

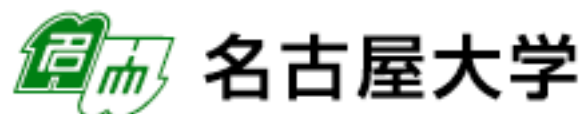
Exploring for walking technicolor from QCD

Yasumichi Aoki [Kobayashi-Maskawa Institute(KMI), Nagoya University]

for the LatKMI collaboration

- QUCS 2012 symposium @ Nara -

Dec. 16, 2012



LatKMI collaboration

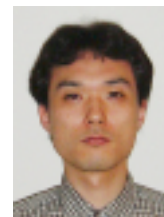
- YA, T.Aoyama, M.Kurachi, T.Maskawa, K.Nagai, H.Ohki,



E. Rinaldi, K.Yamawaki, T.Yamazaki



名古屋大学



A. Shibata



Walking Technicolor (WTC)

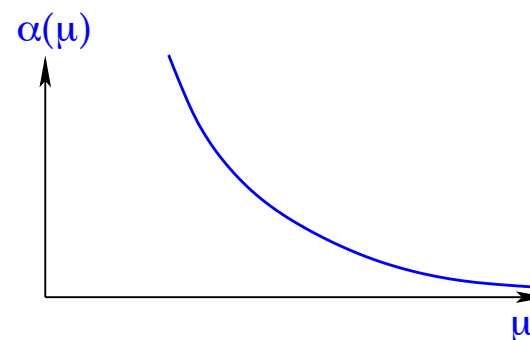
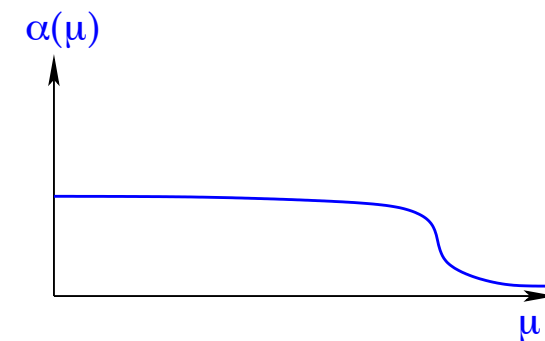
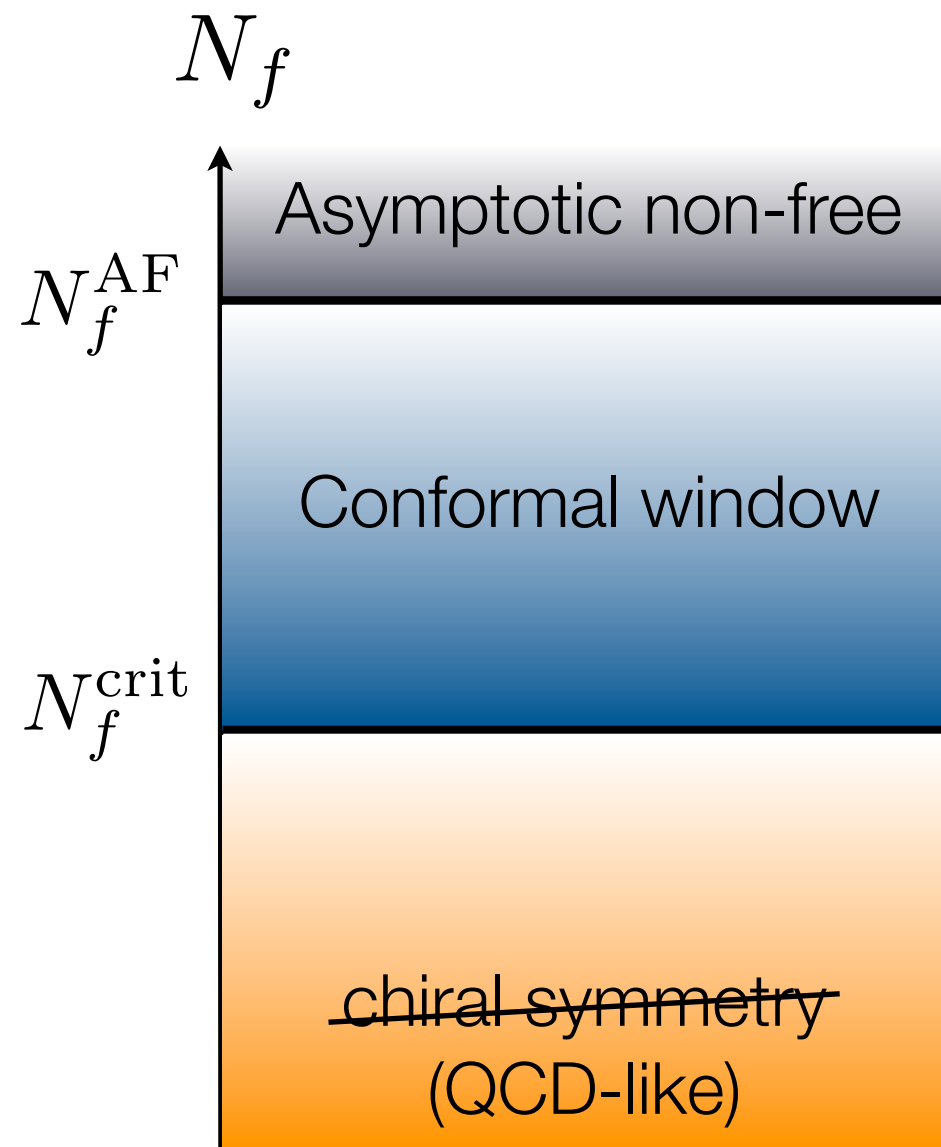
- a candidate of the new physics beyond the Standard Model of particles
- could replace Higgs sector of the Standard Model
 - Higgs sector is a low energy effective theory of WTC
- free from the gauge hierarchy problem (naturalness)
- gives explanation of the electro-weak gauge symmetry breaking,
 - thus origin of mass of the elementary particles
- “Higgs” = pseudo Nambu-Goldstone boson
 - due to breaking of the approximate scale invariance (Dilaton)

Requirements for the successful WTC theory

- spontaneous chiral symmetry breaking
 - running coupling “walks” = slowly changing with $\mu \rightarrow$ nearly conformal
 - large mass anomalous dimension: $\gamma_m \sim 1$
 - light scalar 0^{++} ($m_H = 126 \text{ GeV @ LHC !}$)
 - with input $F_\pi = 246 / \sqrt{N} \text{ GeV}$ (N: # weak doublet in techni-sector)
 - to reproduce W^\pm mass
 - typical QCD like theory: $M_{\text{Had}} \gg F_\pi$ (ex.: QCD: $m_\rho/f_\pi \sim 8$)
 - Naive TC: $M_{\text{Had}} > 1,000 \text{ GeV}$
 - 0^{++} is a special case: pseudo Nambu-Goldstone boson of scale inv.
- ➡ is it really so ?

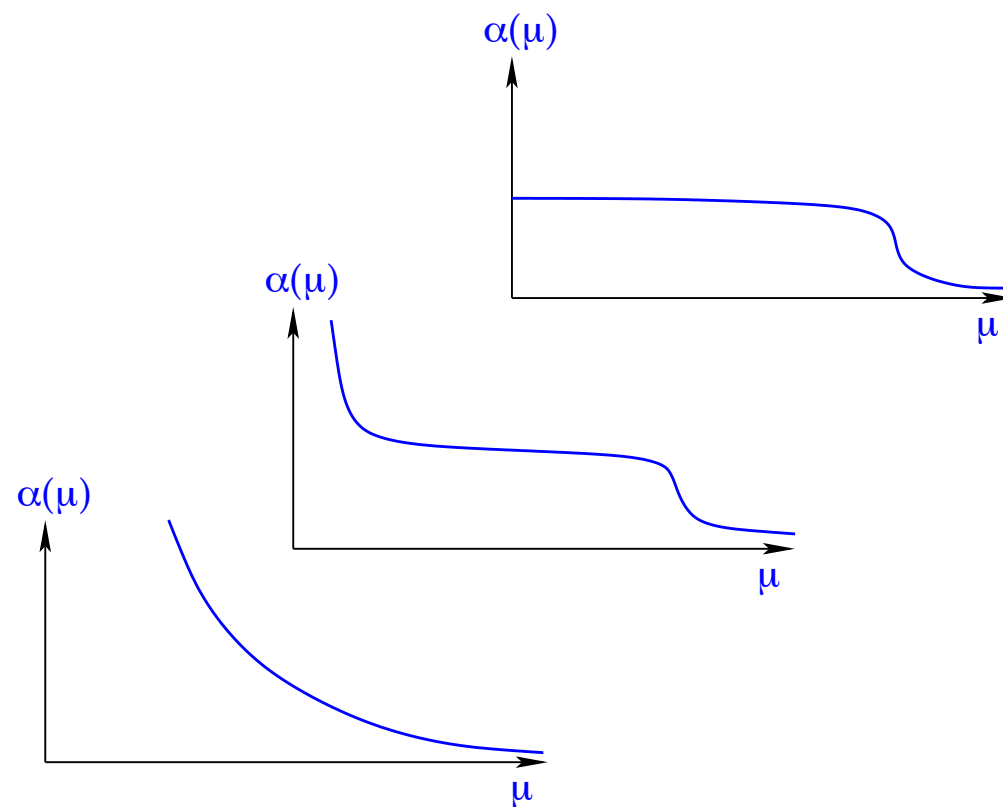
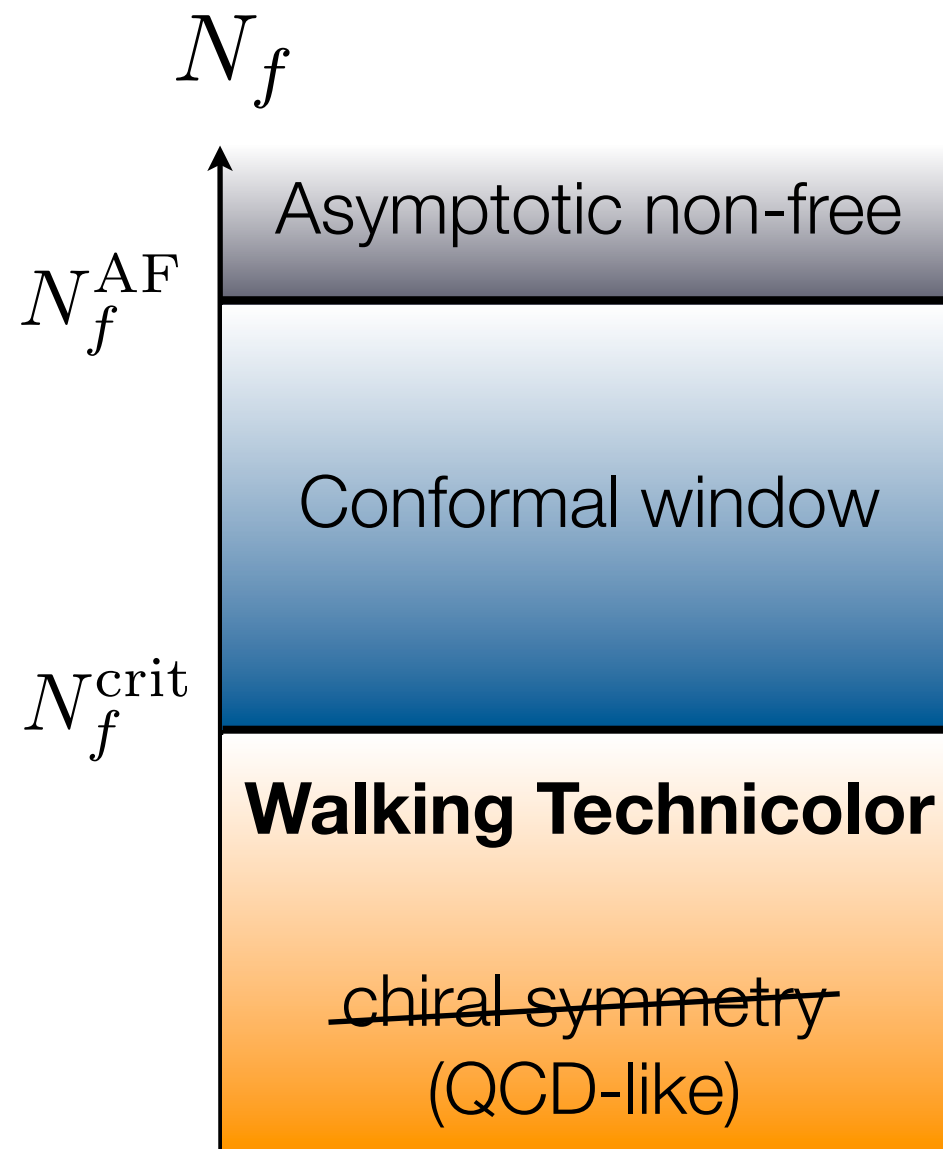
conformal window and walking gauge coupling

- non-Abelian gauge theory with N_f *massless* fermions -



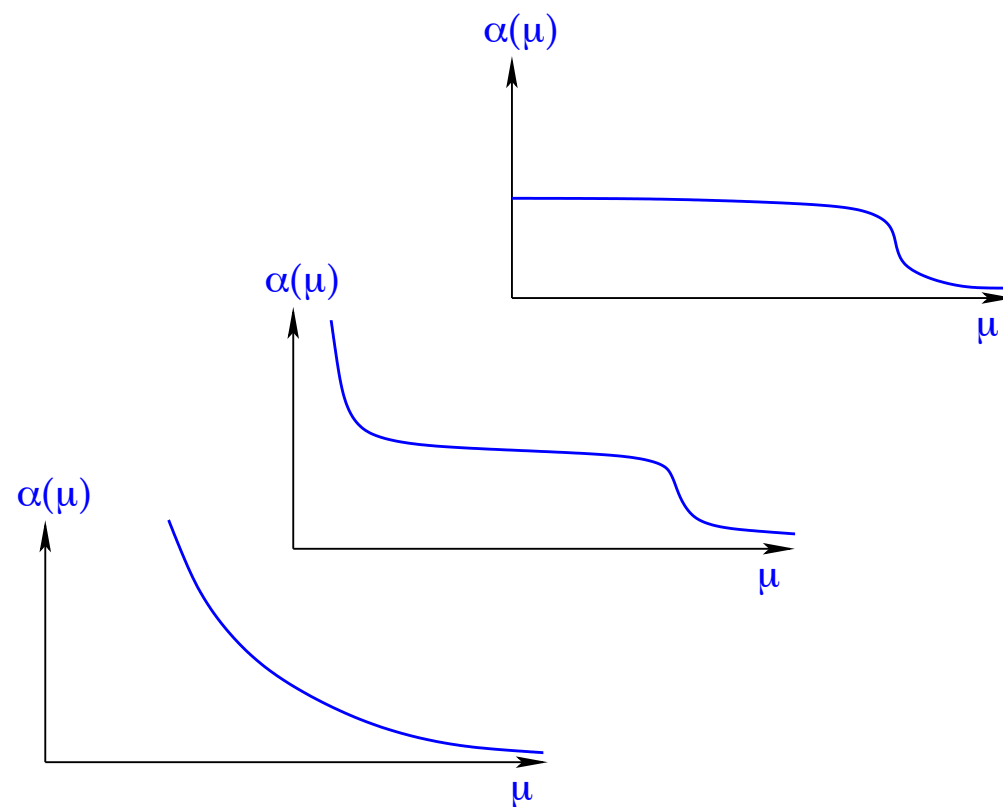
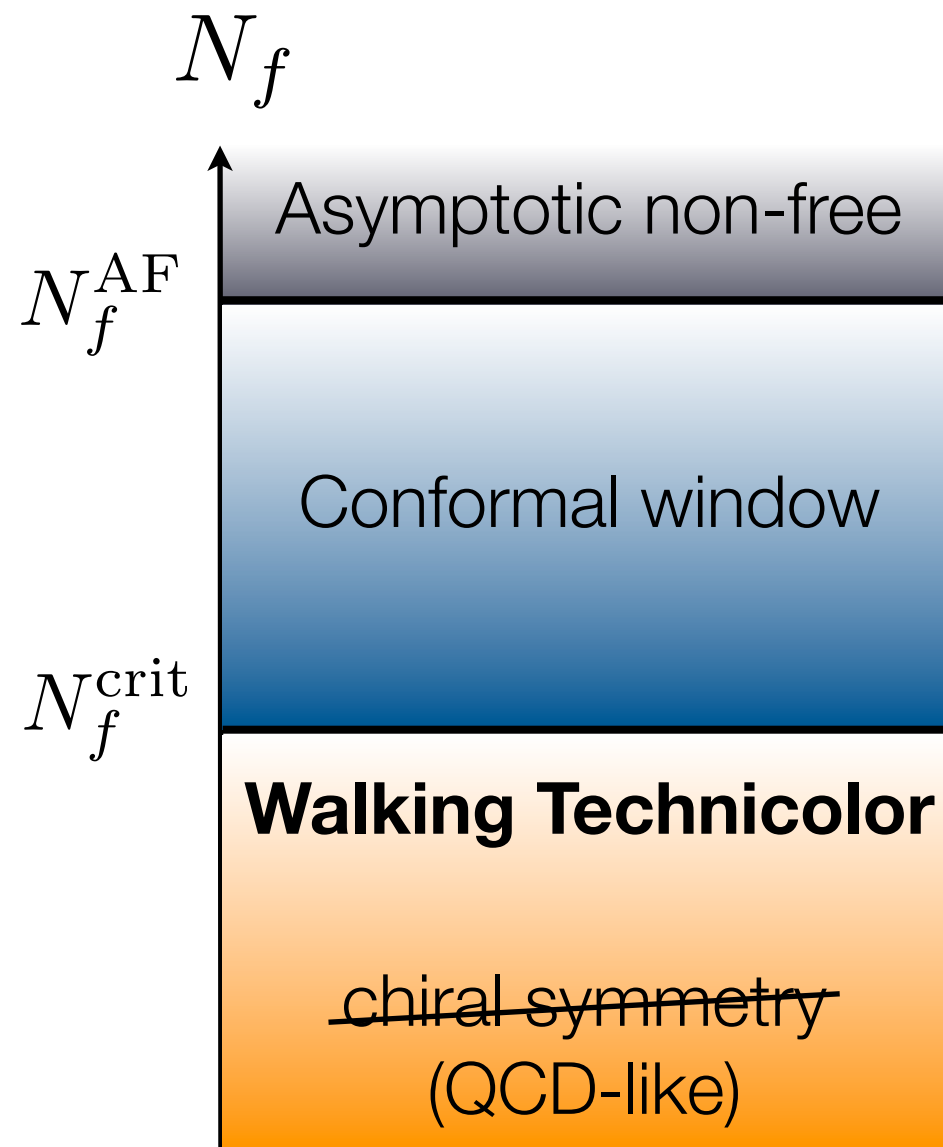
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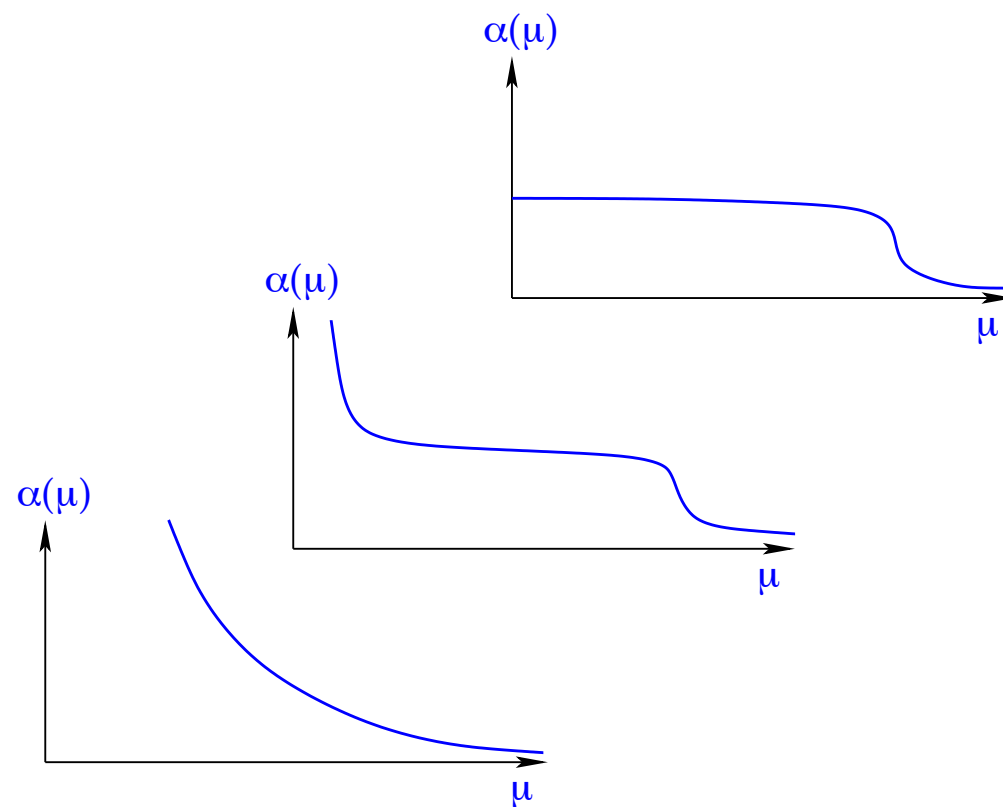
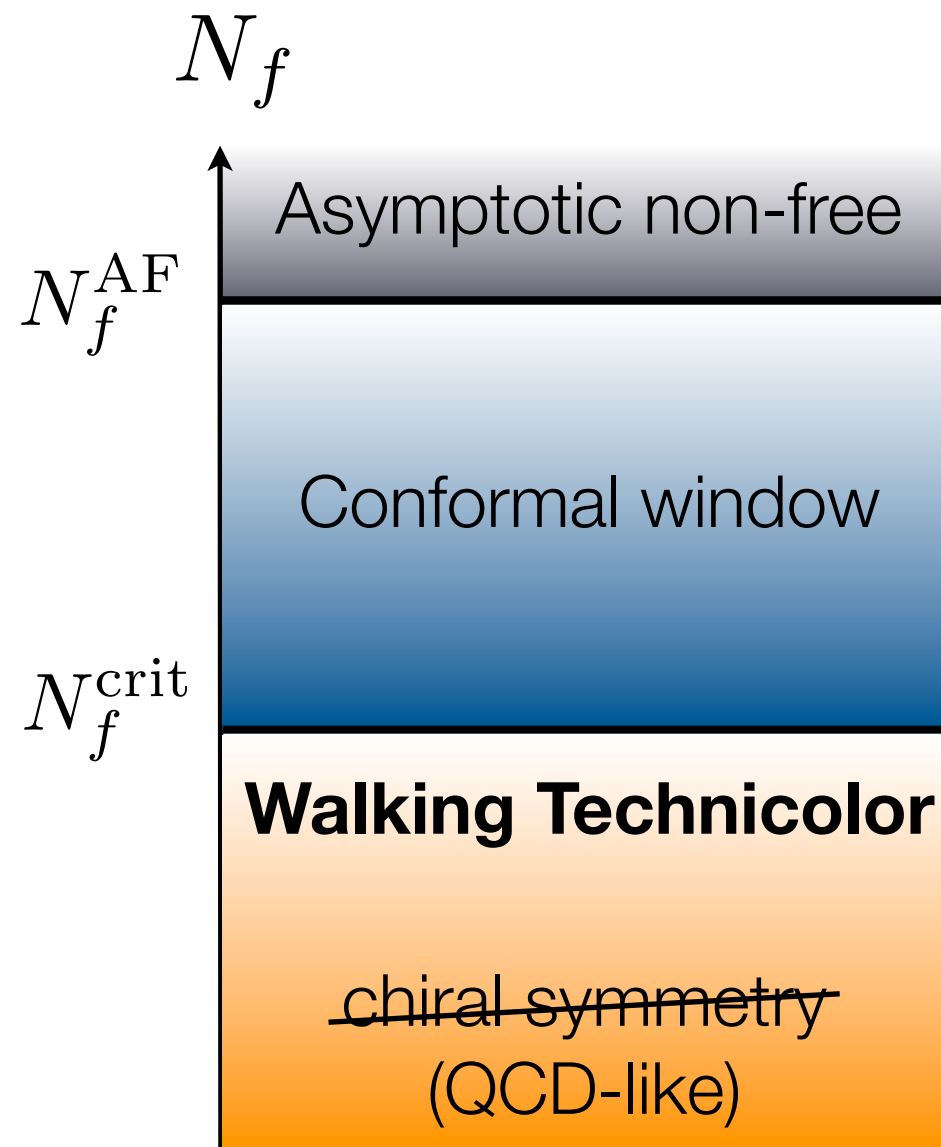
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- Walking Technicolor could be realized just below the conformal window

conformal window and walking gauge coupling

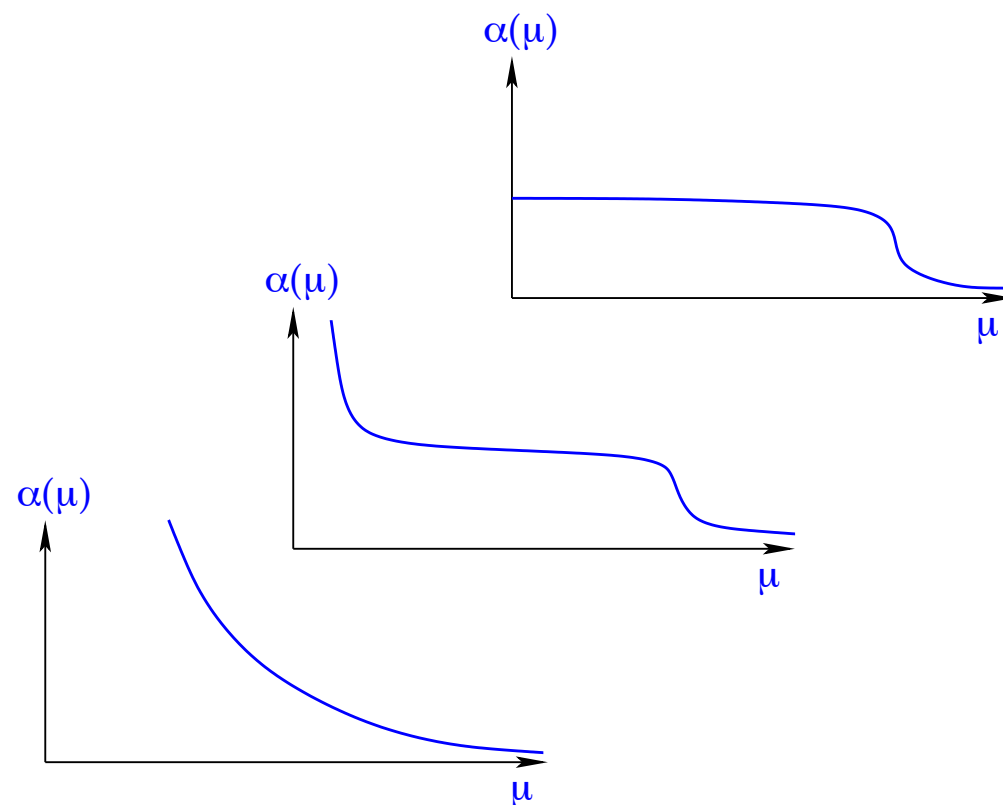
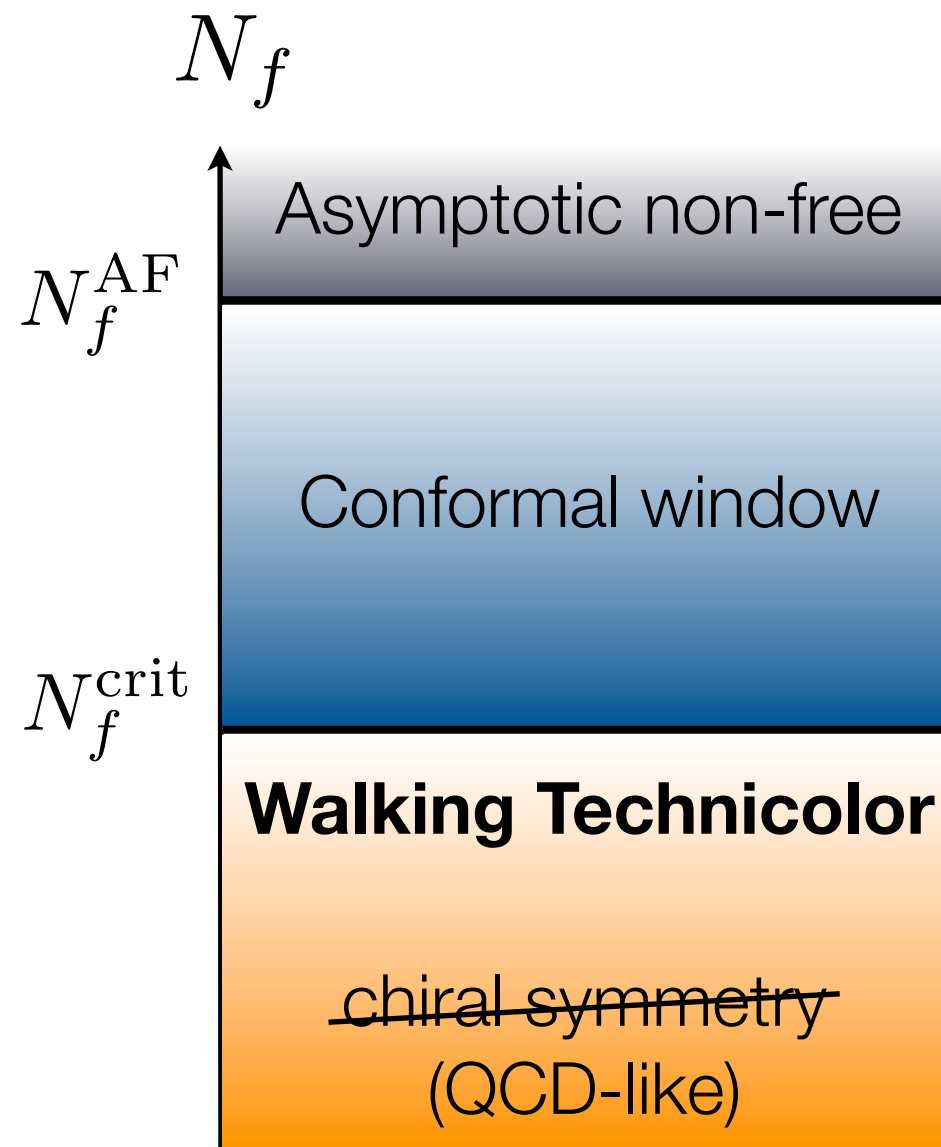
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- Walking Technicolor could be realized just below the conformal window
- crucial information: N_f^{crit} and...

conformal window and walking gauge coupling

- non-Abelian gauge theory with N_f *massless* fermions -

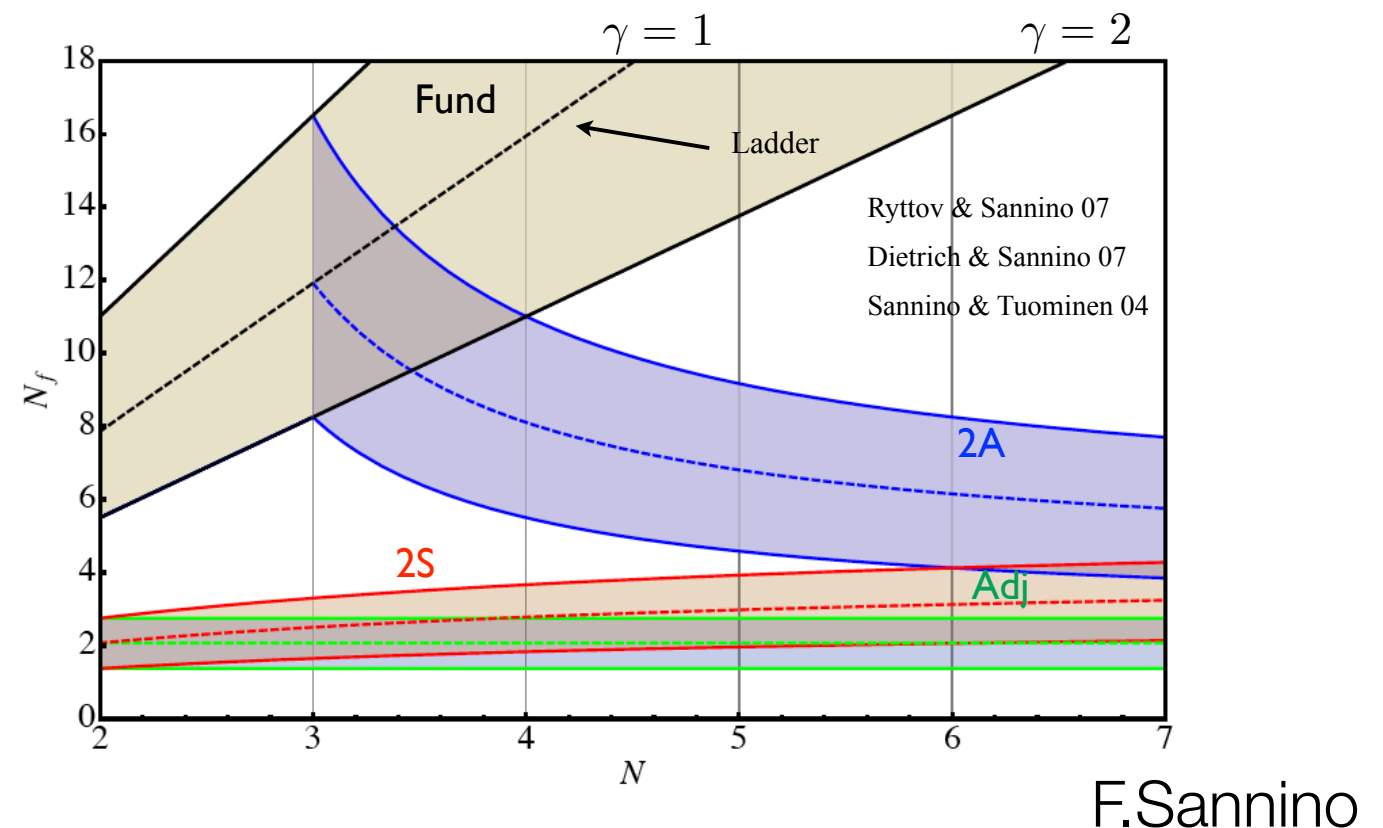


- Walking Technicolor could be realized just below the conformal window
- crucial information: N_f^{crit} and...
- mass anomalous dimension γ & the composite mass spectrum around N_f^{crit}

models being studied:

