素粒子・原子核・宇宙「京からポスト京に向けて」シンポジウム
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重力崩壊型超新星のボルツマン方程式による
ニュートリノ輻射流体計算

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TALK PLAN

1. Introduction

2. Boltzmann-Hydro Code

3. Results for 2D Core-Collapse Simulation

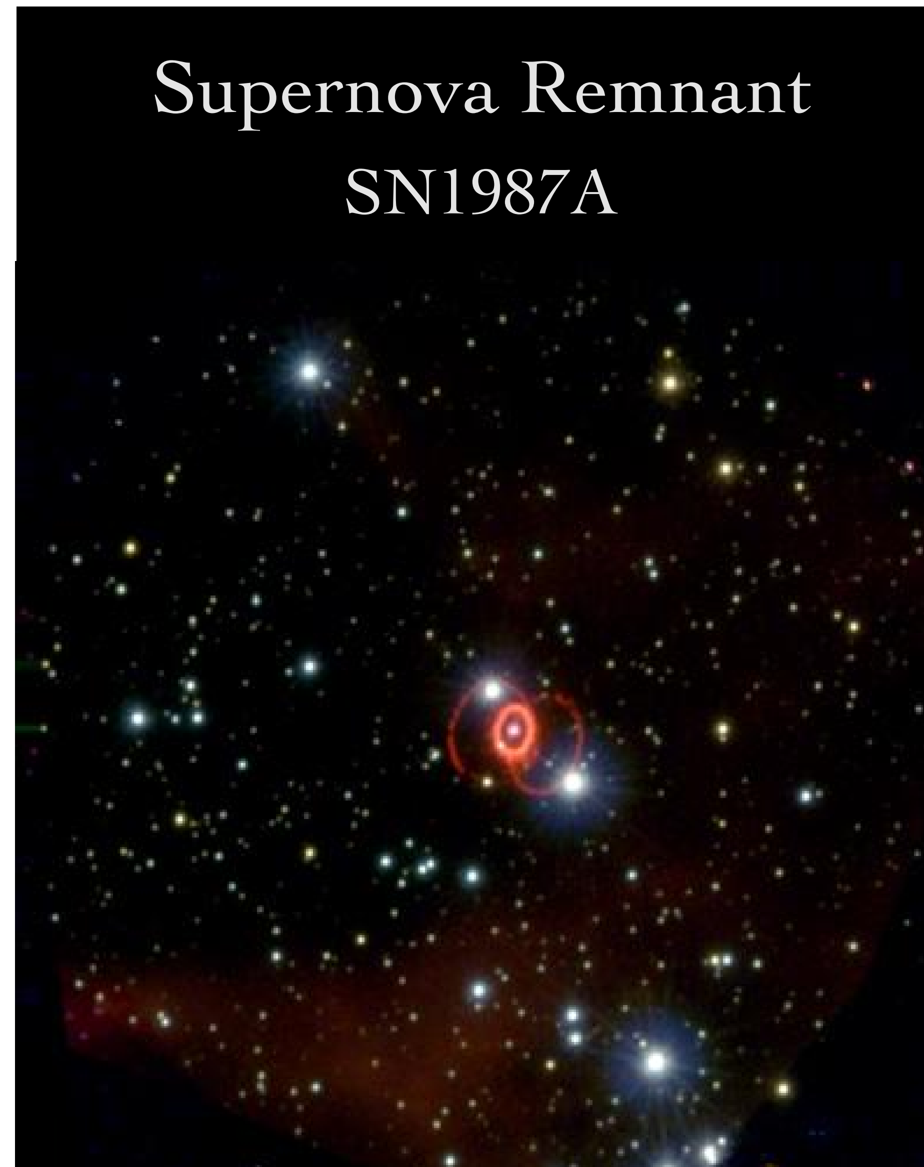
4. Toward 3D Core-Collapse Simulation

5. Summary

Core-Collapse Supernovae

Explosions of massive stars

How do they explode ?



http://imgsrc.hubblesite.org/hu/db/1998/08/images/g/formats/web.jpg

Explosion Scenario

Massive Star (>10$M_\odot$) ➡ Neutron Star(~1.4$M_\odot$)

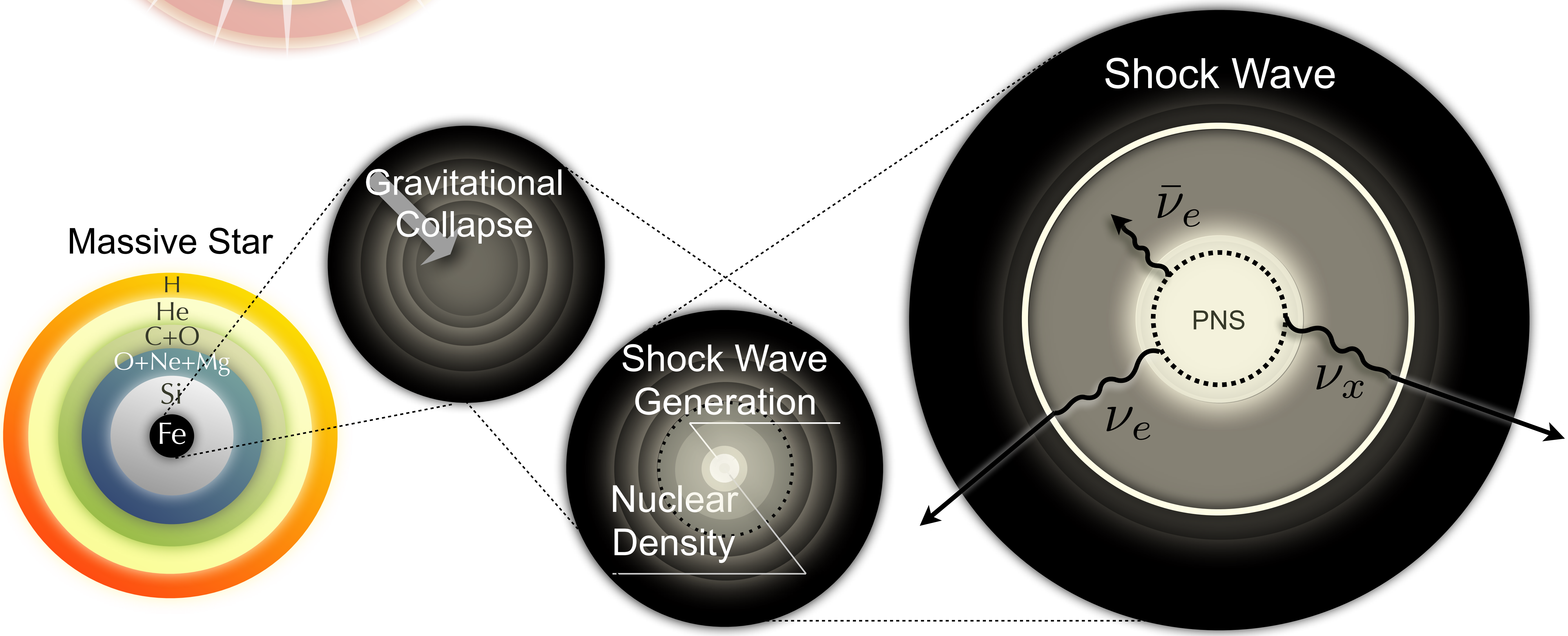
⇓

Gravitational Energy ($E_{grav} \sim 10^{53}$ erg)

↙ ↓ ↘

Neutrino + Photon + Kinetic Energy of Matter

($E_\nu \sim 10^{53}$ erg) ($E_{rad} \sim 10^{49}$ erg) ($E_{kin} \sim 10^{51}$ erg)



Neutrino Heating Mechanism

Boltzmann Equation

Neutrino Transport

On-Shell Condition

$$p^{\mu} p_{\mu} = -m_{\nu}^{2}$$

m_{ν} : a neutrino rest mass

$$f = f(x^{\mu}, p^{i}) \qquad (\mu = 0, 1, 2, 3\,; \quad i = 1, 2, 3)$$

$$p^{\mu} \frac{\partial f}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau} \frac{\partial f}{\partial x^{i}} = \left(\frac{\delta f}{\delta \tau} \right)_{\text{collision}}$$

f : distribution function of neutrinos

x^{μ}: space time coordinates

p^{μ} : the four-momentum of a neutrino $\left(\equiv \frac{\partial x^{\mu}}{\partial \tau} \right)$

τ : the affine parameter of the neutrino trajectory

※ the metric of (−1 +1 +1 +1) ,

※ the unit of $c = G = 1$

Geodesic Equation

$$\frac{dp^{\alpha}}{d\tau} + \Gamma^{\alpha}_{\beta\gamma} p^{\beta} p^{\gamma} = 0$$

Ricci Rotation Coefficient

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{\partial x'^{\alpha}}{\partial x^{a}} \frac{\partial x^{b}}{\partial x'^{\beta}} \left[\frac{\partial}{\partial x^{b}} \left(\frac{\partial x^{a}}{\partial x'^{c}} \right) + \begin{Bmatrix} a \\ b\ c \end{Bmatrix} \frac{\partial x^{c}}{\partial x'^{\gamma}} \right]$$

Affine Connection

$$\begin{Bmatrix} c \\ a\ b \end{Bmatrix} = \frac{1}{2} g^{dc} \left(\frac{\partial g_{ad}}{\partial x^{b}} + \frac{\partial g_{bd}}{\partial x^{a}} - \frac{\partial g_{ab}}{\partial x^{d}} \right)$$

$g_{\mu\nu}$: metric tensor

Collision Term

Weak Interaction

$$\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} = \left[\frac{\delta f}{\delta \tau}\right]_{\text{emis-abs (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{scat (s)}} + \left[\frac{\delta f}{\delta \tau}\right]_{\text{pair (s)}}$$

Emission/Absorption

Electron Capture

$$e^- + p \longleftrightarrow \nu_e + n \quad \text{[ecp]}$$

Anti-Electron Capture

$$e^+ + n \longleftrightarrow \bar{\nu}_e + p \quad \text{[aecp]}$$

Electron Capture on nuclei

$$e^- + A \longleftrightarrow \nu_e + A' \quad \text{[eca]}$$

Scattering

Neutrino-Nucleon scattering

$$\nu + N \longleftrightarrow \nu + N \quad \text{[nsc]}$$

Neutrino-Nuclei scattering

$$\nu + A \longleftrightarrow \nu + A \quad \text{[csc]}$$

Neutrino-Electron scattering

$$\nu + e \longleftrightarrow \nu + e \quad \text{[esc]}$$

Pair Process

Electron-positron pair process

$$e^- + e^+ \longleftrightarrow \nu_i + \bar{\nu}_i \quad \text{[pap]}$$

Nucleon-nucleon bremsstrahlung

$$N + N \longleftrightarrow N + N + \nu_i + \bar{\nu}_i \quad \text{[nbr]}$$

Neutrino Number Density

s : species ($s = \nu_e,\ \bar{\nu}_e,\ \nu_x$)

$$\Gamma_s \equiv \int \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{\Gamma} \equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$$

Neutrino Energy Density ($\mu = 0$)

Radiation Pressure ($\mu = 1, 2, 3$)

$$G_s^\mu \equiv \int p_s^\mu \left(\frac{\delta f}{\delta \tau}\right)_{\text{collision (s)}} d^3\mathbf{p} \qquad \boxed{G^\mu} \equiv \sum_s G_s^\mu$$

Euler Equations

Hydrodynamics

Baryon mass conservation: $\left(\rho u^{\mu}\right)_{;\mu} = 0$

Energy and momentum conservation: $\left(T^{\mu\nu}\right)_{;\nu} = \boxed{-G^{\mu}}$ $\quad T^{\mu\nu} \equiv [\rho(1+e)+P]u^{\mu}u^{\nu} - Pg^{\mu\nu}$

Lepton number conservation: $\left(n_e u^{\mu}\right)_{;\mu} = \boxed{-\Gamma}$ $\quad$ EOS table (ρ, Ye, T)

Gravitation: $\mathbf{G}^{\mu\nu} = 8\pi T^{\mu\nu} \quad \Rightarrow \quad \Delta\phi = 4\pi\rho$ (Newton Approx.)

