超弦理論と超対称理論の数値的研究の現状と展望

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３月１２日（木）＠紀尾井フォーラム

弦理論への期待

素粒子論	宇宙論
Standard Model	インフレーションとビッグバン理論
ヒッグス粒子 フェルミオン（3世代） ゲージ群 の起源	インフラトンおよび そのポテンシャルの初期条件 暗黒物質、暗黒エネルギーの起源
階層性問題	宇宙項問題

弦理論によるこれらの問題の解決

目指しているもの

- ブラックホール

ブラックホールの熱力学的性質の微視的起源
量子重力効果
ホーキング輻射と情報喪失パラドックス

- 初期宇宙

宇宙の誕生 (９次元から３次元への相転移)
インフレーション，密度ゆらぎ
標準模型 (を超える理論)の出現

- 超対称性理論

プランクスケールと電弱スケールの間の階層性に対する理解

研究手段と方向性

超対称 { ゲージ理論 / 行列模型 } のモンテカルロ・シミュレーション

- ゲージ・重力対応
  Maldacena ('97), Gubser-Klebanov-Witten('98), …
- 超弦理論、M理論に非摂動的に記述する行列模型
  Banks-Fischler-Shenker-Susskind, ('96)
  Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

→ 行列的な自由度から時空が創発(emerge)

- 超対称ゲージ理論の数値シミュレーション法の発展
  Cohen-Unsal-Kaplan-Katz('03), Sugino('04),
  Hanada-Nishimura-Takeuchi ('07)

研究体制

ゲージ／重力対応の直接検証
西村(KEK)、金児(KEK)、土屋(静岡大) （＋花田、伊敷、島崎、加堂）

タイプIIB行列模型に基づく初期宇宙の研究
土屋（静岡大）、西村(KEK)、東（摂南大） （＋伊藤、Kim、Anagnostopoulos）

２次元、３次元、４次元超対称ゲージ理論
鈴木（九大）、杉野（岡山光量子研）、松浦（慶応大） （＋花田）

2/24 (火)　KEKに集まって、半日議論を行った。
そのときの議論に基づいて、検討状況の報告を行う。

Plan of the talk

1. はじめに

2. ゲージ／重力対応の直接検証

3. 行列模型に基づく初期宇宙の研究

4. まとめと展望

1. ゲージ／重力対応の直接検証

Direct test of gauge/gravity duality

Difficult because the gauge theory side is strongly coupled.

D0 brane case Itzhaki-Maldacena-Sonnenschein-Yankielowicz ('98)

1d SYM with 16 supercharges

Monte Carlo simulation

- internal energy v.s. temperature

  microscopic origin of black hole thermodynamics

  α' corrections, string loop corrections

- Wilson loop

  One can see Schwarzschild radius from gauge theory!

- correlation functions

  confirmation of the GKP-Witten prescription

D0-brane system : 1d SYM with 16 SUSY

$$S_{\rm b} = \frac{1}{g^2} \int_0^\beta dt \ {\rm tr} \left\{ \frac{1}{2} (DX_i(t))^2 - \frac{1}{4} [X_i(t), X_j(t)]^2 \right\}$$

$$S_{\rm f} = \frac{1}{g^2} \int_0^\beta dt \ {\rm tr} \left\{ \frac{1}{2} \Psi_\alpha D \Psi_\alpha - \frac{1}{2} \Psi_\alpha (\gamma_i)_{\alpha\beta} [X_i, \Psi_\beta] \right\}$$

1d gauge theory $\quad D = \partial_t - i\,[A(t), \ \cdot \ ]$

$$\begin{cases} X_j(t) & (j = 1, \cdots, 9) \quad \text{p.b.c.} \\ \Psi_\alpha(t) & (\alpha = 1, \cdots, 16) \quad \text{anti p.b.c.} \end{cases}$$

$T = \beta^{-1}$ temperature

$\lambda = g^2 N$ 't Hooft coupling

$$\lambda_{\rm eff} = \frac{\lambda}{T^3}$$

$\lambda = 1$ (without loss of generality)

- low T ⇨ strongly coupled — dual gravity description
- high T ⇨ weakly coupled — high T exp.

