物理点格子 QCD による ストレンジネス S=-1 セクタのバリオン间力

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Baryon force in strangeness S=-1 sector from physical point lattice QCD H. Nemura¹, for HAL QCD Collaboration S. Aoki², T. Aoyama², T. Doi³, T. M. Doi³, F. Etminan⁴, S. Gongyo³, T. Hatsuda³, Y. Ikeda¹, T. Inoue⁵, T. Iritani³, N. Ishii¹, D. Kawai², T. Miyamoto², K. Murano¹, and K. Sasaki², Hadrons to Atomic nuclei from Lattice QCD ¹Osaka University, ²Kyoto University, ³RIKEN, ⁴University of Birjand, ⁵Nihon University

arXiv:1810.04046 [hep-lat]

Outline

Introduction

Importance of LN-SN tensor force for hypernuclei

Brief introduction of HAL QCD method

Effective block algorithm for various baryon-baryon channels, CPC207,91(2016)[1510.00903]
Preliminary results of LN-SN potentials at nearly physical point; update from [1702.00734]
LN-SN(I=1/2), central and tensor potentials
SN(I=3/2), central and tensor potentials

Phase shifts of SN(I=3/2) scattering

Summary

Plan of research



Baryon interaction



J-PARC, JLab, GSI, MAMI, ... YN scattering, hypernuclei





Structure and reaction of (hyper)nuclei

Equation of State (EoS) of nuclear matter

Neutron star and supernova







What is realistic picture of hypernuclei?

 $\otimes B(\text{total}) = B(^{4}\text{He}) + B_{\Lambda}(^{5}\text{He})$

A conventional picture: B(total) $= B(^{4}\text{He}) + B_{\Lambda}(^{5}\text{He})$ = 28+3 MeV.



Comparison between d=p+n and core+Y

	5 //// n	³ D 00000 p	n	L=0 Μ α Λ	L=2 ΛΛΥΟ α΄Σ
	$\langle T_S \rangle$	$\langle T_D \rangle$	$\langle V_{NN}(\text{central})\rangle$	$\langle V_{NN}(\text{tensor})\rangle$	$\langle V_{NN}(LS) \rangle$
	(MeV)	(MeV)	(MeV)	(MeV)	(MeV)
AV8	8.57	11.31	-4.46	-16.64	-1.02
G3RS	10.84	5.64	-7.29	-11.46	0.00
	$\langle T_{Y-c} \rangle_{\Lambda} \langle T$	$Y_{-C}\rangle_{\Sigma} + \Delta \langle H_C \rangle$	<pre><vy>(のこり)></vy></pre>	$2 \langle V_{AN-\Sigma N}$ (tensor))>
^{5}He	9.11	3.88+4.68	-0.86	-19.51	
$\Lambda^4 H^*$	5.30	2.43+2.02	0.01	-10.67	
^{4}H	7.12	2.94+2.16	-5.05	-9.22	

HN, Akaishi, Suzuki, PRL89, 142504 (2002).

Rearrangement effect of ⁵He



$$H = \sum_{i=1}^{A} \left(m_{i}c^{2} + \frac{p_{i}^{2}}{2m_{i}} \right) - T_{CM} + \sum_{i
$$H_{core} = \sum_{i=1}^{A-1} \frac{p_{i}^{2}}{2m_{N}} - \frac{\left(\sum_{i=1}^{A-1} p_{i}\right)^{2}}{2(A-1)m_{N}} + \sum_{i$$$$

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PRL 94, 202502 (2005)

PHYSICAL REVIEW LETTERS



FIG. 1. Λ and $\Lambda\Lambda$ separation energies of A = 3 - 6, S = -1 and -2 s-shell hypernuclei. The Minnesota NN, D2' YN, and mND_S YY potentials are used. The width of the line for the experimental B_{Λ} or $B_{\Lambda\Lambda}$ value indicates the experimental error bar. The probabilities of the N Ξ , $\Lambda\Sigma$, and $\Sigma\Sigma$ components are also shown for the $\Lambda\Lambda$ hypernuclei.