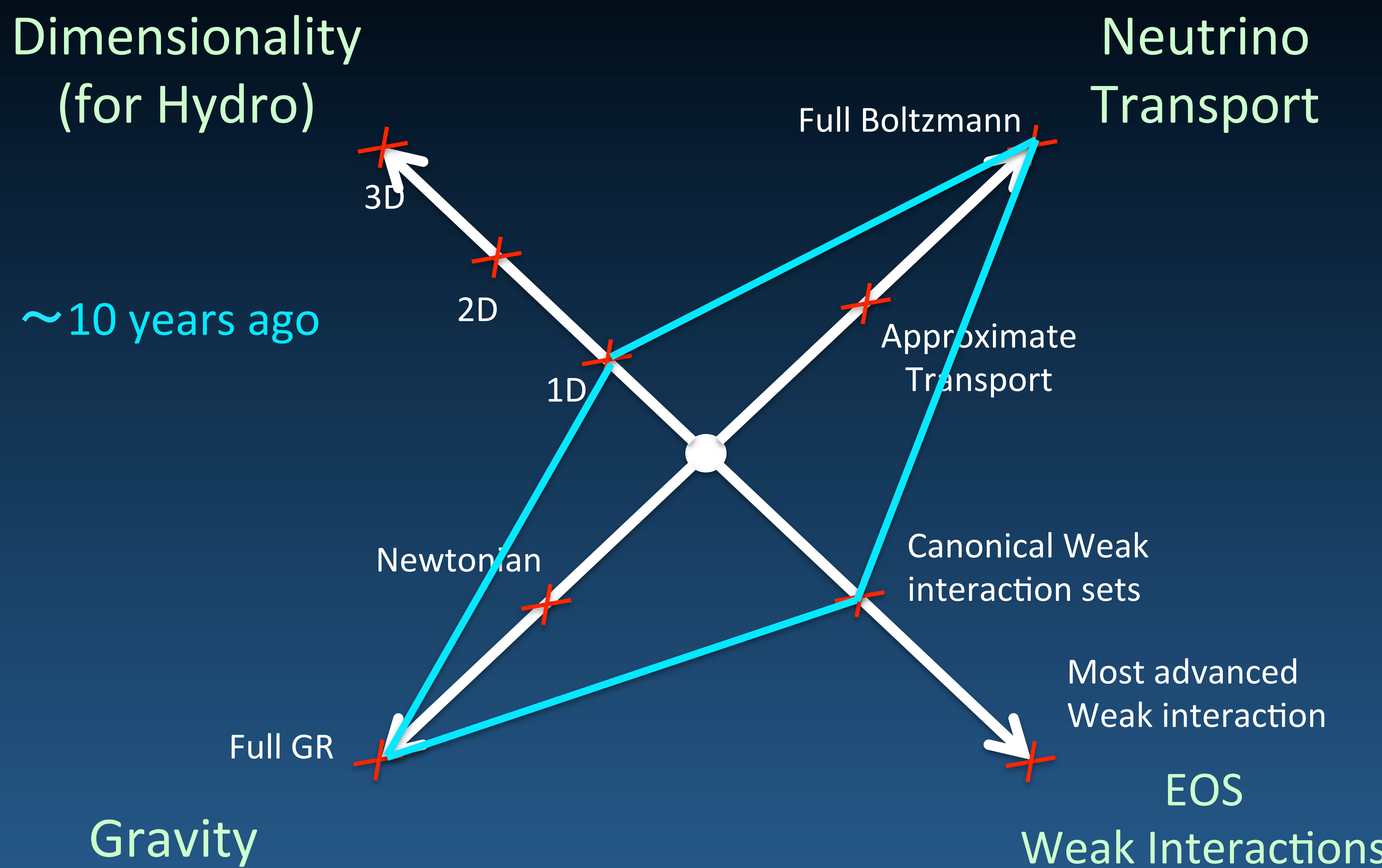
ρ : barion density $\quad u$: velocity $\quad T^{\mu\nu}$: energy-momentum tensor $\quad P$: matter pressure

e : specific internal energy density $\quad g_{\mu\nu}$: metric tensor $\quad n_e$: electron number density

$\mathbf{G}^{\mu\nu}$: Einstein Tensor $\quad G^0$: neutrino radiation energy, G^i : neutrino radiation pressure,

$\Gamma \equiv \Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}$: deleptonization rate $\quad Y_e$: electron fraction ($\equiv$ electron number / barion number)

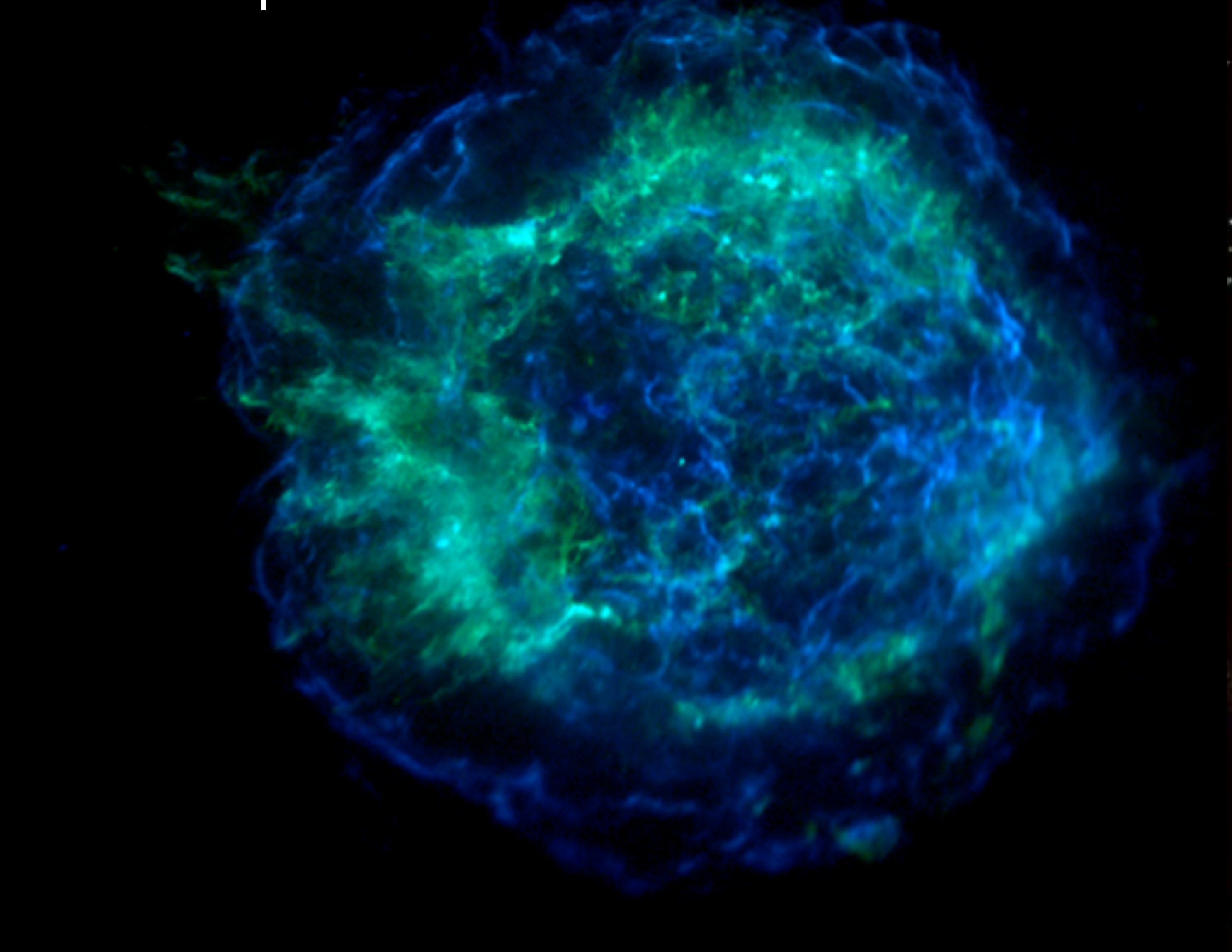
- Current Status of supernova simulations -



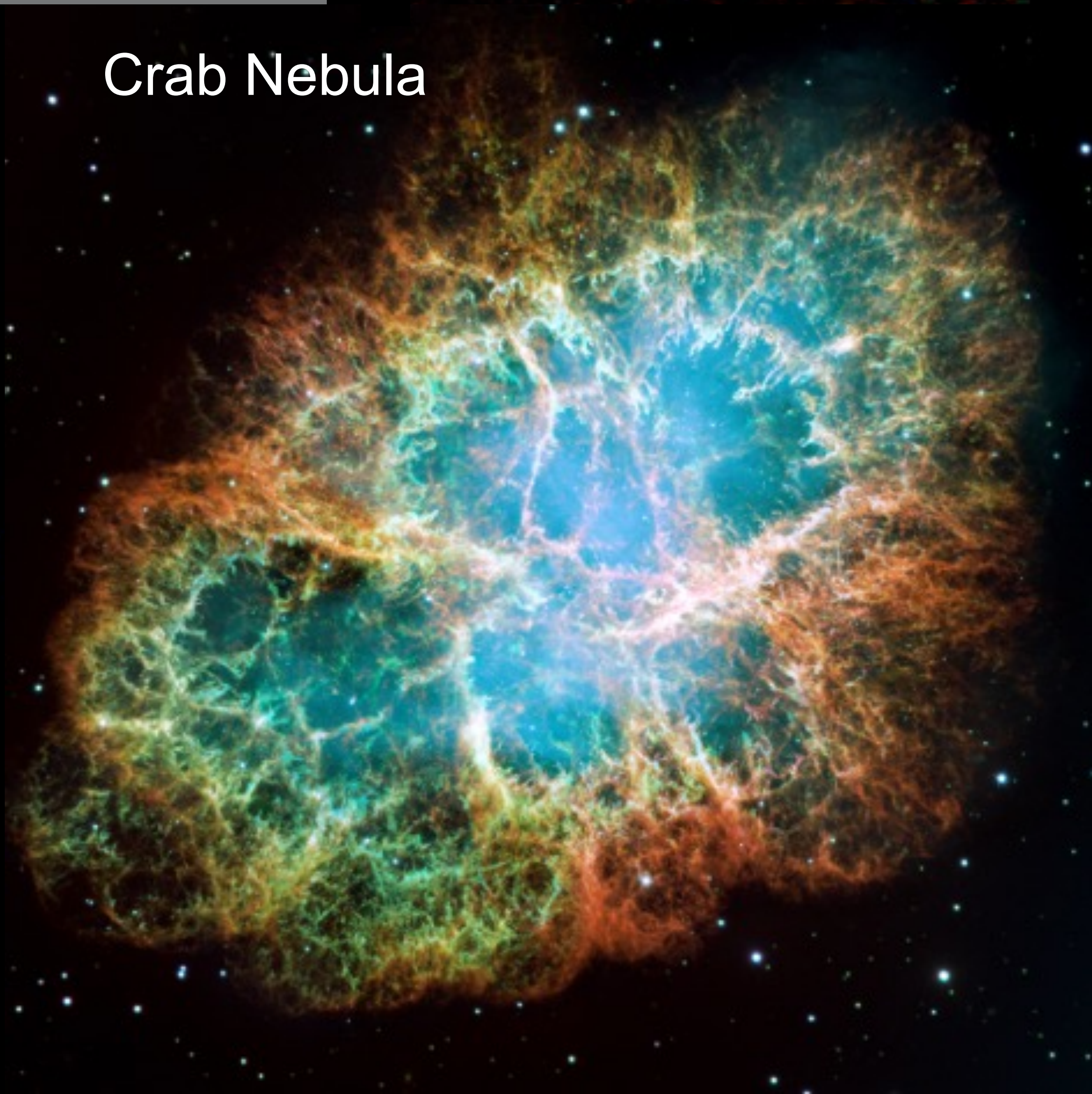
(by Nagakura)

