原子核におけるα粒子凝縮状態の研究

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素核宇融合による計算基礎物理学の進展
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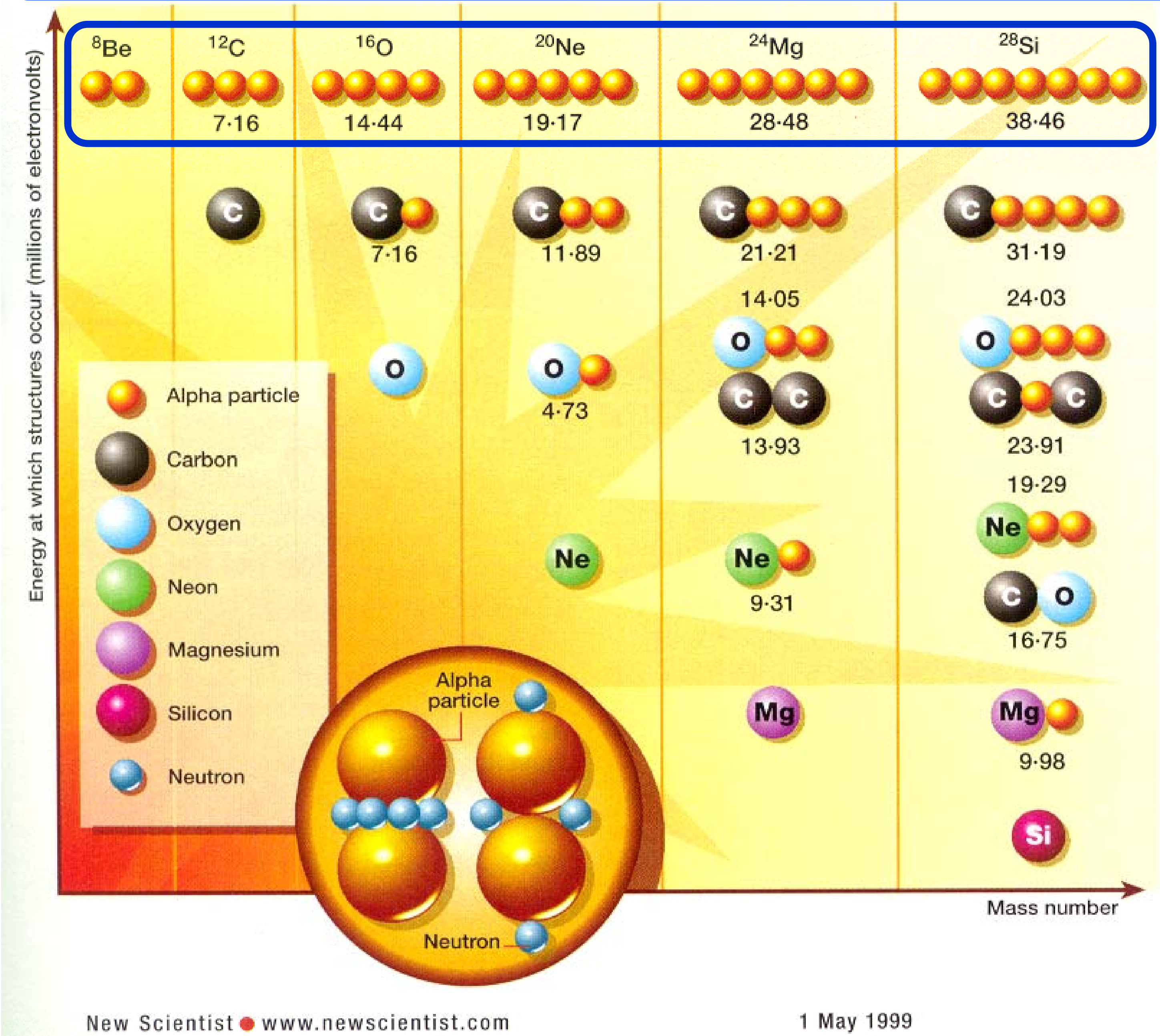
@合歓の里, 2011年12月3日ー5日.

Prediction of cluster states in light nuclei (Ikeda Diagram)

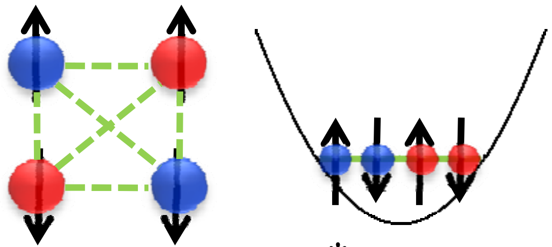


Classified according to the Threshold Rule.

K. Ikeda et al., PTP suppl. Extra num., 464 (1968).

The most tightly bound light cluster

α particle (quartet)

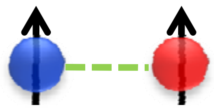


E/A ~7 MeV E^* ~20 MeV

stiff

The most elemental subunit in nuclear cluster structures.

Pair (deuteron) is less bound in nuclear system.

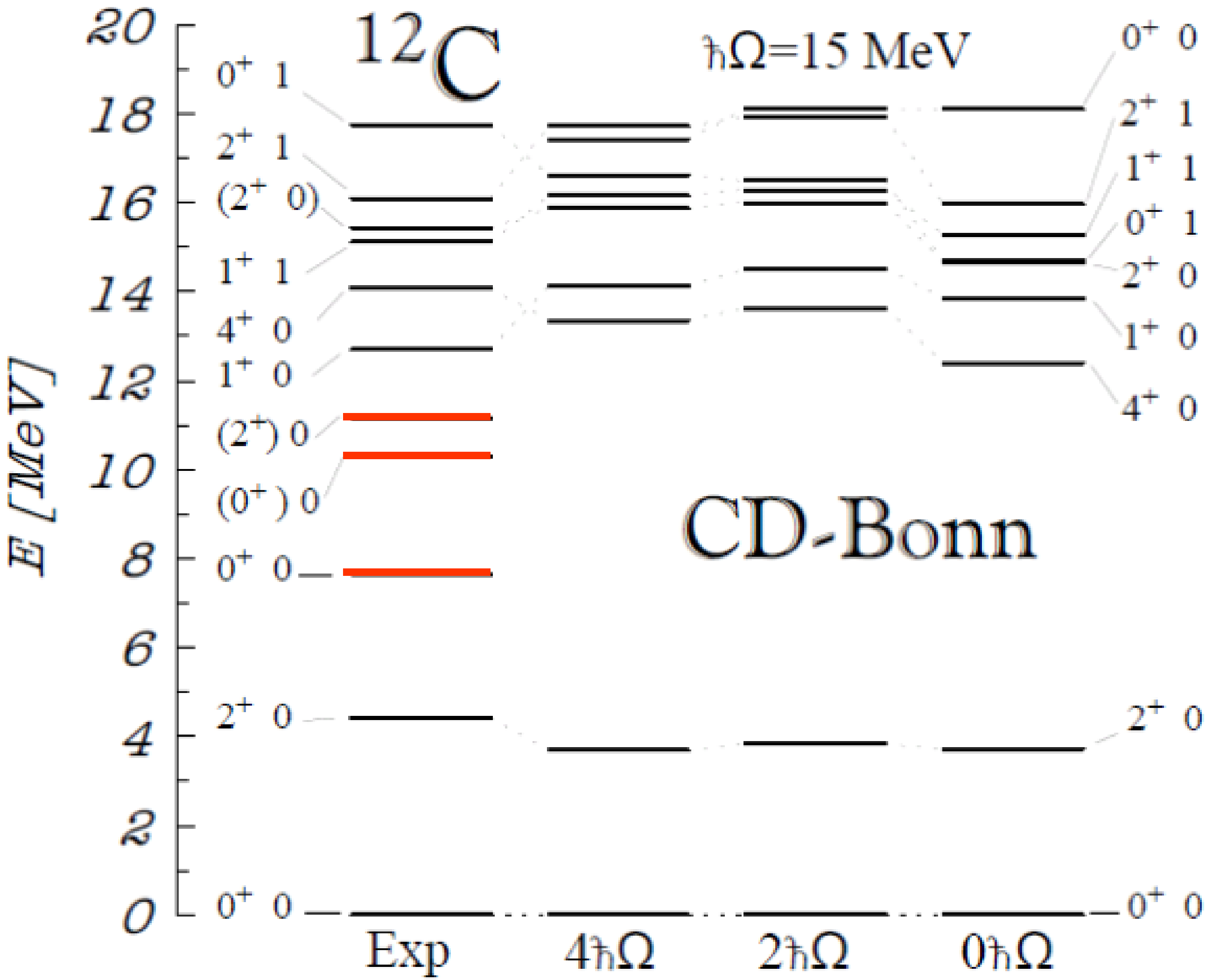


E/A ~1 MeV

Typical mysterious 0^+ states in nuclear structure problem

0_2^+ state of ^{12}C (Hoyle state) **indispensable to ^{12}C production in stars**