- SU(3)
 - fundamental: $N_f=6, 8, 10, 12, 16$
 - sextet: $N_f=2$
- SU(2)
 - adjoint: $N_f=2$
 - fundamental: $N_f=8$
- SU(4)
 - decuplet: $N_f=2$

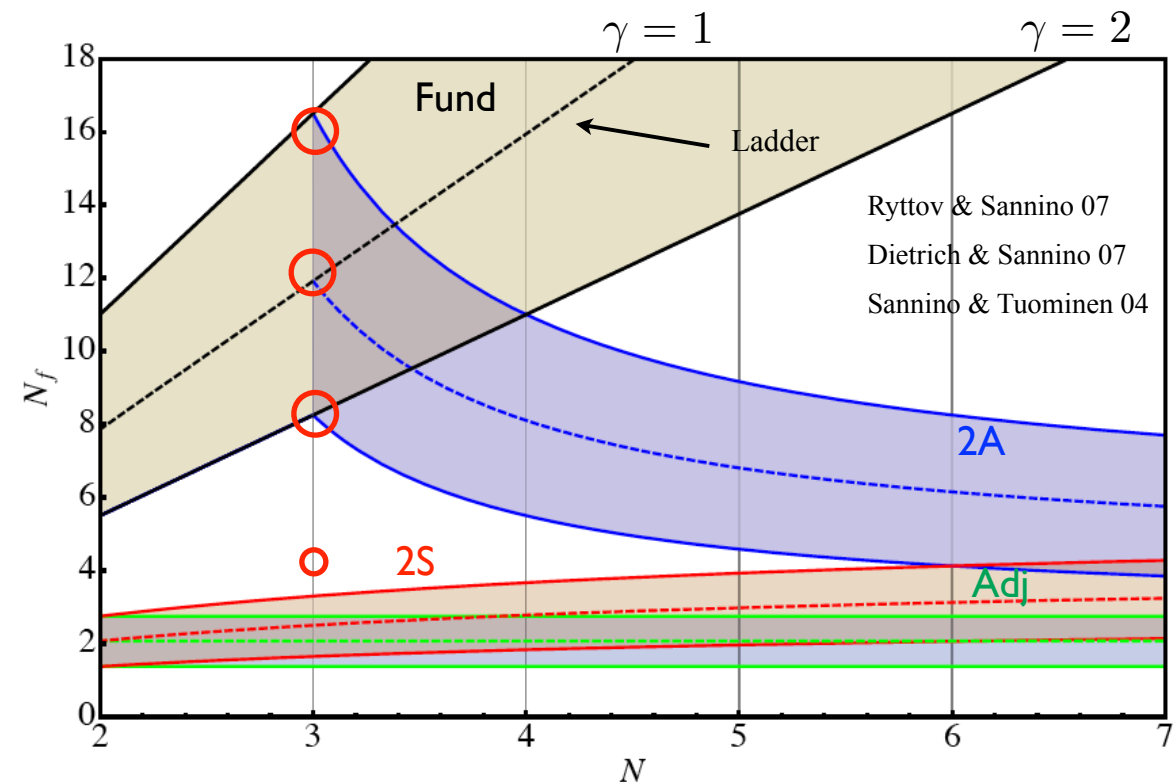
SU(N) Phase Diagram



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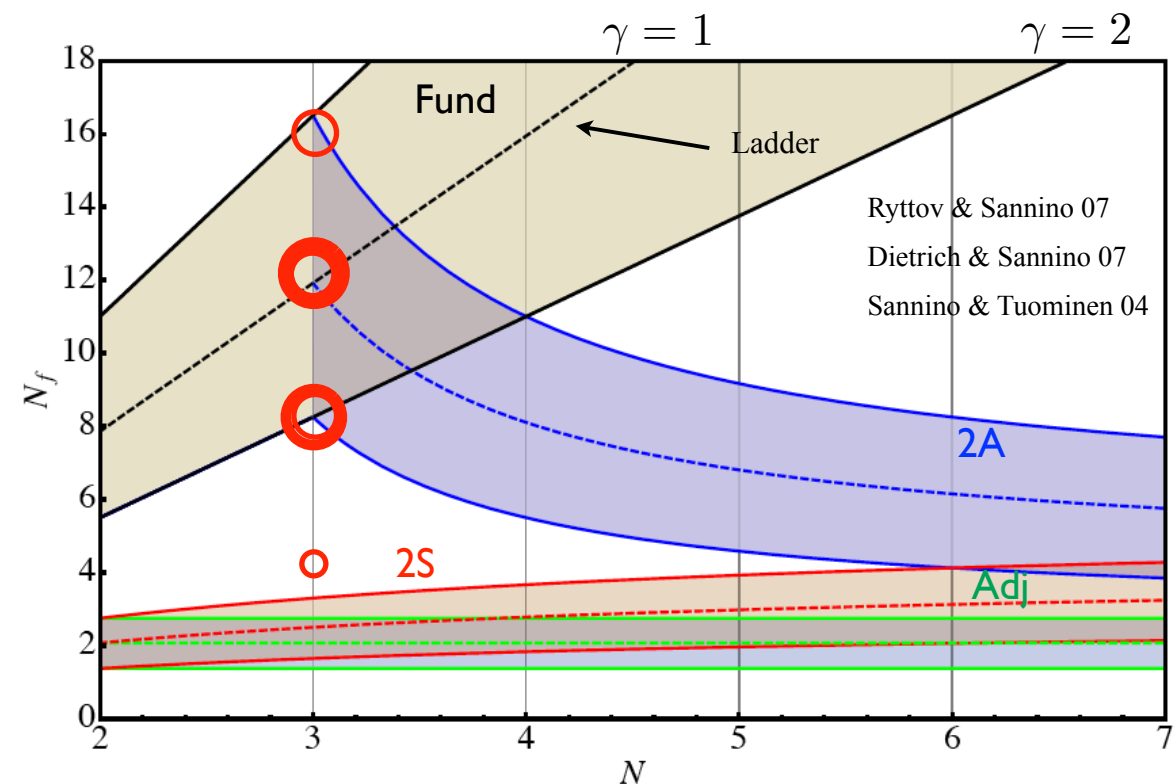
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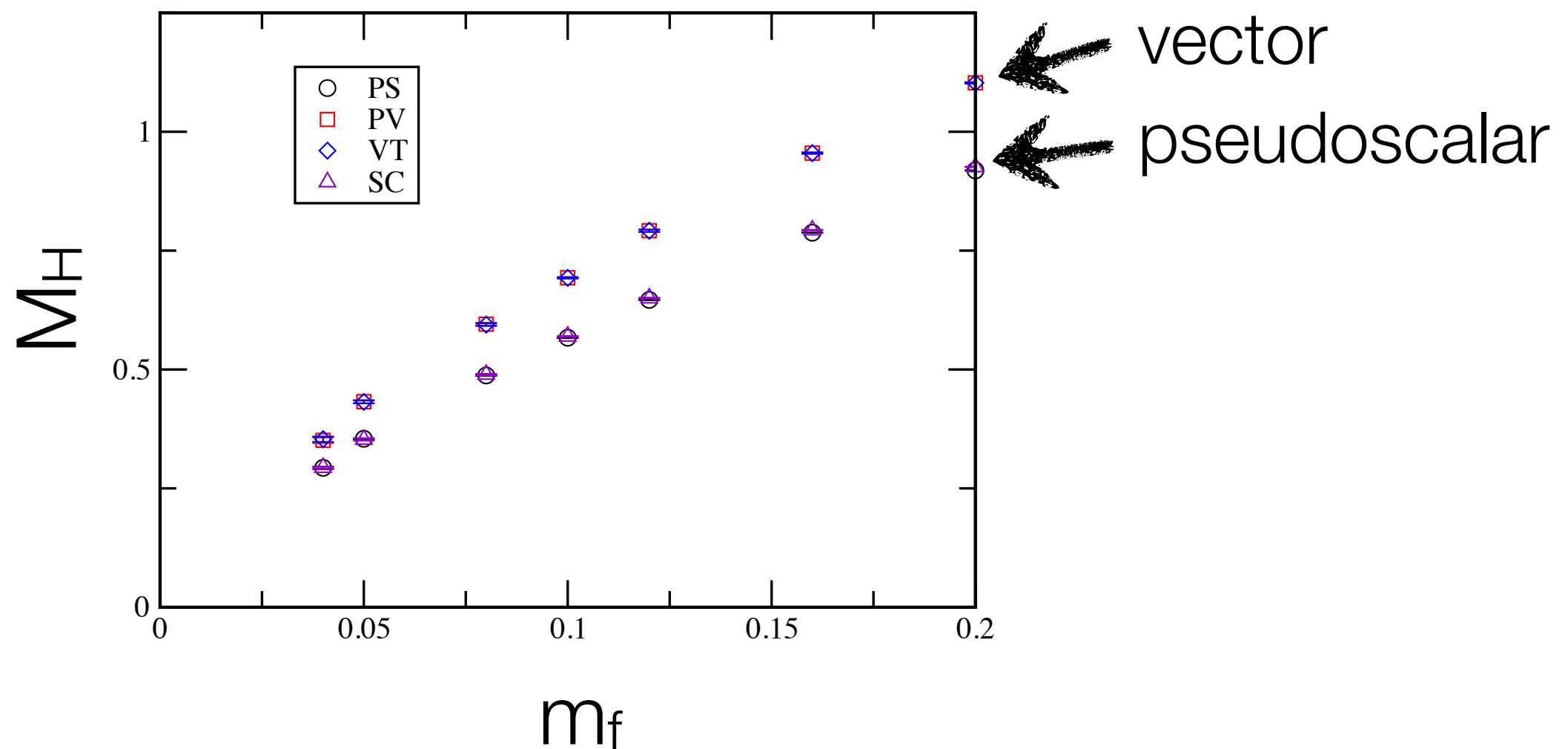


Simulation

- Fermion Formulation: HISQ (Highly Improved Staggered Quarks)
 - being used for state-of-the-art QCD calculations / MILC,...
- Gauge Field Formulation: tree level Symanzik gauge
- $N_f=4$: $\beta=6/g^2=3.7$, $V=L^3 \times T$: $L/T=2/3$; $L=12, 16$
- $N_f=8$: $\beta=6/g^2=3.8$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30, 36$
- $N_f=12$ (two lattice spacings): [LatKMI collab. PRD86 (2012) 054506]
 - $\beta=6/g^2=3.7$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.04 \leq m_f \leq 0.2$
 - $\beta=6/g^2=4.0$, $V=L^3 \times T$: $L/T=3/4$; $L=18, 24, 30$, $0.05 \leq m_f \leq 0.24$
- using MILC code v7, with modification: HMC and speed up in MD

staggered flavor symmetry for $N_f=12$ HISQ

- comparing masses with different staggered operators for π & ρ for $\beta=3.7$



- excellent staggered flavor symmetry, thanks to HISQ

Hadron spectrum: m_f -response in mass deformed theory

- IR conformal phase:
 - coupling runs for $\mu < m_f$: like $n_f=0$ QCD with $\Lambda_{\text{QCD}} \sim m_f$
 - multi particle state : $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$ (criticality @ IRFP)
- $S \chi$ SB phase:
 - ChPT
 - at leading: $M_\pi^2 \propto m_f$, ; $F_\pi = F + c m_f$

a crude study using ratios

- conformal scenario:

- $M_H \propto m_f^{1/(1+\gamma_m^*)}$; $F_\pi \propto m_f^{1/(1+\gamma_m^*)}$ for small m_f

- ★ $F_\pi/M_\pi \rightarrow \text{const.}$ for small m_f

- ★ $M_\rho/M_\pi \rightarrow \text{const.}$ for small m_f

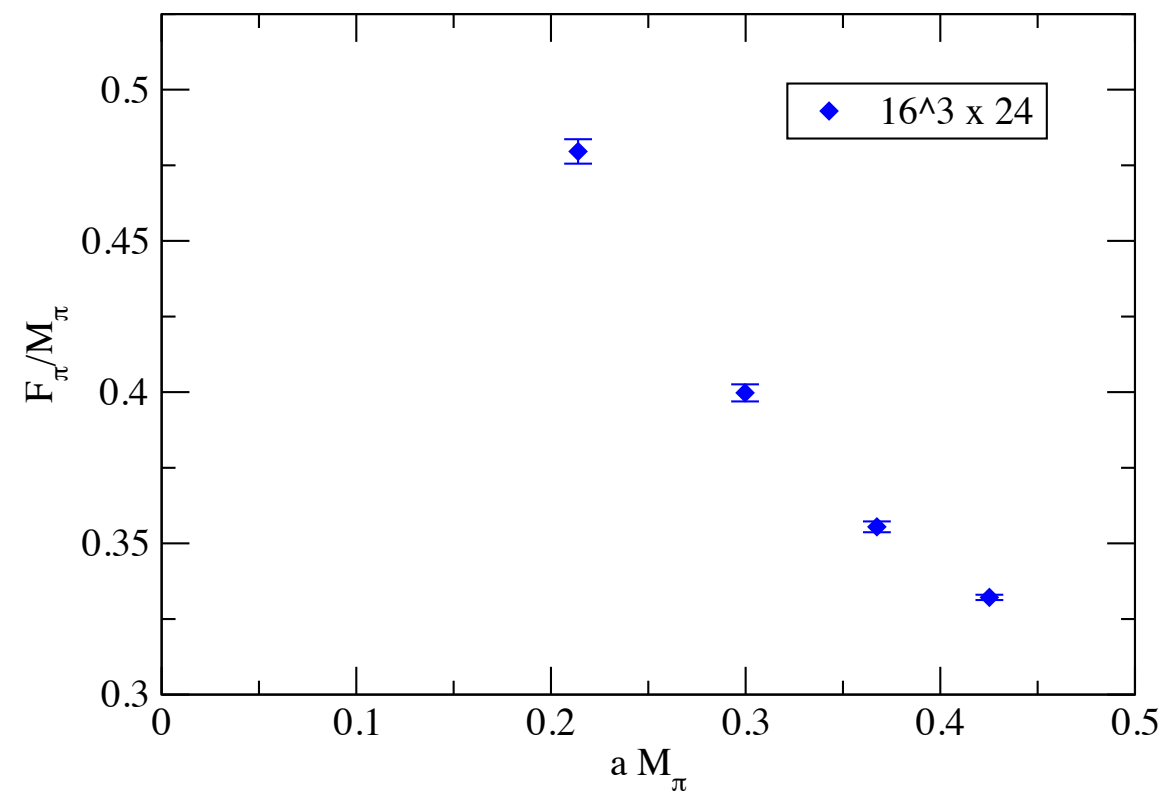
- chiral symmetry breaking scenario:

- $M_\pi^2 \propto m_f$, ; $F_\pi = F + c' M_\pi^2$ for small m_f

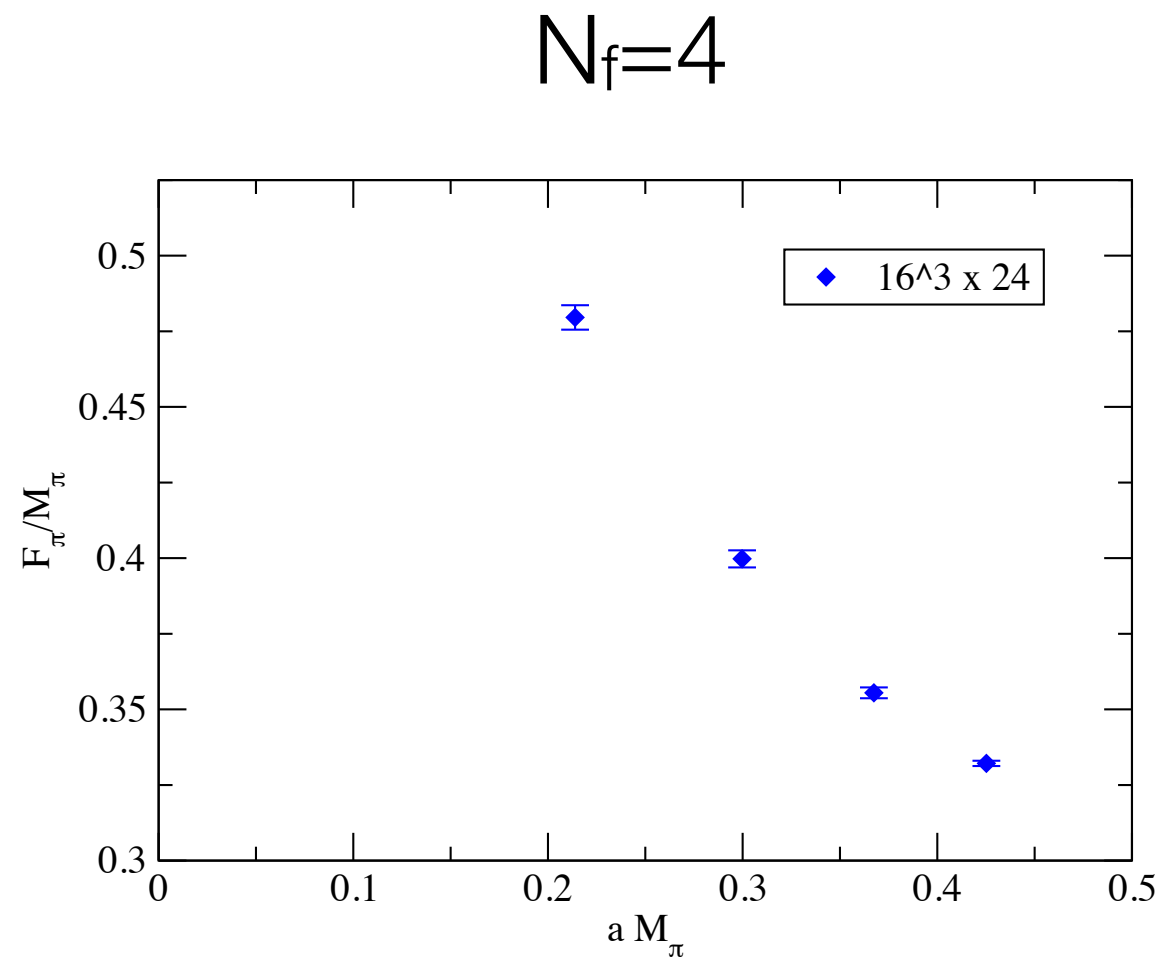
- ★ $F_\pi/M_\pi \rightarrow \infty$ for $m_f \rightarrow 0$

a crude analysis: F_π/M_π vs M_π

$N_f=4$

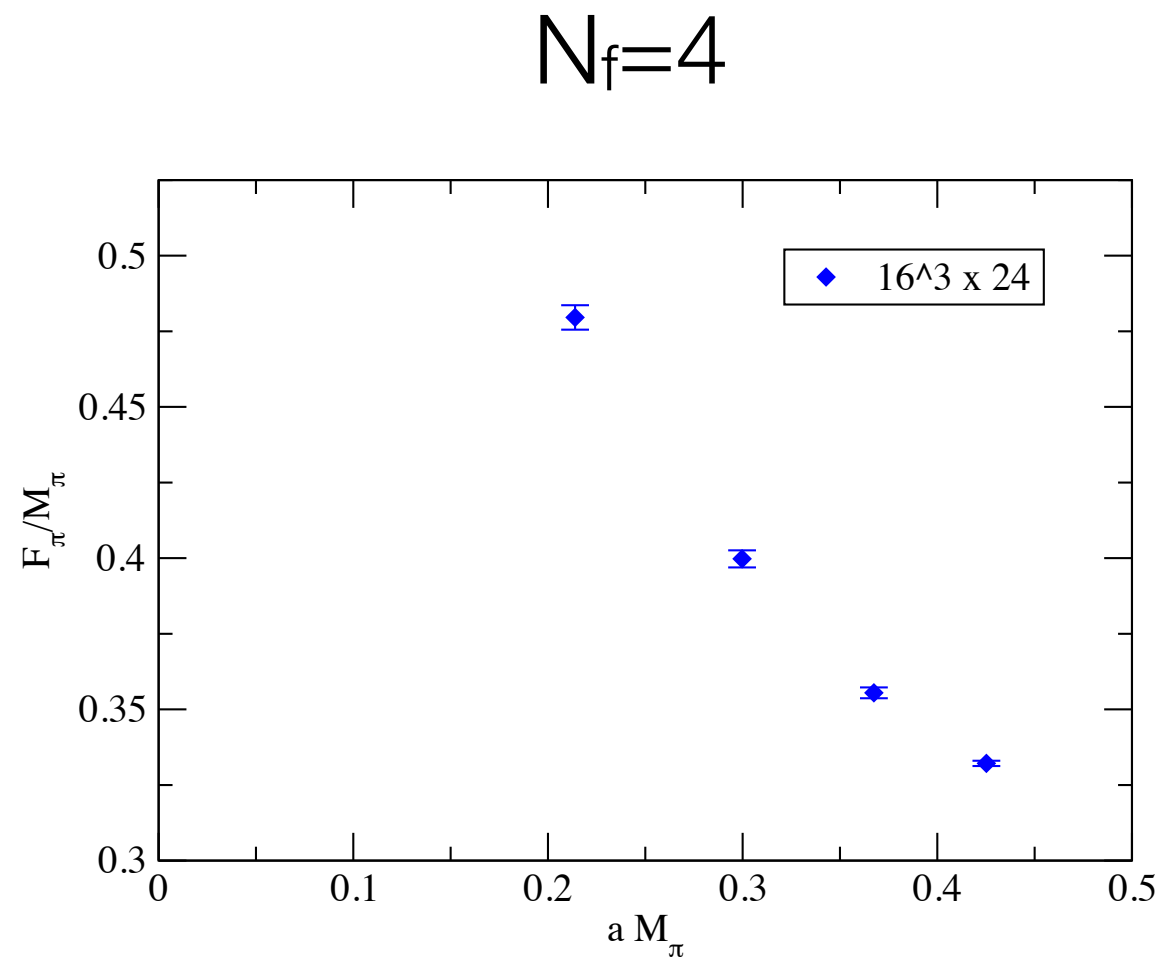


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- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)

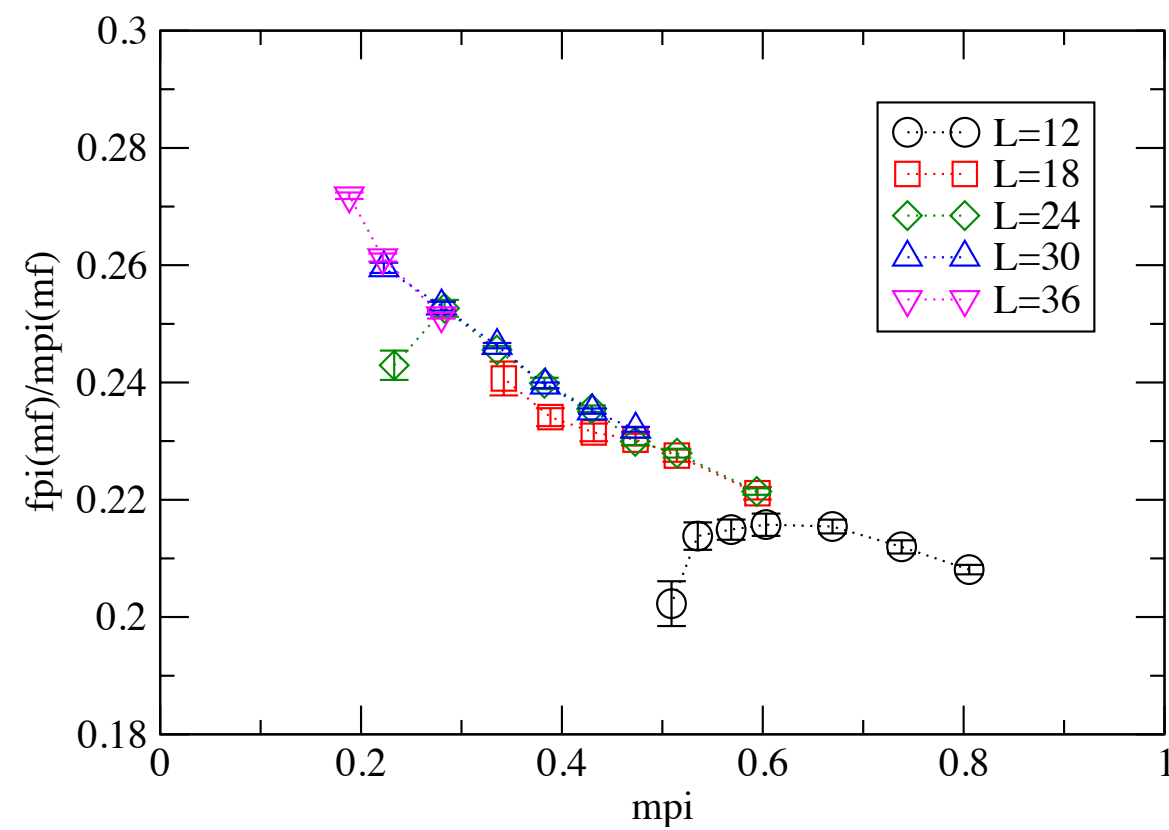
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- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)
- spontaneous chiral symmetry breaking

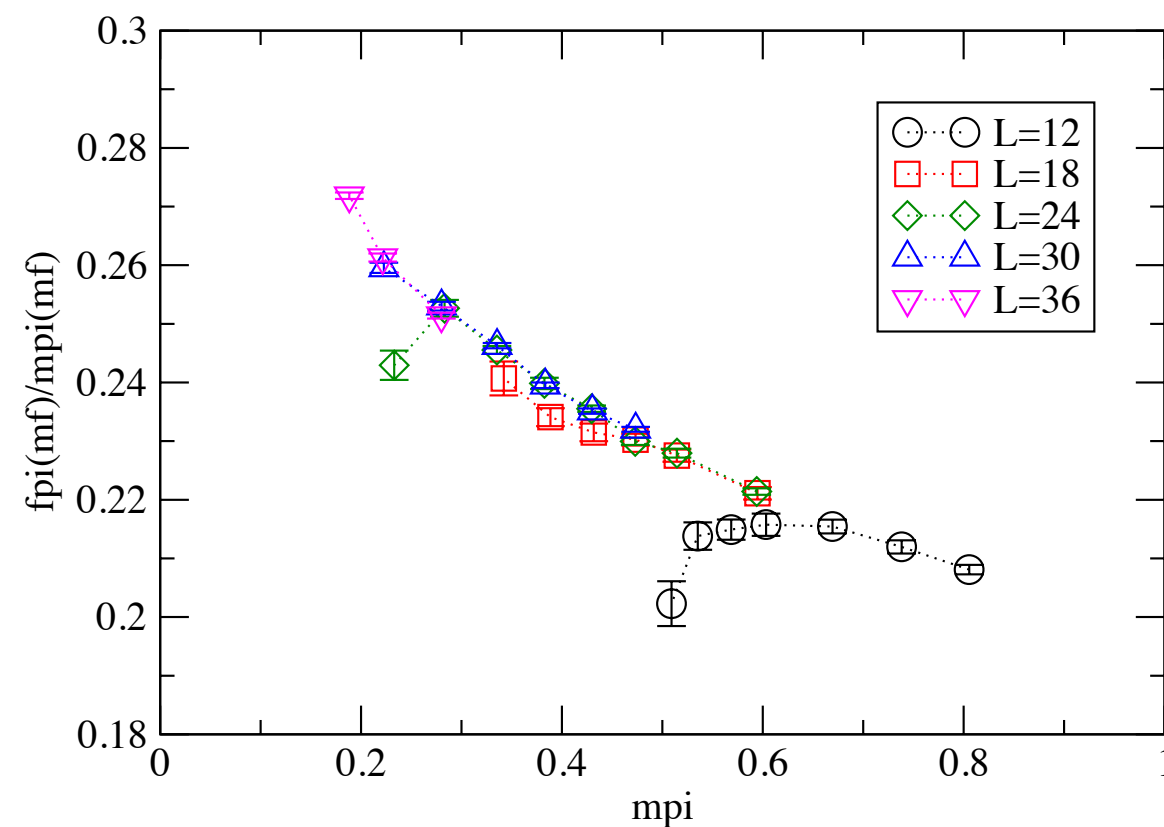
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$N_f=8$