Non-Spherically Symmetric Explosion

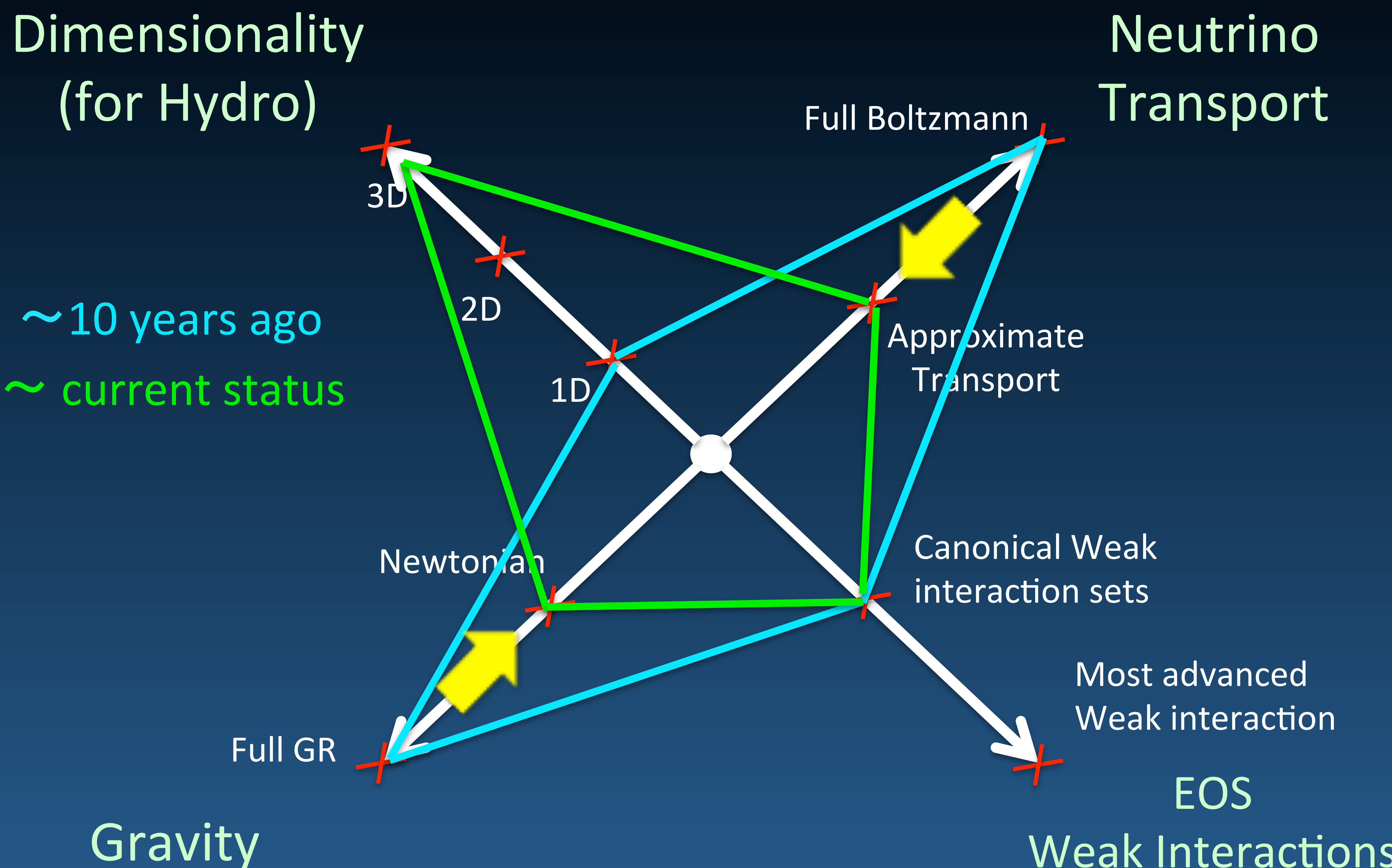
Cassiopeia A



Crab Nebula



- Current Status of supernova simulations -



(by Nagakura)

Various Approximations for Multi-D Neutrino Transfer

- **Ray-by-Ray Approach** (MPA, Oak Ridge, Kotake-Takiwaki-Suwa)

  Neutrino-Advection is essentially considered under spherical symmetry.

- **Moment method**
  (MPA, Kyoto, Caltech, Basel (Kuroda))

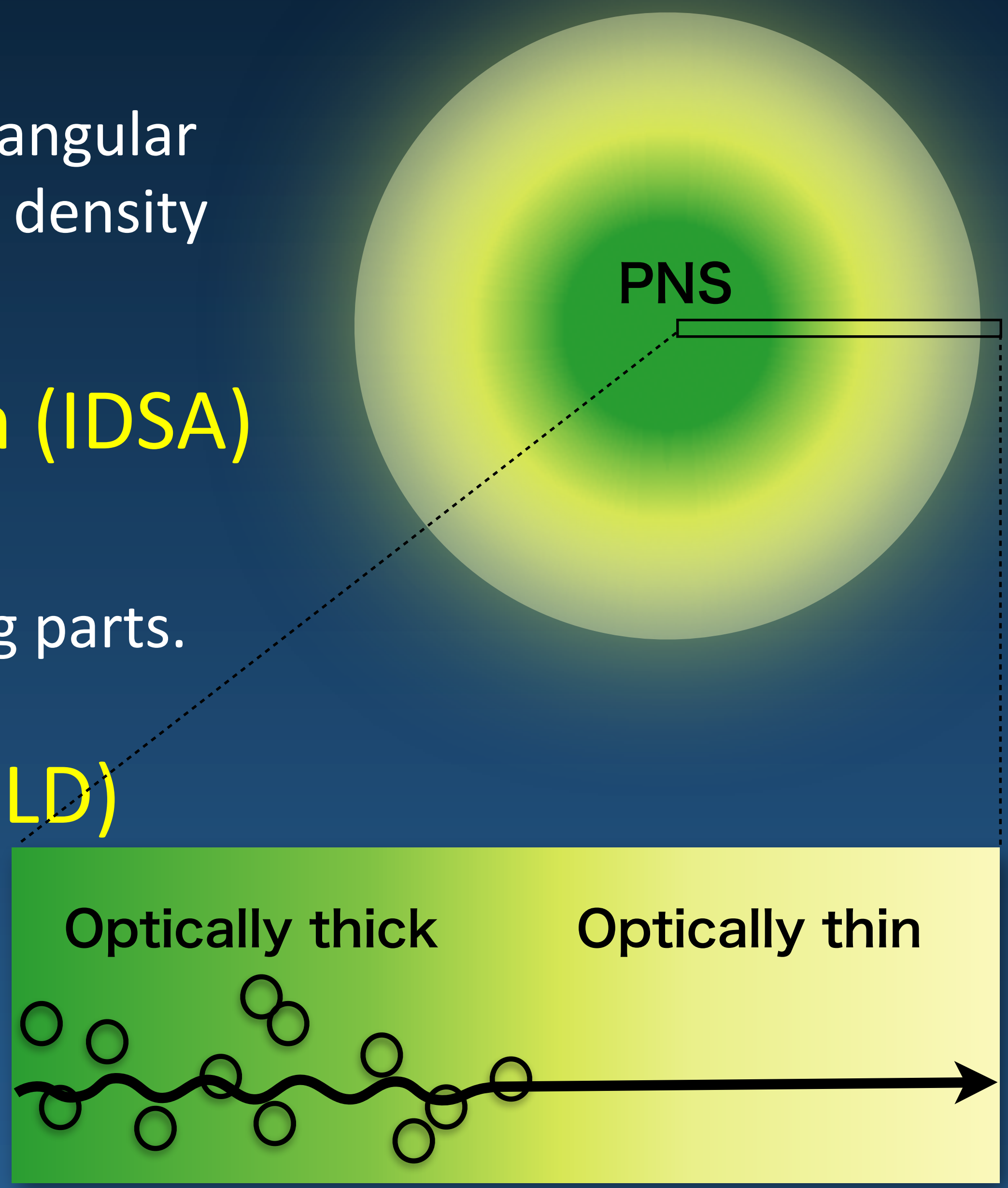
  Multiplied by cos θν and integrated with the neutrino angular directions, the time-evolution equations of the energy density and the energy flux can be obtained.

- **Isotropic Diffusion Source Approximation (IDSA)**
  (Basel, Kotake-Takiwaki-Suwa)

  Neutrinos are decomposed into trapped and streaming parts.

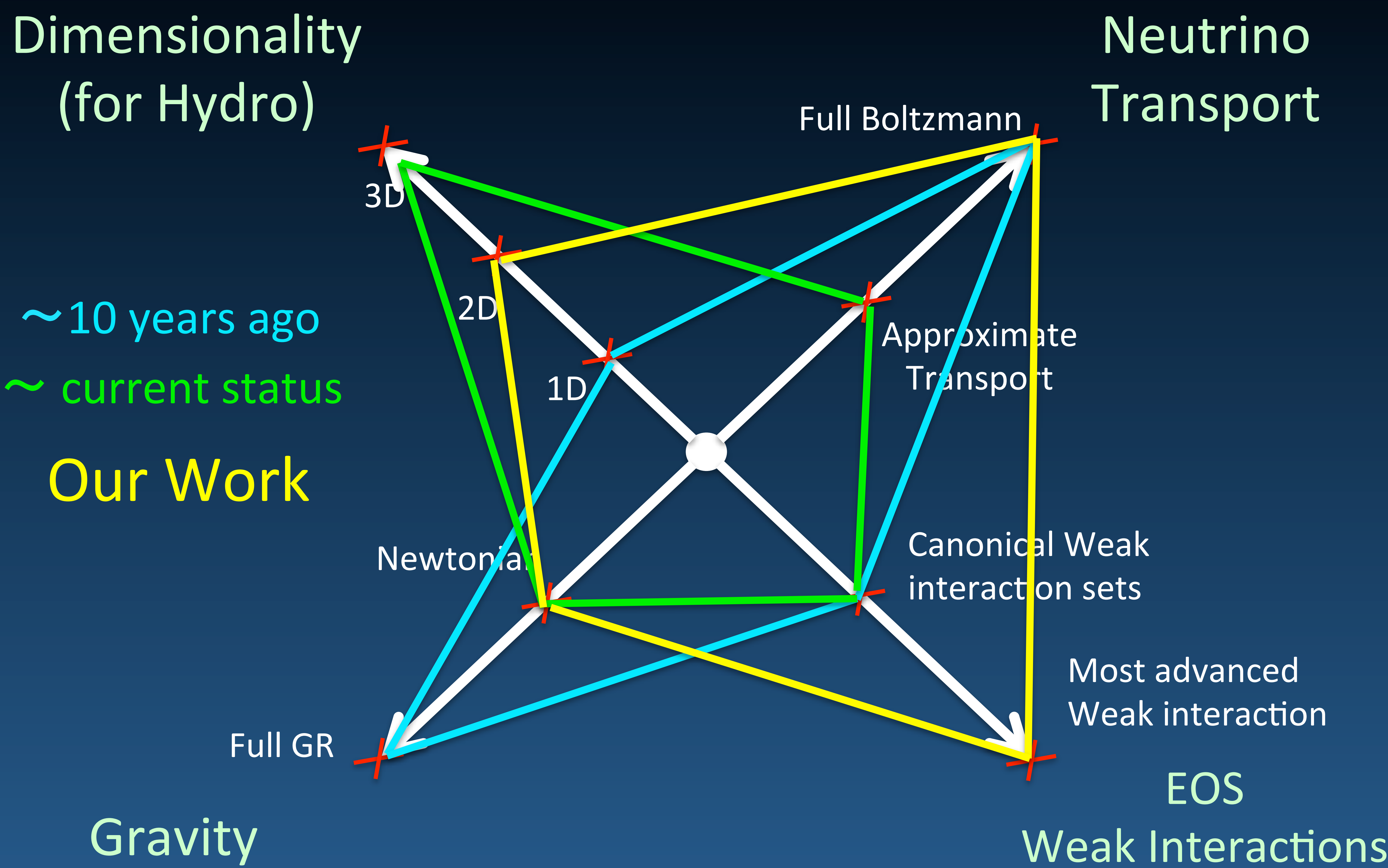
- **Multi-Group Flux-Limited-Diffusion (MGFLD)**
  (Oak Ridge, Princeton, Caltech)

  The energy flux can be determined to automatically switch between optically thick limit and thin limit with the transport cross section.



