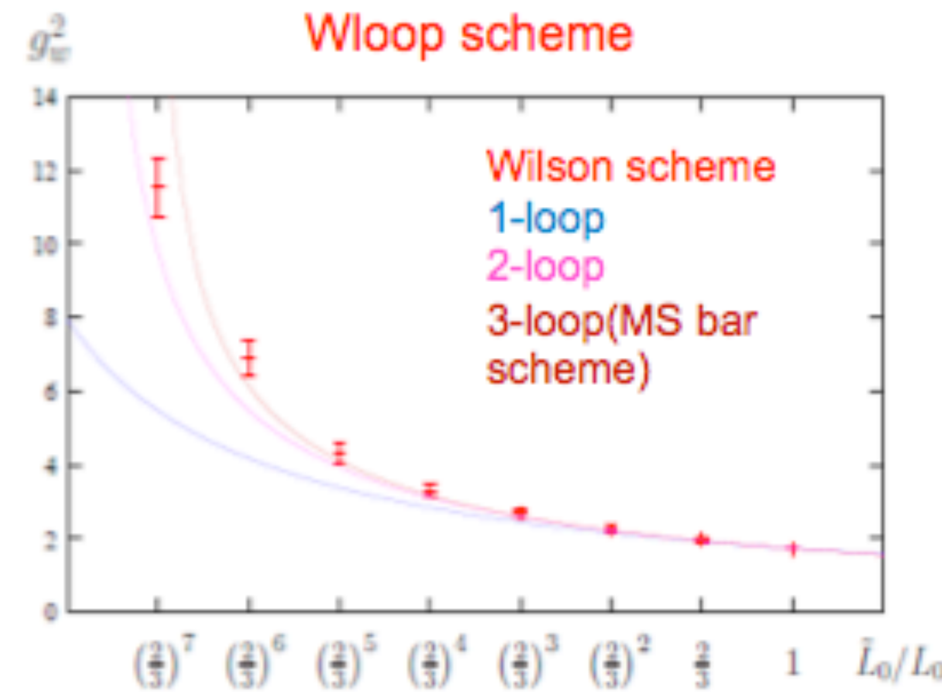
We can apply this method to any renormalization schemes on the lattice.

Running of the renormalized coupling constant in Quenched QCD

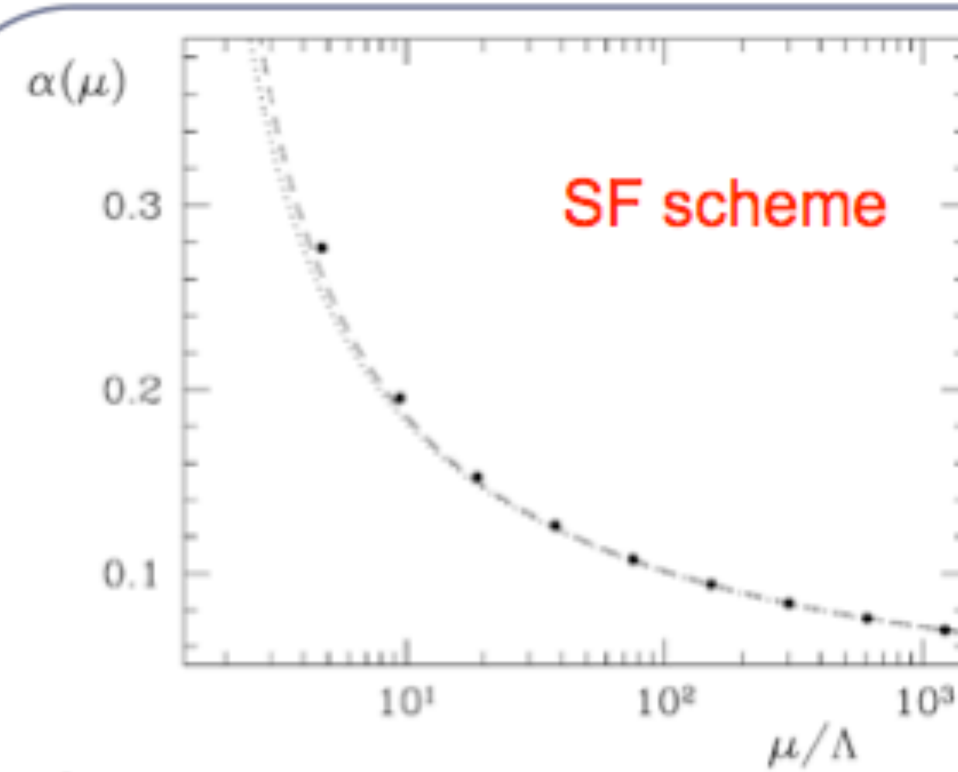
TPL scheme



Wloop scheme



arXiv : 0902.3768 (hep-lat)



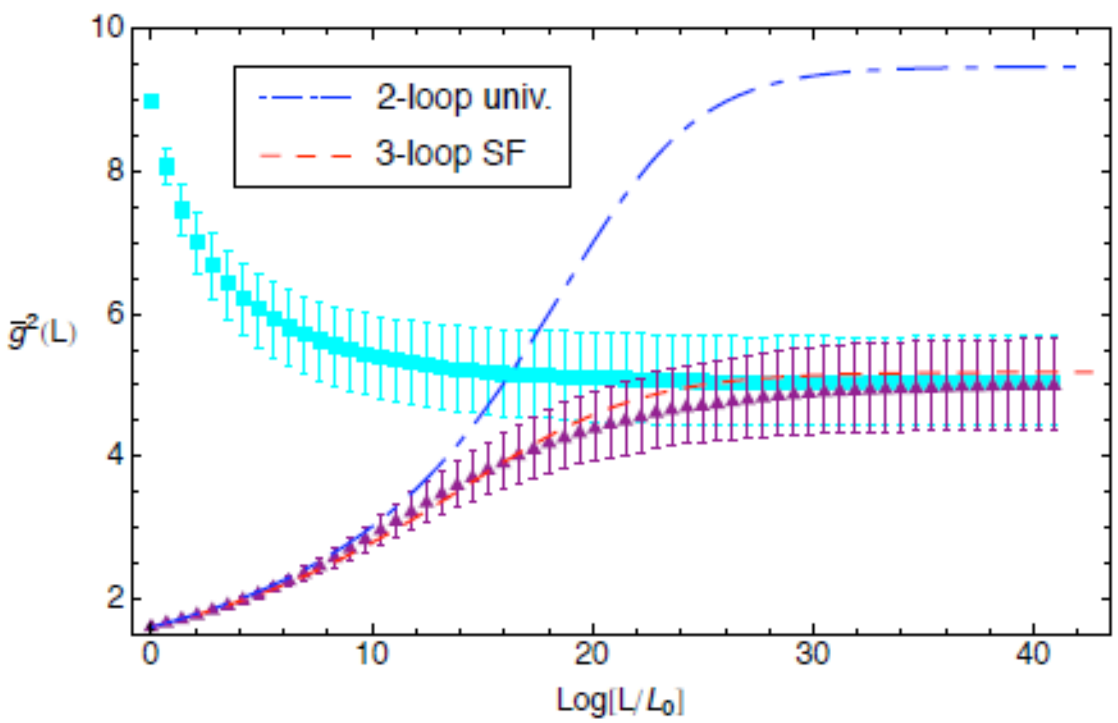
Capitani et.al:NPB B544 : 669-698

Step scaling method

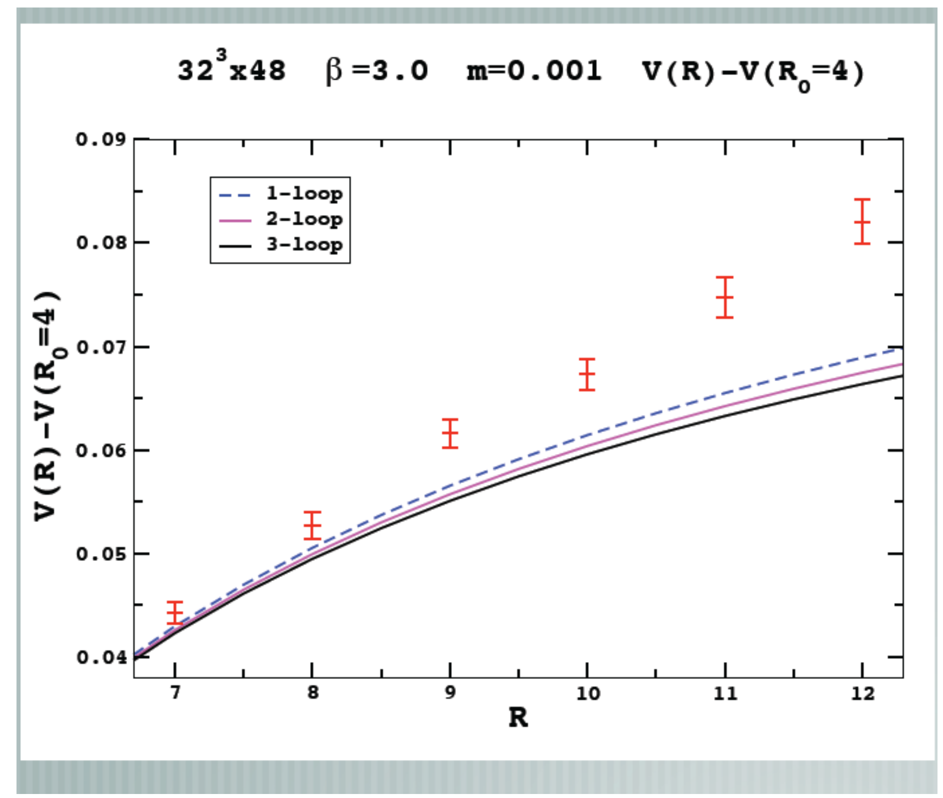
Running coupling constant(Nf=12)

Appelquist et al. (SF scheme)
Phys.Rev.D79:076010,2009

Taking constant extrapolation in (a/L)



Fodor et al. (potential scheme)
PoS LAT2009:055,2009, talk at Lattice2010

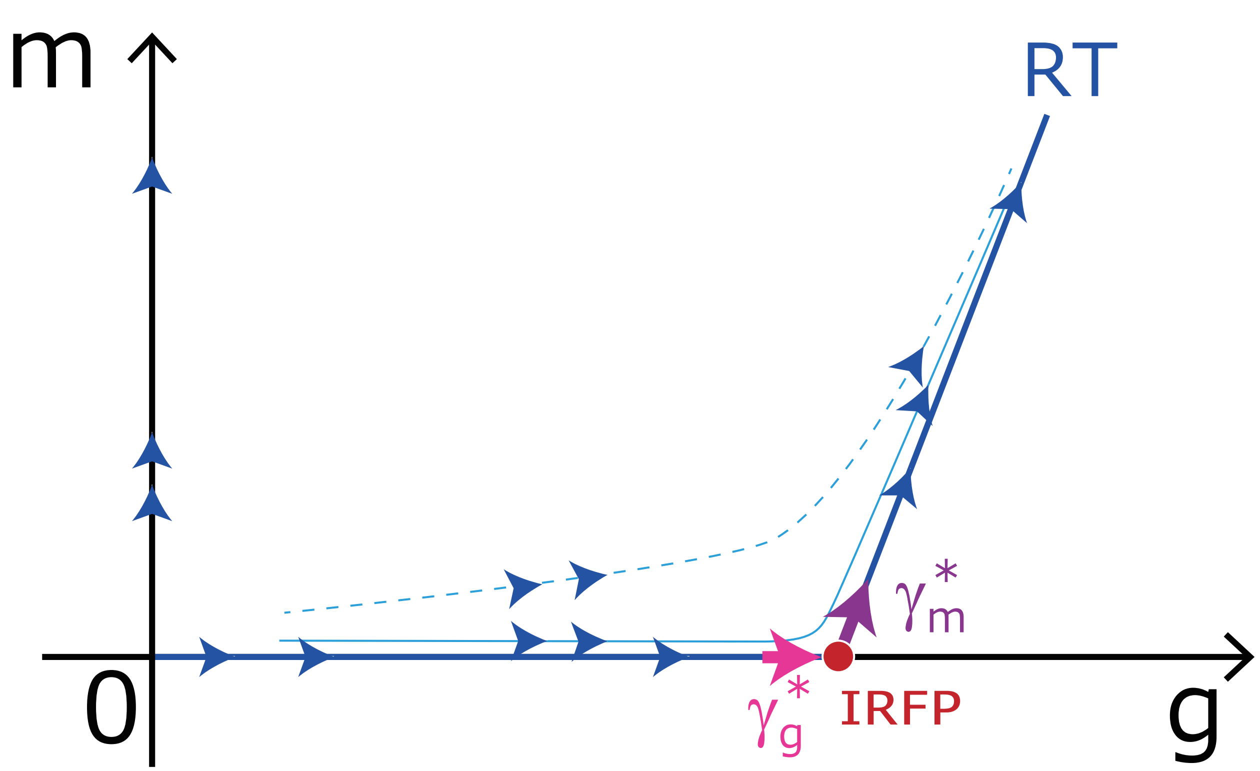


Plot: Slide of K.Holland's talk at Lattice2010

The continuum extrapolation was not considered.

(O(a) effects depends on the renormalization scheme)

Several renormalization schemes and universality



scheme transformation

$$g_1 \to g_2 = f(g_1)$$

$f(g_1)$ is an analytic fn. of g_1

beta fn. $$\beta(g_2) = \frac{\partial f(g_1)}{\partial g_1}\beta(g_1)$$

The existence of the fixed point is scheme independent.

Note that the renormalized coupling constant at the FP depends on the scheme.

$$g_2^* = f(g_1^*)(\neq g_1^*)$$

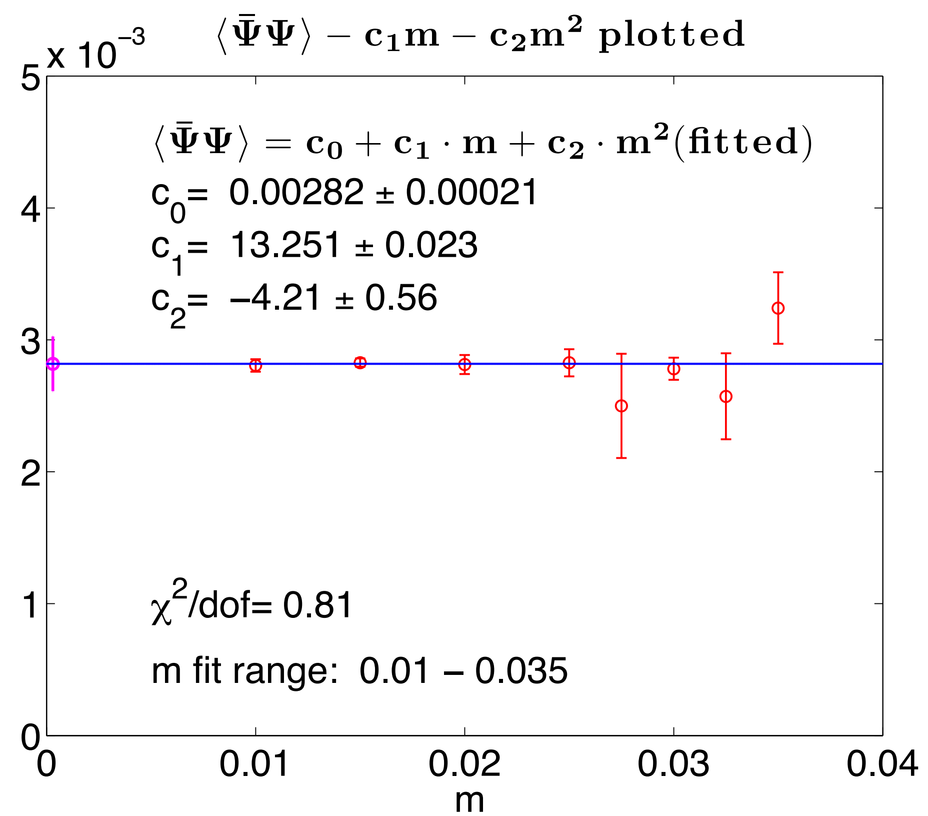
The critical exponents (γ_g^*, γ_m^*) around the fixed point are scheme independent.