Ab initio non-core shell model calculation



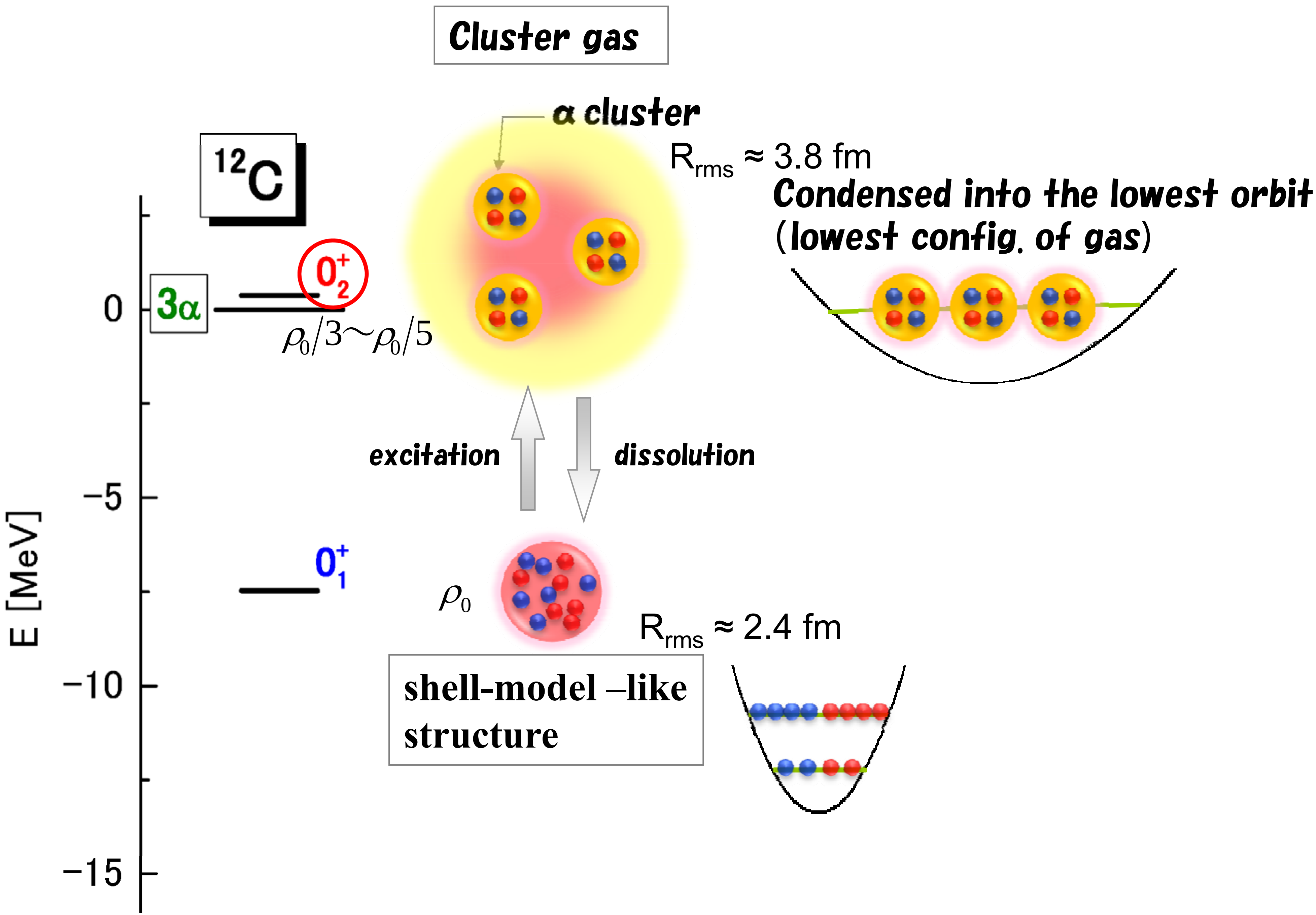
P. Navratil et al., PRL 84, 5728 (2001).

0_2^+ , 2_2^+ , 0_3^+ states : missing

0_2^+ state: excitation energy is not lower than 20 MeV

The typical excited states which resist a shell model description

First example of alpha cond. state



First example of α condensate state in finite nuclei

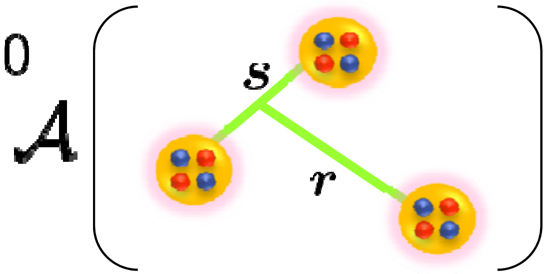
RGM (Full 3α) vs 3α cond. (3α confined in 0S orbit)

$\mathcal{A}$: antisymmetrizer acting on 12 nucleons

$$\langle \phi^3(\alpha)|H - E|\mathcal{A}[\chi(\boldsymbol{s},\boldsymbol{r})\phi^3(\alpha)]\rangle = 0$$

M. Kamimura, NPA 351, 456 (1981).

RGM w.f.



	Exp.	RGM
Energy (MeV)	7.65	7.74
α decay width (eV)	8.7±2.7	7.7
$M(0_2^+ \rightarrow 0_1^+)$ (fm^2)	5.4±0.2	6.7
$B(E2: 0_2^+ \rightarrow 2_1^+)$ ($e^2\,fm^4$)	13±4	5.6

First example of α condensate state in finite nuclei

<u>RGM (Full 3α) vs 3α cond. (3α confined in 0S orbit)</u>

The Solution of 3α RGM eq. of motion is almost equivalent to the 3α cond. w.f

The full 3α problem gives the 3α condensate w.f. as its solution!

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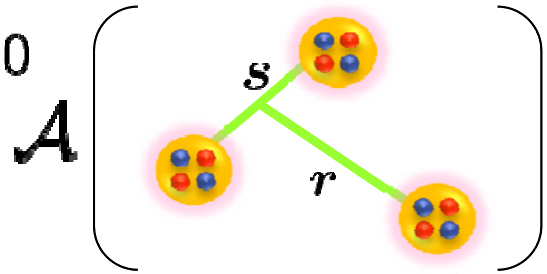


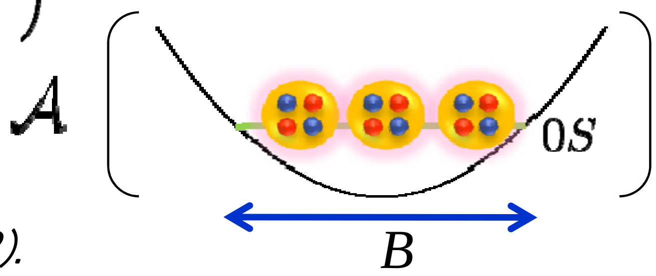
$$\chi = \prod_{i=1}^{3} \exp\left(-\frac{2}{B^2}(\boldsymbol{X}_i - \boldsymbol{X}_G)^2\right)$$

X_i : com coordinate of the i-th α

X_G : total com coordinate

Y. F. et al., PRC 67, 051306(R) (2003).

3α cond. w.f.



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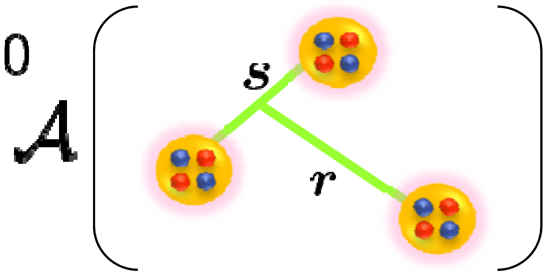

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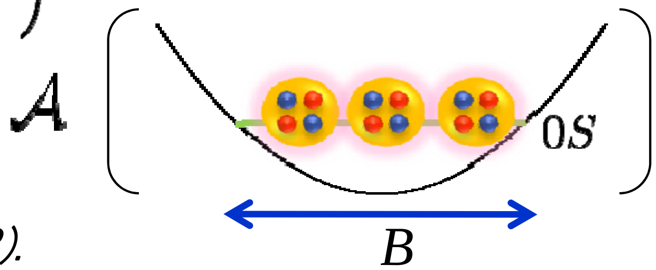
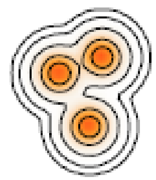