(by Nagakura)

- Current Status of supernova simulations -



(by Nagakura)

Core-Collapse Supernovae Simulations
for $11.2M_\odot$ and $15M_\odot$ using <u>Boltzmann-Hydro Code</u>

from collapse (1D) to shock revival/stalled (2D)

$N_r \times N_\theta \times N_\varphi \times N_\varepsilon \times N_{\theta\nu} \times N_{\varphi\nu}$ = 384x128x1x20x10x6

Using the most sophisticated neutrino transport code,
we confirm whether explosion occur or not.

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Boltzmann Equation

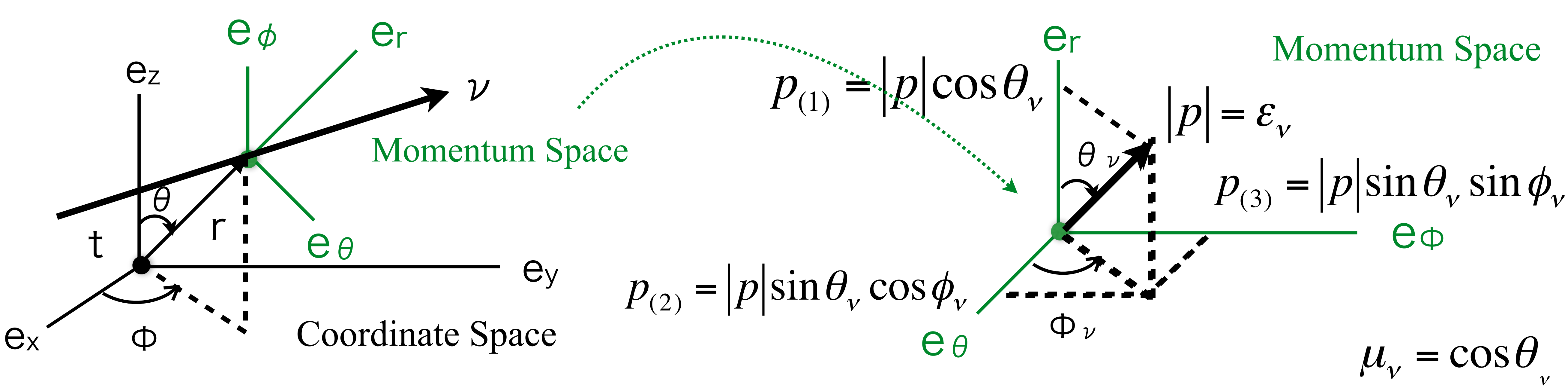
(Sumiyoshi and Yamada 2012)

$$f = f(t, r, \theta, \phi, \varepsilon_\nu, \mu_\nu, \phi_\nu) \qquad ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\frac{\partial f}{\partial t} + \cos\theta_\nu \frac{\partial f}{\partial r} + \frac{\sin\theta_\nu \cos\theta_\nu}{r}\frac{\partial f}{\partial \theta} + \frac{\sin\theta_\nu \sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial \phi}$$

$$-\frac{\sin\theta_\nu}{r}\frac{\partial f}{\partial \theta_\nu} - \frac{\cos\theta}{\sin\theta}\frac{\sin\theta_\nu \sin\phi_\nu}{r}\frac{\partial f}{\partial \phi_\nu} = \left(\frac{\delta f}{\delta t}\right)_{\text{collision}}$$

Spherical Coordinate System



※ the unit of $c = G = 1$

Two Energy Grids Approach for Momentum Space

(Nagakura et al. 2014)

Collision term is calculated in the rest frame using Lagrangian Remapped Grid (LRG)

Advection term is calculated in the laboratory frame using Laboratory Fixed Grid (LFG)

$$\left(\frac{\delta f}{\delta t}\right)_{\text{collision}}^{\text{laboratory}} = \frac{d\lambda}{dt}\left(\frac{\delta f}{\delta \lambda}\right)_{\text{collision}} = \frac{d\lambda}{dt}\frac{d\tau}{d\lambda}\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision}}^{\text{restframe}} = \frac{\varepsilon_\nu^{\text{lb}}}{\varepsilon_\nu^{\text{rf}}}\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision}}^{\text{restframe}} = D^{\text{lb}}\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision}}^{\text{restframe}}$$

↑ Doppler factor

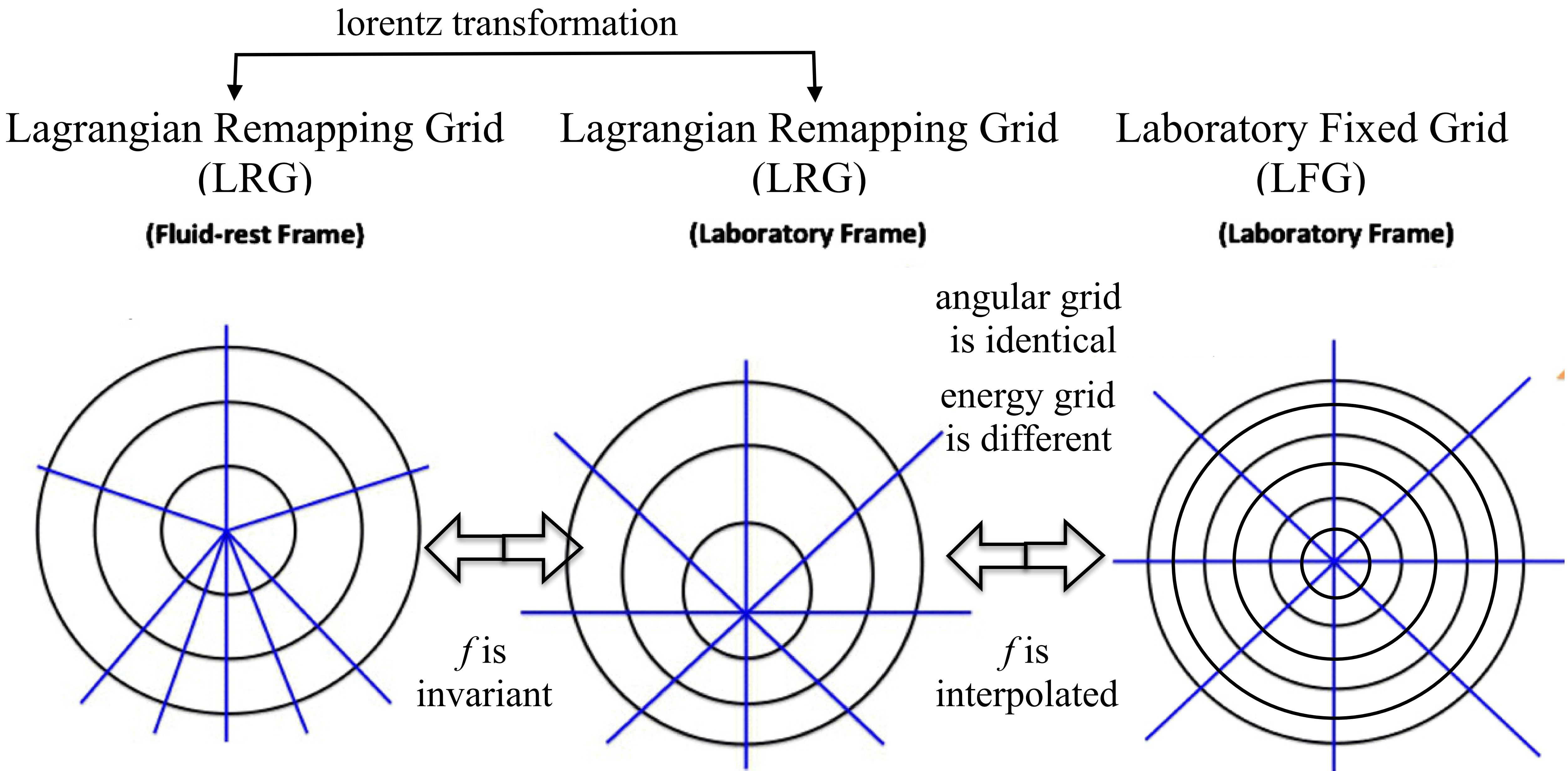
Boltzmann Equation

$$\frac{\partial f}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}\left(r^2 f\right) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}\left(f\sin\theta\right) + \frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f}{\partial\phi}$$

$$+\frac{1}{r}\frac{\partial}{\partial\mu_\nu}\left(f(1-\mu_\mu^2)\right) - \frac{\sqrt{1-\mu_\mu^2}}{r}\frac{\cos\theta}{\sin\theta}\frac{\partial}{\partial\phi_\nu}\left(f\sin\phi_\nu\right) = D^{\text{lb}}\left(\frac{\delta f}{\delta \tau}\right)_{\text{collision}}^{\text{restframe}}$$

Two Grids Approach for Momentum Space

(Nagakura et al. 2014)



Right-hand side of Boltzmann equation (collision term) is calculated by using this frame

the fully-implicit method is impossible

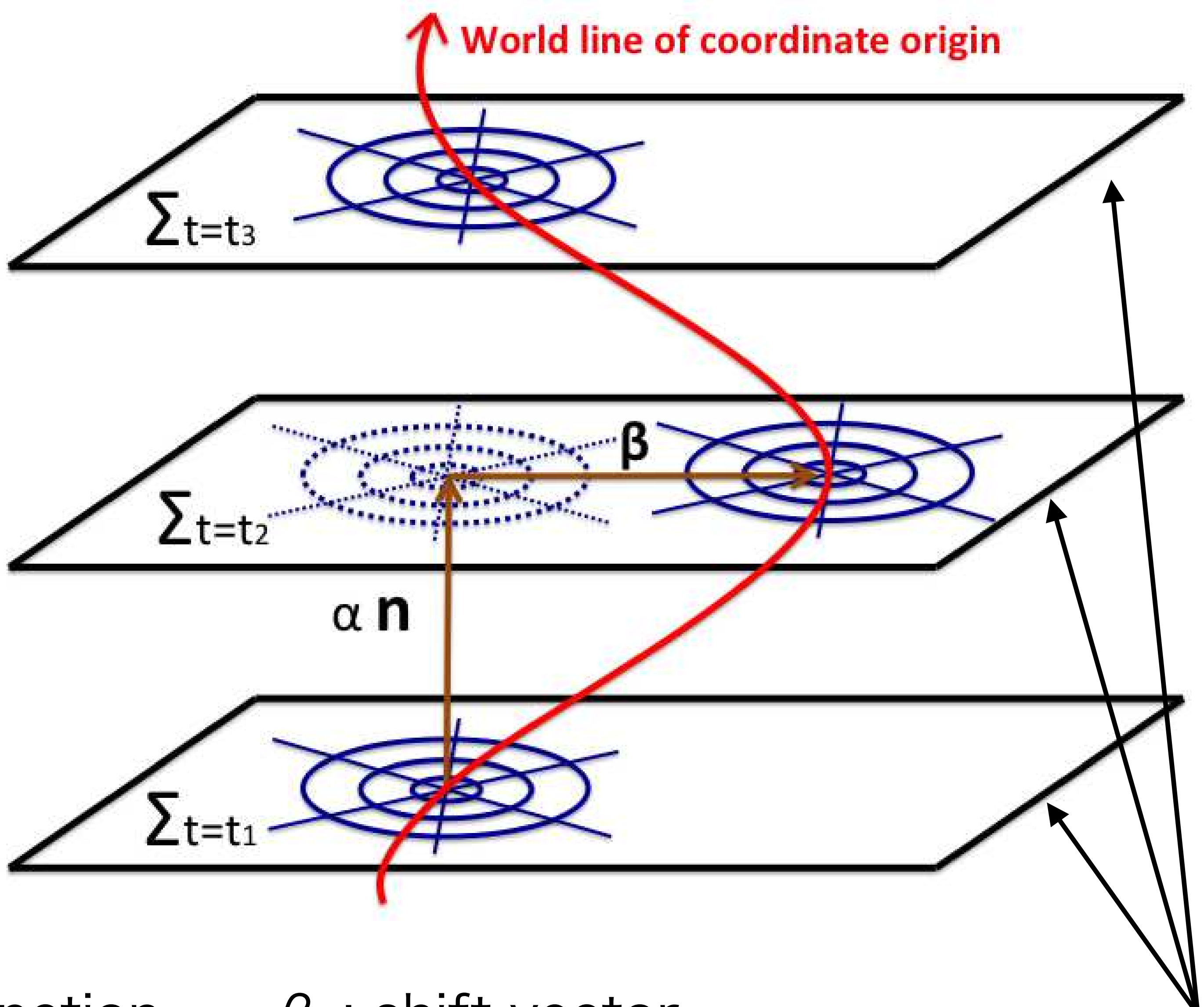
Left-hand side of Boltzmann equation (advection term) is calculated with this frame

Moving Mesh

(Nagakura et al. 2016)

Proto-neutron star moves by non-spherically symmetric distribution of the matter around it.

