YN Potentials of Strangeness S=-1 from Lattice QCD

H. Nemura[1],
for HAL QCD Collaboration
S. Aoki[1], T. Doi[2], T. Hatsuda[3], Y. Ikeda[4], T. Inoue[5],
N. Ishii[1], K. Murano[6], and K. Sasaki[1],

[1]Center for Computational Science, University of Tsukuba, Japan
[2]Center for Nuclear Study, University of Tokyo, Japan
[3]Theoretical Research Division, Nishina Center RIKEN, Japan
[4]Department of Physics, Tokyo Institute of Technology, Japan
[5]College of Bioresouce Science, Nihon University, Japan
[6]Strangeness Nuclear Physics, Nishina Center RIKEN, Japan

Outline

- Introduction
- Formulation --- potential (central + tensor)
- Numerical results:
  - $N\Lambda$ force (V_C + V_T)
  - $N\Sigma$ (I=3/2) force (V_C + V_T)
- Recent improvement for Vc and VT
- Summary and outlook

Introduction:

- Study of hyperon-nucleon (*YN*) and hyperon-hyperon (*YY*) interactions is one of the important subjects in the nuclear physics.
  - Structure of the neutron-star core,
    - Hyperon mixing, softning of EOS, inevitable strong repulsive force,
  - H-dibaryon problem,
    - To be, or not to be,
- The project at J-PARC:
  - Explore the multistrange world,
- However, the phenomenological description of *YN* and *YY* interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological *NN* potential.

The purposes of this work

- *NY* forces from lattice QCD

- Spin dependence
- Potential (central + tensor)

- Numerical calculation:
  - Full lattice QCD by using N_F=2+1 PACS-CS full QCD gauge configurations with the spatial lattice volume (2.86 fm)^3
  - We also use the N_F=2+1 gauge configurations by CP-PACS/JLQCD, with the spatial lattice volume (1.93 fm)^3

Formulation

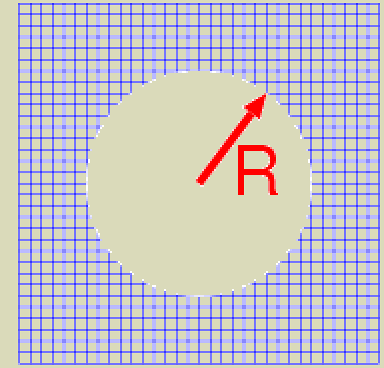
i) basic procedure:

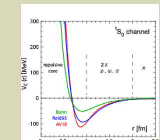
asymptotic region

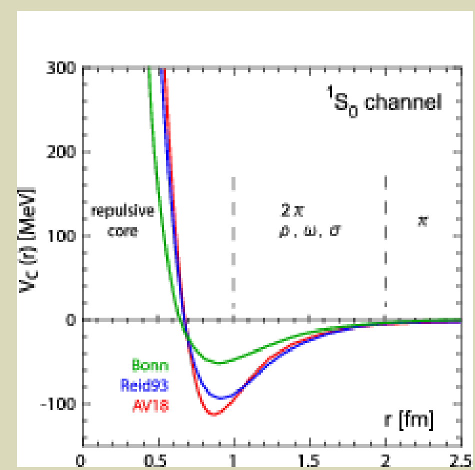
--> phase shift

ii) advanced (HAL's) procedure: interacting region

--> potential





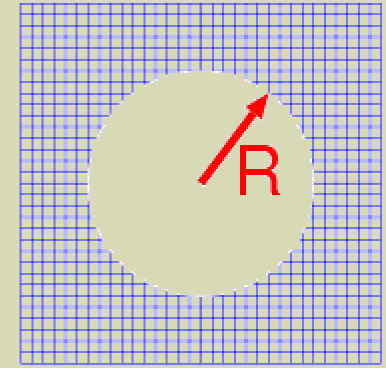


Formulation

i) basic procedure:

An example of

Luscher's formula



asymptotic region

(or temporal correlation)

--> scattering energy

--> phase shift

$$E = \frac{k^2}{2\mu}$$

$$k\cot\delta_0(k) = \frac{2}{\sqrt{\pi}L} Z_{00}(1;(kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1;q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n}\in Z^3} \frac{1}{(n^2-q^2)^s} \qquad \Re s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991).

Aoki, et al., PRD71, 094504 (2005).

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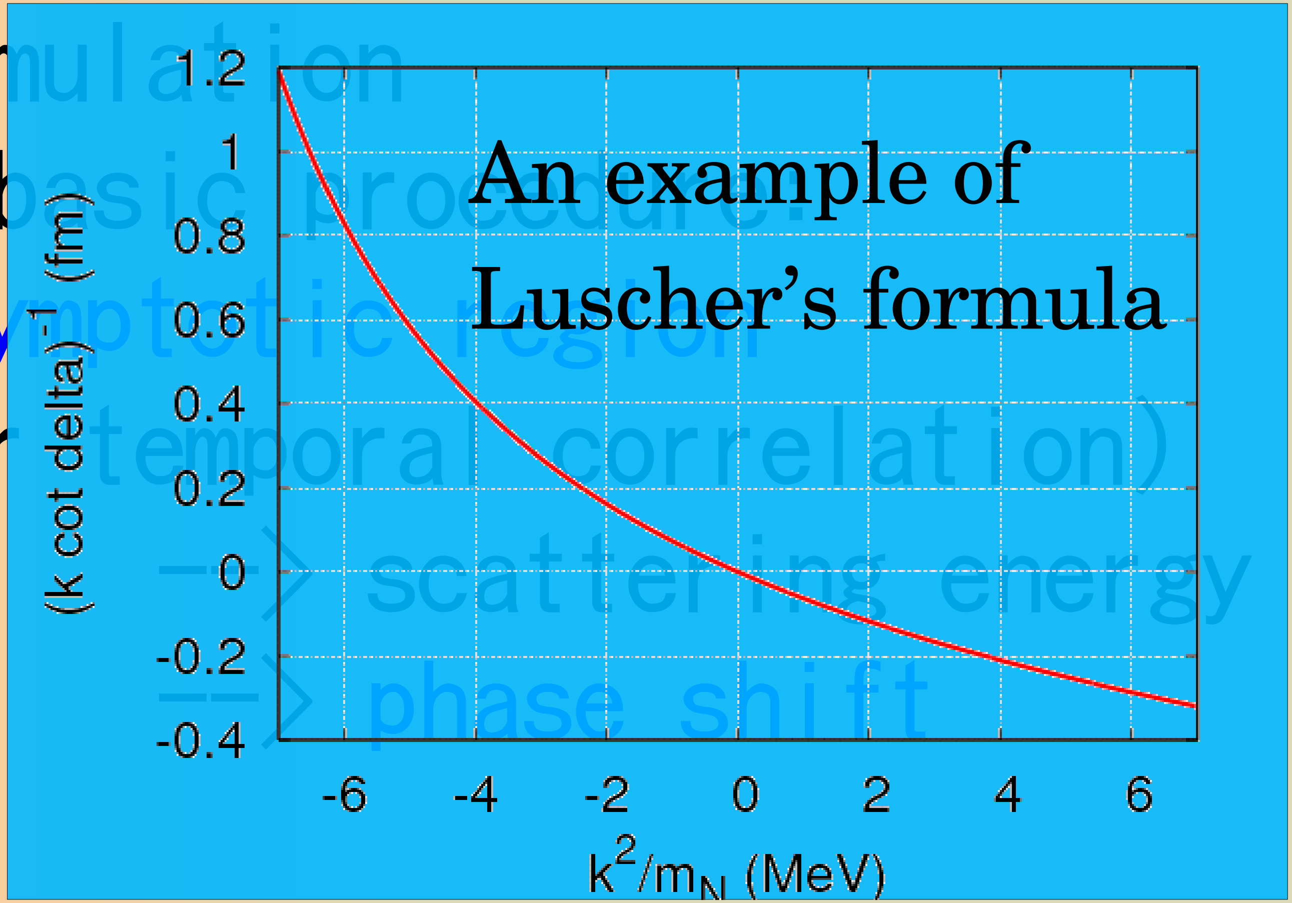

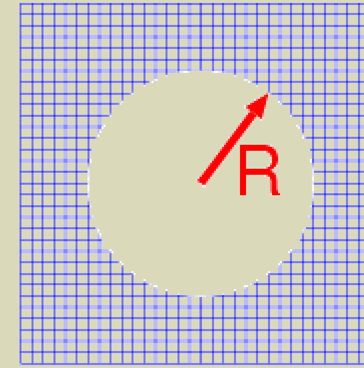

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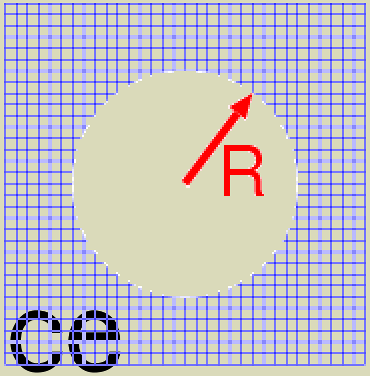
HAL formulation

ii) advanced procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential



Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., arXiv:0805.2462[hep-ph].

NOTE:

> Potential is not a direct experimental observable.

> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

HAL formulation

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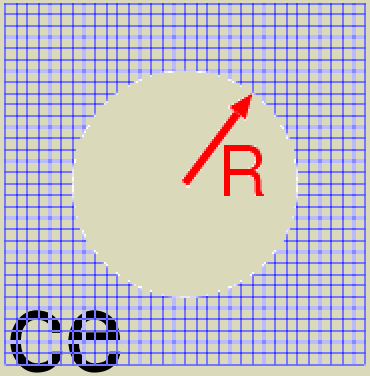
make better use of the lattice output ! (wave function)



interacting region

--> potential

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> Phase shift
> Nuclear many-body problems

An improved recipe for lattice potential:

- cf. Ishii (HAL QCD, 2010); Talk tomorrow.