(2),(3) Chiral symmetryと Mass deformed theory

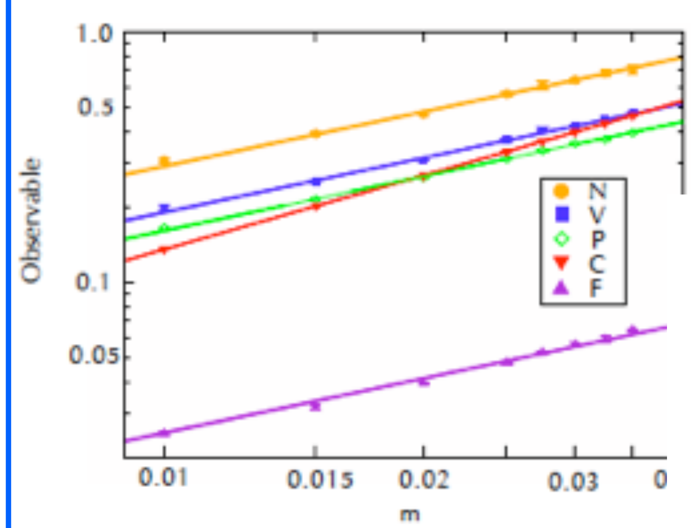
Z.Fodor et al.: Phys.Lett.B703:348-358,2011.

measured mass spectrum and chiral condensate at beta=2.2
in several lattice sizes and fermion bare masses.

Comparison between two hypotheses



Also Jin and Mawhinney's work

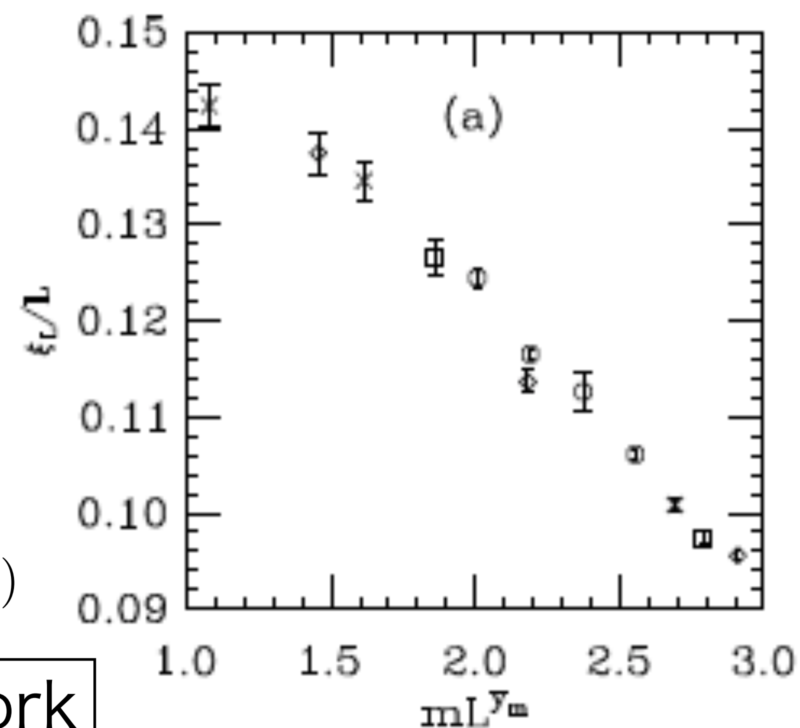


DeGrand:
PRD84 (2011) 116901

$$\frac{1}{M_X L} = F(m_q L^{1+\gamma_m^*})$$

Appelquist et. al:
PRD84(2011)054501

$$M_X \sim m_q^{\frac{1}{1+\gamma_m^*}} + m_q$$



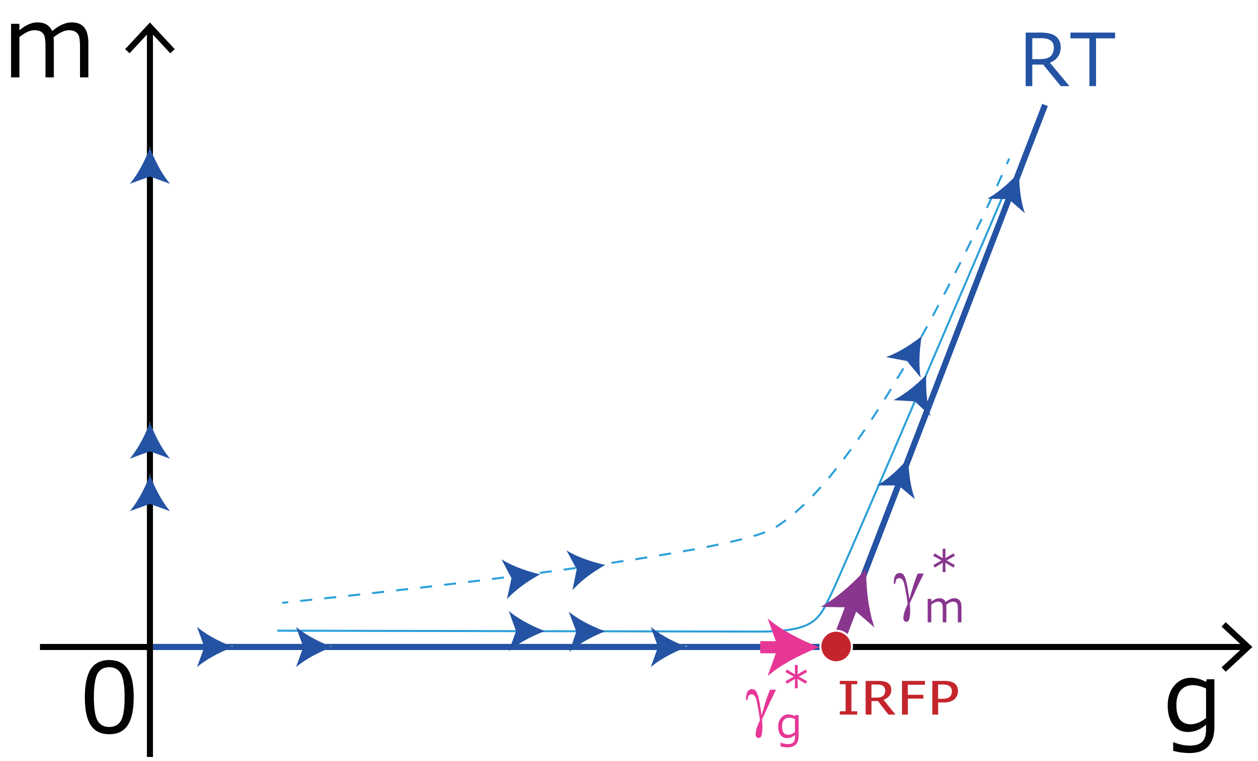
Also LatKMI's work

Both chiral broken hypothesis and conformal hypothesis work well.
It might be hard to show the existence of IRFP from the fit quality.

Mass deformed conformal theory in the continuum limit

If only the mass op. is the relevant op. around the nontrivial fixed point,
a scaling of the mass of the composite op. is described by the mass anomalous dim.

Miransky scaling $M_X \sim m_q^{\frac{1}{1+\gamma_m^*}}$



Remarks

(1) Around fixed point

(Tuning the values of g and ma is needed.
We need two independent observables)

(2) Continuum extrapolation

Fit fn on the lattice

$$M_X = C_X m^{1/(1+\gamma^*)} + D_X m$$

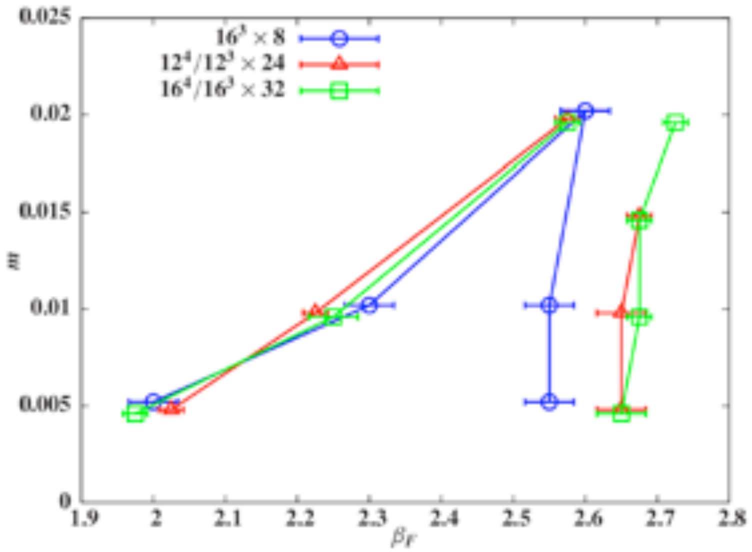
(If gamma is small, then the fit becomes difficult.
If gamma is large, then the other operator might become relevant or marginal.)

(3) Finite volume effect

2loop prediction:

$$\gamma_g^* = 0.36 \quad \gamma_m^* = 0.77$$

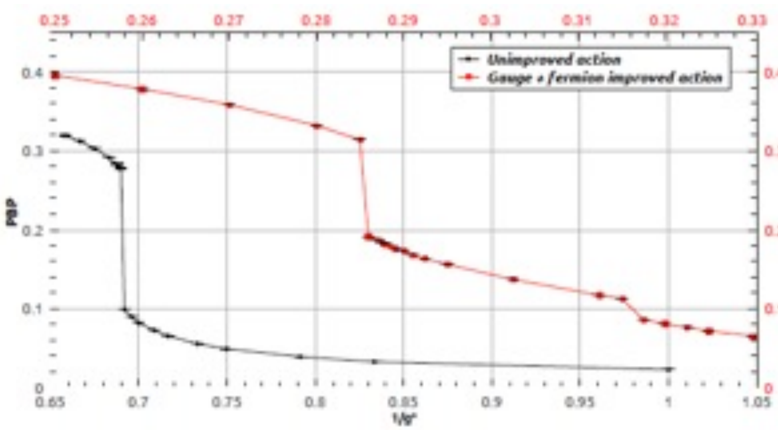
Phase structure on the lattice



Cheng, Hasenfratz and Schaich:
PRD85 (2012) 094509

HYP smearing

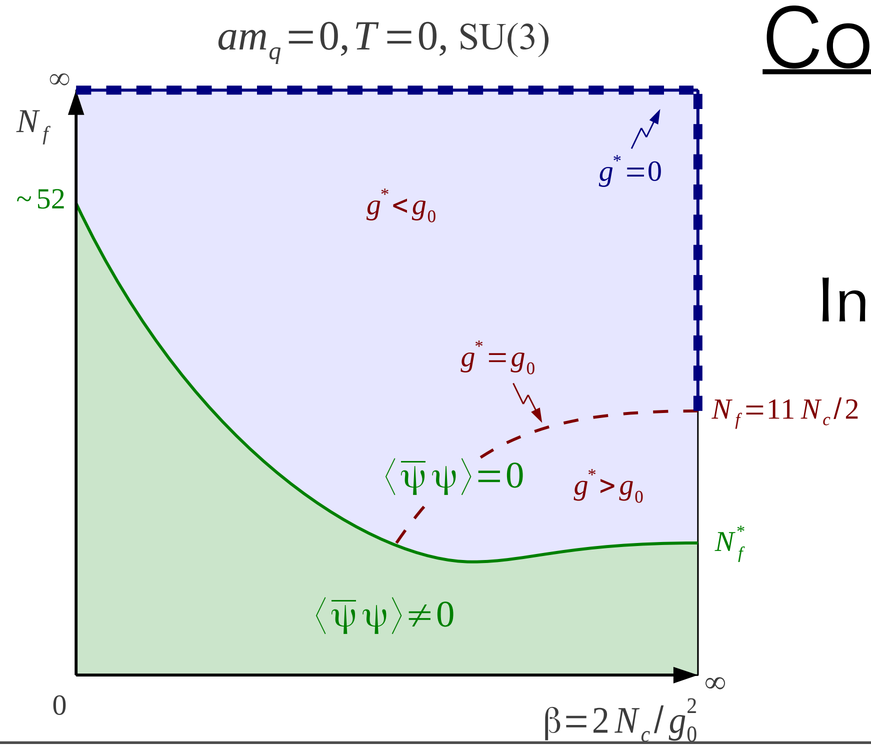
In the intermediate region the shift symmetry is broken.



Deuzeman, Lombardo, da Silva and Pallante:
PLB720(2013)358

Naik improvement
next-to-nearest neighbor terms are no longer irrelevant
and indeed modify the pattern observed without improvement.

Conjectured phase diagram



de Forcrand, Kim and Unger:
JHEP 1302(2013)051

In the strong coupling limit, the chiral symmetry is broken Nf<52.

In the studies on the phase structure,
a careful parameter search is important for
each lattice setup.

(4) Volume scaling for the Dirac eigenmodes

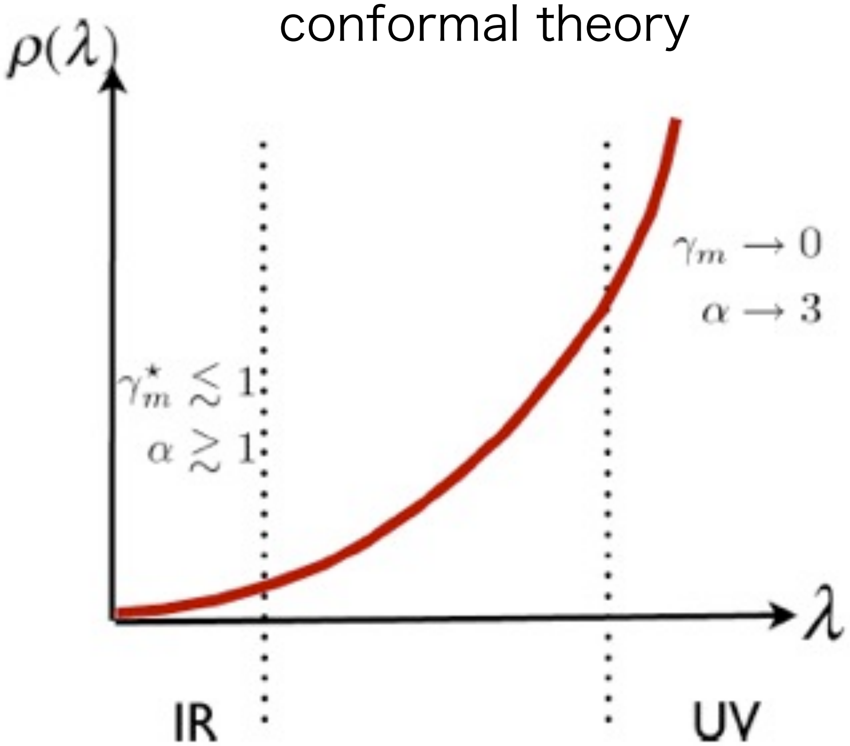
In the massless theory, 1/L gives IR cutoff. Instead of the mass, there is a scaling of (1/L). In the chiral limit, a single observable suffice to find the IR conformality.