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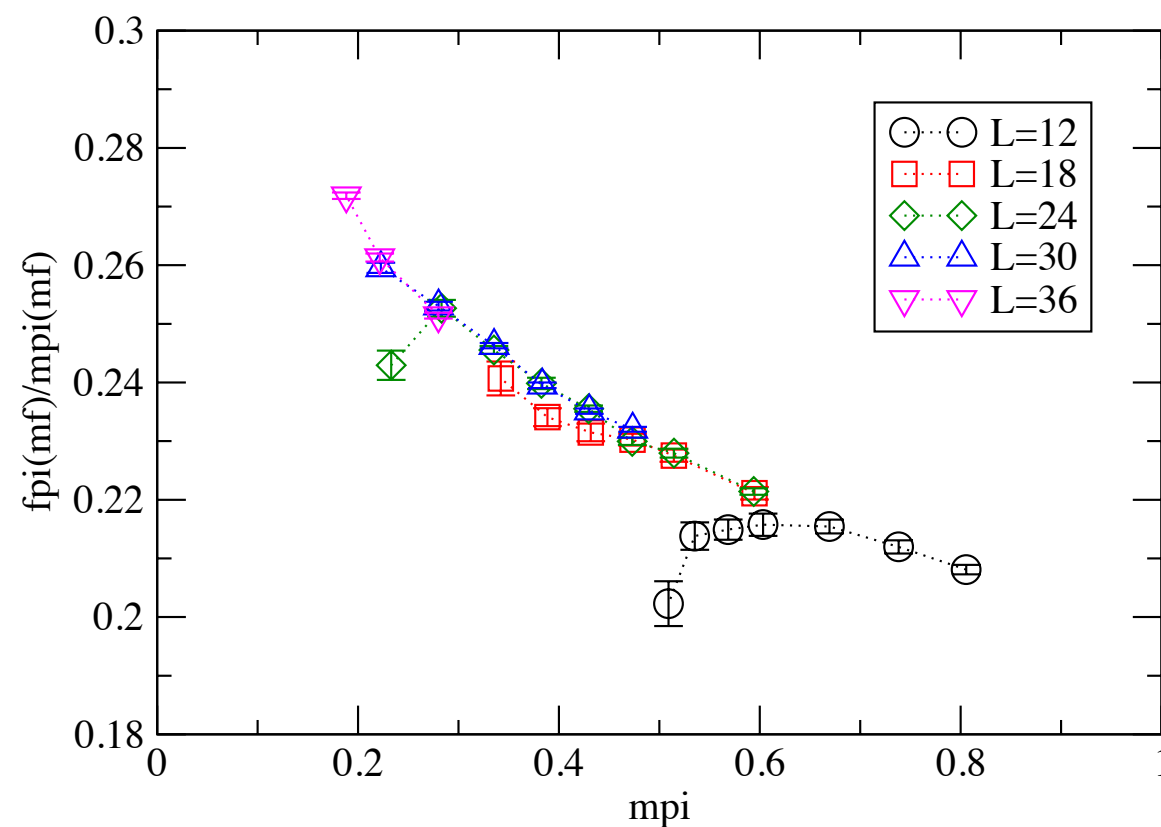
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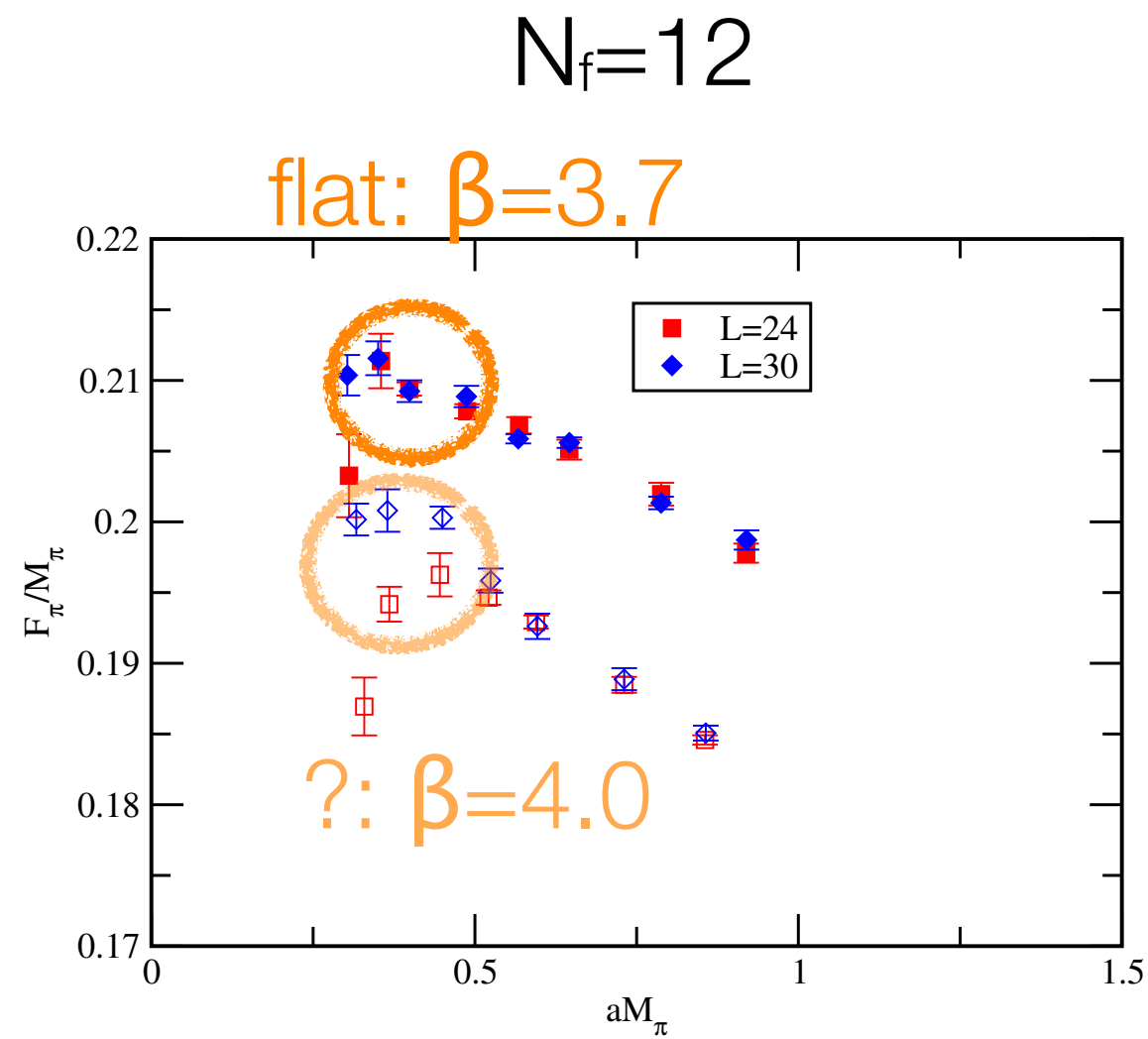
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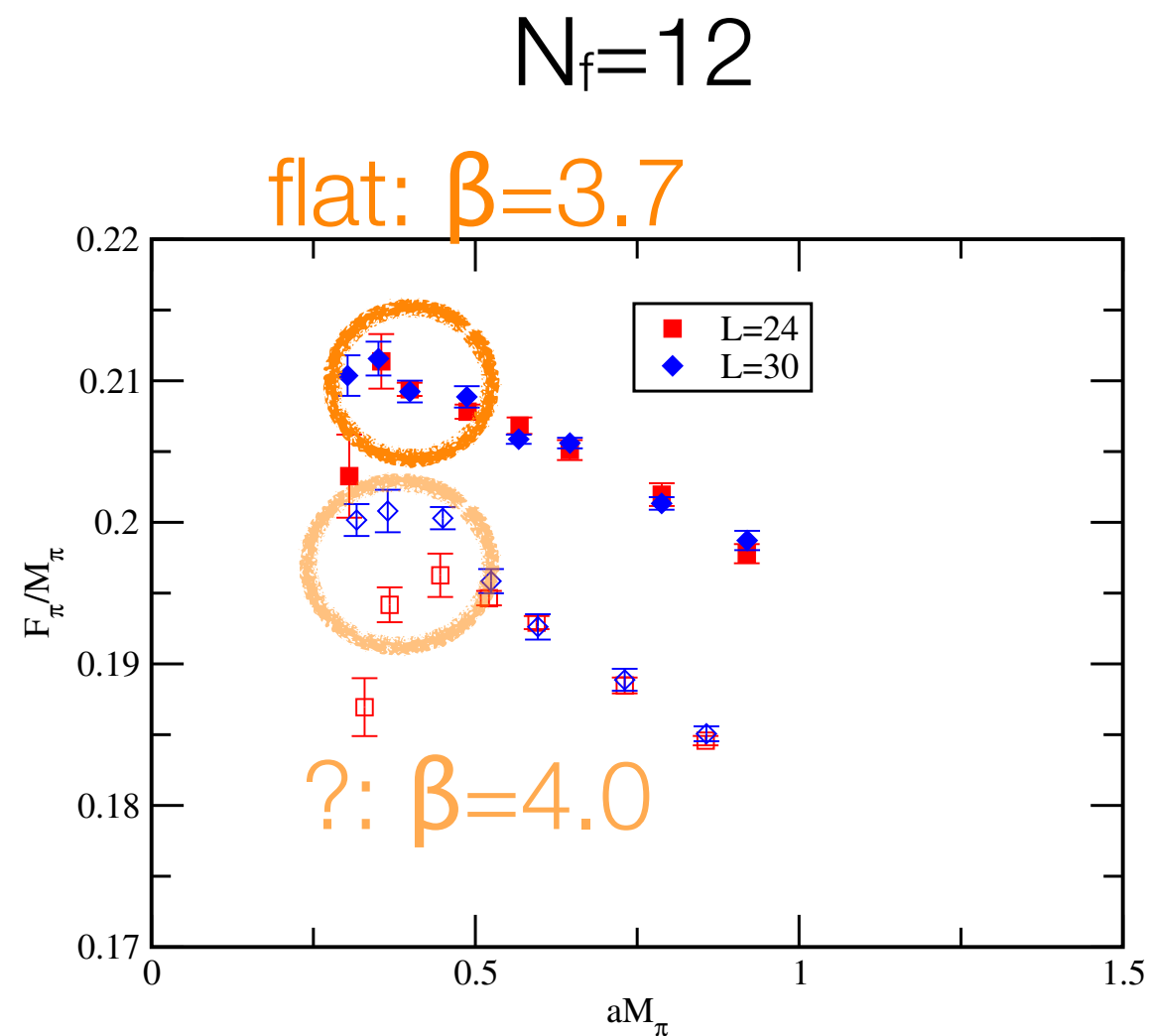


- tends to diverge towards the chiral limit ($M_\pi \rightarrow 0$)
- spontaneous chiral symmetry breaking, likely

a crude analysis: F_π/M_π vs M_π

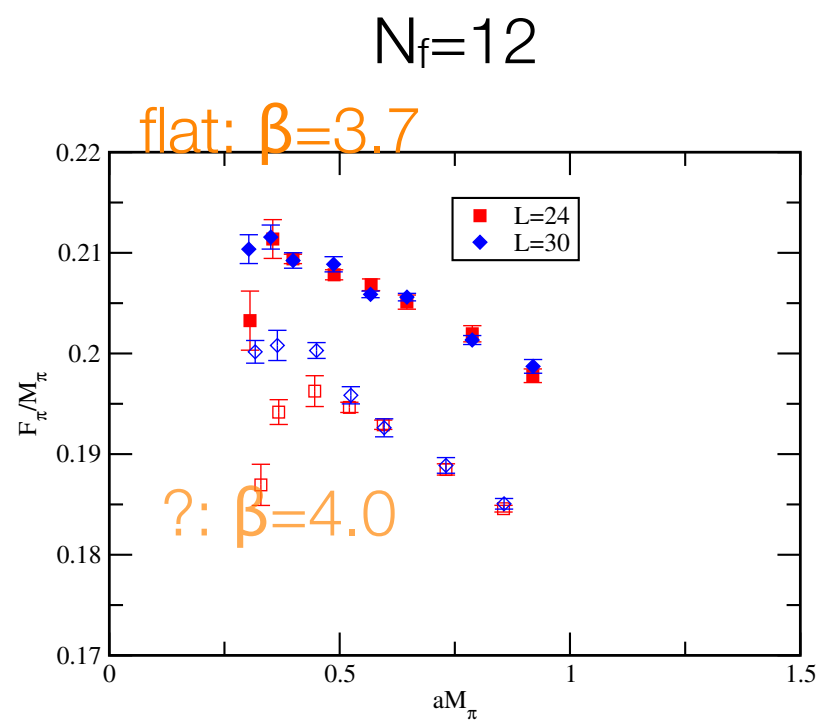


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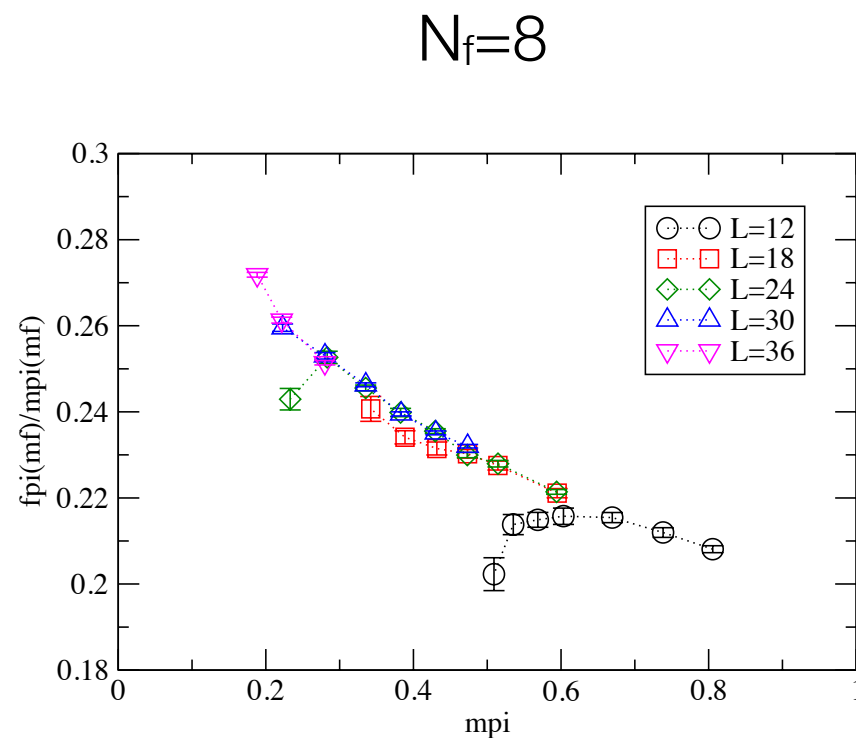


- $\beta=3.7$: small mass: consistent with conformal scenario
- $\beta=4.0$: volume likely too small to discuss the scaling

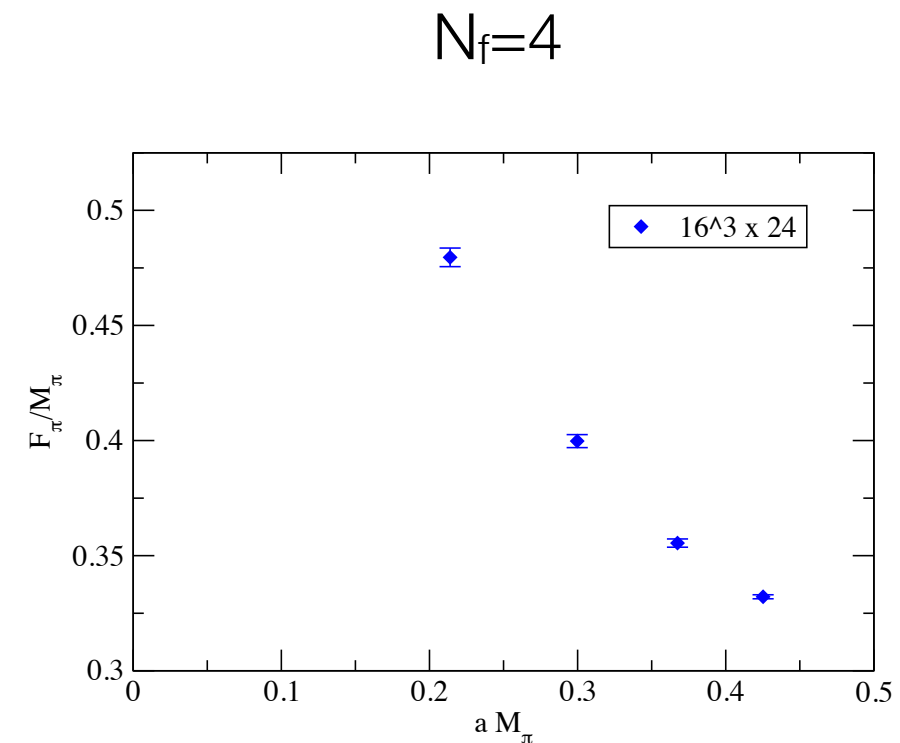
a crude analysis: F_π/M_π vs M_π leads to a likely scenario



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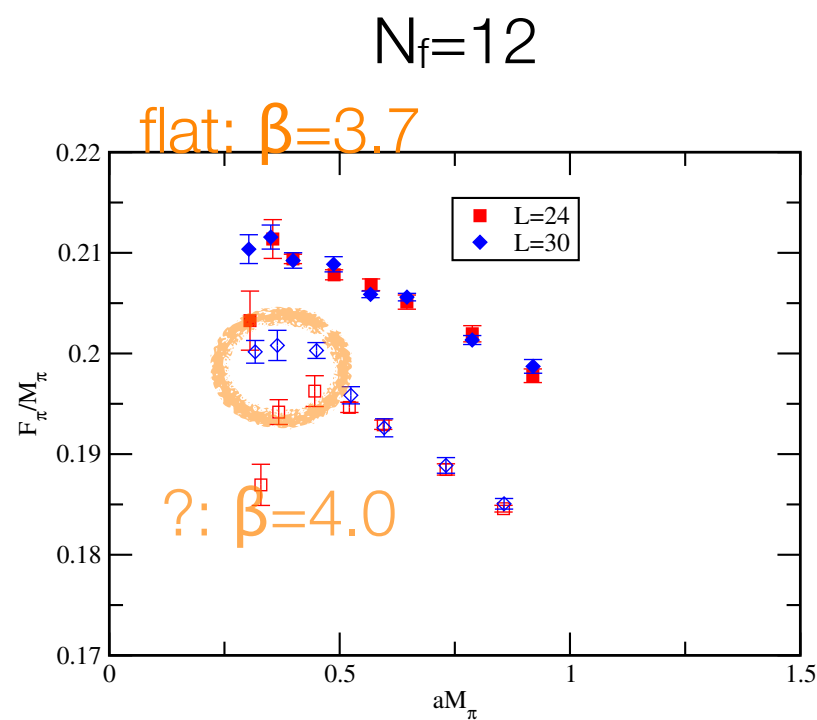


- ~~chiral symmetry~~

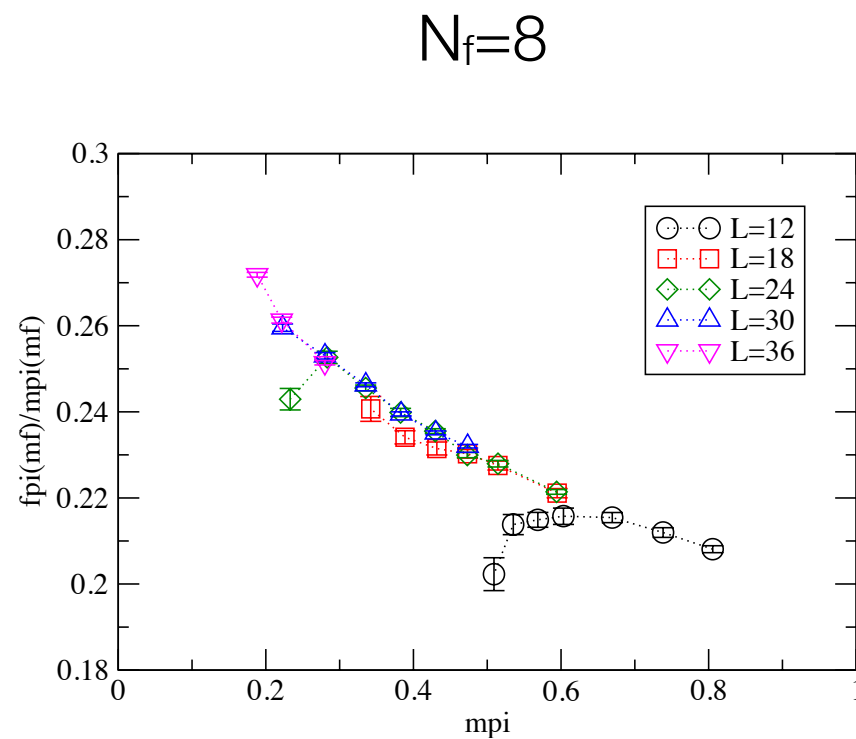


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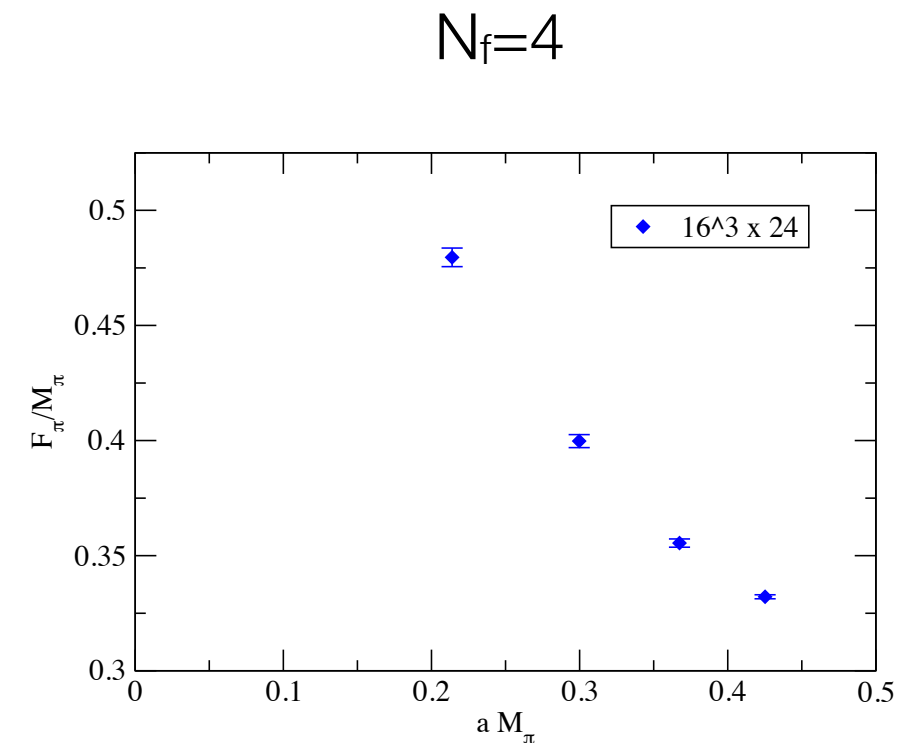
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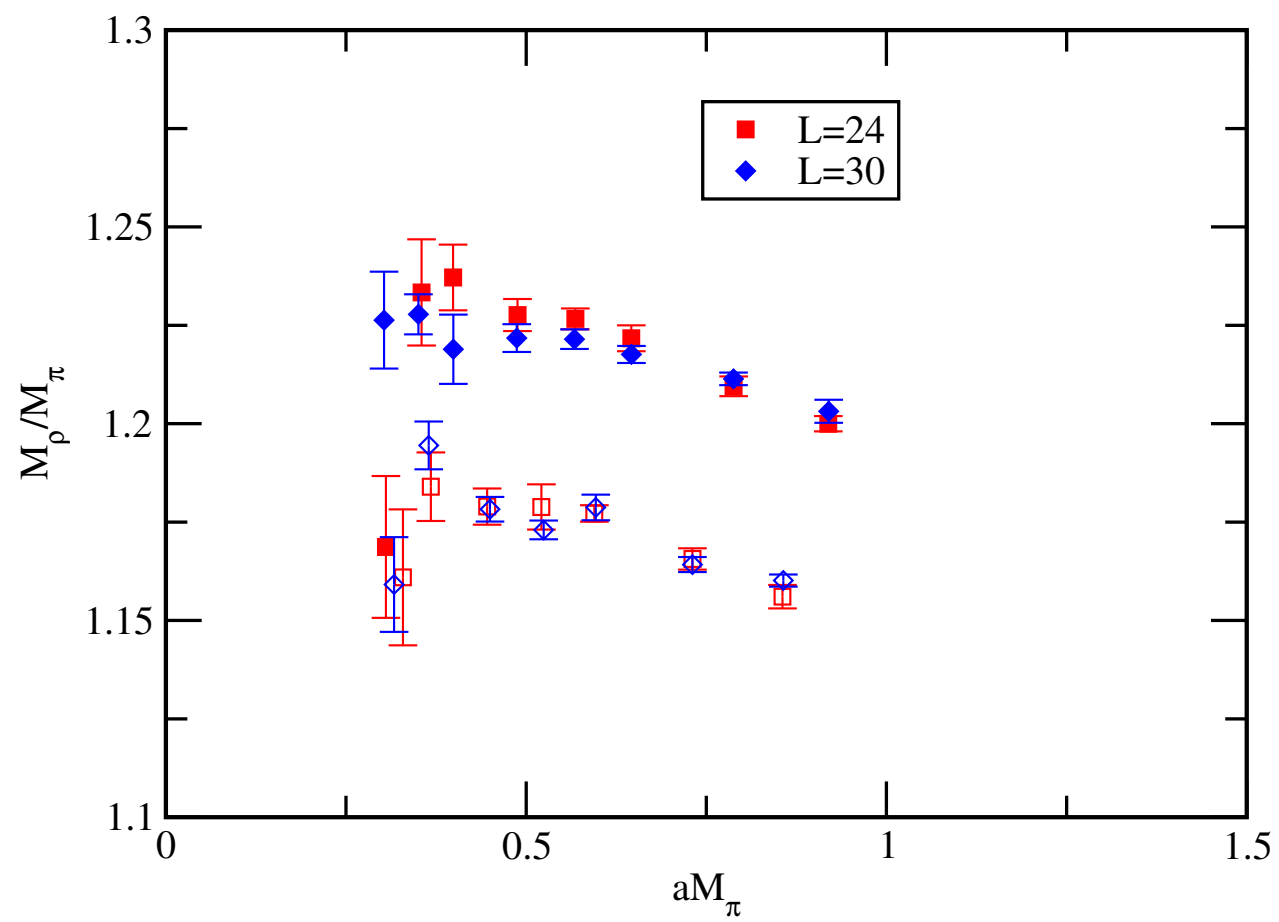
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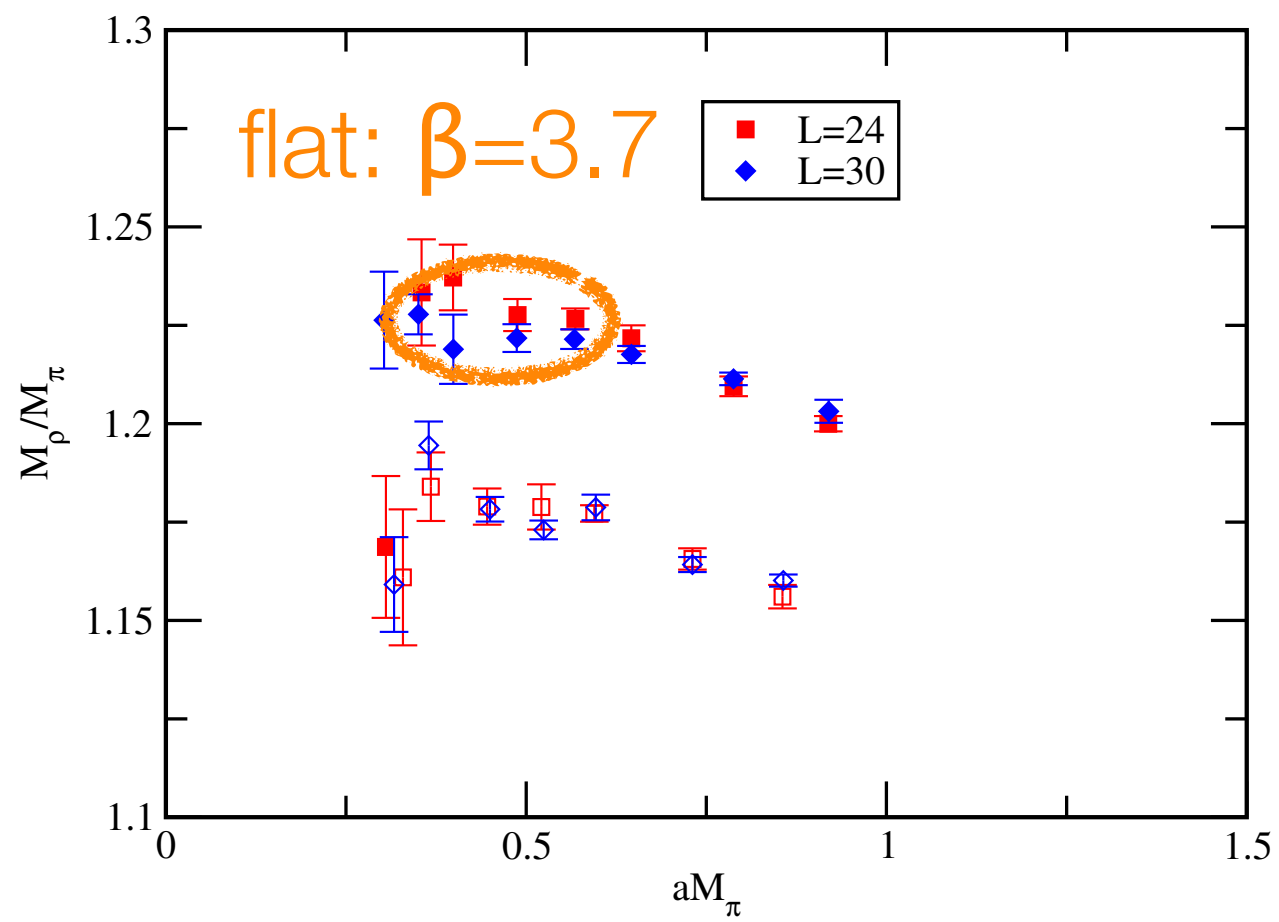
a crude analysis: M_ρ/M_π vs M_π

$N_f=12$: HISQ



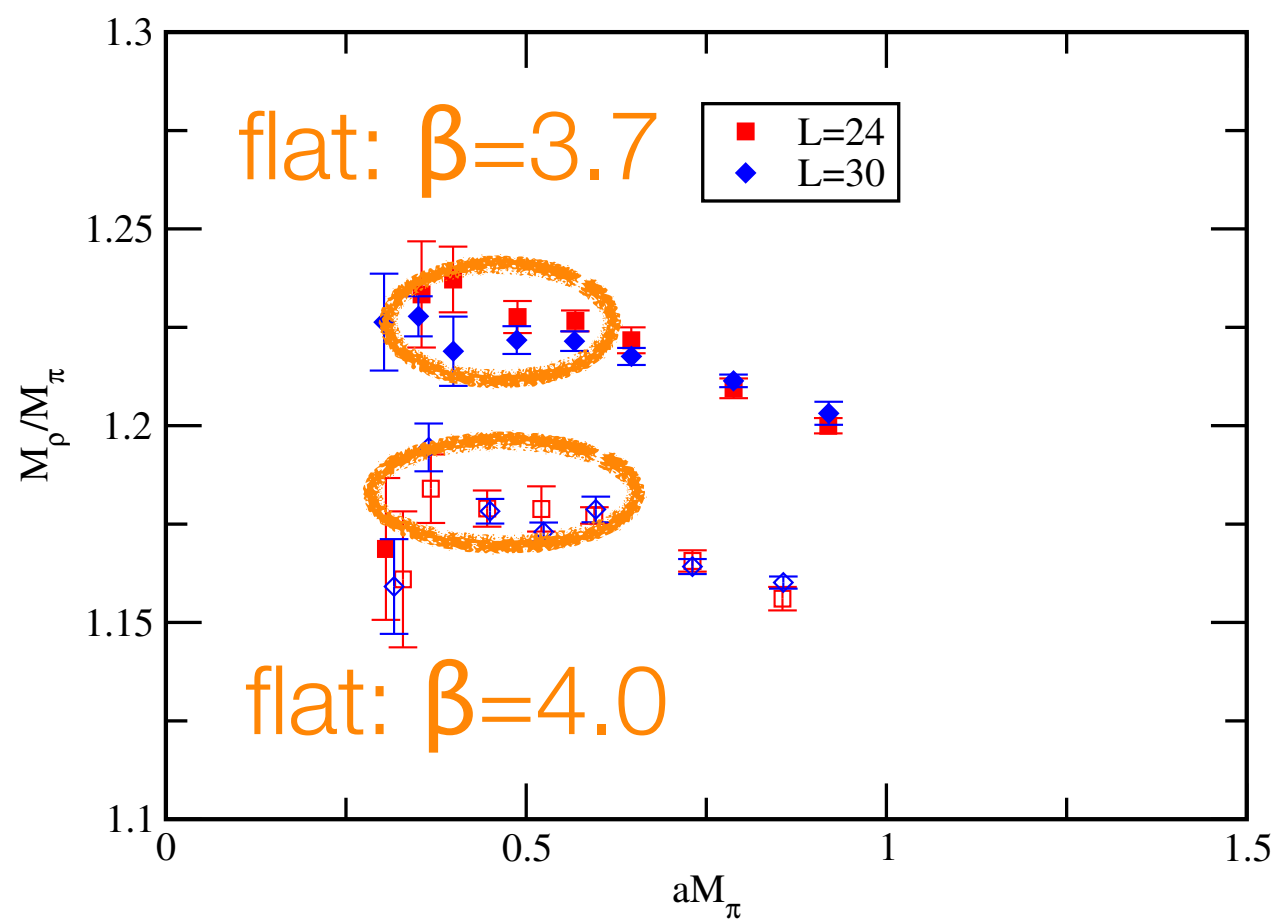
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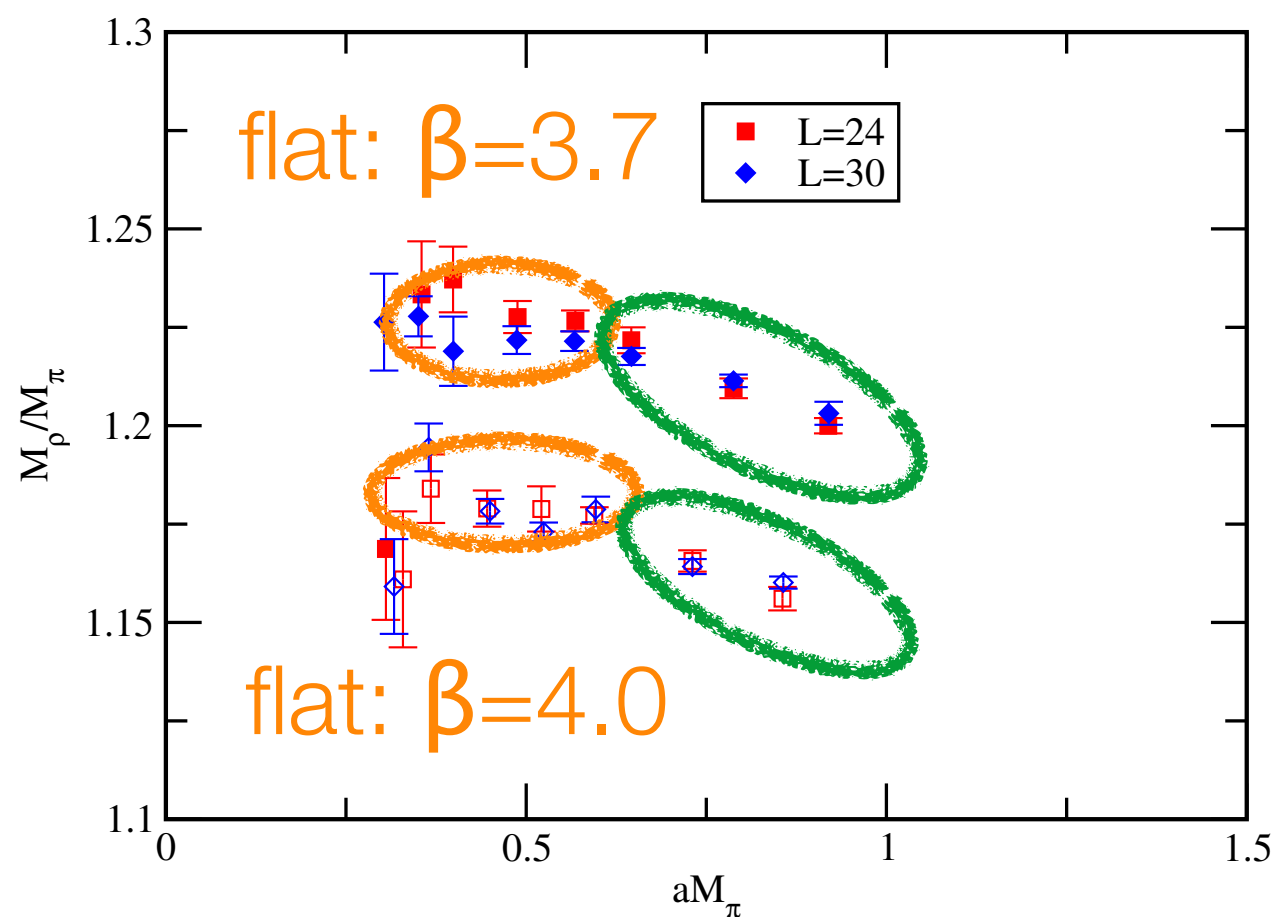
$N_f=12$: HISQ