- Take account of the temporal correlation as well as the spatial correlation of the NBS amplitude in terms of the R-correlator:

$$R(t,\vec{r}) = \frac{C_{YN}(t,\vec{r})}{C_Y(t)C_N(t)}$$

$$R(t+\Delta t,\vec{r}) = e^{-\Delta t H} R(t,\vec{r})$$
$$= (1-\Delta t H) R(t,\vec{r})$$

- Time-dependent effective Schroedinger eq.:

$$-\frac{\partial}{\partial t} R(t,\vec{r}) = H R(t,\vec{r})$$

An improved recipe for NY potential:

- cf. Ishii (HAL QCD, 2010); Talk tomorrow.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r})+\int d^3r' U(\vec{r},\vec{r}\,')R(t,\vec{r}\,')=-\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\to \frac{k^2}{2\mu}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}\,')=V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}\,')$$

- A general expression of the potential:

$$V_{NY}=V_0(r)+V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y)$$
$$+V_T(r)S_{12}+V_{LS}(r)(\vec{L}\cdot\vec{S}_+)$$
$$+V_{ALS}(r)(\vec{L}\cdot\vec{S}_-)+O(\nabla^2)$$

A recipe for NΛ potential:

.

- The equal time BS wave function with angular momentum (J,M) on the lattice,

$$\phi^{(JM)}_{\alpha\beta}(\vec{r}) = \sum_{\vec{x}} \left\langle 0 \middle| p_\alpha(\vec{r}+\vec{x})\,\Lambda_\beta(\vec{x}) \middle| p\Lambda\,;k\,,JM \right\rangle$$

$$p_\alpha(x) = \varepsilon_{abc}(u_a(x)\,C\gamma_5\,d_b(x))\,u_{c\alpha}(x)\,,$$

$$\Lambda_\alpha(x) = \varepsilon_{abc}\left\{(d_a C\gamma_5 s_b)\,u_{c\alpha} + (s_a C\gamma_5 u_b)\,d_{c\alpha} - 2(u_a C\gamma_5 d_b)\,s_{c\alpha}\right\}$$

- The 4-point $N\Lambda$ correlator on the lattice,

$$F^{(JM)}_{\alpha\beta}(\vec{x},\vec{y},t-t_0) = \left\langle 0 \middle| p_\alpha(\vec{x},t)\,\Lambda_\beta(\vec{y},t)\,\overline{\Theta^{(JM)}_{p\Lambda}}(t_0) \middle| 0 \right\rangle$$

$$= \sum_n A^{(JM)}_n \left\langle 0 \middle| p_\alpha(\vec{x})\,\Lambda_\beta(\vec{y}) \middle| E_n \right\rangle e^{-E_n(t-t_0)}$$

$\overline{\Theta^{(JM)}_{p\Lambda}}(t_0)$ wall source at $t = t_0$

A recipe for NΛ potential:

- cf. Ishii (HAL QCD, 2010); Talk tomorrow.

- Calculate the 4-point $N\Lambda$ correlator on the lattice,

$$\phi_{N\Lambda}(x-y)e^{-E(t-t_0)} \propto \langle p_\alpha(x,t)\Lambda_\beta(y,t)\overline{\Lambda_\beta}(0,t_0)\overline{p_\alpha}(0,t_0)\rangle$$

  - Which has the physical meanings of,
    - Create a $N\Lambda$ state and making imaginary time evolution, in order to have the lowest state of the $N\Lambda$ system.
    - Take the R-correlator $R(t-t_0, \boldsymbol{x}-\boldsymbol{y})$, which can be understood as a wave function of the non-relativistic quantum mechanics.

- Obtain the effective central potential from the effective Schroedinger equation.

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + V(r)\right)R(t,\vec{r}) = -\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\rightarrow \quad V(r) = \frac{-\frac{\partial}{\partial t}R(t,\vec{r})}{R(t,\vec{r})} + \frac{\hbar^2}{2\mu}\frac{\nabla^2 R(t,\vec{r})}{R(t,\vec{r})}$$

A recipe for NY potential: (contd.)

- For $J = 1$, ϕ comprises *S*-wave and *D*-wave,

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

where,

$$|\phi_S\rangle = \mathcal{P}|\phi\rangle = (1/24)\sum_{\mathcal{R}\in O}\mathcal{R}|\phi\rangle$$

$$|\phi_D\rangle = Q|\phi\rangle = (1-\mathcal{P})|\phi\rangle$$

- Therefore, we have 2-component Schrödinger eq.

S-wave:

$$\mathcal{P}(T + V_C + V_T S_{12})|\phi\rangle = -\partial/\partial t\, \mathcal{P}|\phi\rangle$$

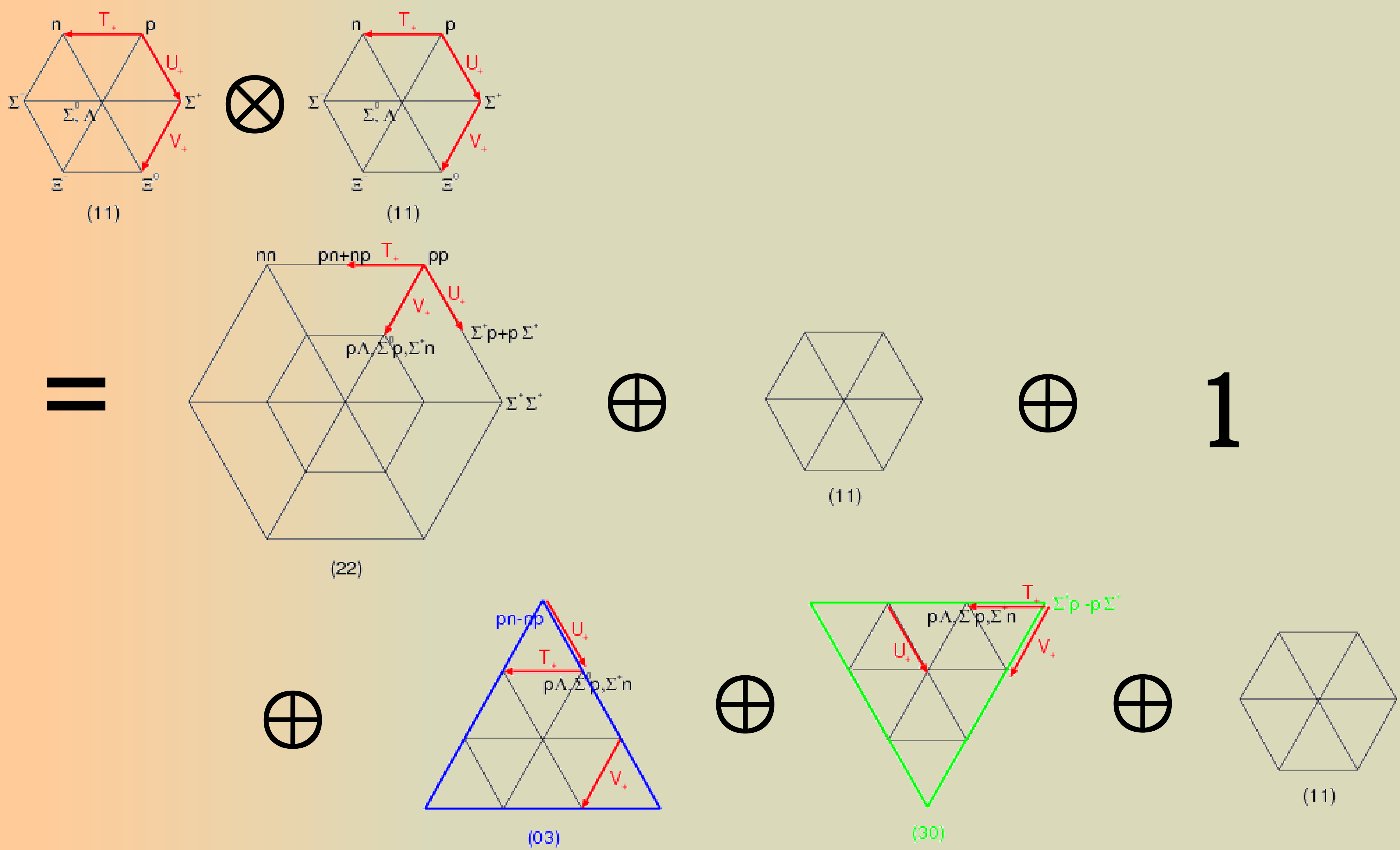
D-wave:

$$Q(T + V_C + V_T S_{12})|\phi\rangle = -\partial/\partial t\, Q|\phi\rangle$$

- Obtain the $V_C(r)$ and the $V_T(r)$ simultaneously.

