高エネルギー天体現象のための現実的核力に基づく核物質状態方程式

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1. Introduction

The aim of this study is

To construct *a new nuclear Equation of State (EOS)*

for supernova (SN) simulations

based on the realistic nuclear force.

The nuclear EOS plays an important role for astrophysical studies.

1. Lattimer-Swesty EOS : *The compressible liquid drop model* (NPA 535 (1991) 331)
2. Shen EOS : *The relativistic mean field theory* (NPA 637 (1998) 435)

- K. Nakazato EOS : (PRD 77(2008) 103006)
- G. Shen EOS : (PRC 83 (2010) 015806)
- C. Ishizuka EOS : (J. Phys. G 35(2008)085201)
- M. Hempel EOS : (NPA 837 (2010) 210)
- S. Furusawa EOS : (APJ 738 (2011) 178)

These EOSs are based on **phenomenological models** for uniform matter.

*There is **no** nuclear EOS based on **the microscopic many-body theory**.*

We aim at **a new EOS for SN** with the variational method.

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

EOS constructed with *the cluster variational method*

CLEAR

Non-uniform Nuclear Matter

We are here.

EOS constructed with

the Thomas-Fermi (TF) calculation

CLEAR

Completion of a Nuclear EOS table for SN simulations

Density ρ : $10^{5.1}$ ⦿ ρ_m ⦿ $10^{16.0}$g/cm^3 110 point

Temperature T : 0 ⦿ T ⦿ 400 MeV 92 point

Proton fraction x : 0 ⦿ x ⦿ 0.65 66 point

2. EOS for Uniform Nuclear Matter

The Nuclear Hamiltonian

$$H = H_2 + H_3$$

Two-body Hamiltonian

$$H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{i<j}^{N} V_{ij}$$

the AV18 two-body nuclear potential

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k}^{N} V_{ijk}$$

the UIX three-body nuclear potential

We assume the Jastrow wave function.

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_F$$

Φ_F: The Fermi-gas wave function at zero temperature

$P_{ts}{}^{\mu}$: Spin-isospin projection operators

f_{ij} : Correlation function

$$f_{ij} = \sum_{t=0}^{1} \sum_{\mu} \sum_{s=0}^{1} \left[f_{Cts}^{\mu}(r_{ij}) + s f_{Tt}^{\mu}(r_{ij}) S_{Tij} + s f_{SOt}^{\mu}(r_{ij}) (\boldsymbol{L}_{ij} \cdot \boldsymbol{s}) \right] P_{tsij}^{\mu}$$

Central **Tensor** **Spin-orbit**

Two-Body Energy

E_2/N is the expectation value of H_2 with the Jastrow wave function in *the two-body cluster approximation*.

$$\frac{E_2}{N}(\rho, x) = \frac{< H_2 >_2}{N}$$

ρ : Total nucleon number density

ρ_p : Proton number density $\quad x = \rho_p/\rho$: Proton fraction

E_2/N is minimized with respect to $f_{Cts}{}^{\mu}(r)$, $f_{Tt}{}^{\mu}(r)$ and $f_{SOt}{}^{\mu}(r)$ with the following two constraints.

1. Extended Mayer's condition

$$\rho \int \left[F^{\mu}_{ts}(r) - F^{\mu}_{Fts}(r)\right] d\mathbf{r} = 0$$

$F_{ts}{}^{\mu}(r)$: Radial distribution functions

$F_{Fts}{}^{\mu}(r)$: $F_{ts}{}^{\mu}(r)$ for the degenerate Fermi gas

2. Healing distance condition

Healing distance

$$r_h = a_h r_0$$

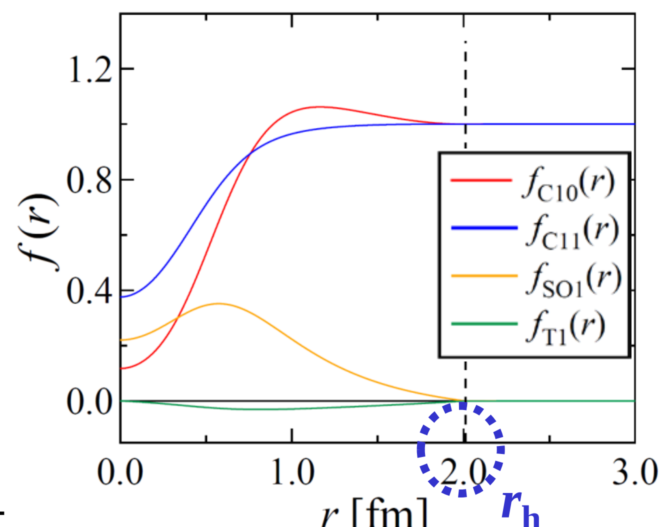
a_h : adjustable parameter

Mean distance between nucleons

$$r_0 = \left(\frac{3}{4\pi\rho}\right)^{1/3}$$

a_h is determined so that E_2/N reproduces the results by APR(Akmal, Pandharipande and Ravenhall)