3α clustering also appears starting without assumption of α's by FMD & AMD

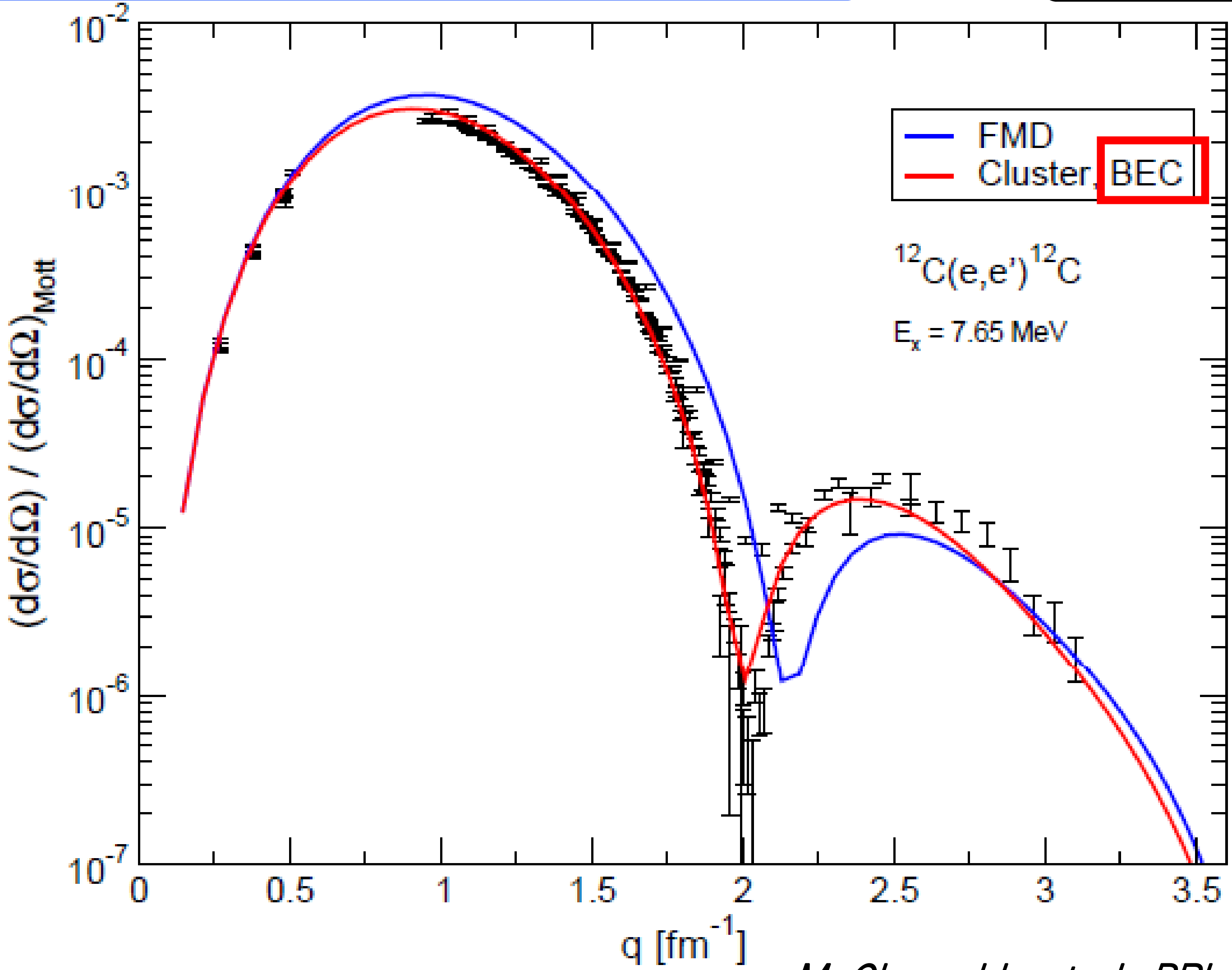
M. Chernykh, T. Neff et al., PRL 94, 032501 (2007).

Y. Kanada-En'yo, PTP 117, 655 (2007).



Electron Scattering Data ($0_1^+ \rightarrow 0_2^+$)

© T. Neff



M. Chernykh. et al., PRL 98, 032501 (2007)

Very nice reproduction by THSR w.f. (BEC)

``BEC'' from Y.F.et al., EPJA 28, 259(2006)

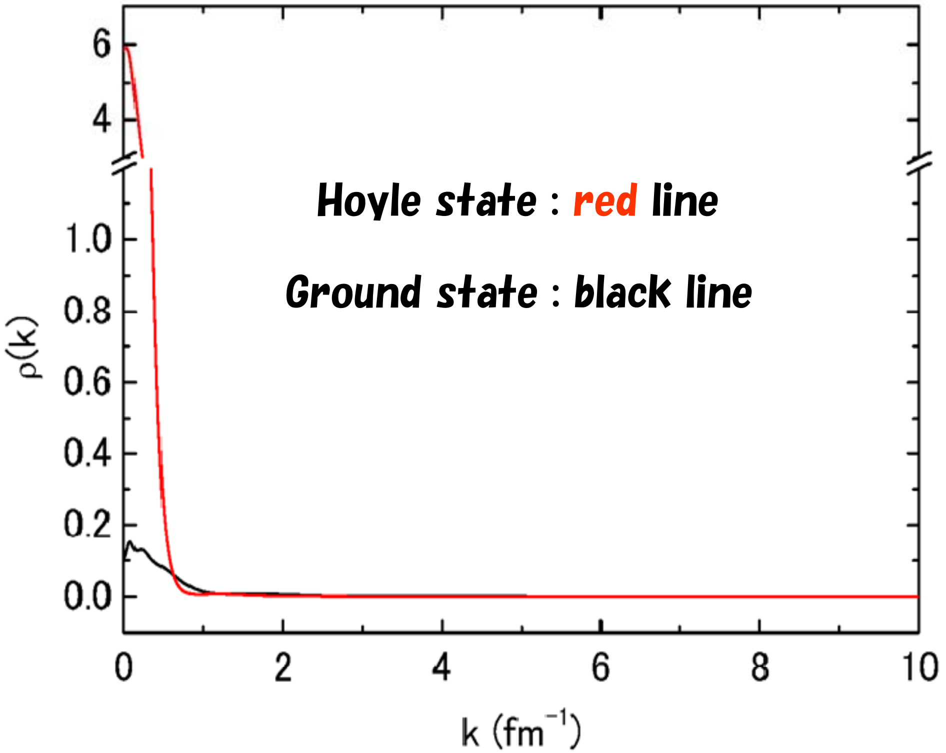
Direct information of alpha condensation for the Hoyle state

via 3α OCM(Orthogonality Condition Model)

$$\rho(k) = \int dr dr' \frac{e^{-ik\cdot r}}{(2\pi)^{3/2}} \rho(r, r') \frac{e^{ik\cdot r'}}{(2\pi)^{3/2}}$$

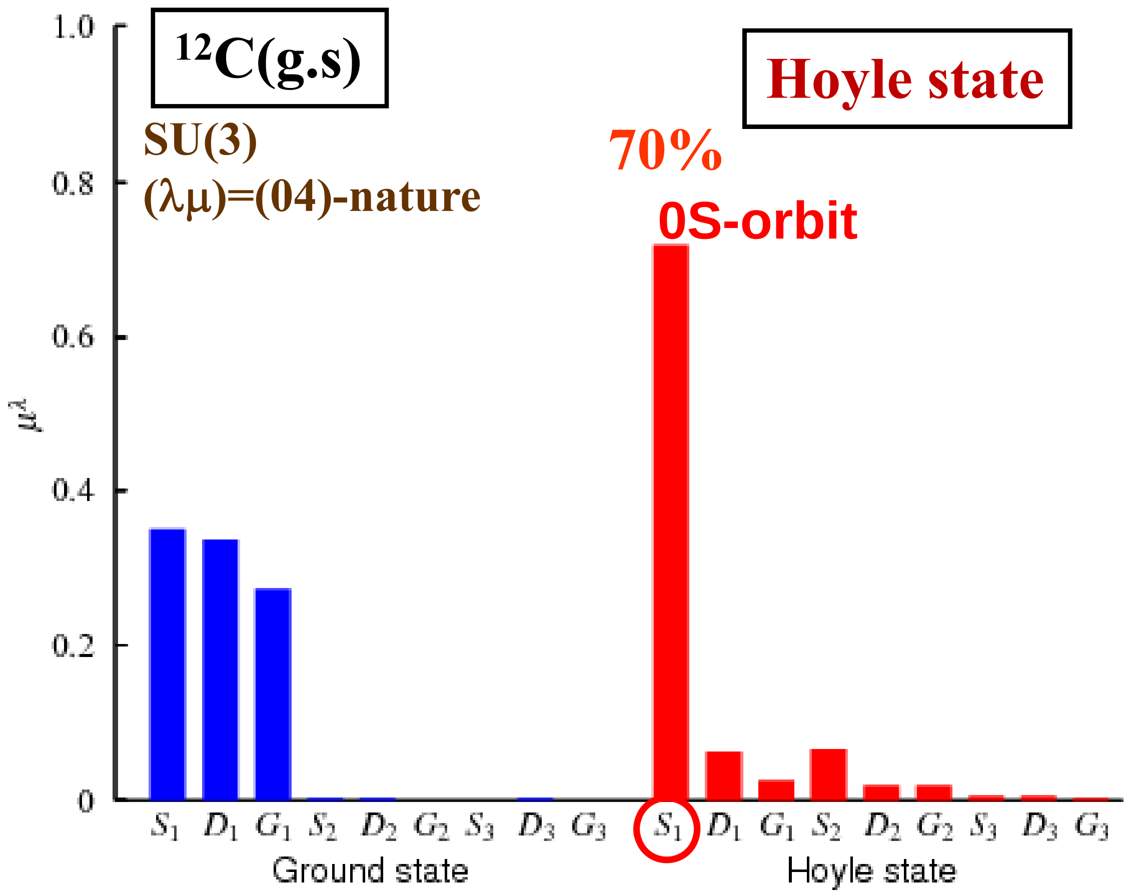
$$\int dr' \rho(r, r') \phi(r') = \mu \ \phi(r)$$