Del Debbio and Zwicky, PRD82(2010)014502
Patella,PRD86(2012)025006
de Forcrand, Kim and Unger, JHEP 1302(2013)051

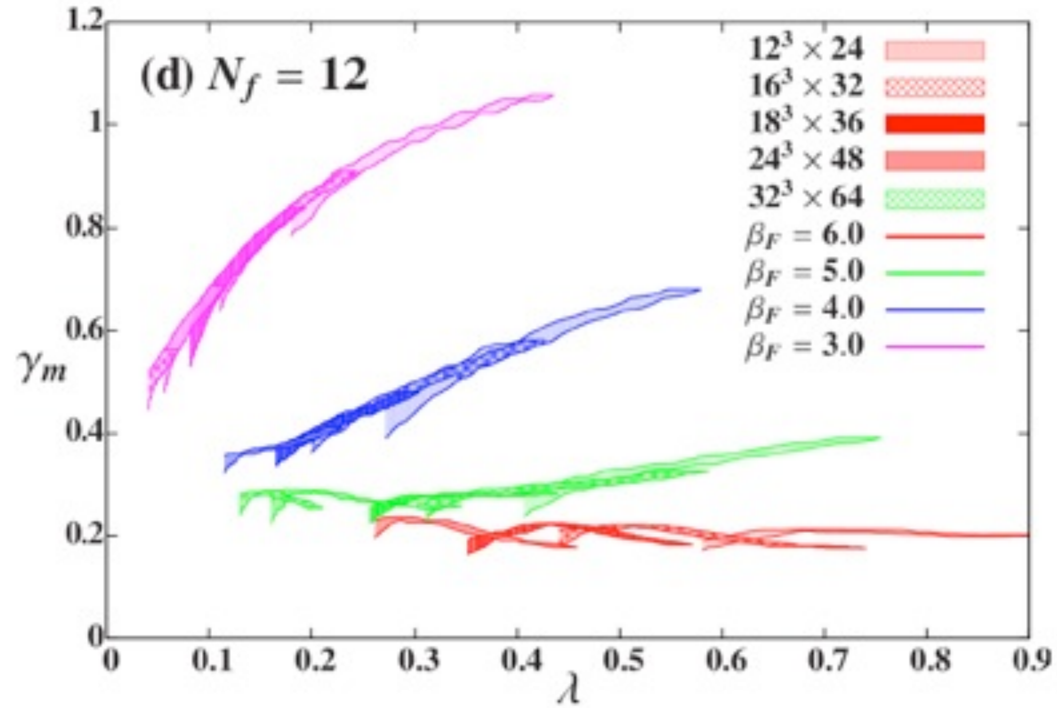
Dirac eigenvalues $D(x,y)\psi_\lambda = i\lambda\psi_\lambda$ At tree level: $\lambda^2 = 4\sum \sin^2\frac{\hat{k}}{2} \sim (\frac{a}{L})^2$

$$\nu(\lambda) = V\int_{-\lambda}^{\lambda}\rho(\omega)d\omega \propto V\lambda^{\frac{4}{1+\gamma_m(\lambda)}}$$

eigenvalue density in IR conformal theory



Cheng, Hasenfratz, Petropulos and Schaich, JHEP1307(2013)061



IR limit

$\lambda \to 0$ ➡ $L \to \infty$

$\gamma_m(\lambda) \to \gamma_m^*$

Nf=8 still questionable.
walking?
IRFP with the other relevant op.?

Studying the volume dependence is important.

(5) Correlation fn. of nearly conformal theory

Ishikawa, Iwasaki, Nakayama and Yoshie: arXiv:1301.4785

two point fn. of a meson state $$G_H(t) = \sum_x \langle \bar{\psi}\gamma_H\psi(x,t)\bar{\psi}\gamma_H\psi(0)\rangle$$

conformal theory (massless, continuum) $$G_H(t) = \tilde{c}\,\frac{1}{t^{\alpha_H}} \qquad \alpha_H = 3 - 2\gamma^*$$

confined theory

$$G_H(t) = c_H\ \exp(-m_H t)$$

(nearly) conformal theory

$$G_H(t) = \tilde{c}_H \frac{\exp(-\tilde{m}_H t)}{t^{\alpha_H}}$$

To extract the mass spectrum, we have to use the Yukawa-type fit fn.

Our result

PTEP (2013) 083B01

Simulation detail

Hybrid Monte Carlo algorithm

Wilson gauge action+ naive staggered fermion

beta=4.0--100 on (L/a)^4 lattice

L/a=6,8,10,12,16,20

Twisted boundary condition for x,y directions

Link variable $U_\mu(x+\hat{\nu}L/a) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger$ $\quad \mu = x,y,z,t$, $\nu = x,y$

Fermion $\psi_\alpha^a(x+\hat{\nu}L/a) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu)^\dagger_{\beta\alpha}$

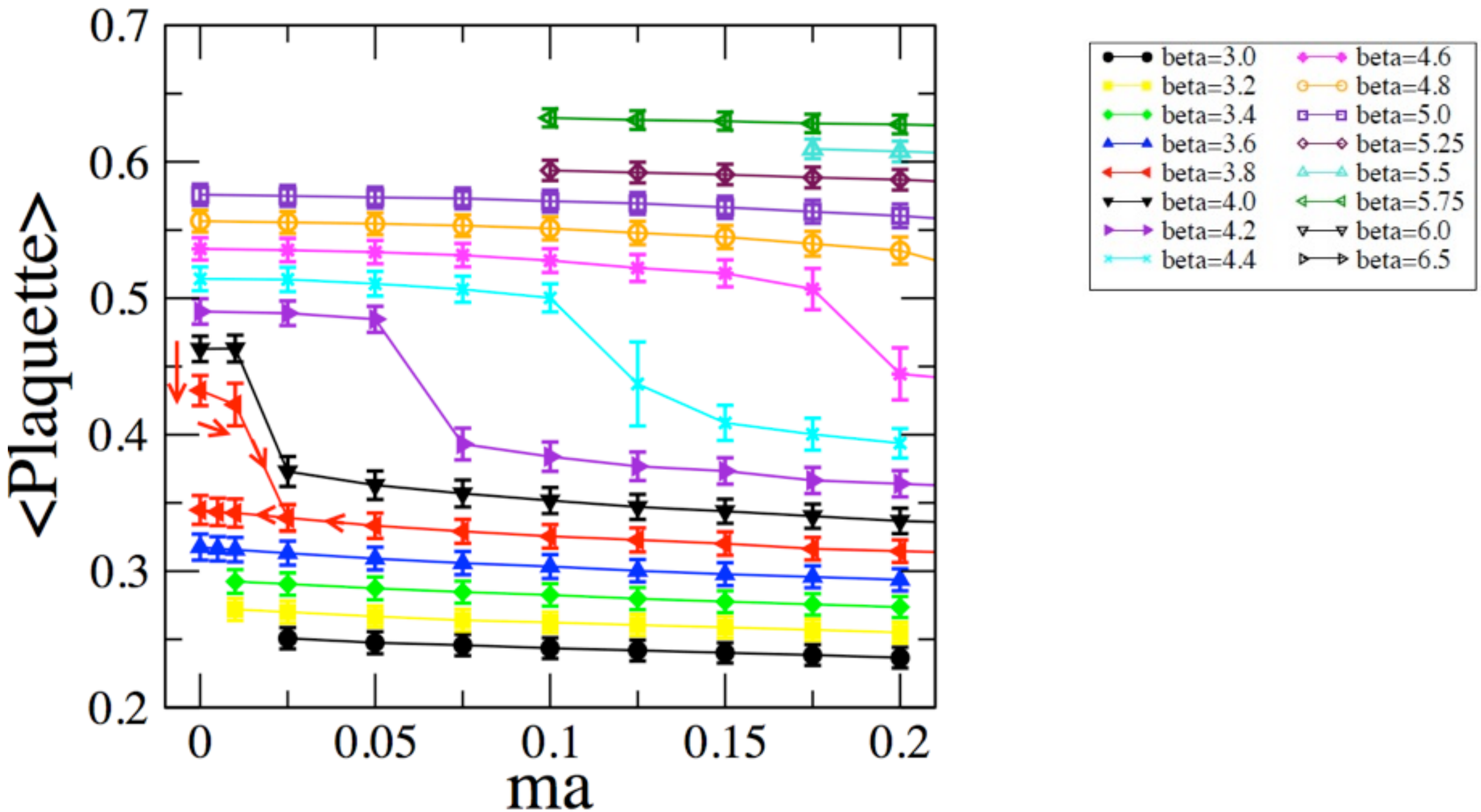
Ω_ν is twist matrices (3x3 complex matrix)

$$\Omega_\nu\Omega_\nu^\dagger = \mathbb{I}, (\Omega_\nu)^3 = \mathbb{I}, \mathrm{Tr}[\Omega_\nu] = 0, \Omega_x\Omega_y = e^{i2\pi/3}\Omega_y\Omega_x$$

Phase diagram in the lattice setup

In our simulation set up,

there is a bulk phase transition in small mass region.



$$(L/a)^4 = 4^4$$

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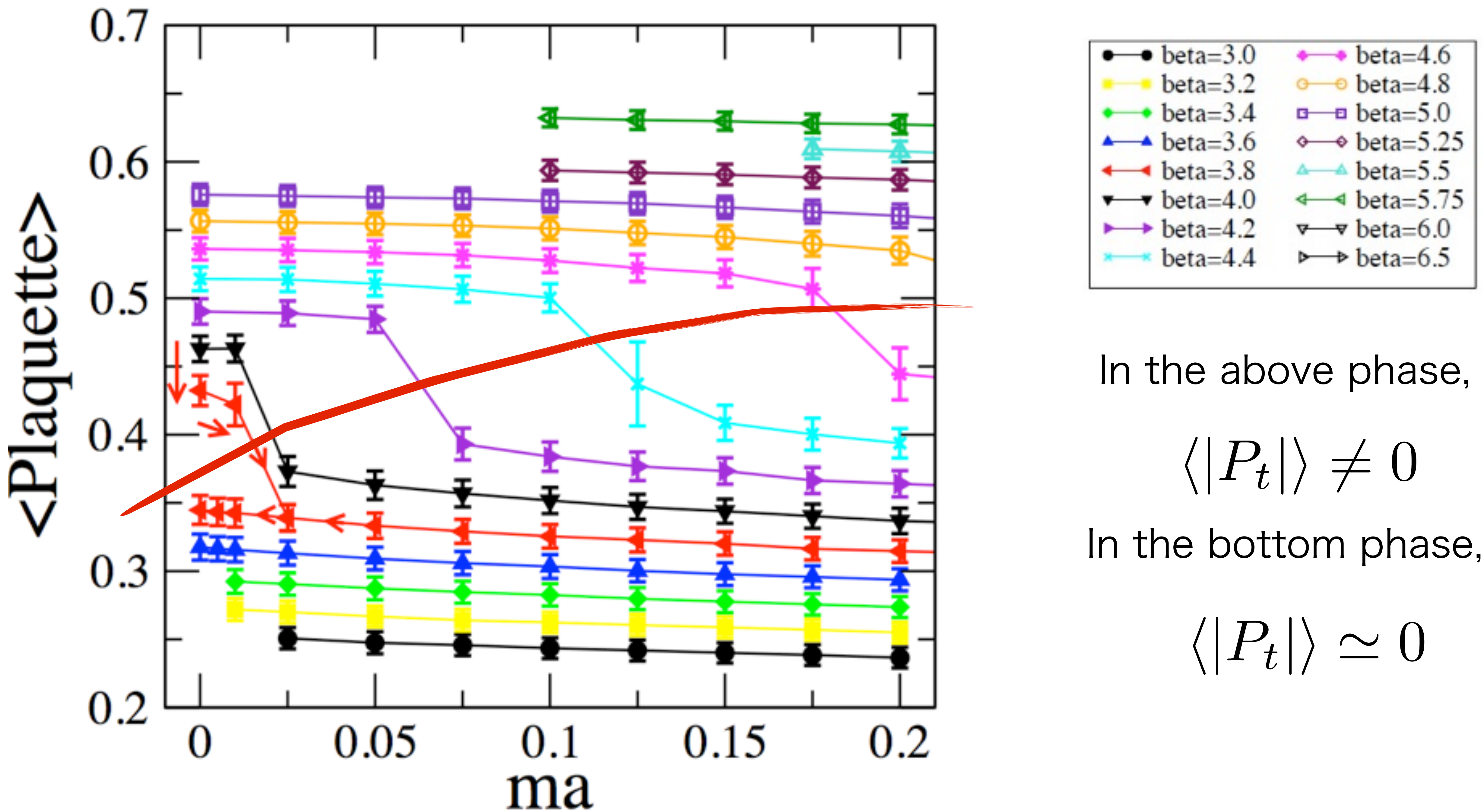


$$(L/a)^4 = 4^4$$

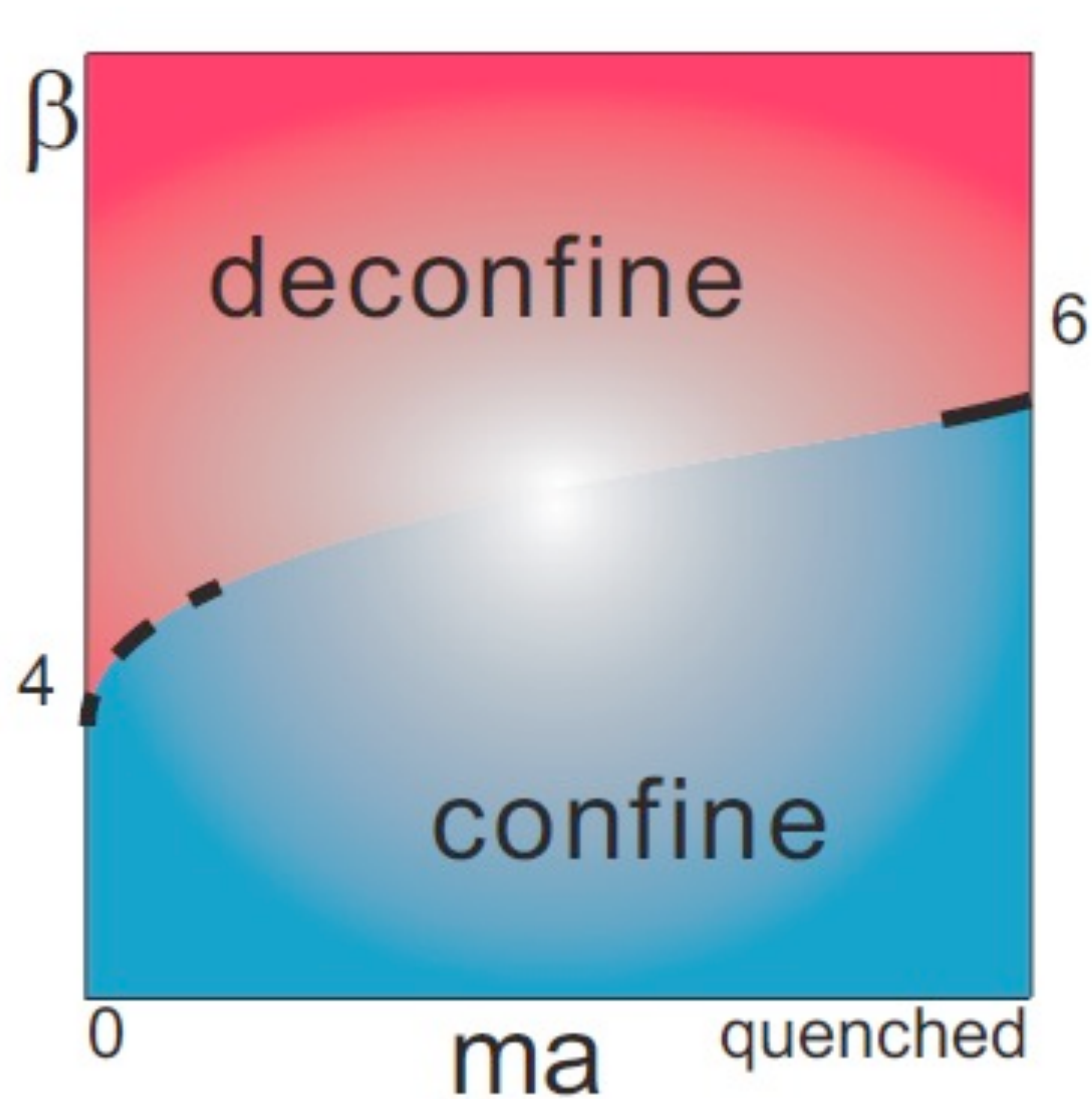
In the above phase,

$$\langle |P_t| \rangle \neq 0$$

In the bottom phase,

$$\langle |P_t| \rangle \simeq 0$$

Phase diagram for SU(3) Nf=12 naive staggered fermion with the twisted boundary condition.