APR : PRC58(1998)1804

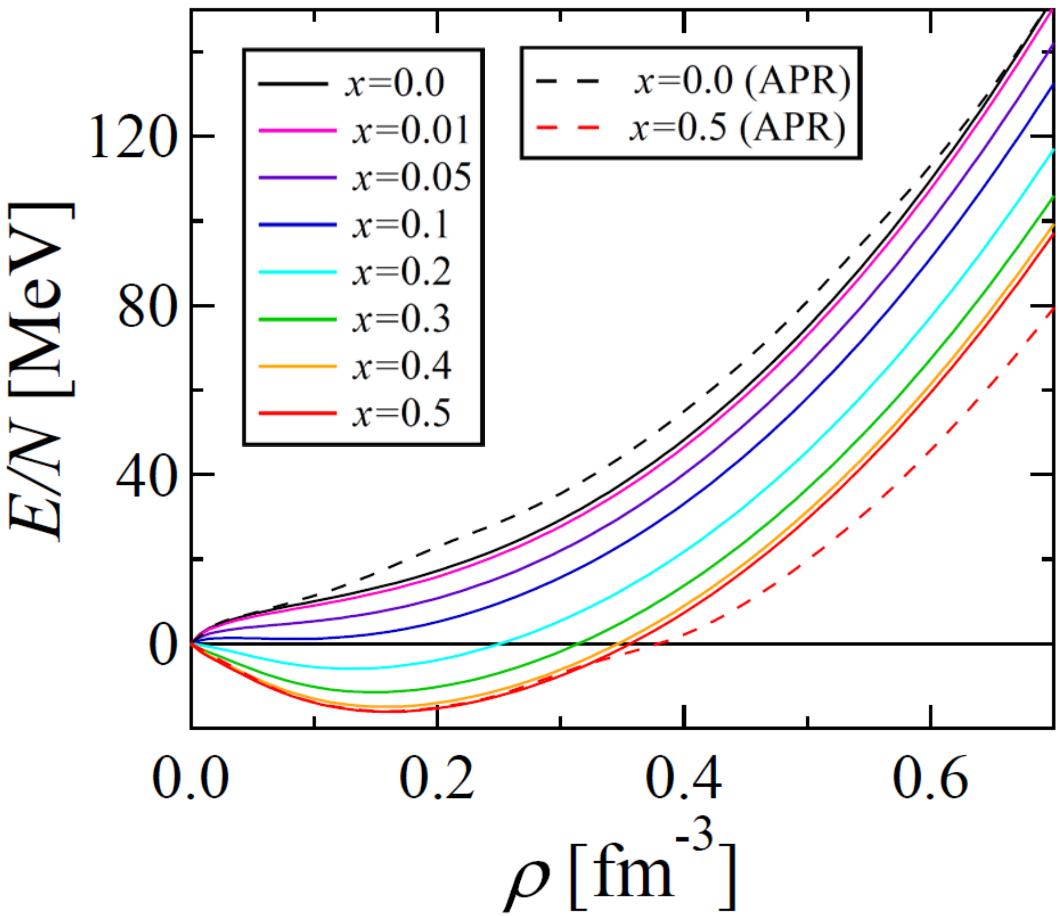


Total Energy per Nucleon at Zero Temperature

$$\frac{E}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$

Three Body Energy

Constructed with the expectation value of H_3 with the Fermi gas wave function

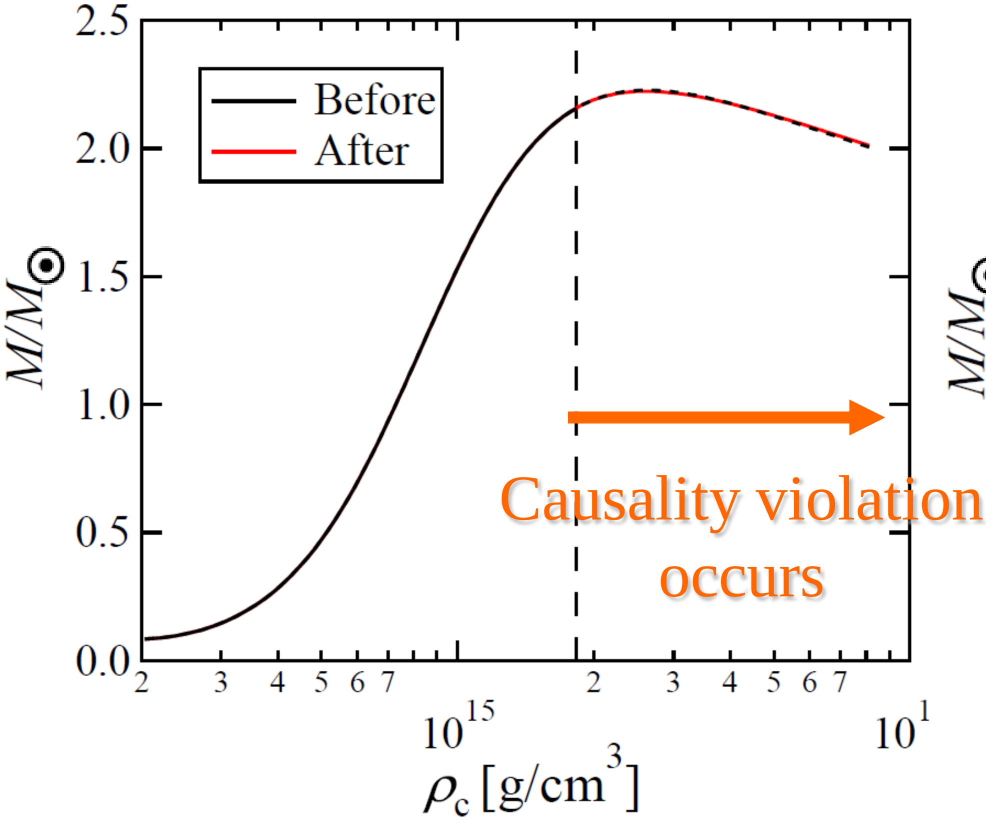


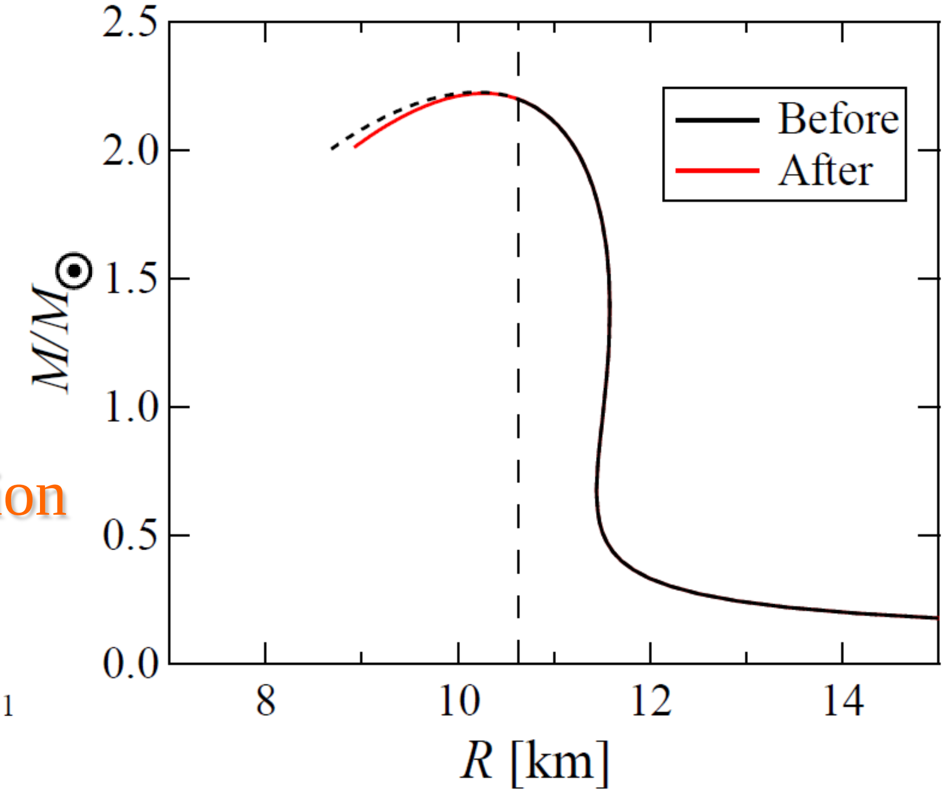
Parameters of E_3/N are determined so that

TF calculation for atomic nuclei reproduces the gross feature of the experimental data.

ρ_0[fm^{-3}]	E_0[MeV]	K[MeV]	E_{sym}[MeV]
0.16	–16.1	240	30.0

Application to Neutron Star





Effect of causality violation is small.

