格子 QCD における核子２体計算の現状と展望

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Refs HAL Coll.,

JHEP 1610(2016)101[arXiv:1607.06371], **PoS(Lattice2016)107**[arXiv:1610.09779], **PoS(Lattice2016)109**[arXiv:1610.09763], **PoS(Lattice2015)089**[arXiv:1511.05246].

• S. Aoki, K. Sasaki, D. Kawai, T. Miyamoto (YITP) • T. Doi, T. Hatsuda (RIKEN) • T. Inoue (Nihon Univ.) • N. Ishii, Y. Ikeda, K. Murano (RCNP)

• H. Nemura (Univ. of Tsukuba) • S. Gongyo (Univ. of Tours) • F. Etminan (Univ. of Birjand)

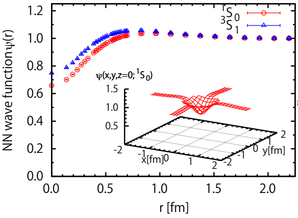
Goal: Hadron Interaction based on QCD

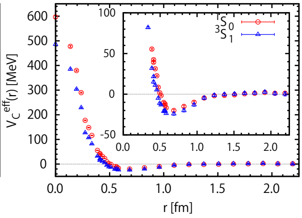
1. **Lüscher's finite volume method**
   energy shift of two-particle in **"box"** ▶ phase shift

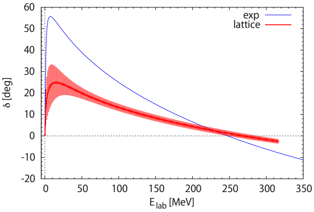
$$\Delta E_L = 2\sqrt{k^2+m^2} - 2m \implies k\cot\delta(k) = \frac{1}{\pi L}\sum_{\boldsymbol{n}\in\mathbb{Z}^3}\frac{1}{|\boldsymbol{n}|^2-(kL/2\pi)^2}$$

2. **HAL QCD method**
   "spatial correlations" ▶ interaction

Nambu-Bethe-Salpeter Wavefunc. → Potential → phase shift





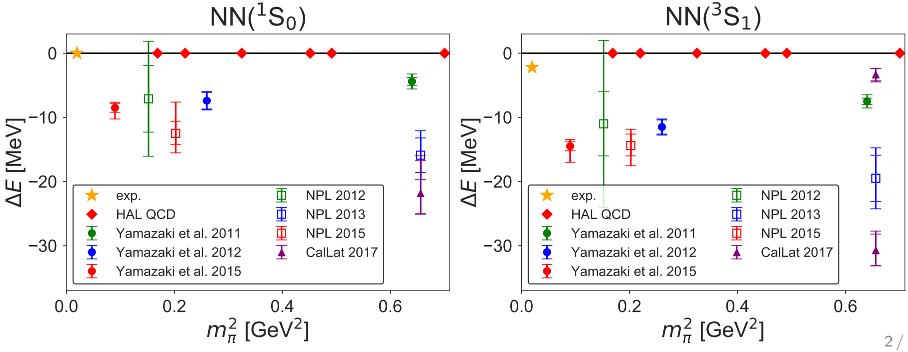


NN Systems from Lattice QCD

	"Lüscher"		HAL QCD	phys. point
dineutron ($^1S_0$)	**bound**	⇔	**unbound**	**unbound**
deuteron ($^3S_1$)	**bound**	⇔	**unbound**	**bound**

- Yamazaki et al., NPL Coll., CalLat — deeply bound state
- HAL QCD Coll. — weak attractive and unbound

⇒ **inconsistencies** between two methods, which is correct?



1. Lüscher's Finite Volume Method (Direct Method)
2. HAL QCD Method
3. Diagnosis of the Direct Method
4. Summary

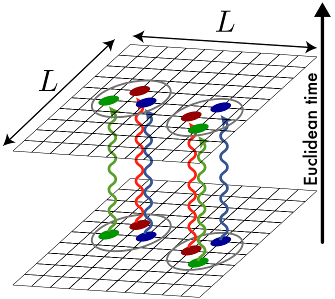
Lüscher's Finite Volume Method

- "**energy shift**" **in finite box** L^3

$$\Delta E_L = E_{BB} - 2m_B = 2\sqrt{k^2 + m_B^2} - 2m_B$$

$\Rightarrow$ **phase shift** $\delta(k)$

$$k \cot \delta(k) = \frac{1}{\pi L} \sum_{\boldsymbol{n} \in \mathbb{Z}^3} \frac{1}{|\boldsymbol{n}|^2 - (kL/2\pi)^2}$$



$\Uparrow$ **THEORY**

$\Downarrow$ **PRACTICE — "Direct Method"**

- **measure**: plateau in **effective mass**

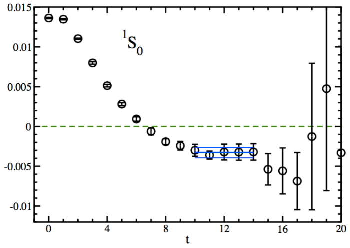
$$\Delta E_{\text{eff}}(t) = \log \frac{R(t)}{R(t+1)} \to \Delta E_L$$

and $\Delta E_L \to -\text{B.E.}\ (L \to \infty)$

$$R(t) = \frac{G_{BB}(t)}{\{G_B(t)\}^2} \to \exp\left[-\left(E_{BB} - 2m_B\right) t\right]$$

with $G_{BB}(t)(G_B(t))$: BB(B) correlators

- NN(^{1}S$_0$) (Yamazaki et al. '12)



Difficulties in Multi-Baryons

- Lüscher's method requires **ground state saturation**

$$G_{NN}(t) = c_0 \exp(-E_0^{(NN)} t) + c_1 \exp(-E_1^{(NN)} t) + \cdots \simeq c_0 \exp(-E_0^{(NN)} t)$$

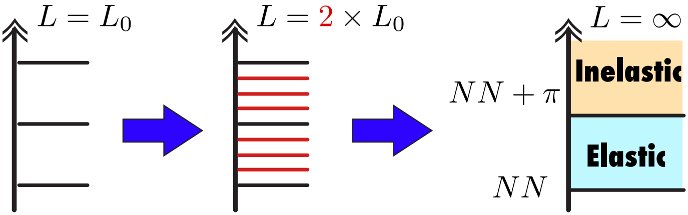
- precise measurement $\Delta E \ll m_B \sim \mathcal{O}(1\ \mathrm{GeV})$

$$E_0^{(NN)} - 2m_B = \Delta E \sim \mathcal{O}(10\ \mathrm{MeV})$$

- **S/N** problem: [mass number A] $\times$ [light quark] $\times$ [$t \to \infty$]

$$S/N \sim \exp\left[-A \times (m_N - (3/2)m_\pi) \times t\right]$$

- **smaller gap of scattering state:** $\Delta E \sim \vec{p}^{\,2}/m \sim \mathcal{O}(1/L^2)$