Boltzmann-Hydro equation in the 3+1 formalism of general relativity (GR)



α: lapse function β_i: shift vector

n: unit vector normal to the spatial hyper-surface with t = constant

Metric of Moving Mesh

(Nagakura et al. 2016)

$$ds^2 = (-\alpha^2 + \beta^k \beta_k)dt^2 + 2\beta_i dt dx^i + \gamma_{ij} dx^i dx^j$$

$$\alpha = 1 \quad \beta^i = \bar{V}^i \quad \gamma_{rr} = 1 \quad \gamma_{\theta\theta} = r^2 \quad \gamma_{\phi\phi} = r^2 \sin\theta^2.$$

α: lapse function β_i: shift vector γ_{ij}: spatial 3-metric

How to determine Vi

$$V^{i(n)} = \frac{P^{i(n)}}{M^{(n)}},$$

$$P^{i(n)} \equiv \int \rho^{(n)} v_o^{i(n+1)} dV_{\rm PNS},$$

$$M^{(n)} \equiv \int \rho^{(n)} dV_{\rm PNS},$$

$$\bar{V}^{i(n+1)} = \bar{V}^{i(n)} + \frac{d\bar{V}^{i(n)}}{dt} \Delta t^{(n)},$$

$$\frac{dV^{i(n)}}{dt} = \frac{(V^{i(n)} - V^{i(n-1)})}{\Delta t^{(n-1)}}$$

$$\frac{d\bar{V}^{i(n)}}{dt} = \frac{dV^{i(n)}}{dt} + C^{(n)} + D^{(n)},$$

$$C^{(n)} \equiv (V^{i(n)} - \bar{V}^{i(n)})/T,$$

$$D^{(n)} \equiv X_m^{i(n)}/T^2,$$

P^i : momentum of PNS M^i : mass of PNS V^i : kick velocity of PNS

X_m^i : deviation of origin from PNS v_0^i : 3-velocity measured in the O-frame

dV_{PNS} : angle-averaged ρ > 10^{13} g cm^{-1} T : recovering time (=0.1ms)

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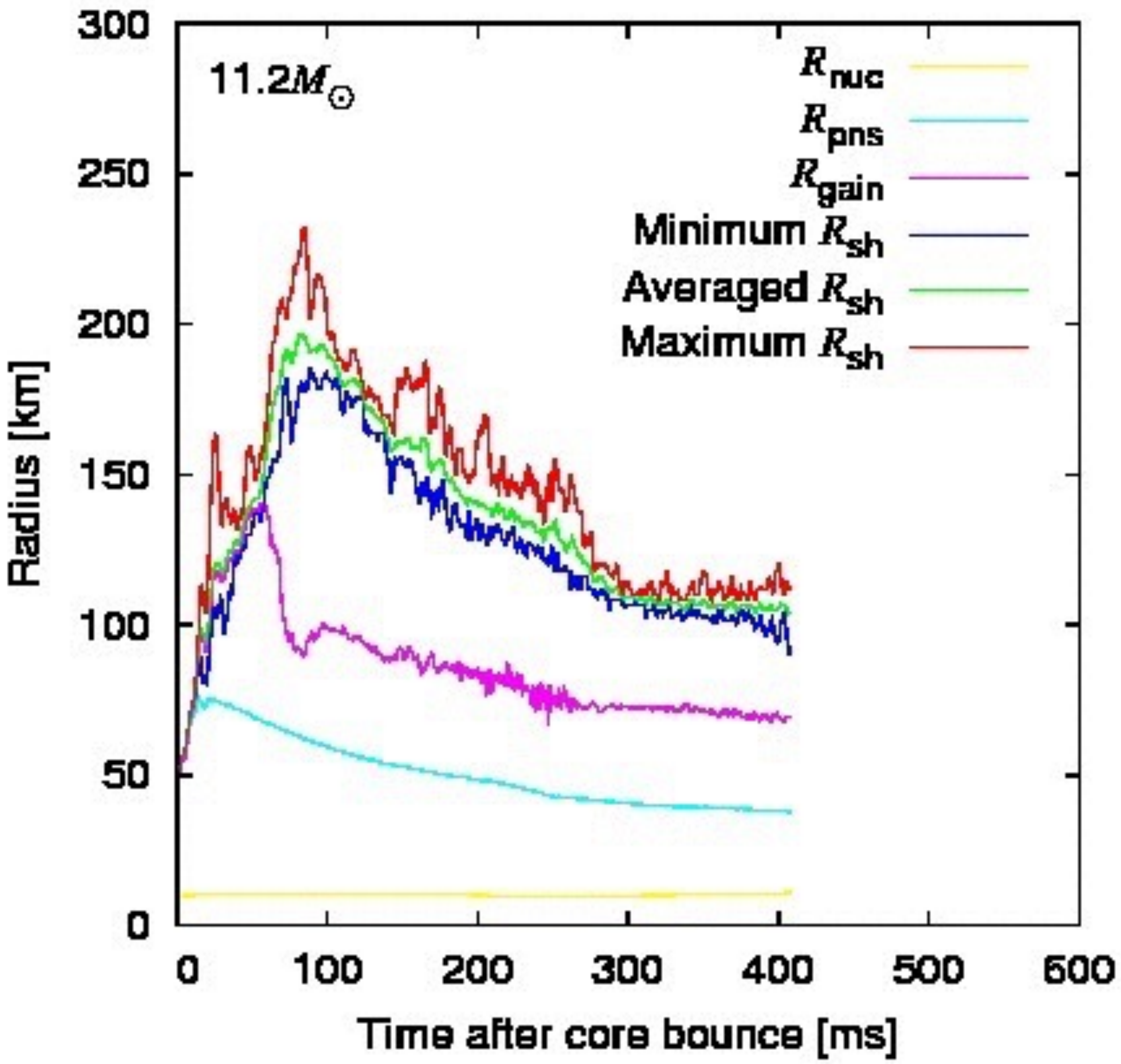
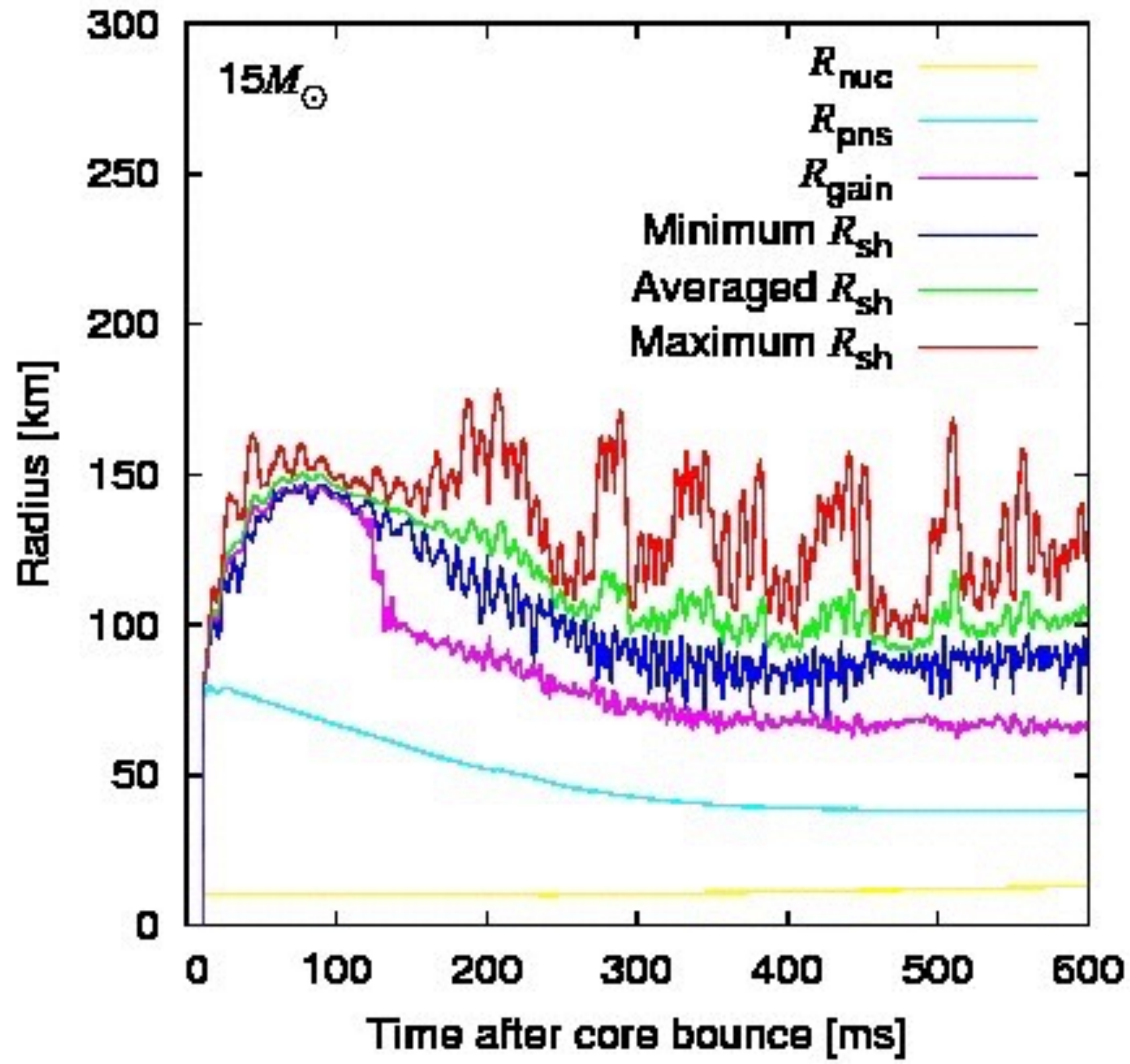
Time Evolution of Shock Radius

Both 11.2Msol and 15Msol progenitors do not explode.