Momentum distribution of α-particle



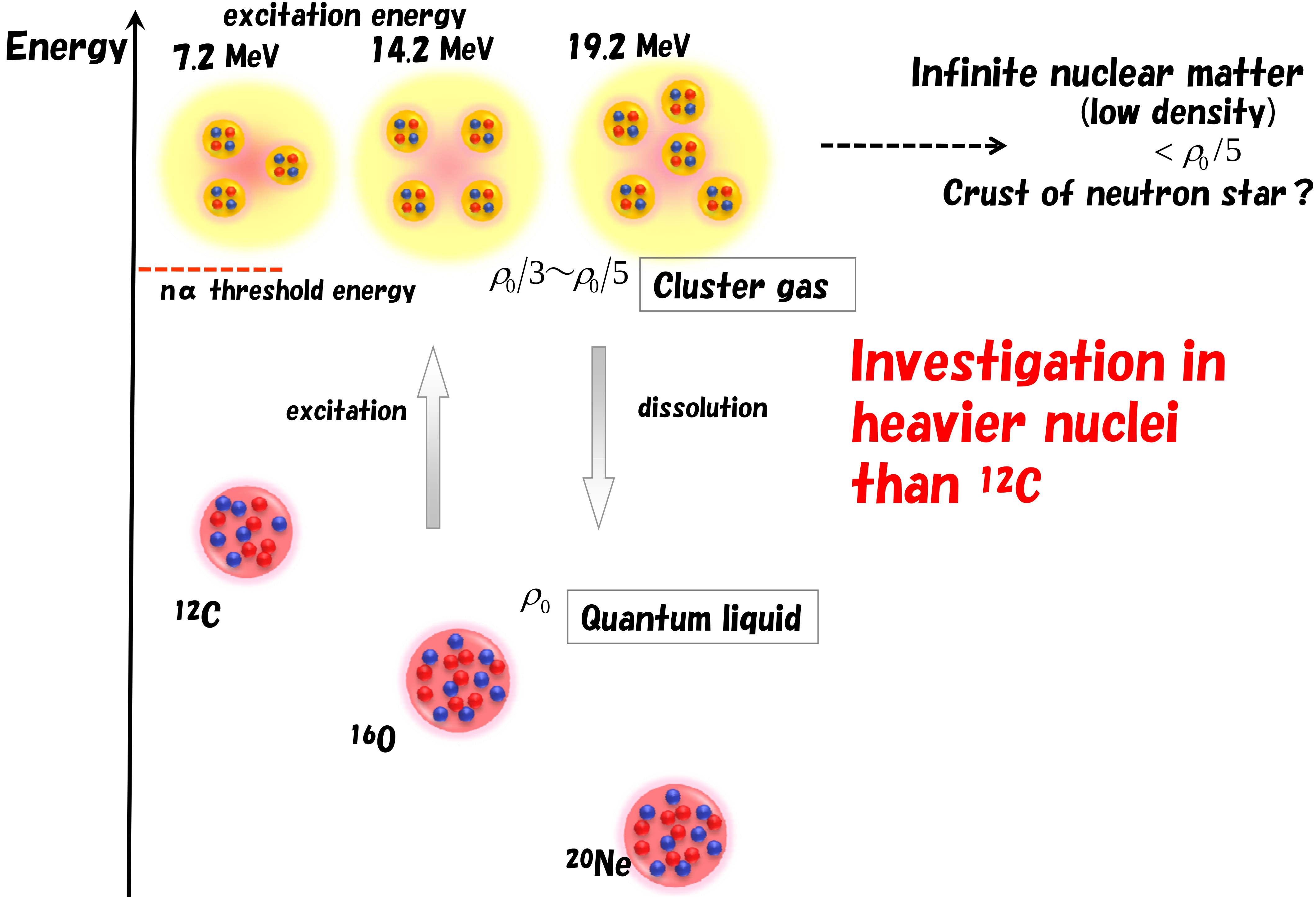
δ function -like peak around zero momentum

Occupation probability of single α-orbit



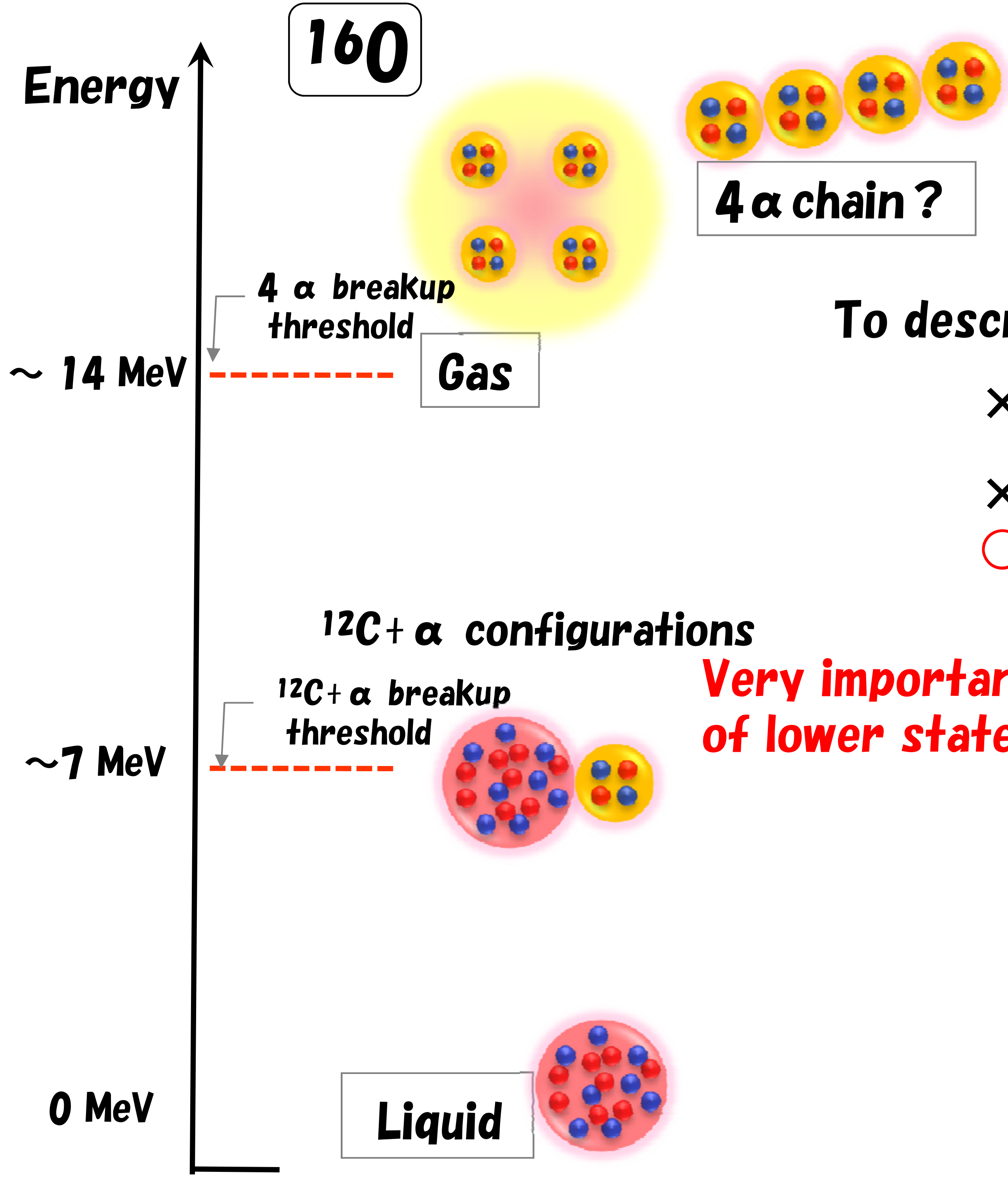
T. Yamada and P. Schuck, EPJA 26, 185 (2005).

``gas phase'' in finite nuclei



Investigation in heavier nuclei than ^{12}C

Analogue to the Hoyle state in ^{16}O?