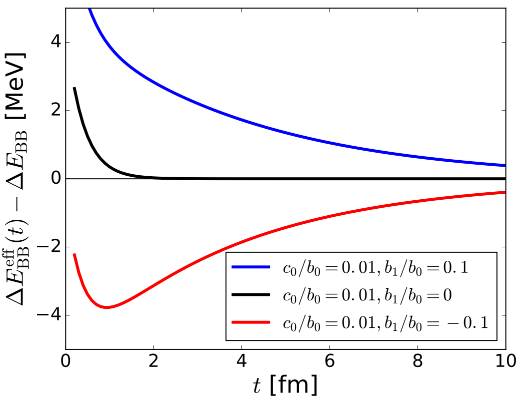
Contamination of Scattering State and Fake Plateau

example

$$R(t) = b_0 e^{-\Delta E_{\rm BB} t} + b_1 e^{-\delta E_{\rm el} t} + c_0 e^{-\delta E_{\rm inel} t}$$

$\delta E_{\rm el} - \Delta E_{\rm BB} = 50$ MeV $\sim \mathcal{O}(1/L^2)$, $\delta E_{\rm inel} - \Delta E_{\rm BB} = 500$ MeV $\sim \mathcal{O}(\Lambda_{\rm QCD})$

- g.s. saturation $\Delta E^{\rm eff}_{\rm BB}(t) - \Delta E_{\rm BB} \to 0$
- elastic saturation $t \sim 1$ fm



Contamination of Scattering State and Fake Plateau

example with **NOISE**

$$R(t) = b_0 e^{-\Delta E_{\rm BB} t} + b_1 e^{-\delta E_{\rm el} t} + c_0 e^{-\delta E_{\rm inel} t}$$

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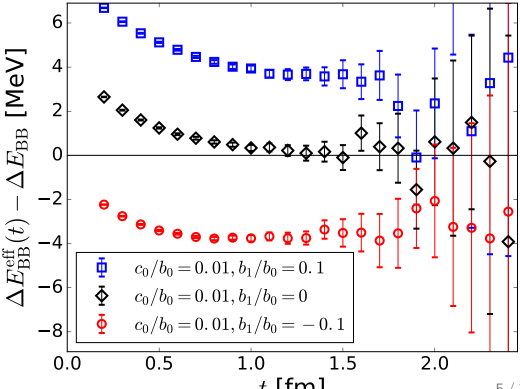
- g.s. saturation $\Delta E^{\rm eff}_{\rm BB}(t) - \Delta E_{\rm BB} \to 0$
- elastic saturation $t \sim 1$ fm
- few % of contamination

$\Rightarrow$ **"mirage"** of plateau around $t \sim 1 - 1.5$fm



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$\Rightarrow$ ground state can be checked by **quark source dependence**

different source $\Leftrightarrow$ different mixing

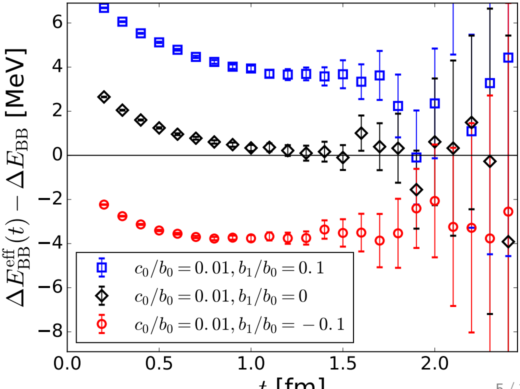
- g.s. saturation
  $\Delta E^{\rm eff}_{\rm BB}(t) - \Delta E_{\rm BB} \to 0$

- elastic saturation $t \sim 1$ fm

- few % of contamination

$\Rightarrow$ **"mirage"** of plateau

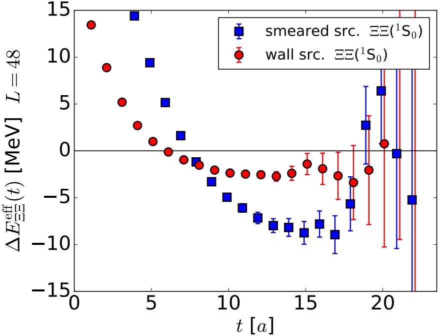
around $t \sim 1 - 1.5$fm



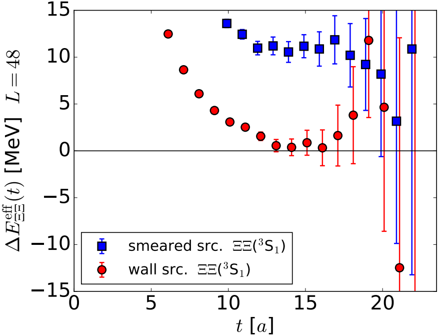
Energy Shift of ΞΞ: Smeared Src. vs. Wall Src.

$\Delta E_L^{\rm eff}(t) \longrightarrow \Delta E_L$??? — depends on quark source (smeared or wall)

$\Xi\Xi({}^1\mathrm{S}_0)$ at $48^3 \times 48$



$\Xi\Xi({}^3\mathrm{S}_1)$ at $48^3 \times 48$



- source dependence suggests these plateaux are **"fake" signal**

$L \to \infty$	**Smeared src.**	**Wall src.**
$\Delta E_{\Xi\Xi}({}^1\mathrm{S}_0)$	< 0 **bound**	$\simeq 0$ **unbound**
$\Delta E_{\Xi\Xi}({}^3\mathrm{S}_1)$	> 0 **unphysical**	$\simeq 0$ **unbound**

$(m_\pi = 0.51$ GeV$)$

Phase Shift Analysis — Sanity Check

Lüscher's formula

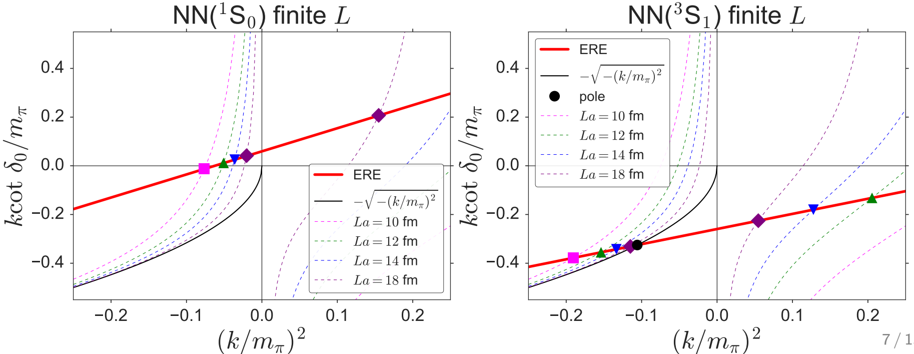
$$k \cot \delta_0(k) = \frac{1}{\pi L} \sum_{\mathbf{n}\in\mathbb{Z}^3} \frac{1}{|\mathbf{n}|^2 - (kL/2\pi)^2}$$