- $\beta=3.7$ & 4.0 : small mass (wider than F_π): consistent with hyper scaling (HS)

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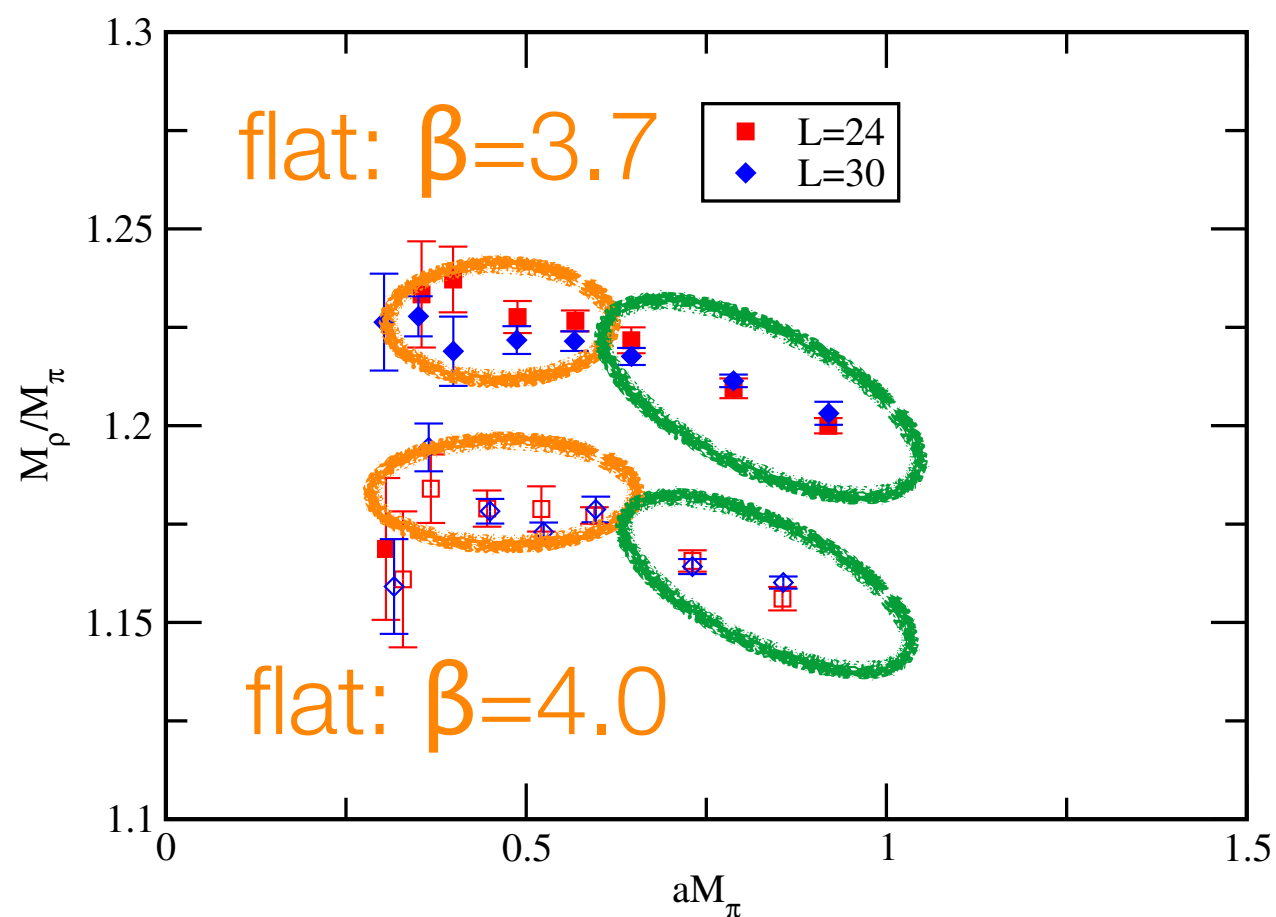
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- mass dependence at the tail is due to non-universal mass correction to HS

a crude analysis: M_ρ/M_π vs M_π

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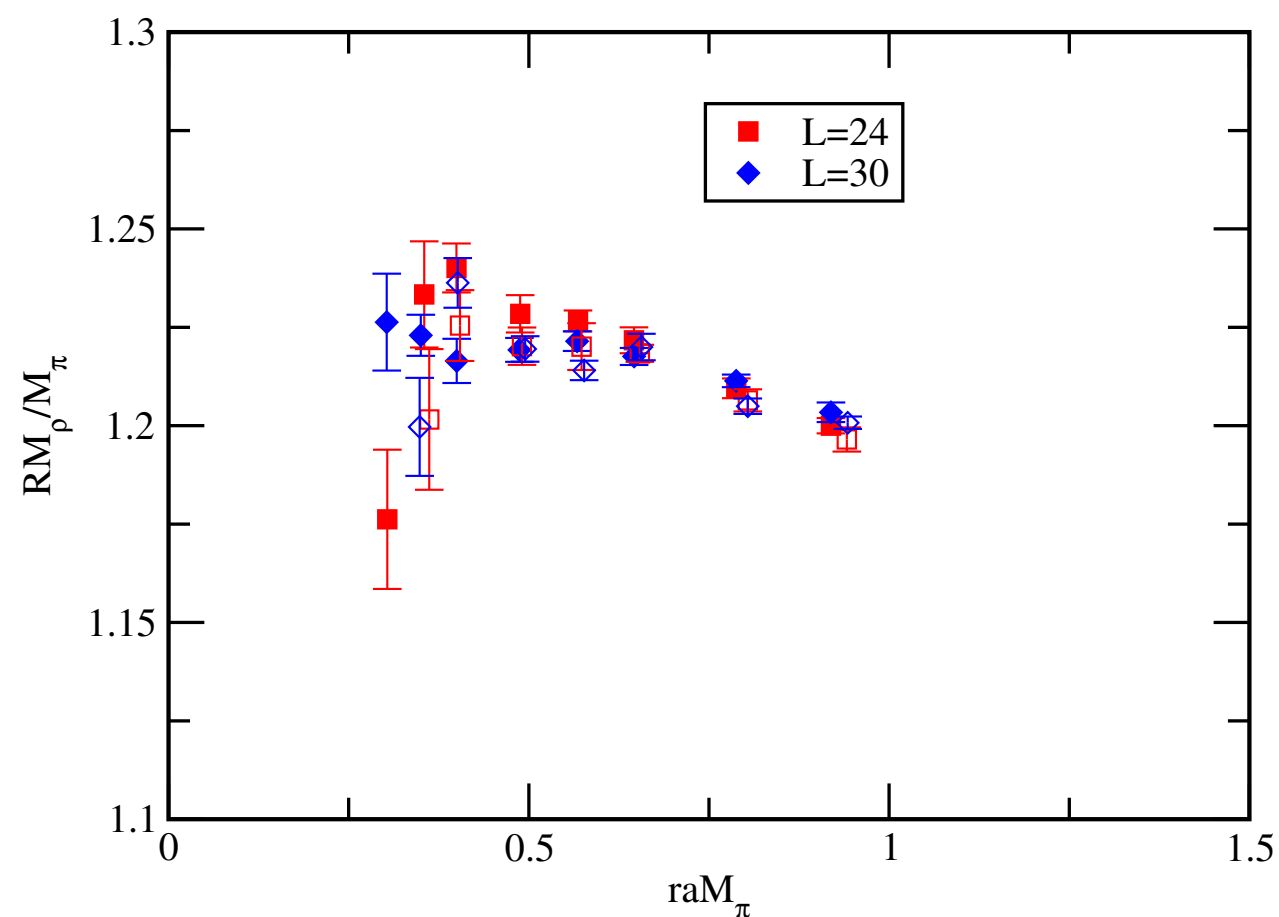


- one may attempt to perform a matching
- assuming $(am)^2$ error is small

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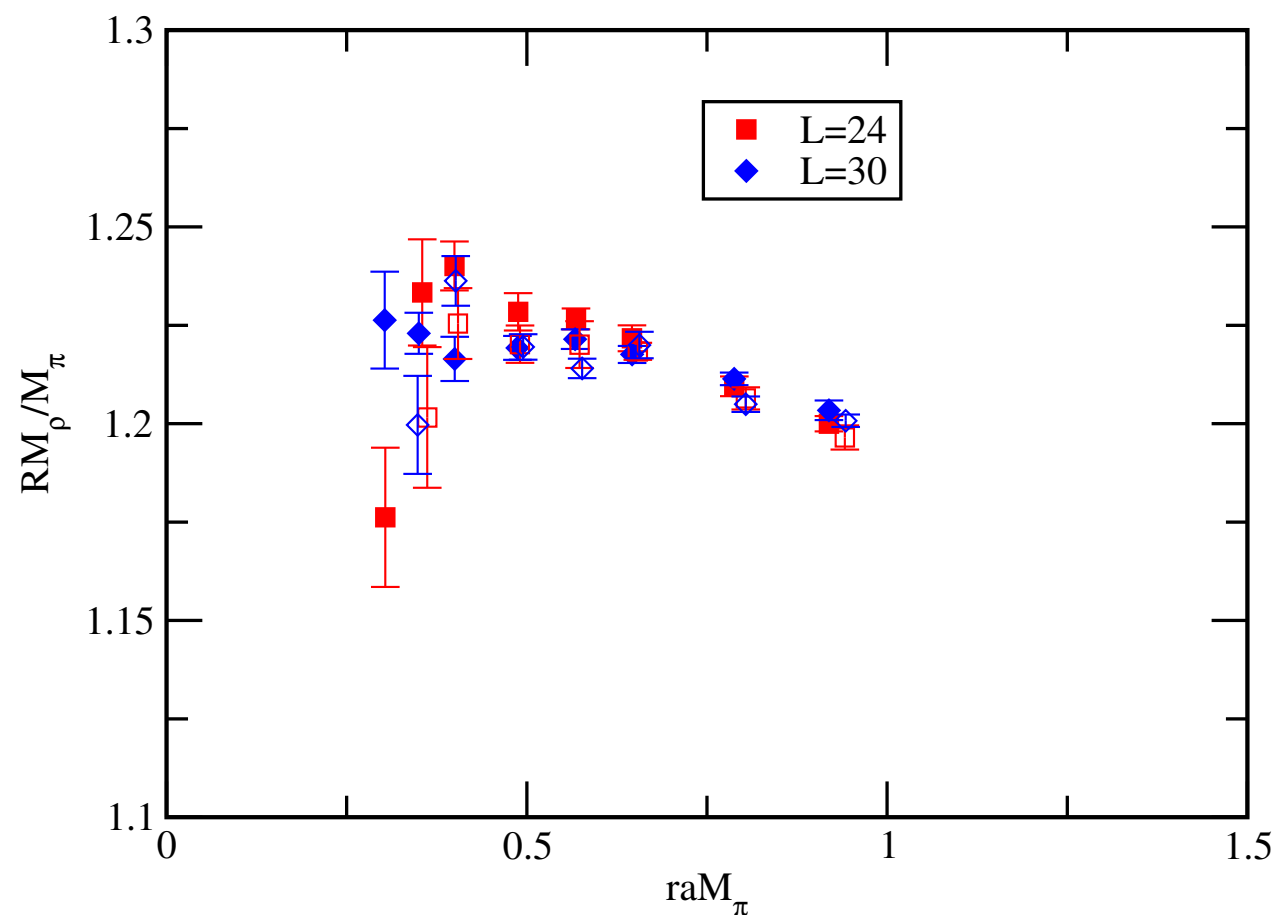


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- ➔ $a(\beta=3.7) / (\beta=4.0) > 1$

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- one may attempt to perform a matching
- assuming $(am)^2$ error is small
- ➔ $a(\beta=3.7) / (\beta=4.0) > 1$
- movement: correct direction in asymptotically free domain !

- $\beta=3.7$ & 4.0 : small mass (wider than F_π): consistent with hyper scaling (HS)
- mass dependence at the tail is due to non-universal mass correction to HS

conformal (finite size) scaling

- Scaling dimension at IR fixed point [Wilson-Fisher]; Hyper Scaling [Miransky]
- mass dependence is described by anomalous dimensions at IRFP
 - quark mass anomalous dimension γ^*
 - operator anomalous dimension
- hadron mass and pion decay constant obey same scaling

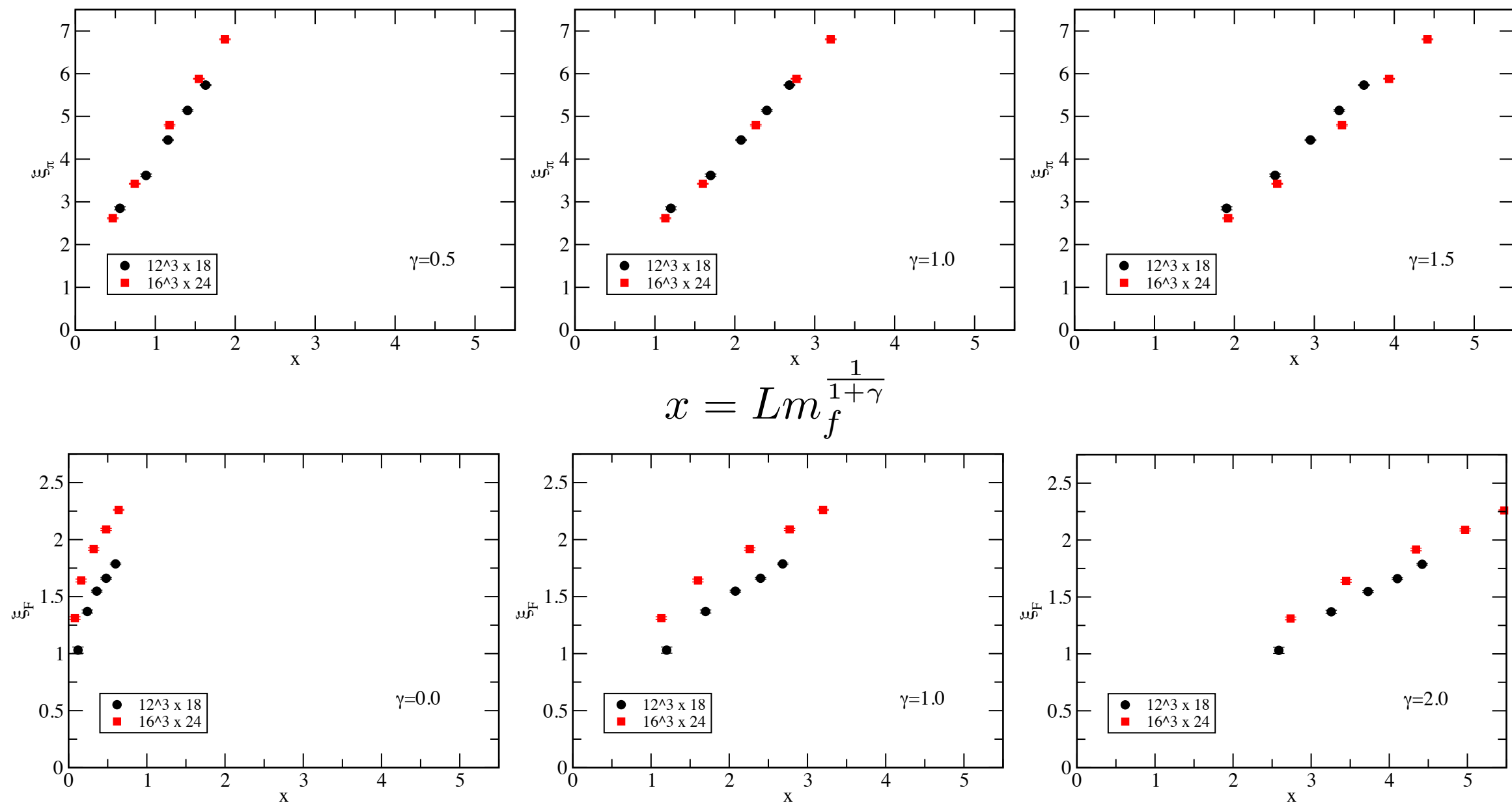
$$M_H \propto m_f^{\frac{1}{1+\gamma^*}} \qquad F_\pi \propto m_f^{\frac{1}{1+\gamma^*}}$$

- **finite size scaling** in a L^4 box (DeGrand; Del Debbio et al)

- scaling variable: $x = L m_f^{\frac{1}{1+\gamma^*}}$

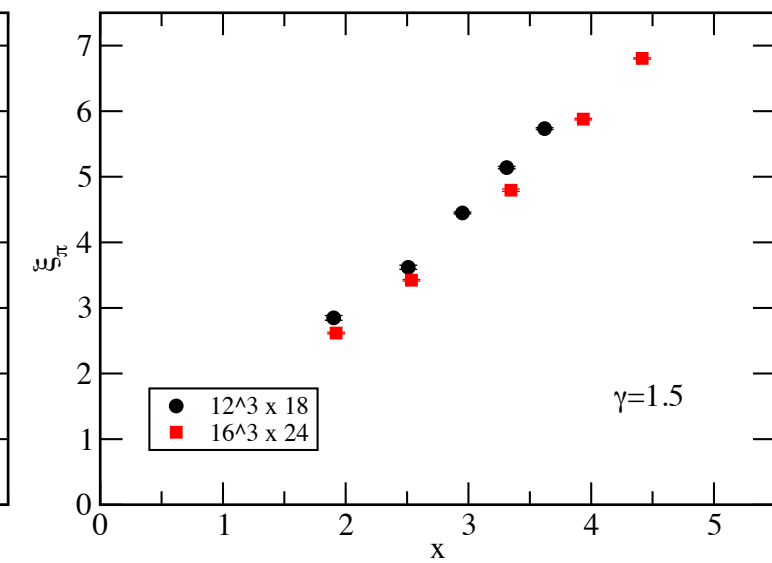
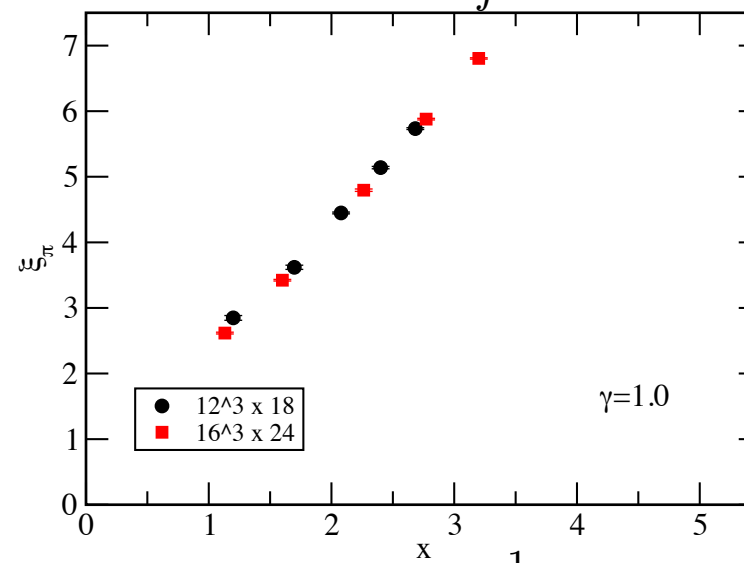
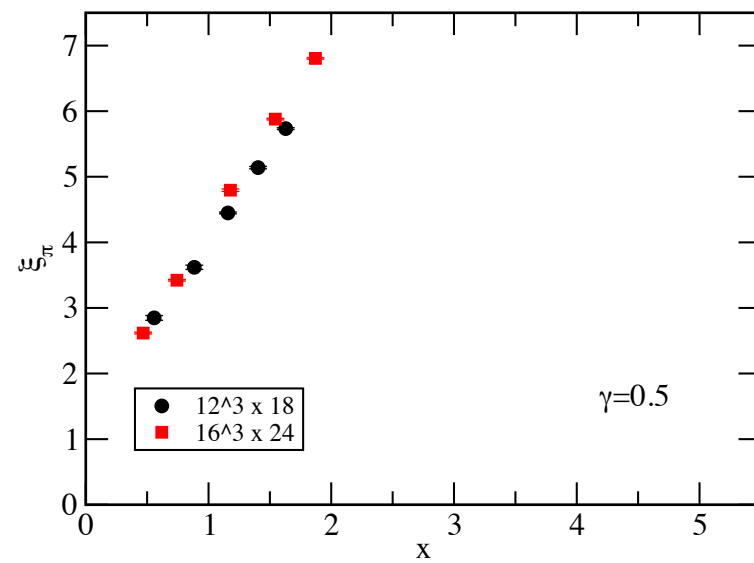
$$L \cdot M_H = f_H(x) \qquad L \cdot F_\pi = f_F(x)$$

$N_f=4$ see if data align at some γ

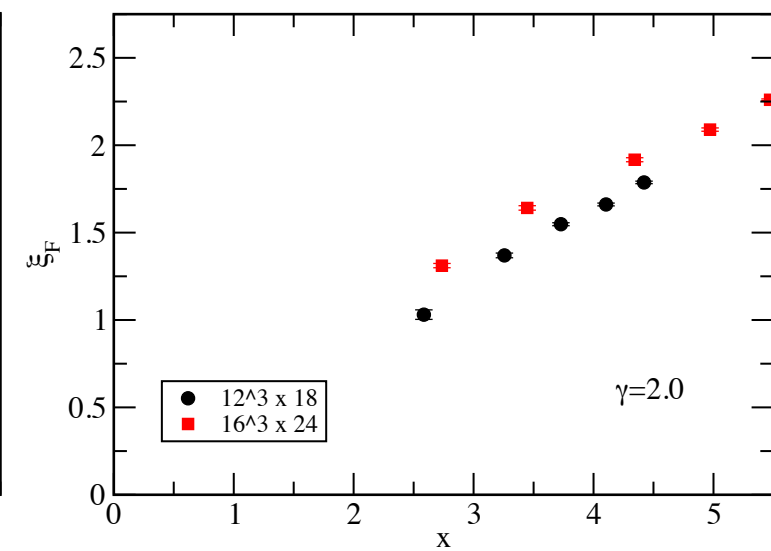
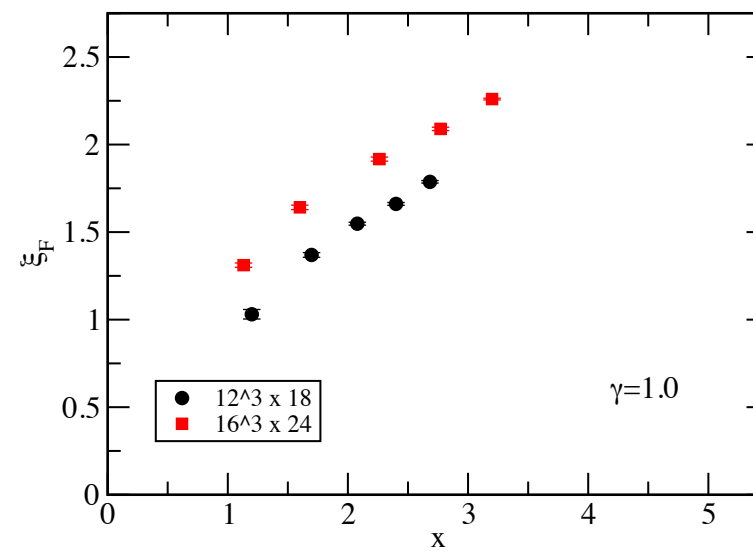
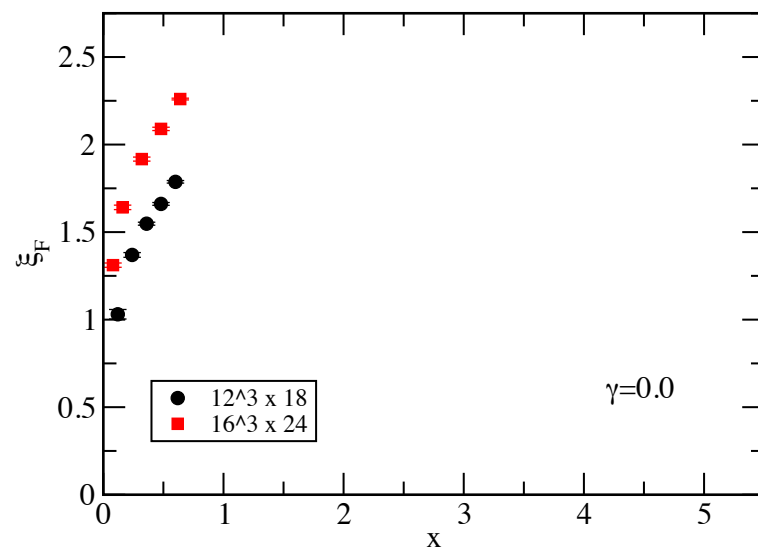


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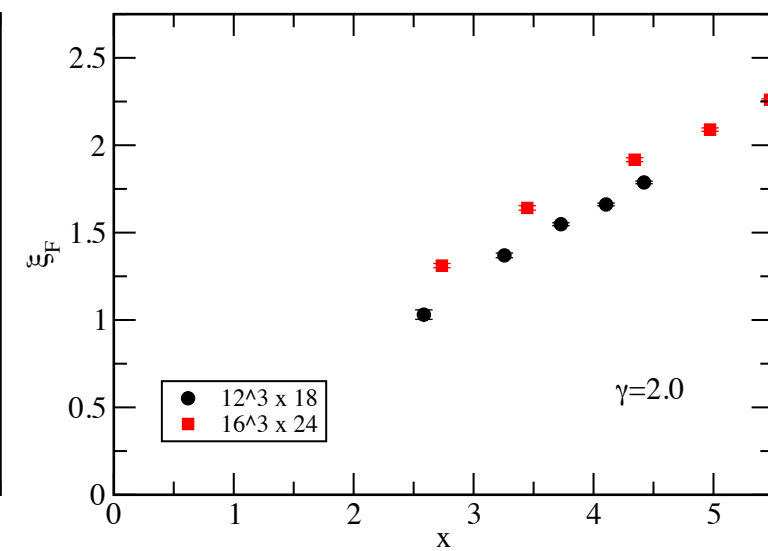
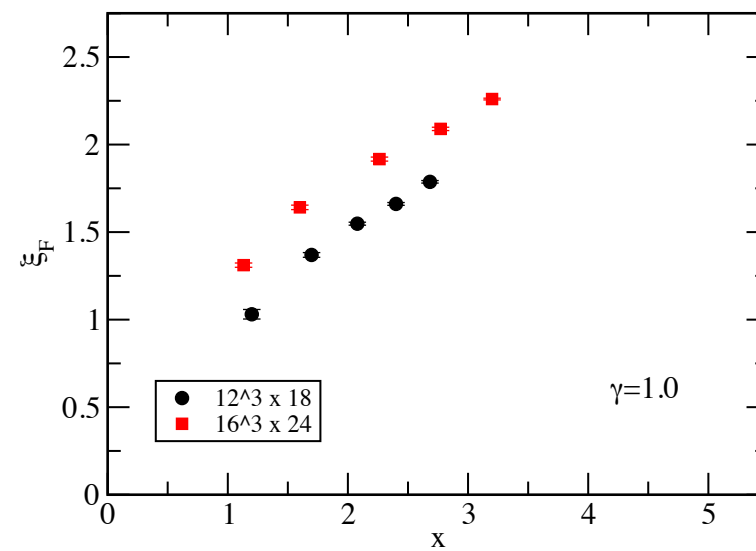
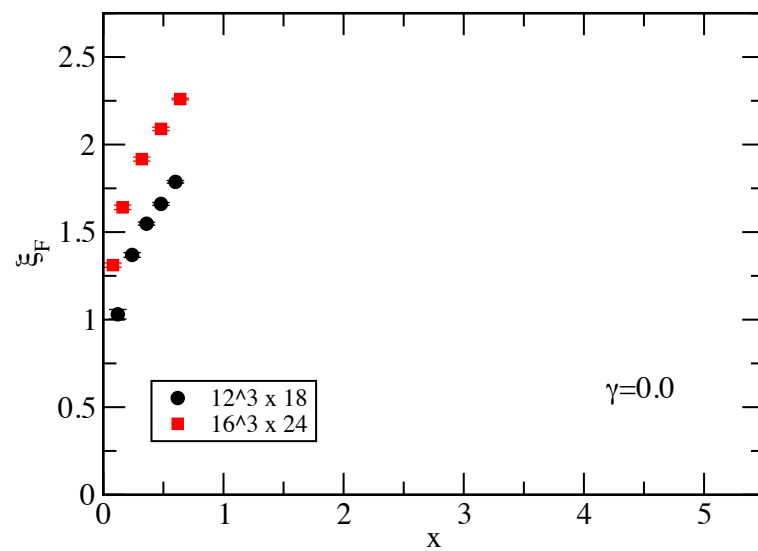
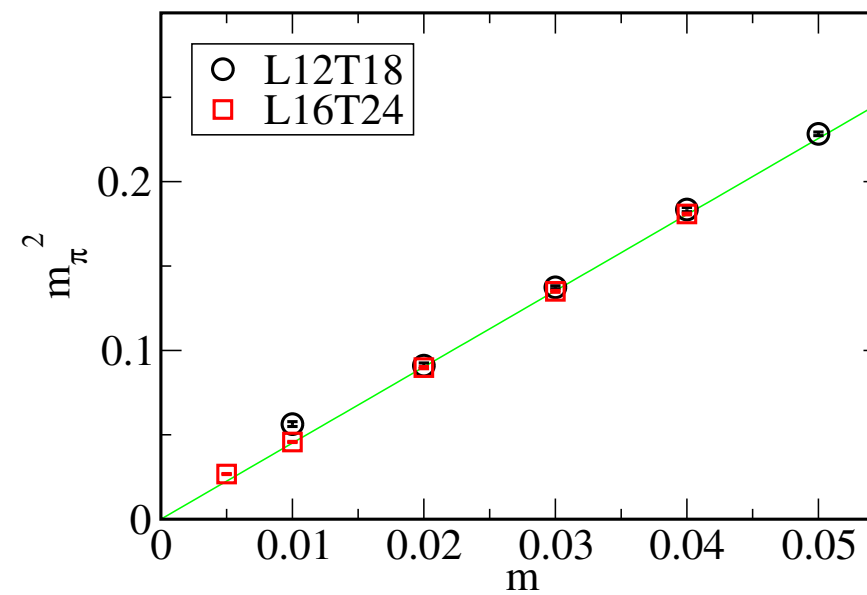
$$M_\pi L \propto m_f^{1/2} L$$



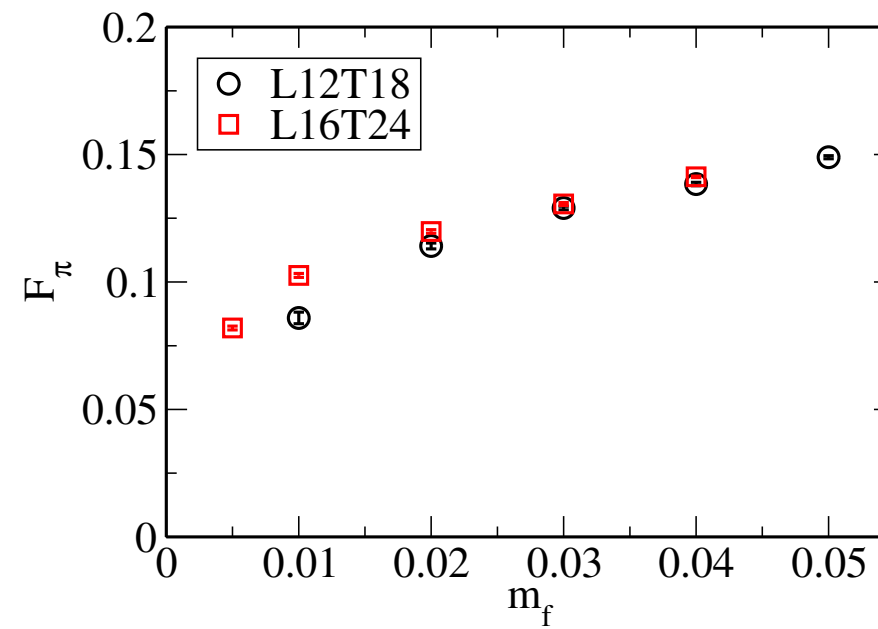
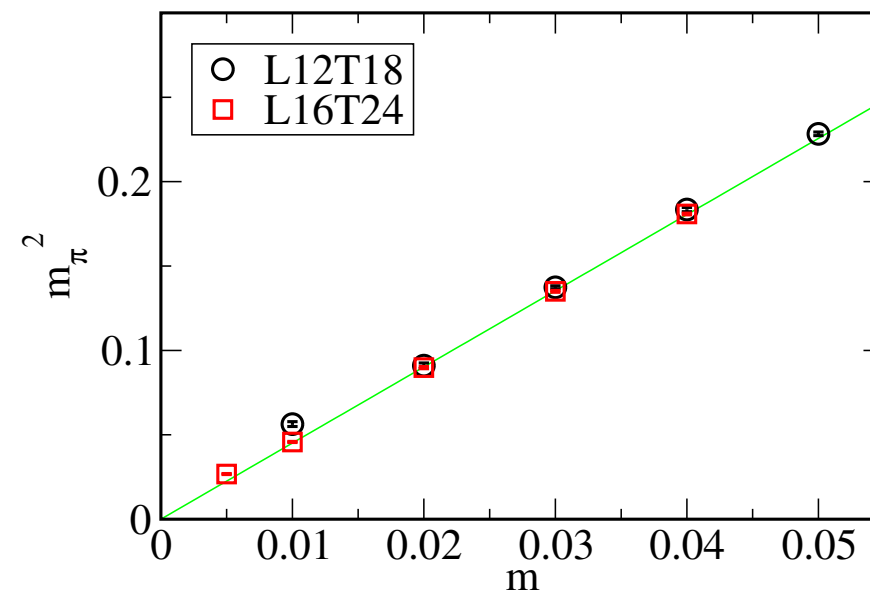
$$x = L m_f^{\frac{1}{1+\gamma}}$$