To describe 4α cluster states

× mean field model

× ^{12}C+α cluster model

○ 4α cluster model

Very important : simultaneous description of lower states(g.s.、^{12}C+α)

Fully solving 4 α-particles relative motions (4α OCM)

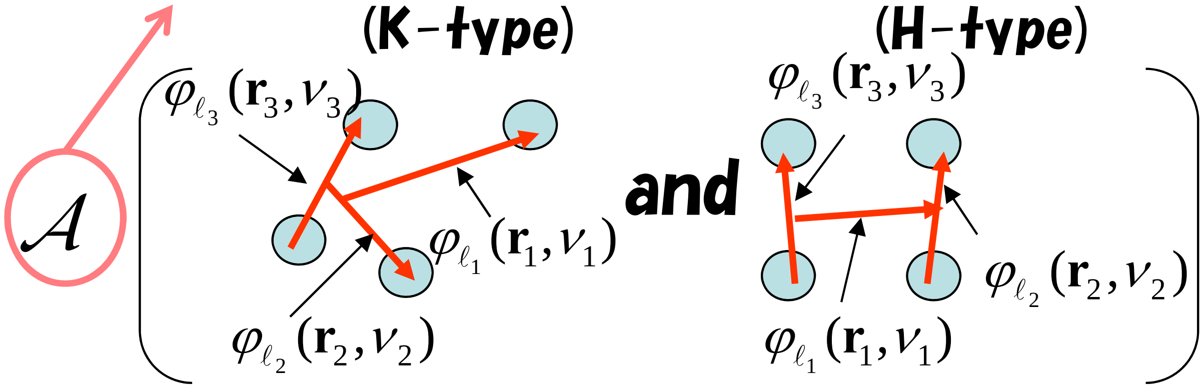
Present: Larger model space

$$\varphi_{\ell m}(\mathbf{r},\nu) = N_\ell(\nu) r^\ell \exp\left(-\nu r^2\right) Y_{\ell m}(\mathbf{r})$$

Gaussian basis (GEM)

E. Hiyama et al. Prog. Part. Phys. 51, 223(2003).

Approximately taken into account



Adopted angular momentum channels: $[[l_1,l_2], l_3]$ ($l_3+l_2+l_1 \leqq 8$) (up to now, $\leqq 5$)

Including $l_3, l_2, l_1 = 4$

Total w.f.

$$\Psi_{\rm OCM}(J_k^\pi) = \sum_{\{l\}\{\nu\}} A^{(k)}_{l_1,l_2,l_{12},l_3}(\nu_1,\nu_2,\nu_3)\hat{S}\left[\left[\varphi_{\ell_1}(\boldsymbol{r}_1,\nu_1),\varphi_{\ell_2}(\boldsymbol{r}_2,\nu_2)\right]_{l_{12}},\varphi_{\ell_3}(\boldsymbol{r}_3,\nu_3)\right]_J$$

$A^{(k)}_{l_1,l_2,l_{12},l_3}(\nu_1,\nu_2,\nu_3)$: Determined by diagonalizing Hamiltonian

Hamiltonian of 4α OCM

$$H = T + \sum_{i<j}\left[V_{2\alpha}(r_{ij}) + V_{2\alpha}^{Coul}(r_{ij})\right] + V_{3\alpha} + V_{4\alpha} + V_{Pauli}$$

Pauli blocking operator on α - α motions

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{2n+\ell<4} \sum_{ij} \left| u_{n\ell}(r_{ij}) \right\rangle \left\langle u_{n\ell}(r_{ij}) \right|$$

Pauli forbidden state: h.o.w.f.

2-body force (folding MHN force)

$$V_{2\alpha}(r) = \sum_n V_n^{(2)} \exp\left(-\beta_n^{(2)} r^2\right)$$

Coulomb force

$$V_{2\alpha}^{Coul}(r) = \frac{4e^2}{r} \operatorname{erf}(ar)$$

Phenomenological 3-body force (repulsive)

$$V_{3\alpha} = V^{(3)} \sum_{i<j<k} \exp\left[-\beta(r_{ij}^2 + r_{jk}^2 + r_{ki}^2)\right]$$

$$V^{(3)} = 87.5 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Phenomenological 4-body force (repulsive)

$$V_{4\alpha} = V^{(4)} \exp\left[-\beta(r_{12}^2 + r_{13}^2 + r_{14}^2 + r_{23}^2 + r_{24}^2 + r_{34}^2)\right]$$

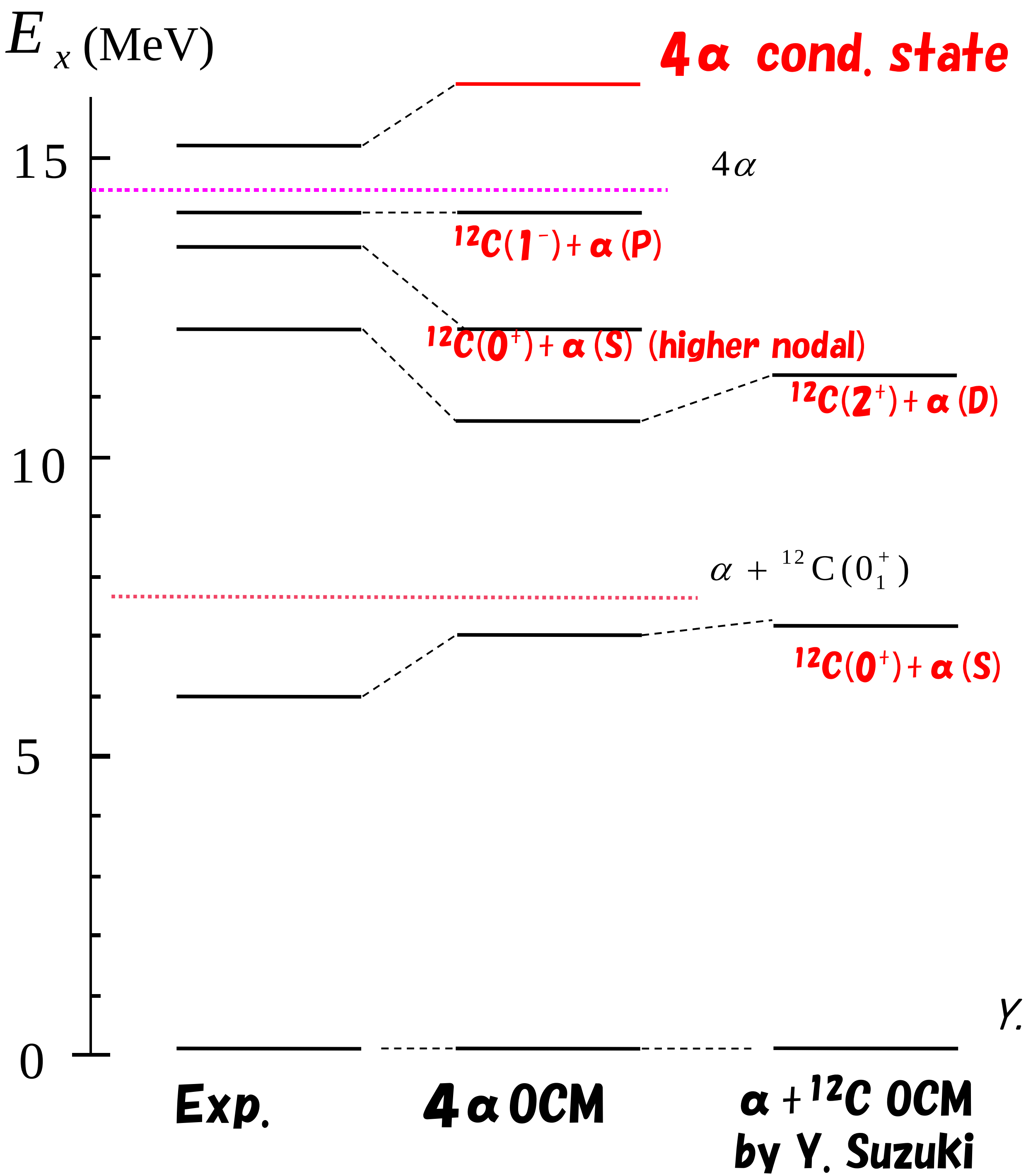
$$V^{(4)} = 12000 \text{ MeV}, \quad \beta = 0.15 \text{ fm}^{-2}$$

Energies from 4α threshold

	Cal. (MeV)	Exp. (MeV)
^{12}C(g.s.)	−7.32	−7.28
^{12}C(2_1^+)	−4.88	−2.84
^{12}C(4_1^+)	2.06	6.43
^{12}C(0_2^+)	0.70	0.38
^{16}O(g.s.)	−14.2	−14.44

$$\left|\langle V_{3\alpha}\rangle\right|, \left|\langle V_{4\alpha}\rangle\right| < \frac{7}{100}\left|\langle V_{2\alpha}\rangle\right|$$

0^+ spectra, rms radii, monopole matrix elements



0_4^+ state: T. Wakasa, Y. F. et al., PLB 653, 173 (2007).