Effective Range Expansion

$$k \cot \delta_0(k) \simeq \frac{1}{a_0} + \frac{1}{2} r_{\text{eff}} k^2 + \cdots$$

cf. pole at $k_0 \cot \delta_0(k_0) = i k_0$

physical quark mass ($m_\pi = 140$ MeV)



Phase Shift Analysis — Sanity Check

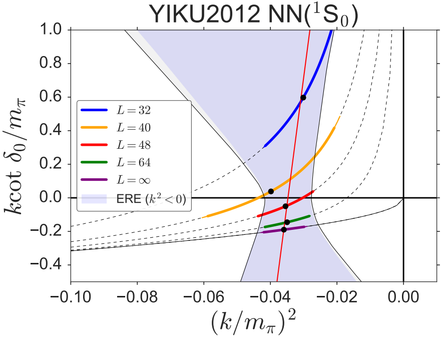
Effective Range Expansion

$$k \cot \delta_0(k) \simeq \frac{1}{a_0} + \frac{1}{2} r_{\rm eff} k^2 + \cdots$$

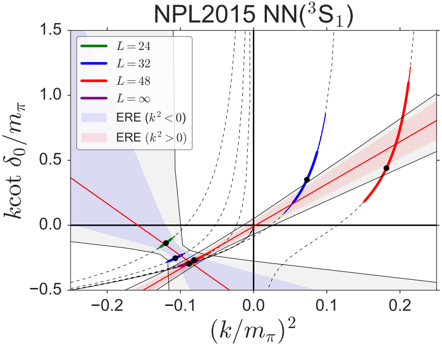
▶ **ALL previous studies show anomalous ERE.**

- NN interactions are anomalous at unphysical pion mass.

singular ERE ($r_{\rm eff} \sim \pm\infty$)



inconsistent ERE



YIKU2012: Yamazaki et al (2012), NPL2015: NPL Coll. (2015)

Phase Shift Analysis — Sanity Check

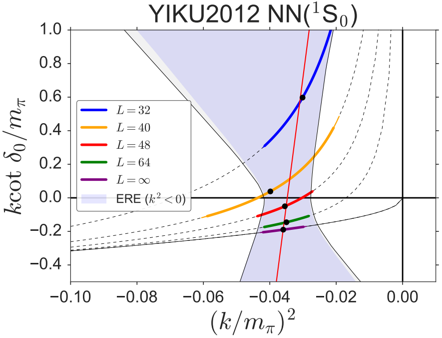
Effective Range Expansion

$$k \cot \delta_0(k) \simeq \frac{1}{a_0} + \frac{1}{2} r_{\mathrm{eff}} k^2 + \cdots$$

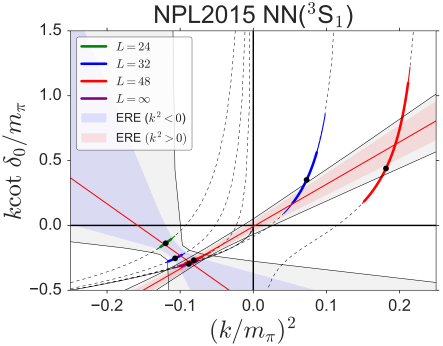
▶ **ALL previous studies show anomalous ERE.**

- ~~NN interactions are anomalous at unphysical pion mass.~~
- These results are **WRONG** due to fake plateaux.

singular ERE ($r_{\mathrm{eff}} \sim \pm\infty$)



inconsistent ERE



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2 HAL QCD Method

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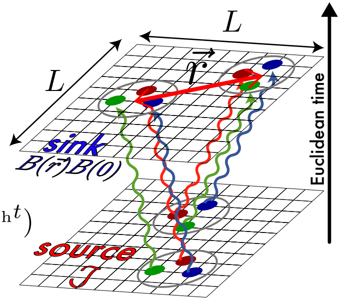
4 Summary

Time-dependent HAL QCD Method

■ Nambu-Bethe-Salpeter wave function



$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$

$$= \sum_n A_n \psi_{W_n}(\vec{r}) e^{-(W_n - 2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th} t})$$

■ scattering states share **the same potential** $U(r,r')$

they are *not contaminations*, but **signals**

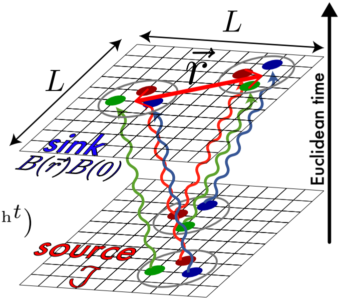
$$[E_{W_0} - H_0]\,\psi_{W_0}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_{W_0}(\vec{r'})$$

$$[E_{W_1} - H_0]\,\psi_{W_1}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_{W_1}(\vec{r'})$$

$$[E_{W_2} - H_0]\,\psi_{W_2}(\vec{r}) = \int d\vec{r'} U(\vec{r},\vec{r'})\psi_{W_2}(\vec{r'})$$

$$\vdots$$

Time-dependent HAL QCD Method



■ **Nambu-Bethe-Salpeter wave function**

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$
$$= \sum_n A_n \psi_{W_n}(\vec{r}) e^{-(W_n - 2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th} t})$$

■ $R(r,t)$ satisfies

$$\left[\frac{1}{4m_B}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0\right] R(\vec{r},t) = \int d\vec{r'} U(\vec{r},\vec{r'}) R(\vec{r'},t)$$

with **elastic** saturation — exponentially easier than g.s. saturation

▶ **"potential"** by velocity expansion of $U(r,r') \simeq V(r)\delta(r-r')$

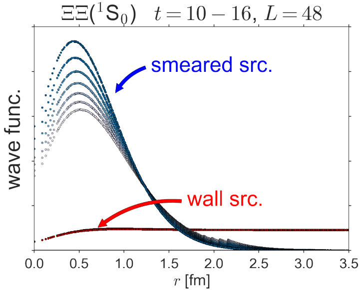
$$V(\vec{r}) = \frac{1}{4m_B}\frac{(\partial/\partial t)^2 R(\vec{r},t)}{R(\vec{r},t)} - \frac{(\partial/\partial t)R(\vec{r},t)}{R(\vec{r},t)} - \frac{H_0 R(\vec{r},t)}{R(\vec{r},t)}$$