Free Energy at Finite Temperatures

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11) (A. Mukherjee et al., PRC 75(2007) 035802)

Free Energy

$$\frac{F}{N} = \frac{E_0}{N} - T\frac{S_0}{N}$$

$\frac{E_0}{N}$: Approximate Internal Energy

$\frac{S_0}{N}$: Approximate Entropy

S_0/N is expressed with the averaged occupation probabilities $n_i(k)$

Approximate Internal Energy

$$\frac{E_0}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$

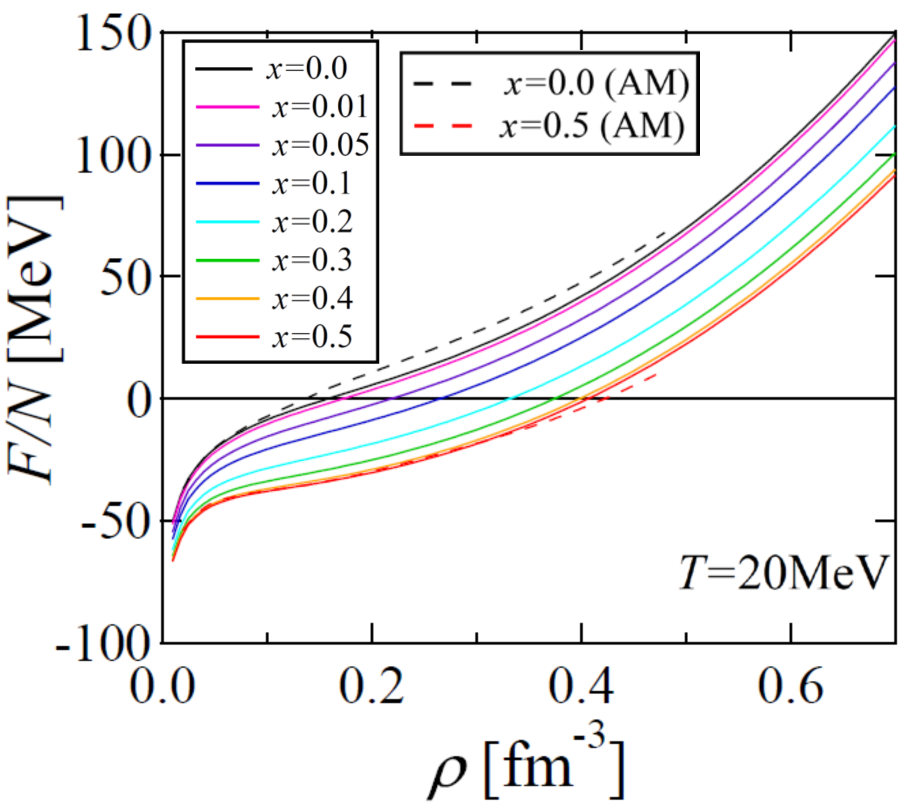
chosen to be the same as at 0 MeV

E_2/N : Expectation value of H_2 with the Jastrow-type wave function at finite temperature.