Y. F. et al., PRL 101, 081502 (2008).

0^+ spectra, rms radii, monopole matrix elements

Large monopole matrix element can be the evidence of cluster states.

T. Yamada, Y. F. et al., PTP120, 1139 (2008).

		Experimental data				4α OCM		
	E_x [MeV]	R [fm]	M(E0) [fm^2]	Γ [MeV]		R [fm]	M(E0) [fm^2]	Γ [MeV]
0^+_1	0.00	2.71				2.7		
0^+_2	6.05		3.55			3.0	3.9	
0^+_3	12.1		4.03			3.1	2.4	
0^+_4	13.6		no data	0.6		4.0	2.4	0.60
0^+_5	14.0		3.3	0.185		3.1	2.6	0.20
0^+_6	15.1		no data	0.166		5.6	1.0	0.14

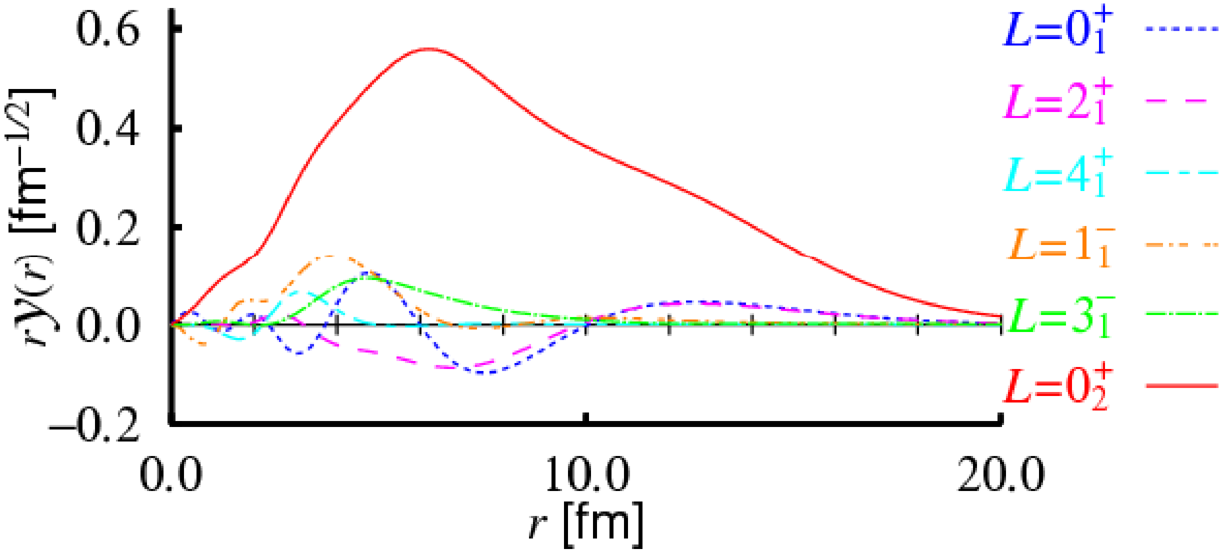
over 15% of total EWSR

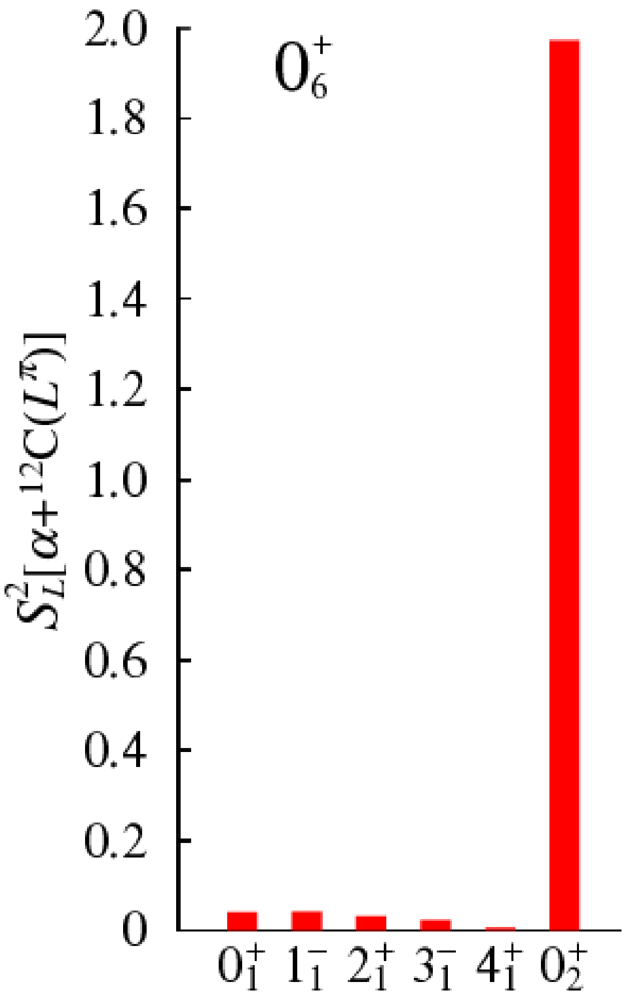
20% of total EWSR

S-factor : $^{12}C+\alpha$ and $^{8}Be+^{8}Be$ components for the 0_6^+ state

$$r\times\mathcal{Y}_{IL,J=0}(r)=r\times\sqrt{\frac{4!}{3!1!}}\left\langle\left[\frac{\delta(r-r')}{rr'}Y_L(\hat{\boldsymbol{r}}')\Psi_{\mathrm{OCM}}(^{12}\mathrm{C}(I))\right]_0\middle|\Psi_{\mathrm{OCM}}(0_k^+)\right\rangle$$

$$S_{IL}^2(J=0)=\int dr\left(r\times\mathcal{Y}_{IL,J=0}(r)\right)^2$$





abscissa: Spin of ^{12}C

Analogue to the Hoyle state

$\alpha+^{12}C$(Hoyle) configuration is dominant. → **4α condensate**

^{12}C(Hoyle): 3α condensate

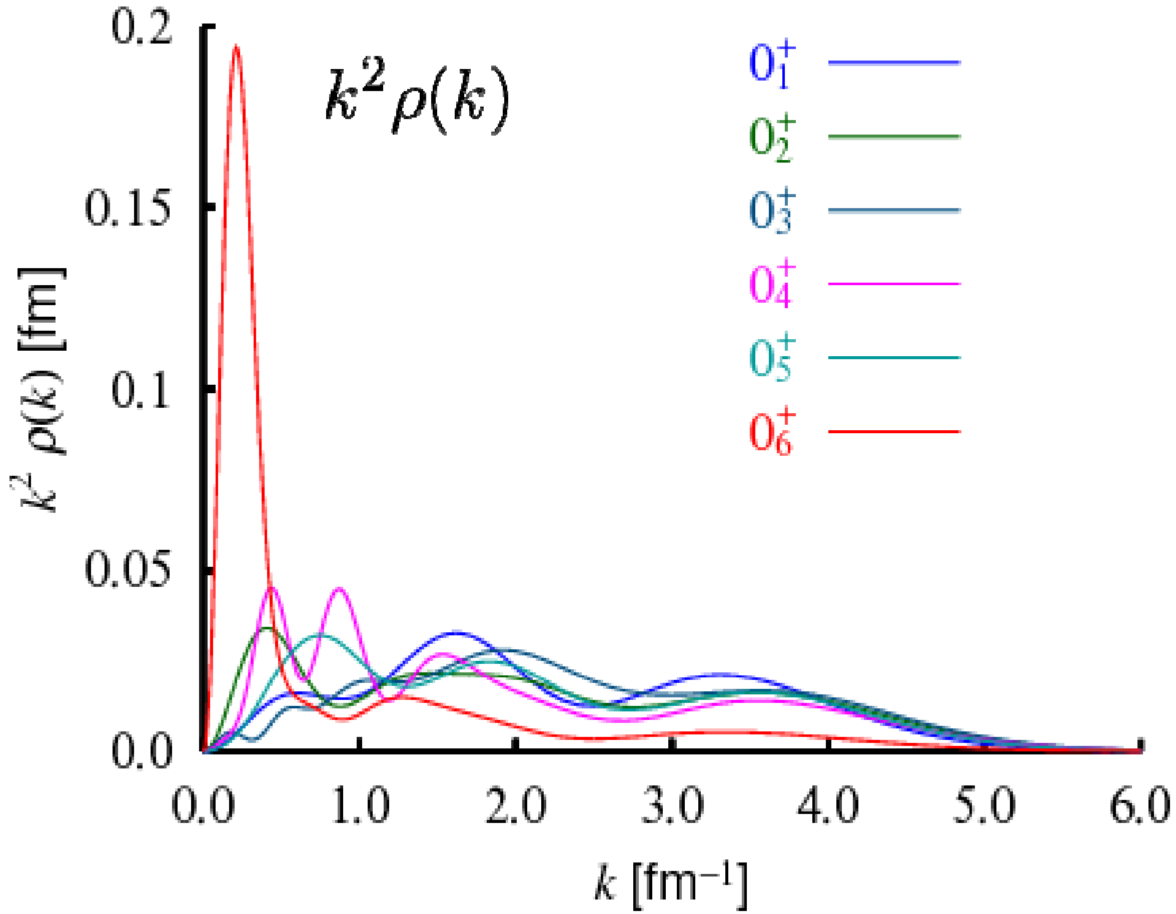
Momentum distributions of the α particles

$$\rho(k) = \int d\boldsymbol{r} d\boldsymbol{r}' \frac{e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}}{(2\pi)^{3/2}} \rho(\boldsymbol{r},\boldsymbol{r}') \frac{e^{i\boldsymbol{k}\cdot\boldsymbol{r}'}}{(2\pi)^{3/2}}$$