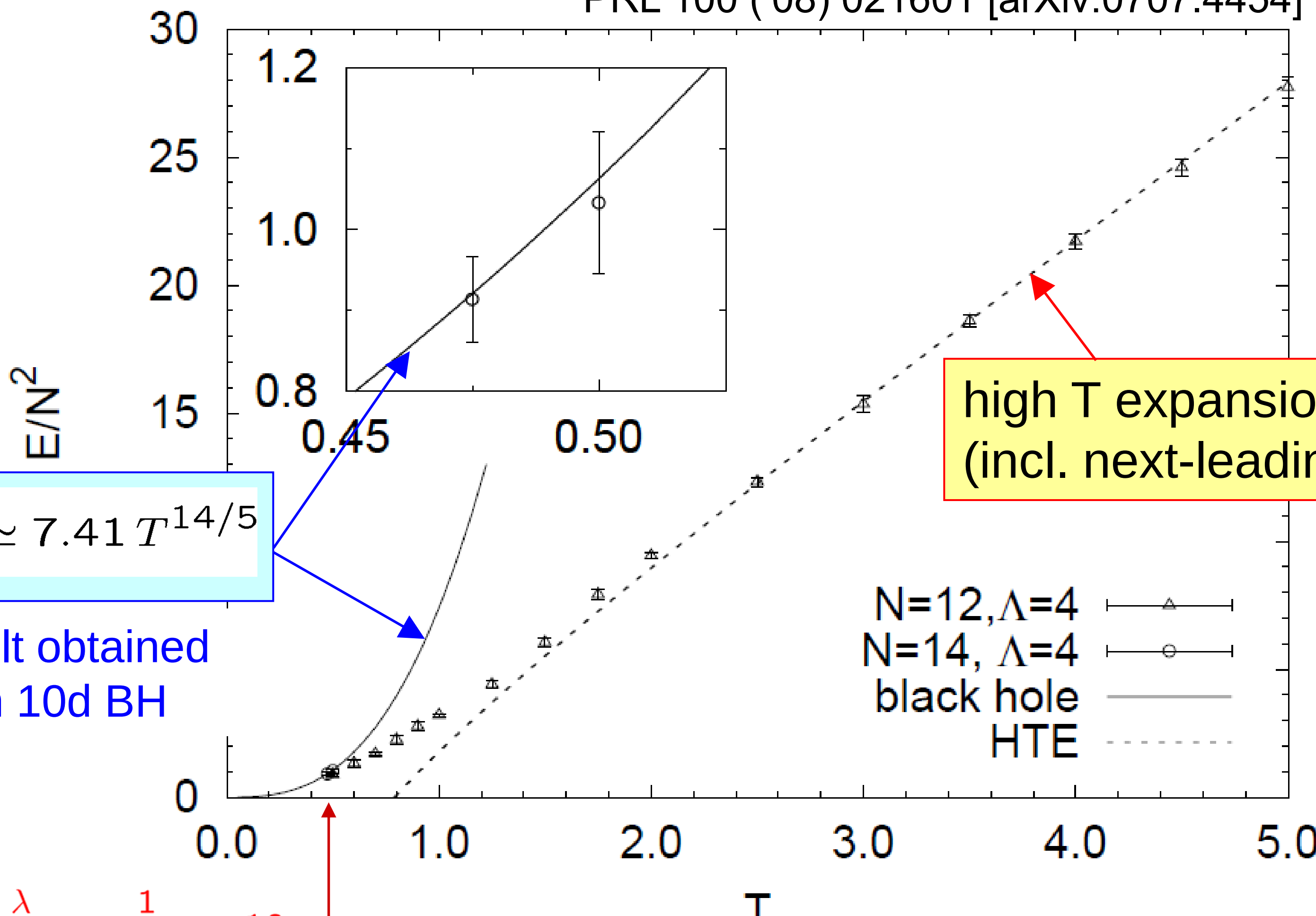
Kawahara-J.N.-Takeuchi ('07)

