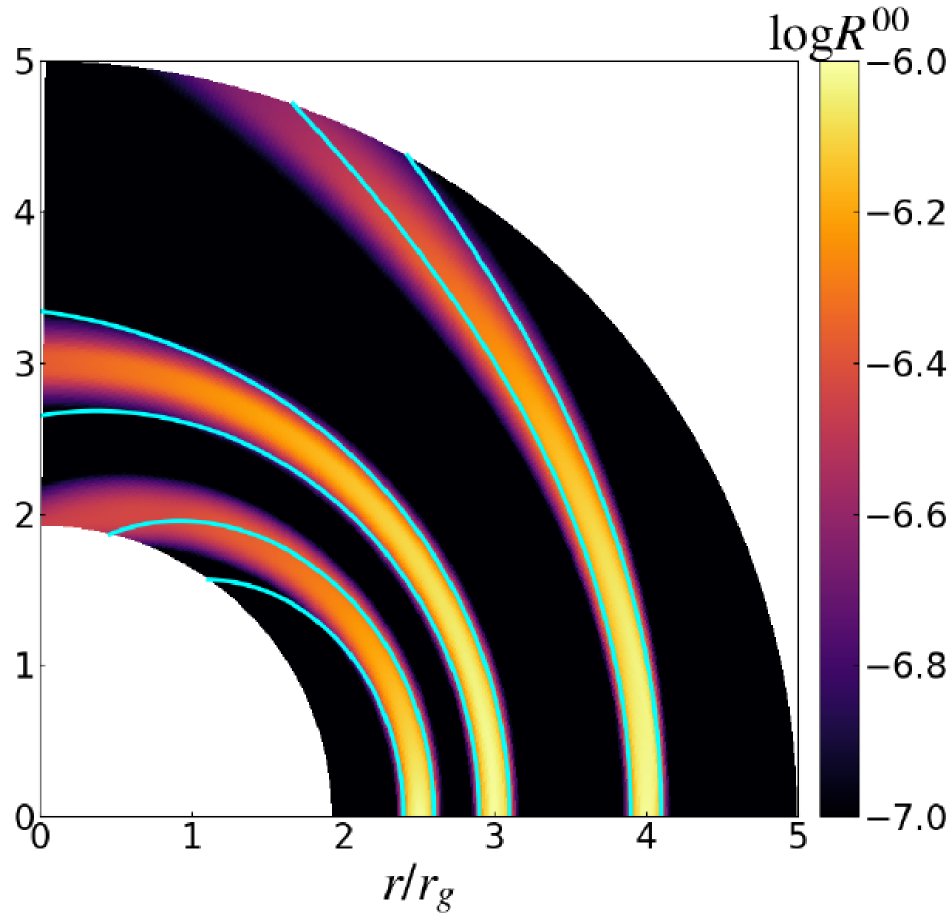
一般相対論的RMHDコード開発



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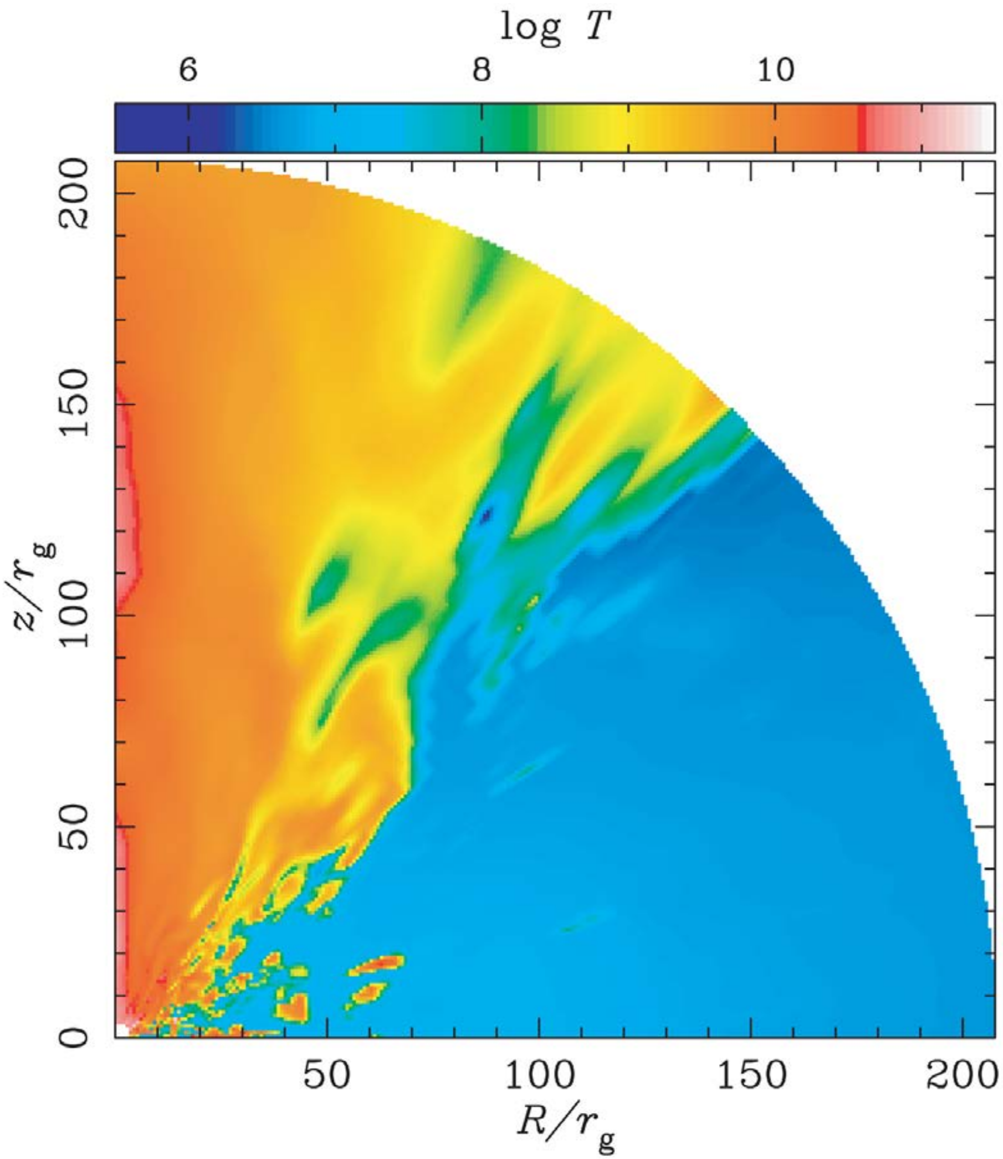
Radiation Hydrodynamics Simulations



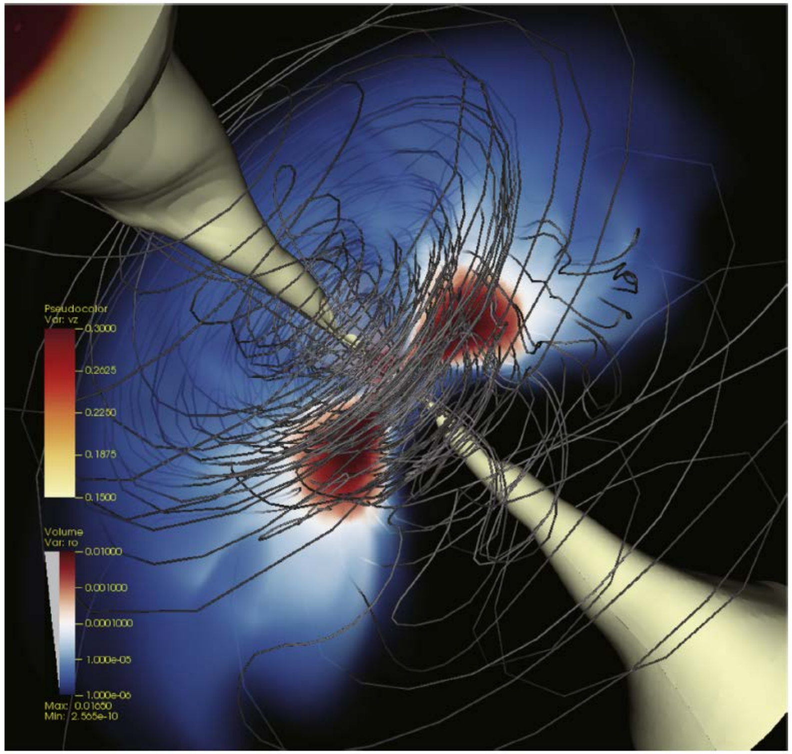
Ohsuga et al. (2005)

Ohsuga et al. (2005) conducted 2D radiation HD simulations of the supercritical accretion flows around a black hole

- Hot outflow with velocity of 0.1c is formed around the rotation axis by strong radiation pressure
- The radiation is important for the outflow formation and structure of the accretion flow

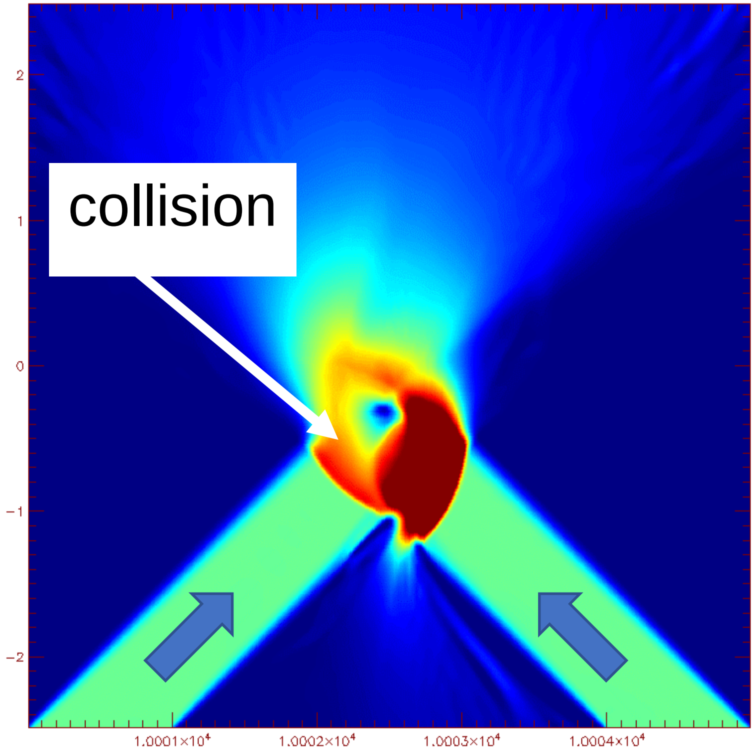
General Relativistic Radiation MHD Simulations (M1 method)

Takahashi et al. (2016) carried out 3D general relativistic radiation MHD simulations of accretion flows.
The jet is formed by the radiation pressure and collimated by the Lorentz force.