$N_f=4$ see if data align at some γ

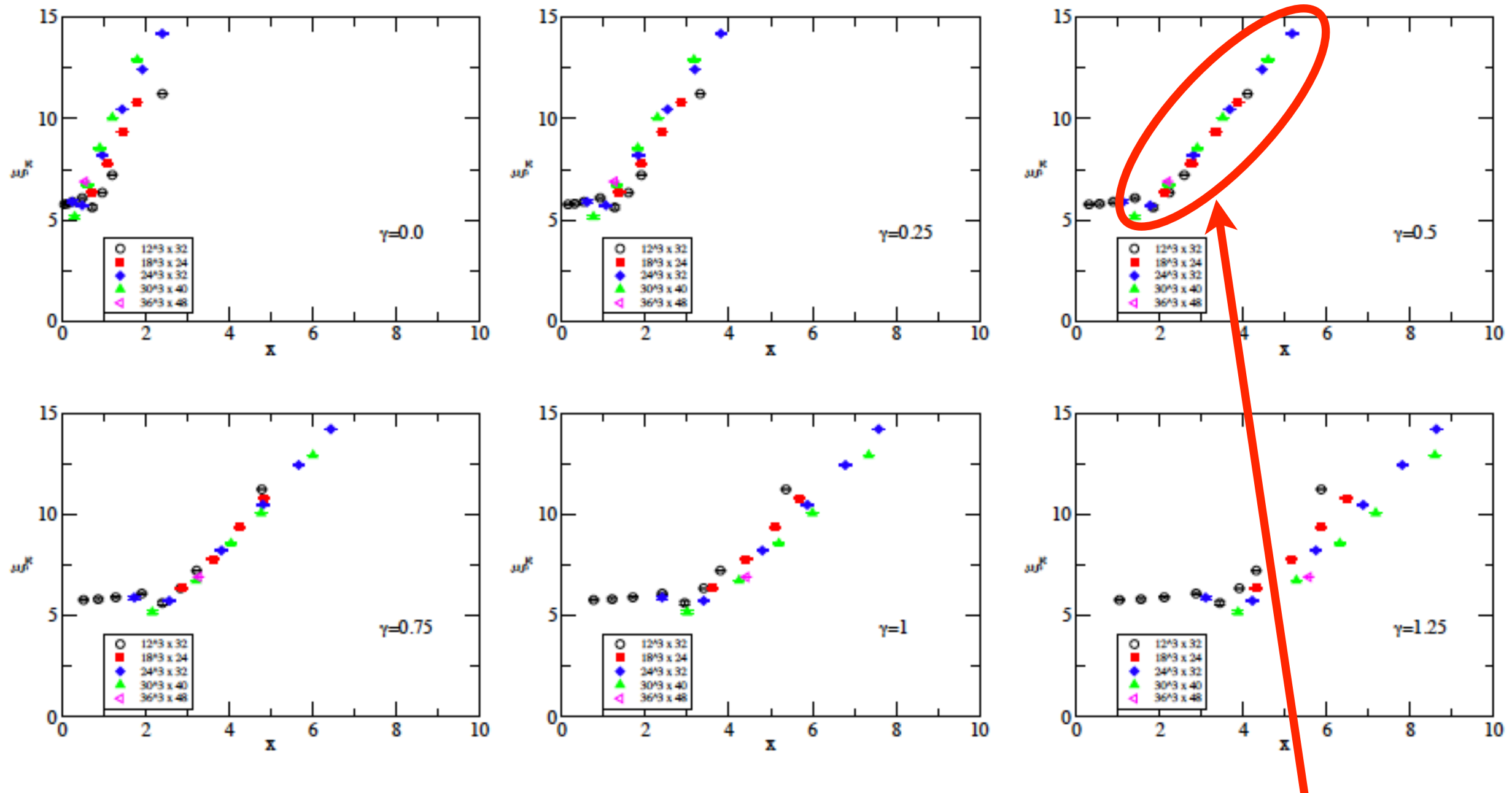


$N_f=4$ see if data align at some γ



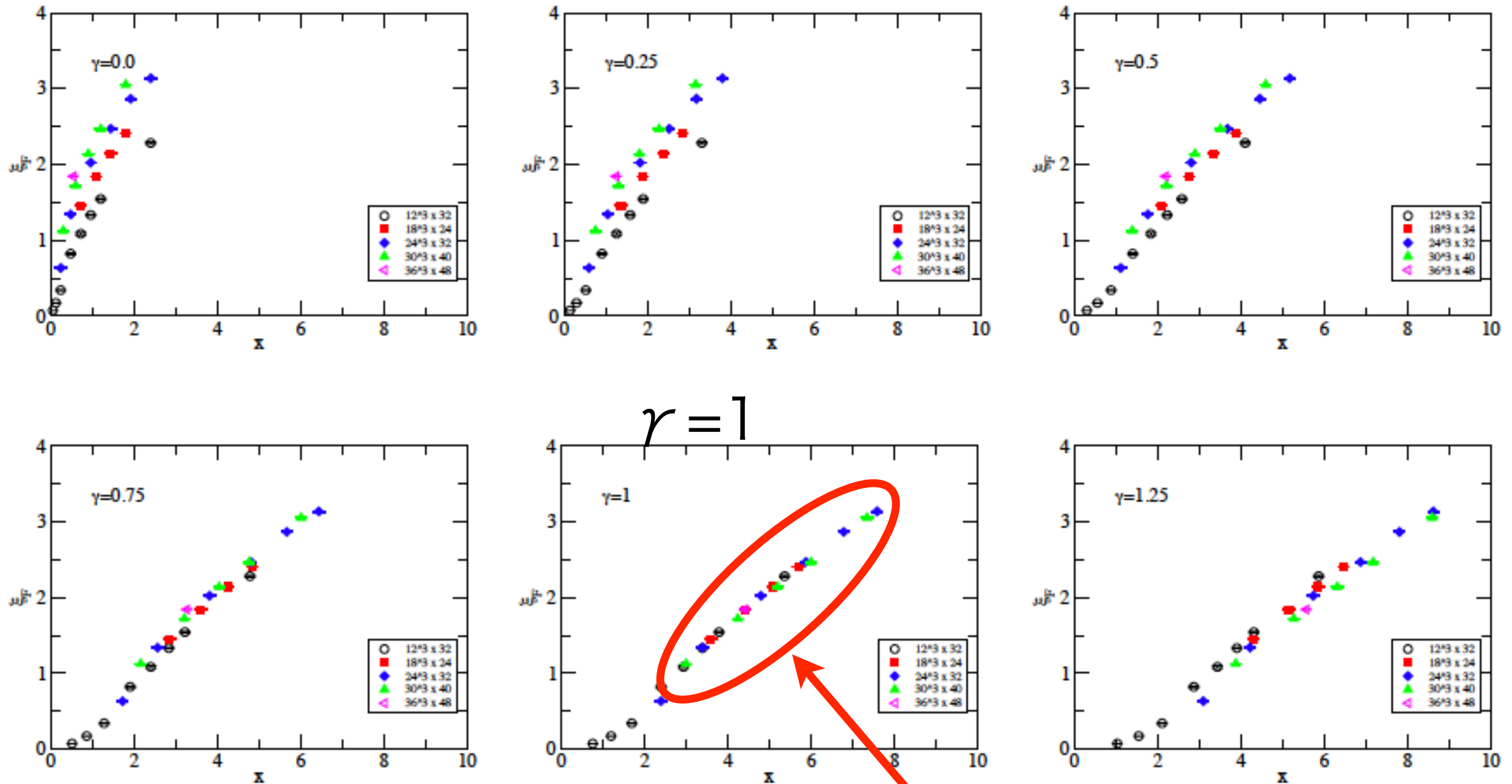
$N_f=8$ see if data align at some γ : M_π

$\gamma=0.5$



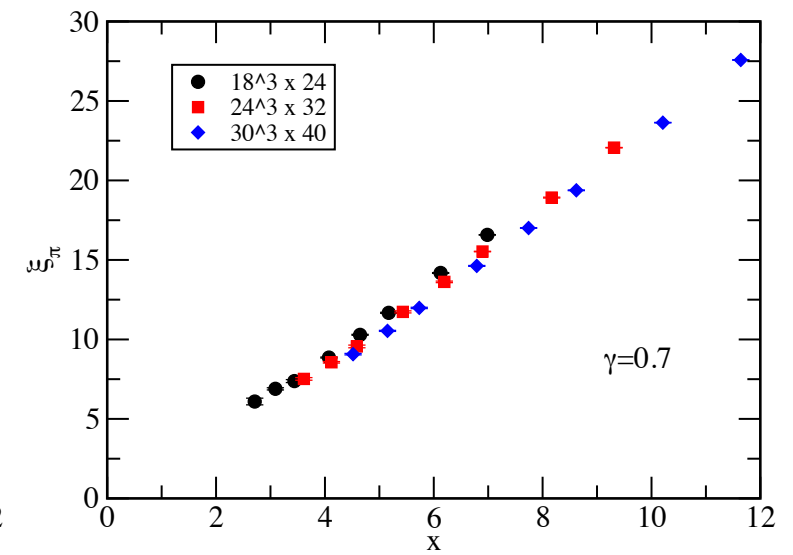
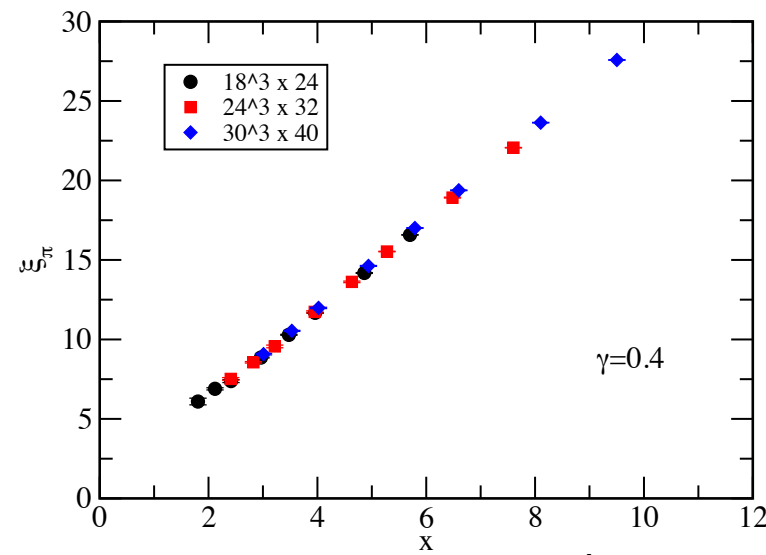
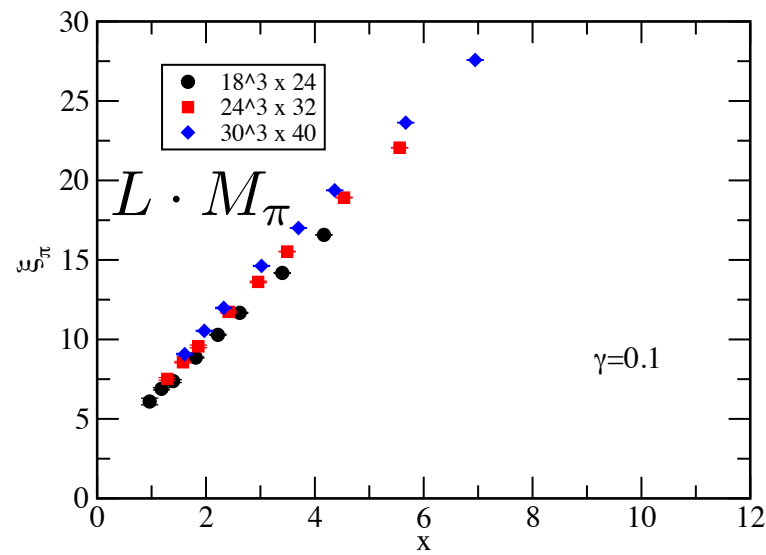
good alignment

$N_f=8$ see if data align at some γ : F_π

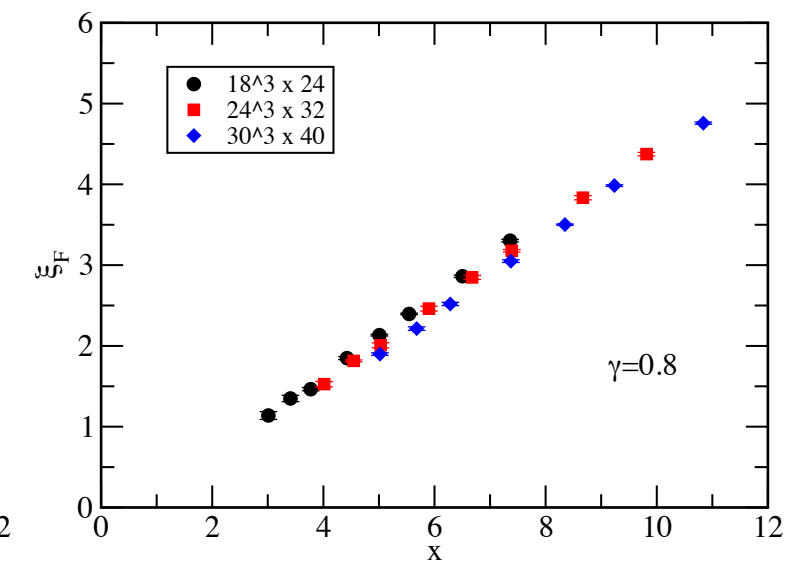
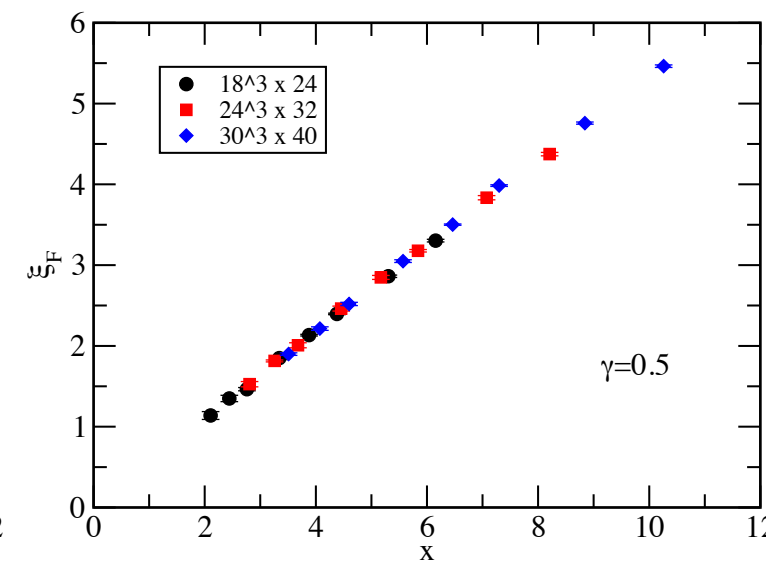
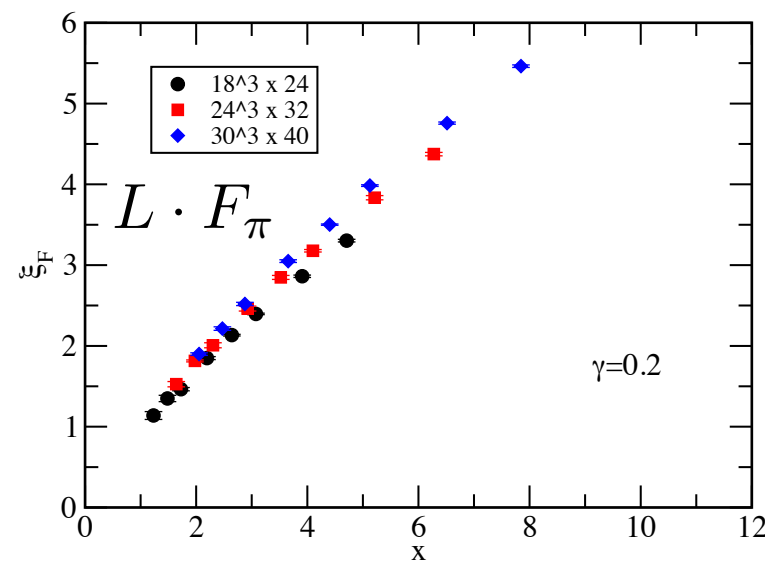


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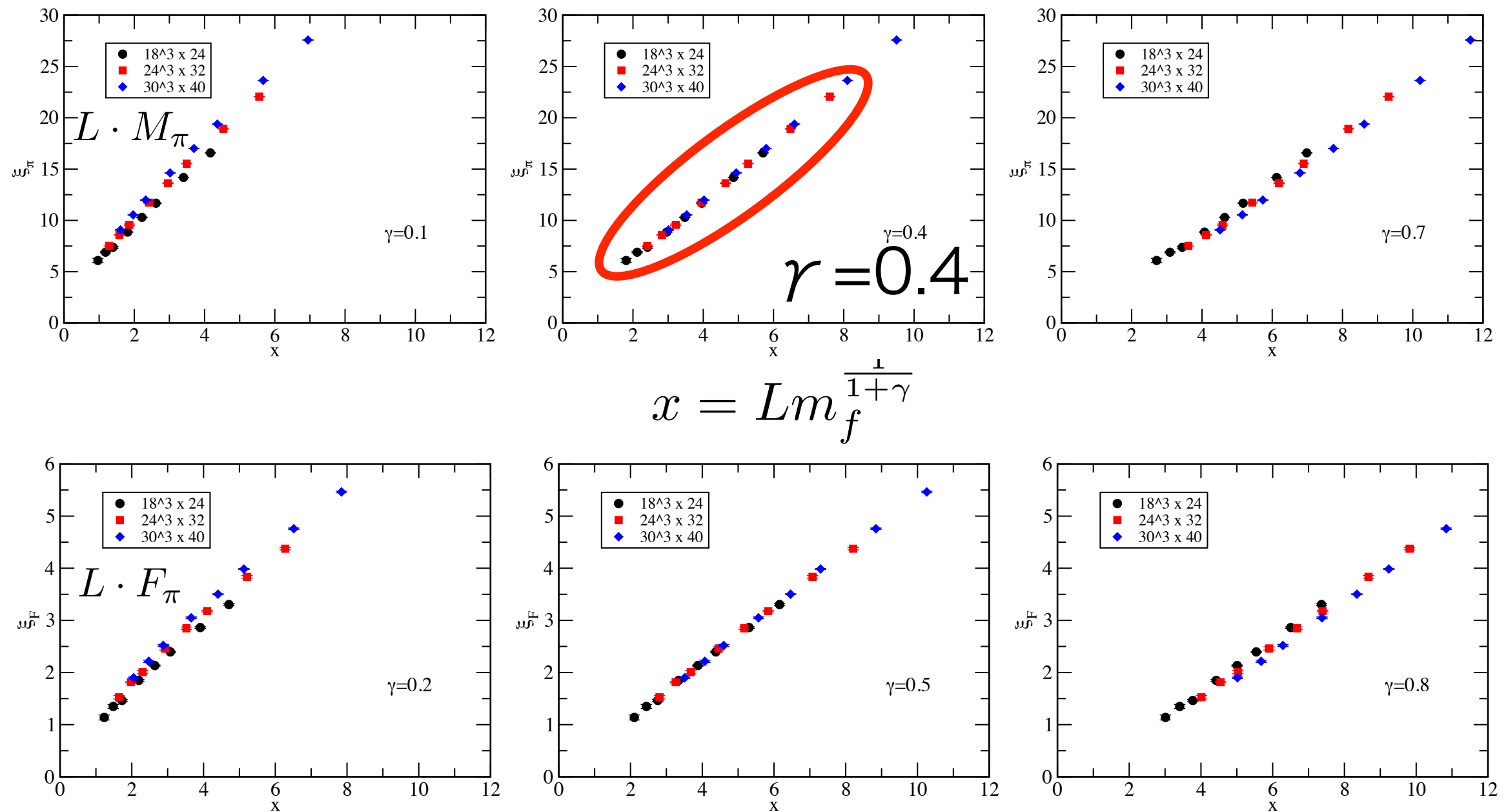
$N_f=12$ see if data align at some γ



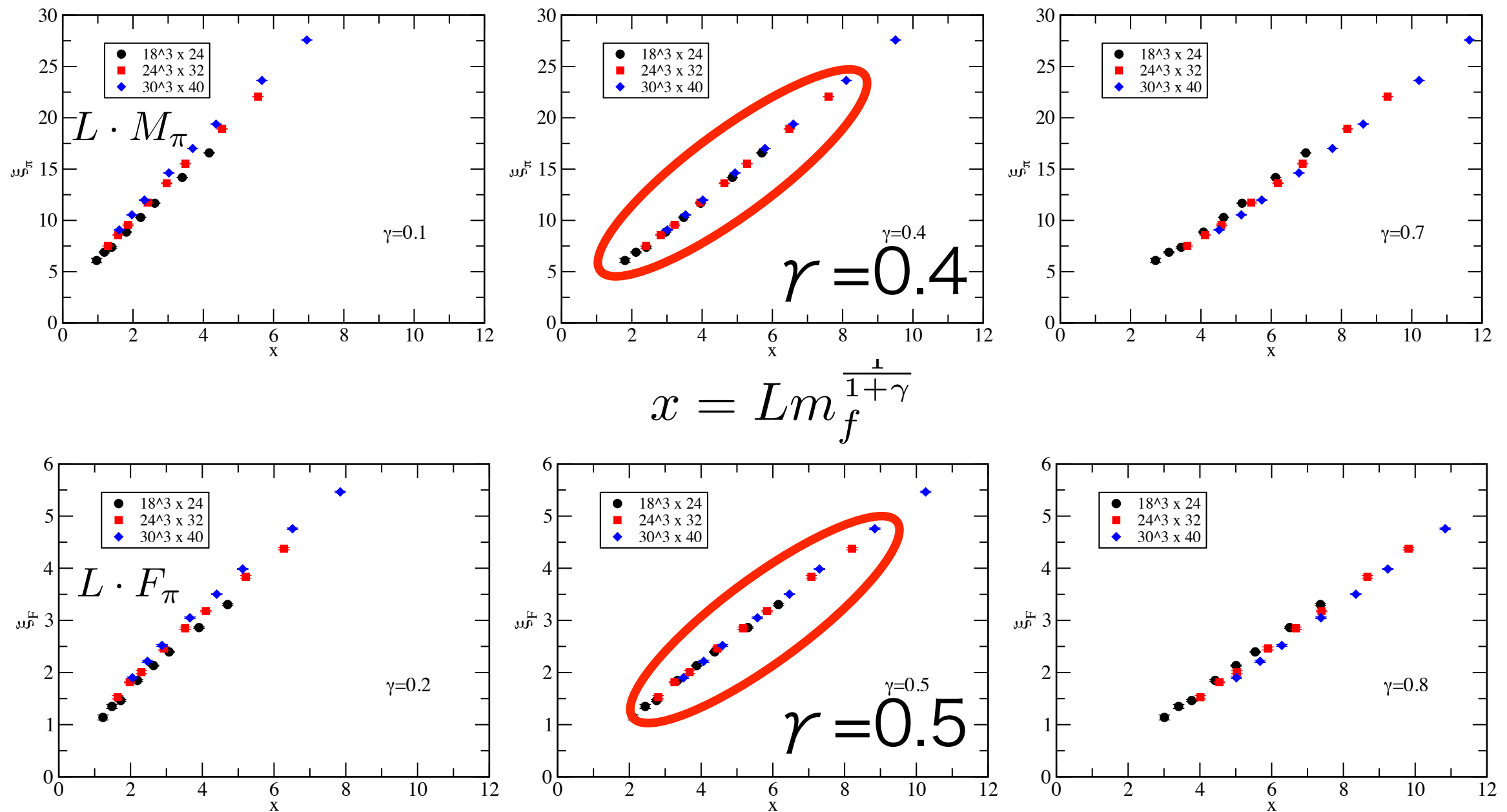
$$x = L m_f^{\frac{1}{1+\gamma}}$$



$N_f=12$ see if data align at some γ



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measure of the “alignment”
without resorting to a model

measure of the “alignment” without resorting to a model

- γ of optimal alignment will minimize:

$$P_p(\gamma) = \frac{1}{\mathcal{N}} \sum_K \sum_{j \notin K} \frac{|\xi_p^j - f_p^{(K)}(x_j)|^2}{\delta^2 \xi_p^j}$$

- $\xi_p = LM_p$ for $p = \pi, \rho$; $\xi_F = LF_\pi$
- $f_p(x)$: interpolation linear
 - (quadratic for a systematic error)
- if ξ^j is away from $f(x_i)$ by $\delta \xi^j$ as average $\rightarrow P=1$
- optimal γ from the minimum of P
- similar definition of the measure: DeGrand, Giedt & Weinberg

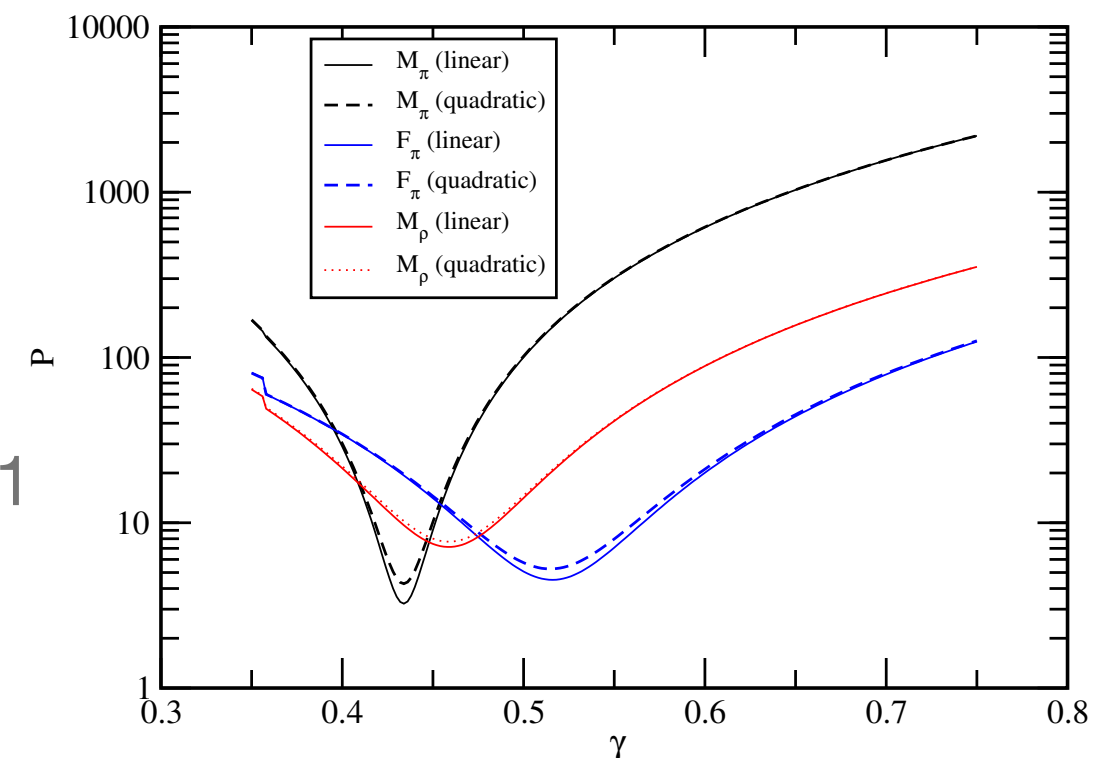
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$P(\gamma)$ for M_π, F_π, M_ρ at $\beta=3.7$



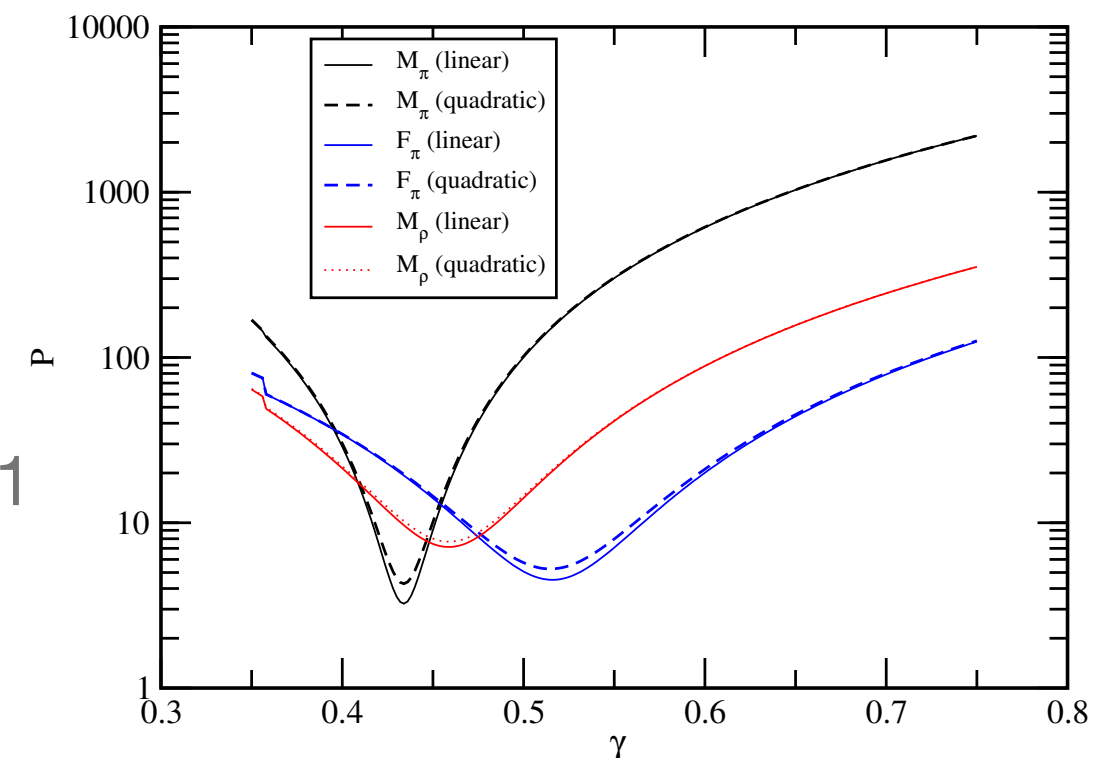
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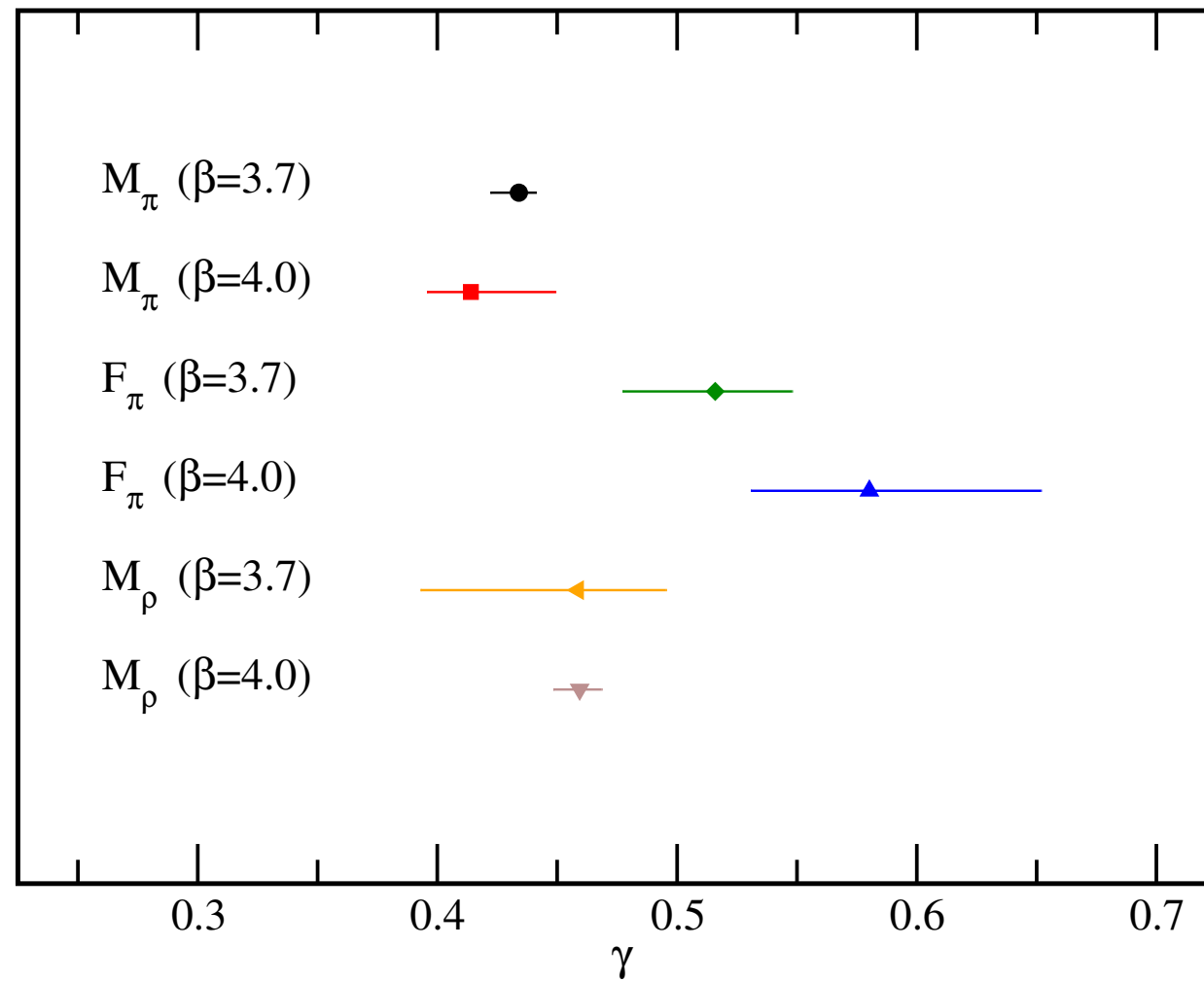
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- similar definition of the measure: DeGrand, Giedt & Weinberg
- systematic error due to small L , large m estimated by examining the x and L range dependence