Internal energy

$$E = \frac{\partial}{\partial \beta}(\beta \mathcal{F})$$

free energy

Anagnostopoulos-Hanada- J.N.-Takeuchi,
PRL 100 ('08) 021601 [arXiv:0707.4454]



high T expansion
(incl. next-leading order)

$$\frac{E}{N^2} \simeq 7.41\, T^{14/5}$$

result obtained from 10d BH

$$\lambda_{\text{eff}} = \frac{\lambda}{T^3} = \frac{1}{0.46^3} \approx 10$$

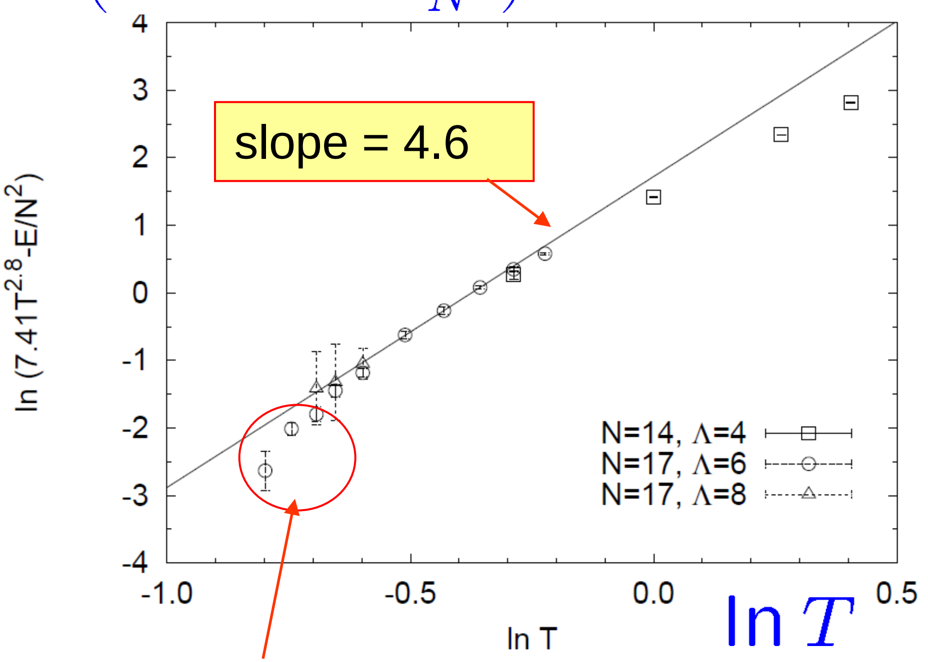
Comparison including α' corrections

Hanada-Hyakutake-J.N.-Takeuchi,
PRL 102 ('09) 191602 [arXiv:0811.3102]

$$\frac{E}{N^2} = 7.41\, T^{14/5} - C\, T^{23/5}$$

α' corrections

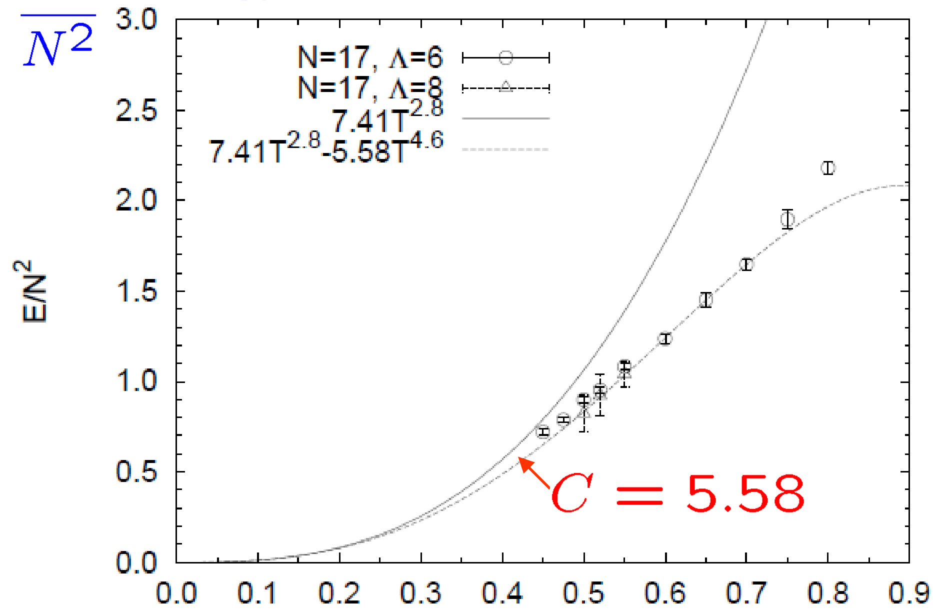
$\ln\left(7.41\, T^{14/5} - \frac{E}{N^2}\right)$