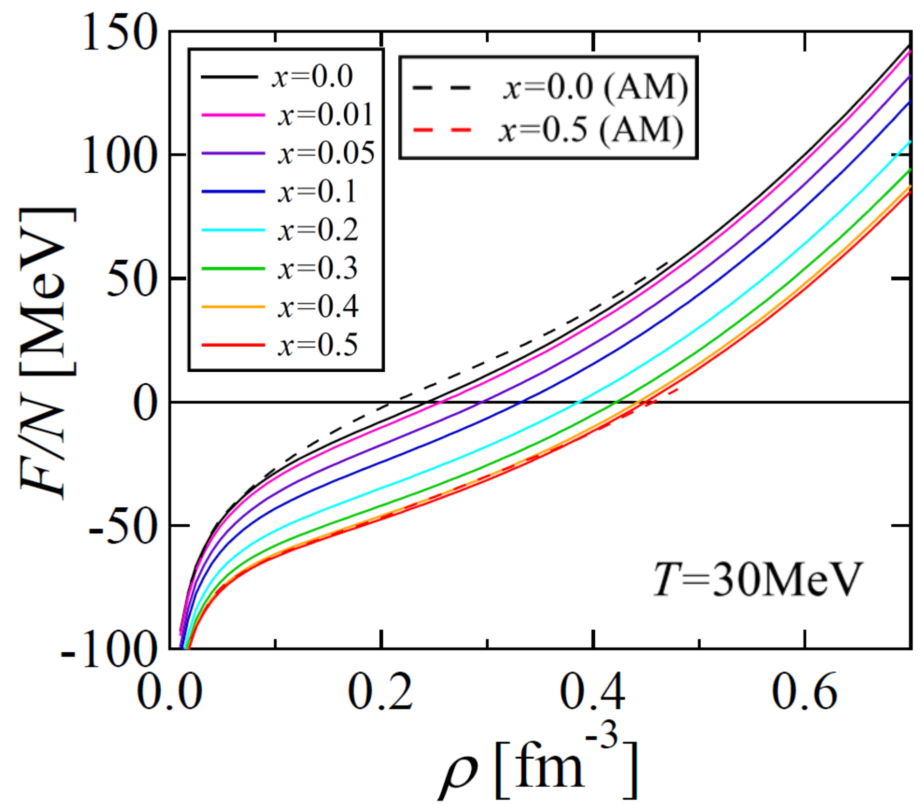
$$\Psi(T) = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right] \Phi_F(n_p(k), n_n(k))$$

Φ_F: The Fermi-gas wave function expressed with $n_i(k)$

Free Energy per Nucleon at Finite Temperatures



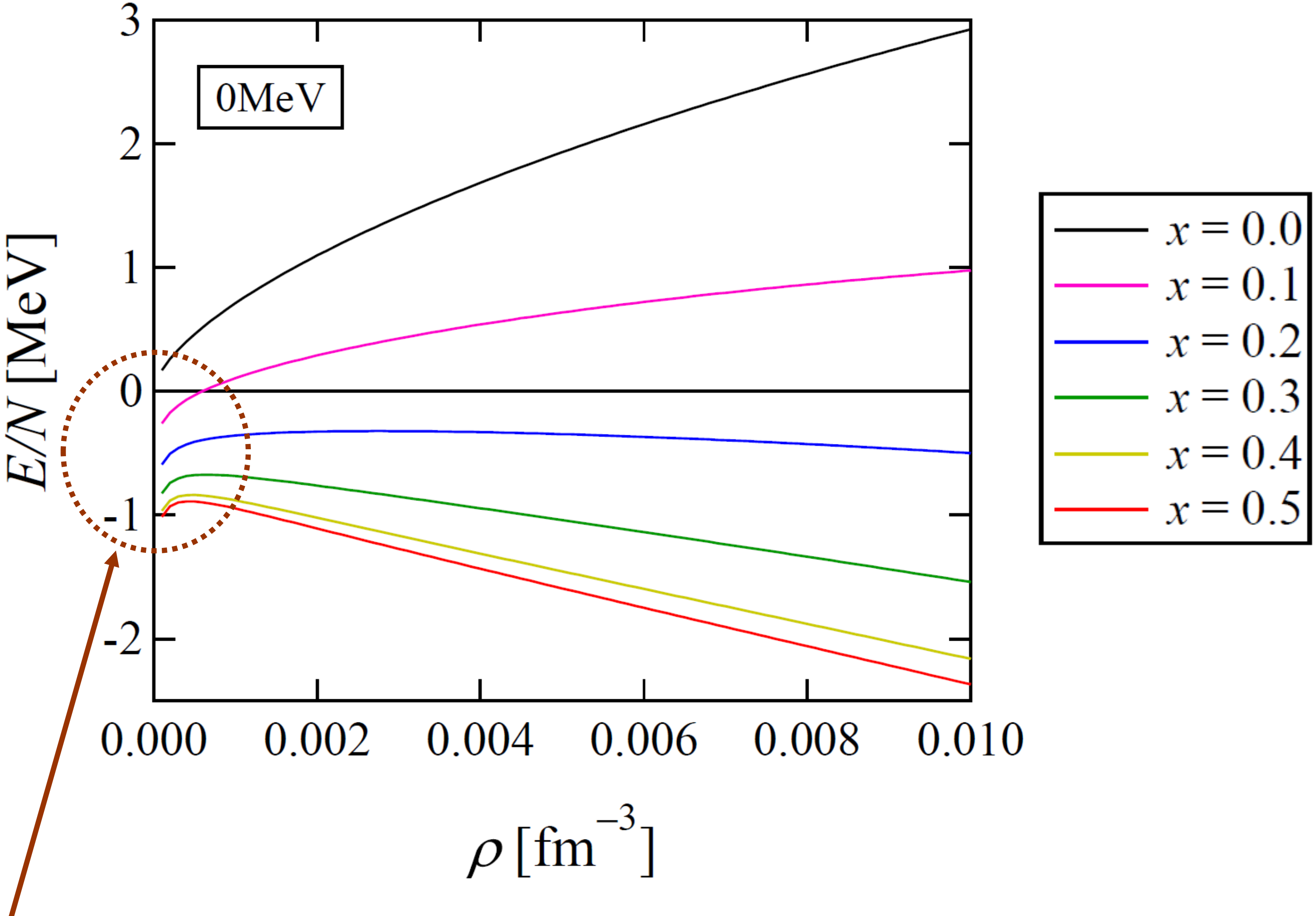
Free energy per nucleon at T=20MeV



Free energy per nucleon at T=30MeV

AM: A. Mukherjee, PRC 79(2009) 045811.

