Spin-Orbit force from Lattice QCD

(格子QCDによるLS力の計算)

Keiko Murano (RIKEN) for HAL QCD Coll.

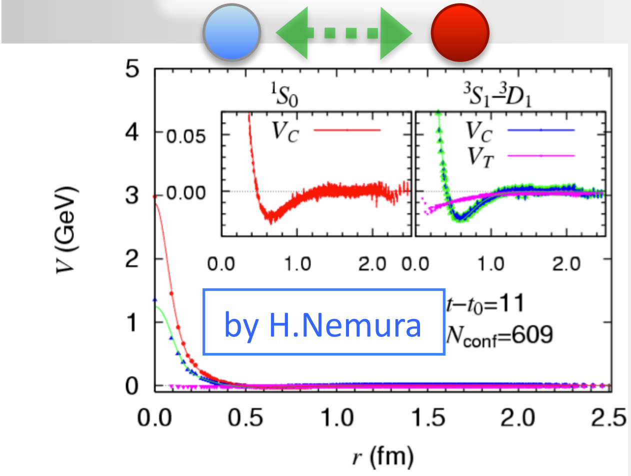
S. Aoki, B. Charron, T.Doi, T. Hatsuda, Y. Ikeda
T.Inoue, N.Ishii,
H. Nemura, K. Sasaki, M. Yamada

素核宇融合による計算基礎物理学の進展 - ミクロとマクロのかけ橋の構築
-2011 年 12 月 3 日合歓の郷

Hadrons to Atomic nuclei HAL from Lattice QCD

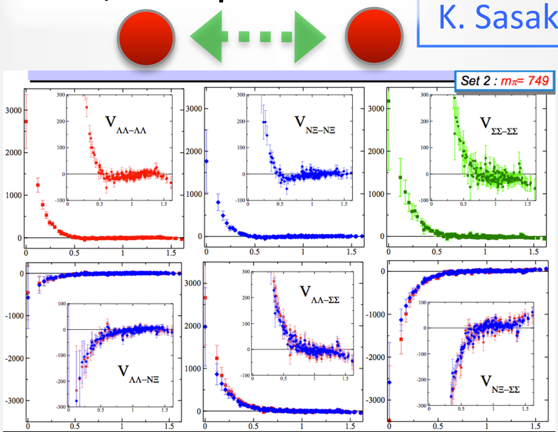
S.Aoki T.Doi T.Hatsuda Y.Ikeda T.Inoue N.Ishii
H.Nemura K.Sasaki for HAL QCD Coll.

Y-N potential

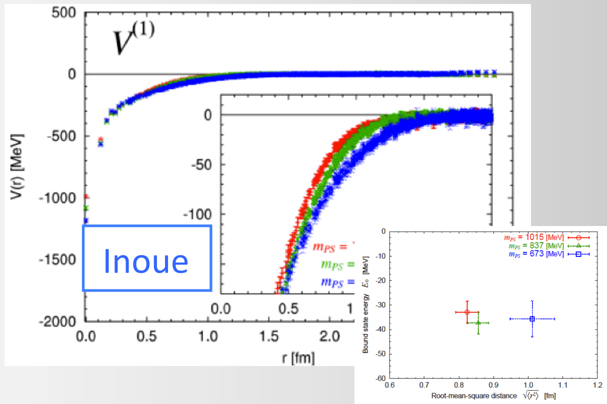


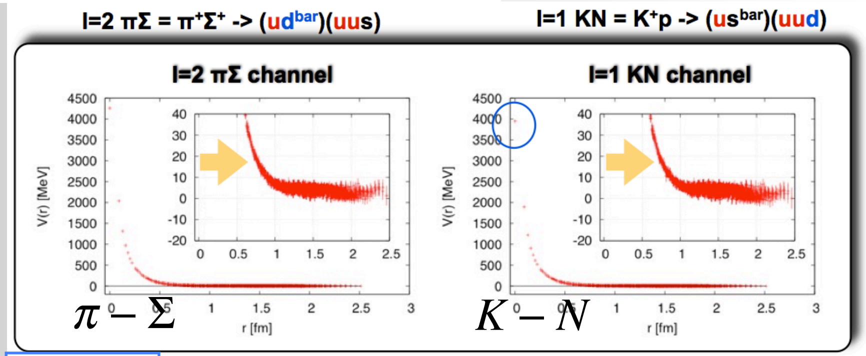
Y-N, Y-Y potential

K. Sasaki



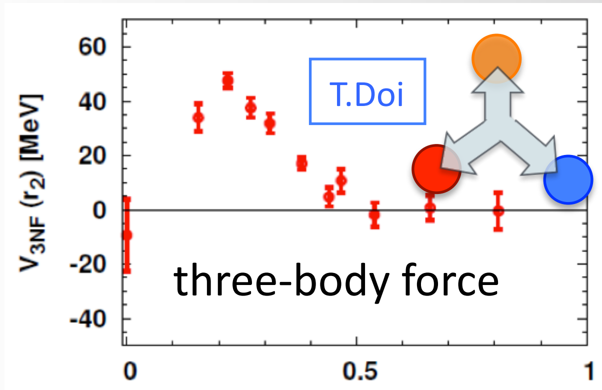
H dibaryon in SU(3)





Y. Ikeda

meson-baryon potential



$Vc^{(+)}$ $V_T^{(+)}$ has been calculated from Lattice QCD.

However, $V_{LS}^{(+)}$, $Vc^{(-)}$, $V_T^{(-)}$, $V_{LS}^{(-)}$ is not yet.

derivation: S.Aoki, T.Hatsuda, N.Ishii, PTP123(2010)89

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^{+}\phi(\vec{r};E)$$

$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^{-}\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

This work

In this work we have extended HAL metod to LS force.

LS force with parity minus sector in NN system is calculated.

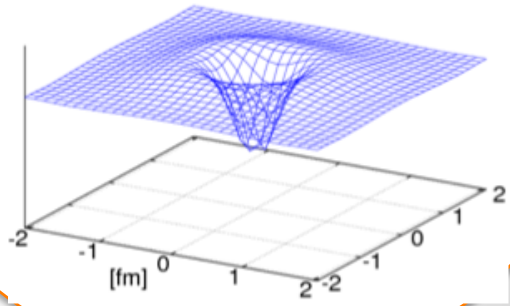
ex) $^1S_0$ (L=0 S=0) case

$$S_{12} = \begin{cases} \neq 0 & : S=1 \\ =0 & : S=0 \end{cases} \qquad \left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E) = \left[Vc^{(+)}(r) + V_T^{(+)}(r)S_{12} + V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]\phi(\vec{r};E)$$

Effective Scrodinger Equation:

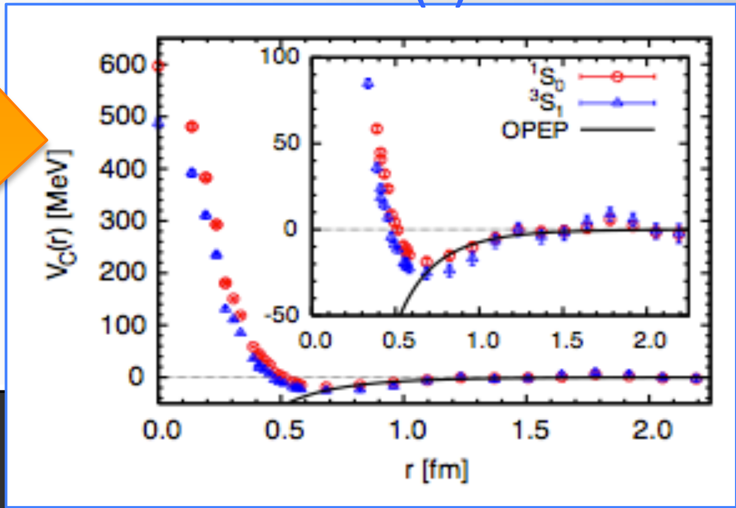
$$\left(\frac{\Delta}{m_N}+E\right)\phi(\vec{r};E) = Vc(\vec{r})\,\phi(\vec{r};E)$$

from Lattice QCD



$$Vc(r) = \left(\frac{1}{m_N}\frac{\Delta\phi(r;E)}{\phi(r;E)}+E\right)$$

Solve about Vc(r)



Ishii,Aoki,Hatsuda
Phys. Rev. Lett. 99, 022001 (2007)

Extension for LS force with S=1 system.