$\ln T$

finite cutoff effects

$\frac{E}{N^2}$



$C = 5.58$

T

MC data at $T \lesssim 0.7$ can be nicely fitted with $C = 5.58$

1/N corrections

⟺ string loop corrections

fixed by explicit calculations on the string theory side (Y. Hyakutake, Prog.Theor.Exp.Phys. 033B04, 2014)

$$\frac{E}{N^2} = 7.41T^{2.8} - 5.58T^{4.6} + \frac{1}{N^2}(-5.76T^{0.4} + aT^{2.2} + \ldots) + \frac{1}{N^4}(bT^{-2.6} + cT^{-0.8} + \ldots) + \mathcal{O}\left(\frac{1}{N^6}\right)$$

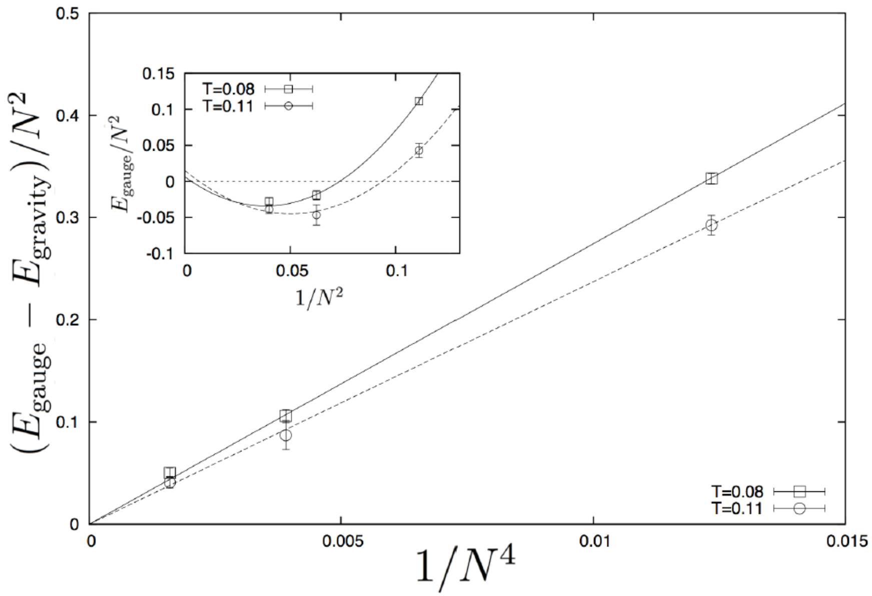
We can test it by looking at:

$$\frac{E}{N^2} - \left(7.41T^{2.8} - 5.58T^{4.6} - 5.76\frac{T^{0.4}}{N^2}\right) \text{ v.s. } \frac{1}{N^4}$$

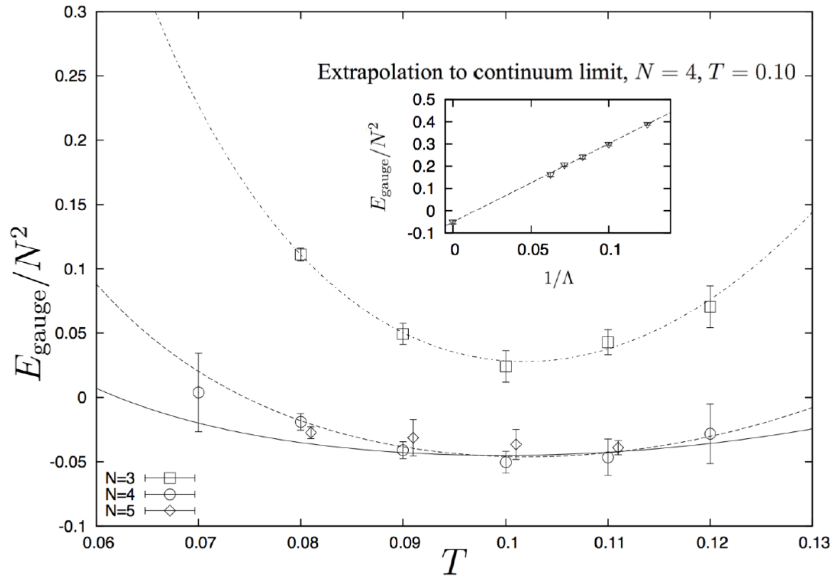
Loop corrections of closed strings

Ishiki-Hanada-Hyakutake-J.N.: arXiv:1311.5607 [hep-th]
Published in Science 344 (2014) 882-885