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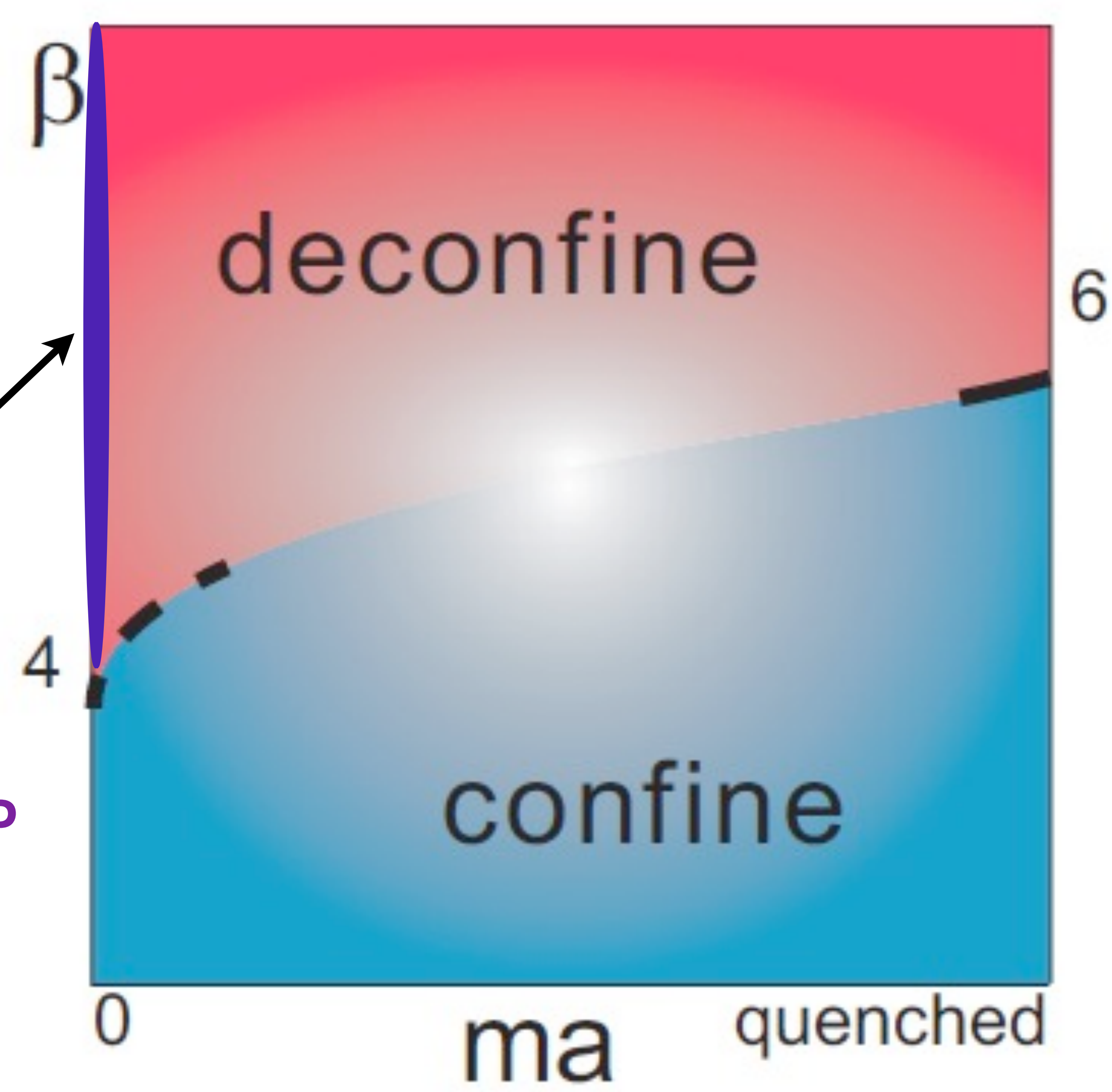


Our simulation
to search for the IRFP

We also see that the chiral symmetry is preserved in this region.

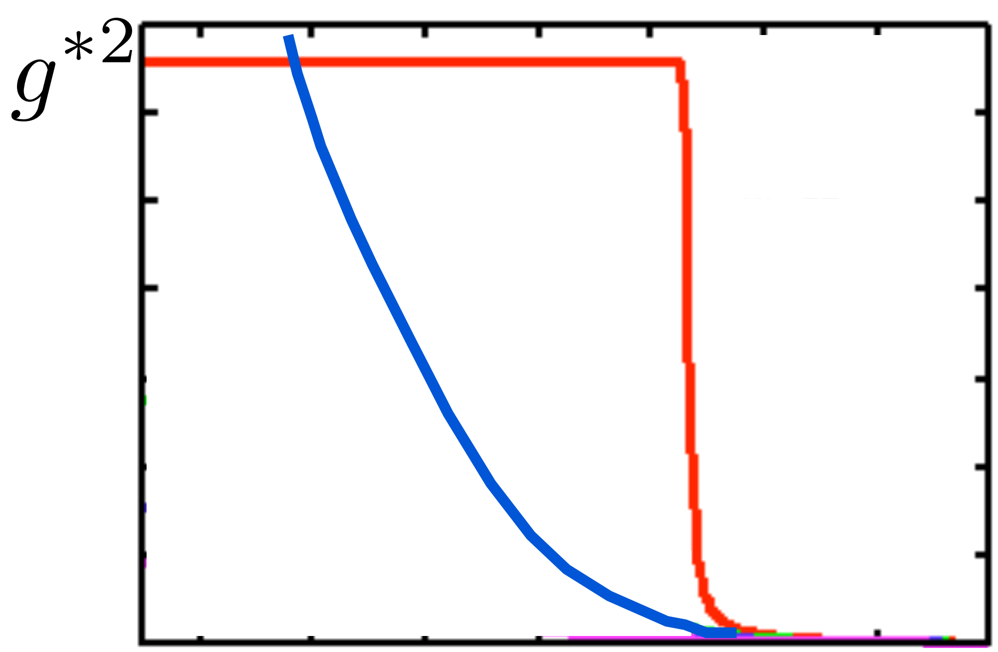
Running coupling

Measuring the growth ratio

Observe the growth ratio of renormalized coupling constant
to see the precise running behavior.

running coupling constant

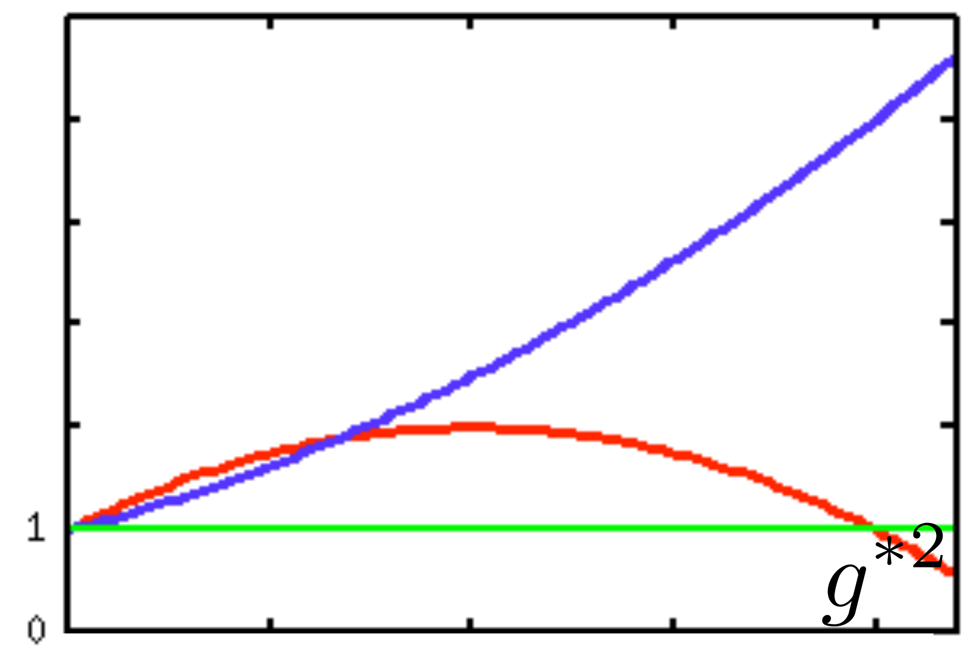
g_R^2



$\ln(L_0/L)$

growth ratio

$\sigma(u)/u = g_R^2(1/sL)/g_R^2(1/L)$



$g_R^2(\mu = 1/L)$

systematic error is accumulated

systematic error is not accumulated

Twisted Polyakov loop (TPL) scheme

Examples of renormalization scheme

Schroedinger functional scheme
Wilson loop scheme
Twisted Polyakov Loop scheme
Wilson flow scheme....

} no O(a/L) error scheme

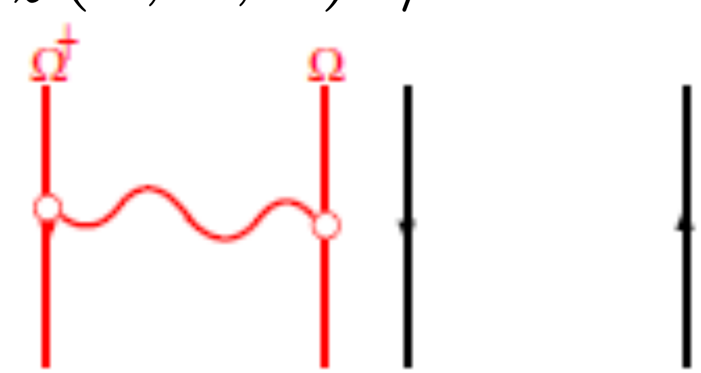
Definition of Twisted Polyakov loop (TPL) scheme on the lattice

de Divitiis, Frezotti, Gaugnelli and Petronzio, NPB422(1994)382

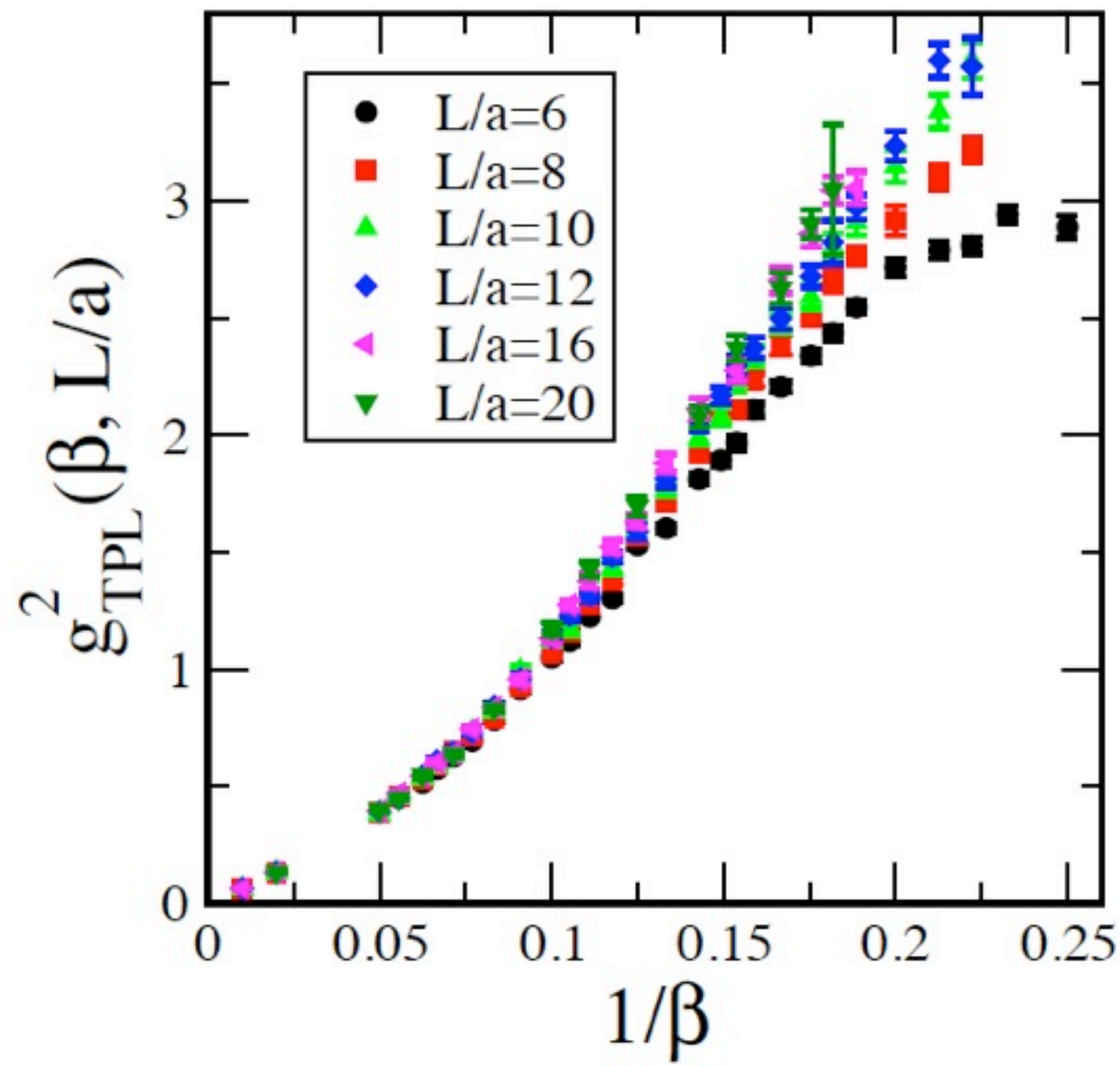
$$g^2_{\mathrm{TPL}} = \lim_{a \to 0} \frac{1}{k_{\mathrm{latt}}} \frac{\langle \sum_{y,z} P_x(y,z,L/2a) P_x(0,0,0)^\dagger \rangle}{\langle \sum_{x,y} P_z(x,y,L/2a) P_z(0,0,0)^\dagger \rangle}$$

k_{latt} is determined by the tree level value to satisfy

$$g^2_{\mathrm{TPL}}\big|_{\mathrm{tree}} = g_0^2$$



Raw data in TPL scheme



2-3 % statistical error.