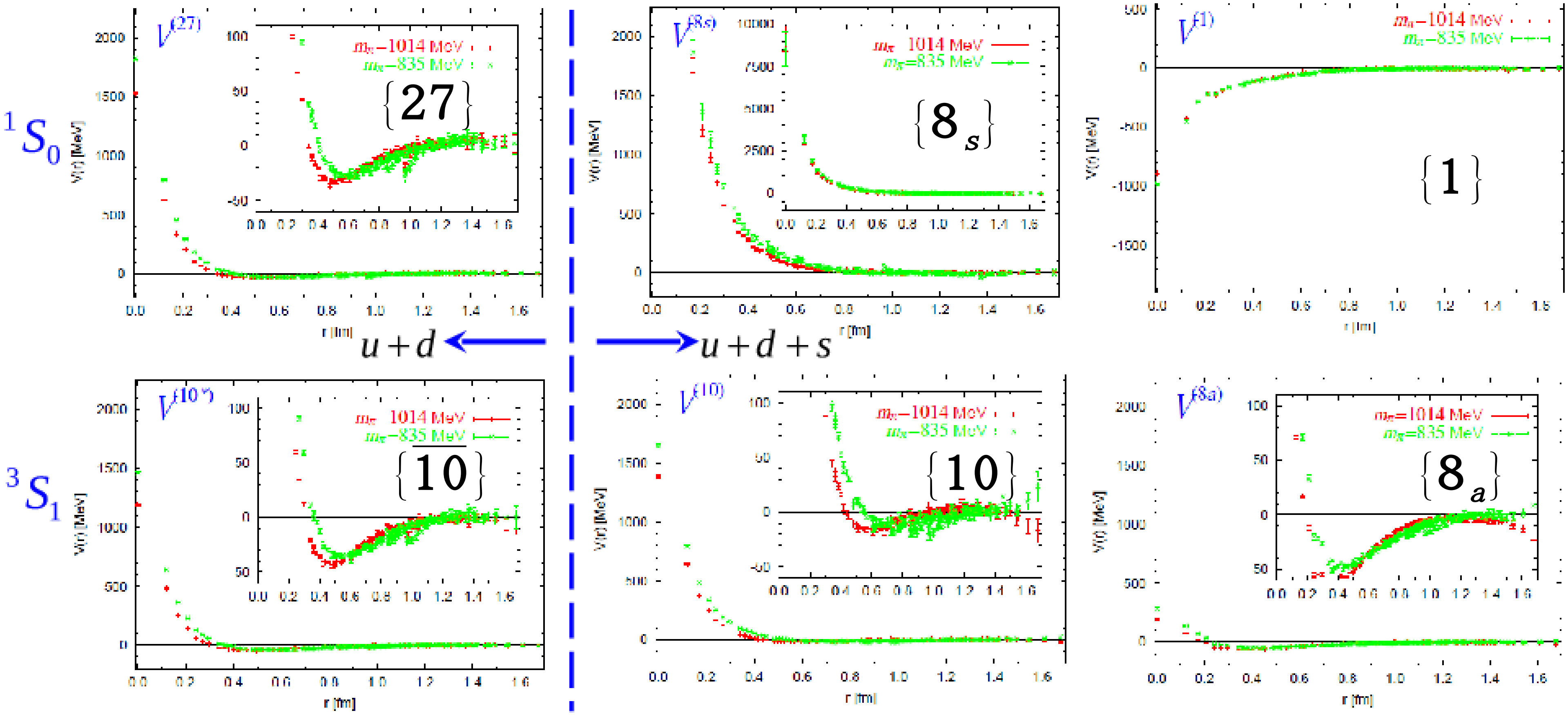
Numerical results:

$$8 \otimes 8 = 27 \oplus 8_s \oplus 1 \oplus \overline{10} \oplus 10 \oplus 8_a$$



> SU(3) limit

Aim: A systematic study of short range baryon-baryon interactions



- Strong flavor dependence
  - Strong repulsive core for flavor $\mathbf{8_s}$ representation.
  - All distance attraction for flavor **1** representation.
  - Weak repulsive core for flavor $\mathbf{8_a}$ representatin.
- This dependence is consistent with quark Pauli blocking picture.

Inoue, et al. (HAL QCD), PTP**124**, 591 (2010).

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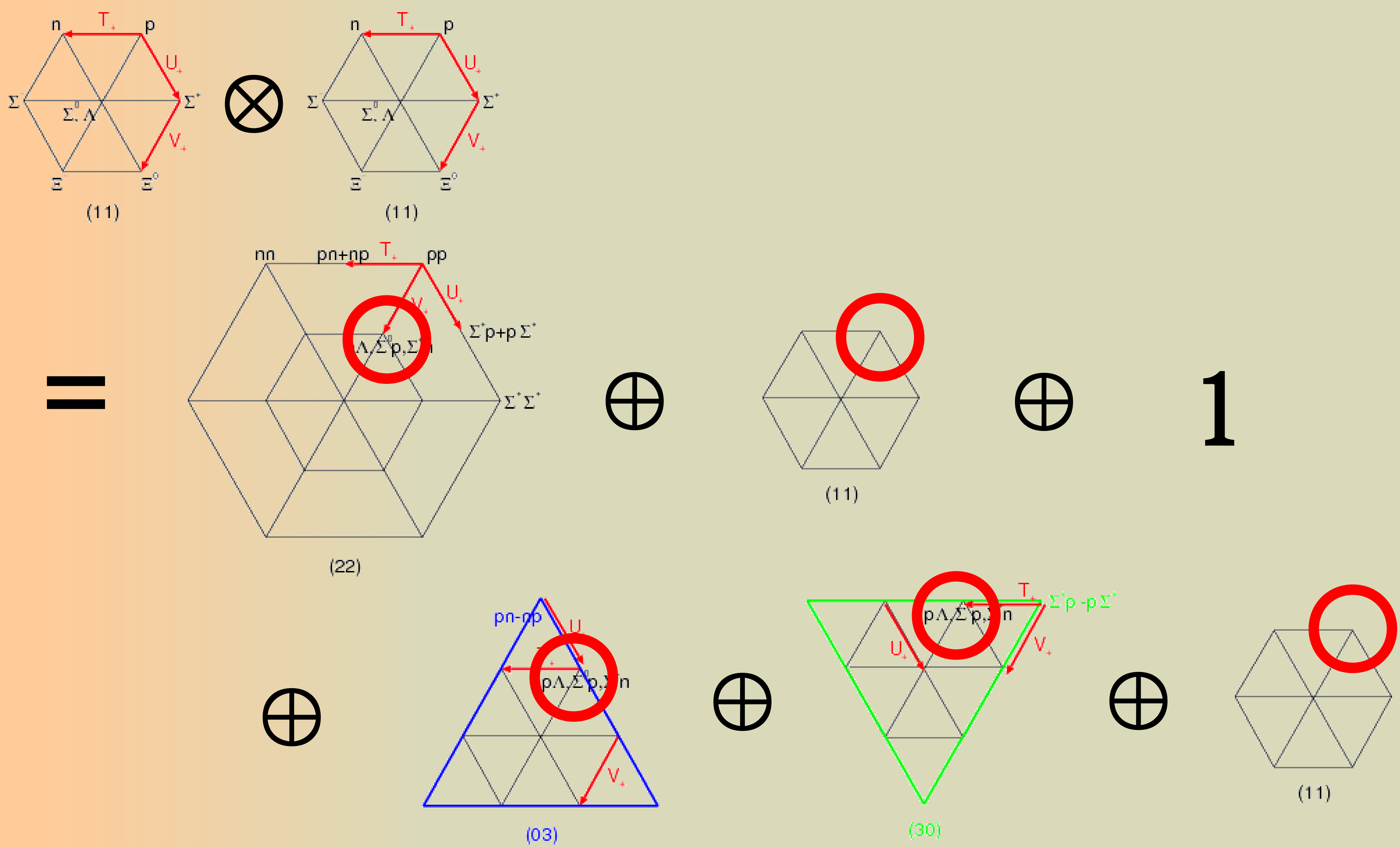
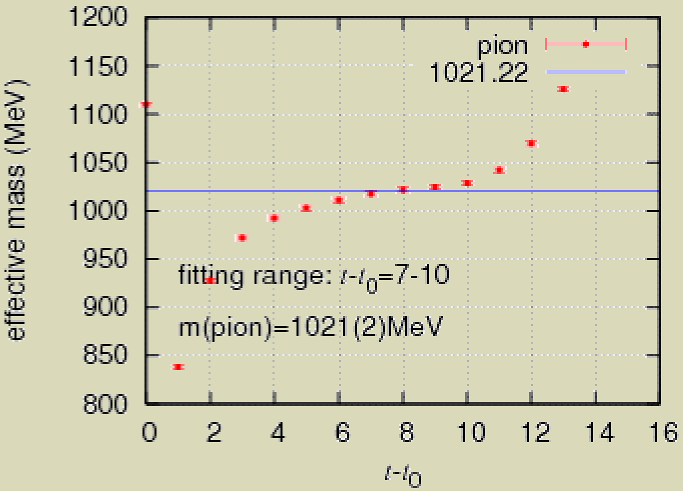
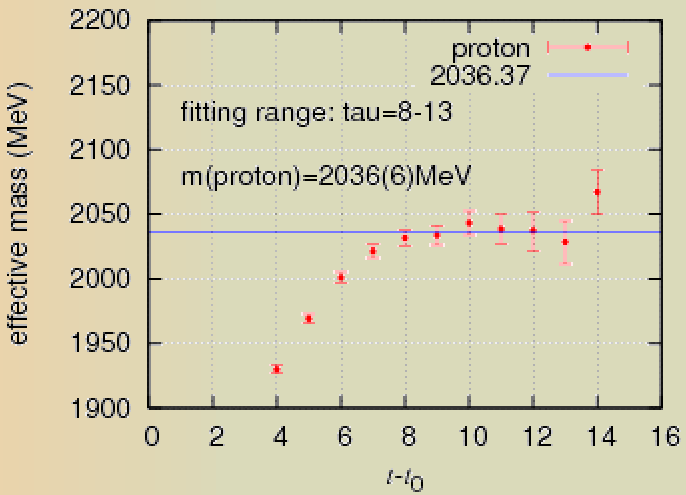
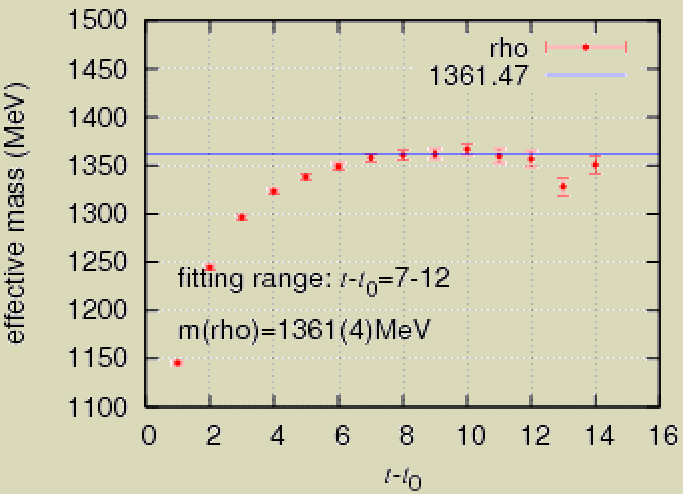