Uniform Nuclear Matter at Low Density



$E/N \rightarrow$ 0MeV is not reproduced in the limit of $\rho \rightarrow 0$ fm^{-3}

Because of the deuteron clustering

Improvement of Cluster Variational Method

Healing distance condition

$$r_h = a_h r_0$$

Mean distance between nucleons

$$r_0 = \left(\frac{3}{4\pi\rho}\right)^{1/3}$$

a_h : adjustable parameter

New healing distance condition

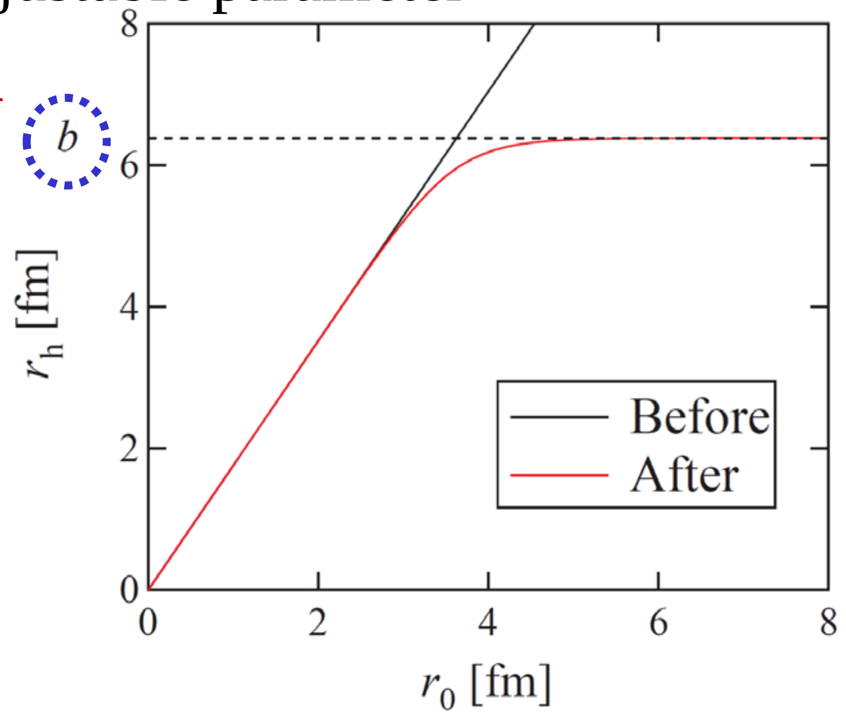
$$r_h = \frac{a r_0}{[1 + (a r_0 / b)^c]^{\frac{1}{c}}}$$