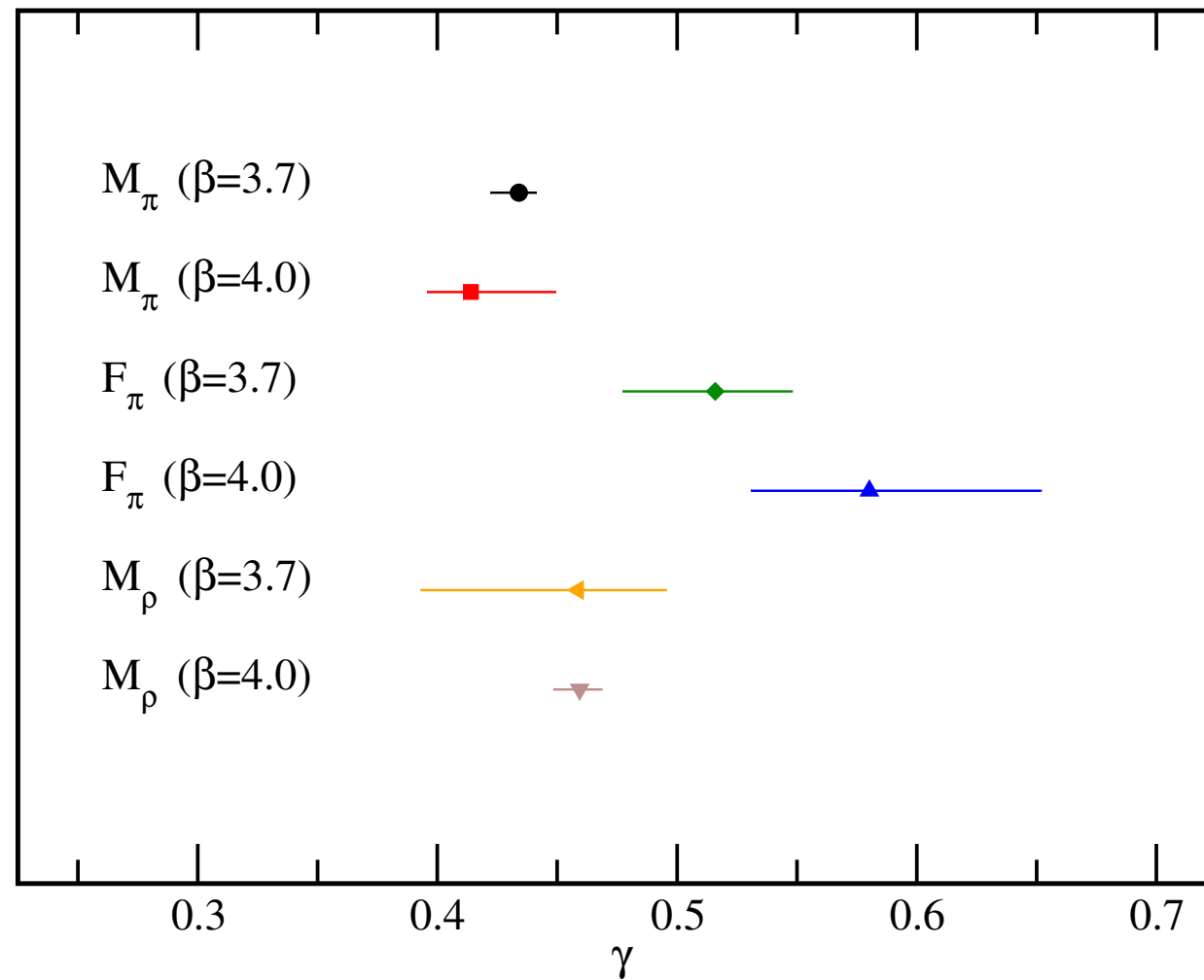
$P(\gamma)$ for M_π, F_π, M_ρ at $\beta=3.7$



summary of γ from $P(\gamma)$ for $N_f=12$

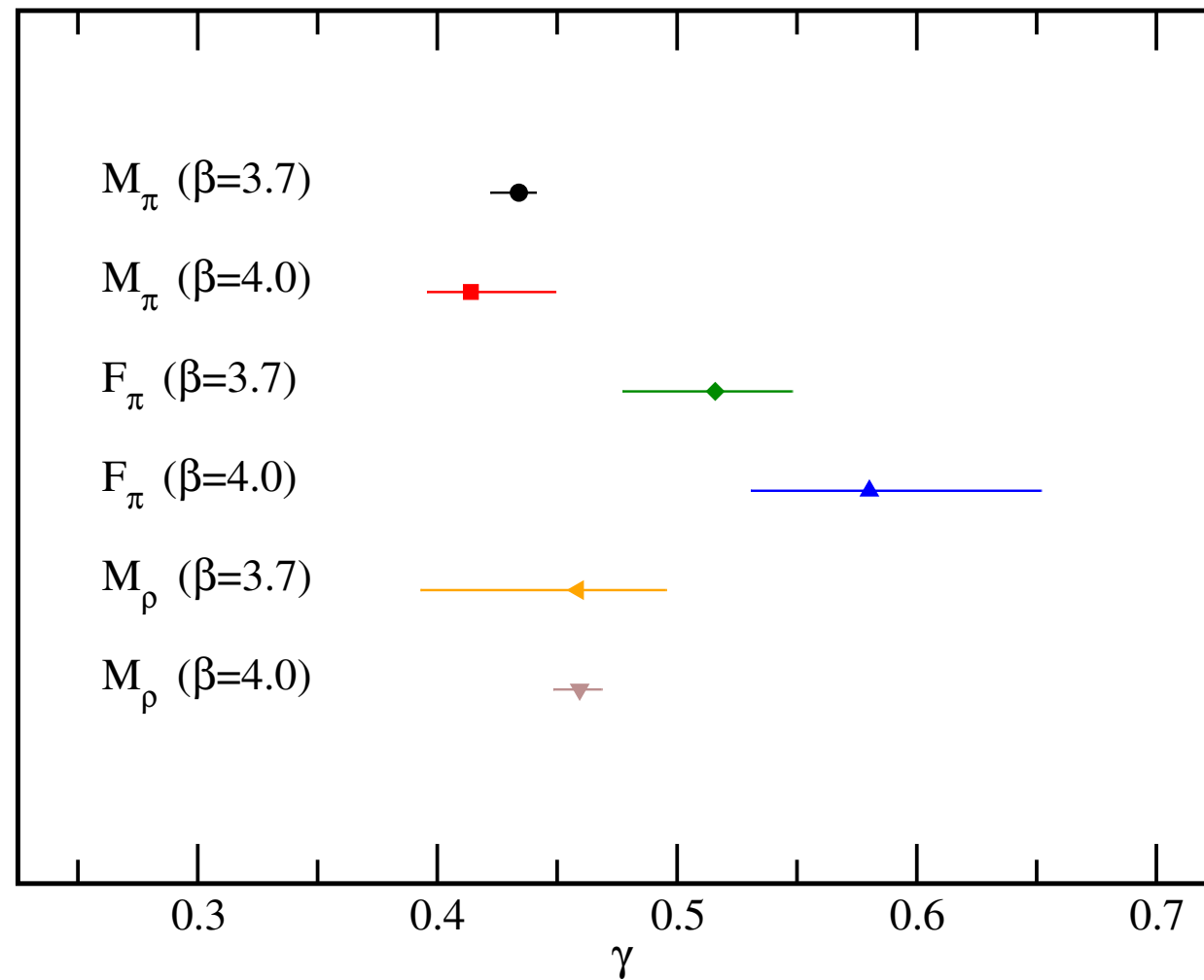


summary of γ from $P(\gamma)$ for $N_f=12$



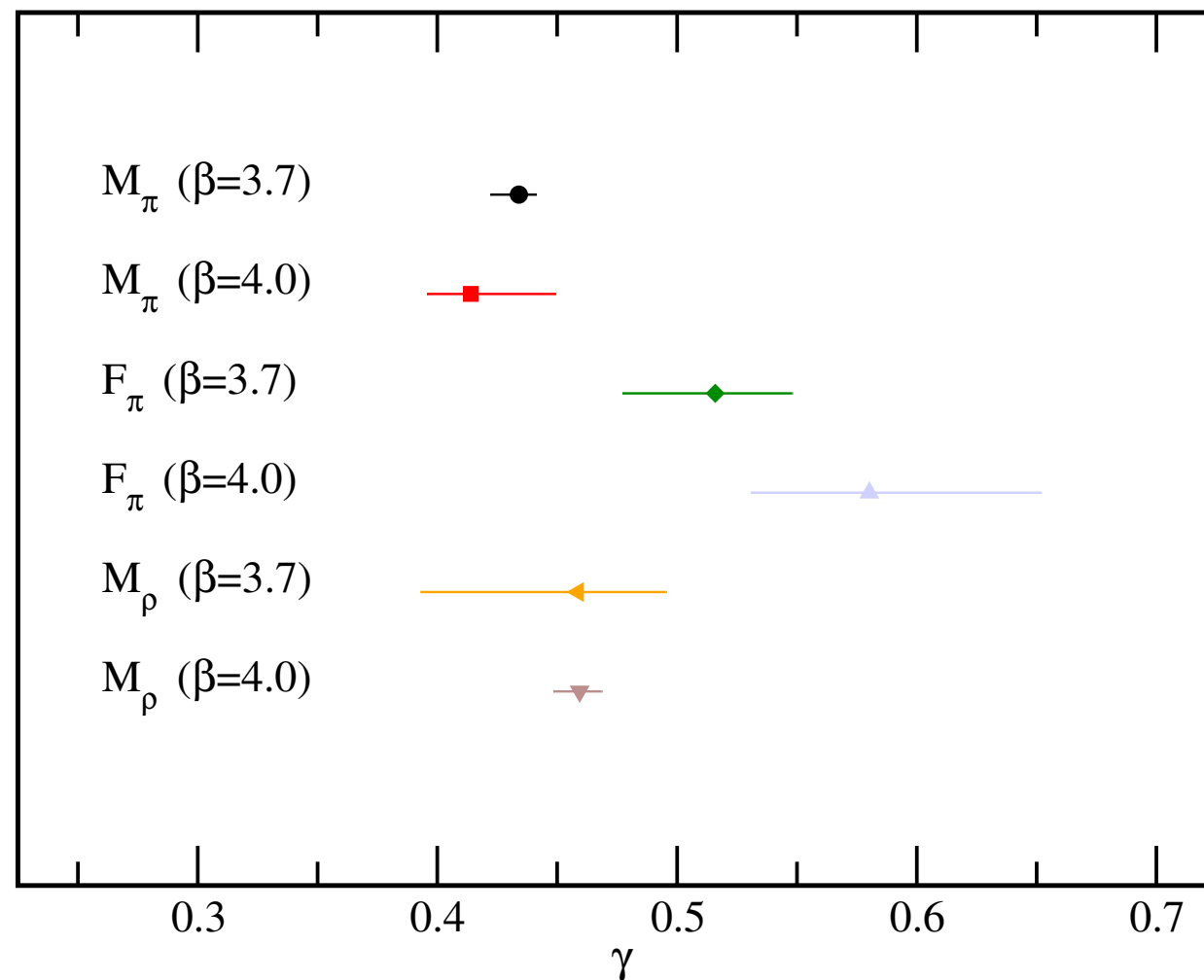
- γ : consistent with 2σ level except for F_π at $\beta=4.0$

summary of γ from $P(\gamma)$ for $N_f=12$



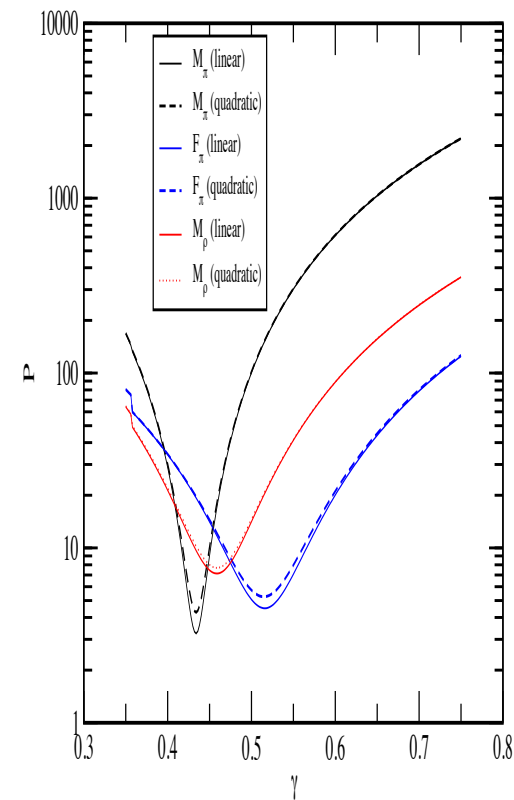
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- remember: F_π at $\beta=4.0$ speculated to be out of the scaling region

summary of γ from $P(\gamma)$ for $N_f=12$



- γ : consistent with 2 σ level except for F_π at $\beta=4.0$
- remember: F_π at $\beta=4.0$ speculated to be out of the scaling region
- universal low energy behavior: good with $0.4 < \gamma^* < 0.5$

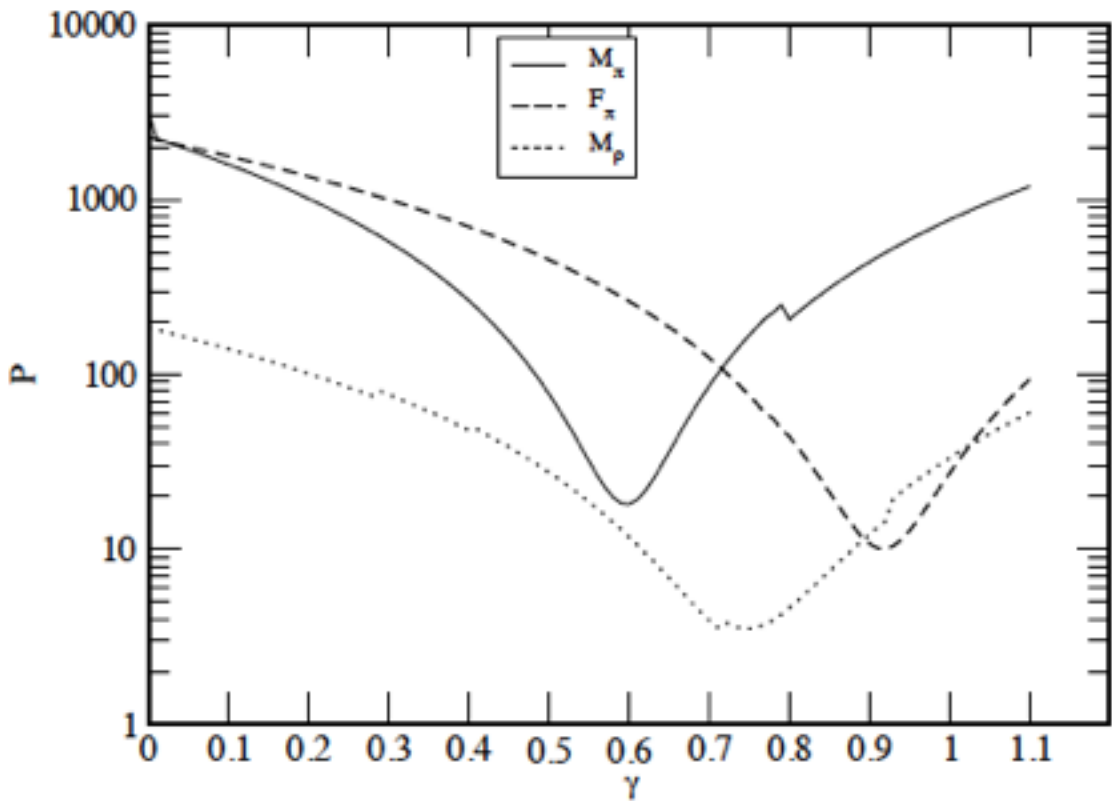
$P(\gamma)$ analysis for $N_f=8$



$N_f=12$

| quantity | γ |
|----------|-----------|
| M_π | 0.434(4) |
| F_π | 0.516(12) |
| M_ρ | 0.459(8) |

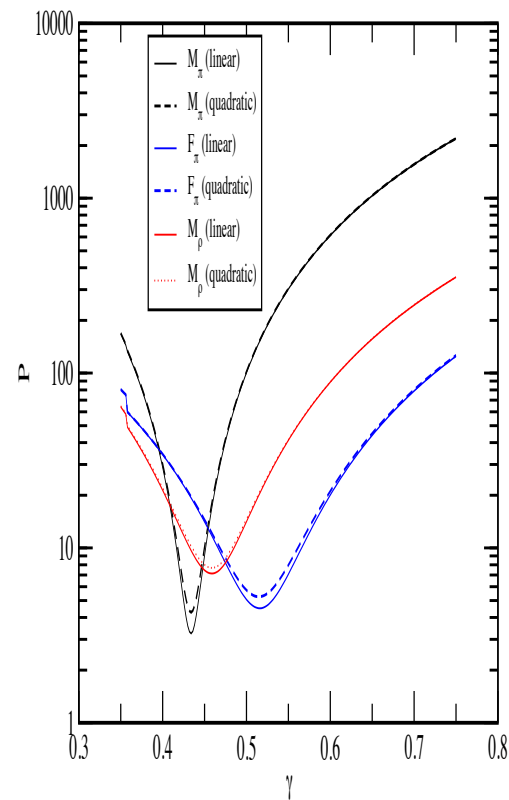
statistical error only



$N_f=8$

| quantity | γ |
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| M_π | 0.593(2) |
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$P(\gamma)$ analysis for $N_f=8$



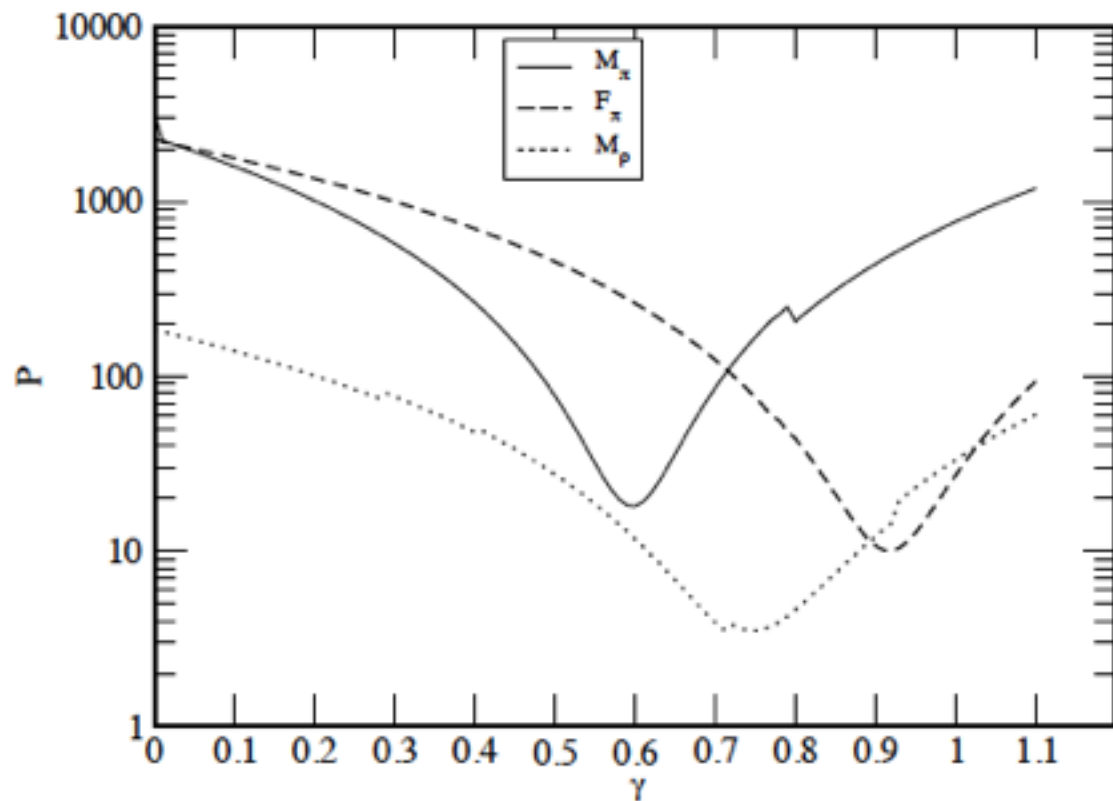
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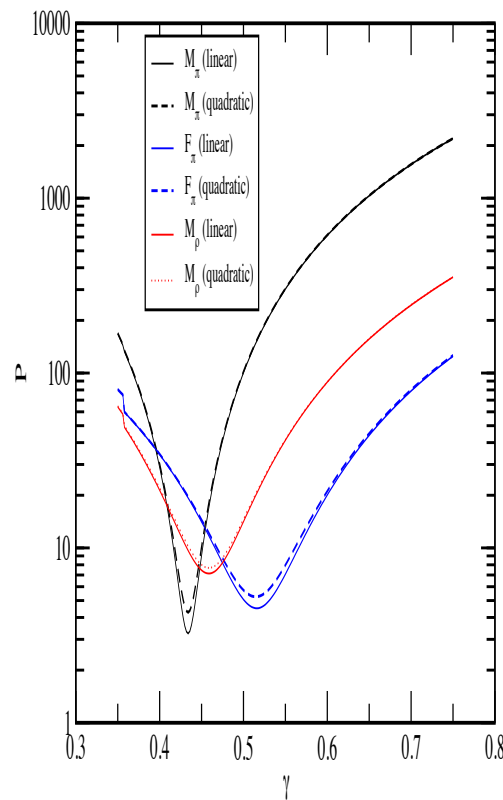
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- Optimal γ obtained for each quantity

$P(\gamma)$ analysis for $N_f=8$



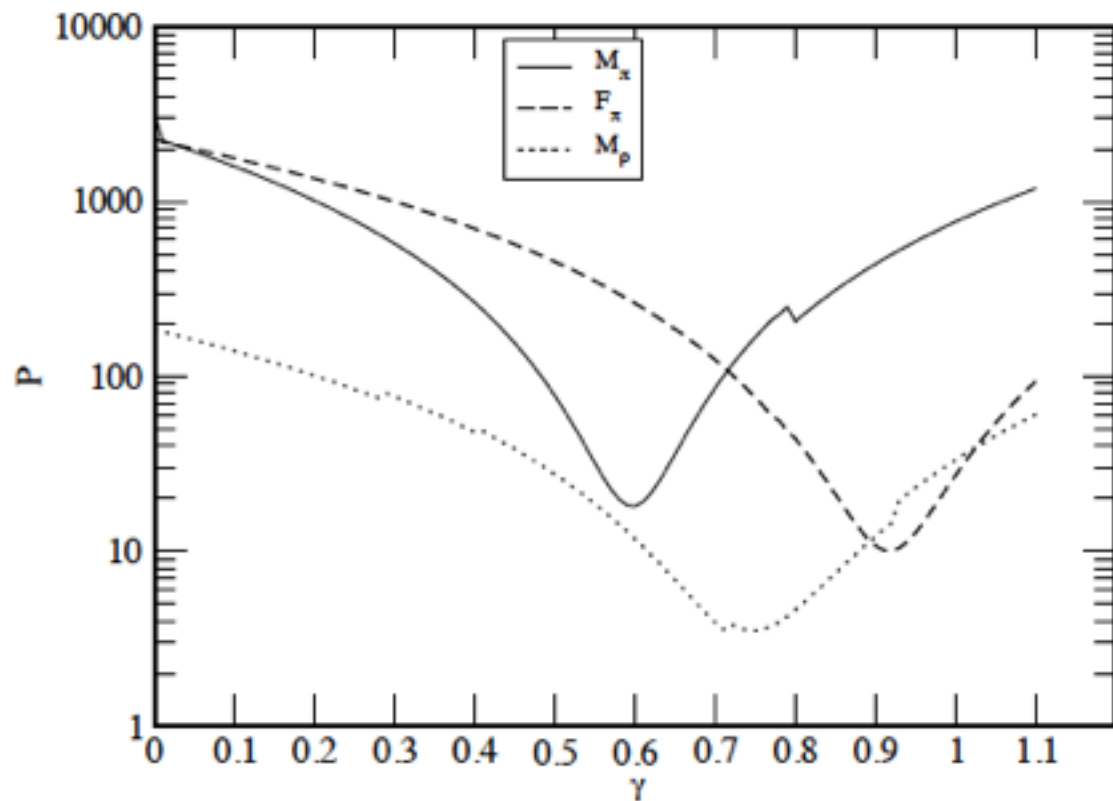
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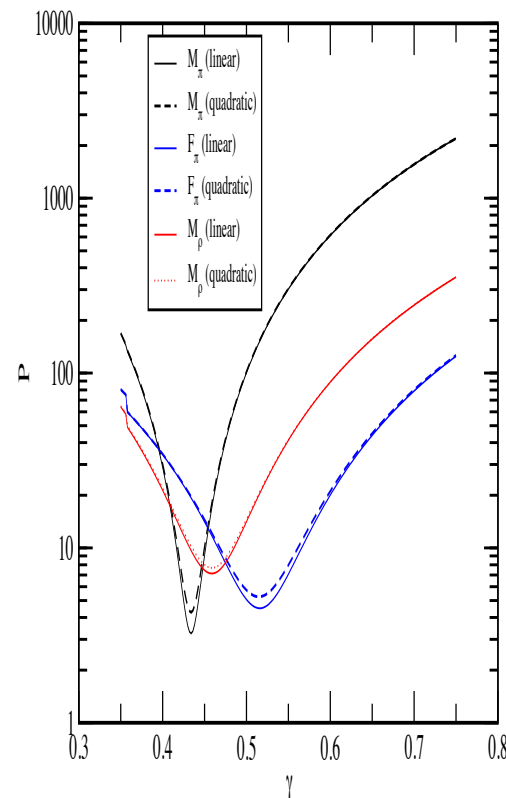
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- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality

$P(\gamma)$ analysis for $N_f=8$



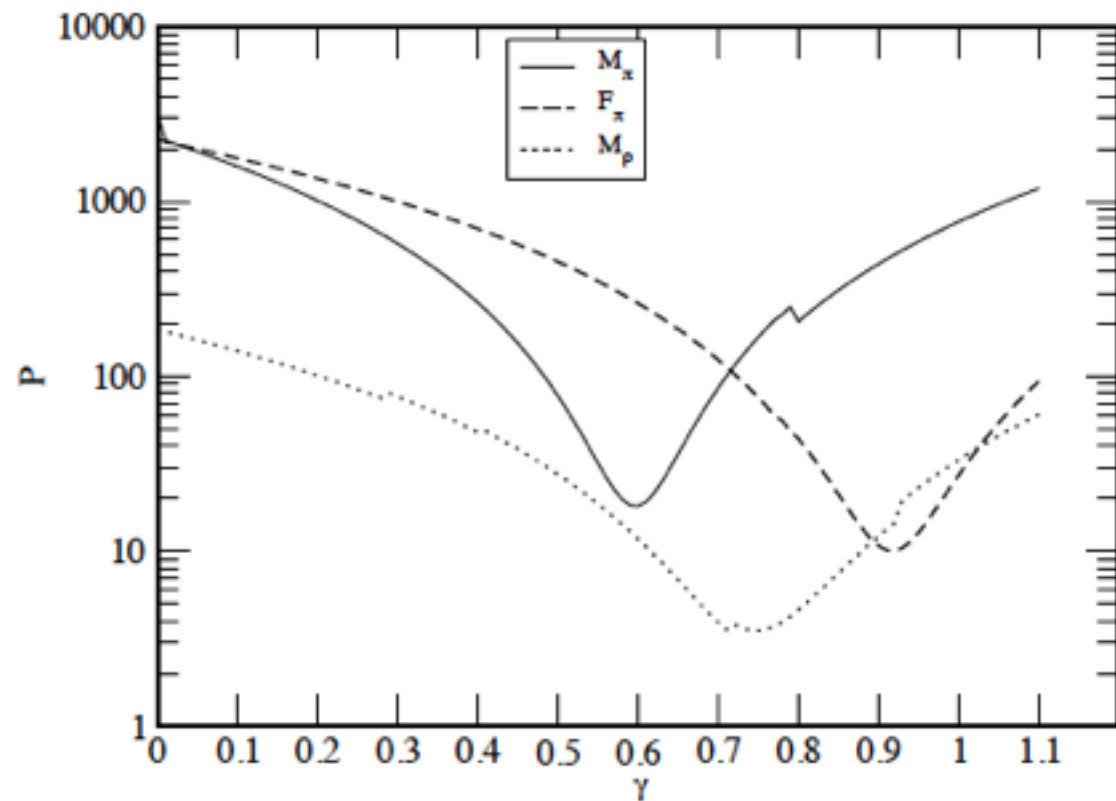
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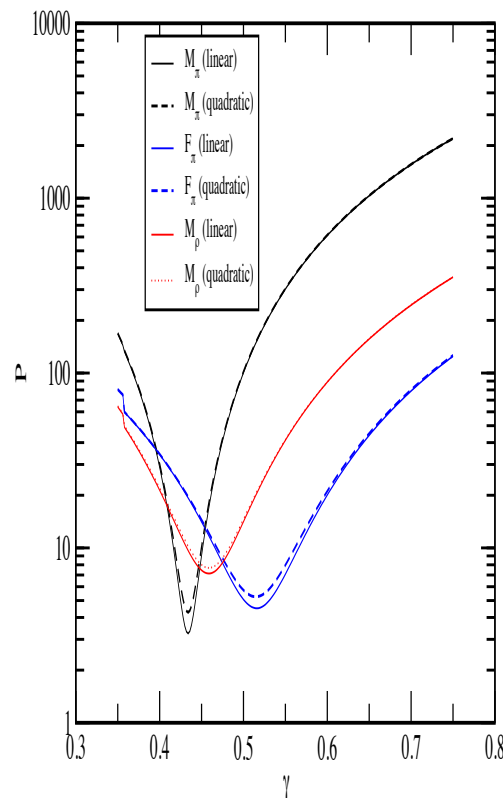
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- Optimal γ obtained for each quantity
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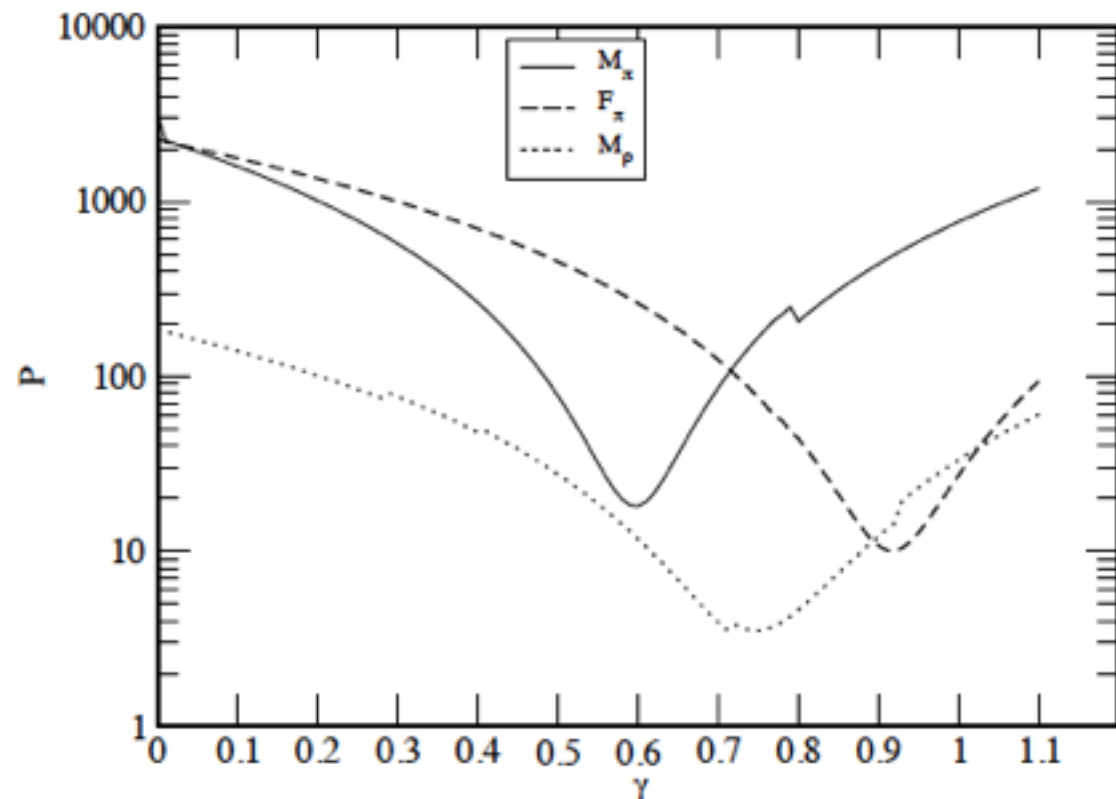
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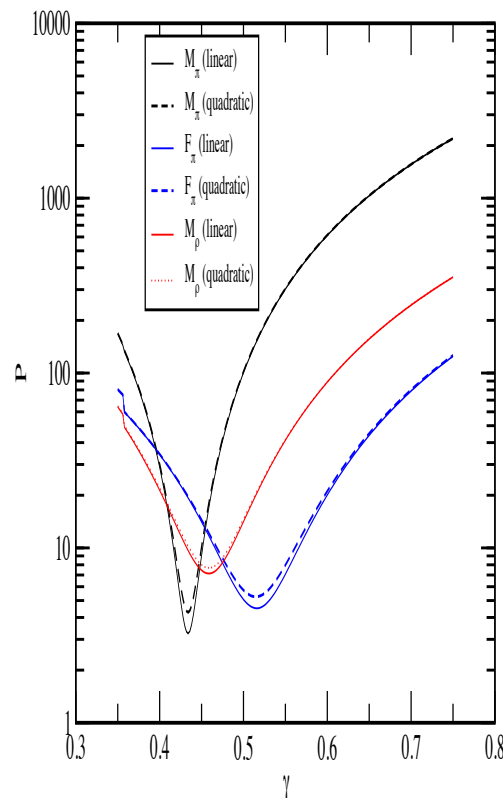
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- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality
- scaling \rightarrow remnant conformality
- remember: ~~chiral symmetry~~