An exercise calculation by using N_F=2+1 CP-PACS+JLQCD gauge configurations:

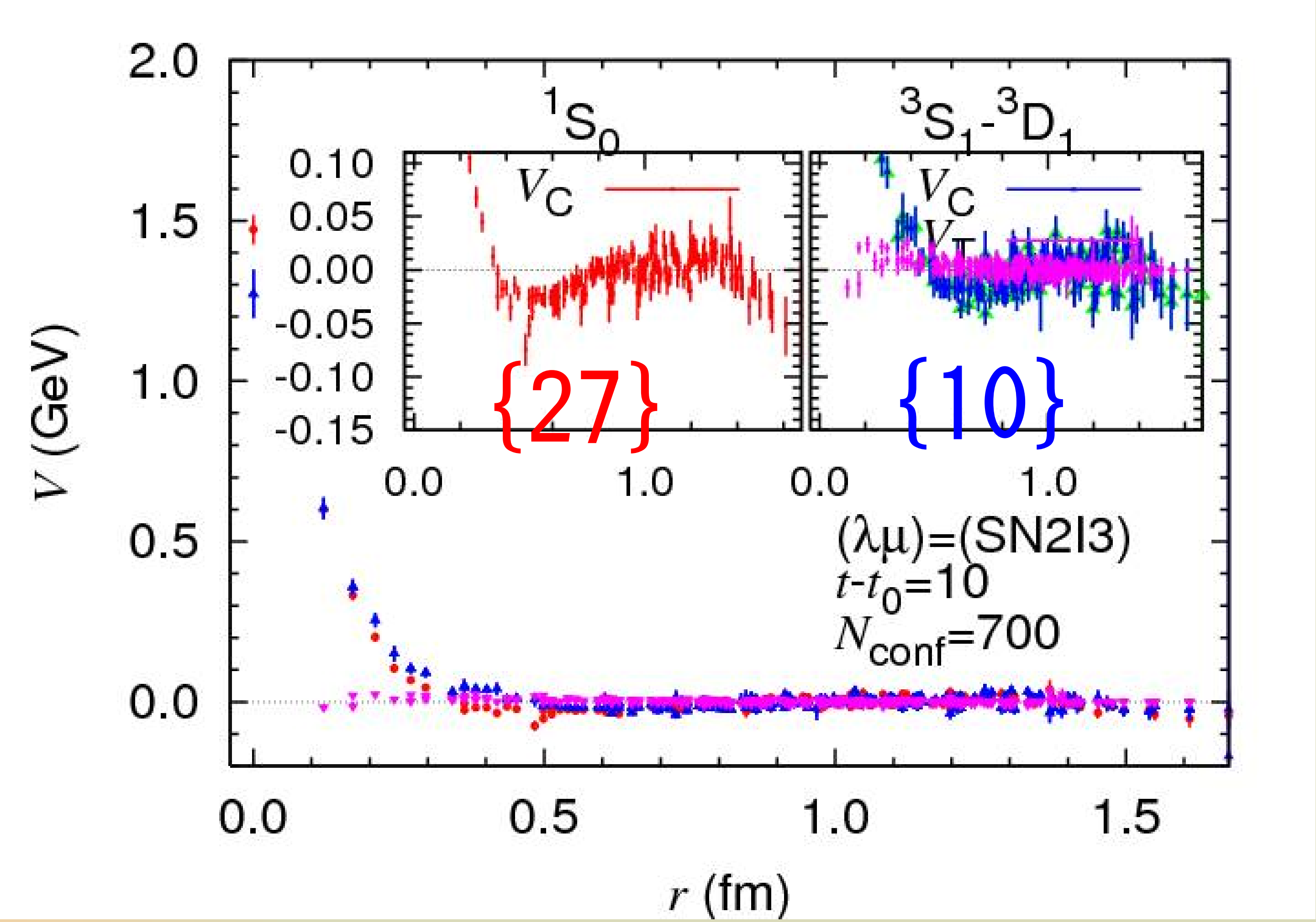
- RC16x32_B1830Kud013710Ks013710C1761
  - a=0.1209(fm)
  - Wall source, point sink
  - 700 gauge configs.







	1S0	3S1-3D1
ΛN	{27}+{8s}	{8a}+{10*}
ΣN(I=1/2)	{27}+{8s}	{8a}+{10*}
ΣN(I=3/2)	{27}	{10}



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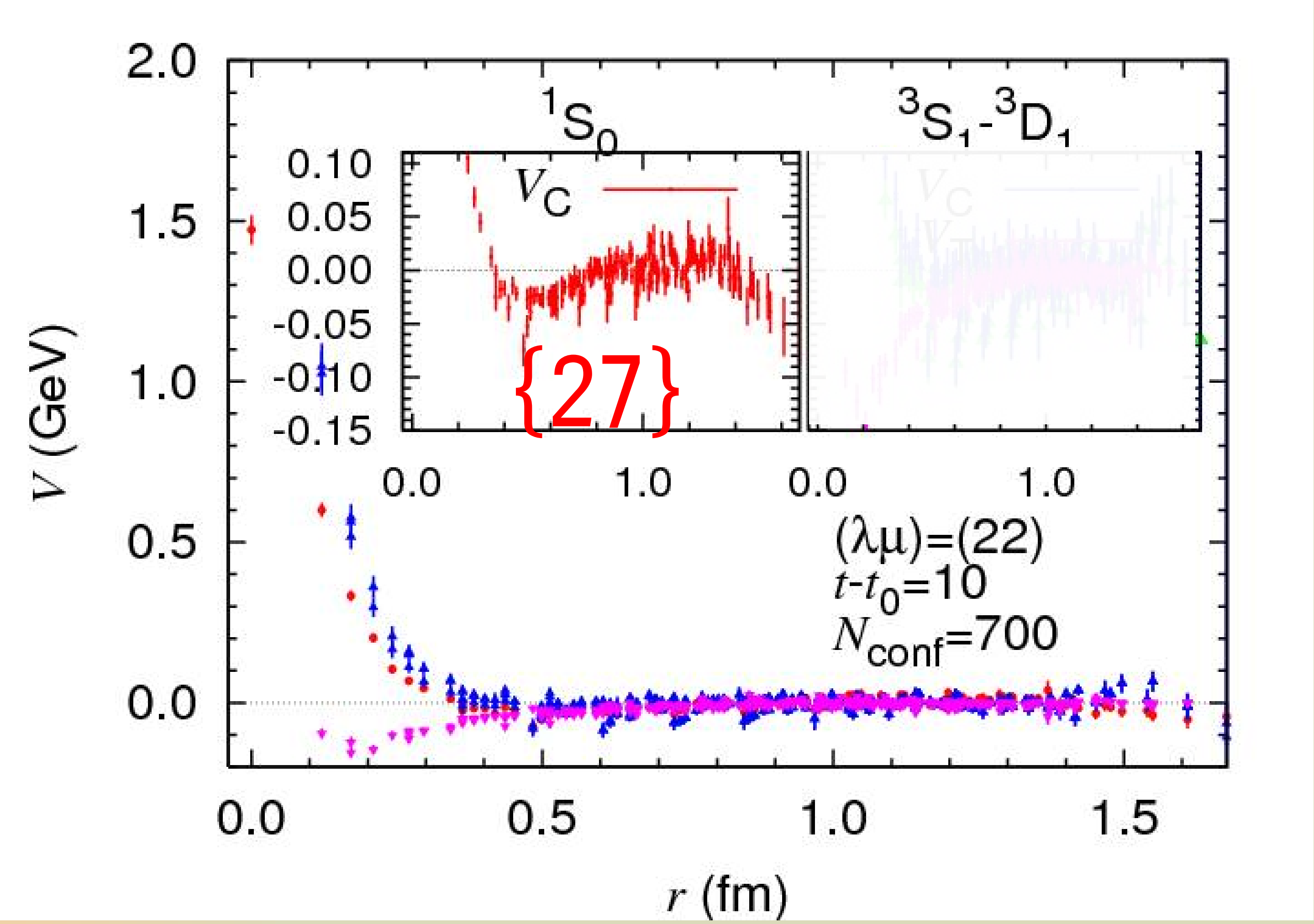