# of Trj is 64,400- 1,892,800.

Fitting fn. for beta interpolation

$$g^2_{TPL}(\beta, L/a) = \frac{6}{\beta} + \sum_{j=1}^{N} \frac{C_j(L/a)}{\beta^{j+1}}$$

s=1.5 step scaling

L/a=6 -> L/a=9

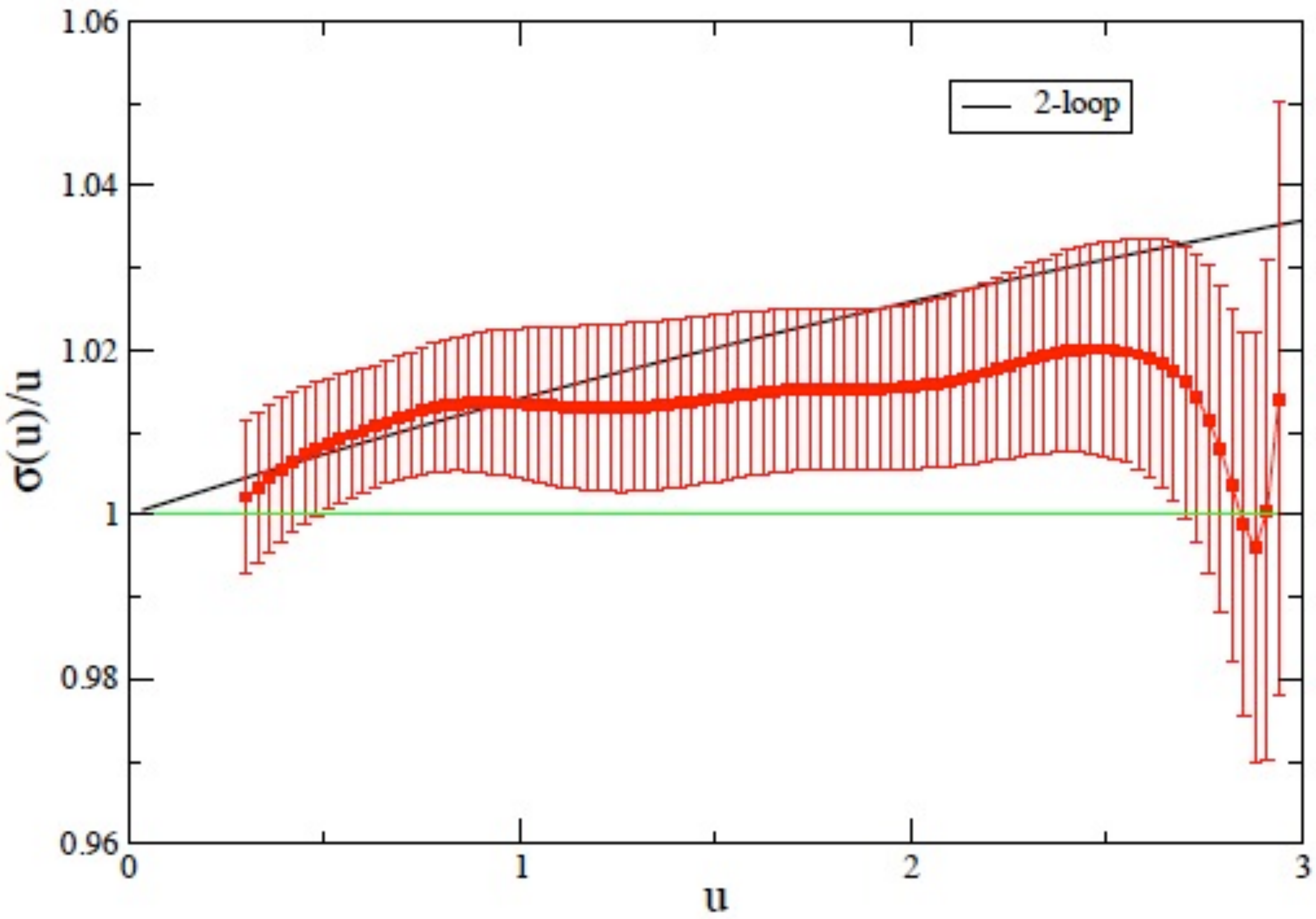
L/a=8 -> L/a=12

L/a=10 -> L/a=15

L/a=12 -> L/a=18

For L/a = 9, 15 and 18,
we estimate values of g2 for a given beta
by the linear interpolation in (a/L)2.

Growth ratio of TPL coupling
(global fit analysis)



TPL coupling shows the fixed point around

$$g^{*2}_{\mathrm{TPL}} \sim 2.7$$

This is the first zero point of the beta function from the asymptotically free region,
it must be IR fixed point.

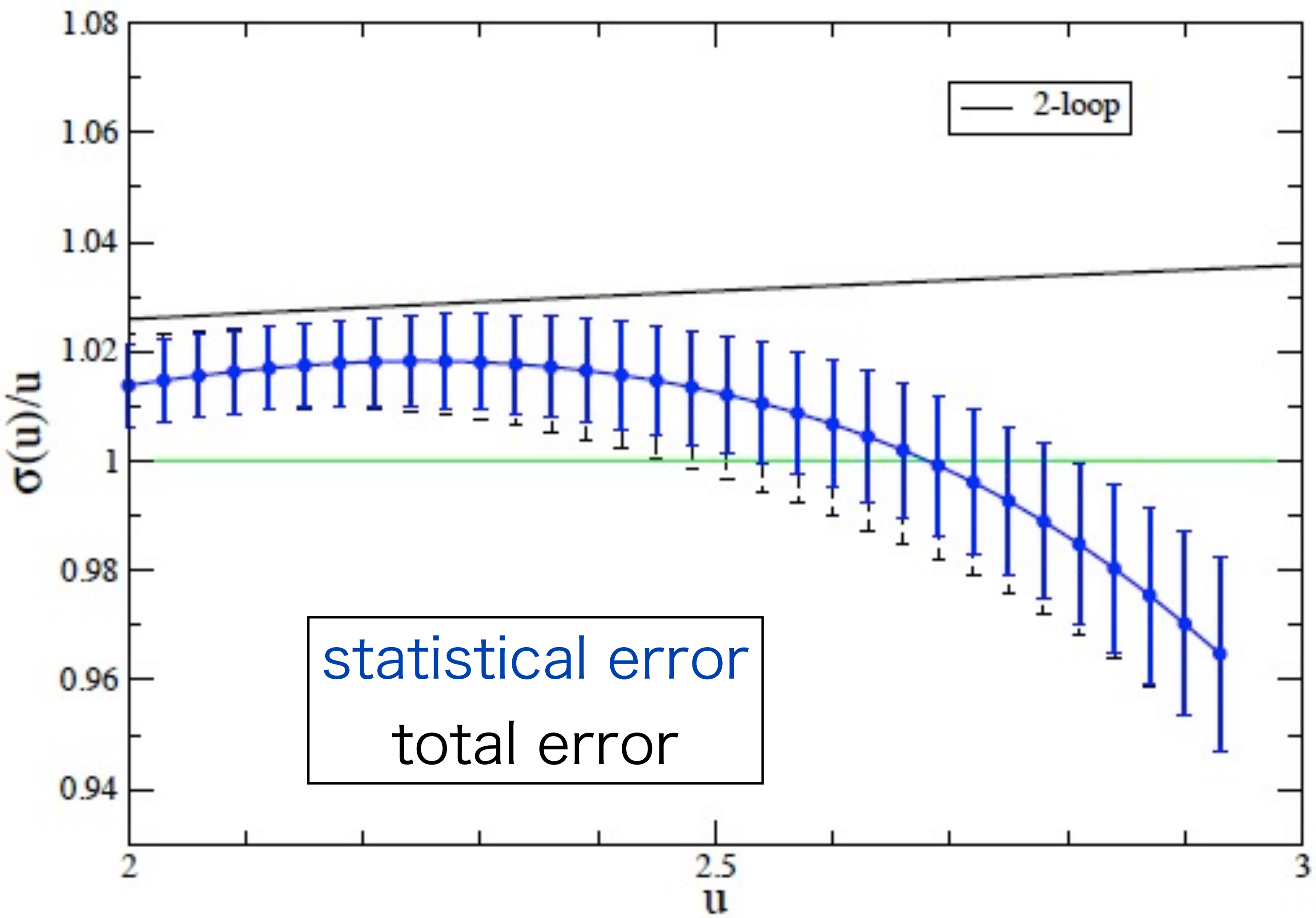
Unfortunately, the growth ratio with errorbar does not cross over the unity line.

Local fit analysis

Focus on the low beta region (u>2.0)

Add the data (more than 30 data points)

Growth ratio of TPL coupling
(local fit analysis)



$$g^{*2}_{\rm TPL} = 2.69 \pm 0.14({\rm stat.})^{0}_{-0.16}({\rm syst.})$$

Around the fixed point, the beta fn. can be approximated by the linear fn.

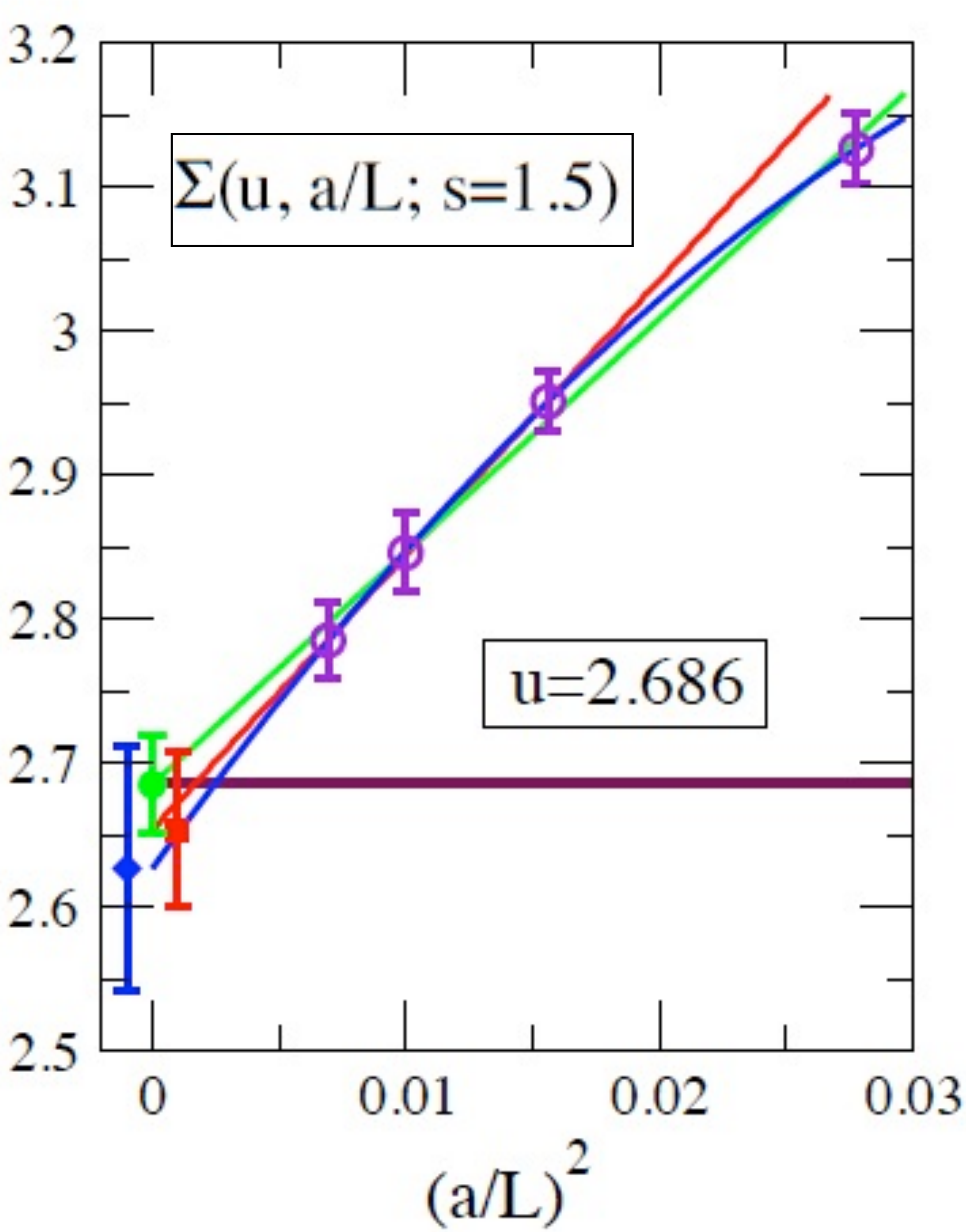
$$\beta(g^2) \sim \gamma_g^*(g^{2*} - g^2)$$

Our result

$$\gamma_g^* = 0.57^{+0.35}_{-0.31}({\rm stat.})^{0}_{-0.16}({\rm syst.})$$

SF scheme	2 loop at $g^{2*} = 9.4$	4 loop (MS bar)
$\gamma_g^* = 0.13 \pm 0.03$	$\gamma_g^* = 0.36$	$\gamma_g^* = 0.28$

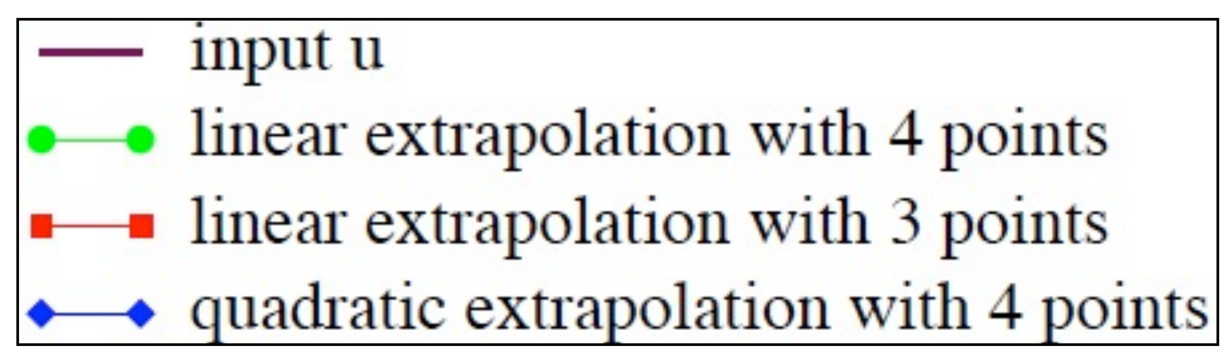
Continuum extrapolation



s=1.5 step scaling

L/a=6 -> L/a=9
L/a=8 -> L/a=12
L/a=10 -> L/a=15
L/a=12 -> L/a=18

2 loop prediction in this region is $\sigma(u = 2.69) \sim 2.78$



The systematic error is small in the strong coupling region in this scheme.

(Fit range dependence and “s” (step scaling parameter) dependence are also small.)

Is there an IR fixed point in SU(3) Nf=12 theory?

Ishikawa, Iwasaki, Nakayama, Yoshie (phase structure, correlation fn.)