Lattice QCD calculation



Multi-hadron on lattice i) basic procedure: asymptotic region --> phase shift ii) HAL's procedure: interacting region --> potential





Multi-hadron on lattice Lattice QCD simulation $L = -\frac{1}{4} G^{a}_{\mu\nu} G^{a\mu\nu} + \bar{q} \gamma^{\mu} (i \partial_{\mu} - g t^{a} A^{a}_{\mu}) q - m \bar{q} q$ $\langle O(\bar{q}, q, U) \rangle = \int dU d \bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U)$ $= \int dU \det D(U) e^{-S_{U}(U)} O(D^{-1}(U))$ $= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$ $\rightarrow \langle \underbrace{\mathbf{v}}_{p\Lambda} (t) \underbrace{\mathbf{v}}_{p\Lambda} (t_0) \rangle \rangle$ $p\Lambda$

Multi-hadron on lattice i) basic procedure: asymptotic region (or temporal correlation) --> scattering energy --> phase shift $=\frac{1}{2u}$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$
$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in Z^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re s > \frac{3}{2}$$

Luscher, NPB354, 531 (1991). Aoki, et al., PRD71, 094504 (2005).



Calculate the scattering state

Multi-hadron on lattice ii) HAL's procedure: make better use of the lattice output ! (wave function) interacting region --> potential Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010).

NOTE:

> Potential is not a direct experimental observable.
> Potential is a useful tool to give (and to reproduce)
the physical quantities. (e.g., phase shift)

Multi-hadron on lattice ii) HAL's procedure: make better use of the lattice output ! (wave function) interacting region --> potential Ishii, Aoki, Hatsuda, PRL99, 022001 (2007); ibid., PTP123, 89 (2010). > Phase shift
> Nuclear many-body problems

The potential is obtained at moderately large imaginary time; no single state saturation is required.

¹The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg$ $1/m_{\pi} \sim 1.4$ fm In addition, no single state saturation between the ground state and the excited states with respect to the relative motion, e.g., $t - t_0 \gg (\Delta E)^{-1} = ((2\pi)^2/(2\mu(La)^2))^{-1} \simeq 8.0$ fm, s required for the HAL QCD method[13]. **RECIPE:** Compute the 4pt correlator $F_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0) = \sum_{\vec{v}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t)B_{2,\beta}(\vec{X},t)\overline{\mathscr{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle,$ (2.3)Take into account the threshold energy differences for coupled-channel system $R_{\alpha\beta_{J}IM}^{\langle B_{1}B_{2}\overline{B_{3}B_{4}}\rangle}(\vec{r},t-t_{0}) = e^{(m_{B_{1}}+m_{B_{2}})(t-t_{0})}F_{\alpha\beta_{J}IM}^{\langle B_{1}B_{2}\overline{B_{3}B_{4}}\rangle}(\vec{r},t-t_{0})$ $= \sum_{\vec{x}} A_n \sum_{\vec{x}} \left\langle 0 \left| B_{1,\alpha}(\vec{x} + \vec{r}, 0) B_{2,\beta}(\vec{x}, 0) \right| E_n \right\rangle e^{-(E_n - m_{B_1} - m_{B_2})(t-t_0)} + \underbrace{O(e^{-(E_{th} - m_{B_1} - m_{B_2})(t-t_0)}_{\text{inelastic}} (2.4) \\ \text{inelastic}_{\text{inelastic}} (2.4) \\ \text{Obtain the potential by using the appropriate equation(s); For spin-singlet,} \\ \left(\frac{\nabla^2}{2\mu_{\lambda}} - \frac{\partial}{\partial t} \right) R_{\lambda\varepsilon}(\vec{r}, t) \simeq V_{\lambda\lambda'}^{(LO)}(\vec{r}) \theta_{\lambda\lambda'} R_{\lambda'\varepsilon}(\vec{r}, t), \text{ with } \theta_{\lambda\lambda'} = e^{(m_{B_1} + m_{B_2} - m_{B_1'} - m_{B_2'})(t-t_0)}.$ For spin-triplet, the "tensor force" becomes active $\begin{cases} \mathscr{P} \\ \mathscr{Q} \end{cases} \times \left\{ V_{\lambda\lambda'}^{(0)}(r) + V_{\lambda\lambda'}^{(\sigma)}(r) + V_{\lambda\lambda'}^{(T)}(r)S_{12} \right\} \theta_{\lambda\lambda'}R_{\lambda'\varepsilon}(\vec{r}, t-t_0) = \begin{cases} \mathscr{P} \\ \mathscr{Q} \end{cases} \times \left\{ \frac{\nabla^2}{2\mu_{\lambda}} - \frac{\partial}{\partial t} \right\} R_{\lambda\varepsilon}(\vec{r}, t-t_0), \end{cases}$ Where $\begin{cases} R(\vec{r}; {}^{3}S_{1}) = \mathscr{P}R(\vec{r}; J = 1) \equiv \frac{1}{24} \sum_{\mathscr{R} \in O} \mathscr{R}R(\vec{r}; J = 1), \\ R(\vec{r}; {}^{3}D_{1}) = \mathscr{Q}R(\vec{r}; J = 1) \equiv (1 - \mathscr{P})R(\vec{r}; J = 1). \end{cases}$ (2.6)In the lowest few orders, we have $V(\vec{r}, \vec{\nabla}_r) = V^{(0)}(r) + V^{(\sigma)}(r)\vec{\sigma}_1 \cdot \vec{\sigma}_2 + V^{(T)}(r)S_{12} + V^{(LS)}(r)\vec{L} \cdot (\vec{\sigma}_1 \pm \vec{\sigma}_2) + O(\nabla^2),$ (2.5)