▶ **This method does not require the ground state saturation.**

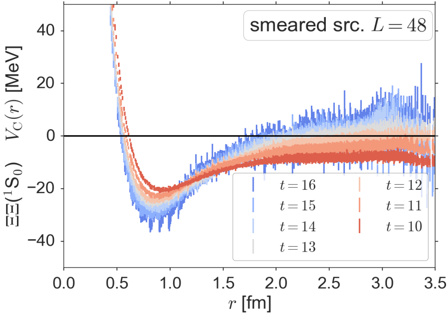
HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

NBS wavefunction: $R^{\rm smear}(r,t)$ or $R^{\rm wall}(r,t)$

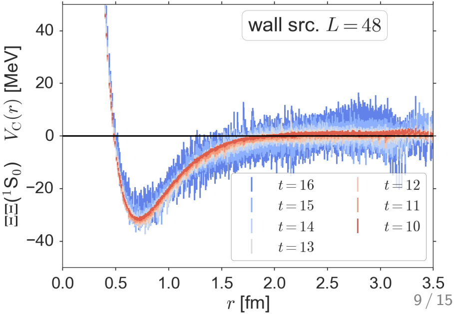
$$V_c(r) = \frac{1}{4m}\frac{(\partial^2/\partial t^2)R}{R} - \frac{(\partial/\partial t)R}{R} - \frac{H_0 R}{R}$$



■ **smeared src.** t-dependent

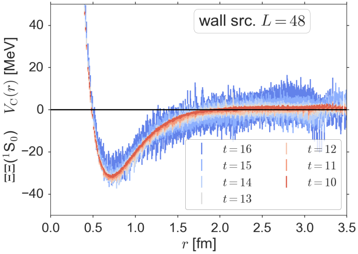


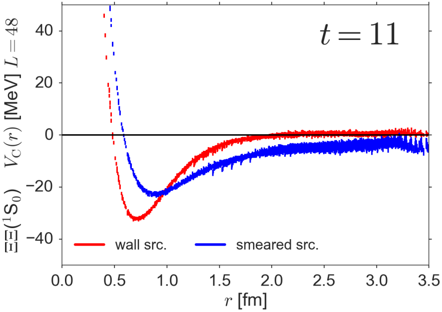
■ **wall src.** t-independent

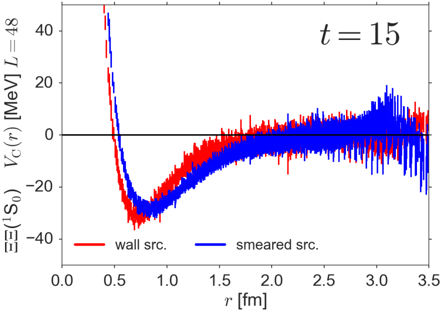


HAL: Potential of $\Xi\Xi(^1S_0)$ Smeared Src. vs Wall Src.

- **wall src.** — good convergence
- **smeared src.** — t-dep.
- **smeared src.** $\longrightarrow$ **wall src.** for large t







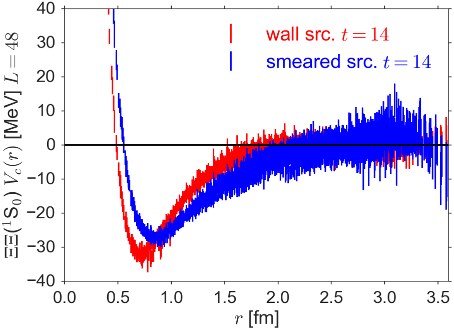
Residual Diff. of Pot.: Next Leading Order Correction

- LO $\Rightarrow$ $U(r,r') = [V_{\text{eff}}(r)]\delta(r-r')$
- NLO $\Rightarrow$ $U(r,r') = [V_{\text{LO}}(r) + V_{\text{NLO}}(r)\nabla^2]\delta(r-r')$

▶ **HAL method works well**

— good convergence in non-locality of $U(r,r')$ for low energy,
NLO correction appears in **smeared src.**

☐ Leading order approximation



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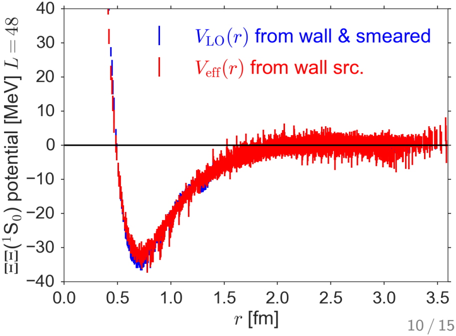


HAL and Lüscher: Energy Shift from Potential

- HAL QCD works well **w/o g.s. saturation problem**

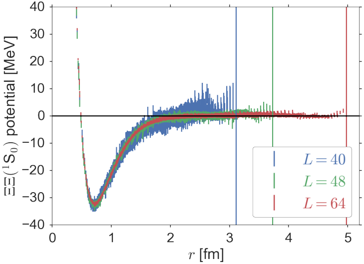
  HAL QCD potential $\Longrightarrow$ true **"energy shift"** in finite volume

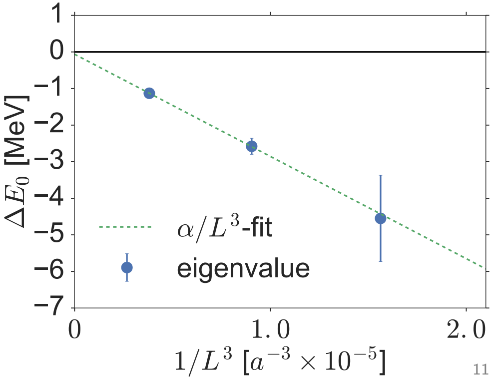
▶ Eigenequation in finite volume L^3 with HAL QCD potential $V(\vec{r})$

$$[H_0 + V]\,\psi = \Delta E \psi$$

☐ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \Longrightarrow$ **scattering** by **Lüscher's formula**