Takahashi et al. (2016)

M1 method



Basic Equations for Fluid

Mass conservation equation

$$\partial_t(\sqrt{-g}\rho u^t) + \partial_i(\sqrt{-g}\rho u^i) = 0$$

Energy momentum conservation equation

$$\partial_t(\sqrt{-g}T^t_\mu)+\partial_i(\sqrt{-g}T^i_\mu) = \sqrt{-g}T^\kappa_\lambda\Gamma^\lambda_{\mu\kappa}+\sqrt{-g}\boxed{G_\mu}$$

Induction equation

$$\partial_t(\sqrt{-g}B^t) = -\partial_j[\sqrt{-g}(b^j u^i - b^i u^j)]$$

Radiation force

$$\boxed{G^\mu} = -\rho(\kappa_a+\kappa_s)R^{\mu\nu}u_\nu - \rho(\kappa_s R^{\alpha\beta}u_\alpha u_\beta + \kappa_a 4\pi B)u^\mu$$

Radiation Transfer Equation

$$\frac{\partial I}{\partial t} + \boldsymbol{n} \cdot \frac{\partial I}{\partial \boldsymbol{x}} = 0 \quad \xrightarrow{\times \int \mathrm{d}\Omega} \quad \frac{\partial E_r}{\partial t} + \frac{\partial F^i}{\partial x^i} = 0$$

$$\frac{\partial I}{\partial t} + \boldsymbol{n} \cdot \frac{\partial I}{\partial \boldsymbol{x}} = 0 \quad \xrightarrow[\times \boldsymbol{n} \int \mathrm{d}\Omega]{} \quad \frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^i} = 0$$

- In order to solve this equation, we need to obtain the radiation pressure $P^{ij} = \mathbb{D} E_r$
- M1 method assumes that the Eddington tensor is below in order to close these equations (Gonzalez et al. 2007)

$$\mathbb{D} = \frac{1-\chi}{2}\mathbb{I} + \frac{3\chi - 1}{2}\boldsymbol{n} \otimes \boldsymbol{n}, \quad \chi = \frac{3 + 4||\boldsymbol{f}||^2}{5 + 2\sqrt{4 - 3||\boldsymbol{f}||^2}}, \quad \boldsymbol{f} = \frac{\boldsymbol{F}_\mathrm{r}}{cE_\mathrm{r}}$$

Radiation Transfer Equation

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$$\xrightarrow[\times \boldsymbol{n} \int \mathrm{d}\Omega]{} \quad \frac{\partial F^i}{\partial t} + \frac{\partial P^{ij}}{\partial x^i} = 0$$

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Simple flowchart

Boltzmann equation

$$
\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left[\left(e^\alpha_{(0)}+\sum_{i=1}^{3} l_{(i)} e^\alpha_{(i)}\right)\sqrt{-g} I\right]
+\frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\sin\bar\theta\,\omega_{(\bar\theta)} I\right)+\frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\phi}\left(\omega_{(\bar\phi)} I\right)+\omega_{(0)} I
= -\gamma(1-\boldsymbol{v}\cdot\boldsymbol{l})\rho(\kappa_{\rm abs}+\kappa_{\rm sca})I + \gamma^{-3}(1-\boldsymbol{v}\cdot\boldsymbol{l})^{-3}\rho\left(\frac{j_0}{4\pi}+\kappa_{\rm sca}\frac{E_{\rm com}}{4\pi}\right)
$$

Energy/momentum equation for radiation

$$
\partial_t(\sqrt{-g} R^t_\mu)+\partial_i(\sqrt{-g} R^i_\mu) = \sqrt{-g} R^\kappa_\lambda \Gamma^\lambda_{\mu\kappa} - \sqrt{-g} G_\mu
$$

Boltzmann equation

Update the specific intensity

Eddington tensor

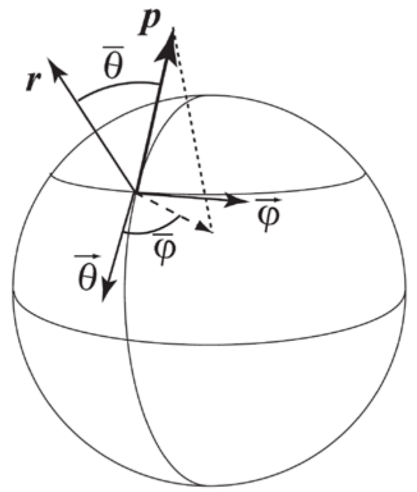
$$
D^{ij} = \int I n^{(i)} n^{(j)} \mathrm{d}\Omega \Big/ \int I \mathrm{d}\Omega
$$

Momentum equation

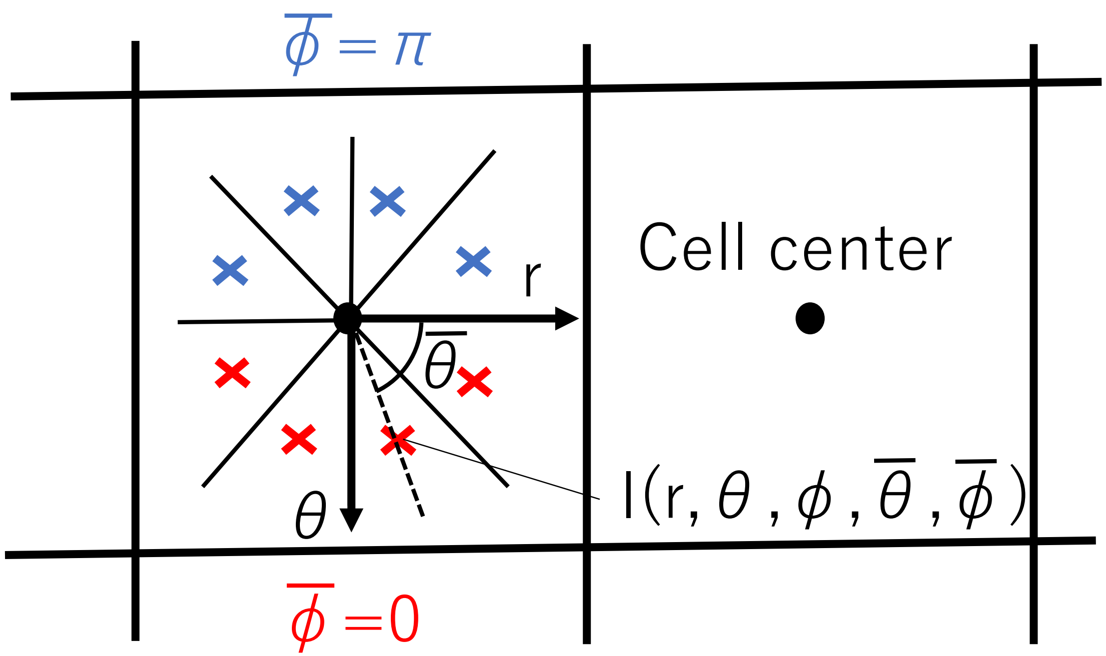
Update the radiation energy and radiation energy flux

Boltzmann Equation

$$
\begin{aligned}
&\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left[\left(e^\alpha_{(0)}+\sum_{i=1}^{3} l_{(i)} e^\alpha_{(i)}\right)\sqrt{-g} I\right] \\
&+\frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\sin\bar\theta\,\omega_{(\bar\theta)} I\right)+\frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\phi}\left(\omega_{(\bar\phi)} I\right)+\omega_{(0)} I \\
&= -\gamma(1-\boldsymbol{v}\cdot\boldsymbol{l})\rho(\kappa_{\rm abs}+\kappa_{\rm sca}) I + \gamma^{-3}(1-\boldsymbol{v}\cdot\boldsymbol{l})^{-3}\rho\left(\frac{j_0}{4\pi}+\kappa_{\rm sca}\frac{E_{\rm com}}{4\pi}\right)
\end{aligned}
$$



Shibata et al. (2014)