An improved recipe for NY potential: ©cf. Ishii (HAL QCD), PLB712 (2012) 437.

Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^{2}R(t,\vec{r}) + \int d^{3}r' U(\vec{r},\vec{r}')R(t,\vec{r}') = -\frac{\partial}{\partial t}R(t,\vec{r})$$

$$U(\vec{r},\vec{r}') = V_{NY}(\vec{r},\nabla)\delta(\vec{r}-\vec{r}')$$
***** A general expression of the potential:
$$V_{NY} = V_{0}(r) + V_{\sigma}(r)(\vec{\sigma}_{N}\cdot\vec{\sigma}_{Y})$$

$$+ V_{T}(r)S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_{+})$$

$$+ V_{ALS}(r)(\vec{L}\cdot\vec{S}_{-}) + O(\nabla^{2})$$

Determination of baryon-baryon potentials at nearly physical point

Effective block algorithm for various baryon-baryon correlators

HN, CPC207,91(2016), arXiv:1510.00903(hep-lat)

Numerical cost (# of iterative operations) in this algorithm $1 + N_{c}^{2} + N_{c}^{2} N_{a}^{2} + N_{c}^{2} + N_{c}^{2$ In an intermediate step: $(N_{c}! N_{\alpha})^{B} \times N_{\mu}! N_{d}! N_{s}! \times 2^{N_{A}+N_{\Sigma^{0}}-B} = 3456$ In a naïve approach: $(N_{c} ! N_{a})^{2B} \times N_{\mu} ! N_{d} ! N_{s} ! = 3,981,312$ p⁽¹⁾ p⁽²⁾ p⁽⁴⁾ A⁽¹⁾ р⁽³⁾ $\Lambda^{(4)}$ p⁽⁵⁾ Λ⁽⁵⁾ $\Lambda^{(2)}$ A⁽⁶⁾ (ud)u (ud)u (ds)u (ds)u (ud)u (ds)u (ds)u (-1)^σ=(-) $(-1)^{\sigma} = (+)$ (-1)^σ=(+) (-1)^o=(+) (-1)⁶=(-) (-1)^a=(-)

Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

Make better use of the computing resources!

HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]], (See also arXiv:1604.08346)

Benchmark (from NN to XiXi channels)



numerical results of the correlators of entire 52 channels from NN to $\Xi\Xi$ systems given in Eqs. (32)-(36), over 31 time-slices, 16³ points for spatial, and 2⁴ points for the spin degrees of freedom, obtained by using this effective block algorithm (dot) and by using the unified contraction algorithm (open circle) as a function of one-dimensionally aligned data point $\xi = \tilde{\alpha} + 2(\tilde{\beta} + 2(\tilde{\alpha}' + 2(\tilde{\beta}' + 2(x + 16(y + 16(z + 16(c + 52((t - t_0 + T) \mod T)))))))))$, where $c = 0, \dots, 51$ selects one of the 52 channels. The absolute value of their difference is also shown (triangle).



Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

⊗ APE-Stout smearing (ρ =0.1, n_{stout}=6)

Son-perturbatively O(a) improved Wilson Clover action at β=1.82 on 96³ × 96 lattice



DDHMC(ud) and UVPHMC(s) with preconditioning
K.-I.Ishikawa, et al., PoS LAT2015, 075;
arXiv:1511.09222 [hep-lat].