- potential $V(r)$





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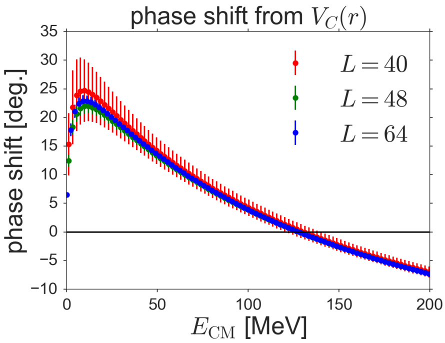
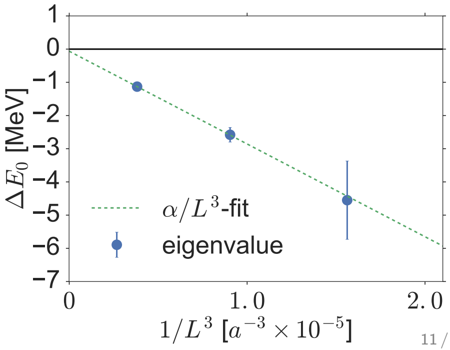
$$[H_0 + V]\,\psi = \Delta E \psi$$

☐ eigenvalue $\Delta E_0 \propto 1/L^3 \longrightarrow 0 \Longrightarrow$ **scattering** by **Lüscher's formula**

$\Longrightarrow$ consistent with **potential analysis** — $\Xi\Xi(^1S_0)$ unbound (at $m_\pi = 0.51$GeV)





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2 HAL QCD Method

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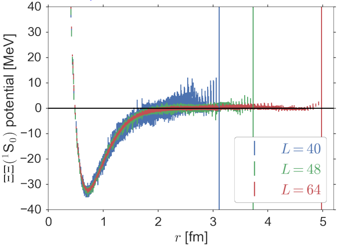
Wavefunc. → Potential → Eigenenergies and Eigenfuncs.

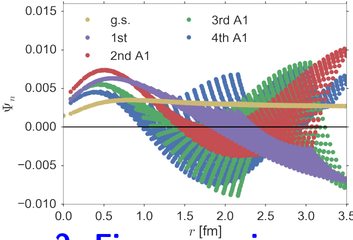
1. Potential



■ **Solve**

Schrödinger eq.
in Finite Volume

2. Eigen-wave functions



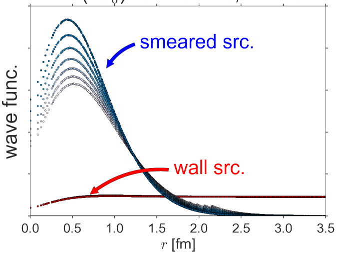
2. Eigen-energies

n	ΔE_n [MeV]
g.s.	-2.58(1)
1st	52.49(2)
2nd	112.08(2)
3rd	169.78(2)
4th	224.73(2)

■ **HAL QCD method** ⇑

0. NBS correlator

$\Xi\Xi(^1S_0)$ $t=10-16$, $L=48$



■ **Feedback**

decompose
by eigenmodes

Contaminations of Excited States in Correlator

HAL pot. ▶ eigenfunc/value Ψ_n, ΔE_n ▶ eigenmode decomposition

$$R^{\mathrm{wall/smear}}(\vec{r},t) = \sum_n a_n^{\mathrm{wall/smear}} \Psi_n(\vec{r}) \exp\left(-\Delta E_n t\right)$$

$$\therefore R(t) \equiv R(\vec{p}=0,t) = \sum_r R(\vec{r},t) = \sum_n b_n^{\mathrm{wall/smear}} e^{-\Delta E_n t}$$

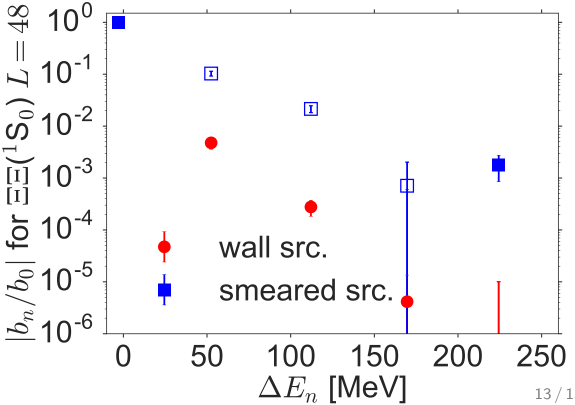
□ ex. **1st excited state**

- **wall source**
  $b_1/b_0 \ll 1$ %
- **smeared source**†
  $b_1/b_0 \simeq -10$ %

- with energy gap
  $E_1 - E_0 \simeq 50$ MeV
  for $L^3 = 48^3$

†unfilled symbols: $b_n/b_0 < 0$

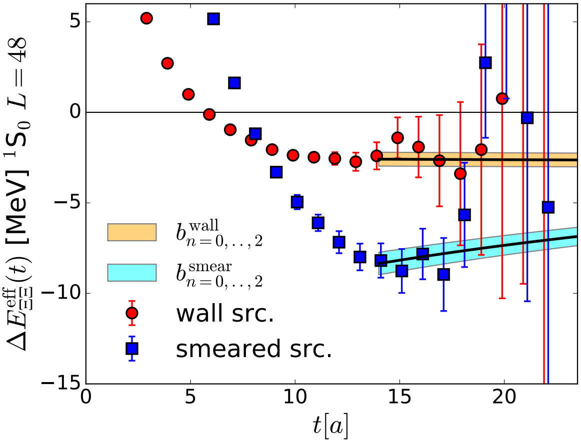
"contamination" of excited states b_n/b_0



Diagnosis of Fake Plateau

$$\Delta E_{\mathrm{eff}}^{\mathrm{wall/smear}}(t) \equiv \log \frac{R(t)}{R(t+1)} = \log \frac{\sum_n b_n^{\mathrm{wall/smear}} \exp\left(-\Delta E_n t\right)}{\sum_n b_n^{\mathrm{wall/smear}} \exp\left(-\Delta E_n (t+1)\right)}$$

- "direct measurement" — reproduced by low-lying states



Diagnosis of Fake Plateau

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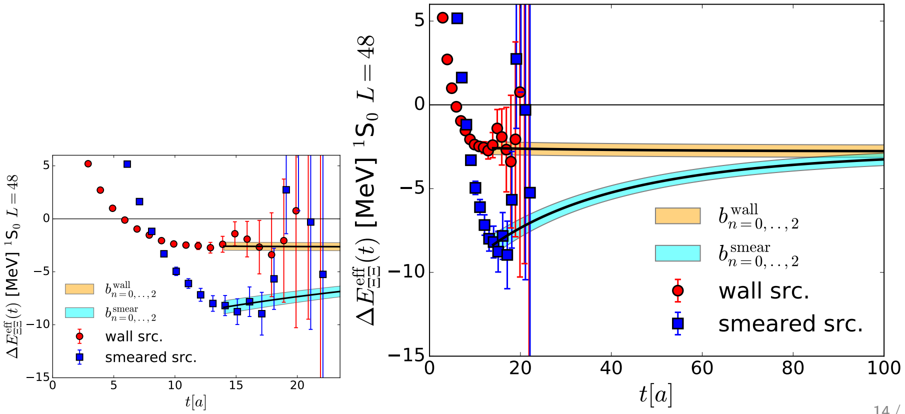
- ■ “direct measurement” — reproduced by low-lying states
- □ **g.s. saturation** of smeared source — **100 lattice units ∼ 10 fm !!!**