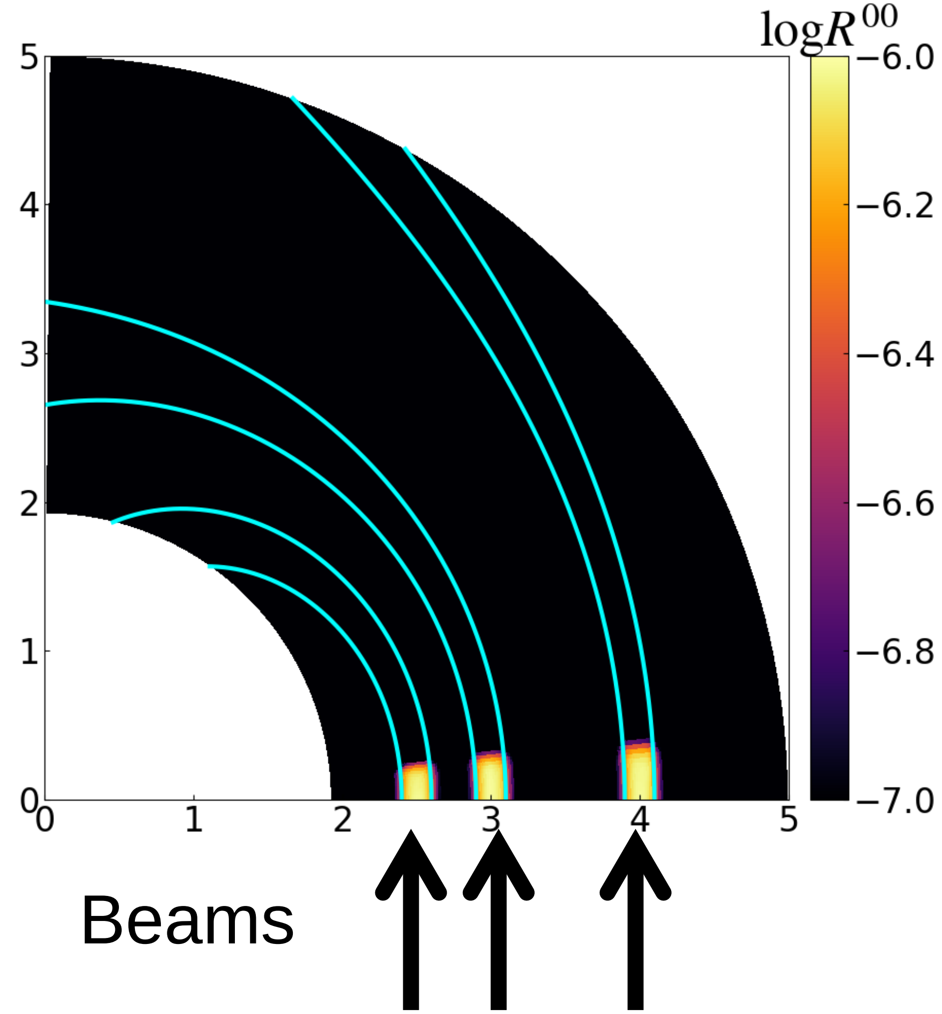


改めて目的

- ブラックホール近傍の輻射輸送をより正確に解く
- 輻射輸送スキームの違いにより、質量降着率や流出率、ジェットの加速にどのような違いが出るのかを明らかにする
  - 先行研究では流体の解き方を統一して調べたものはない
  - このコードでは流体の解き方は同じスキームなので純粋な輻射輸送スキームの比較が可能
- 最終的にはブラックホールのスピンや磁場などを変えて、ジェットの形成条件と速度などを調べていきたい

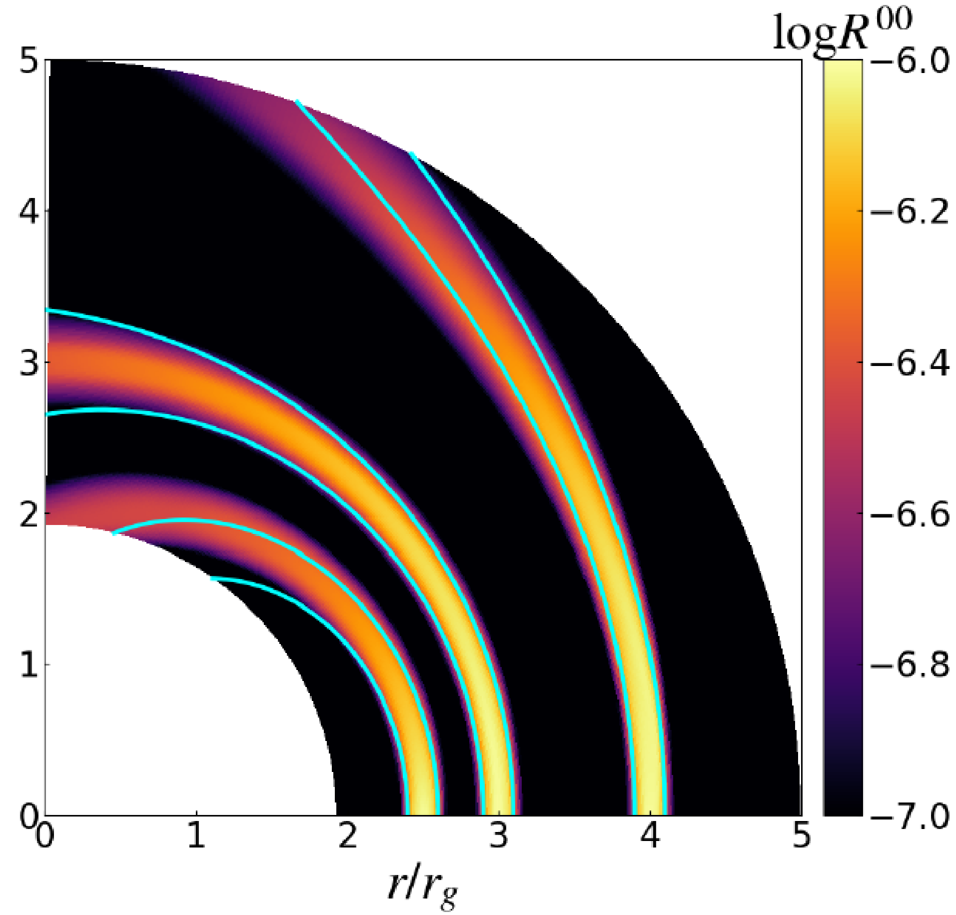
Radiative Transfer around Schwarzschild Black Hole

- Test for advection term and general relativistic effect
- The radius of the circular photon orbit is 3Rg
- Three beams are injected from φ=0 boundary around r = 2.5Rg, 3Rg, and 4Rg
- Beam width is 0.2Rg (6 grids)

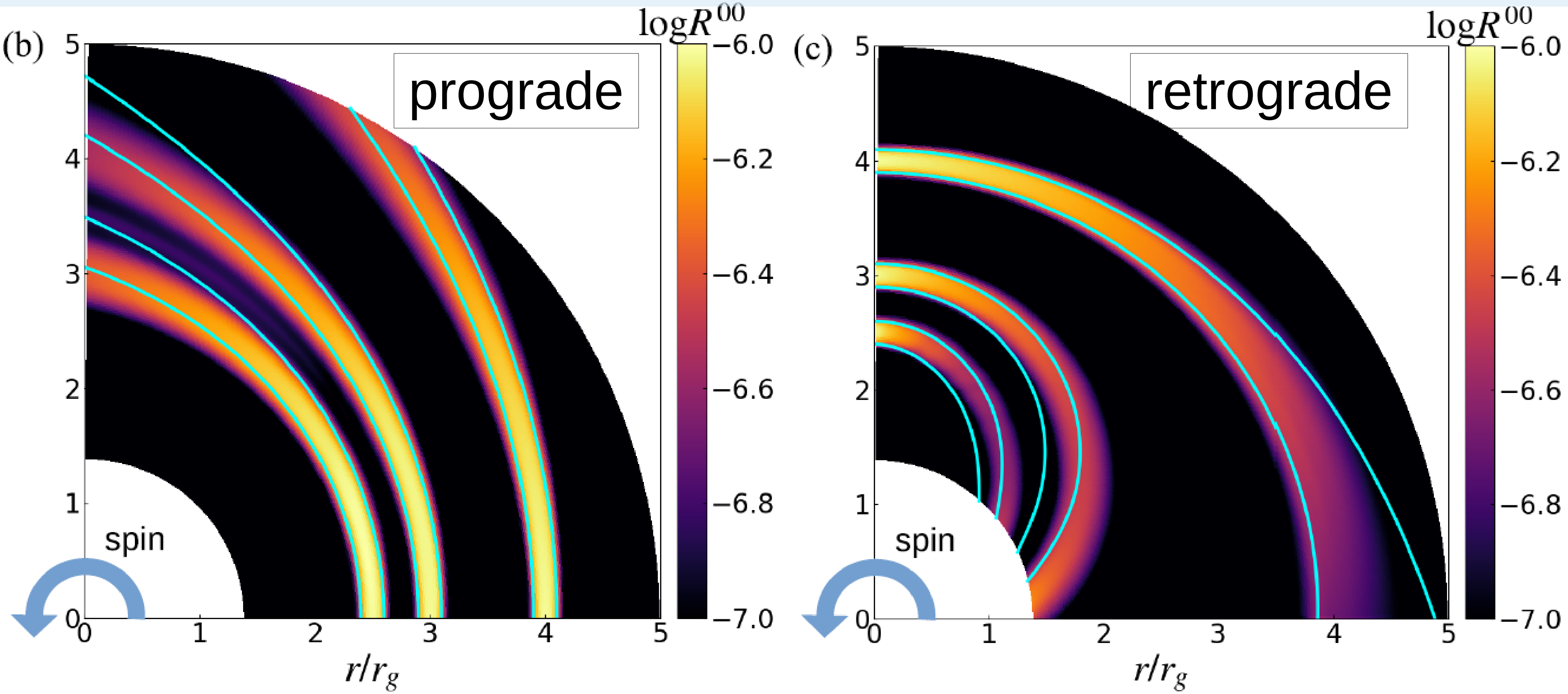


Radiative Transfer around Schwarzschild Black Hole

- Beams are bent by general relativistic effect
- The middle beam has circular orbit
- The inner beam falls into the black hole
- The outer beam can escape from the black hole
- The difference from the geodesic is due to the discreteness of $\Delta\bar{\theta}$~1-2°