Appelquist, Fleming, Neil, M.Lin, Schaich (running coupling, mass spectrum)

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Appelquist, Fleming, Neil, M.Lin, Schaich (running coupling, mass spectrum)

Deuzeman, Lombardo, Pallante, Miura, da Silva (finite temperature)

Cheng, A. Hasenfratz, Petropoulos, Sch... c eigenmodes)

YES

DeGrand (mass spectrum)

LatKMI (mass spectrum)

D.Lin, Ogawa, Ohki, Shintani (running coupling)

Fodor, Holland, Kuti, Nogradi, Schroeder, (running coupling, phase structure, spectrum)

Jin and Mawhinney (phase structure)

相互作用をする赤外固定点の見つけ方

(1) Step scaling 法によるrenormalized couplingの測定

(2) low betaでのchiral symmetry

bulk phaseがあるのでこれだけでは固定点があるかどうかわからない

(3) 質量変形した理論のhyperscaling

mass spectrum

fitのクォリティでは固定点があるかどうかわからない

(4) Dirac固有モードのhyperscaling (Volume-scaling)

質量以外のrelevantな演算子がある場合は？

(5) メソン的演算子の相関関数の形

fitのクォリティでは固定点があるかどうかわからない？

Anomalous dimension

arXiv:1307.6645[hep-lat]

異常次元の求め方

(1) ステップスケーリング法

$$\gamma_m = -\gamma_P$$

Luescher, Weisz and Wolff, NPB 359 (1991) 221
ALPHA collaboration, NPB 544 (1999) 669

(2) 質量変形した理論に対するhyperscaling法

複合状態のmass spectrum

$$M_X \sim c_X m^{\frac{1}{1+\gamma_m^*}}$$

Miransky, PRD59(1999)105003
Luty, JHEP 0904(2009)050
Del Debbio and Zwicky, PRD82(2010)014502

(3) ゼロ質量におけるDirac固有モードのhyperscaling法(Volume-scaling)

$$\nu(\lambda) \sim \lambda^{\frac{4}{1+\gamma_m^*}}$$

Patella,PRD86(2012)025006
Cheng, Hasenfratz, Petropulos and Schaich, JHEP1307(2013)061

(4) メソン的演算子の相関関数の形から求める方法

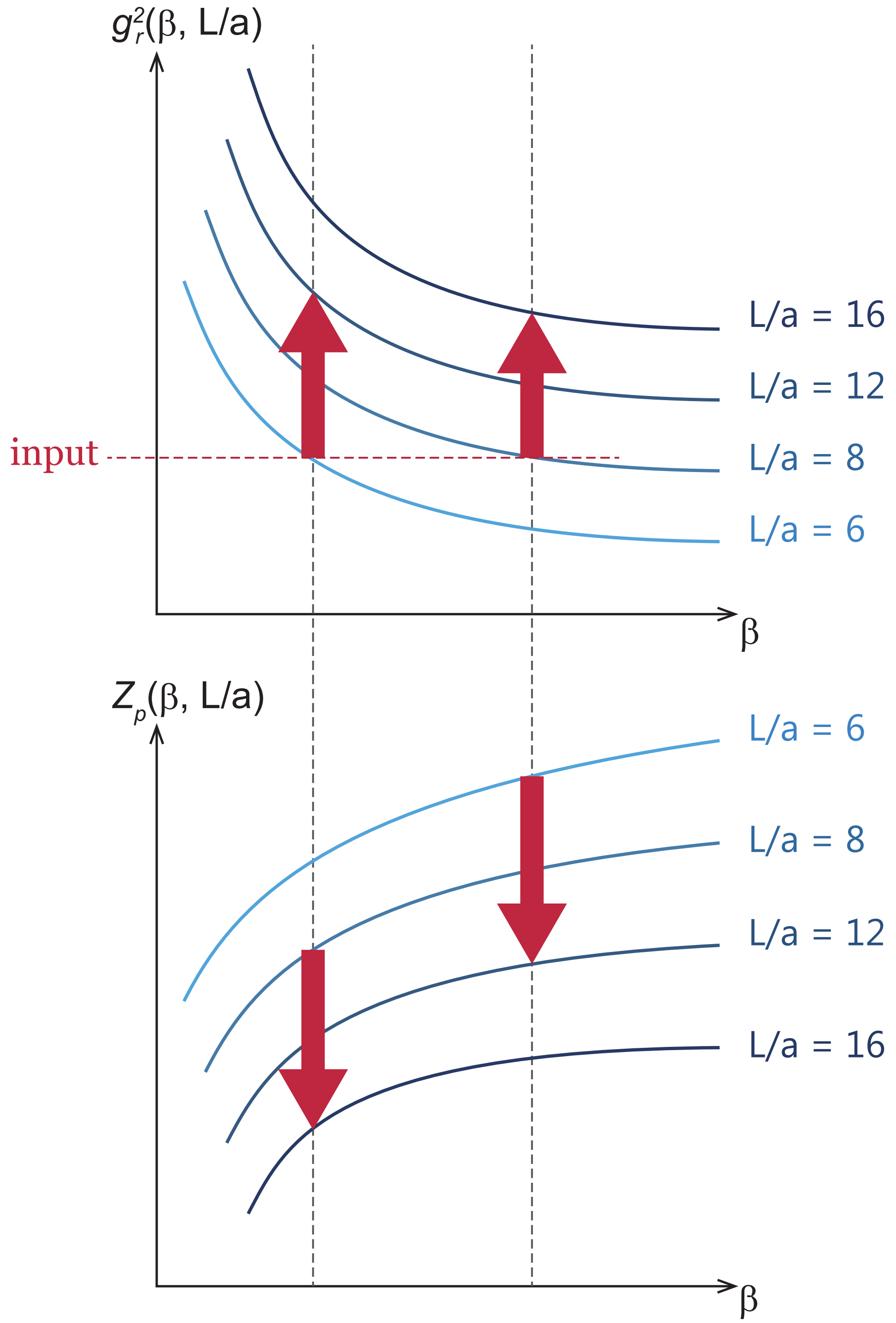
Ishikawa, Iwasaki, Nakayama and Yoshie, PRD87(2013)071503

$$\langle \phi(x)\phi(0) \rangle \sim \frac{c}{|x|^{2(d-\gamma_m^*)}}$$

Step scaling method

- measuring the wave function renormalization constant -

ALPHA collaboration NPB 544 (1999) 669



PCAC relation in QCD

$$Z_m^{-1}$$

$$\partial^\mu (A_R)_\mu = 2mP_R$$
$$= 2(Z_m \cdot m_0)(Z_P \cdot P_0)$$

Mass scaling function

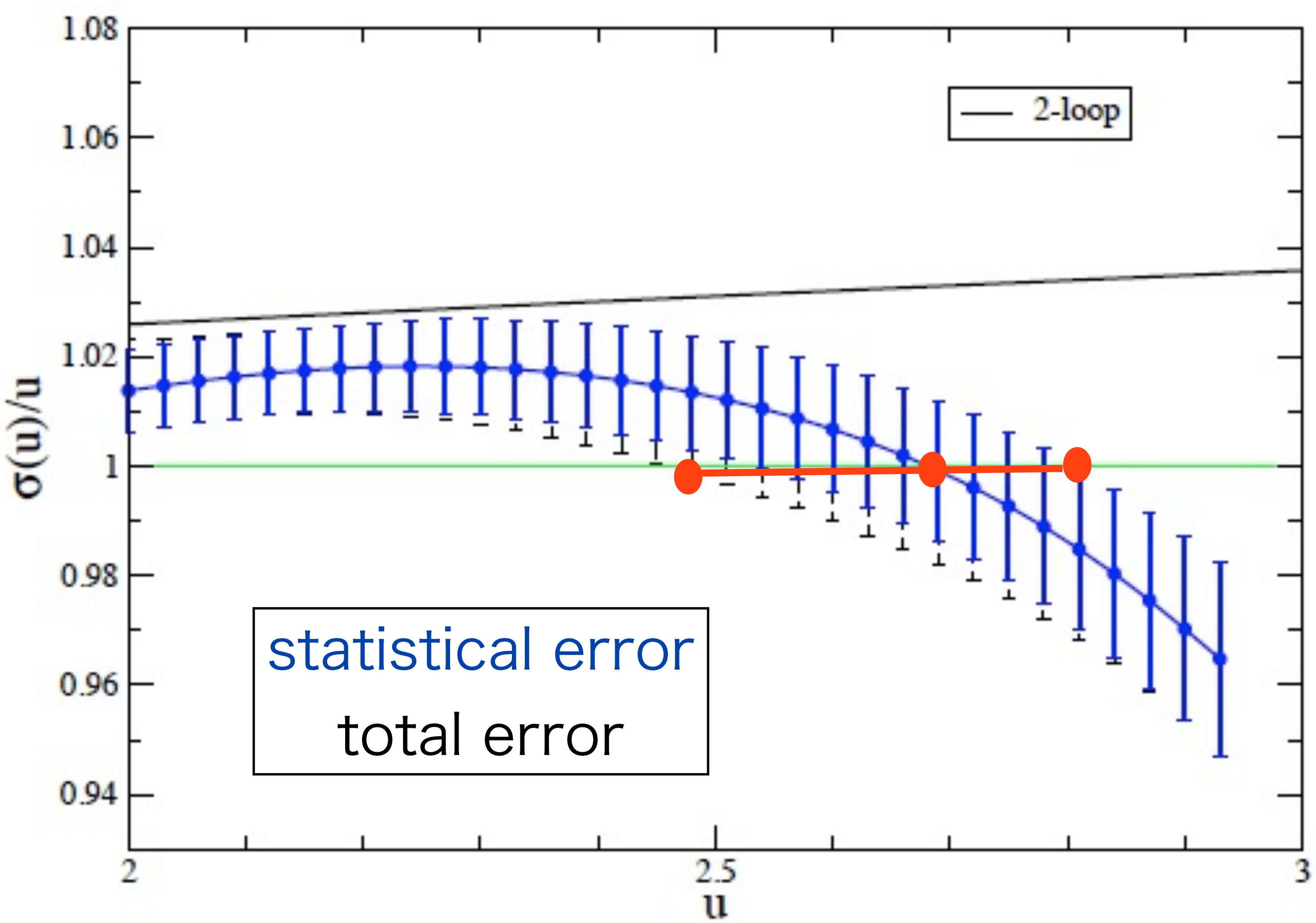
$$\sigma_P(g^2, s) = \lim_{a\to 0} \left. \frac{Z_P(g_o, a/sL)}{Z_P(g_0, a/L)} \right|_{g^2=const}$$

We measure Z factor of pseudo-scalar op. at the IRFP.

A new definition of Z factor

$$Z_P(g_0) \equiv \sqrt{\frac{C_P^{tree}(t)}{C_P(t)}} \qquad \text{at fixed t}$$

固定点を実現する格子セットアップ

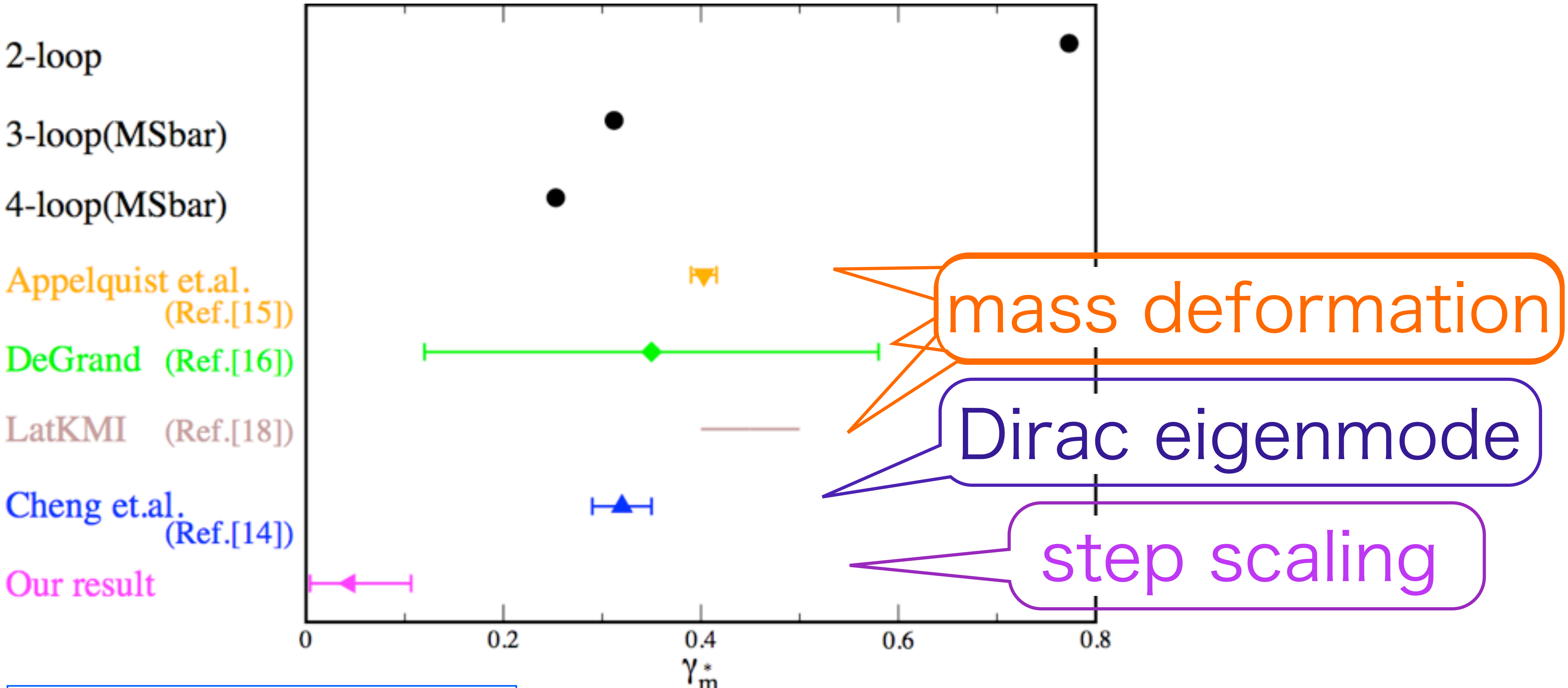


各格子サイズでのbetaの値

L/a	u=2.475	u=2.686	u=2.823
6	5.378	4.913	4.600
8	5.796	5.414	5.181
10	5.998	5.653	5.450
12	6.121	5.786	5.588
16	6.241	5.909	5.709
20	6.296	5.944	5.733

Nf=12赤外固定点での異常次元に関する現状

E.I., arXiv:1307.6645[hep-lat]

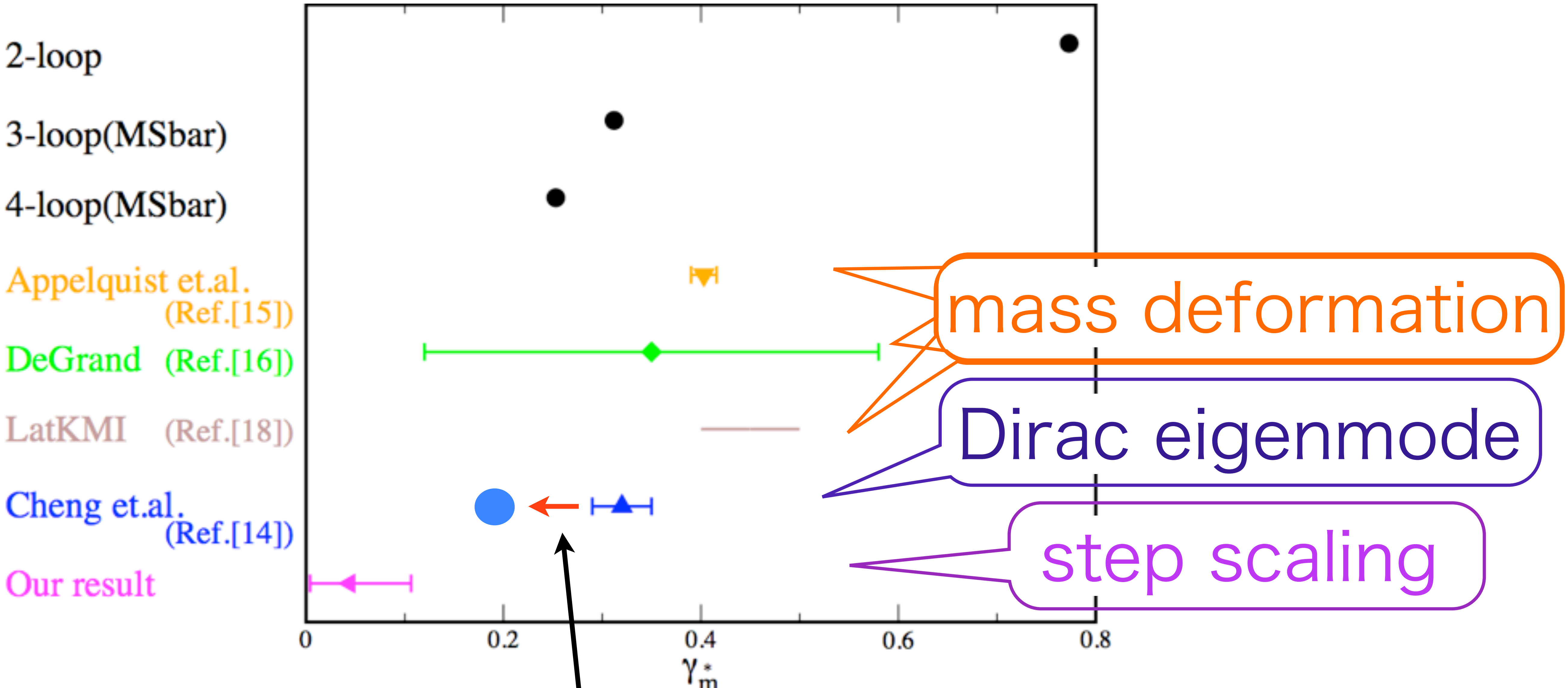


Fodor's data: PLB703 (2011) 348-358
Fit(I): PRD84(2011) 054501
Fit(II): PRD84 (2011) 116901
LatKMI : PRD86 (2012) 054506
Cheng et.al : JHEP1307 (2013) 061
Ours : PTEP (2013)083B01
arXiv: 1307.6645

$$\gamma_m^* = 0.044^{+0.062}_{-0.040}$$

Nf=12赤外固定点での異常次元に関する現状

E.I., arXiv:1307.6645[hep-lat]