Neutrino-Driven Convection

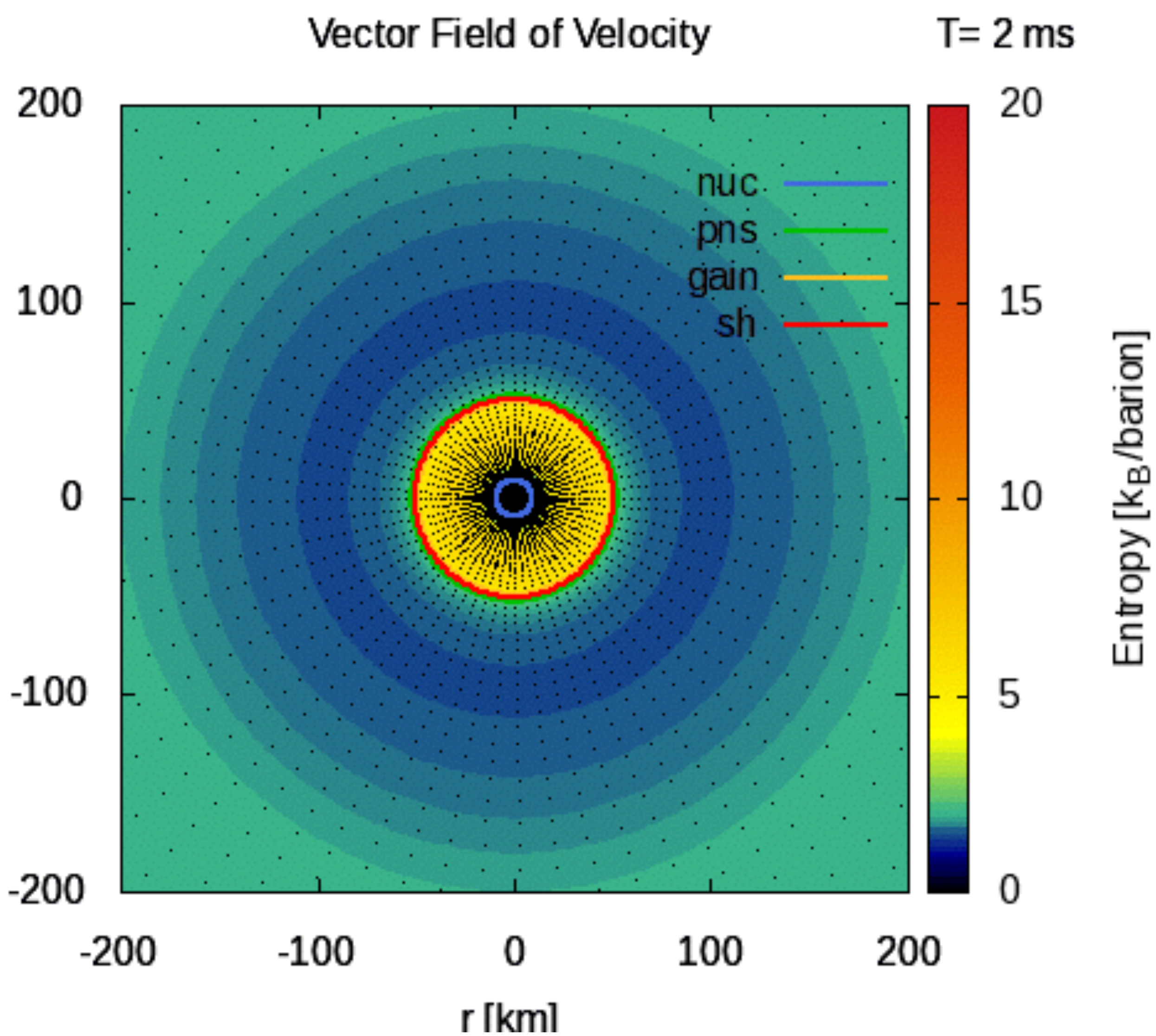
SASI (Standing Accretion Shock Instability)

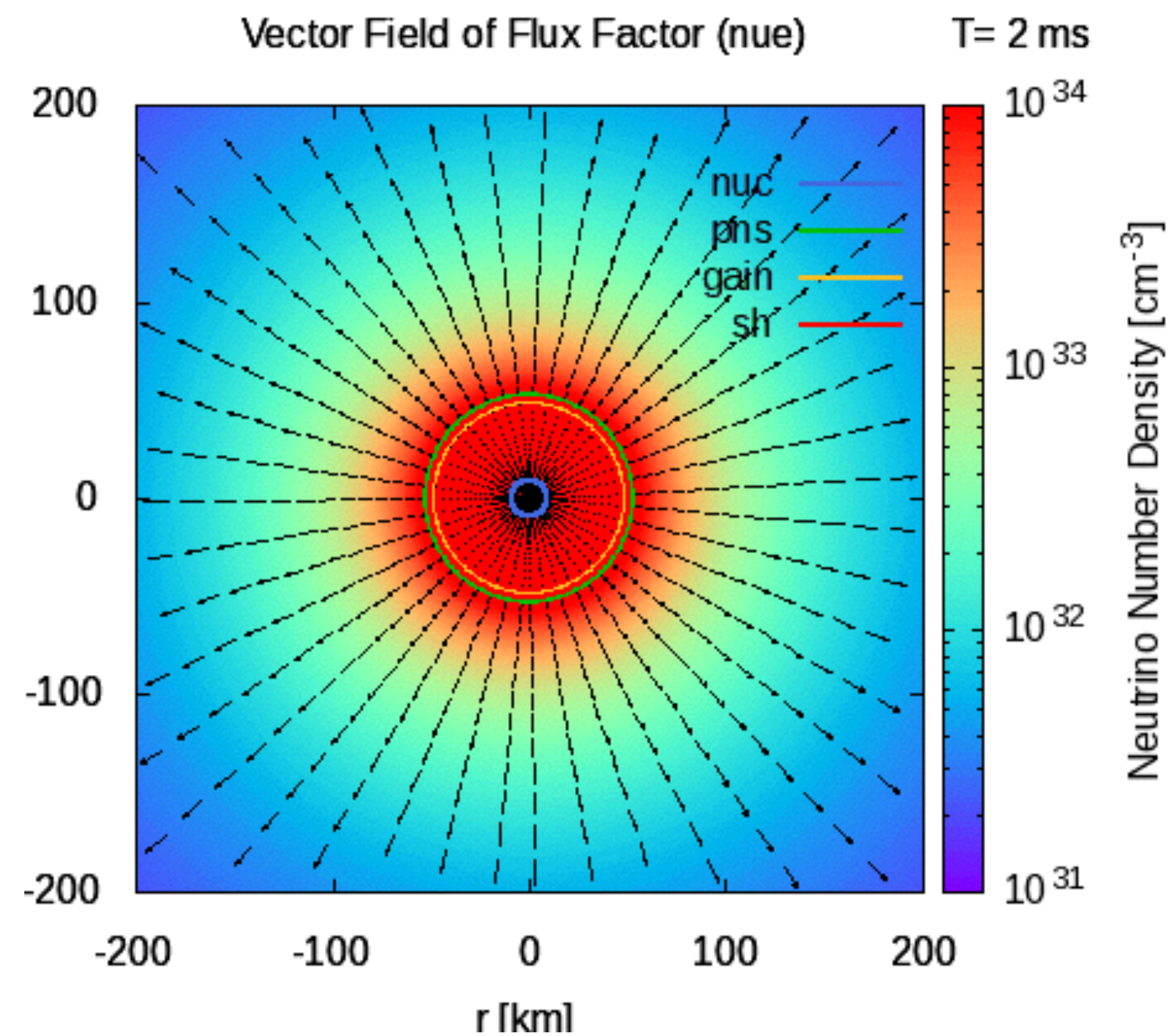
Oscillation amplitude of the shock wave for 11.2Msol is small.

Oscillation amplitude of the shock wave for 15Msol is large.