Radiative Transfer around Kerr Black Hole (a=0.9)

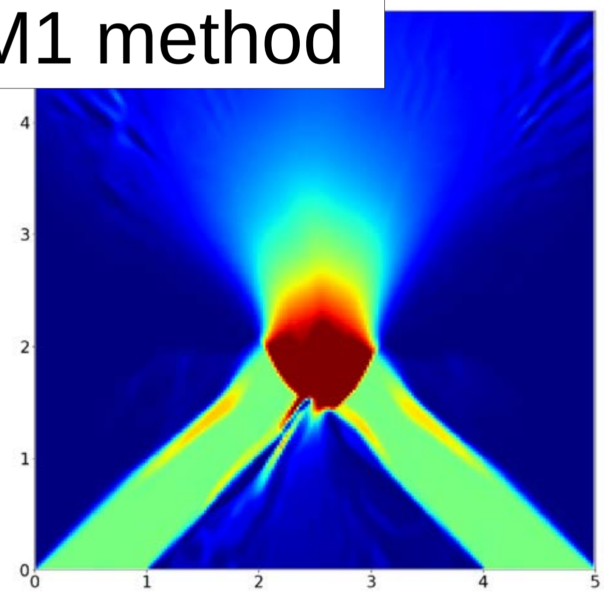


- Other conditions are the same as previous simulation
- For prograde model, the radius of the circular photon orbit is ~Rg
- For retrograde model, it is ~4Rg

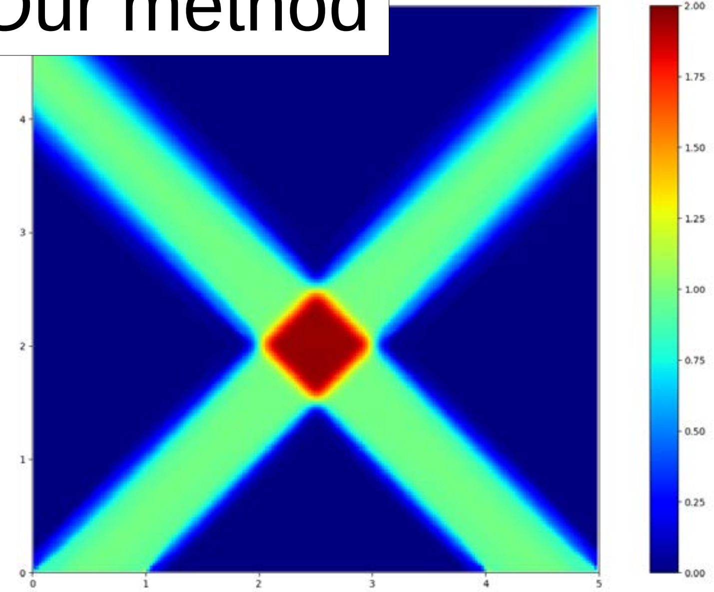
Beam Crossing Test

- Test for advection term
- Grids (x, y, $\bar{\theta}$) = (200, 200, 22)
- M1 method cannot solve the beam crossing since the radiation collides each other
- Our method can solve beam crossing successfully without collision

M1 method

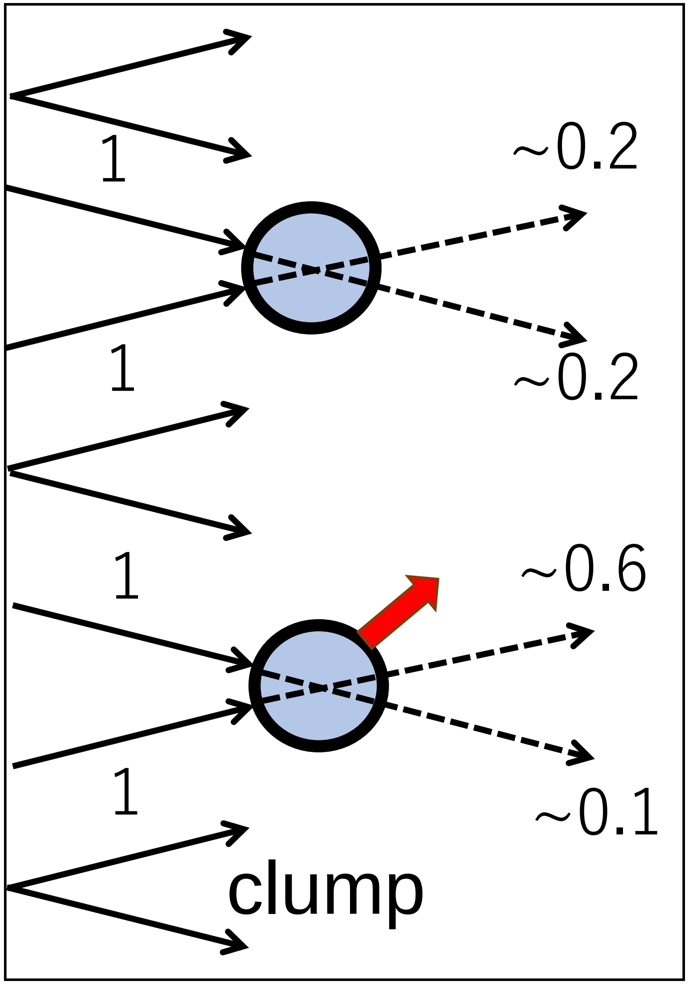


Our method



Shadow Test

- Test for absorption term
- Values of fluid are fixed
- We put two clumps with κabs=1.5
- Bottom clump is moving
- We inject 2 rays from the boundary



$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\alpha}\left[\left(e^\alpha_{(0)}+\sum_{i=1}^{3}l_{(i)}e^\alpha_{(i)}\right)\sqrt{-g}I\right]$$

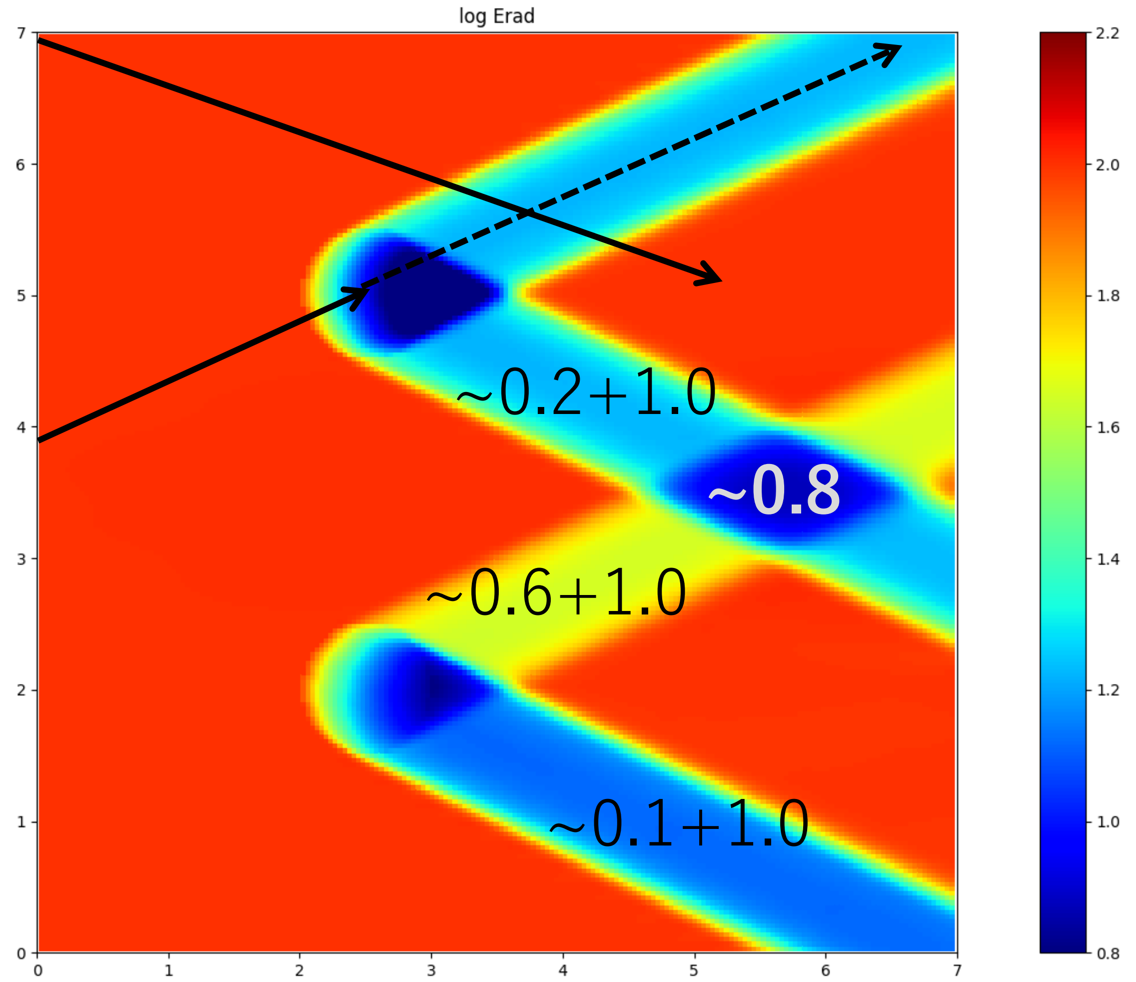
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\sin\bar{\theta}\omega_{(\bar{\theta})}I\right)+\frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\phi}}\left(\omega_{(\bar{\phi})}I\right)+\omega_{(0)}I$$

$$=\boxed{-\gamma(1-\boldsymbol{v}\cdot\boldsymbol{l})\rho(\kappa_{\rm abs}}+\kappa_{\rm sca})I+\gamma^{-3}(1-\boldsymbol{v}\cdot\boldsymbol{l})^{-3}\rho\left(\frac{j_0}{4\pi}+\kappa_{\rm sca}\frac{E_{\rm com}}{4\pi}\right)$$