1 Lüscher's Finite Volume Method (Direct Method)

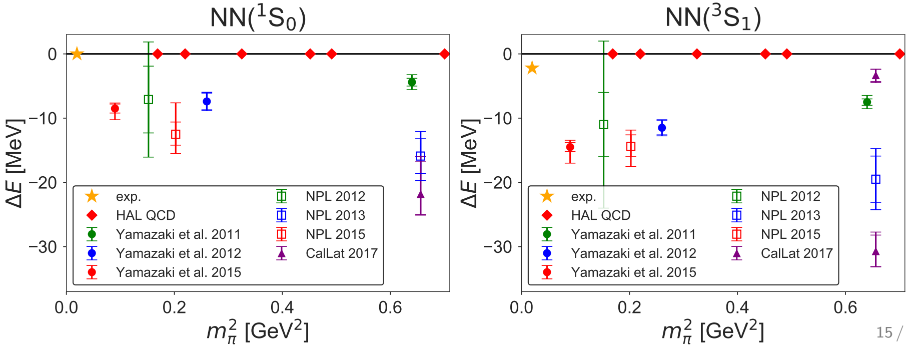
2 HAL QCD Method

3 Diagnosis of the Direct Method

4 Summary

Summary: Baryon Interactions from Lattice QCD

- naïve **"Direct calculation"** of multibaryon system
  — **ground state saturation** is **extremely** difficult
  - scattering states $\Longrightarrow$ **"fake signal"** and **insane** phase shift
- only **HAL QCD method** works well **without g.s. saturation**



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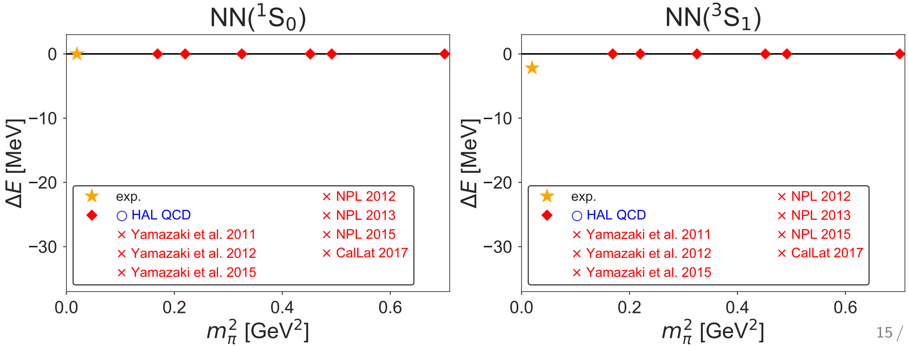

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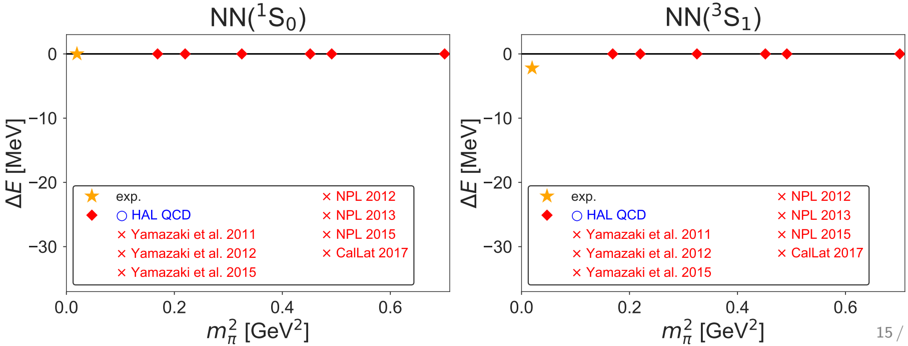
  $\longrightarrow$ "direct method" — S/N $\sim 10^{-25}$

systematic understandings of baryon interactions based on QCD



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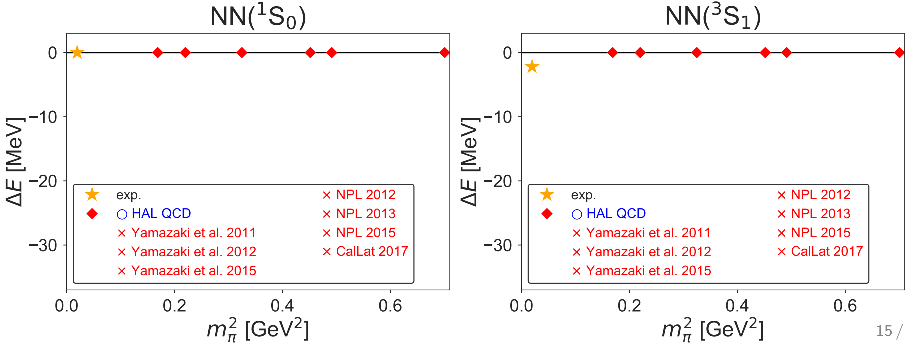
  systematic understandings of baryon interactions based on QCD
- Post-K $\Longrightarrow$ ☐ LS-force, P=(-) channel, 3-body forces, ... $\rightarrow$ EoS



5 Appendix

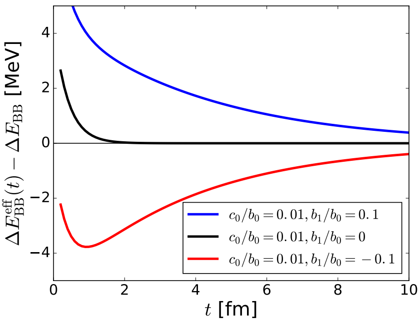
Demo: Contamination of Scattering State

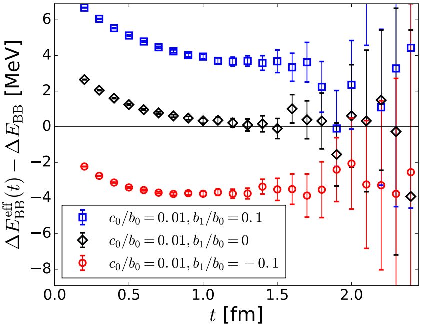
Mock up data

$$R(t) = b_0 e^{-\Delta E_{\rm BB} t} + b_1 e^{-\delta E_{\rm el} t} + c_0 e^{-\delta E_{\rm inel} t}$$

with $\delta E_{\rm el} - \Delta E_{\rm BB} = 50$ MeV $\sim \mathcal{O}(1/L^2)$, $\delta E_{\rm inel} - \Delta E_{\rm BB} = 500$ MeV $\sim \mathcal{O}(\Lambda_{\rm QCD})$