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi^{(-)}(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\phi^{(-)}(\vec{r};E)+\mathcal{O}(\nabla^2)$$

calculation of Vc , VT and VLS

parity minus , S=1 system:

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi^{(-)}(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\phi^{(-)}(\vec{r};E)+\mathcal{O}(\nabla^2)$$

$\phi^{(-)}(\vec{r})=\psi^{(1)}(\vec{r}),\ \psi^{(2)}(\vec{r}),\ \psi^{(3)}(\vec{r})$: linear independent each other

$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{(1)}(\vec{r})=Vc^{(-)}\ \psi^{(1)}(\vec{r})+V_T^{(-)}S_{12}\ \psi^{(1)}(\vec{r})+V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{(1)}(\vec{r})$$

$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{(2)}(\vec{r})=Vc^{(-)}\ \psi^{(2)}(\vec{r})+V_T^{(-)}S_{12}\ \psi^{(2)}(\vec{r})+V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{(2)}(\vec{r})$$

$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{(3)}(\vec{r})=Vc^{(-)}\ \psi^{(3)}(\vec{r})+V_T^{(-)}S_{12}\ \psi^{(3)}(\vec{r})+V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{(3)}$$

NBS wave function $\psi^{(1)}(\vec{r}),\ \psi^{(2)}(\vec{r}),\ \psi^{(3)}(\vec{r})$ are calculated from Lattice QCD.

calculation of Vc , VT and VLS

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi^{(-)}(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\phi^{(-)}(\vec{r};E)+\mathcal{O}(\nabla^2)$$

Solve about Vc, VT, VLS

$$\begin{pmatrix}\left(\nabla^2/2m\right)\psi^{(1)}(\vec{r})\\ \left(\nabla^2/2m\right)\psi^{(2)}(r)\\ \left(\nabla^2/2m\right)\psi^{(3)}(\vec{r})\end{pmatrix}=\begin{pmatrix}\psi^{(1)}(\vec{r}) & S_{12}\ \psi^{(1)}(\vec{r}) & \vec{L}\cdot\vec{S}\ \psi^{(1)}(\vec{r})\\ \psi^{(2)}(\vec{r}), & S_{12}\ \psi^{(2)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(2)}(\vec{r}),\\ \psi^{(3)}(\vec{r}), & S_{12}\ \psi^{(3)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(3)}(\vec{r}),\end{pmatrix}\begin{pmatrix}Vc^{(-)}(r)-E\\ V_T^{(-)}(r)\\ V_{LS}^{(-)}(r)\end{pmatrix}$$

Vc, VT, VLS can be obtained from three wave function

$\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$

calculation of Vc , VT and VLS

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi^{(-)}(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\phi^{(-)}(\vec{r};E)+\mathcal{O}(\nabla^2)$$

Solve about Vc, VT, VLS

$$\begin{pmatrix} \psi^{(1)}(\vec{r}) & S_{12}\ \psi^{(1)}(\vec{r}) & \vec{L}\cdot\vec{S}\ \psi^{(1)}(\vec{r}) \\ \psi^{(2)}(\vec{r}), & S_{12}\ \psi^{(2)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(2)}(\vec{r}), \\ \psi^{(3)}(\vec{r}), & S_{12}\ \psi^{(3)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(3)}(\vec{r}), \end{pmatrix}^{-1} \begin{pmatrix} \left(\nabla^2/2m\right)\psi^{(1)}(\vec{r}) \\ \left(\nabla^2/2m\right)\psi^{(2)}(r) \\ \left(\nabla^2/2m\right)\psi^{(3)}(\vec{r}) \end{pmatrix} = \begin{pmatrix} Vc^{(-)}(r)-E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

Vc, VT, VLS can be obtained from three wave function

$\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$

Here, $\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$ must be ....

- Parity Minus
- linear independent each other
- Orbit alangular momentum $L \neq 0$

$$\begin{pmatrix} \psi^{(1)}(\vec{r}) & S_{12}\ \psi^{(1)}(\vec{r}) & \vec{L}\cdot\vec{S}\ \psi^{(1)}(\vec{r}) \\ \psi^{(2)}(\vec{r}), & S_{12}\ \psi^{(2)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(2)}(\vec{r}), \\ \psi^{(3)}(\vec{r}), & S_{12}\ \psi^{(3)}(\vec{r}), & \vec{L}\cdot\vec{S}\ \psi^{(3)}(\vec{r}), \end{pmatrix}^{-1} \begin{pmatrix} \left(\nabla^2/2m\right)\psi^{(1)}(\vec{r}) \\ \left(\nabla^2/2m\right)\psi^{(2)}(r) \\ \left(\nabla^2/2m\right)\psi^{(3)}(\vec{r}) \end{pmatrix} = \begin{pmatrix} Vc^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

Here, $\psi^{(1)}(\vec{r}), \psi^{(2)}(\vec{r}), \psi^{(3)}(\vec{r})$ must be ....

- Parity Minus
- linear independent each other
- Orbit alangular momentum $L \neq 0$

In this work, we calculated : ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$

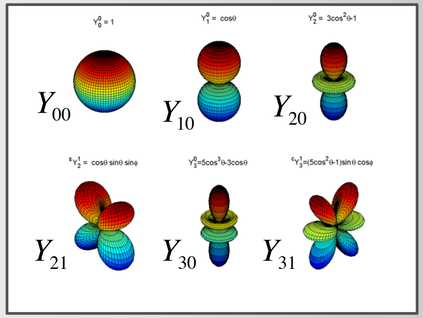
How to construct them ?

How to construct Parity even, with $L \neq 0$

In previous works...

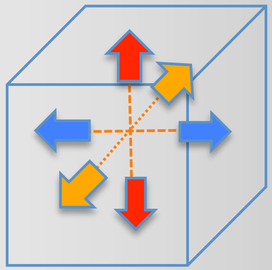
two nucleon in rest frame (wall sorce)

➡
- Parity Plus
- orbital angular momentum L=0



In this work :

- Impose the momentum to the nucleon (quark)
- construct the L=1 (on the cubic group : T1) state with combining 6-different direction of momentum.