Shadow Test

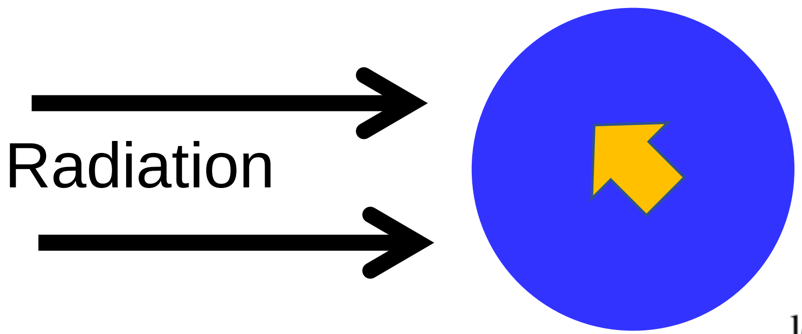
- We can get two shadows
- This result is consistent

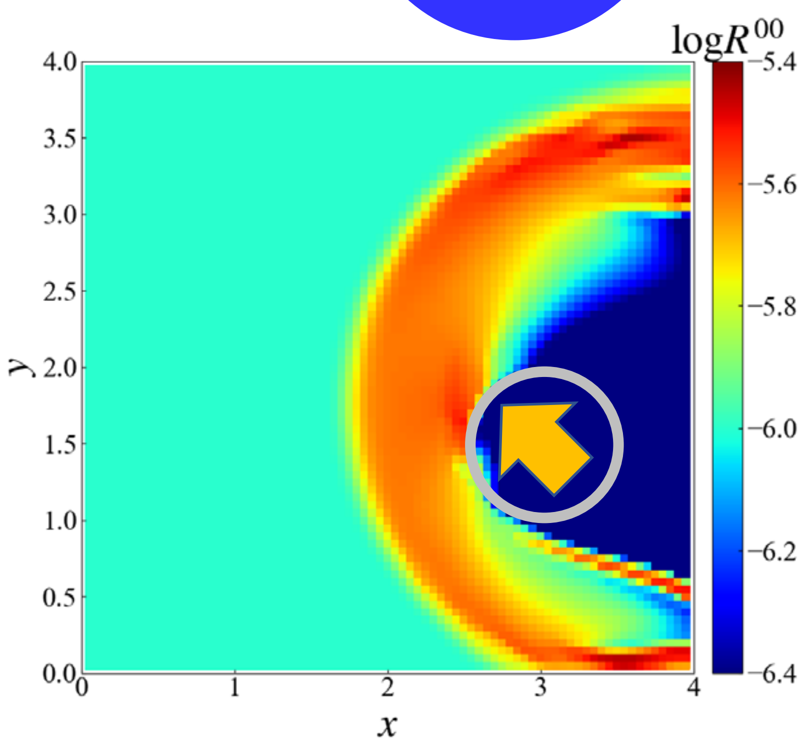
of that for Boltzmann equation



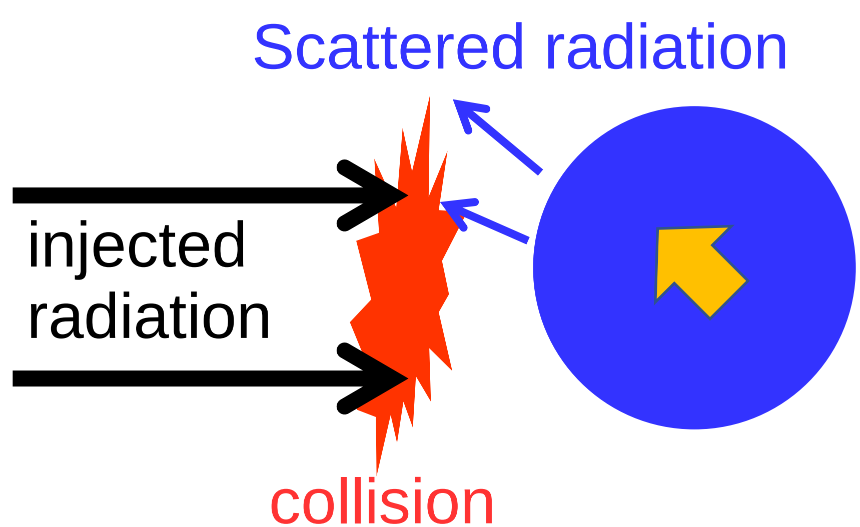
Interaction with optically thick cloud (M1 method)

Optically thick cloud for scattering

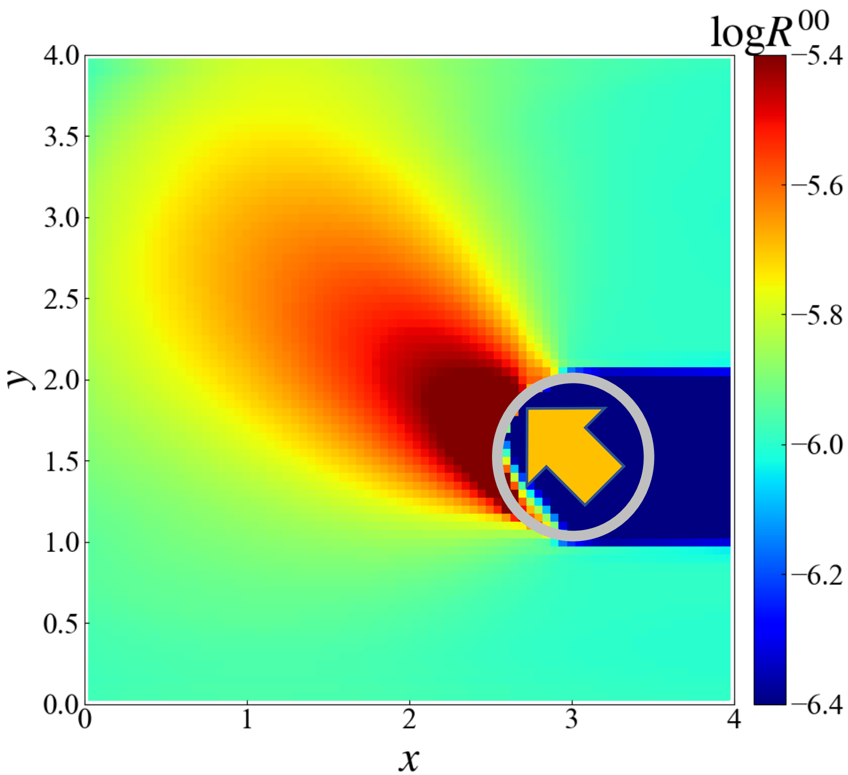


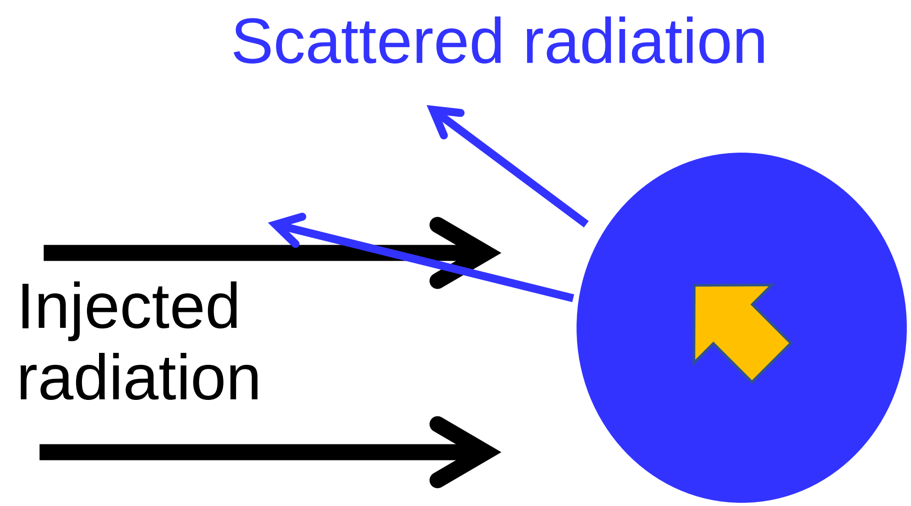


- Test for scattering term
- Optically thick cloud for scattering with velocity of 0.6c locates at (x,y)=(3, 1.5)
- M1 method cannot solve this test



Interaction with optically thick cloud (our method)