$r_h \rightarrow a_h r_0$ **(high density)**

$r_h \rightarrow b$ **(low density)**

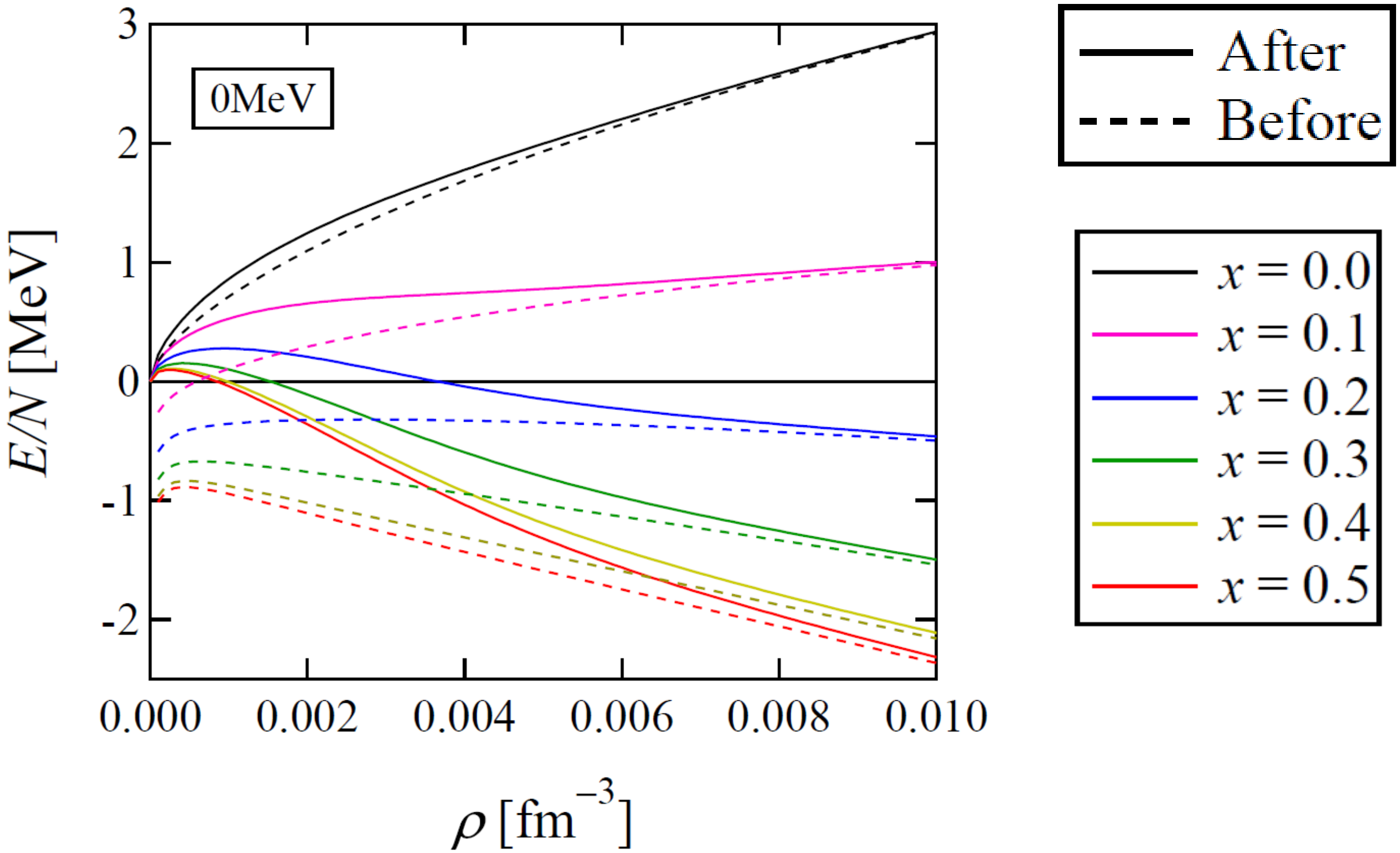
a, b, c : adjustable parameters

$a = 1.76$ $b = 6.38$ fm $c = 10$



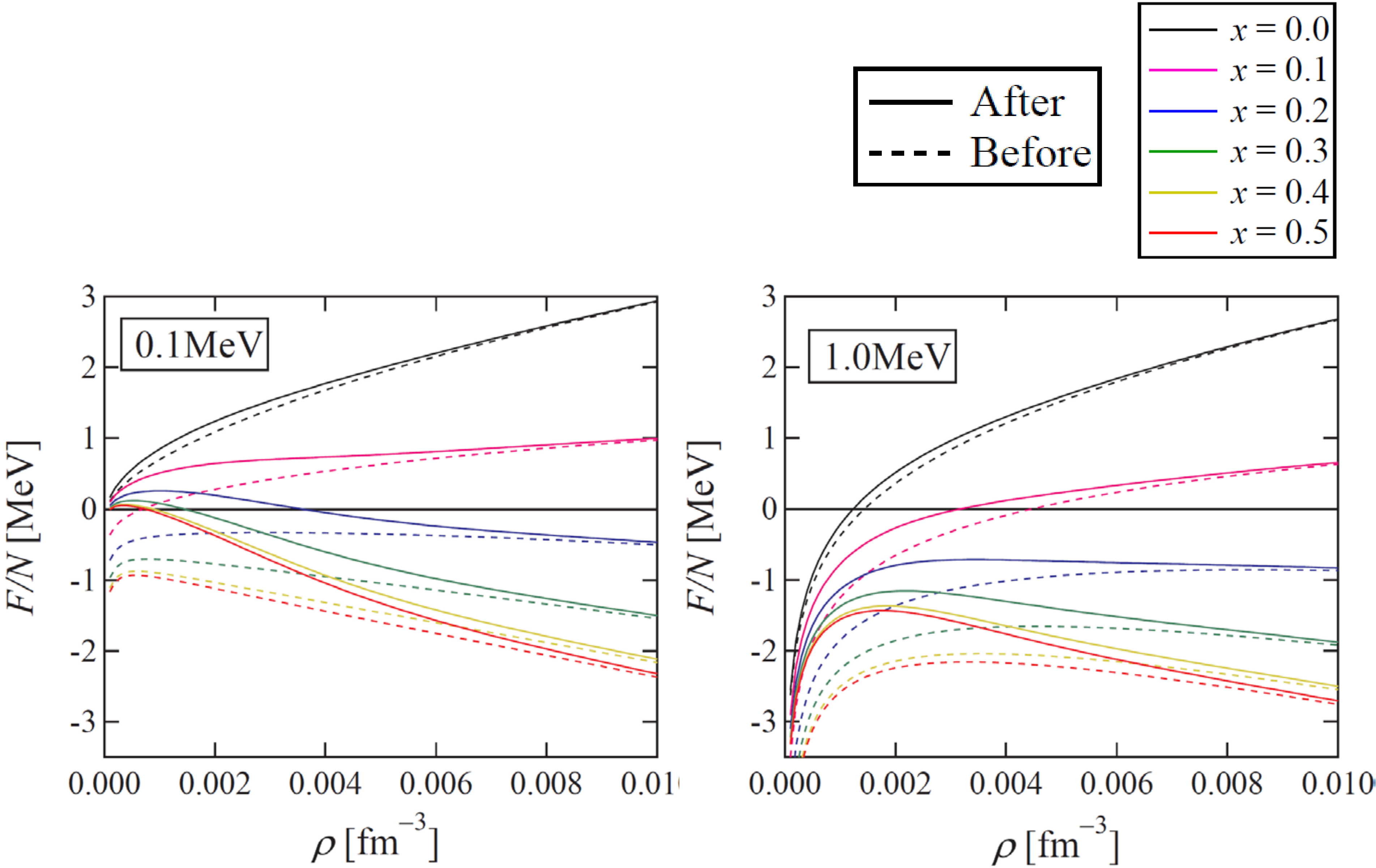
b, c is determined so that TF calculation for atomic nuclei keeps the gross feature.

E/N with New Cluster Variational Method



$E/N \rightarrow$ 0MeV in the limit of $\rho \rightarrow 0$ fm^{-3}

F/N with New Cluster Variational Method



3. Non-uniform Nuclear Matter

We follow the TF method by Shen et. al. (NPA637(1998)435)

Energy in the Wigner-Seitz (WS) cell

Bulk energy **Gradient energy**

$$E = \int d\boldsymbol{r}\varepsilon(n_p(r), n_n(r)) + F_0 \int d\boldsymbol{r} |\nabla n(r)|^2$$
$$+ \frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_p(r) - n_e][n_p(r') - n_e]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_{bcc} \frac{(Ze)^2}{a}$$

Coulomb energy

$F_0 = 68.00$ MeV fm^5

ε : Energy density of uniform nuclear matter

Parameter	Minimum	Maximum	Mesh	Number	
$\log_{10}(T)$ [MeV]	–1.24	1.40	0.12	23 + 1	(0MeV)
$\zeta = (1-2Y_p)^2$	0.0	1.0	0.1	11+2	(ζ= 0.85, 0.95)
n_B [fm^{-3}]	0.0001	0.1600	0.0001	1600	