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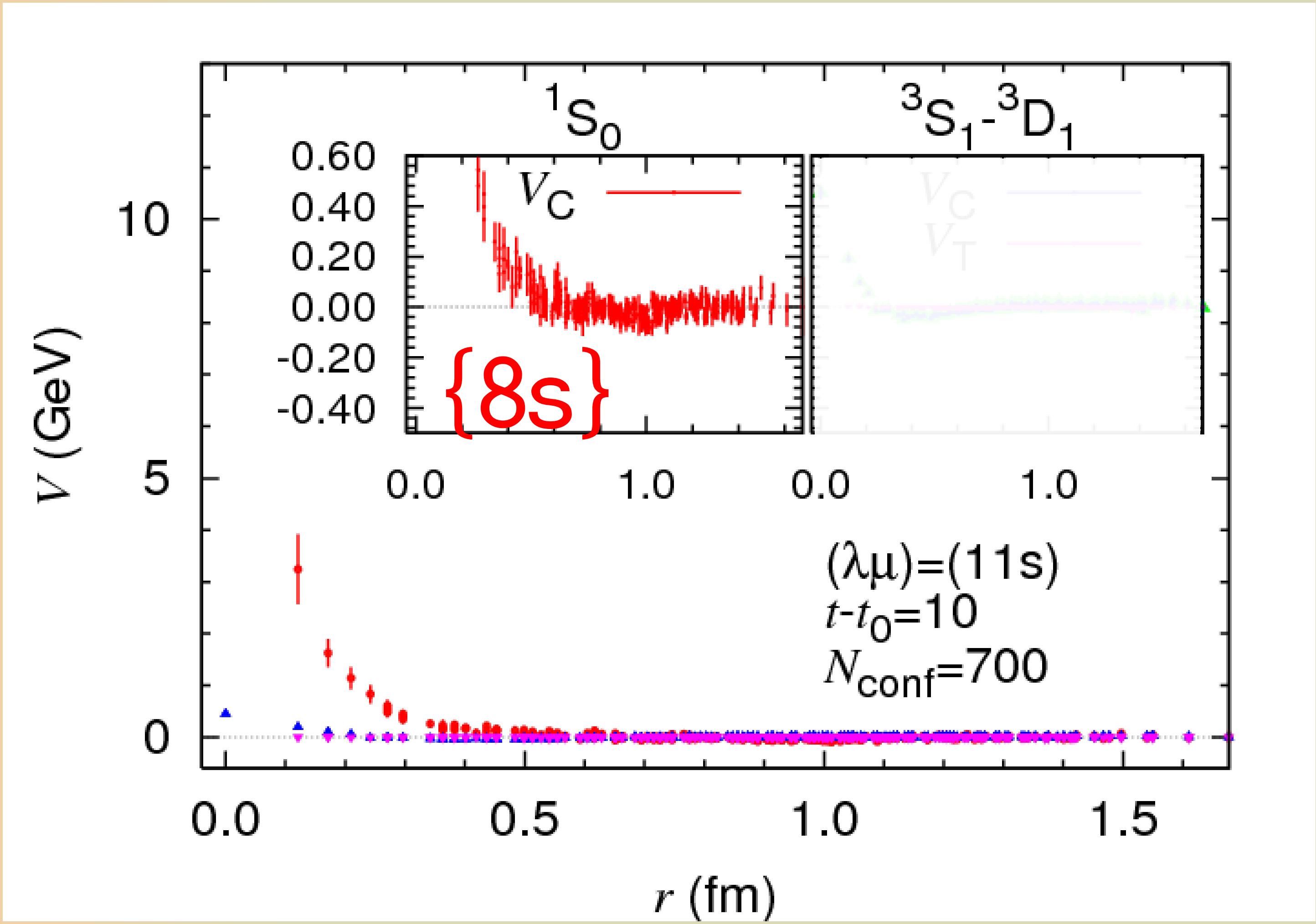

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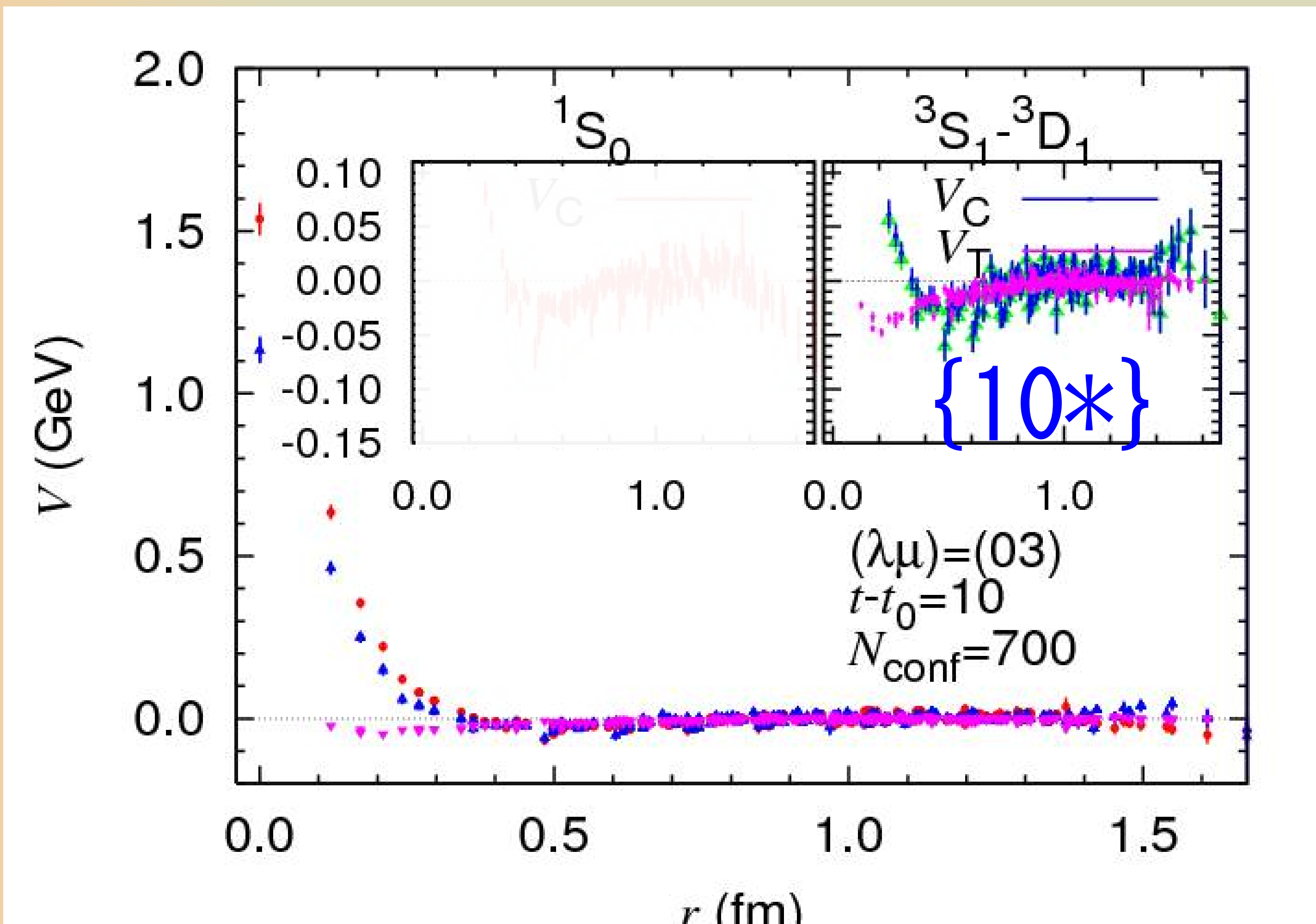

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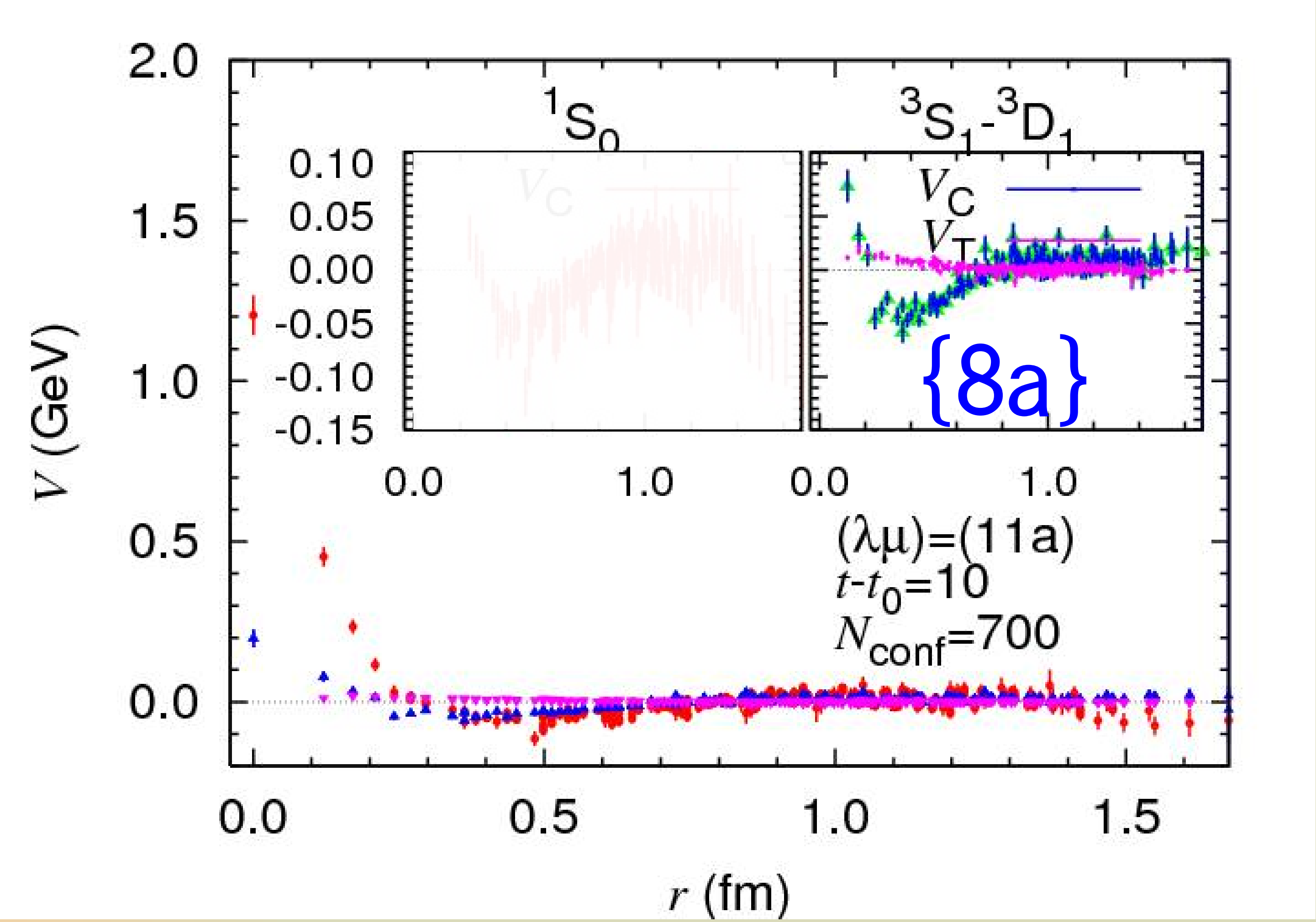