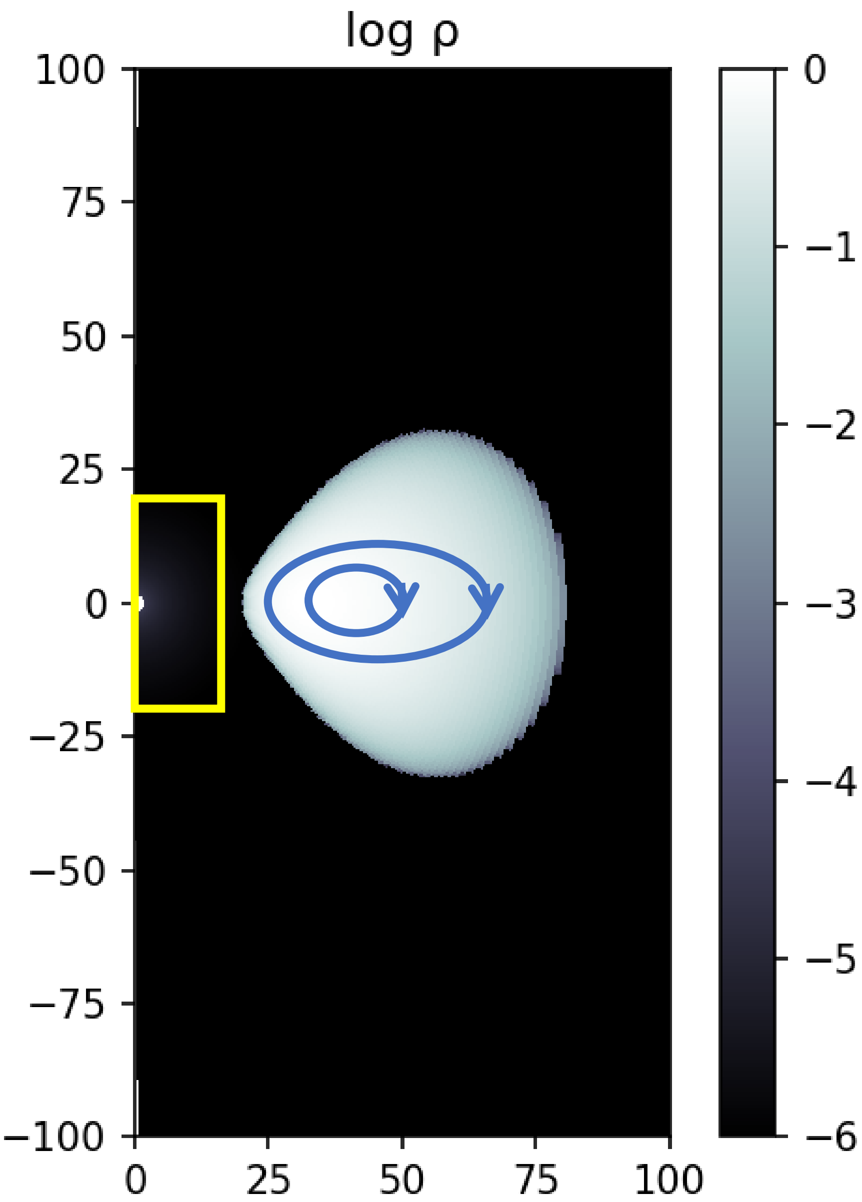




Scattered radiation propagates to left hand side

Our method can solve the interaction of radiation with optically thick cloud for scattering without non-physical collision of radiation.

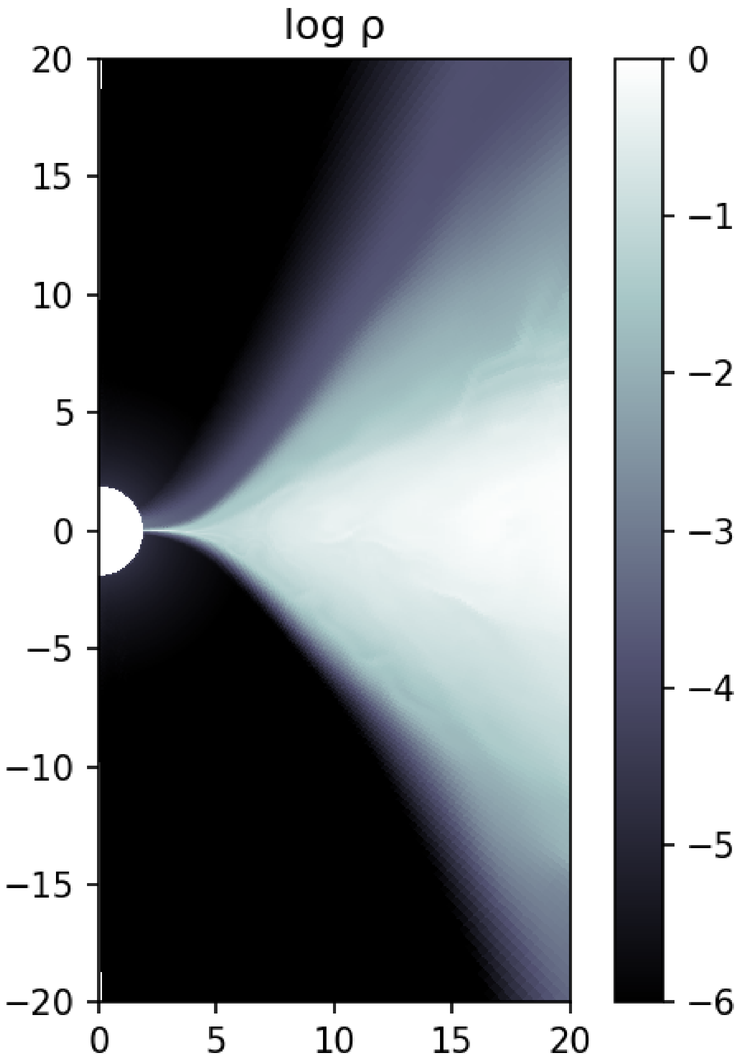
Initial condition



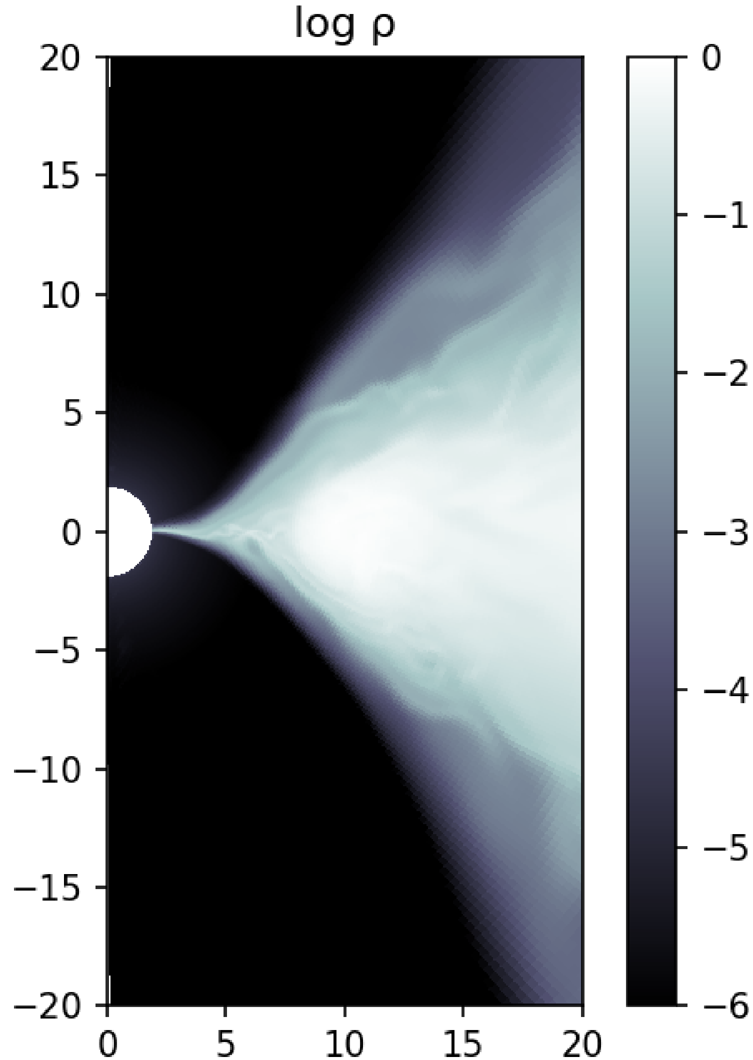
- We start simulations from an equilibrium torus given by Fishbone & Moncrief (1976)
- We assume the weak poloidal magnetic field in the torus
- The radiation energy is assumed to be much small
- $(N_r, N_\theta, N_{\bar{\theta}}, N_{\bar{\varphi}}) = (300, 300, 8, 16)$