- g.s. saturation around $t \to 10$ fm
- fake plateau around $t \sim 1$ fm





Sink Operator Dependence

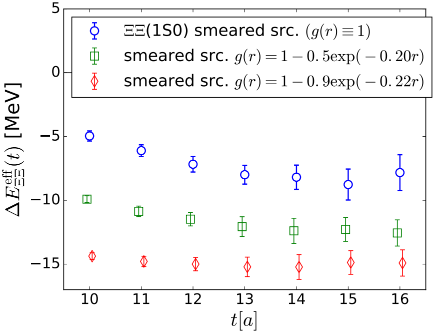
generalized direct method

$$\overline{R}^{(g)}(t) = \sum_r g(r) R(r,t) = \sum_r g(r) \frac{\sum_x \left\langle 0 | B(r+x,t) B(x,t) \overline{\mathcal{J}(0)} | 0 \right\rangle}{\{G(t)\}^2}$$

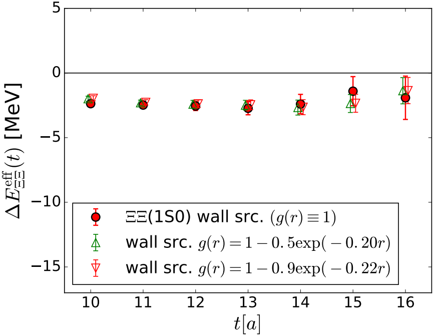
"true g.s." does not depend on $g(r)$

$g(r) = 1 + a \exp(-br)$ type projection

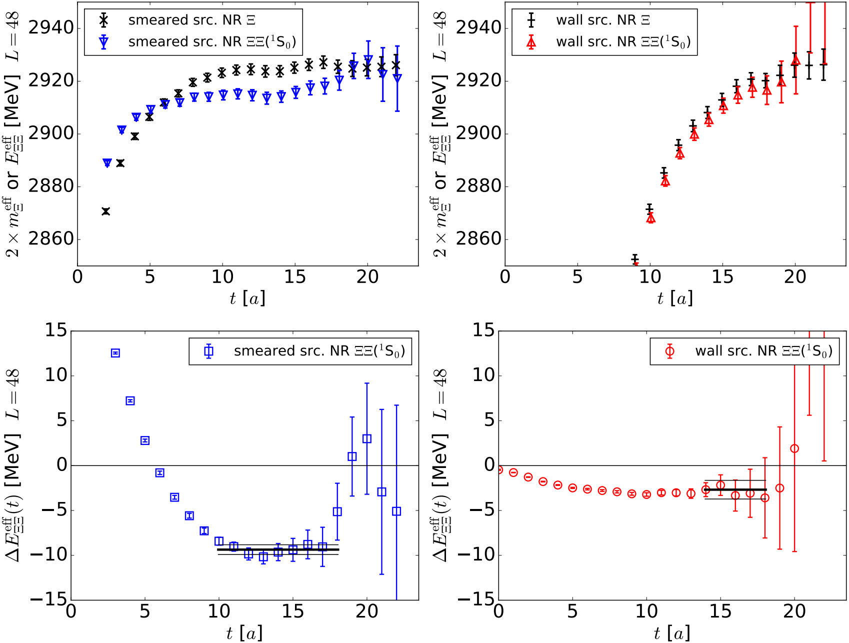
smeared src. — sink dep. plateau



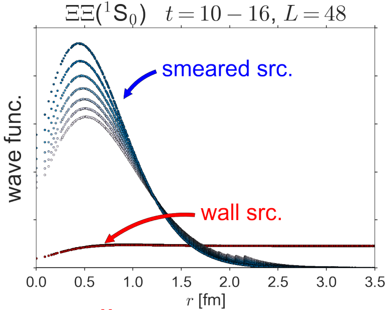
wall src. — sink indep.



$\Delta E_{\mathrm{eff}}(t) = E^{\mathrm{eff}}_{\Xi\Xi}(t) - 2m^{\mathrm{eff}}_{\Xi}(t)$: Smeared Src. vs. Wall Src.



HAL: Wave Function and $\Xi\Xi(^1S_0)$ Potential $V_c(\vec{r})$



- ○ **wall src.** — weak t-dep.
- ○ **smeared. src.** — strong t-dep.

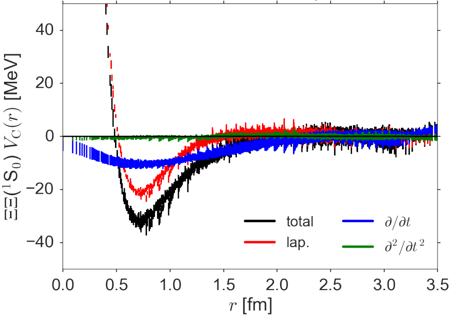
$\Rightarrow$ contribution of excited states

- ● time-dep. HAL method works well

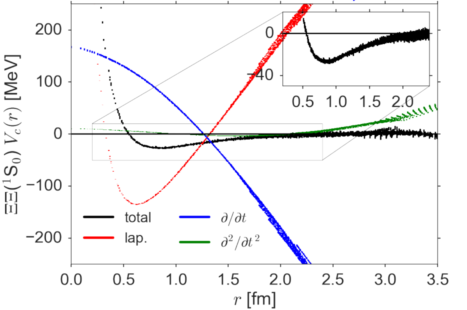
$\Rightarrow$ $\mathcal{O}(100)$ MeV of cancellation

$$V_c(\vec{r}) = -\frac{H_0 R}{R} - \frac{(\partial/\partial t)R}{R} + \frac{(\partial/\partial t)^2 R}{4mR}$$

wall src. $L = 48$, $t = 14$



smeared src. $L = 48$, $t = 14$



(Original) HAL QCD Method

■ **Nambu-Bethe-Salpeter wave function**

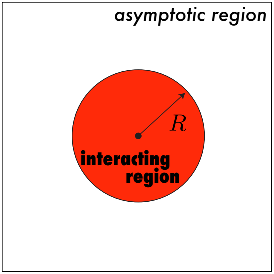
$$\psi_k(\vec{r}) = \langle 0|B(\vec{x}+\vec{r},0)B(\vec{x},0)|BB,W_k\rangle$$