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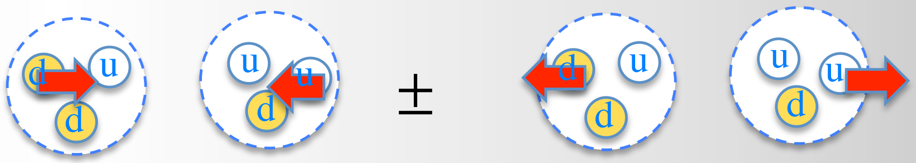
How to construct Parity even, with $L \neq 0$

Parity :

parity plus $\phi(\vec{r})^{P=+} = \phi(\vec{r};+\vec{k}) + \phi(\vec{r};-\vec{k})$

parity minus $\phi(\vec{r})^{P=-} = \phi(\vec{r};+\vec{k}) - \phi(\vec{r};-\vec{k})$



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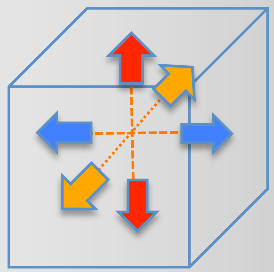

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How to construct Parity even, with $L \neq 0$

Angular momentum:

Projection operator
based on cubic group:
because rotational sym
is broken O(3) ➔ cubic group

$$\frac{2l+1}{4\pi}\int d\theta\, d\phi \;\; D^{(l=L)}(\theta,\phi)^{*} \;\psi(R(\theta,\phi)\;\vec{x})$$

$$\frac{d_{\Gamma}}{24}\sum_{i=0}^{24} \; D_{\mu,\nu}^{(\Gamma)}(g_i)^{*}\psi_{\nu}(R(g_i)\;\vec{x})$$

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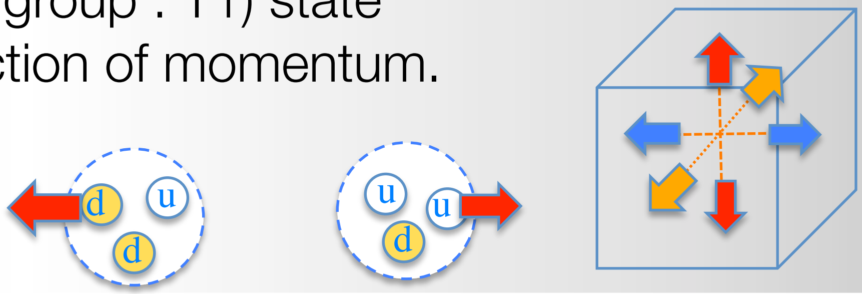


In this work, we calculated : ${}^3P_0$, ${}^3P_1$ and ${}^3P_2$

Numerical Results

Set Up

Japan Lattice Data Grid

Nf=2

Iwasaki gauge + clover fermion

beta=0.195

kappa=0.1375

a=0.1555

1/a = 1271 MeV

L^3 x T = 16 ^3 x 32

mN=2165 MeV

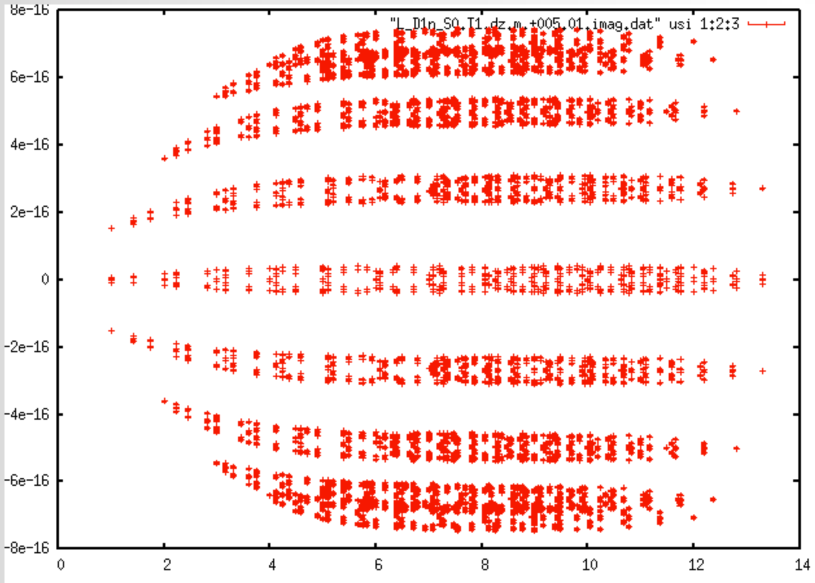
mpi=1136 MeV

heavy quark mass

(calculation cost $\propto$ 1/mq)