Time evolution of the density

M1 method

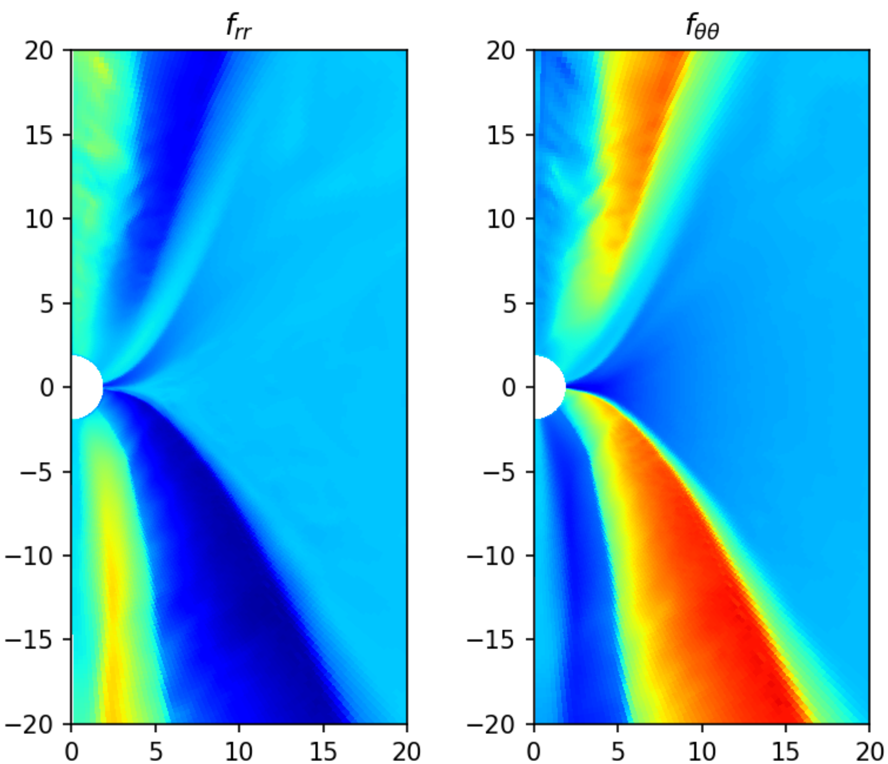


our method

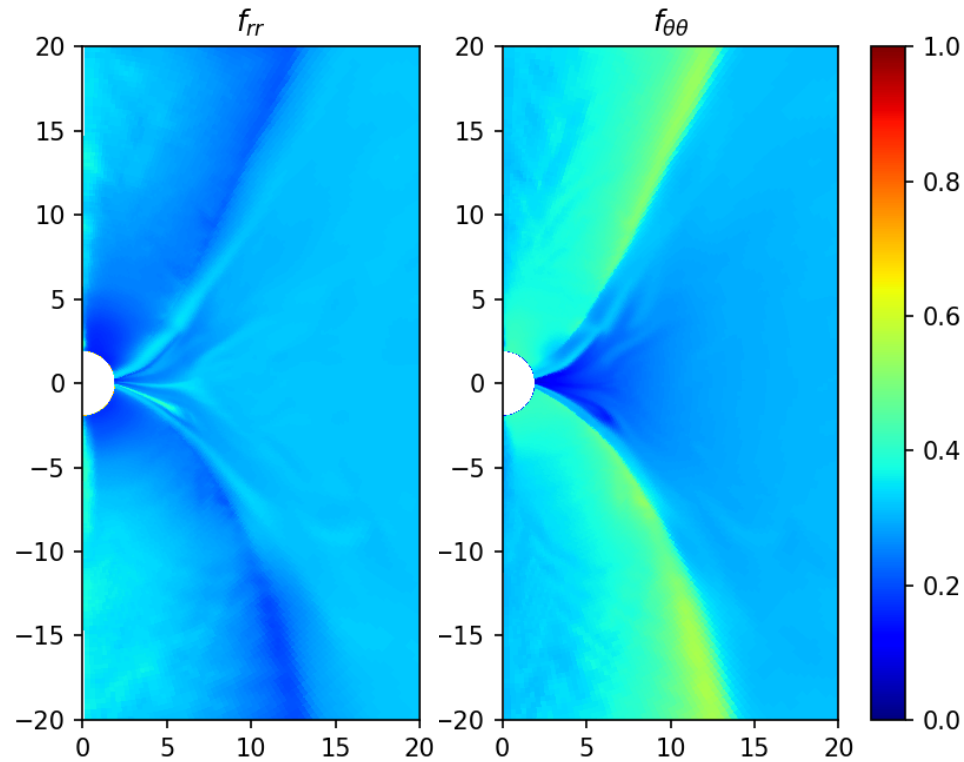


Collision around the rotation axis

M1 method



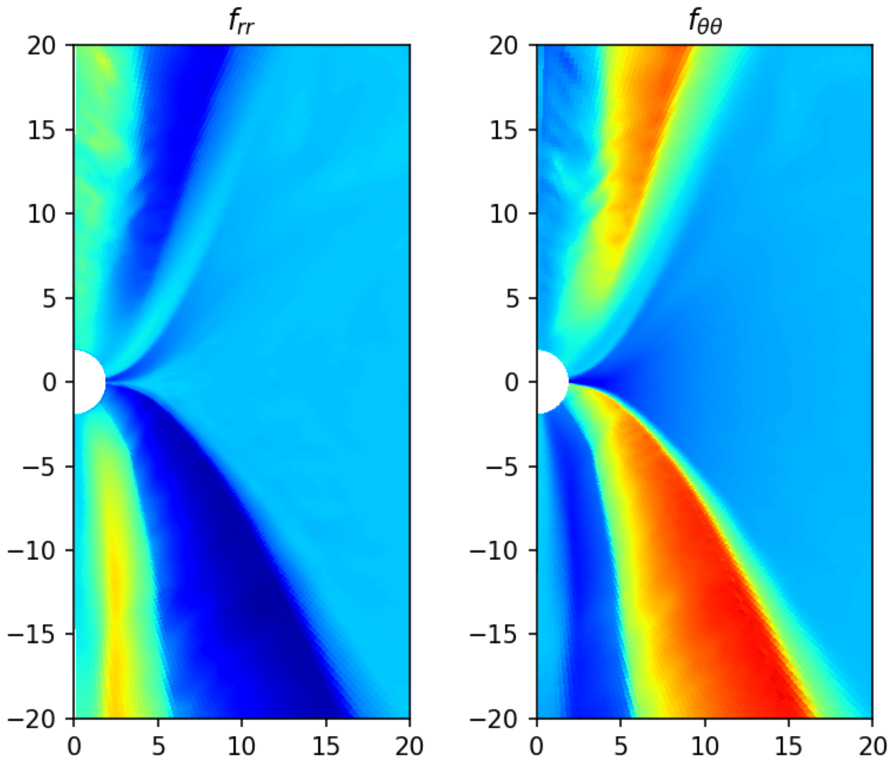
our method



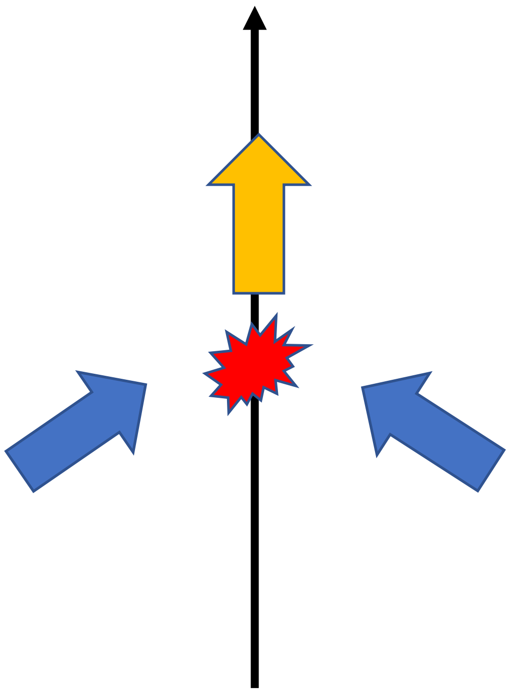
- For our method, the rr-component of the Eddington tensor becomes smaller and θθ-component becomes larger
- This can be due to the collision around the rotation axis

Collision around the rotation axis

M1 method



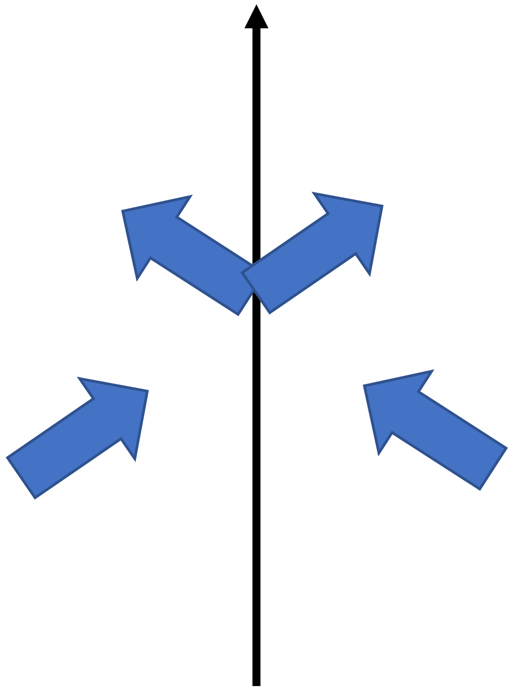
Rotation axis



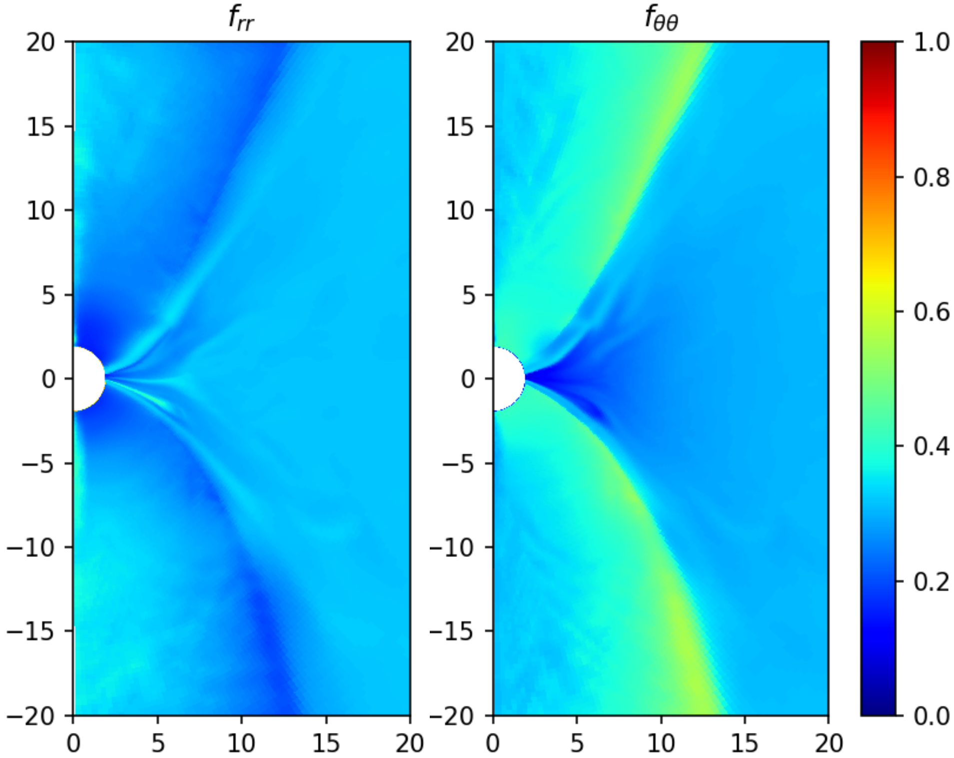
- When the radiation collides around the rotation axis, the flux in θ direction becomes zero
- The rr-component of the Eddington tensor becomes large