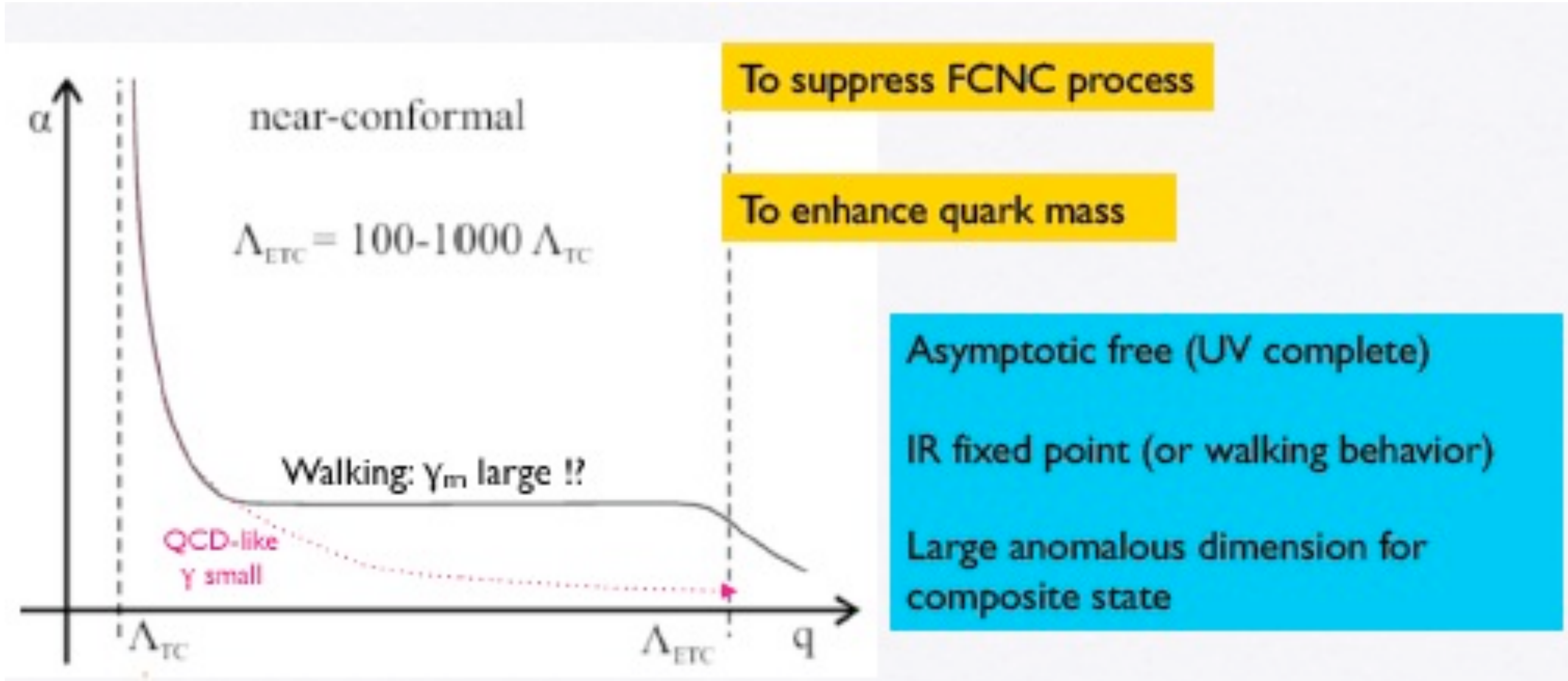


格子間隔を有限に止めて、無限体積極限をとる。

データを更新して、 有限体積効果とbetaの臨界指数の効果を入れたら

我々の値に近づいた。@Lattice2013

phenomenological prediction?!



2-quark and 2-techni-fermion

$$\frac{c_2}{\Lambda_{ETC}^2}\langle\bar{\Psi}\Psi\rangle_{ETC}(\bar{\psi}\psi)$$

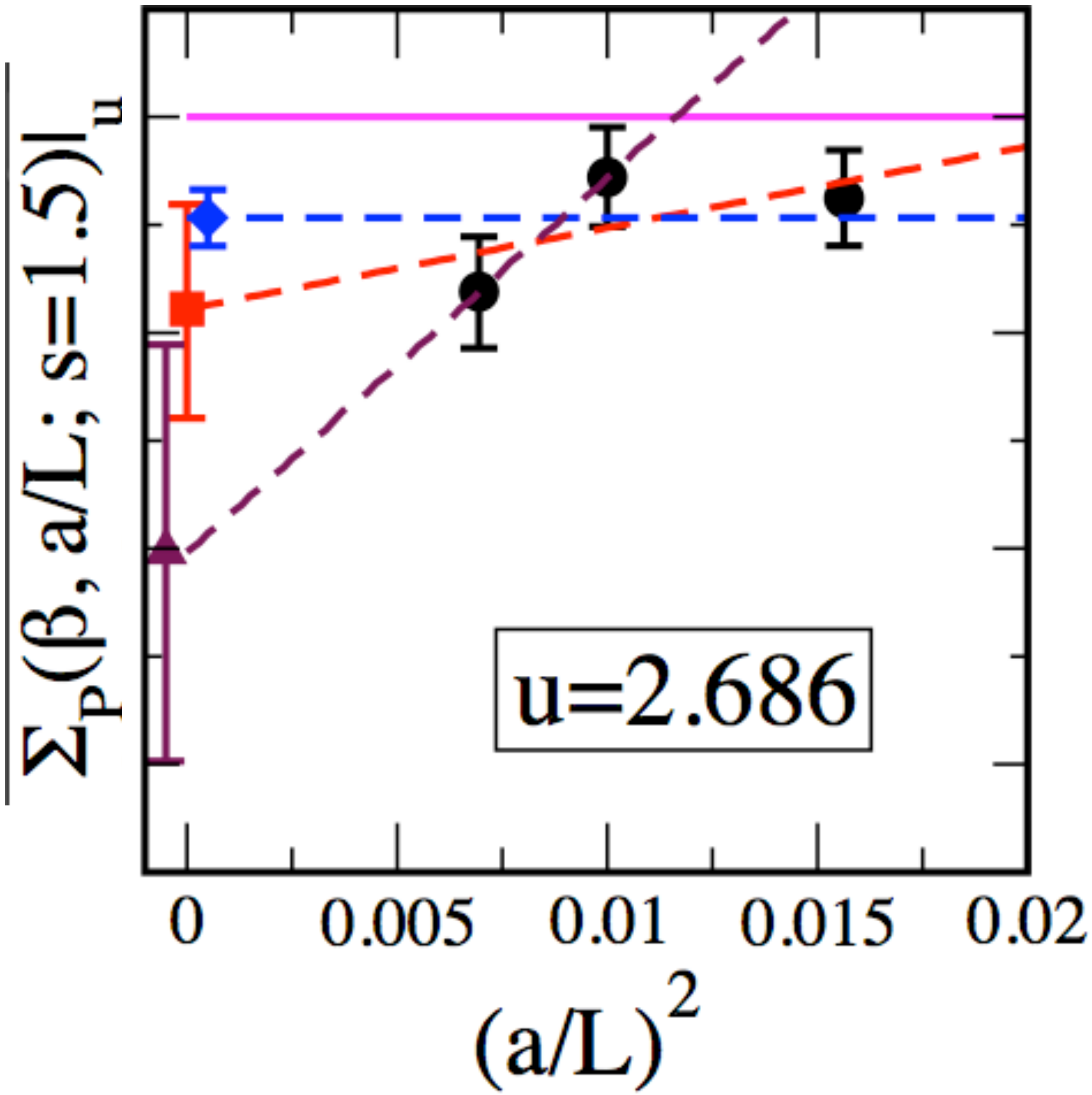
$$M_q \sim \frac{1}{\Lambda_{ETC}^2}\left(\frac{\Lambda_{ETC}}{\Lambda_{TC}}\right)^{\gamma^*}\langle\bar{\Psi}\Psi\rangle_{ETC}$$

Assume that $\Lambda_{TC} \sim$ 1TeV, $\Lambda_{ETC} \sim$1000TeV

$\gamma_m^* = 1$	$M_q \sim$ 1GeV
$\gamma_m^* = 0.5$	$M_q \sim$ 30MeV
$\gamma_m^* = 0.25$	$M_q \sim$ 5MeV
$\gamma_m^* = 0.15$	$M_q \sim$ 2MeV

残った課題 :step scaling法

pseudo-scalarのZ因子の連続極限の様子



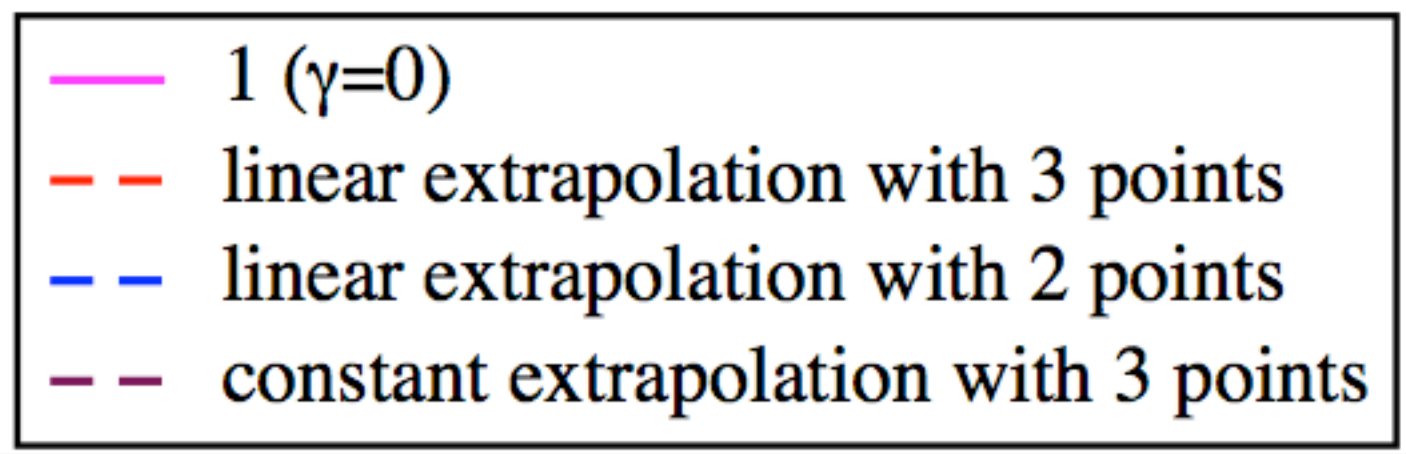
s=1.5 step scaling

L/a=6 -> L/a=9(離散化誤差大きすぎる)