spin-singlet (S=0)
parity minus

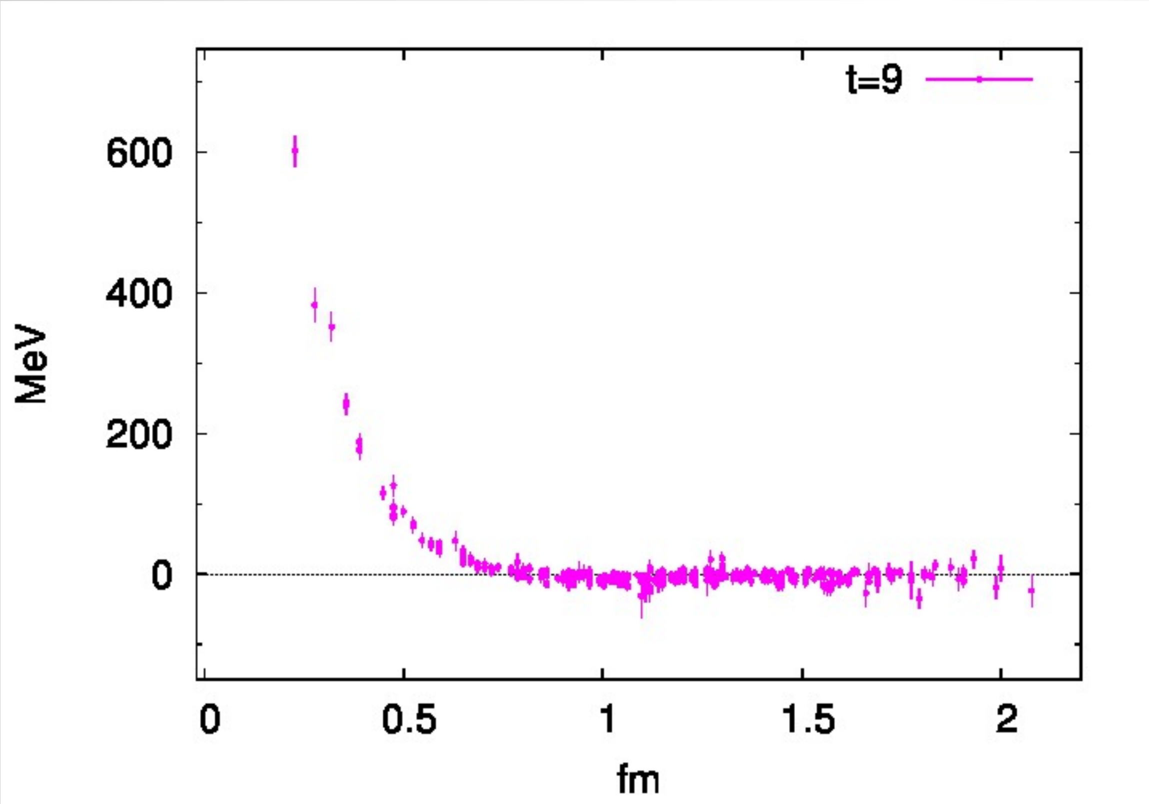
1P1 NBS wave function for S=0



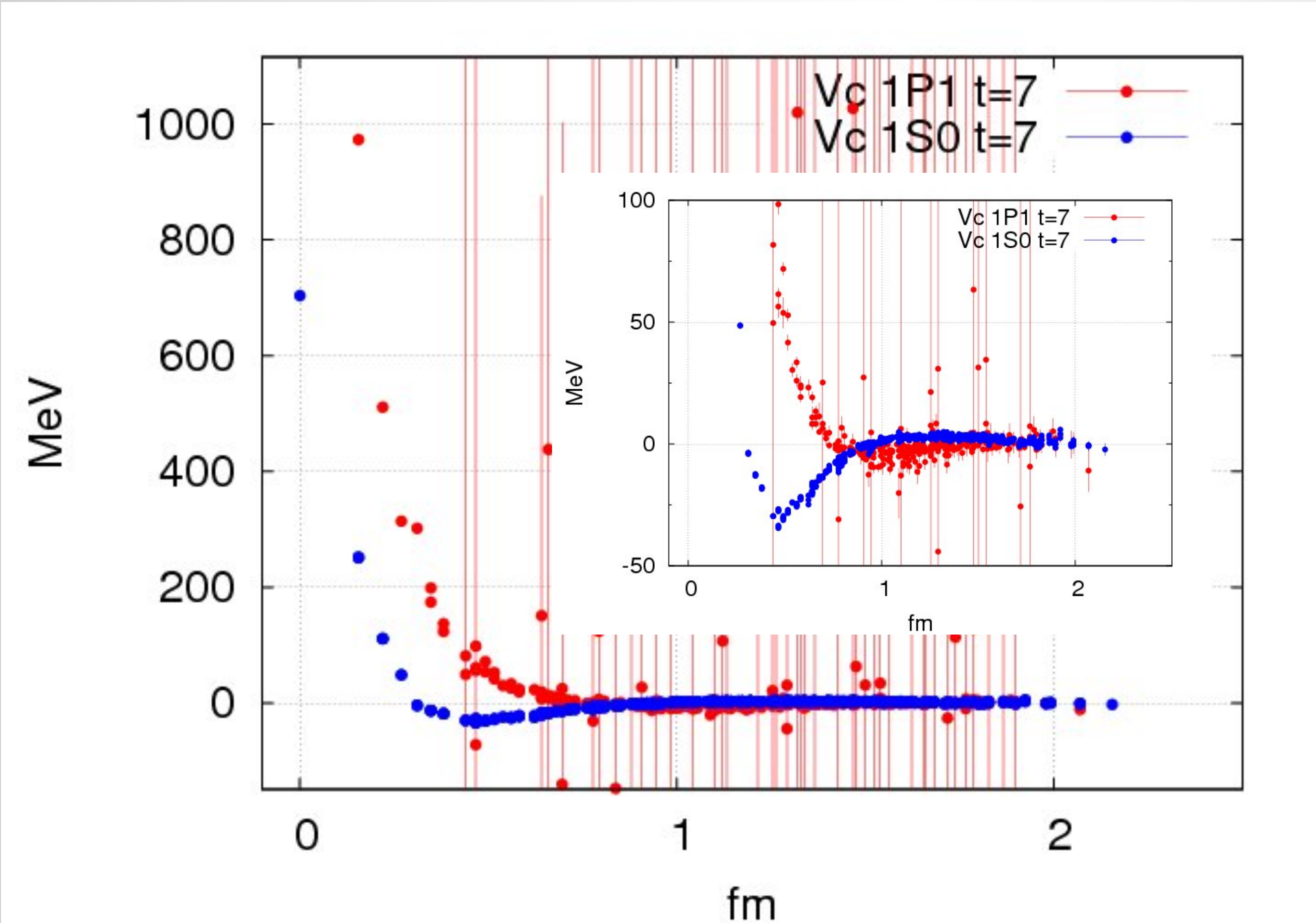
$$\left(\frac{\nabla^2}{m_N}+E\right)\phi^{(-)}(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]\phi^{(-)}(\vec{r};E)$$

$$\Rightarrow Vc_{S=0}^{(-)}(r)=\left(\frac{1}{m_N}\frac{\Delta\phi(r;E)}{\phi(r;E)}+E\right)$$

1P1 (S=0) central potential for parity odd

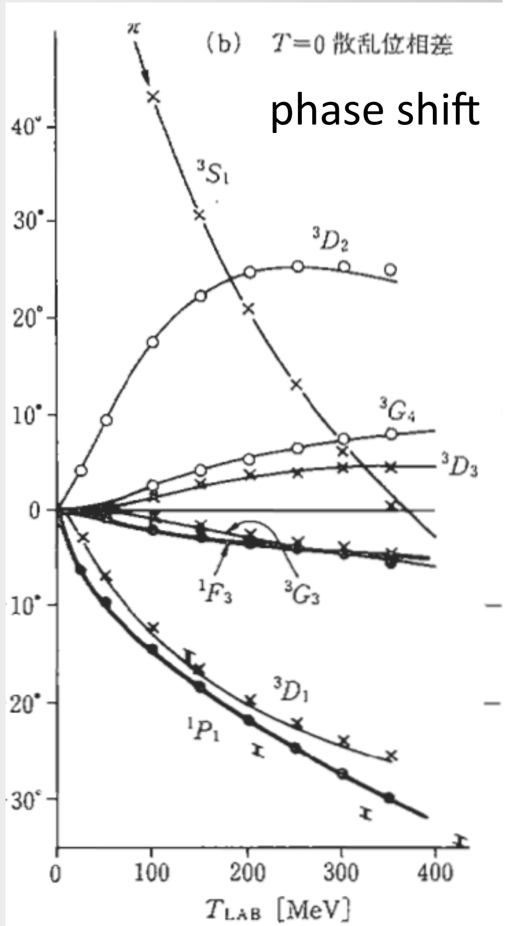


(so3 improved ver, not time-dependent)



Parity Plus

Parity Minus



1P1 cos type source ･･ $E = (2\pi / L)^2 / m_N$

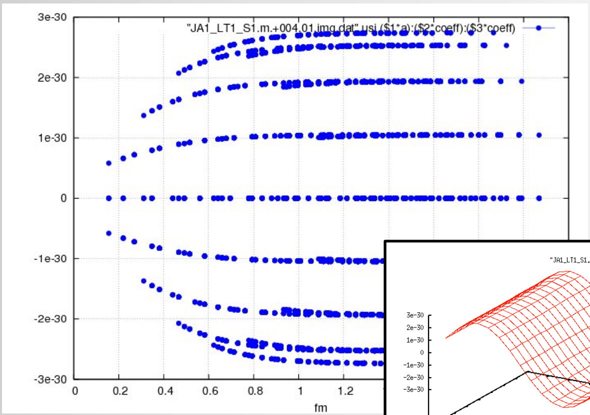
1S0 wall source ･･ shift

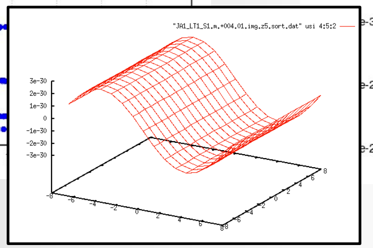
spin-triplet (S=1)
parity minus

results : NBS wave function for S=1

3P0

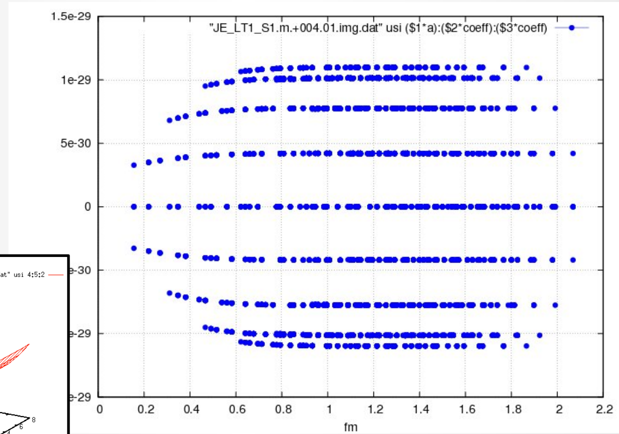
J=1 (A1) L=1 (T1) (imaginary part)