$P(\gamma)$ analysis for $N_f=8$



$N_f=12$

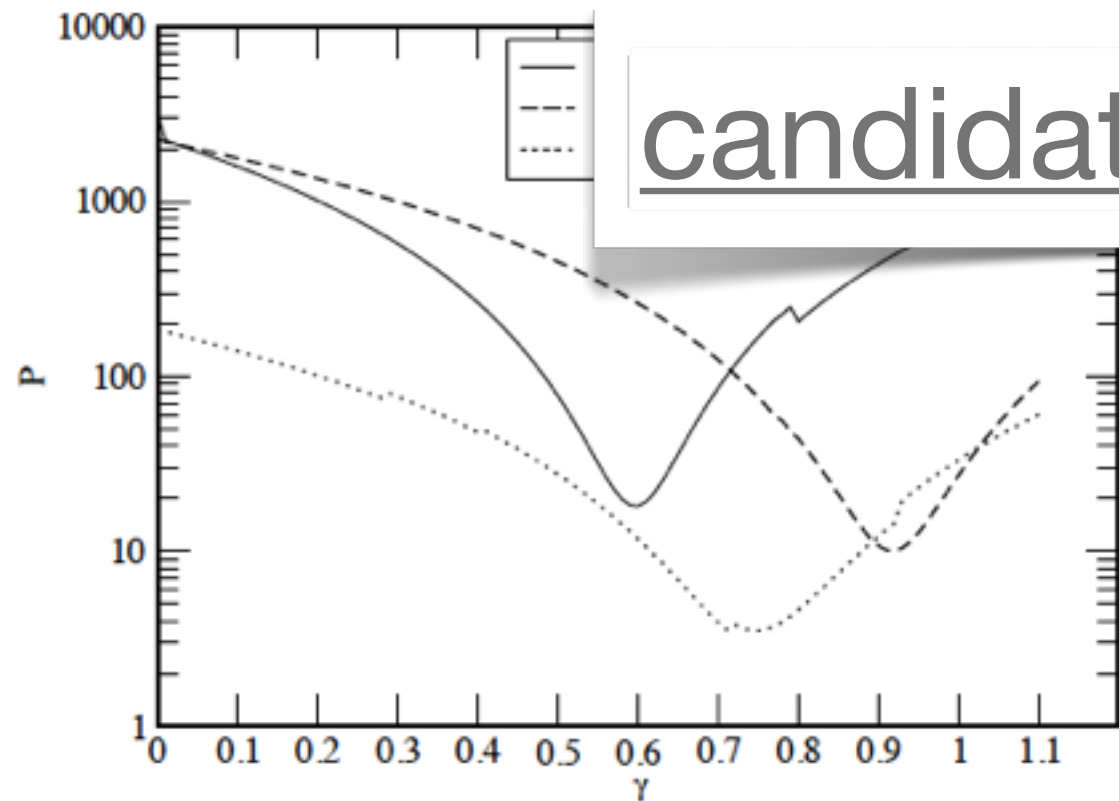
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statistical error only

$N_f=8$

| quantity | γ |
|----------|----------|
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| F_π | 0.5(4) |
| M_ρ | 0.4(10) |

candidate of walking TC



- Optimal γ obtained for each quantity
- γ scattered \rightarrow no exact conformality
- scaling \rightarrow remnant conformality
- remember: ~~chiral symmetry~~

0^{++}

spectrum

[preliminary]

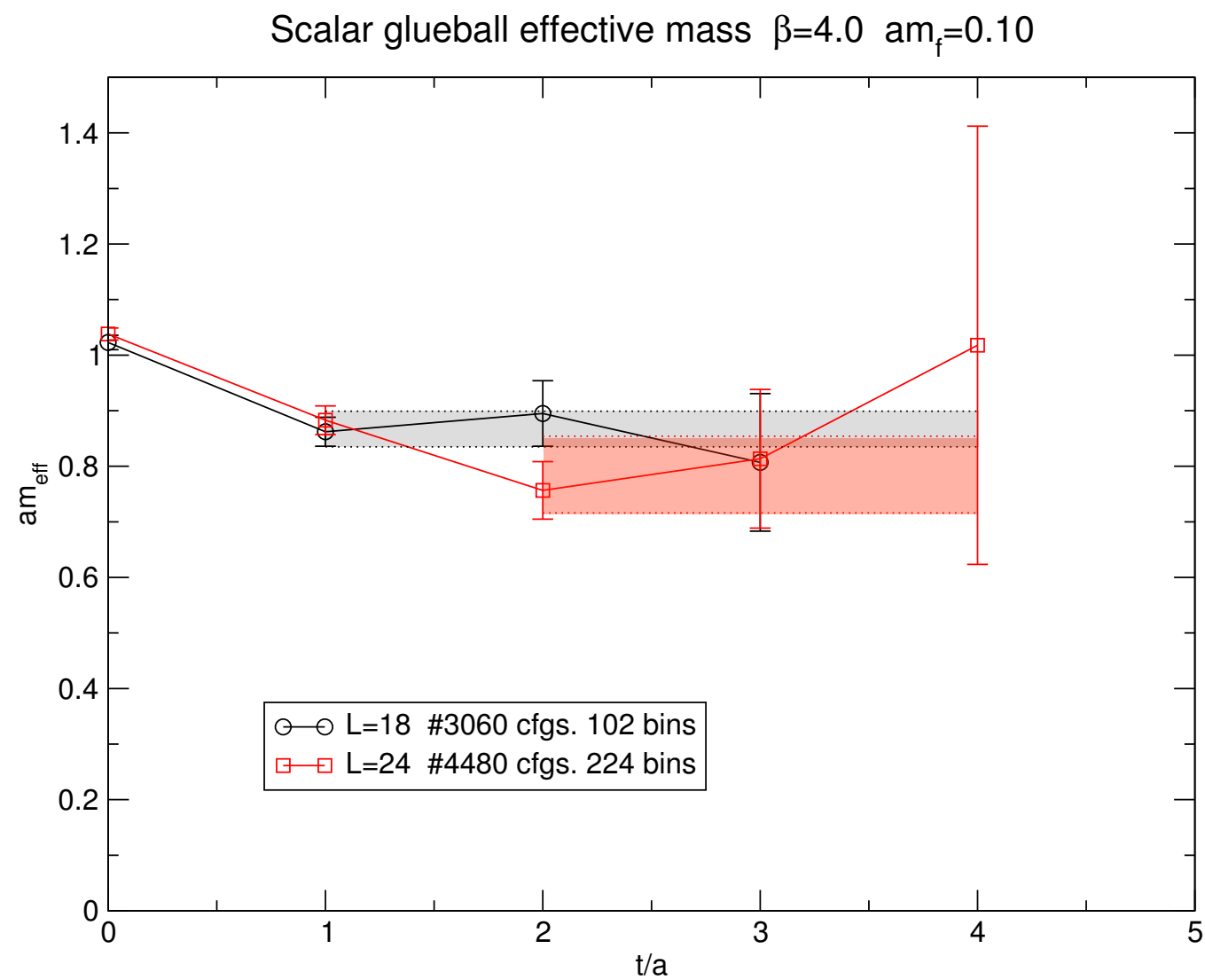
motivation

- adding another quantity to the scaling analysis
- see if light 0^{++} state (\rightarrow Higgs in WTC) emerges
- noisy, thus, difficult quantity in QCD

method

- 0^{++} glueball
 - variational method with many ops. (e.g. E. Gregory et al arXiv:1208.1858)
- flavor singlet scalar from fermion bilinear
 - stochastic estimator with 64 random vectors
- high statistics: a few 1000 ~14000 configurations

Results: scalar glueball

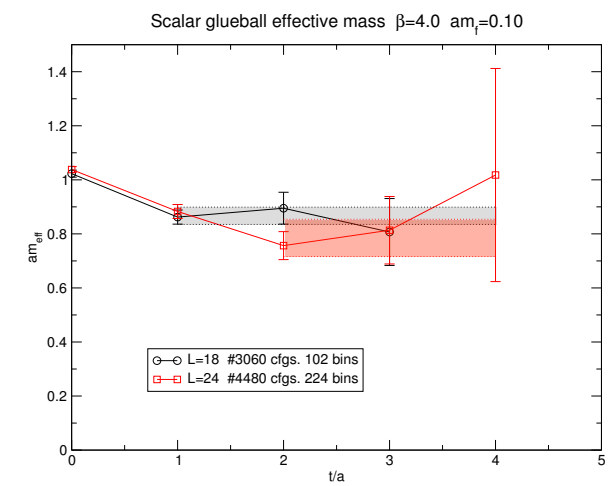


$$m_{0^{++}} > m_{\pi}$$

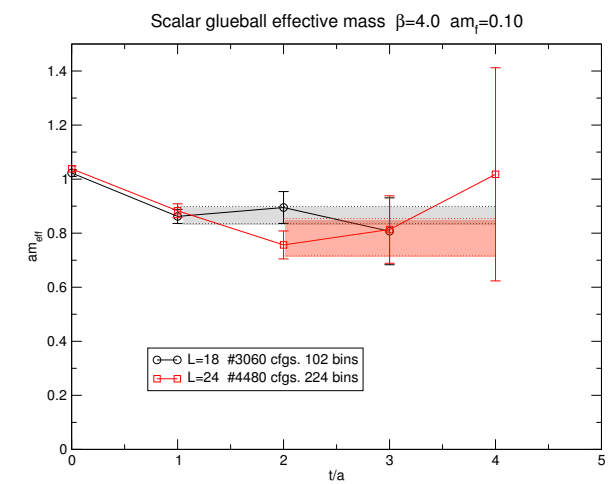
L=18 \rightarrow $am=0.867(32)$

L=24 \rightarrow $am=0.785(69)$

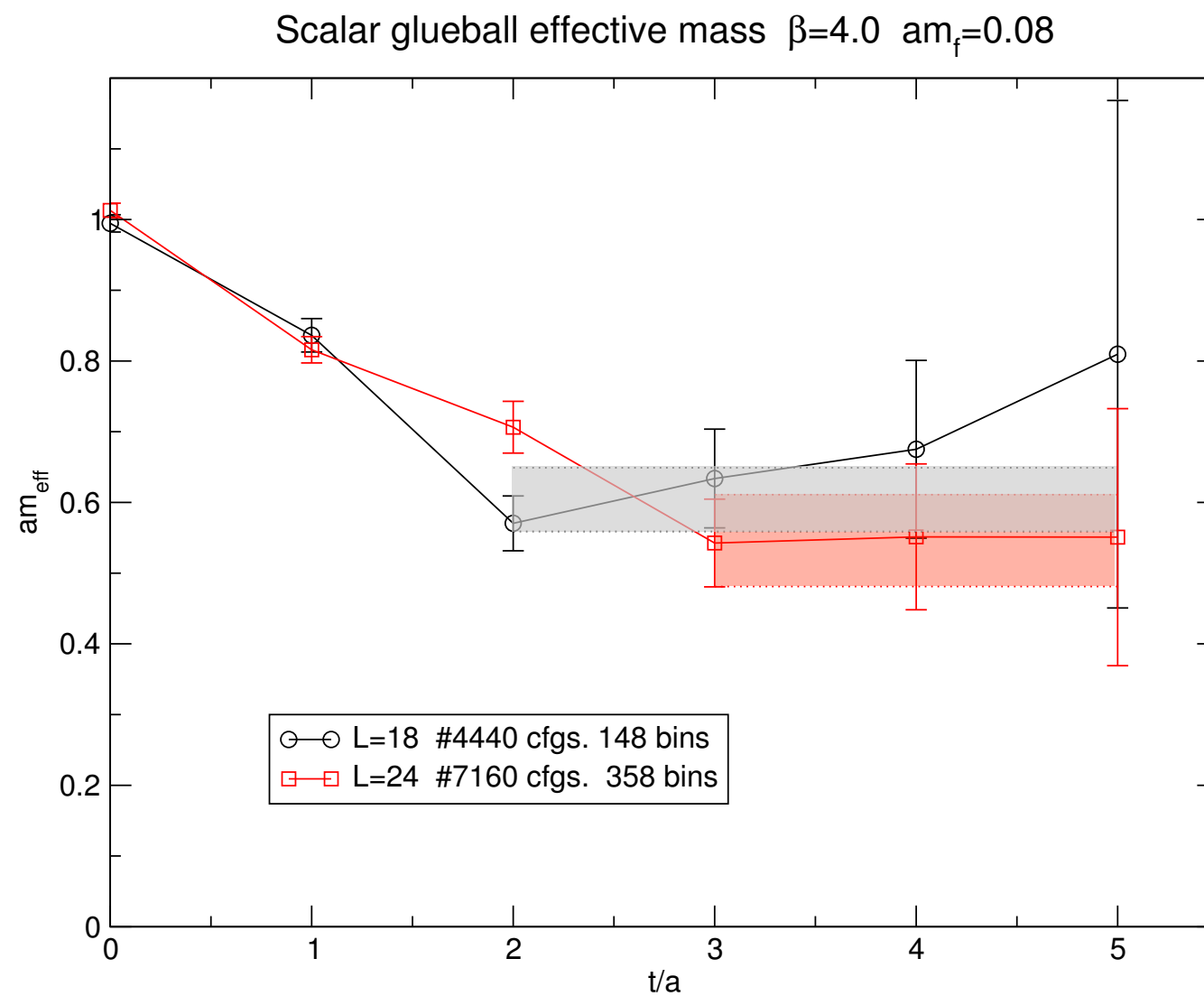
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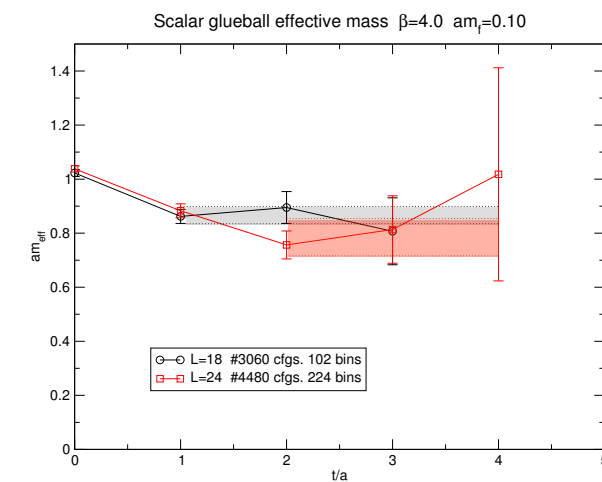
Results: scalar glueball



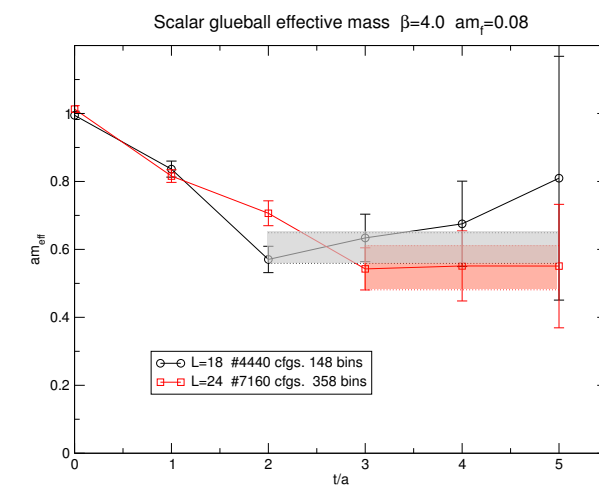
$$m_{0++} \gtrsim m_{\pi}$$

$L=18 \rightarrow am=0.604(45)$

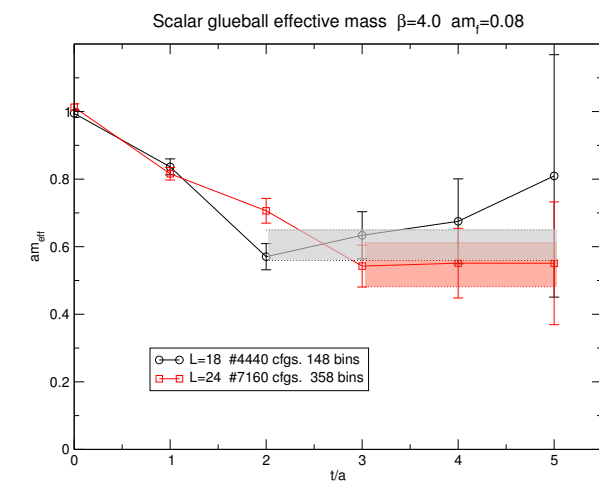
$L=24 \rightarrow am=0.546(65)$



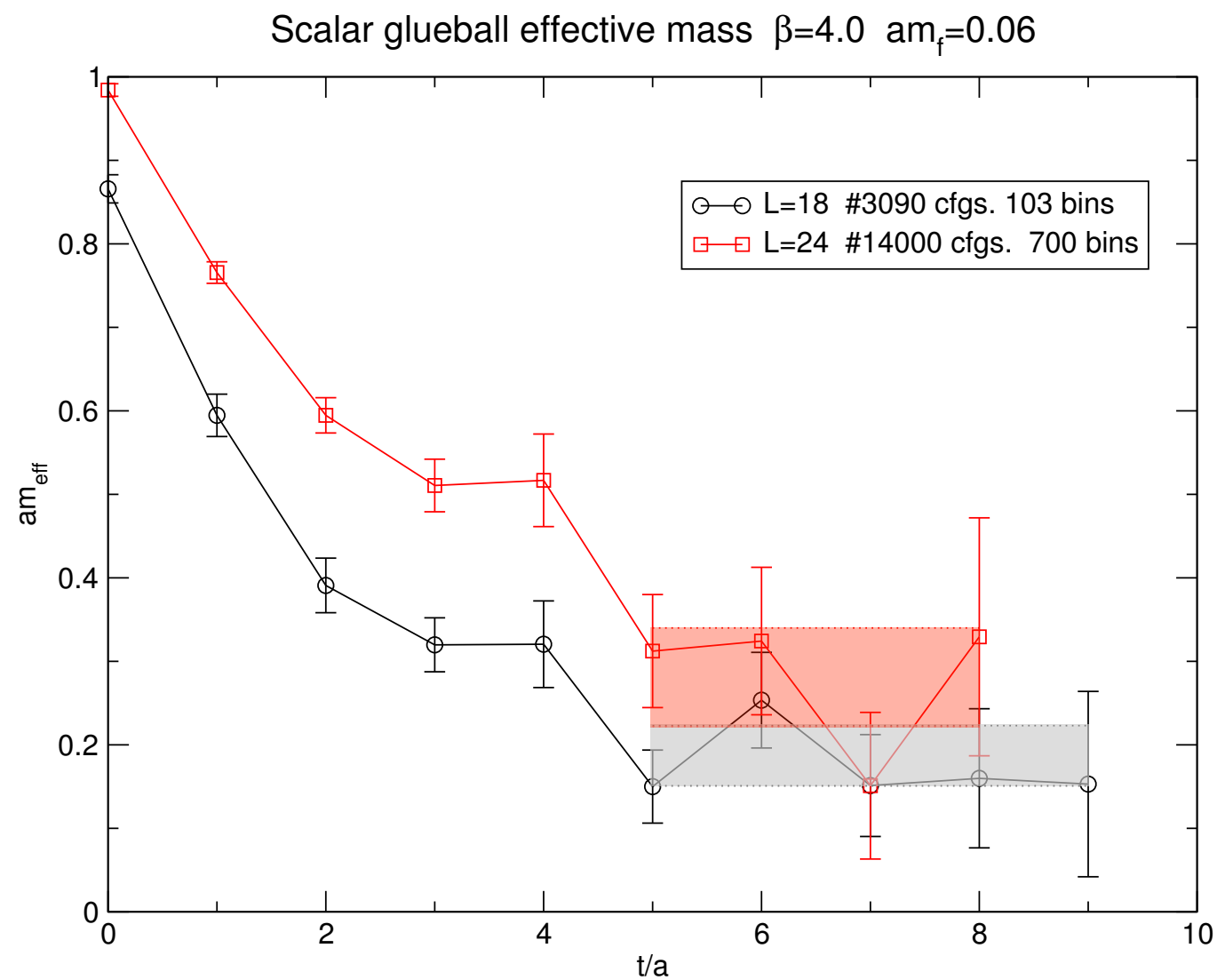
Results: scalar glueball



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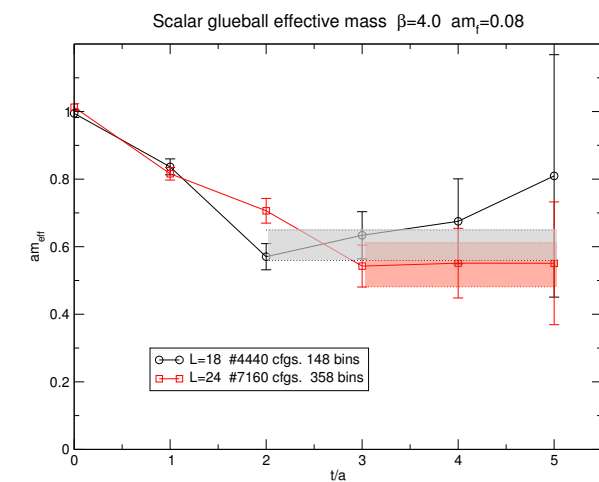
Results: scalar glueball