[●] NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x Nsrc (Nsrc=4 → 20 → 52 → 96 (2015FY+))

LN-SN potentials at nearly physical point

The methodology for coupled-channel V is based on: Aoki, et al., Proc.Japan Acad. B87 (2011) 509. Sasaki, et al., PTEP 2015 (2015) no.11, 113B01. Ishii, et al., JPS meeting, March (2016). Nemura, et al., [1702.00734].

#stat: (this/scheduled in FY2015+) 0.2 0.54 1.00 $t-t_0=5-12, t-t_0=5-14$

 $\begin{array}{cccc} \Lambda N - \Sigma N & (I = 1/2) \\ V_{C} \begin{pmatrix} 1 & S_{0} \end{pmatrix} & V_{C} \begin{pmatrix} 3 & S_{1} - 3 & D_{1} \end{pmatrix} & V_{T} \begin{pmatrix} 3 & S_{1} - 3 & D_{1} \end{pmatrix} \\ \Sigma N & (I = 3/2) \\ V_{C} \begin{pmatrix} 1 & S_{0} \end{pmatrix} & V_{C} \begin{pmatrix} 3 & S_{1} - 3 & D_{1} \end{pmatrix} & V_{T} \begin{pmatrix} 3 & S_{1} - 3 & D_{1} \end{pmatrix} \\ \end{array}$

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Nemura, et al., [1/02.00.1] #stat: (this/scheduled in FY2015+) 0.2 0.54 1.00 2.00

 $\Lambda N - \Sigma N (I = 1/2)$ $t - t_0 = 5 - 12, t - t_0 = 5 - 14$ $V_C ({}^{1}S_0)$ $V_C ({}^{3}S_1 - {}^{3}D_1)$ $V_T ({}^{3}S_1 - {}^{3}D_1)$

 $\sum_{C} (V_{C}(V_{C})) = V_{C}(V_{C}(V_{C}) + D_{1}) = V_{T}(V_{C}(V_{C}) + D_{1})$ $\sum_{C} (V_{C}(V_{C}) + D_{1}) = V_{T}(V_{C}(V_{C}) + D_{1}) = V_{T}(V_{T}(V_{C}) + D_{1})$

Effective mass plot of the single baryon's correlation function



Potentials obtained at t-t0 = 5 to 12 are shown; For the largest statistics, t-t0 = 13 and 14 will also be shown.

TABLE 4

The eigenvalues of the normalization kernel in eq. (3.3) for S = -1two-baryon (BB) system

S = -1

 	<u> </u>			
I	J	BB	Eigenvalues (uncoupled)	Eigenvalues (coupled)
1 2	0	NЛ	1	$0 \frac{10}{9}$
-		NΣ	$\frac{1}{9}$	
$\frac{1}{2}$	1	NΛ	1	$\frac{8}{9}$ $\frac{10}{9}$
		NΣ	1	
$\frac{3}{2}$	0	NΣ	<u>10</u> 9	
<u>3</u> 2	1	NΣ	<u>2</u> 9	

Oka, Shimizu and Yazaki (1987)

Eigenvalues of single and coupled channels are given.





Very preliminary result of LN potential at the $V_{T}(^{-3}S_{1}-^{-3})$ physical point $\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right)$

$$R(\vec{r},t) = \int d^3r' U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdot (\vec{r},t) +$$

V2x2lapl-vte03prg v2x2kk2m-vte03pt.3D1.BBpLwavepLpH1



(8)



Very preliminary result of LN potential at the physical point

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r}, t) = \int d^3r' U(\vec{r}, \vec{r}') R(\vec{r}', t) + O(k^4) = V_{\rm LO}(\vec{r}) R(\vec{r}, t) + \cdot (8)$$





Summary

(I-1) Latest results of LN-SN potentials at nearly physical point. (Lambda-N, Sigma-N: central, tensor)

Statistics approaching to 1.96 => 2.0 (=present/scheduled in 2015) Increasing statistics, the result at large t converges to the result at next-to-smaller t (with poor statistics).

Signals in spin-triplet are relatively going well smoothly. The channels that the quark model predicts strongly repulsive have relatively poor signals; Simultaneous calcs (NN to XiXi) is the point we have to take into account for the comprehensive perspective as well as energy-computing-resource efficiency. (1-2) Effective hadron block algorithm for the various baron-baryon interaction

Comput.Phys.Commun.207,91(2016) [arXiv:1510.00903(hep-lat)] The algorithm will be applied to more wide range problems. Future work:

(II-1) Physical quantities including the binding energies of fewbody problem of light hypernuclei with the lattice YN (and NN) potentials