3P2

J=2 (E) L=1 (T1) (imaginary part)

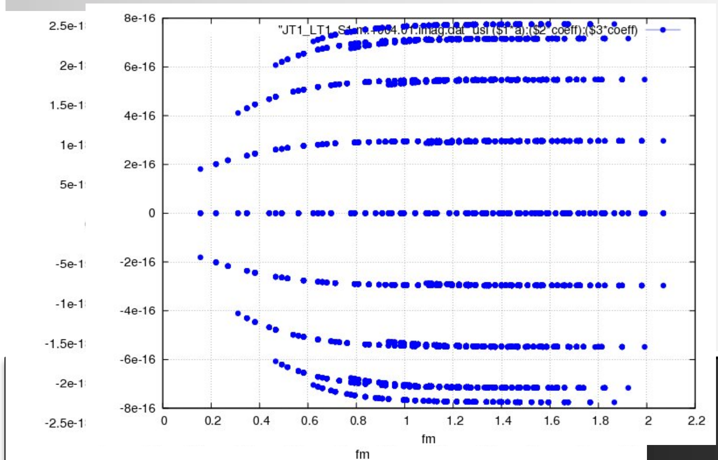


a=0.1555
1/a = 1271 MeV
L^3 x T = 16 ^3 x 32
mN=2165 MeV

$$\psi(\vec{r}) = R(r) Y_{1m}(\theta,\phi)$$

3P1

J=1 (T1) L=1 (T1)



We have obtained

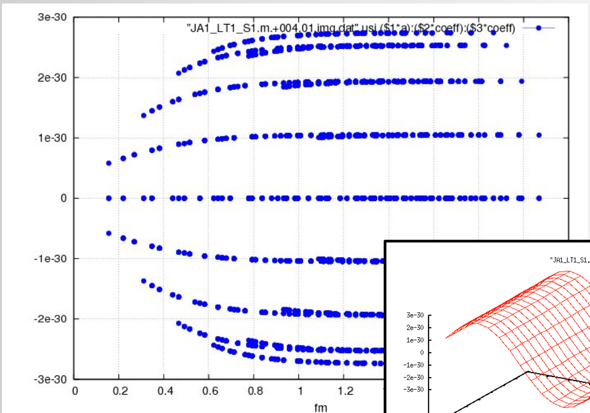
3P0, 3P1, 3P2, 3F2, 3F3

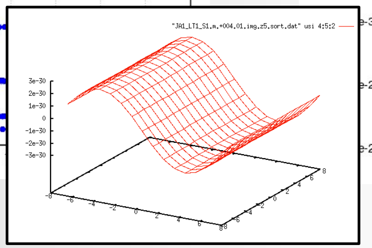
NBS wave functions

results : NBS wave function for S=1

3P0

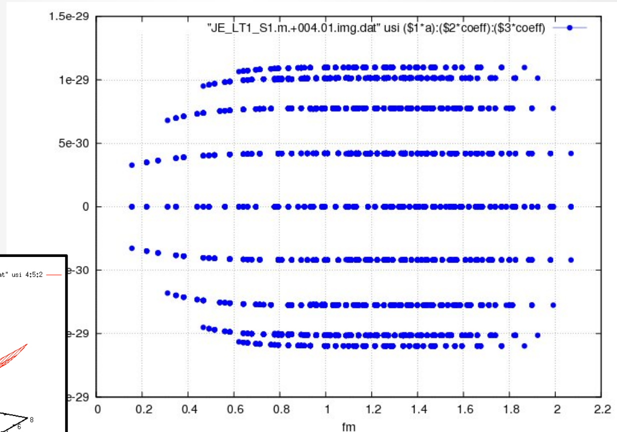
J=1 (A1) L=1 (T1) (imaginary part)





3P2

J=2 (E) L=1 (T1) (imaginary part)



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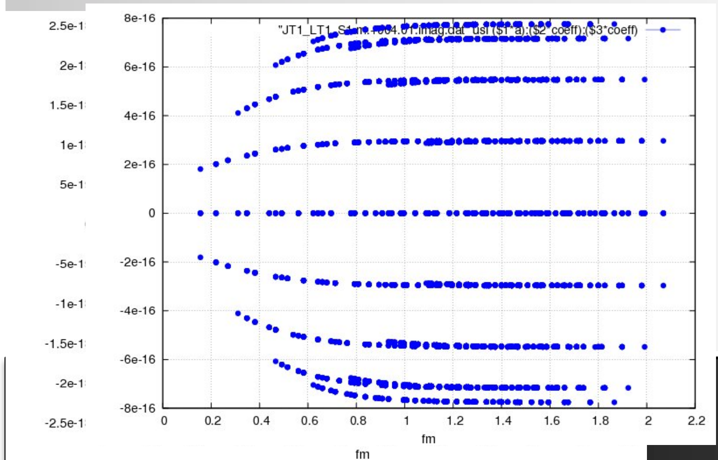
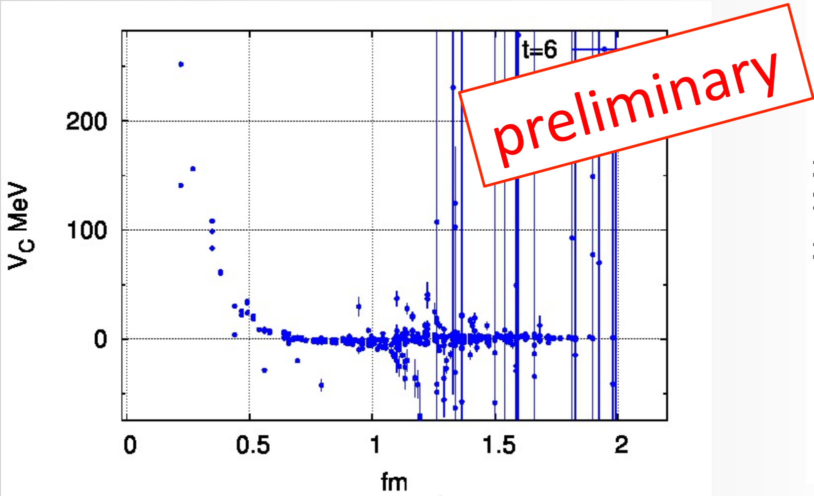