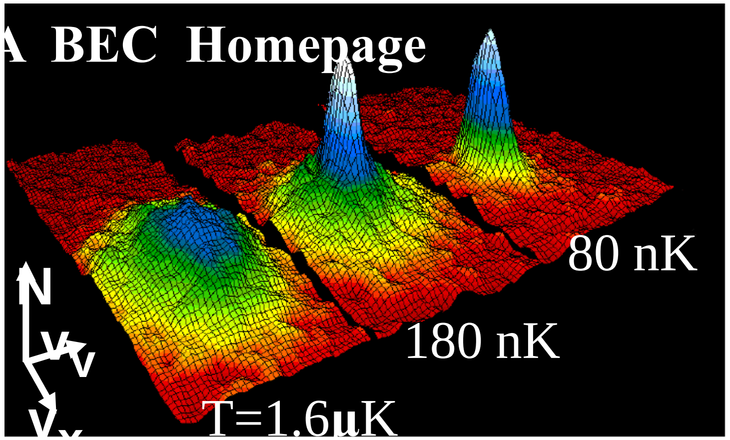
$$\rho(\boldsymbol{r},\boldsymbol{r}') = \frac{1}{4}\sum_{i=1}^{4} \langle \Psi_{\rm OCM}(0_k^+)|\delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r}')\rangle\langle\delta(\boldsymbol{r}_i - \boldsymbol{X}_G - \boldsymbol{r})|\Psi_{\rm OCM}(0_k^+)\rangle$$

$\boldsymbol{r}_i$: coordinate of the i-th α particle

$\boldsymbol{X}_G$: coordinate of total center-of-mass



BEC in the atomic world



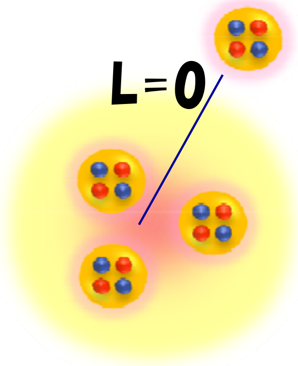
0_6^+ : delta-function-like peak at zero momentum

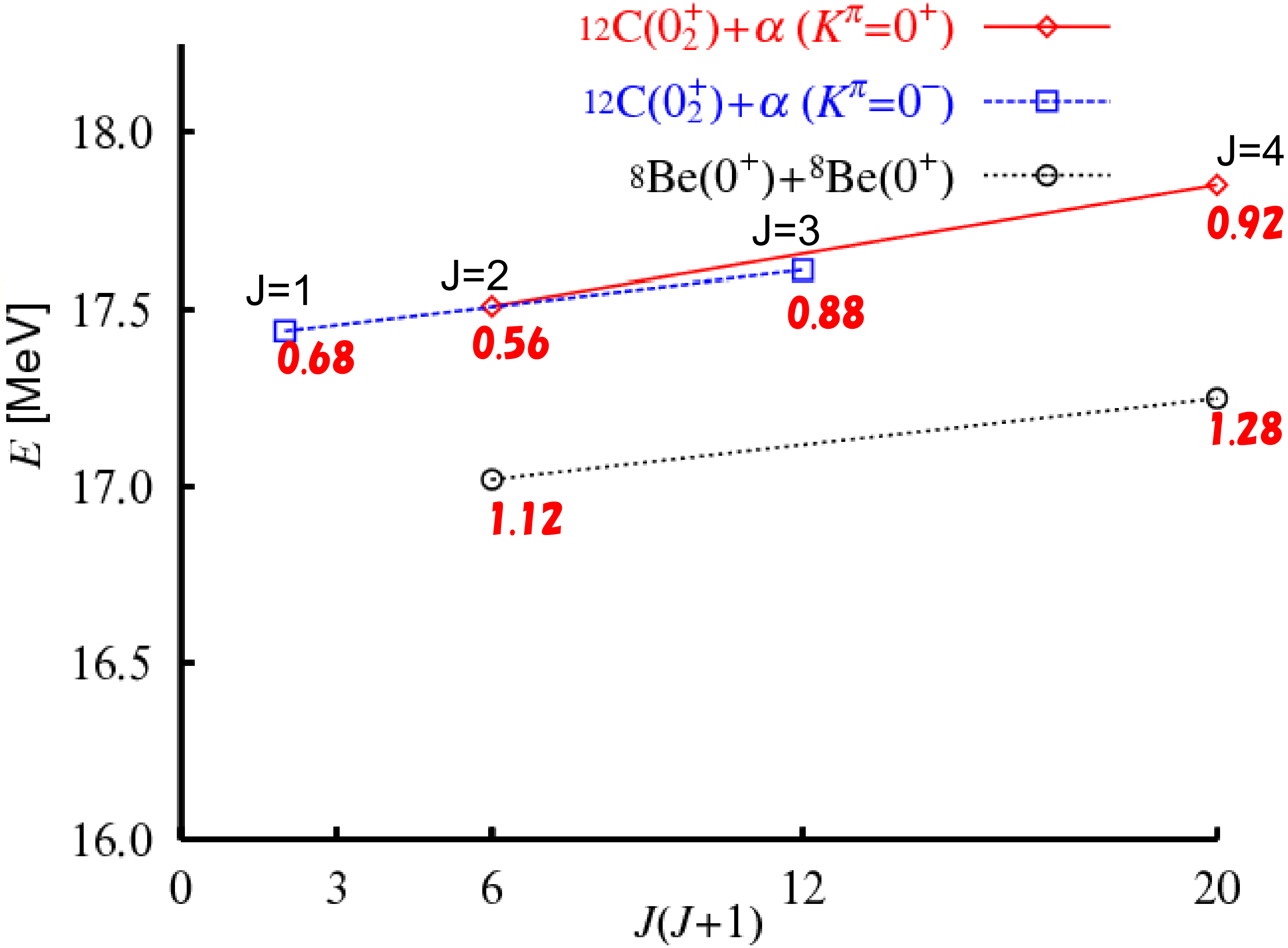
4α condensate state character.

de Broglie w.l. $\lambda = \dfrac{2\pi}{\sqrt{\langle k^2 \rangle}} \geq 20$ fm

Rotational bands of Hoyle + alpha, ^{8}Be+^{8}Be ?

$$E_{exc.} = \frac{\hbar^2}{2\mathcal{M}_I} J(J+1)$$





num.: S^2-factor of alpha + Hoyle

Y.F. S. Ohkubo et al., in preparation.