L/a=8 -> L/a=12

L/a=10 -> L/a=15

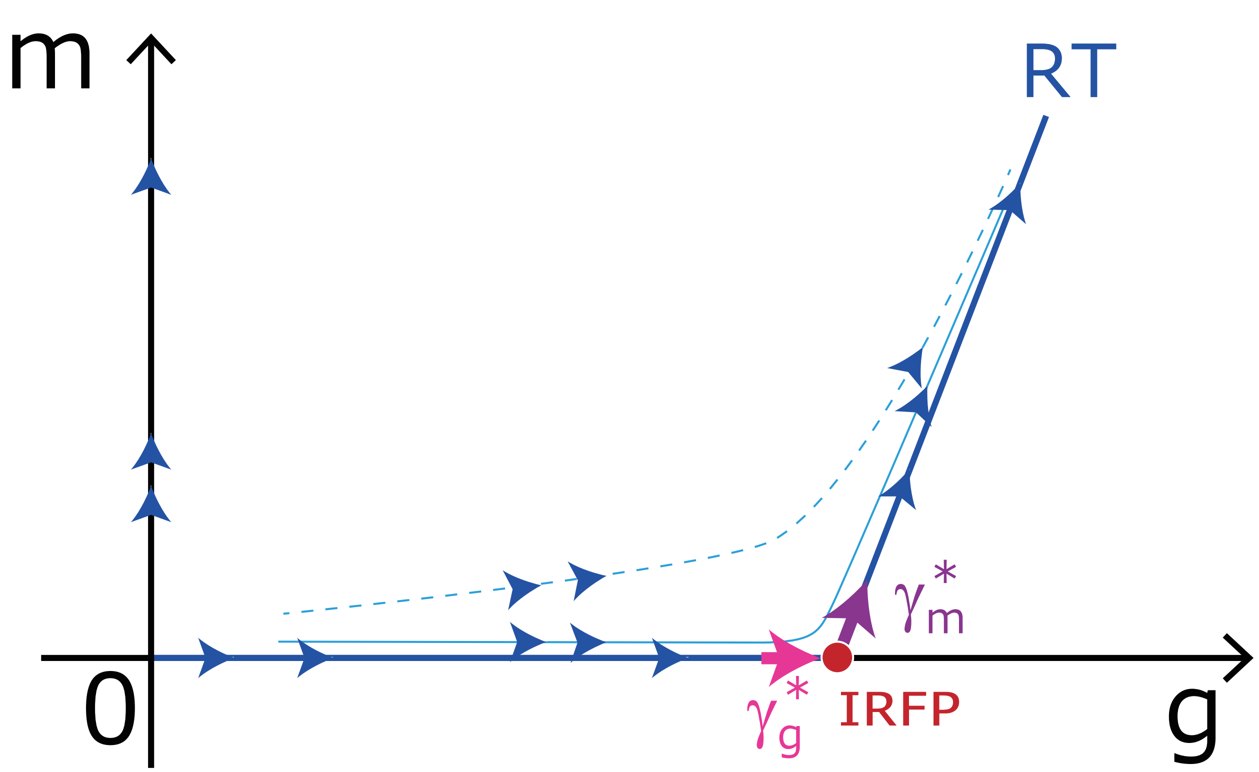
L/a=12 -> L/a=18



3つのデータ点を用いた２パラメータフィットによる外挿では系統誤差の見積もりが不十分

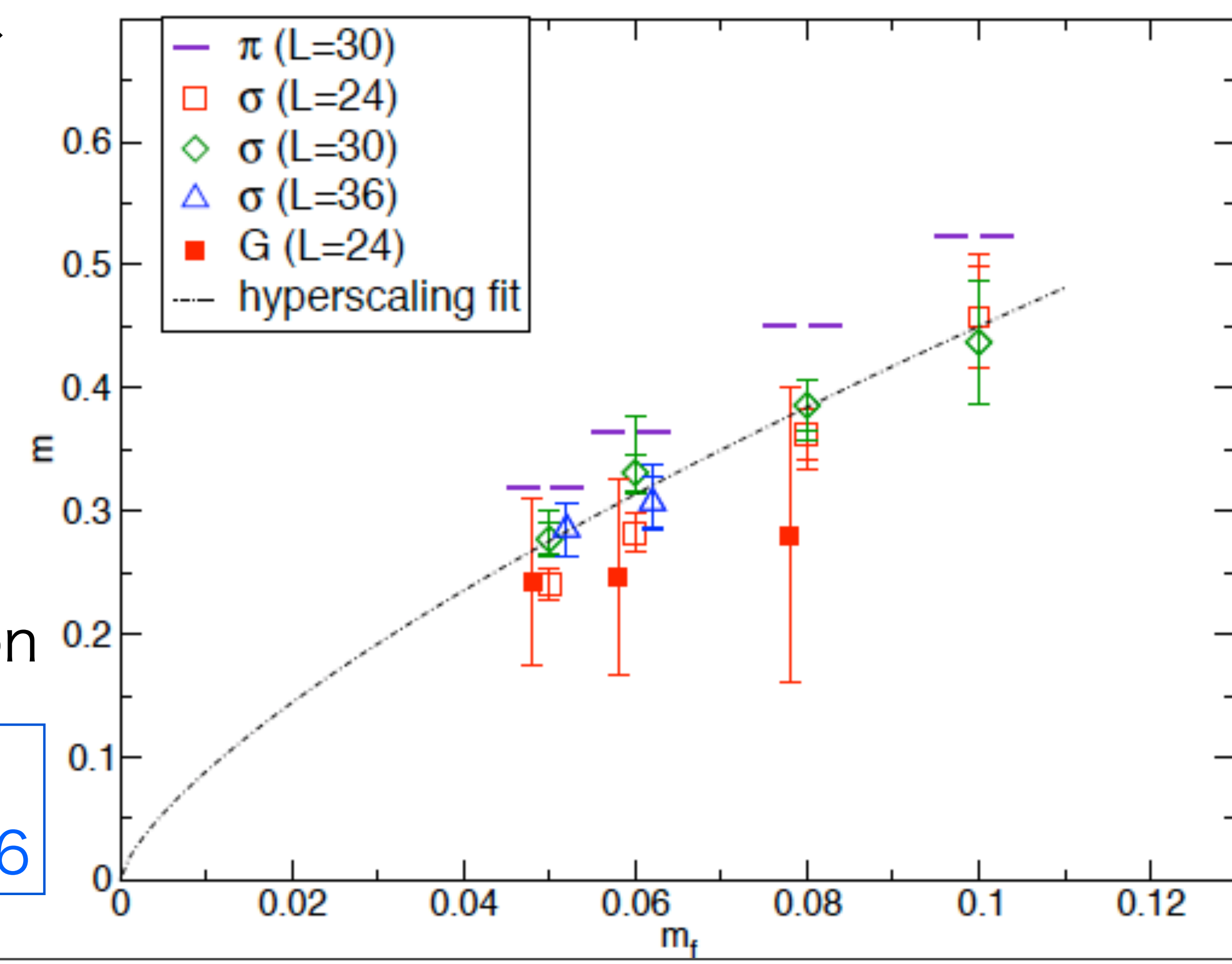
もう一つ大きい格子サイズデータが必要

残った課題 :mass deformed法



- critical betaへのtuning
- massless 近傍へのtuning

$$M_X \sim m_q^{\frac{1}{1+\gamma_m^*}}$$



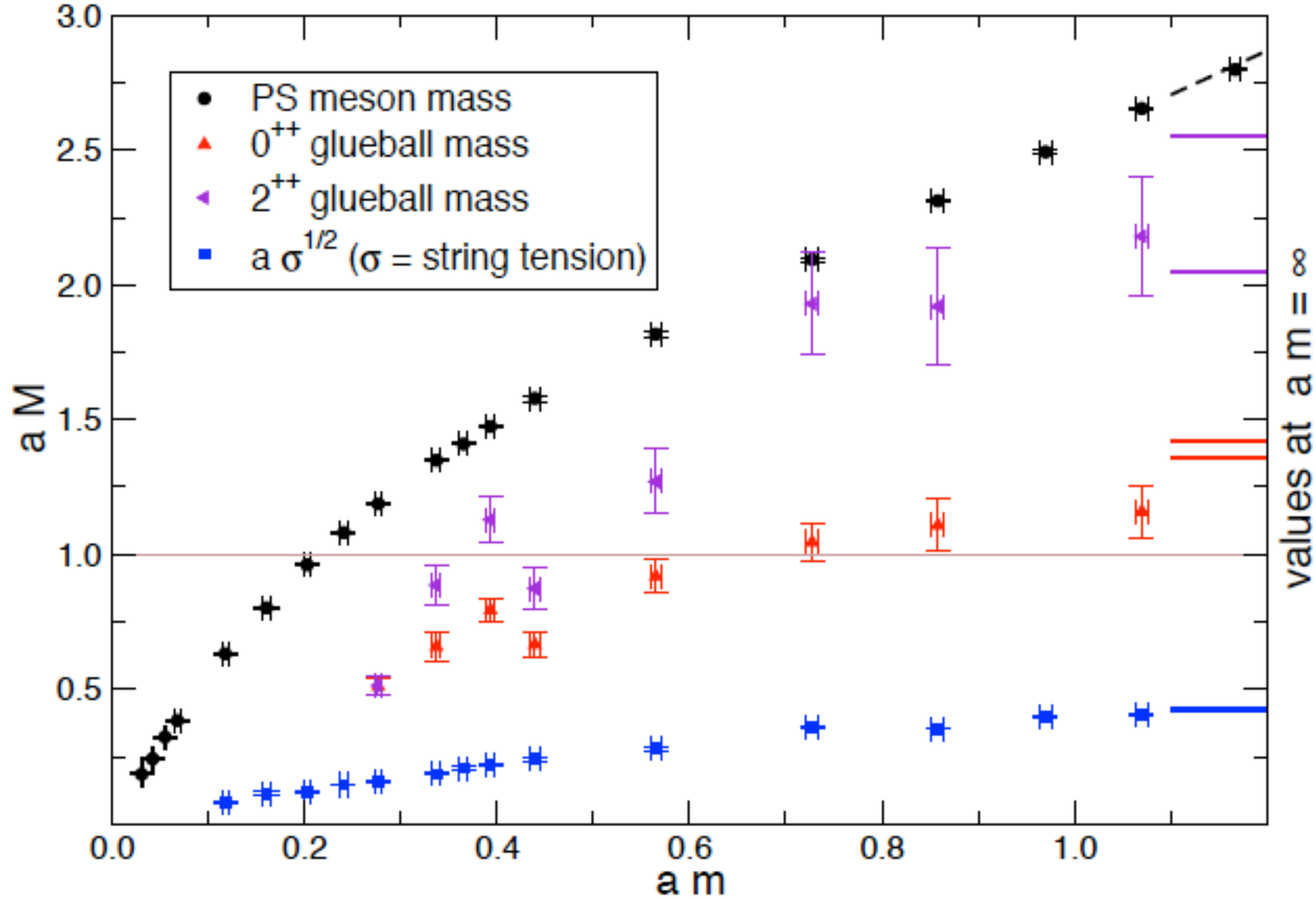
beta=4.0 HISQ action

LatKMI
arXiv:1305.6006

他の理論の場合

mass deformed theoryによるhyperscaling

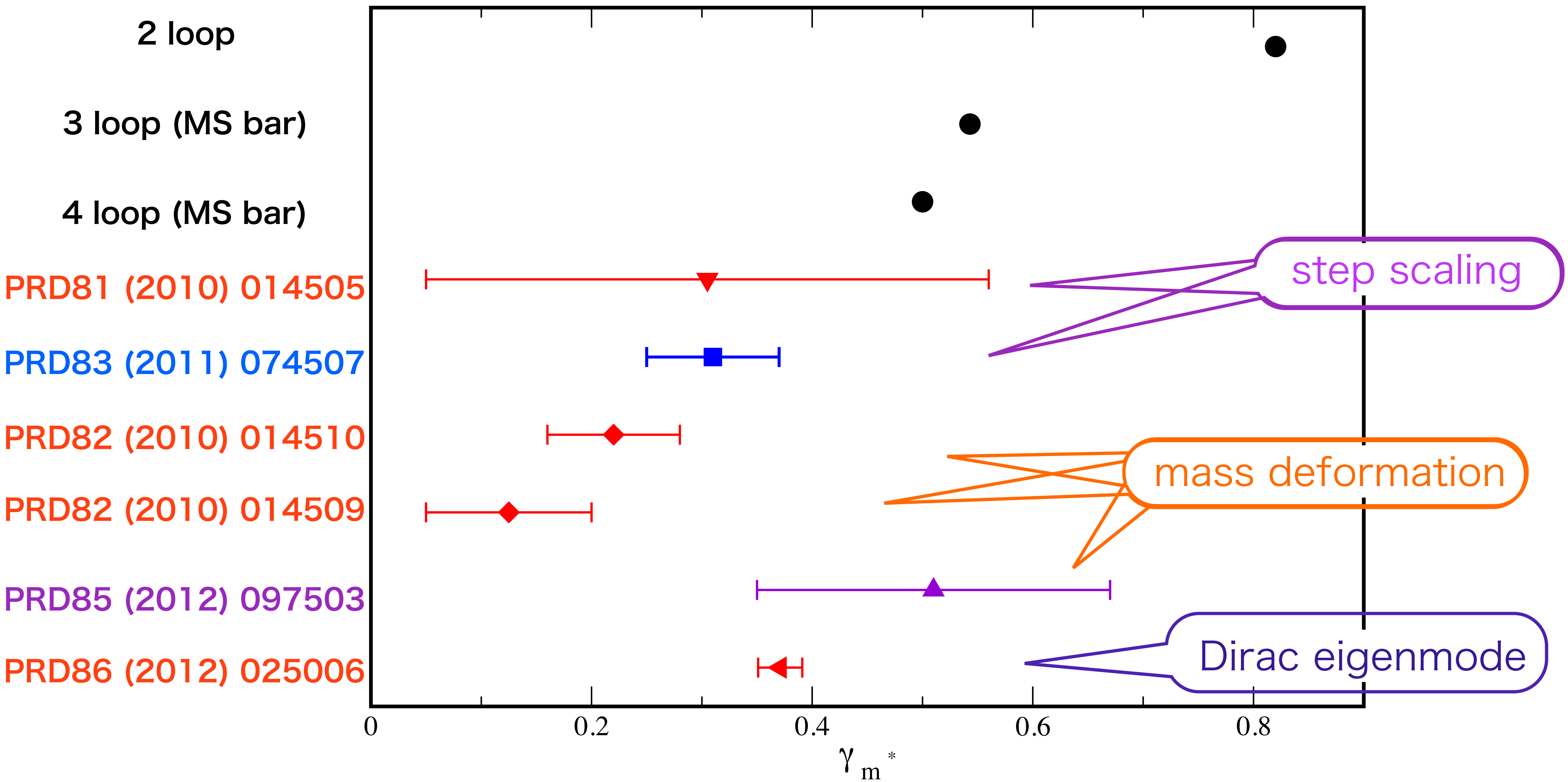
SU(2) Nf=2 adjoint fermions



横軸: PCAC mass

Patella et.al., PoS Lattice2010:068,2010

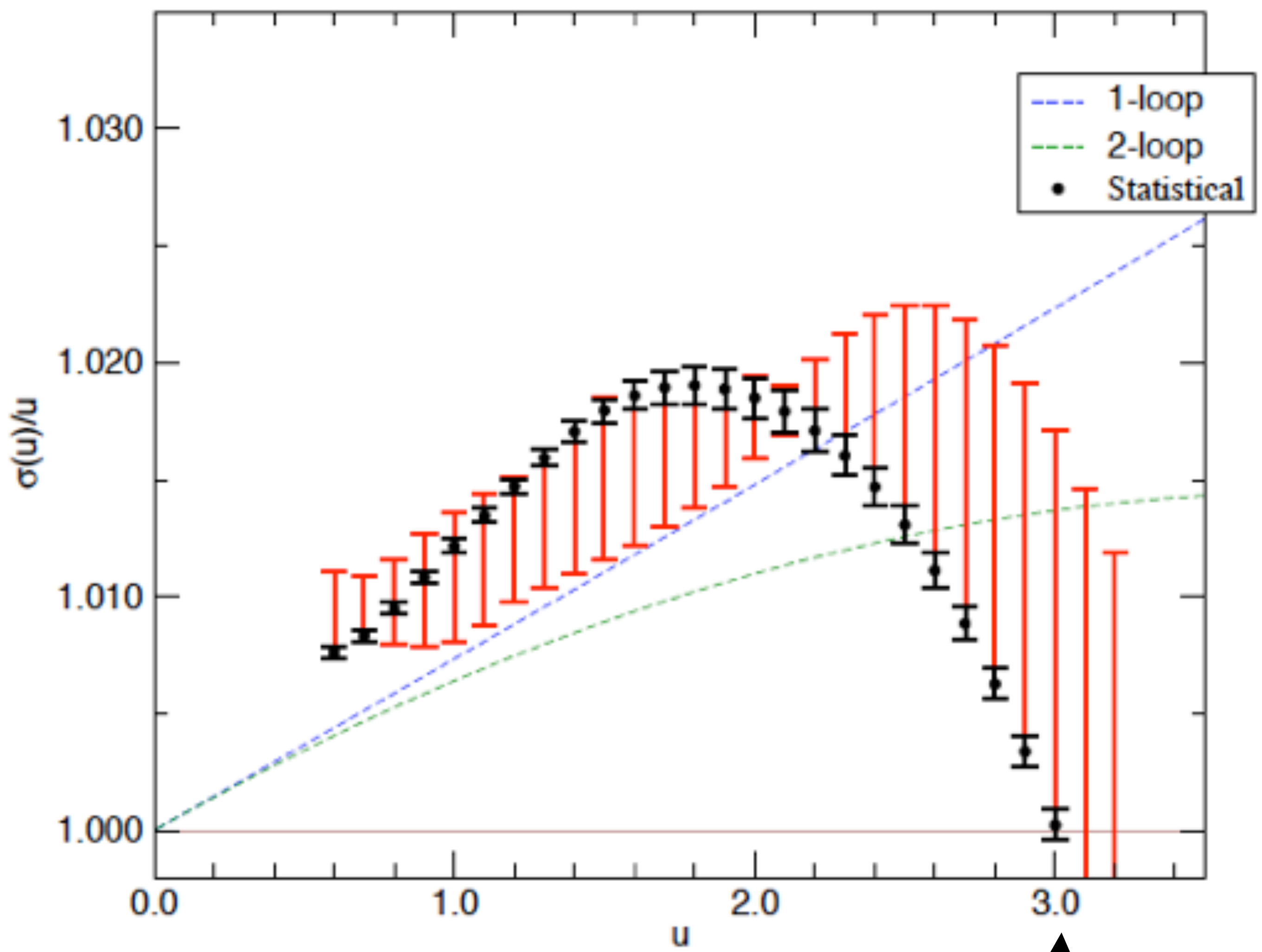
Mass anomalous dim.
SU(2) Nf=2 adjoint fermion
(minimal walking technicolor)



running coupling constant

SU(2) Nf=2 adjoint fermions

Del Debbio et. al., PRD81 (2010) 014505



連続極限の系統誤差はコントロールできていないけど…

beta=2.25

Conclusion and Discussion

The IRFP exists in SU(3) Nf=12 massless theory.
Continuum extrapolation and parameter search are important.

(1) The phenomenological model construction by using the mass anomalous dimension from the lattice simulation.

Nf=12 model must be killed by lattice results. (Also minimal walking technicolor: Del Debbio et.al.)

(2) Spectrum for several modes, who is the lightest state? pseudoscalar? dilaton?

(3) Study on universal quantities (anomalous dimension, “central charge” in 4-dim)

(4) We have to understand the property of conformal field theory on the finite lattice volume. (functional form of correlation fn.)

Lattice precise data can give phenomenological and theoretical information around nontrivial fixed point.