$$\psi(\vec{r}) = R(r) Y_{1m}(\theta,\phi)$$

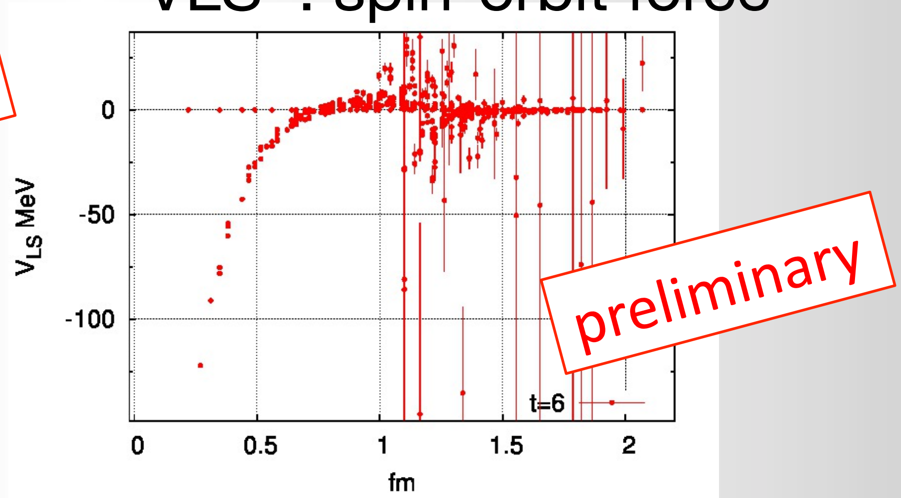
$$\begin{pmatrix} (\nabla^2/2m)\ {}^3P_0(\vec{r}) \\ (\nabla^2/2m)\ {}^3P_1(r) \\ (\nabla^2/2m)\ {}^3P_2(\vec{r}) \end{pmatrix} = \begin{pmatrix} {}^3P_0(\vec{r}) & S_{12}\ {}^3P_0(\vec{r}) & \vec{L}\cdot\vec{S}\ {}^3P_0(\vec{r}) \\ {}^3P_1(\vec{r}), & S_{12}\ {}^3P_1(\vec{r}), & \vec{L}\cdot\vec{S}\ {}^3P_1(\vec{r}), \\ {}^3P_2(\vec{r}), & S_{12}\ {}^3P_2(\vec{r}), & \vec{L}\cdot\vec{S}\ {}^3P_2(\vec{r}), \end{pmatrix} \begin{pmatrix} Vc^{(-)}(r) - E \\ V_T^{(-)}(r) \\ V_{LS}^{(-)}(r) \end{pmatrix}$$

Numerical results of potentials S=1 parity minus

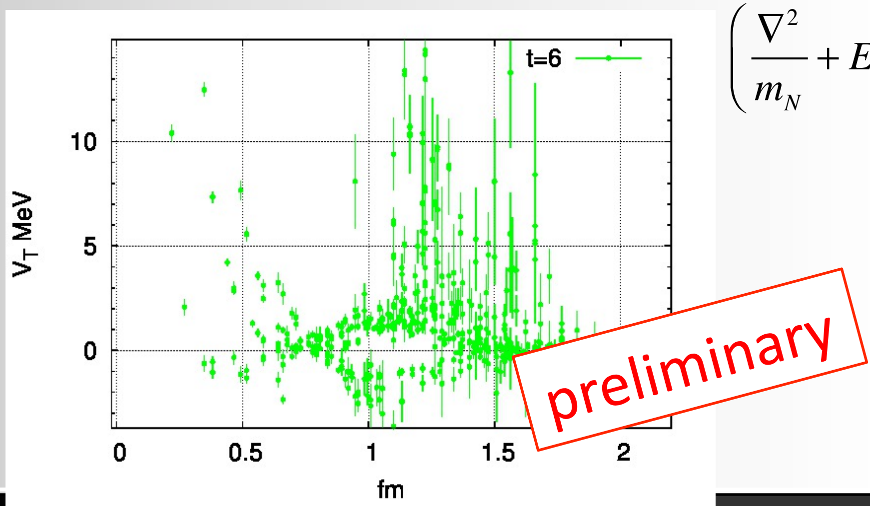
Vc : center force



VLS : spin-orbit force



VT : tensor force



$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^{+}\phi(\vec{r};E)$$

$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^{-}\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

Numerical results of potentials S=1 parity minus

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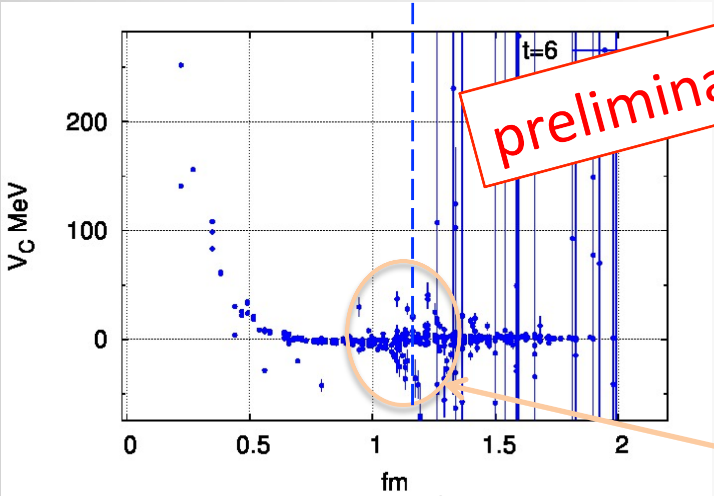
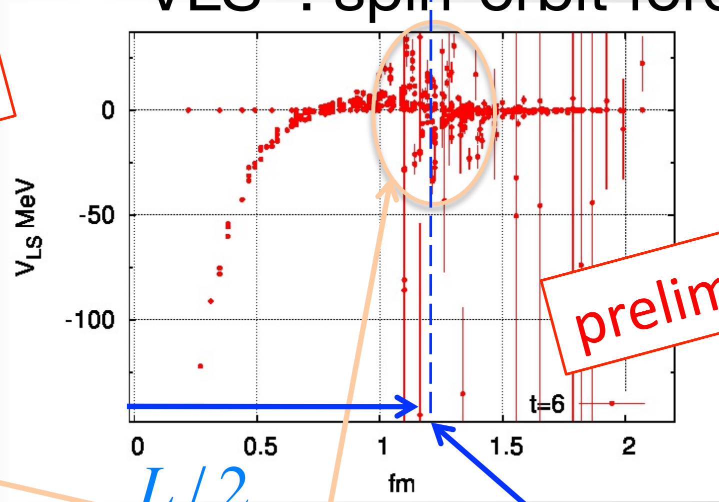
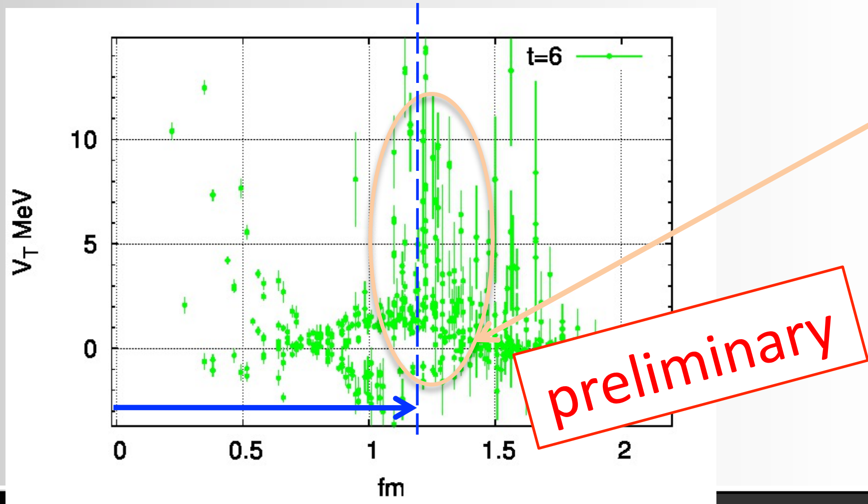


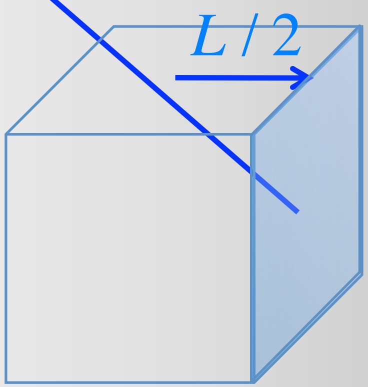
VLS : spin-orbit force



VT : tensor force



there are strong fluctuation.