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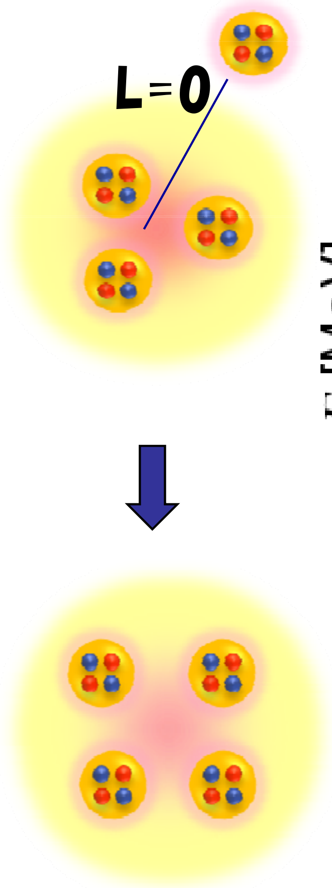
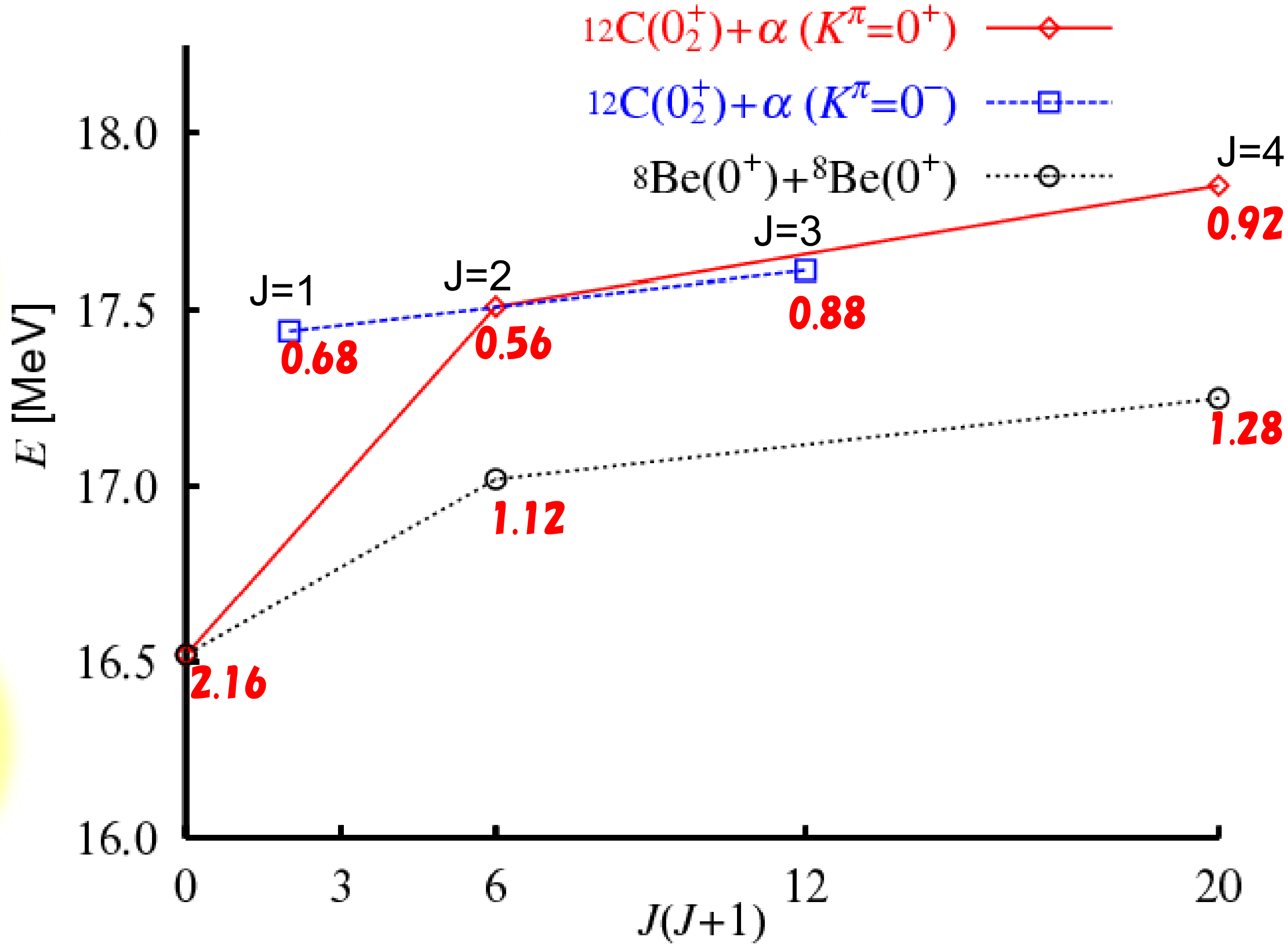




0_6^+ state : energy gain due to condensation
Rotating alpha dropped at the lowest orbit.

Local condensate → complete condensate

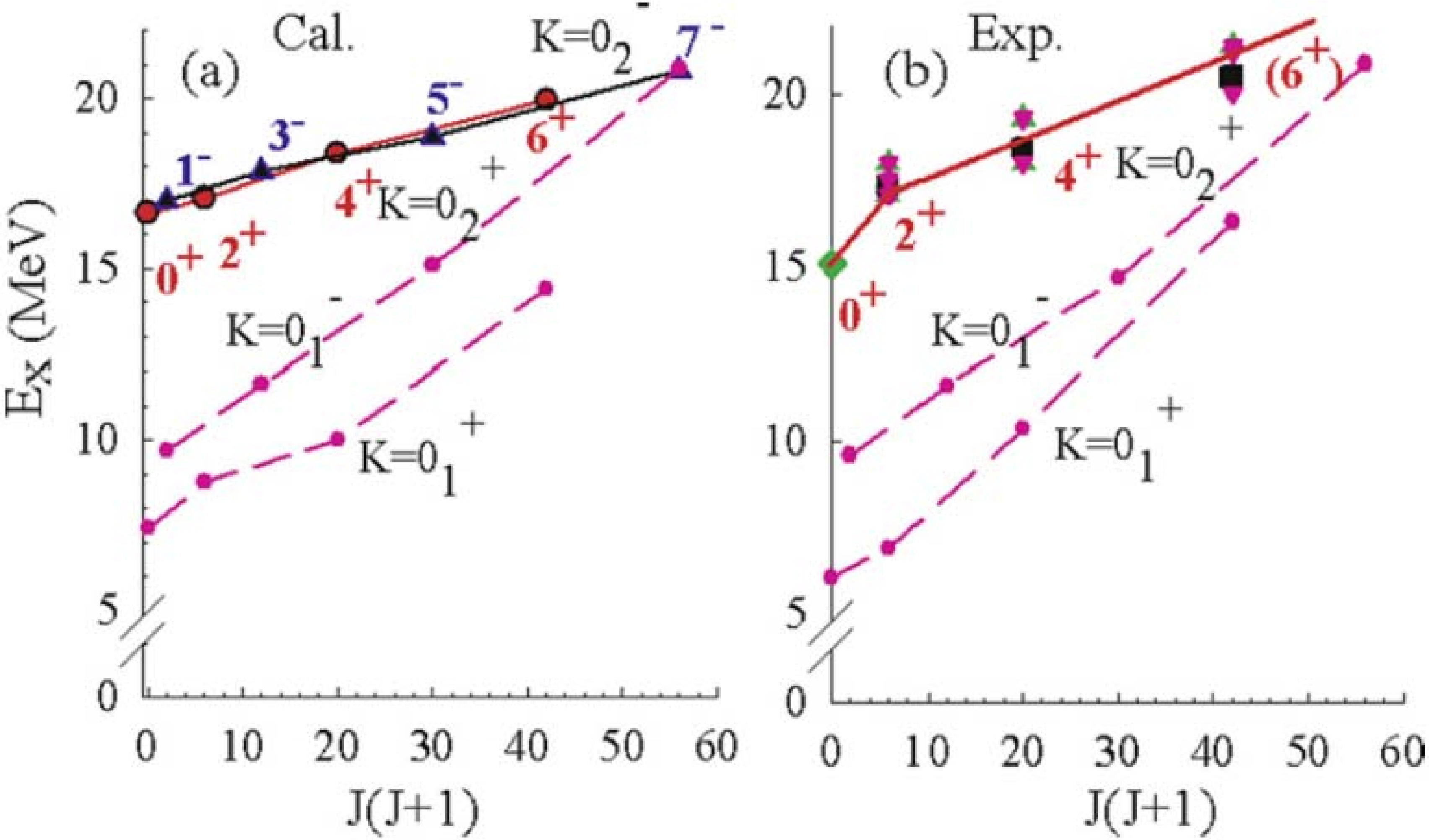
Y.F. S. Ohkubo et al., in preparation.

num.: S^2-factor of alpha + Hoyle

Momentum inertia is reduced.

Signature of superfluidity ??

Rotational band of Hoyle + alpha



Hoyle + alpha, 2-body scattering solutions.

Momentum inertia is reduced.

Signature of superfluidity ??

S. Ohkubo and Y. Hirabayashi, PLB 684, 127 (2010).

Summary

Investigation of loosely bound alpha gas states in heavier nuclei than ^{12}C.

- More α-particle condensate states very likely to exist.

  Analogue state in ^{16}O to the Hoyle state (found with 4αOCM calc.)
  as the sixth 0^+ state
  Assigned to 15.2 MeV state?
  More experimental information is needed.

- Hoyle analogs for non-zero spin states are promising.

  likely Hoyle + alpha rotational band
  sign of condensate
  Problem is continuum mixing

On going issue: beyond bound state approximation
4-alpha CSM (Complex Scaling Method) with T2K-Tsukuba (up to 512cpu' s)

Thanks

to my Collaborators

Taiichi Yamada (Kanto Gakuin Univ.)

Hisashi Horiuchi (RCNP)

Akihiro Tohsaki (RCNP)

Peter Schuck (IPN, Orsay)

Gerd Röpke (Rostock Univ.)

Masaaki Takashina (RCNP)

Tomotsugu Wakasa (Kyushu Univ.)

Wolfram von Oertzen (HMI, Berlin)

Shigeo Ohkubo (Kochi women Univ.)

and for your attention.