Fluid and Neutrinos in the Optically Thin Region

15Msol



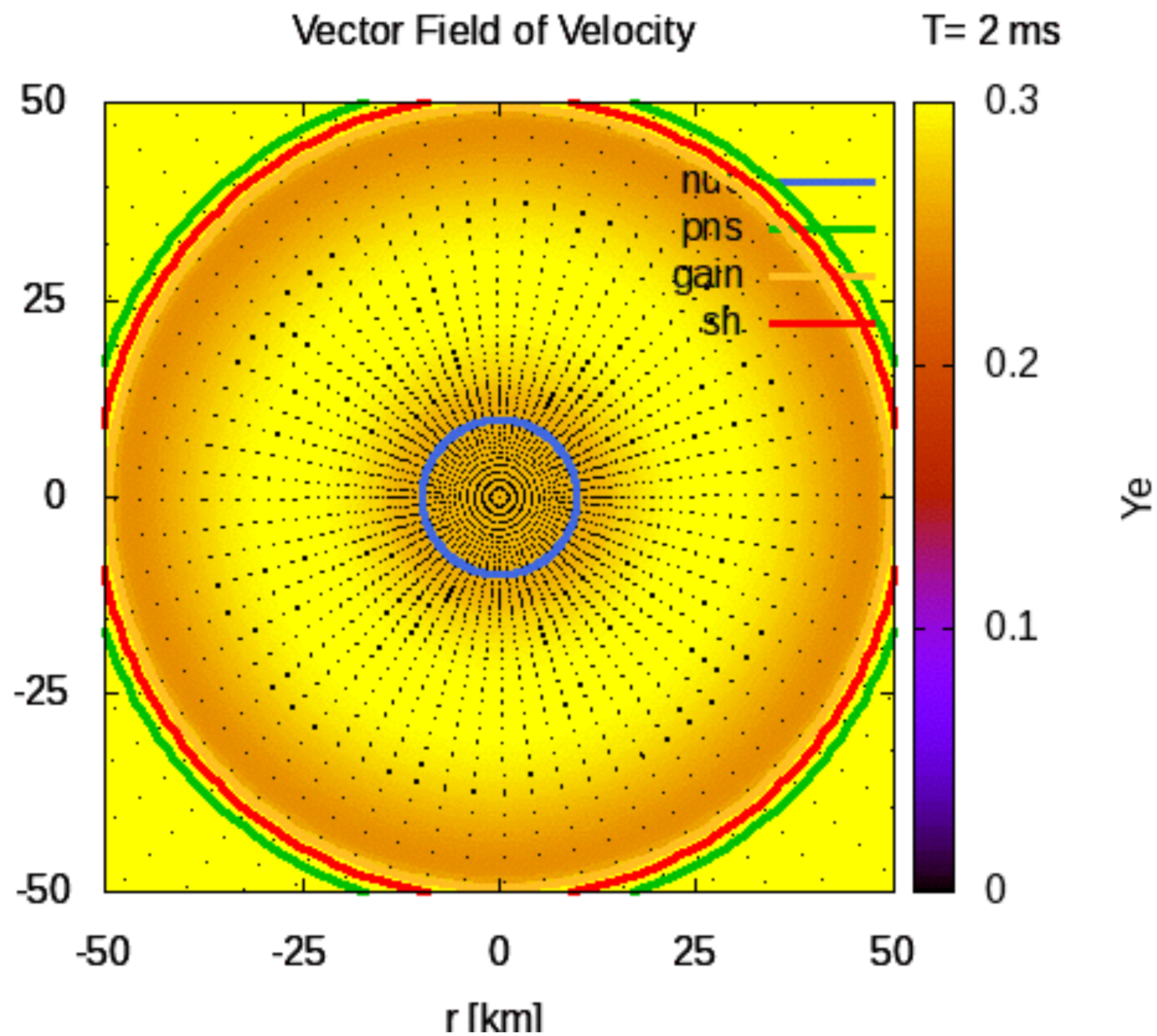


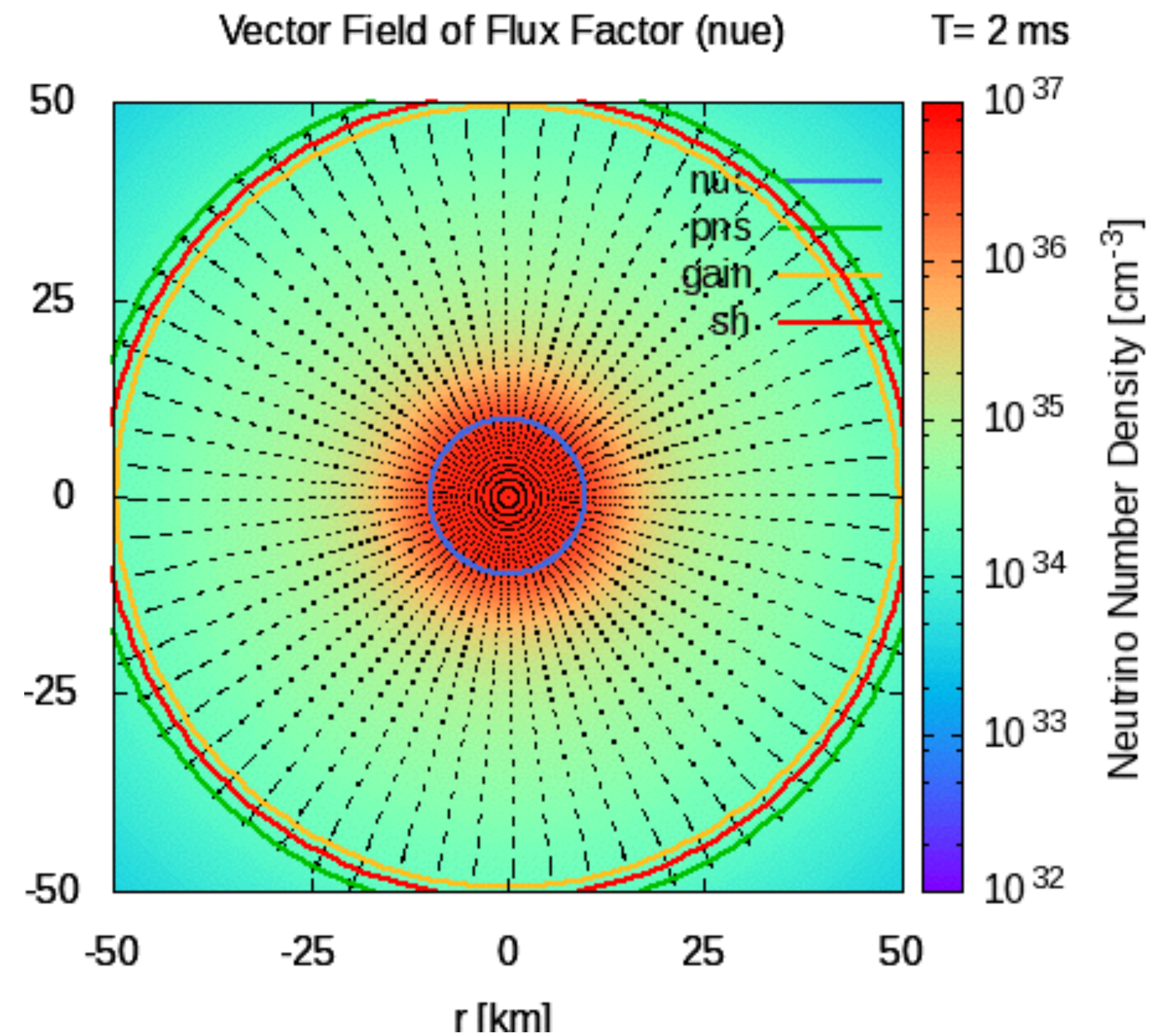
Shock wave vigorously oscillate along the symmetric axis (SASI).

Neutrinos propagate almost along the radial direction.

Fluid and Neutrinos in the Optically Thick Region

15Msol





Convection develop around the negative Ye gradient.

PNS moves along the symmetric axis.

Neutrinos are dragged by matter motions in the proto-neutron star.

Collision term is calculated with the full order of v/c

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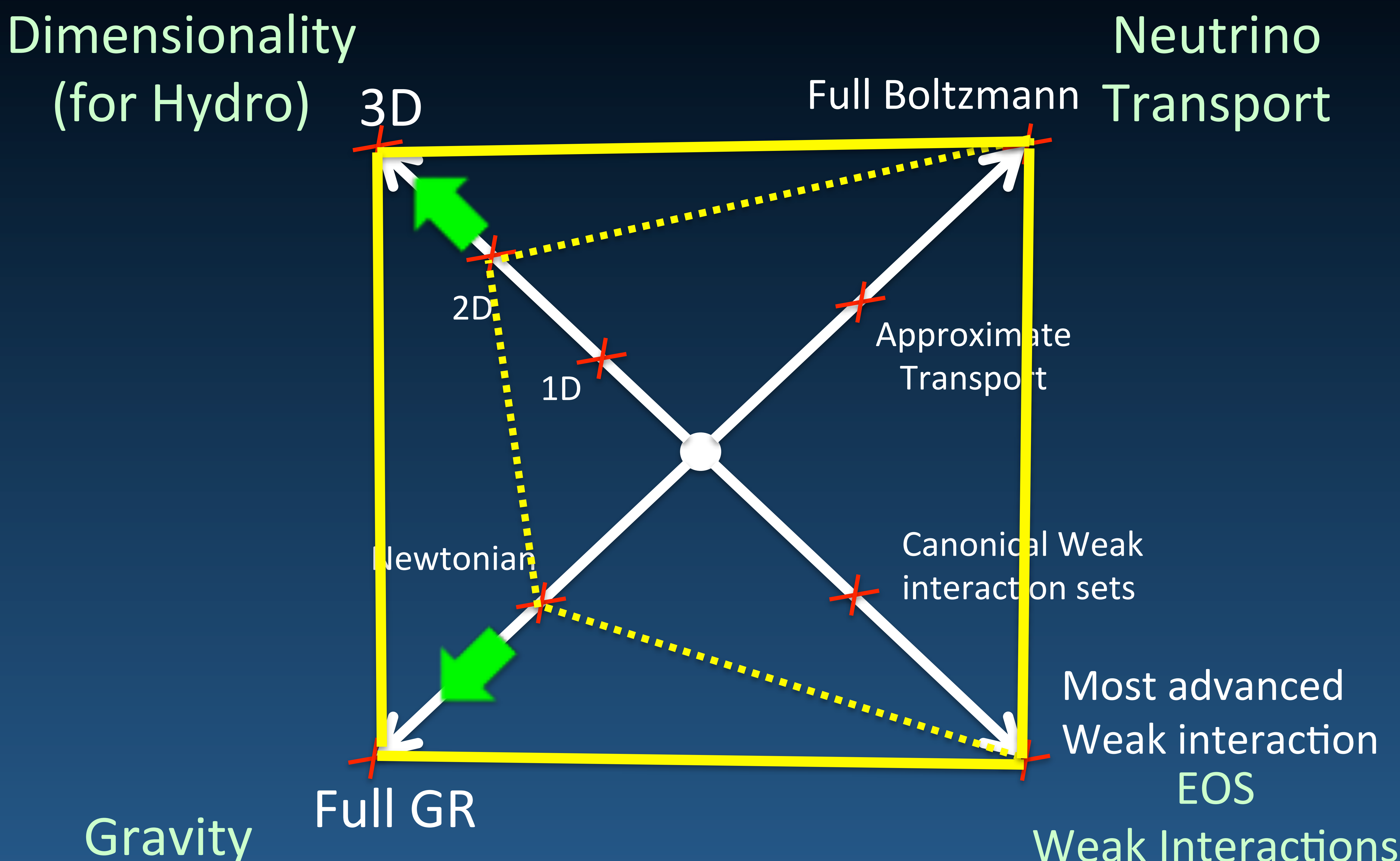
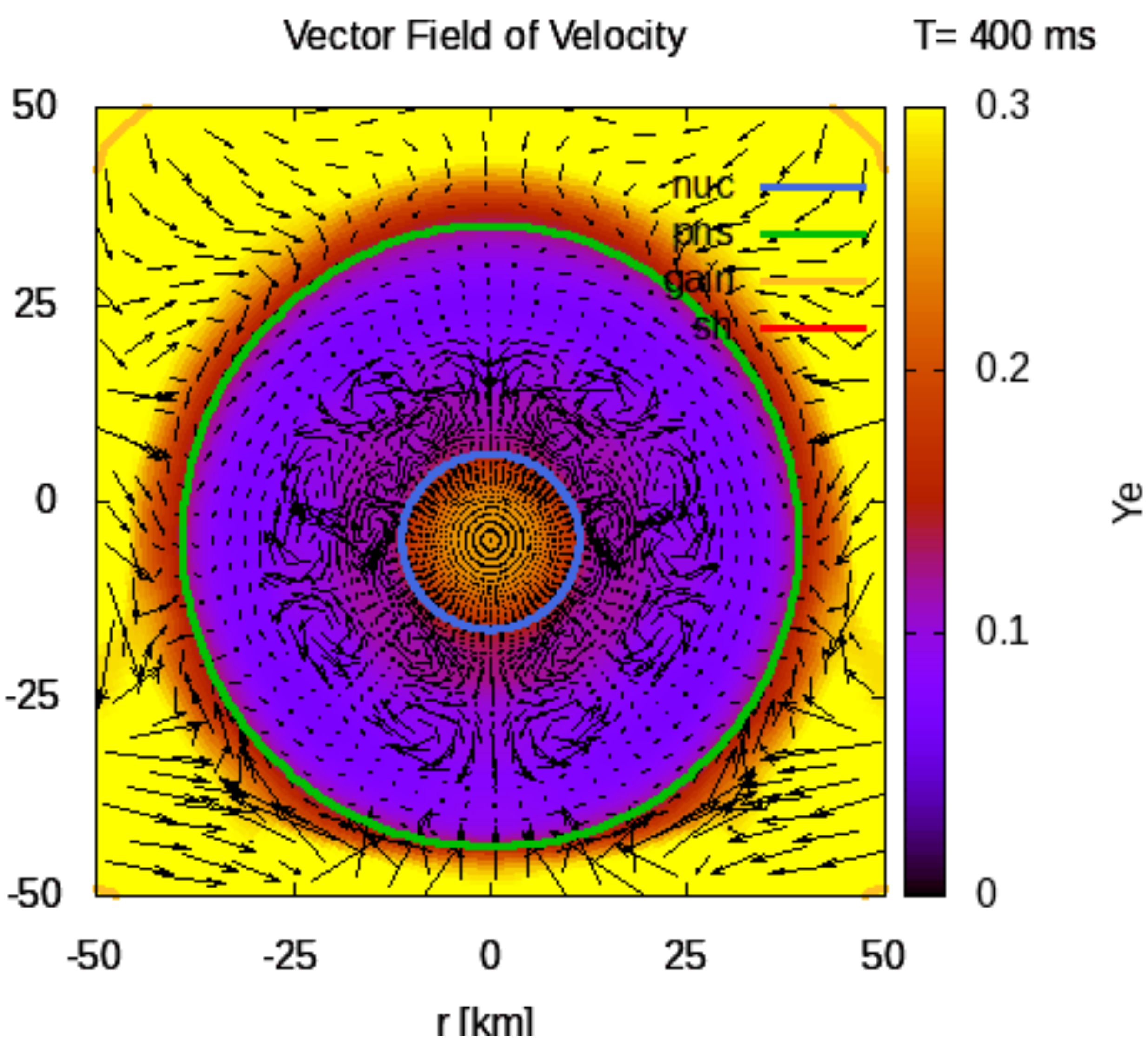
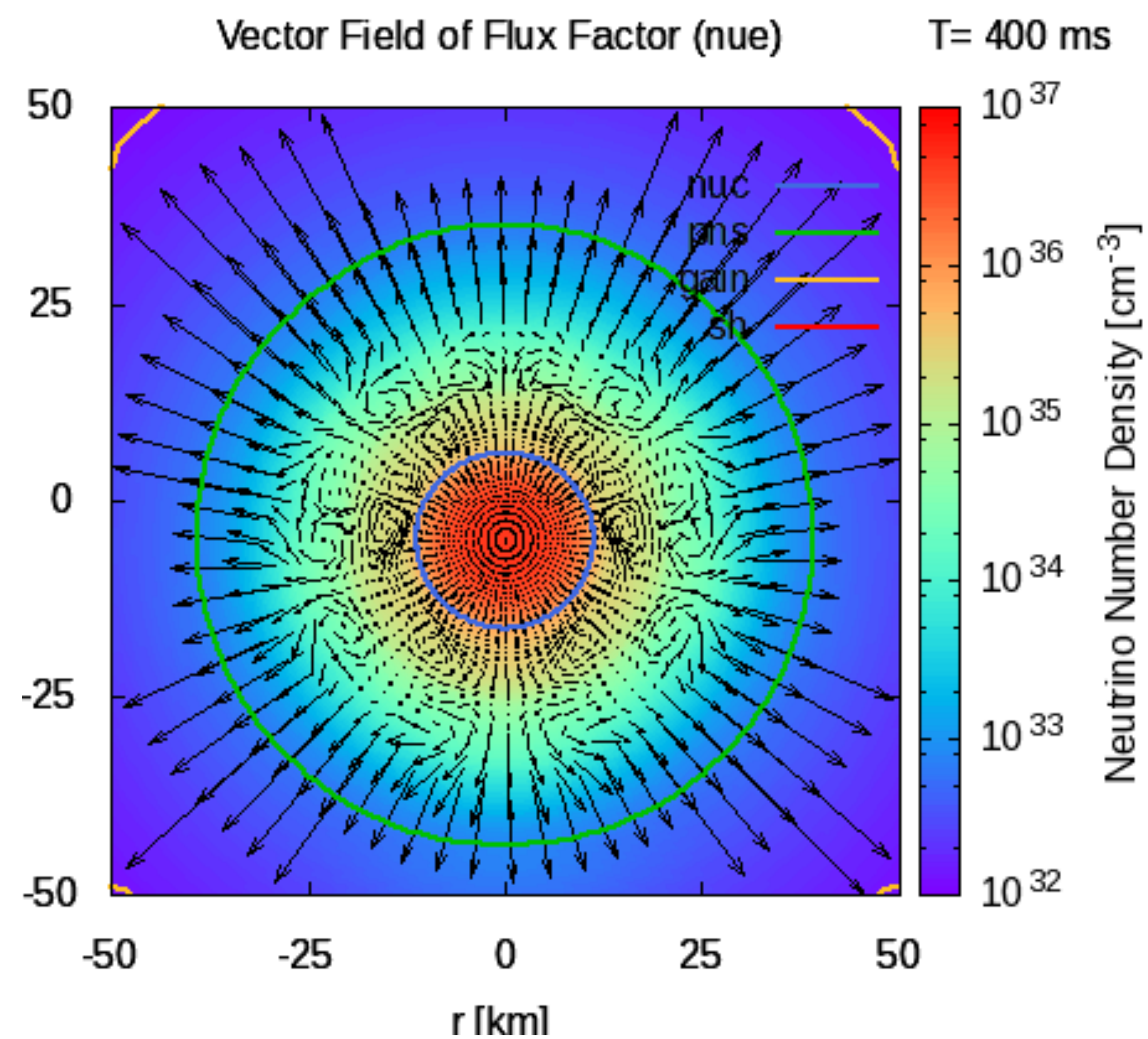
- Towards Exa-Scale Computing -