← caused by boundary.

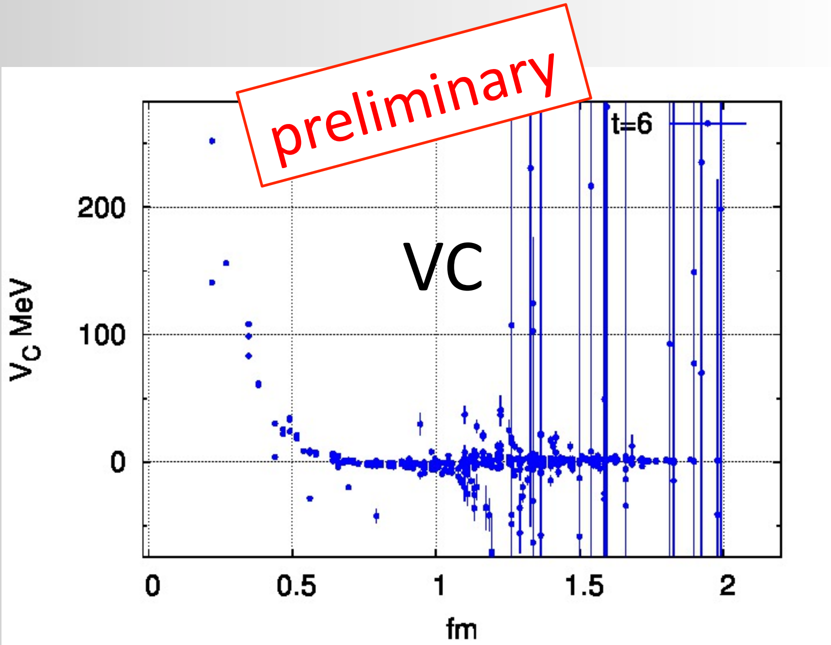
NBS wave has strong finite effect

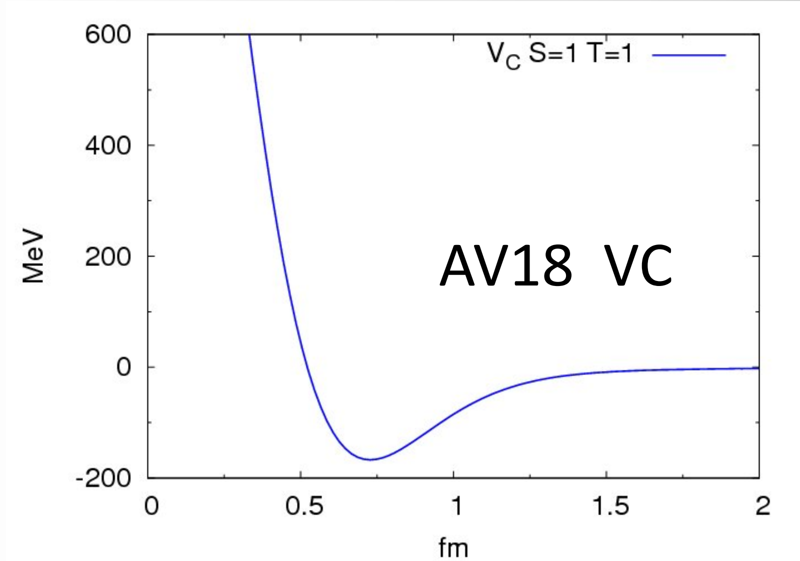
comparison with phenomenological potential (AV18)

central force has repulsive core
(qualitatively same with one with AV18)

Phys. Rev. C **51**, 38 (1995)

R. B. Wiringa, V. G. J. Stoks and R.Schiavilla,





$$v^{R}_{ST}(NN) = v^{c}_{ST,NN}(r) + v^{l2}_{ST,NN}(r)L^2 + v^{t}_{ST,NN}(r)S_{12} + v^{ls}_{ST,NN}(r)\mathbf{L}\cdot\mathbf{S} + v^{ls2}_{ST,NN}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

comparison with phenomenological potential (AV18)

tensor force is weak repulsive force
(qualitatively same with one with AV18)

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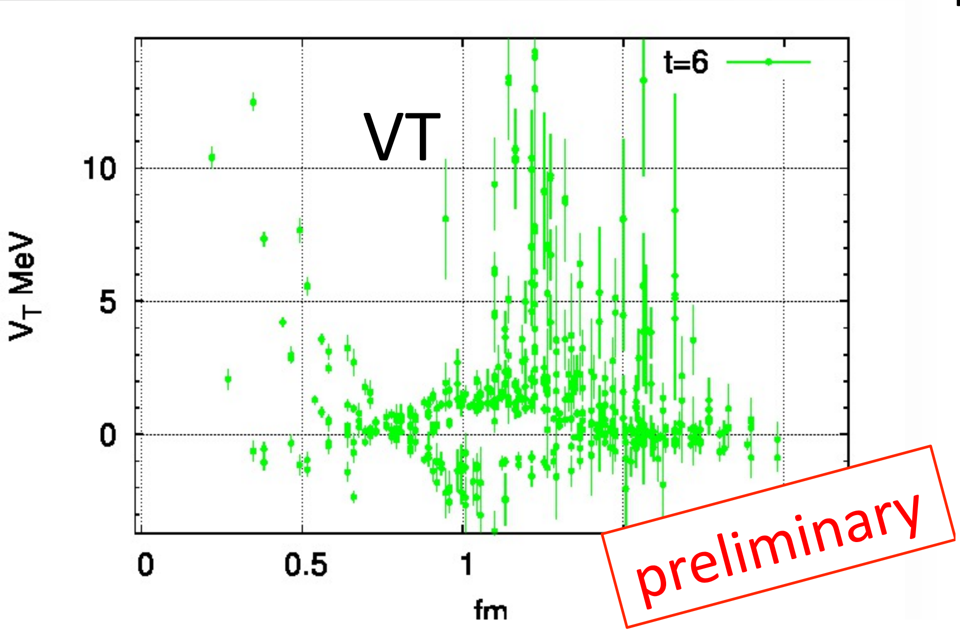
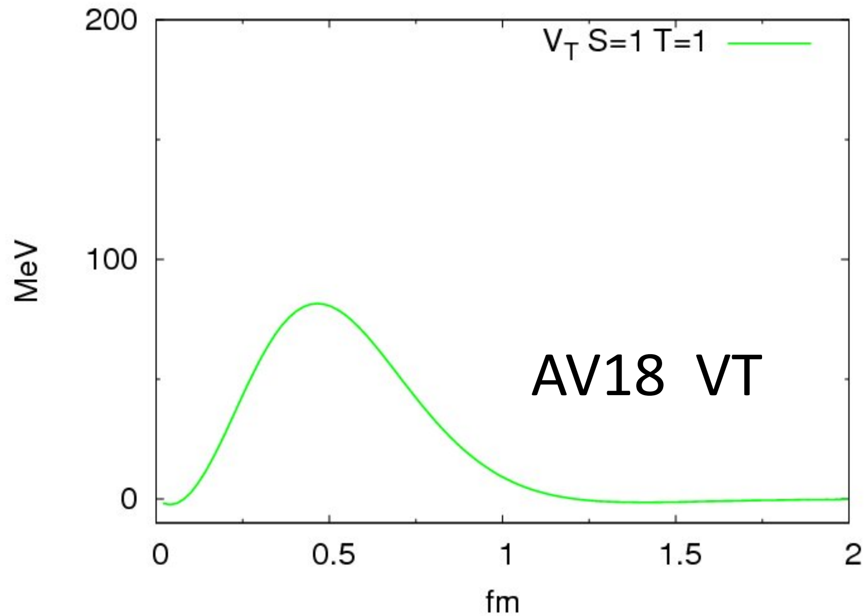
R. B. Wiringa, V. G. J. Stoks and R.Schiavilla,