The main results

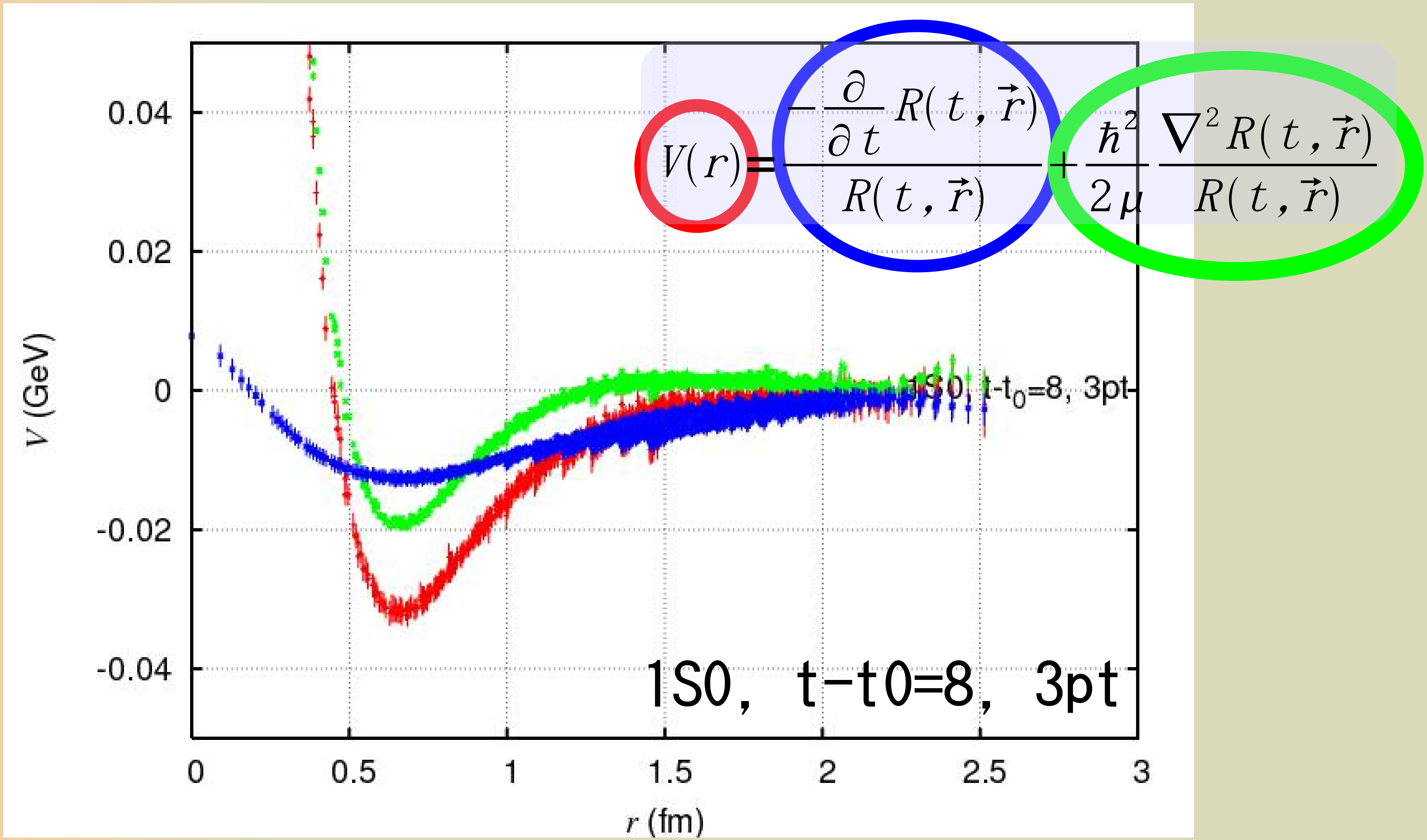
Full QCD calculations by using N_F=2+1 PACS-CS gauge configurations:

- S. Aoki, et al., (PACS-CS Collaboration), PRD**79**, 034503 (2009), arXiv:0807.1661 [hep-lat].
- Iwasaki gauge action at β=1.90 on $32^3 \times 64$ lattice
- O(a) improved Wilson quark action
- $1/a$ = 2.17 GeV (a = 0.0907 fm)

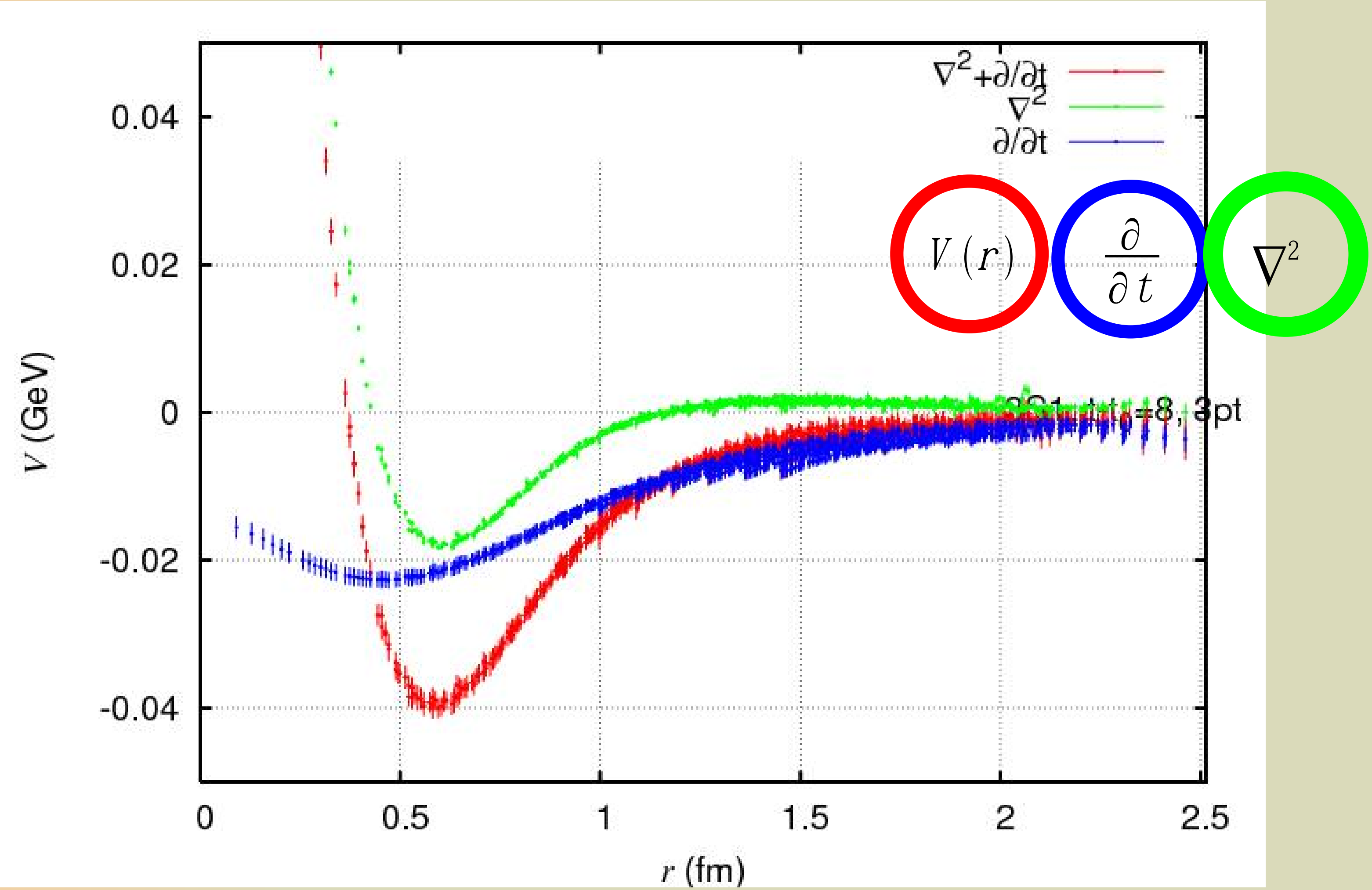
$(\kappa_{ud})_{N_{\rm conf}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_Ξ
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ @present calc (Dirichlet BC along T)								
(0.13700)	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
Exp.	135	770	494	892	940	1116	1190	1320