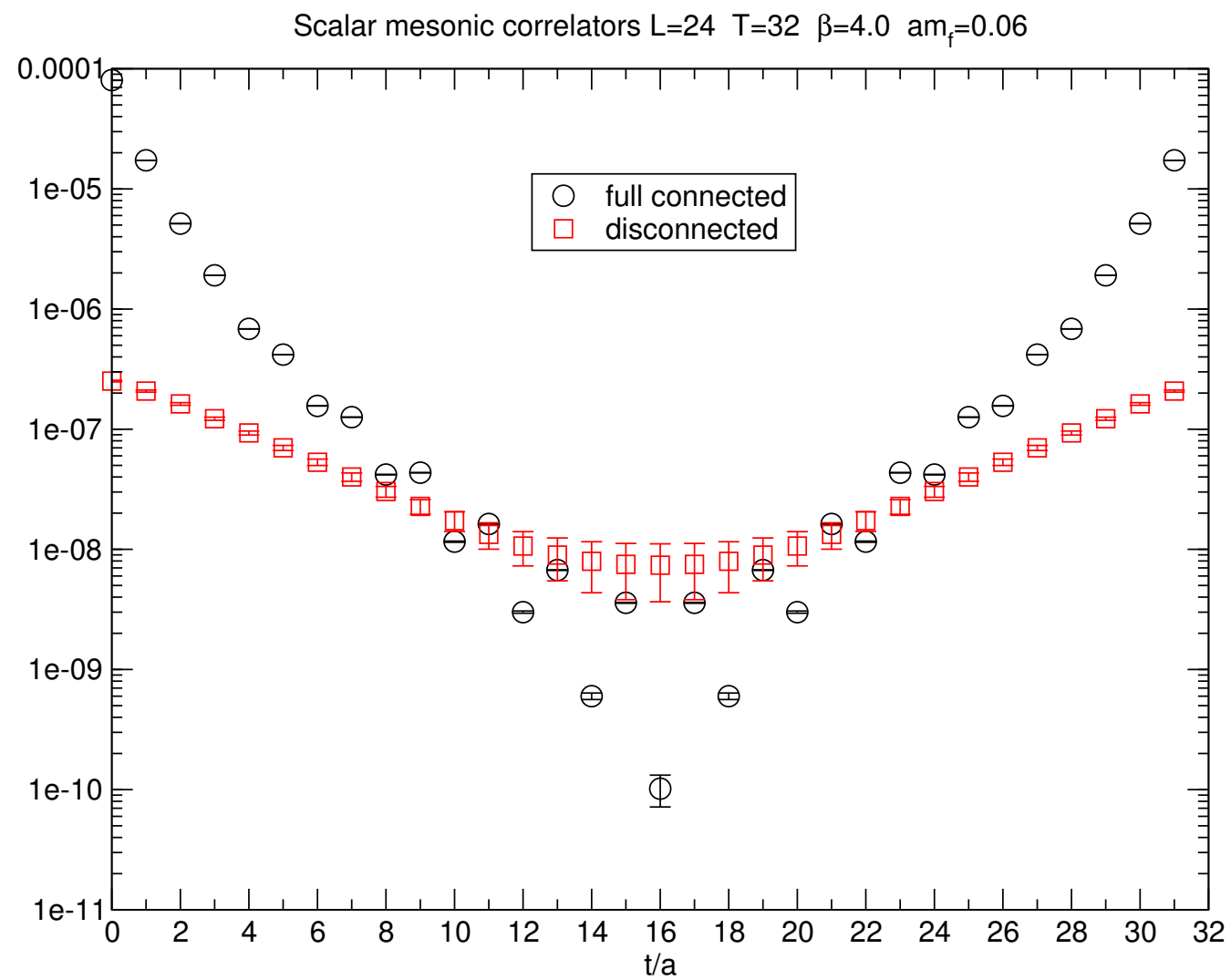
$$m_{0^{++}} \lesssim m_{\pi}$$

$L=18 \rightarrow am=0.187(36)$

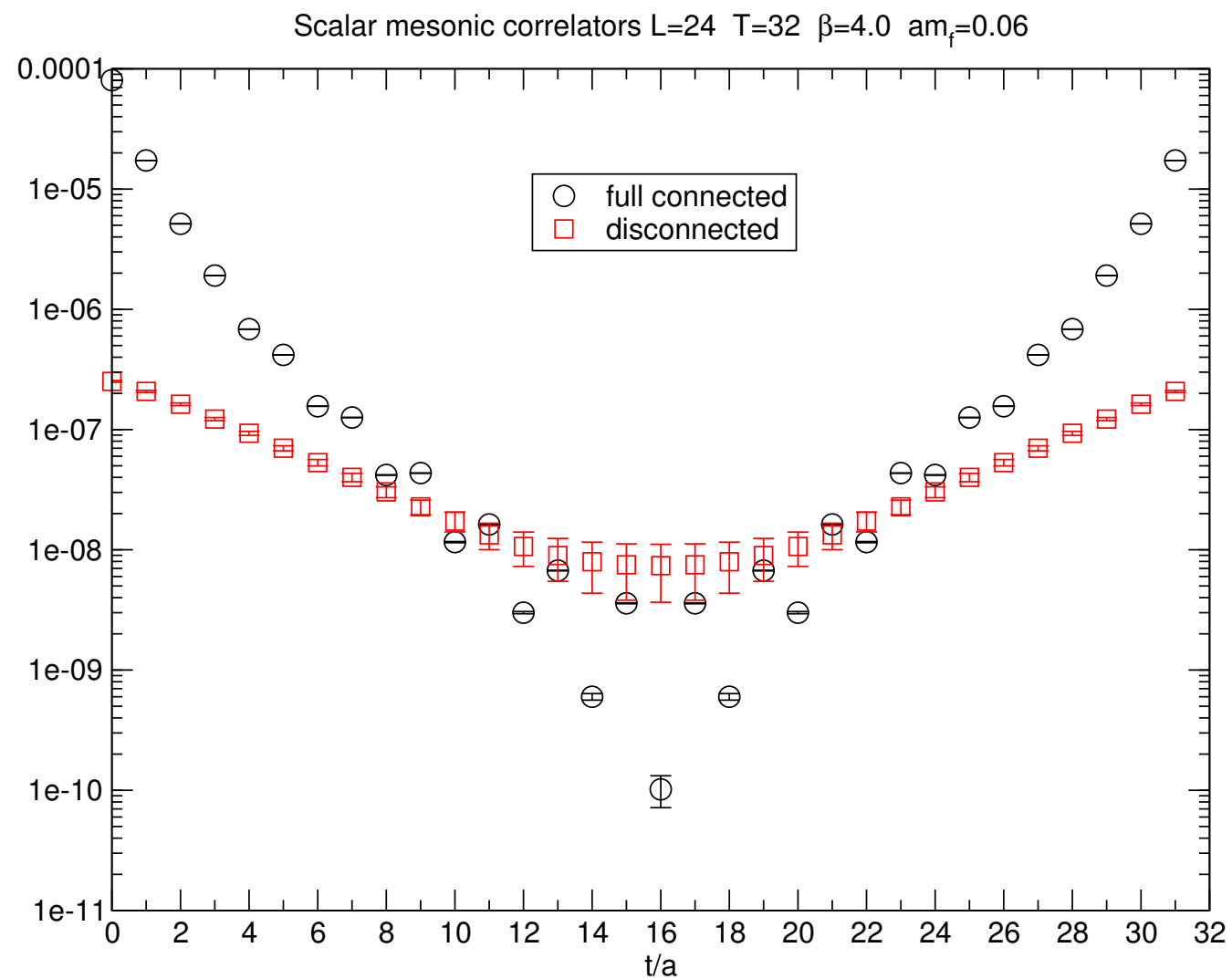
$L=24 \rightarrow am=0.281(59)$



Results: scalar flavour-singlet meson

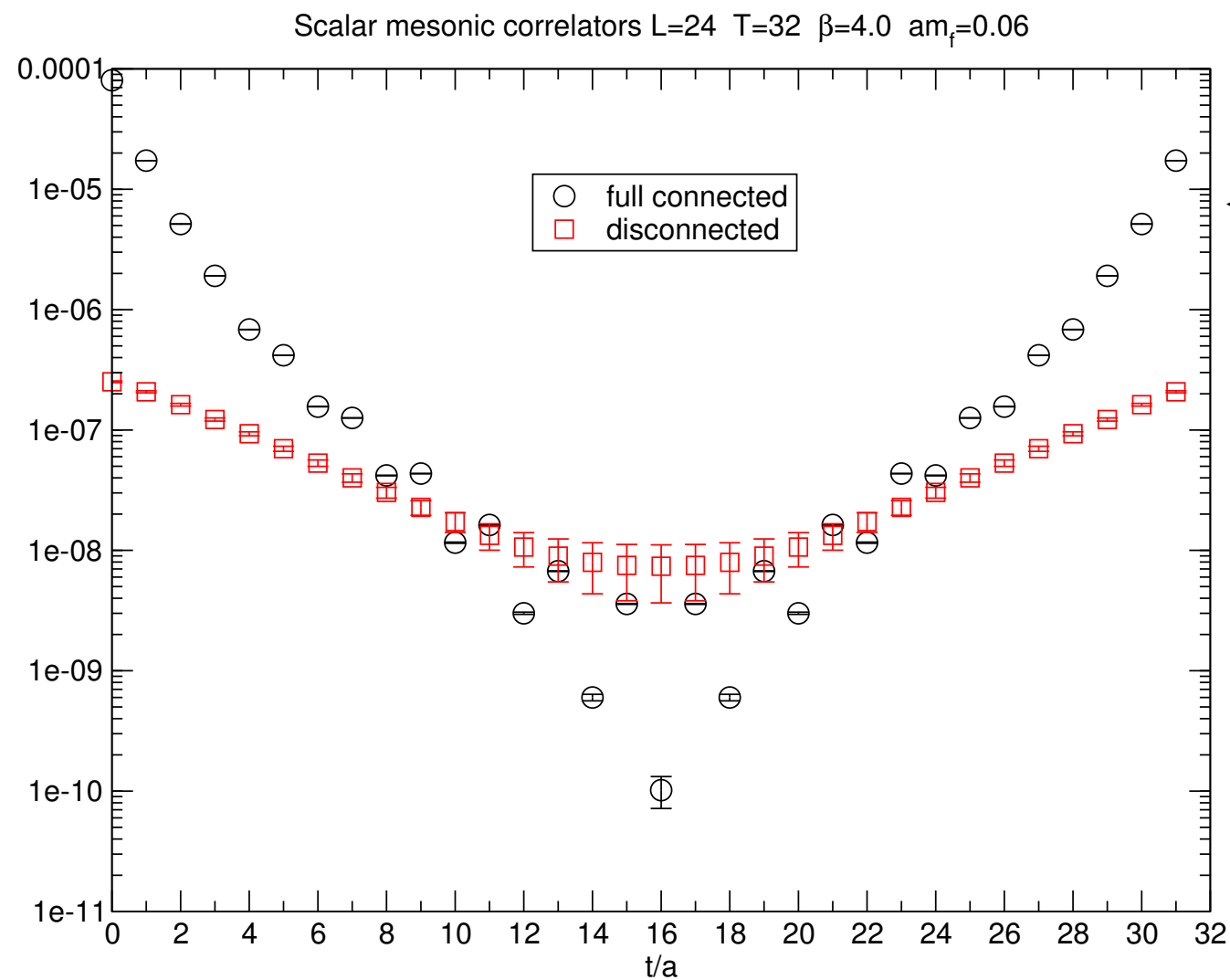


Results: scalar flavour-singlet meson



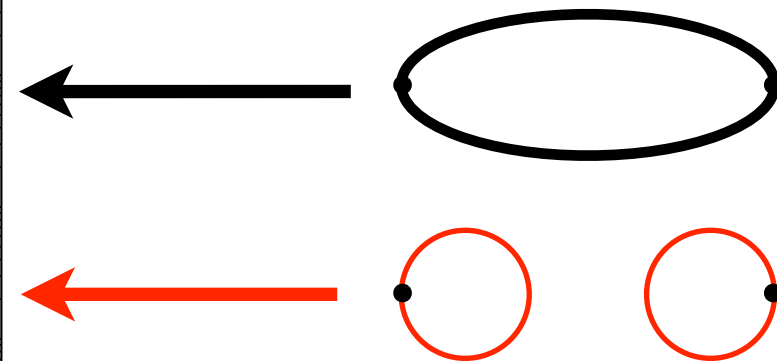
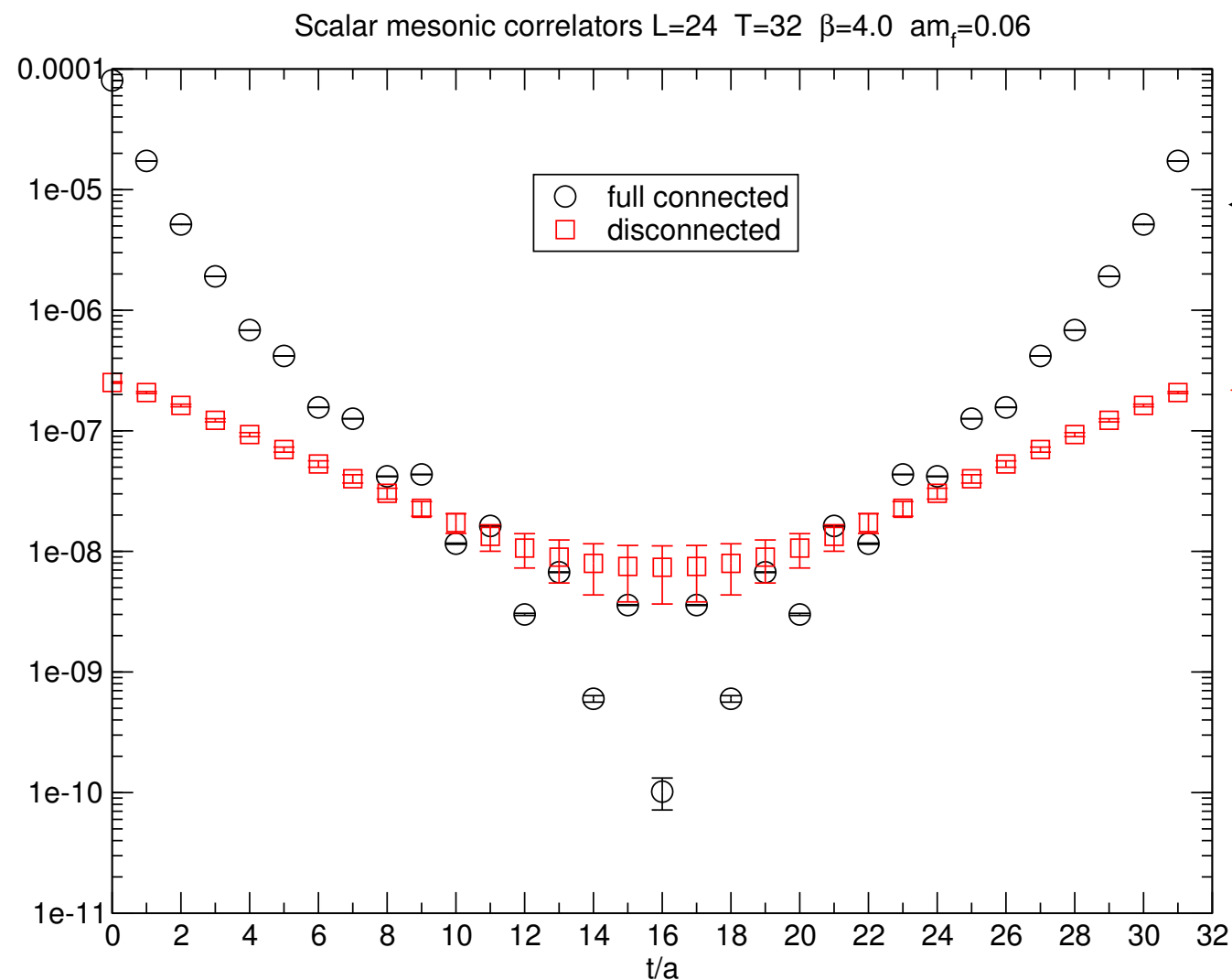
- using 14000 configurations

Results: scalar flavour-singlet meson



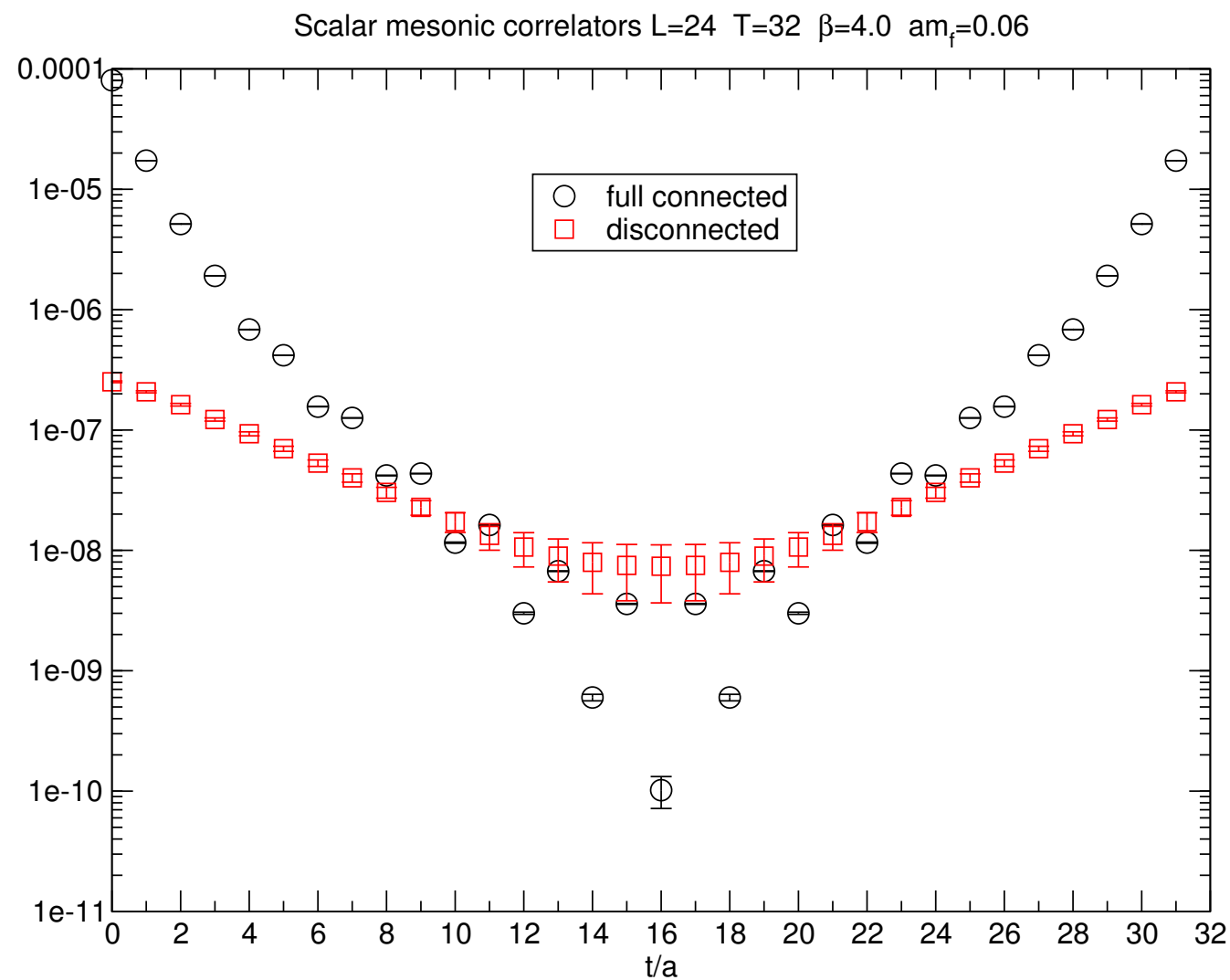
- using 14000 configurations
- 2 stochastic gaussian sources for connected piece

Results: scalar flavour-singlet meson



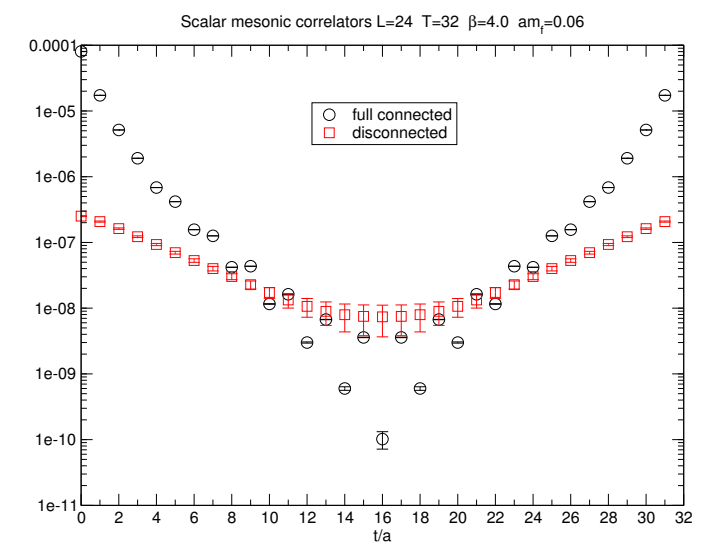
- using 14000 configurations
- 2 stochastic gaussian sources for connected piece
- 64 stochastic gaussian sources for disconnected piece

Results: scalar flavour-singlet meson

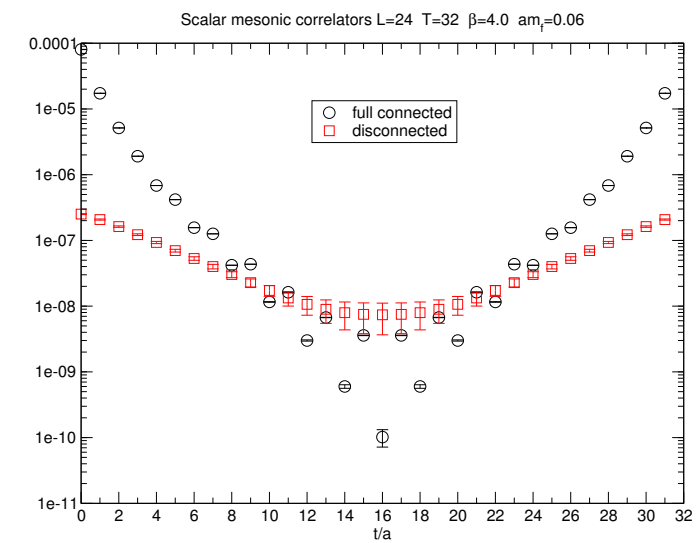


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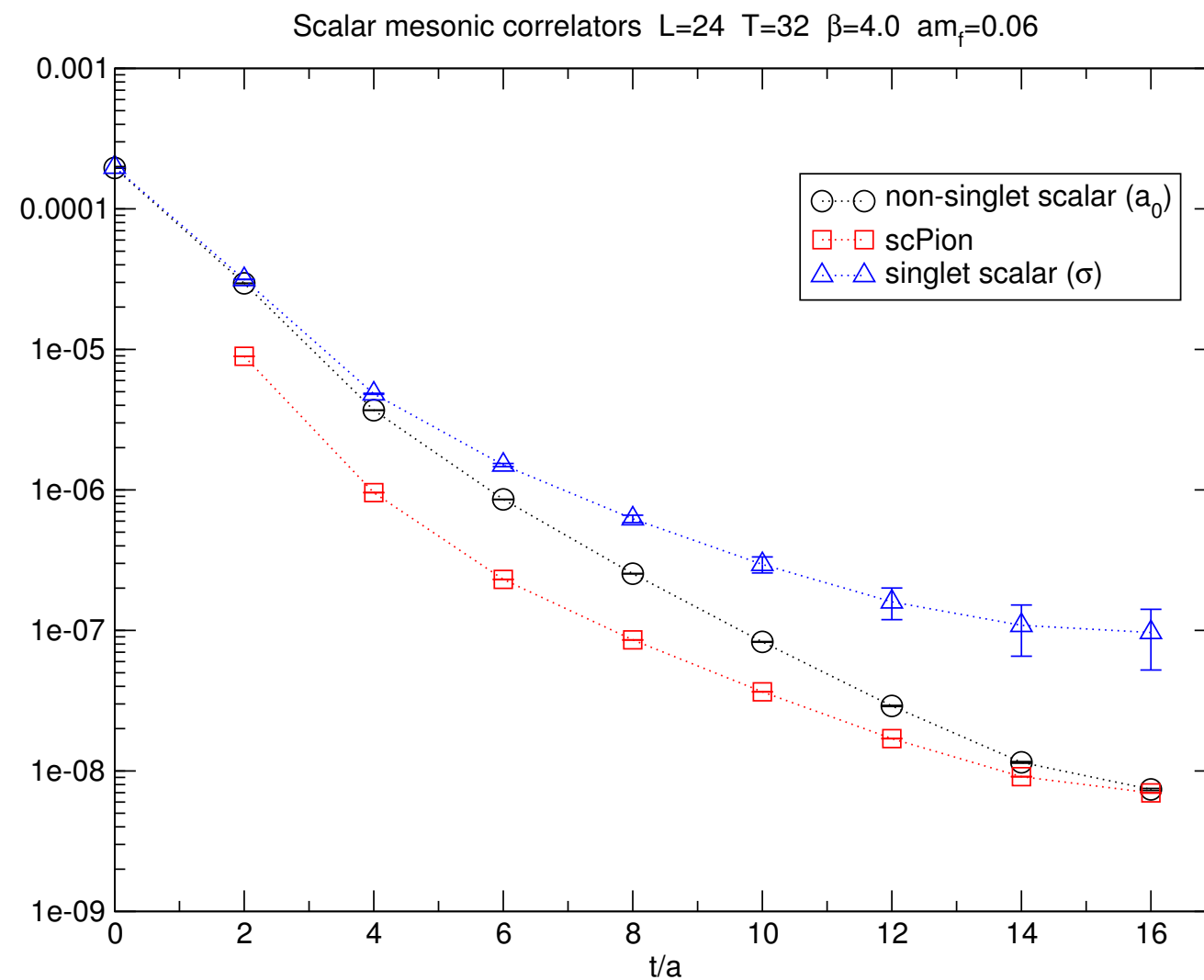
Results: scalar flavour-singlet meson



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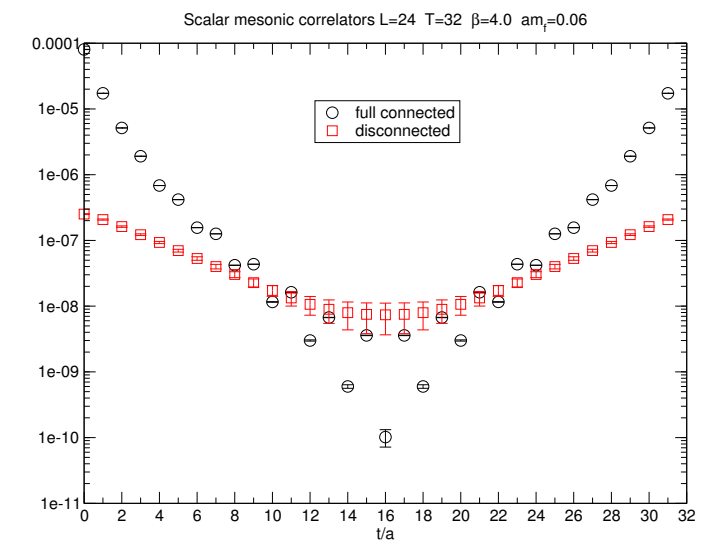
Results: scalar flavour-singlet meson



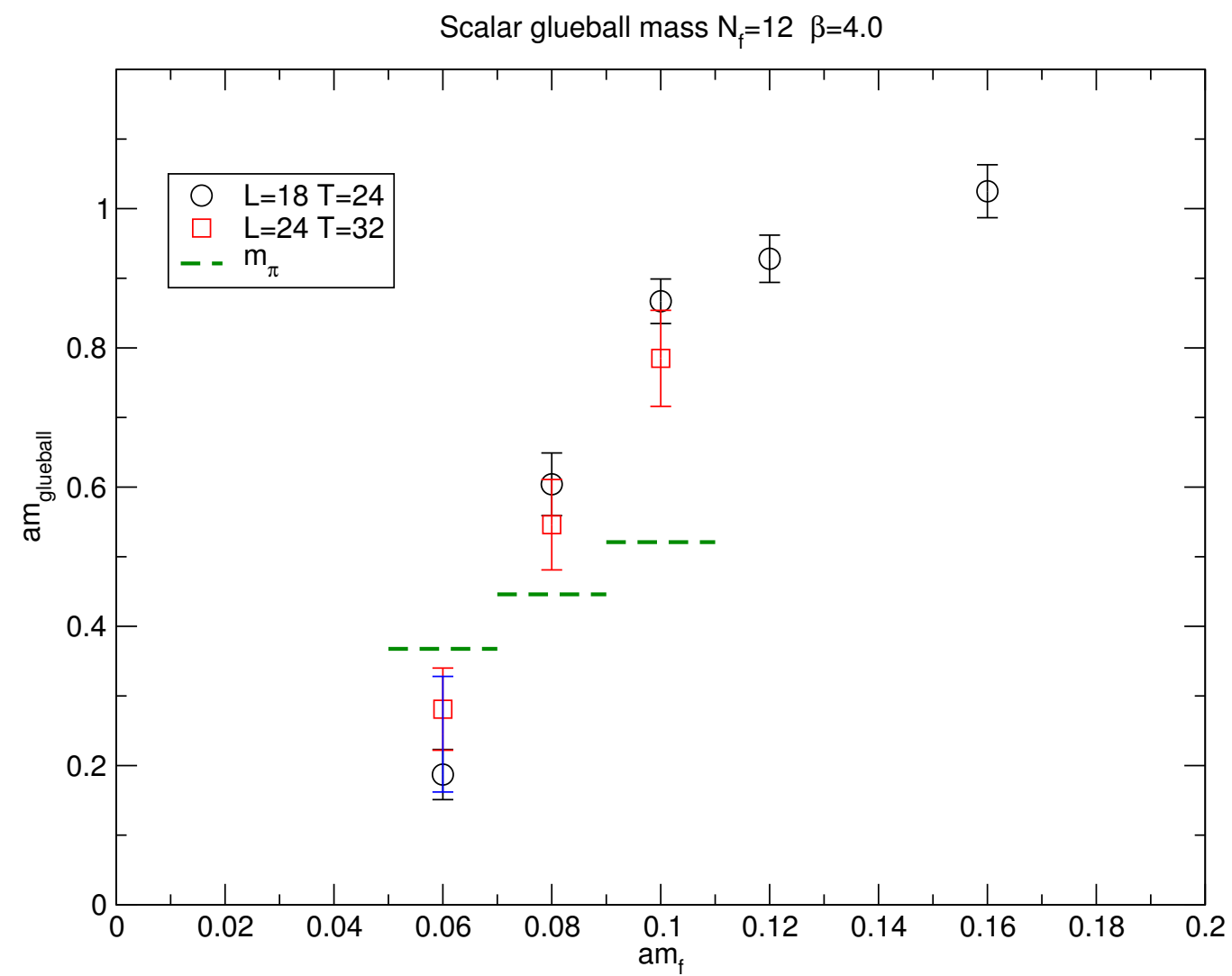
$$C_{0+}(t) = 2C(t) + C(t+1) + C(t-1)$$

$$C_{0-}(t) = 2C(t) - C(t+1) - C(t-1)$$

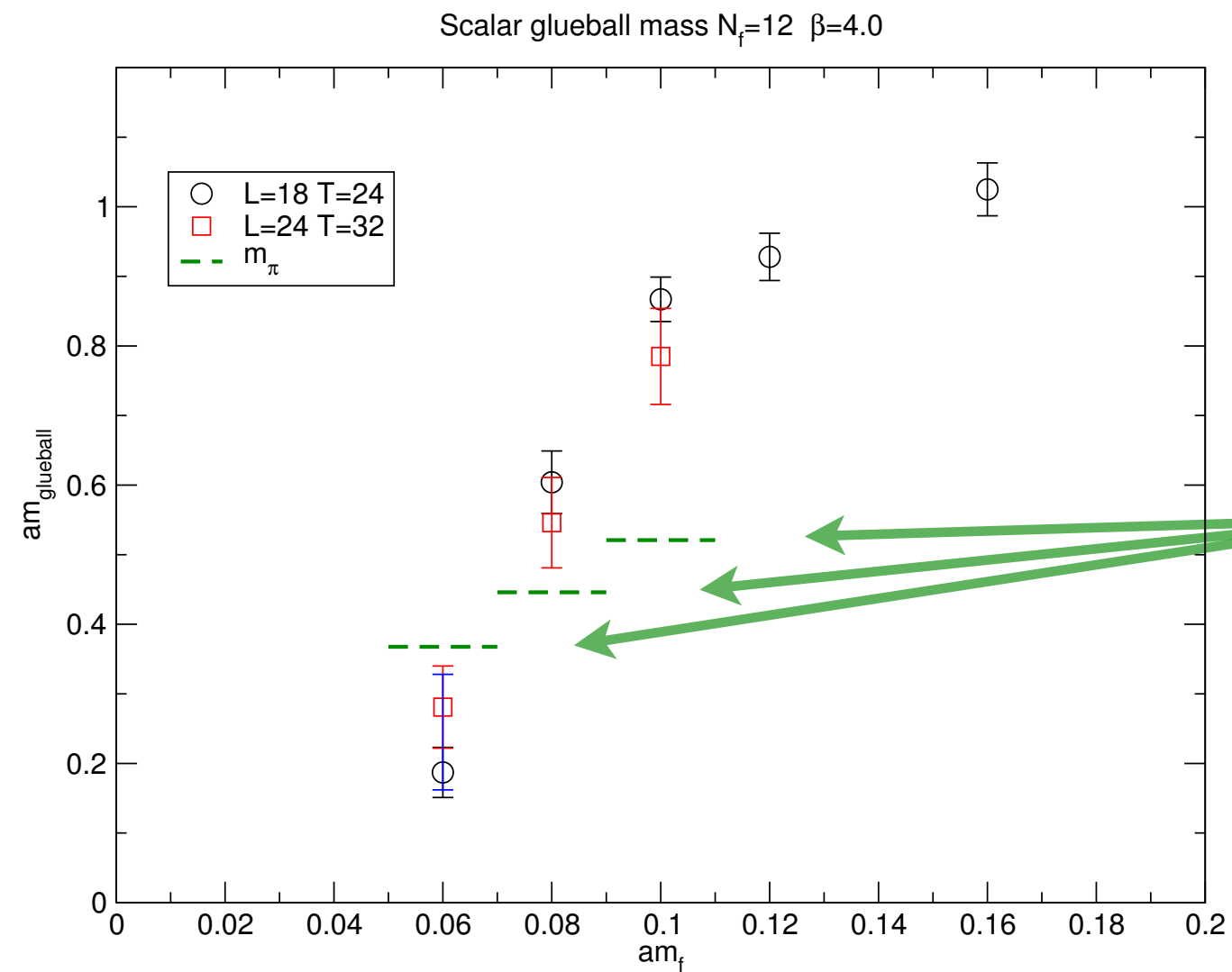
$$C_{\sigma}(t) = -C_{0+}(t) + 3D_{0+}^2(t)$$



Results: summary

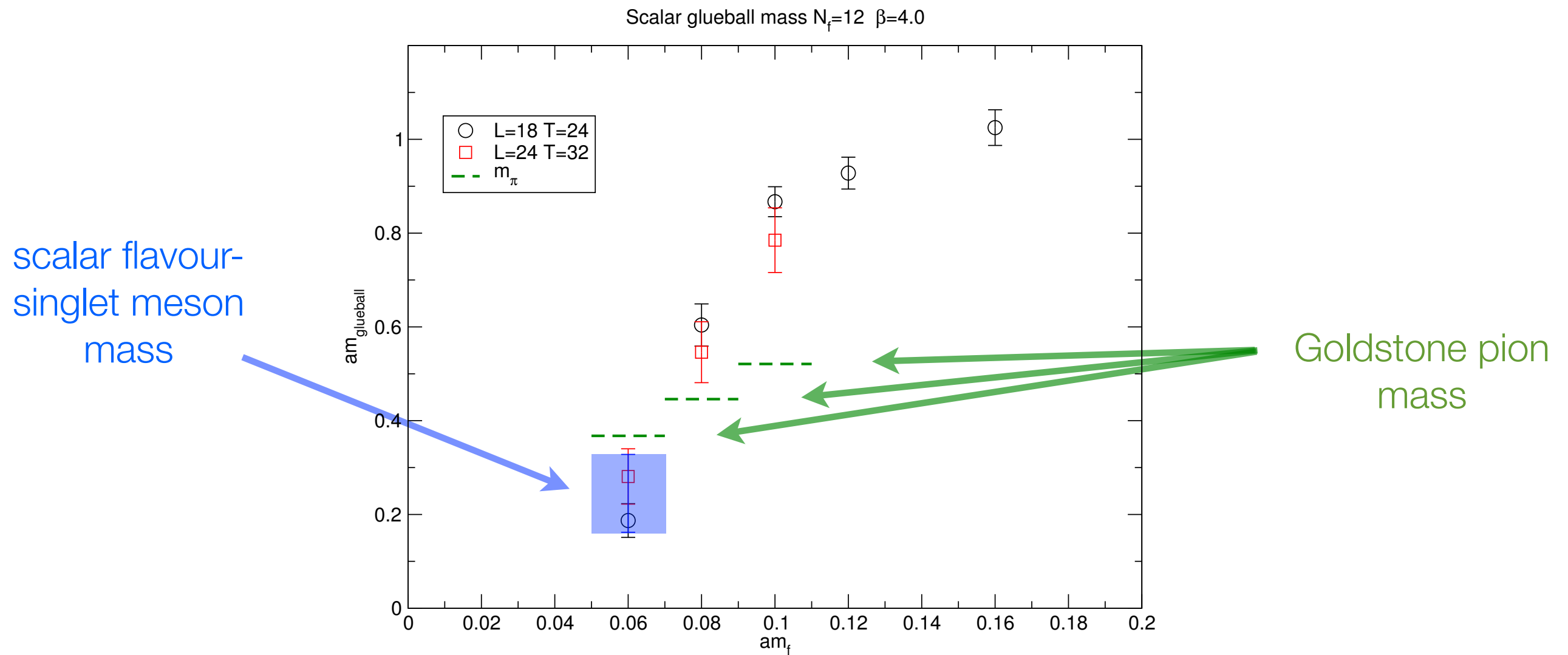


Results: summary

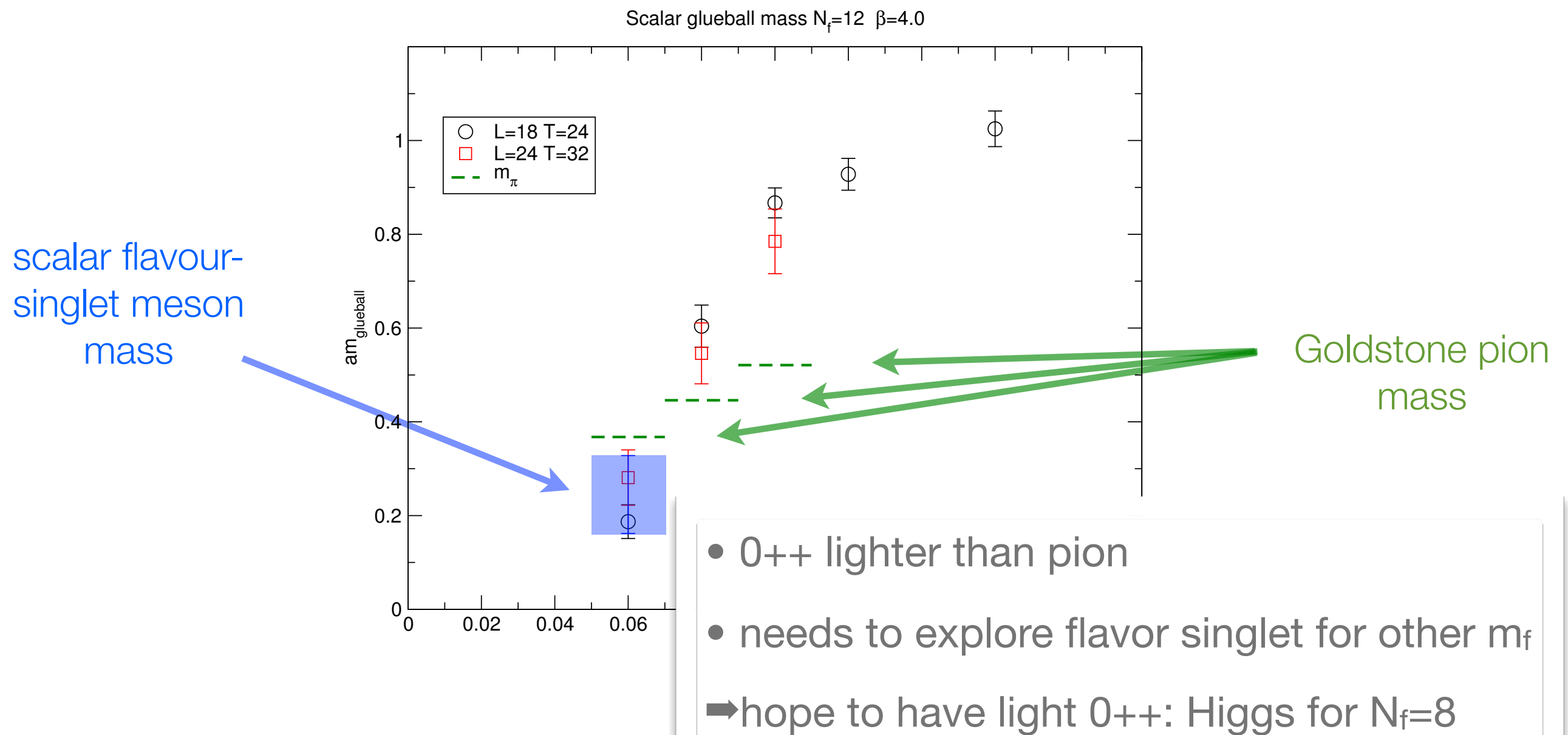


Goldstone pion
mass

Results: summary



Results: summary



Summary and Outlook

- $SU(3)$ gauge theory + N_f fundamental fermions
- $N_f=12$ likely conformal
- $N_f=8$ candidate of WTC
- existence of light 0^{++} is promising!

- $N_f=12$, 0^{++} be continued
- $N_f=8$ large scale simulation
 - detailed chiral analysis for F , m_{Had} , $m_{0^{++}}$
 - anomalous dimension γ (method that does not assume conformality)
 - S parameter...

Thank you for your attention