Collision around the rotation axis

Rotation axis



our method

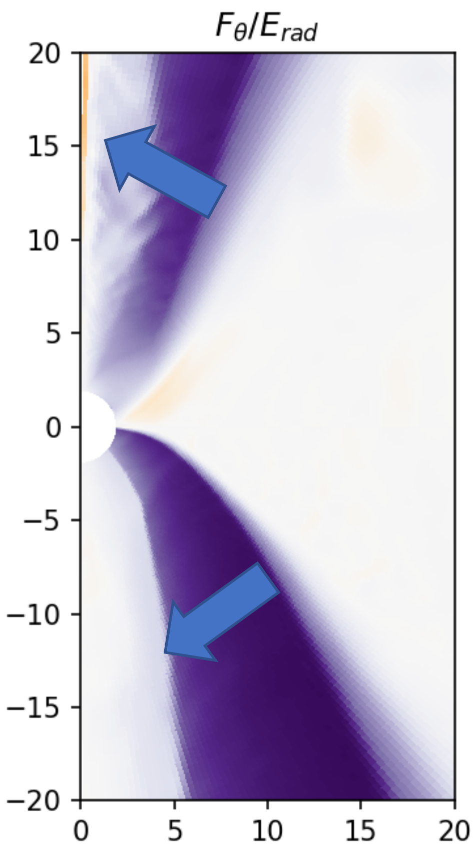


- The rr-component of the Eddington tensor becomes smaller since the specific intensity in θ-direction remains

降着流からの輻射

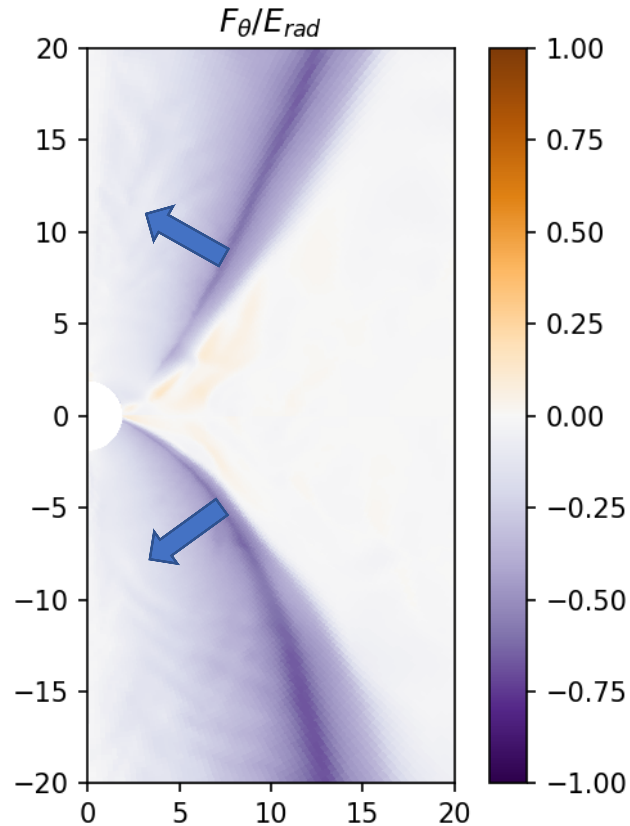
M1 method

Fθ/Erad

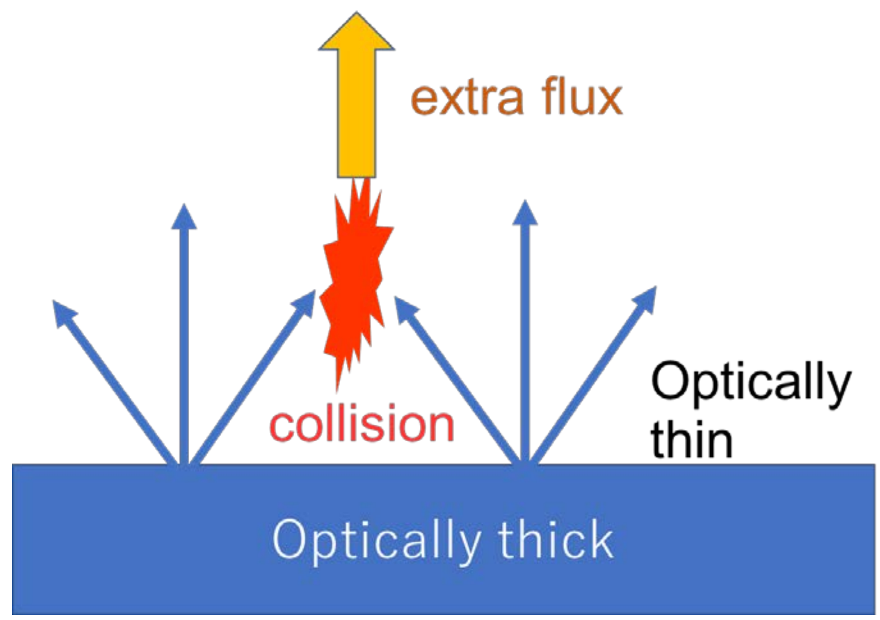


our method

Fθ/Erad

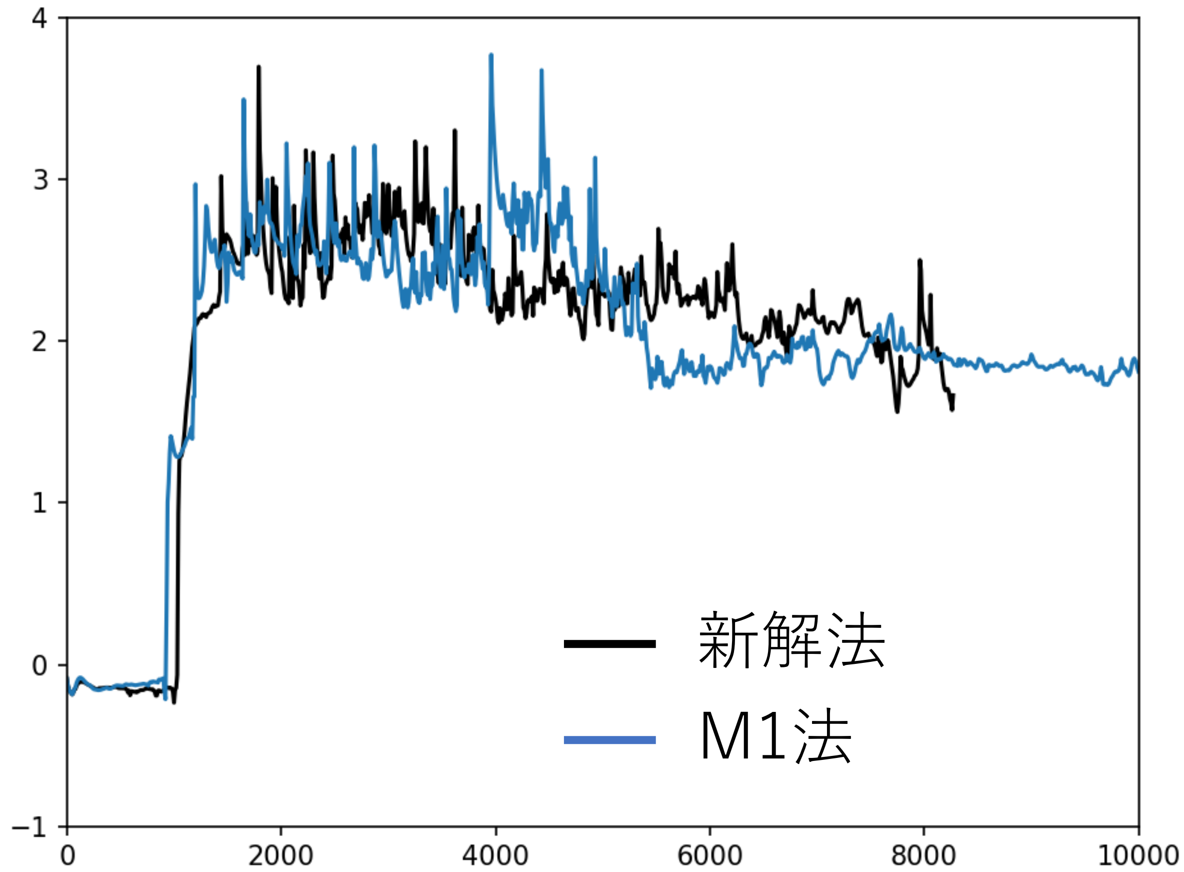


Schematic picture of the generation of extra flux



・M1法の方がθ方向に抜ける輻射エネルギーの割合を過剰評価している

Mass accretion rate



- 全体的な傾向は大きくは変わらないが、時期によっては数倍程度変わる可能性がある
- T=5000Rg/c以降はM1法は降着率がほぼ定常になっているが、新解法ではまだ変動している

Summary

- We perform some test simulations and apply to the accretion flow solving the Boltzmann equation
- Our method is superior to M1 method (e.g. beam crossing, interaction of the optically thick cloud)
- Our scheme can solve the radiation transfer around the rotation axis more exactly
- We will perform simulations with various density, magnetic fields, spin parameter