ΛN potential

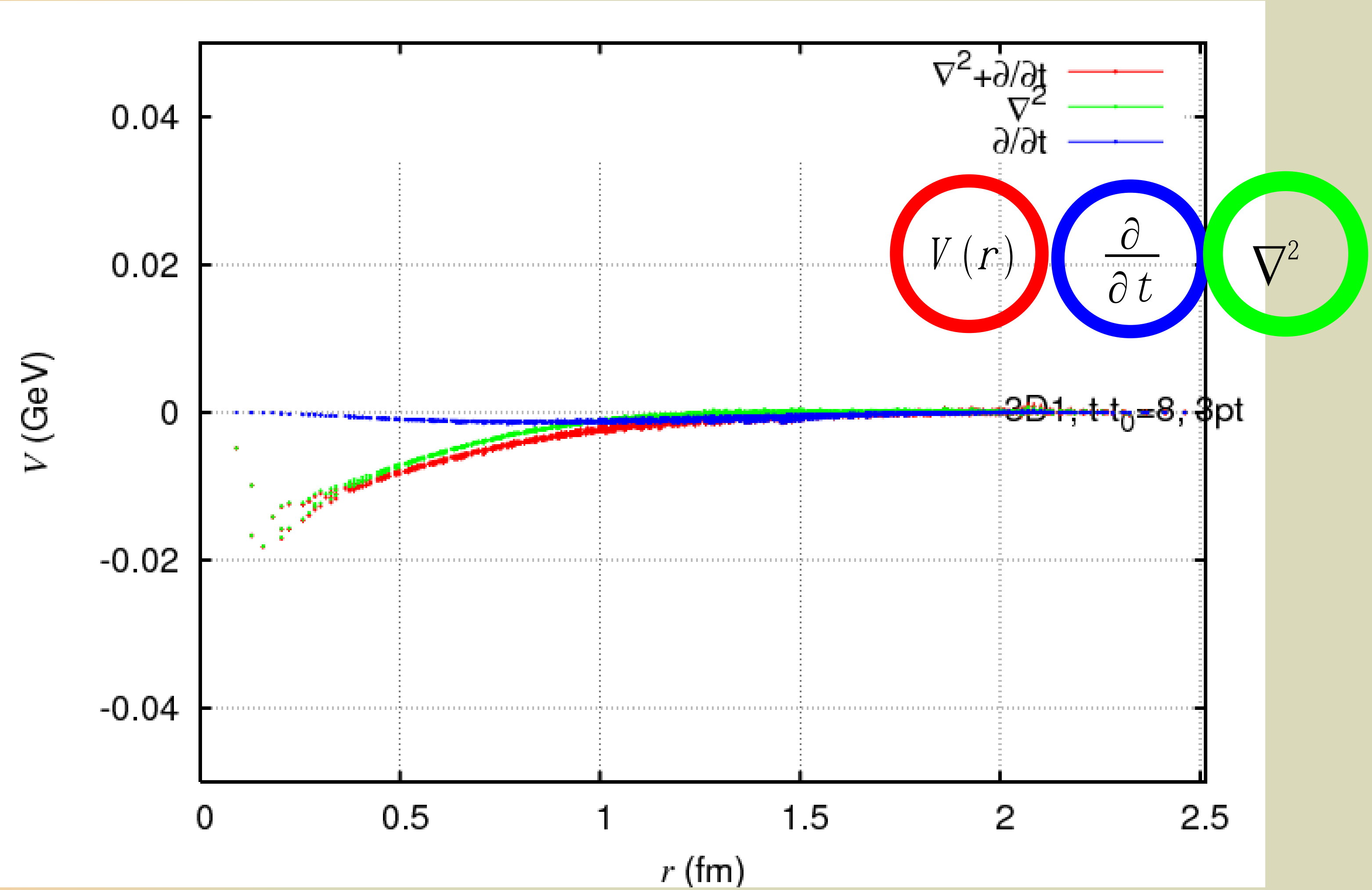
$V_C(\Lambda N;\ 1S0)$



$V_C(\Lambda N;\ 3S1-3D1)$

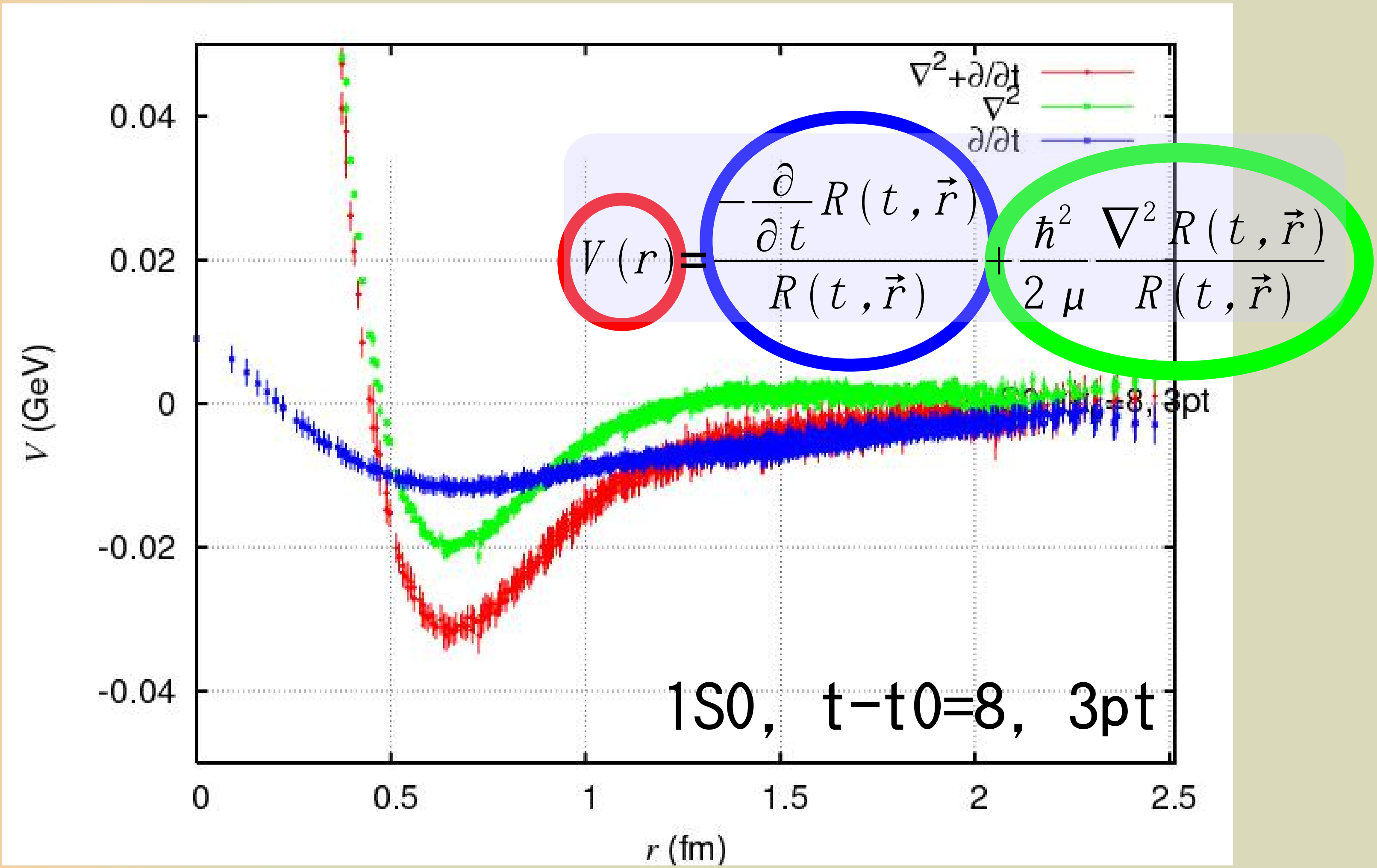


$V_T(\Lambda N;\ 3S1\text{-}3D1)$

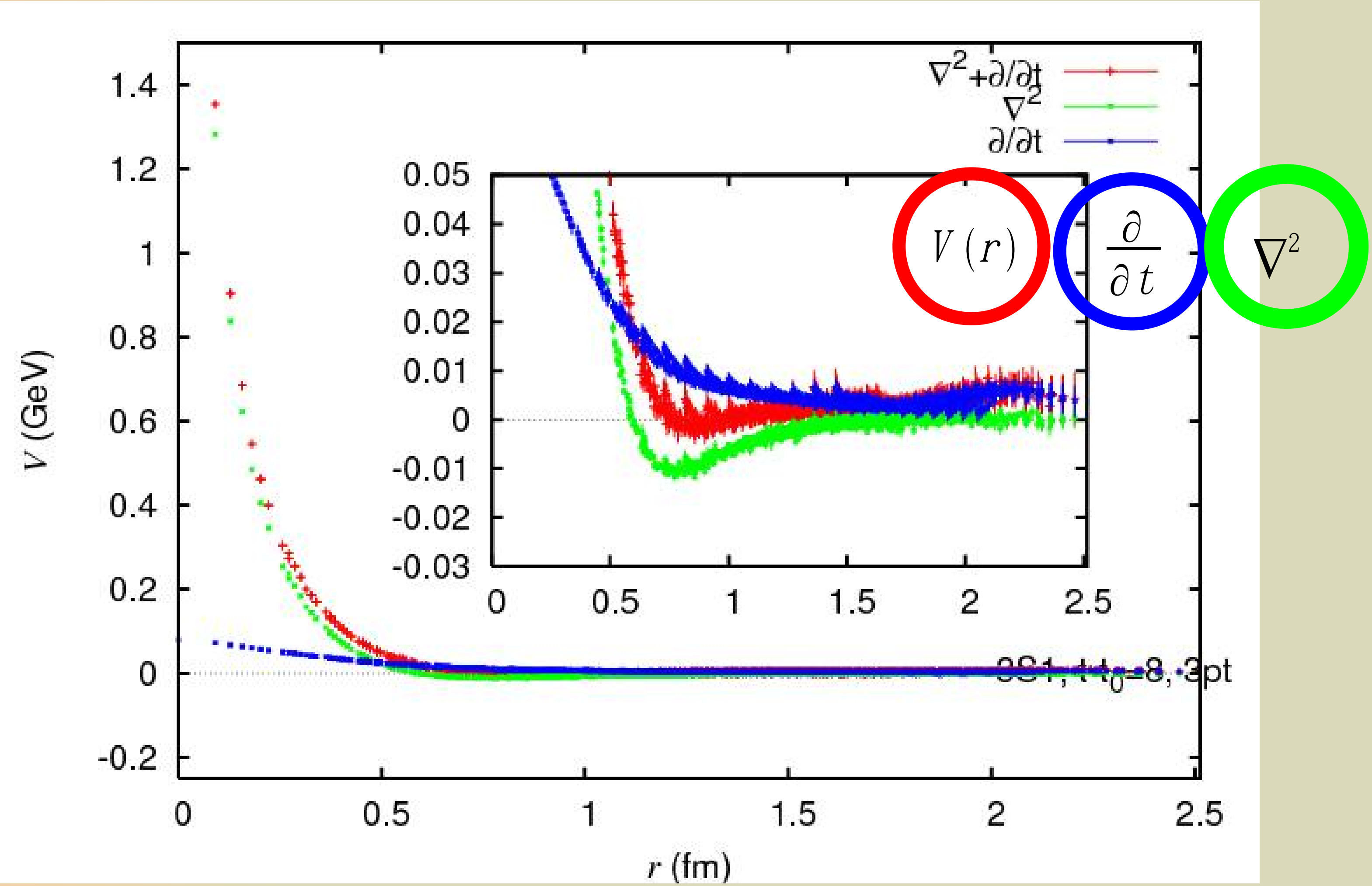


$\Sigma N(I=3/2)$ potential

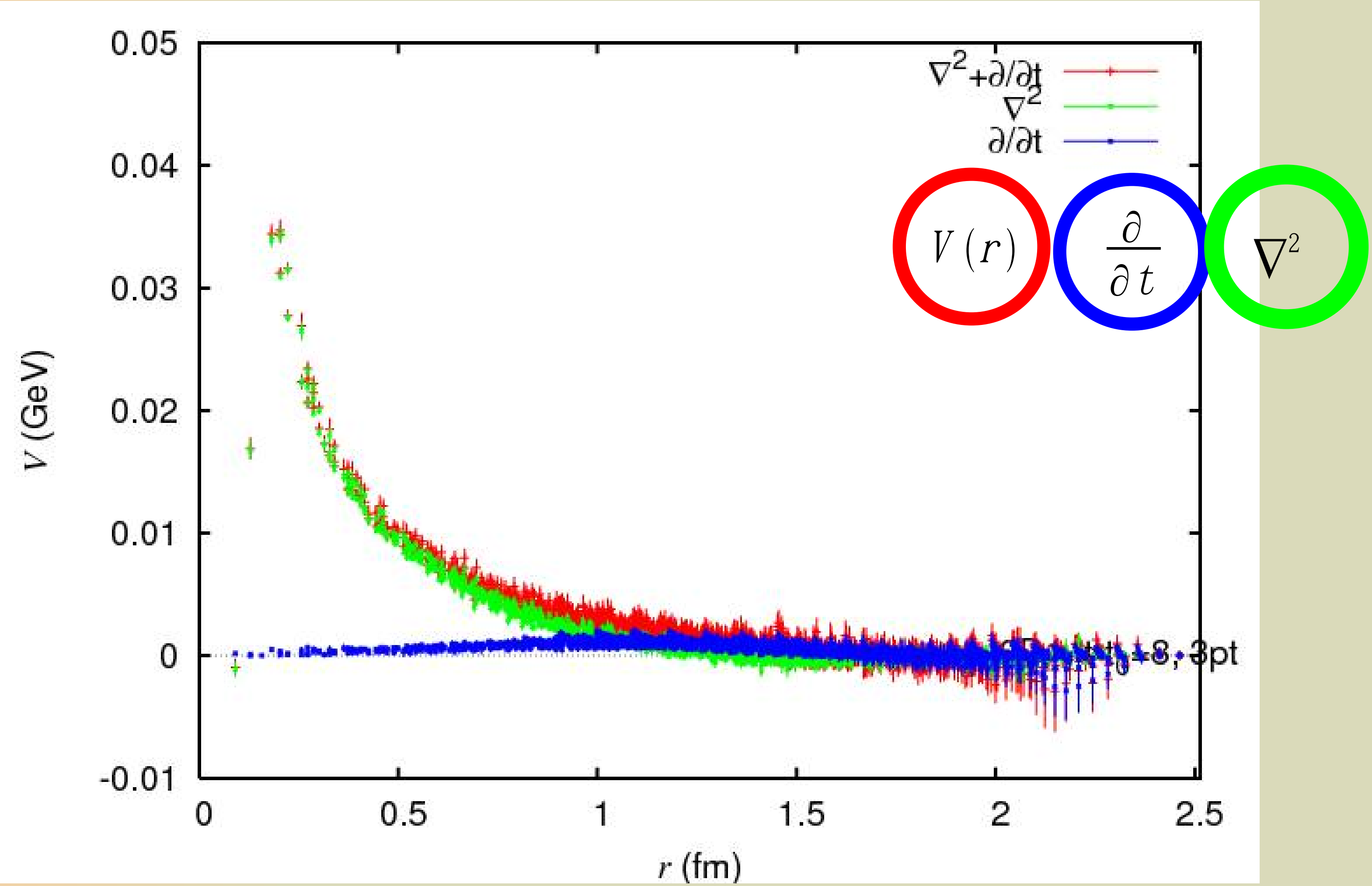
$V_C(\Sigma N(I=3/2);\ 1S0)$



$V_c(\Sigma N(I=3/2);\ 3S1-3D1)$



$V_T(\Sigma N(I=3/2);\ 3S1\text{-}3D1)$



Summary:

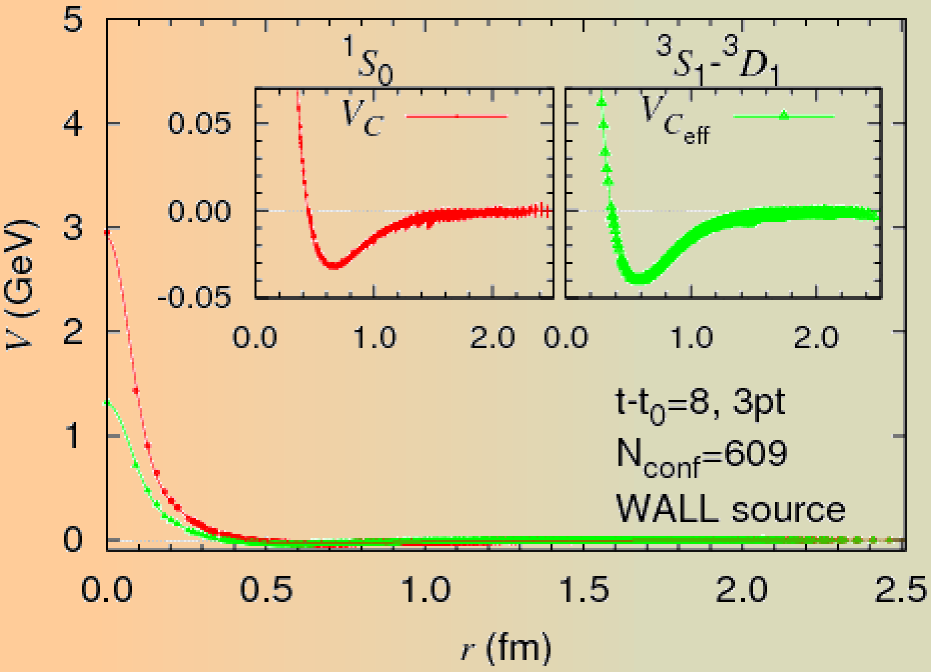
- The lattice QCD study for Lambda-Nucleon and Sigma-nucleon(I=3/2) interactions.
- $p\Lambda$:
  - Central, tensor. For full QCD
  - Time-derivative terms enhance the attractive force.
- Qualitatively similar to well-known nuclear forces.
  - Repulsive at short distance.
  - Attractive well at medium to long distance.

- $N\Sigma$(I=3/2):
  - Central, tensor. For full QCD
  - The 1S0 potential is similar to Lambda-N potential
  - The 3S1 potential is repulsive

Outlook:

- Quark mass dependence.
- Scattering lengths.
    - spin-dependence.
    - Comparison with the hypernuclear data.
- Coupled-channel potential.

Proton-Lambda interaction (preliminary)



→ Parametrized potential

↓

Phase shift