(by Nagakura)

3D Core-Collapse Simulations

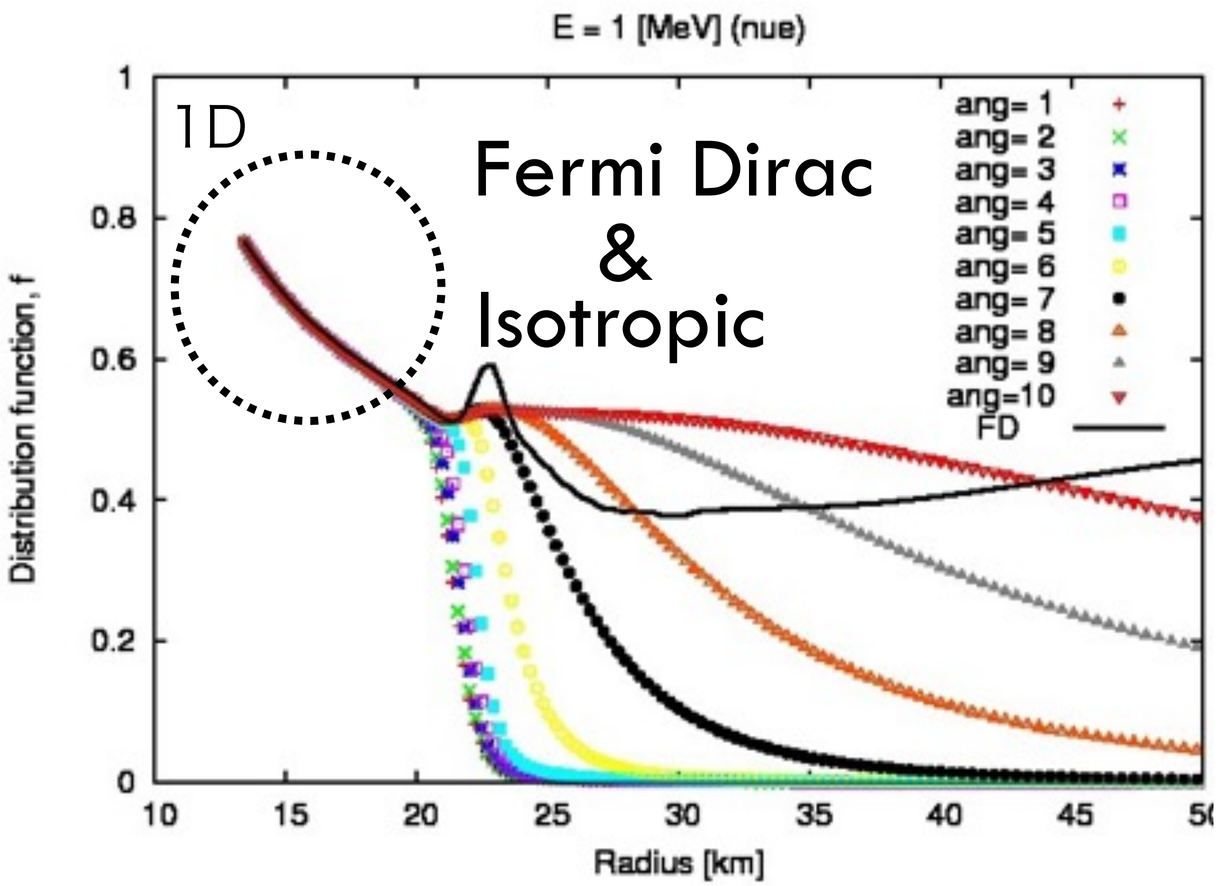


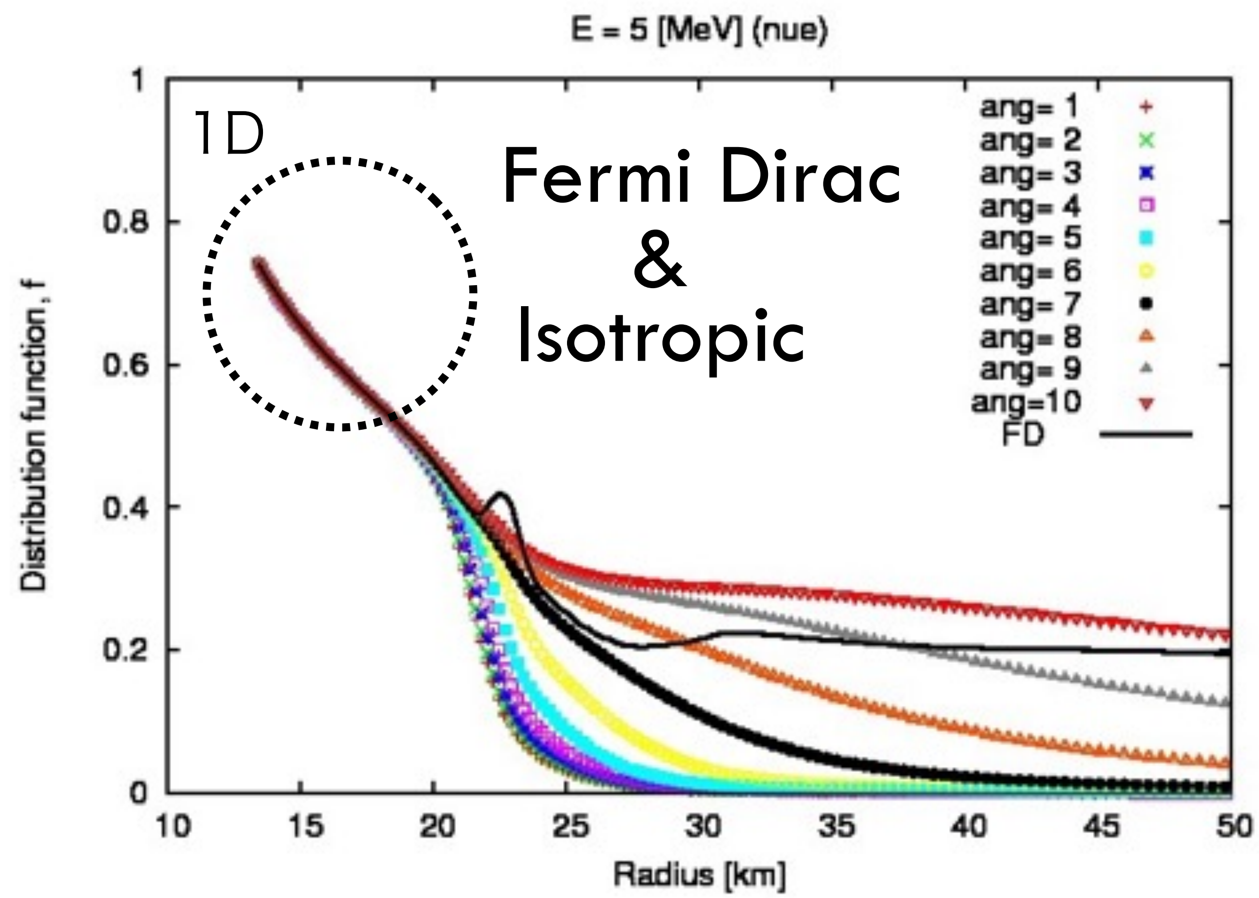


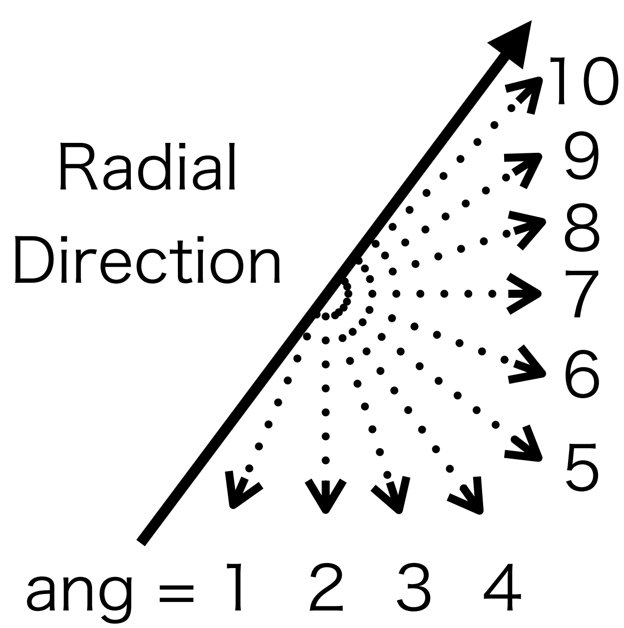
- Courant condition become severe for the spherical coordinate grid for 3D.

- The central region at the baryon density ρ > 10^{14} g cm^{-3} (blue circle) is excised from the computational region.

Inner Boundary Condition for Boltzmann Equation







Inner Boundary

ρ ~10^{14} [g/cm^3]

Fermi Dirac Distribution

$$F_i(E_i) = \frac{1}{1+\exp[(E_i-\mu_i)/kT]}$$

Research Plan for Exa-Scale Computing

1D spherically symmetric simulation ⇒ GR

- EOS, weak reaction rate (2014 -)

2D axisymmetric simulation ⇒ GR

- Non-rotational (2015 - 2016)/ rotational (2016 - 2017) models

  2D neutrino-driven convection and turbulent flow, sloshing mode of SASI, PNS kick

3D simulation ⇒ GR

- Non-rotational/rotational models without the central region (2016 -)

  3D neutrino-driven convection and turbulent flow, sloshing/spiral mode of SASI

- Non-rotational/rotational models with the central region (2018 -)

  PNS kick/spin

+ Validation and improvement of approximative neutrino transport method
+ Gravitational wave and neutrino observations

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Summary

Core-Collapse Supernovae Simulations for 11.2M$_\odot$ and 15M$_\odot$ using Boltzmann-Hydro Code

from collapse (1D) to shock revival/stalled (2D)

- Both 11.2Msol and 15Msol progenitor models do not explode.
- Neutrino-driven convection and SASI appear for 11.2Msol and 15Msol, respectively, in the optically thin region
- Convection triggered by negative Ye gradient develop for both 11.2Msol and 15Msol in the optically thick region.
- Neutrinos are dragged by matter in the optically thick region, while they propagate almost along the radial direction in the optically thin region.

- 3D simulation is the next target for the development of Boltzmann-Hydro code.