$24 \times 13 \times 1600 \backsimeq 500000$ point

Nucleon density distribution

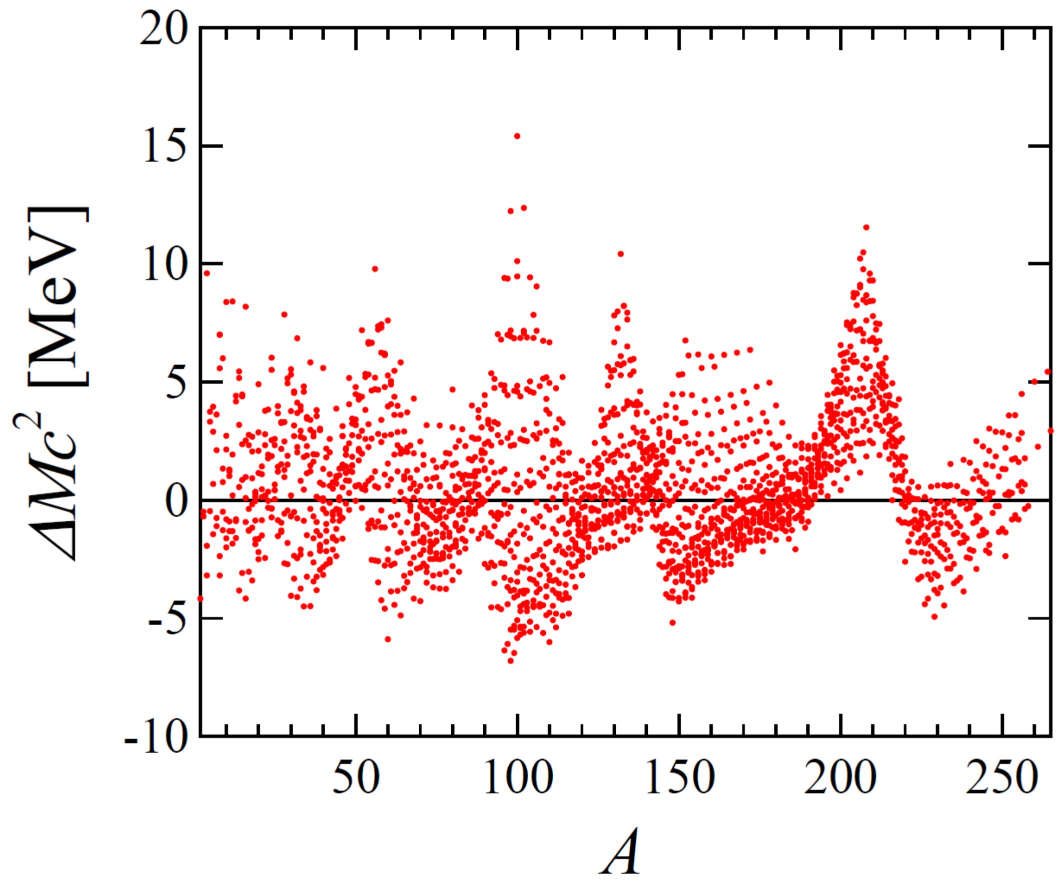
$$n_i(r) = \begin{cases} n_i^{in}[1 - (r/R_i)^{t_i}]^3 + n_i^{out} & (0 \le r \le R_i) \\ n_i^{out} & (R_i \le r \le R_{cell}) \end{cases} \quad (i = \text{p, n})$$

a : Lattice constant

$$V_{cell} = \frac{4\pi R_{cell}^3}{3} = a^3$$

E/V_{cell} is minimized with respect to n_i^{out}, n_i^{in} R_i, t_i, a at given density and proton fraction.