- asymptotic region — $r > R$

$$\psi_k(\vec{r}) \simeq C\frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

- interacting region — $r < R$

$$[E_k - H_0]\,\psi_k(\vec{r}) = \int d\vec{r'}U(\vec{r},\vec{r'})\psi_k(\vec{r'})$$



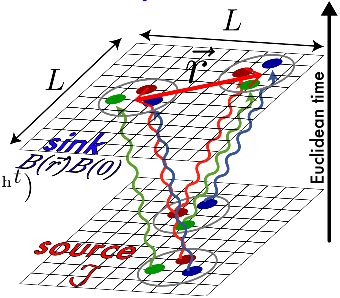
$U(r,r')$: E-independent potential, which is faithful to **the phase shift**

☐ we calculate **4-pt function**

$$R(\vec{r},t) \equiv \frac{\langle 0|T\{B(\vec{x}+\vec{r},t)B(\vec{x},t)\}\overline{\mathcal{J}}(0)|0\rangle}{\{G_B(t)\}^2}$$

$$= \sum_n A_n\psi_{W_n}(\vec{r})e^{-(W_n-2m_B)t} + \mathcal{O}(e^{-\Delta W_{\rm th}t})$$

$$\longrightarrow A_0\psi_{W_0}(\vec{r})e^{-(W_0-2m_B)t}$$



⇒ g.s. saturation is required !!

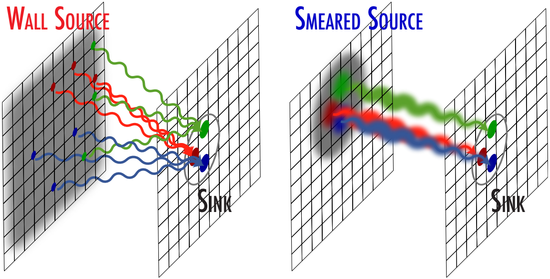
Lattice Setup: Wall Source and Smeared Source

☐ ex. $\Xi\Xi(^1S_0)$ **interaction** from HAL QCD methods

27 multiplet — the same rep. as $NN(^1S_0)$

☐ CHECK **2 quark sources** — mixture of excited states are different

- **wall source**
  standard of HAL QCD
- **smeared source**
  standard of direct method†



■ setup — $2+1$ improved Wilson + Iwasaki gauge†

- lattice spacing: $a = 0.08995(40)$ fm, $a^{-1} = 2.194(10)$ GeV
- lattice volume: $32^3 \times 48$, $40^3 \times 48$, $48^3 \times 48$, and $64^3 \times 64$

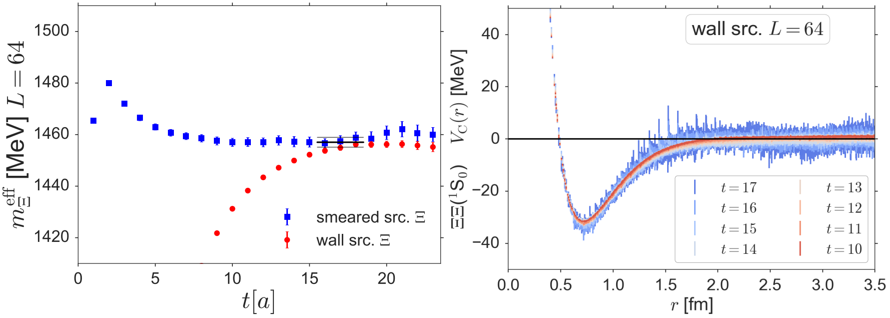
$m_\pi = 0.51$ GeV, $m_N = 1.32$ GeV, $m_K = 0.62$ GeV, $m_\Xi = 1.46$ GeV

† Yamazaki-Ishikawa-Kuramashi-Ukawa, arXiv:1207.4277.

Inelastic Contamination of wall source?

in fact, single baryon saturation of **wall src.** is later than **smeared src.**

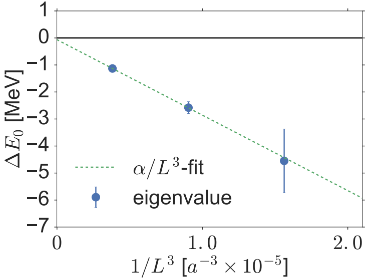
✓ **CHECK** saturation and t-dependence of $V_C(r)$ carefully
— **OK!**

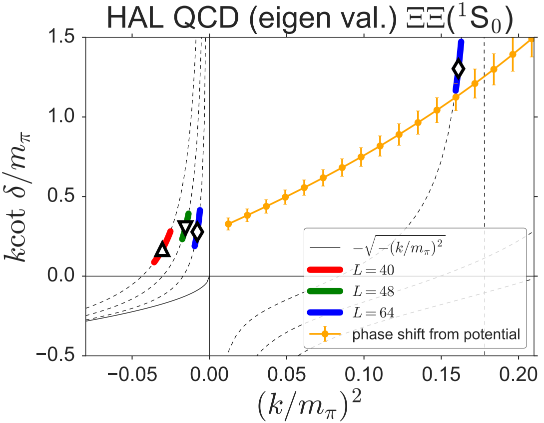


Sanity Check

✓ phase shift from Lüscher's formula shows **reasonable behavior**

⇒ also consistent with results from potential for $k^2 > 0$





Direct method reinforced by HAL method

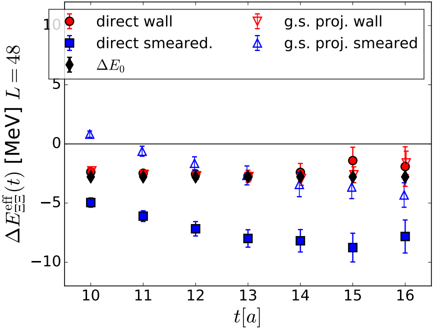
generalized direct method

$$\overline{R}^{(f)}(t) = \sum_r f(r) R(r,t) = \sum_r f(r) \frac{\sum_x \left\langle 0 | B(r+x,t) B(x,t) \overline{\mathcal{J}(0)} | 0 \right\rangle}{\{G(t)\}^2}$$

using $f(r)$ — eigen-wave func. by HAL QCD potential at finite vol.

$\Longrightarrow$ Direct calc. (**wall**/**smeared**) = HAL QCD method

$f(r) = \Psi^{\dagger}_{\mathrm{g.s.}}(r)$



$f(r) = \Psi^{\dagger}_{\mathrm{1st}}(r)$