$$v^R_{ST}(NN) = v^c_{ST,NN}(r) + v^{l2}_{ST,NN}(r)L^2 + v^t_{ST,NN}(r)S_{12} + v^{ls}_{ST,NN}(r)\mathbf{L}\cdot\mathbf{S} + v^{ls2}_{ST,NN}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

comparison with phenomenological potential (AV18)

LS force is strong attractive force
(qualitatively same with one with AV18)

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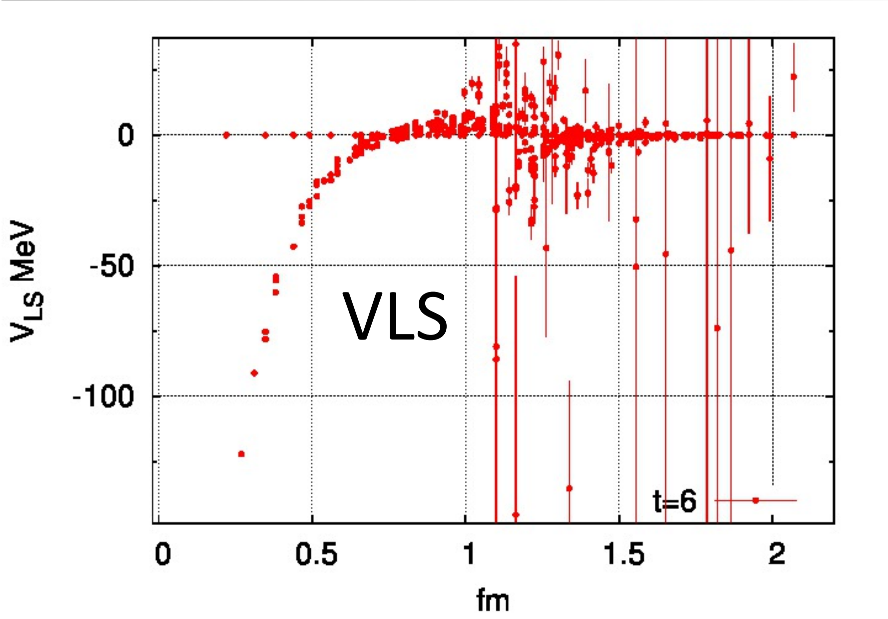
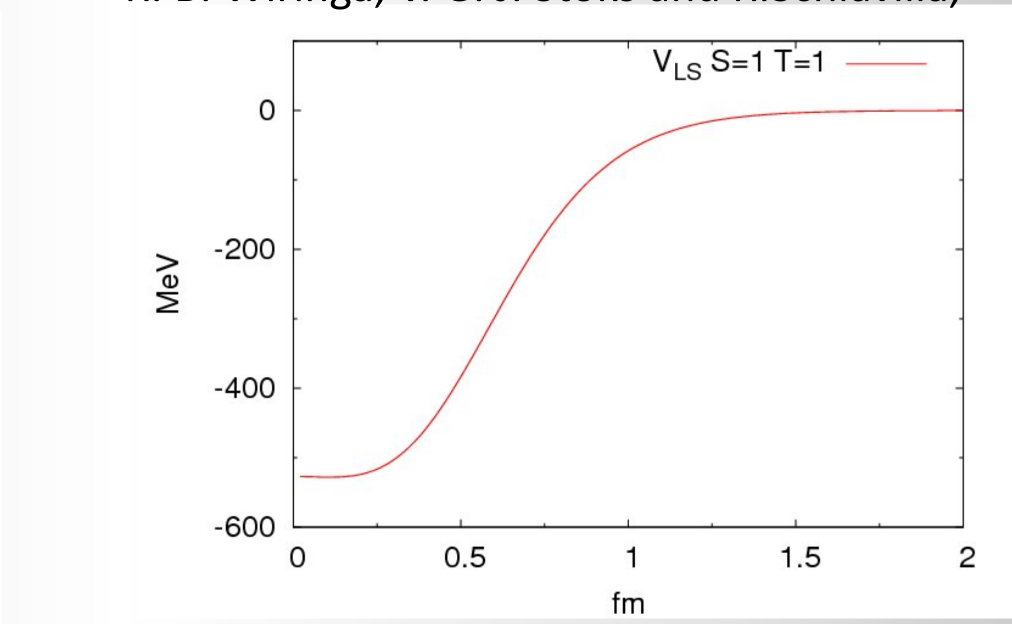
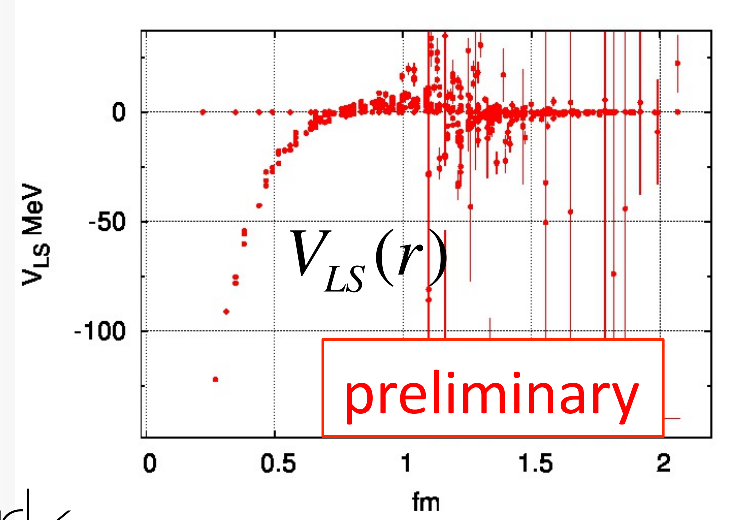




$$v^R_{ST}(NN) = v^c_{ST,NN}(r) + v^{l2}_{ST,NN}(r)L^2 + v^t_{ST,NN}(r)S_{12} + \boxed{v^{ls}_{ST,NN}(r)}\mathbf{L}\cdot\mathbf{S} + v^{ls2}_{ST,NN}(r)(\mathbf{L}\cdot\mathbf{S})^2$$

Summary

We are currently preparing the calculation of potentials including LS forces for parity minus sector.



Future work

- parity plus LS force

- physical point (mpi = 135 MeV)
- applied for YN and YY interaction

will be performed in K-Computer (2012).