TF Calculation for Atomic Nuclei



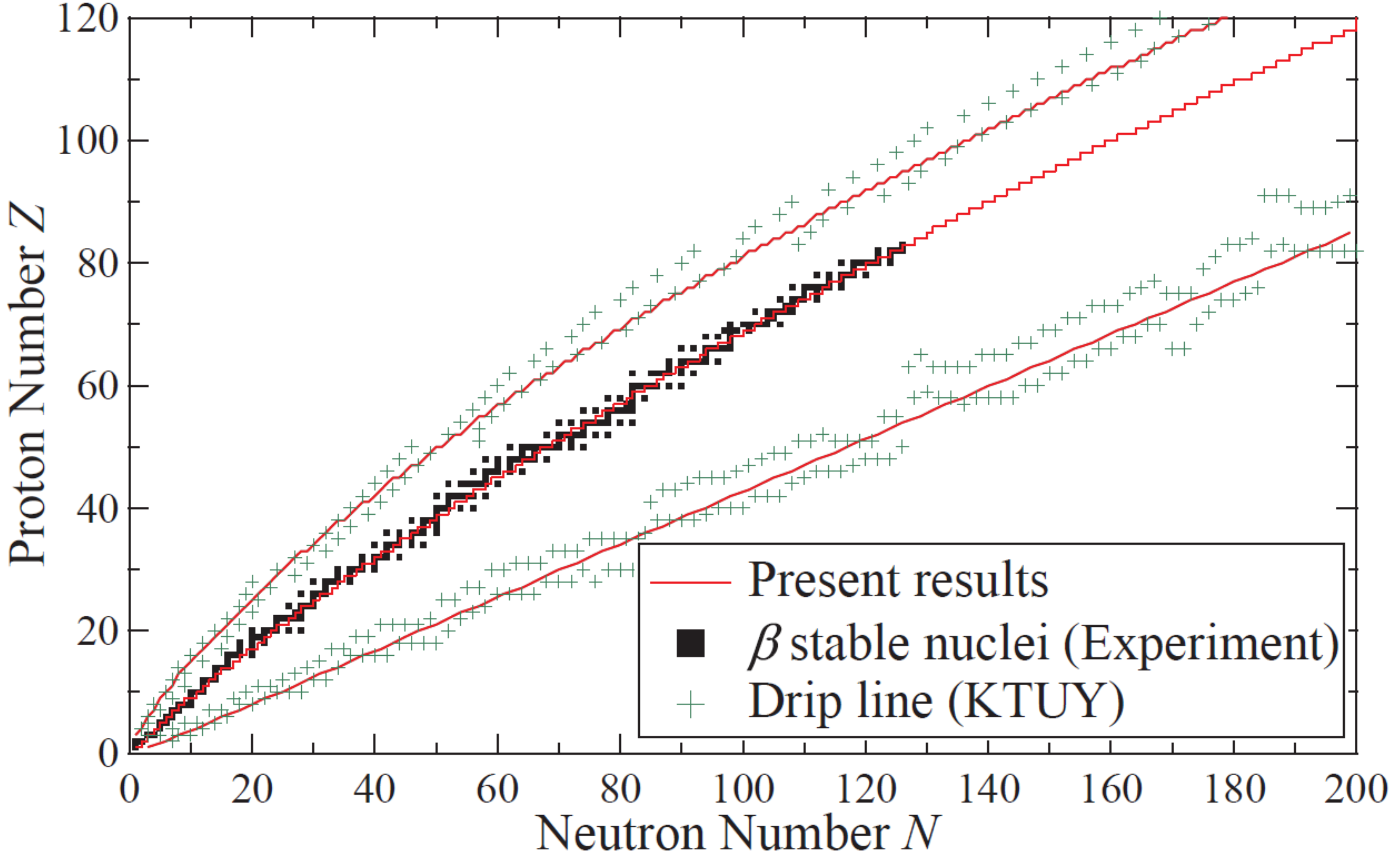
$\triangle M = M_{TF} - M_{exp}$

M_{TF} : Mass by the Thomas-Fermi calculation

M_{exp} : Experimental data

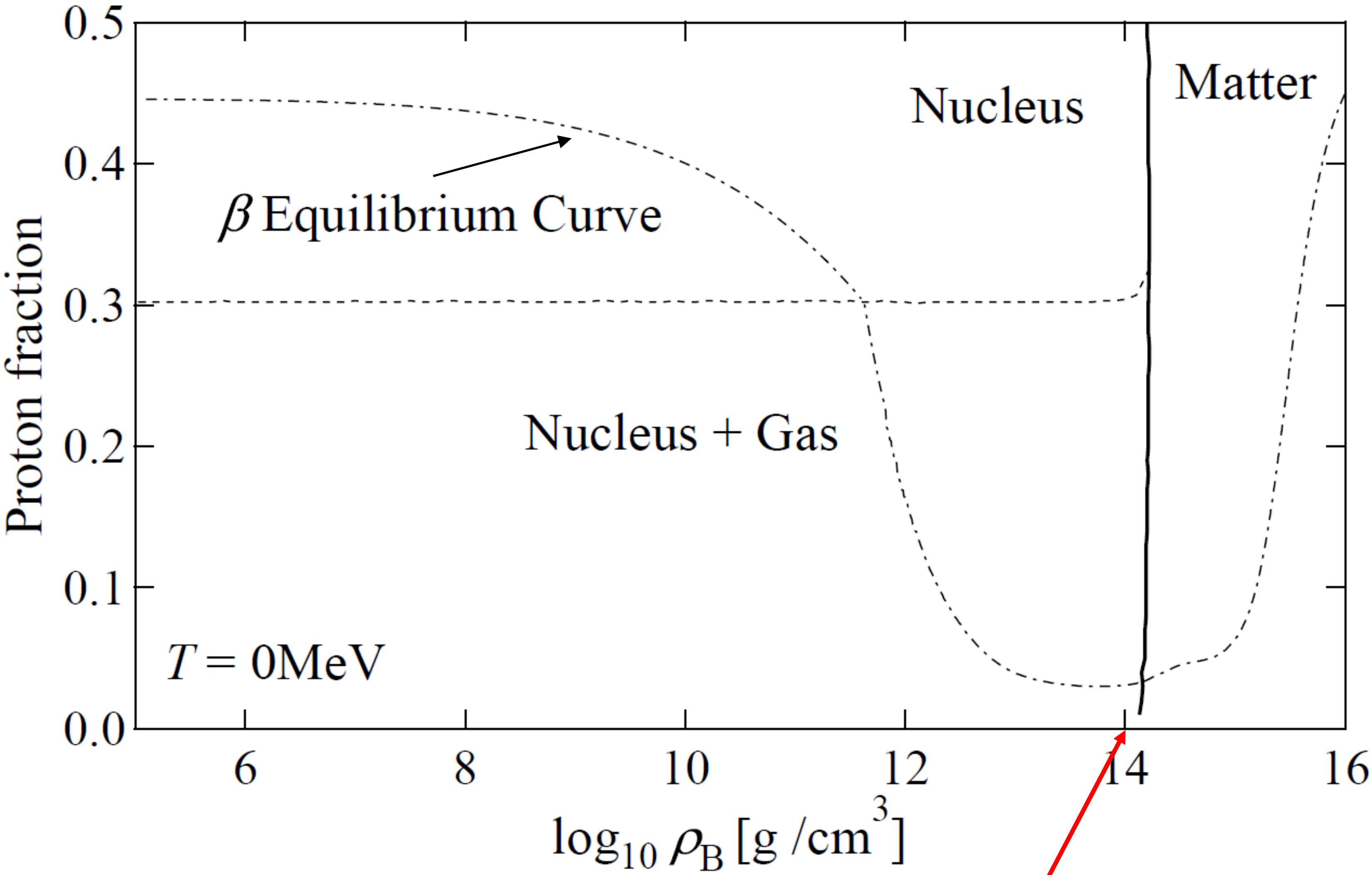
RMS deviation (for 2226 nuclei) 2.99 MeV

TF Calculation for Atomic Nuclei



Our results are in good agreement with
the experimental data and the sophisticated atomic mass formula.

TF Calculation for Non-uniform Nuclear Matter



Phase diagram of nuclear matter at $T = 0$MeV

$\rho_B = 10^{14.23}$ g/cm^3

5. Summary

- The EOS for uniform nuclear matter is constructed with the cluster variational method. (zero and finite temperatures)
- The EOS for non-uniform nuclear matter at zero temperature is calculated with the Thomas-Fermi callculation.

Uniform nuclear matter

Deuteron clustering appears at low density.

E/N at low density is refined for TF calculation.

Non-uniform nuclear matter

Phase diagram at zero temperature is reasonable.

Future Plans

- Construction of the EOS table for non-uniform matter
- Addition of α particle contribution

Construction of the EOS for supernova simulations