$\frac{1}{N^2}\left(E_{\text{gauge}} - E_{\text{gravity}}\right)$



$\frac{1}{N^2} E_{\text{gauge}}$



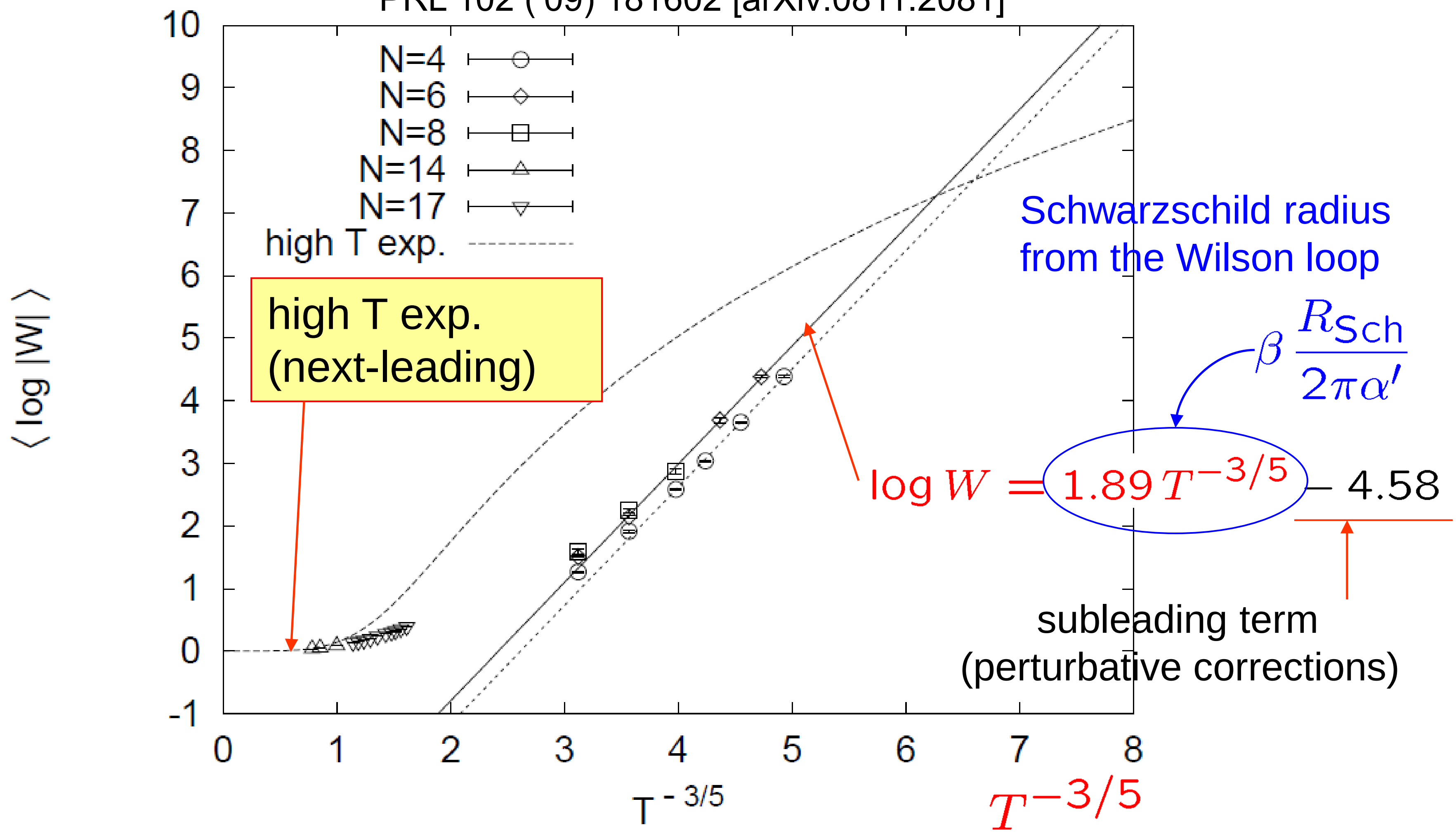
$1/N^4$ T

Monte Carlo results are consistent with string loop corrections!

Wilson loops

$$W = \mathrm{tr}\,\mathcal{P}\exp\left[i\int_0^\beta dt\{A(t) + iX_9(t)\}\right]$$

Hanada-Miwa-J.N.-Takeuchi,
PRL 102 ('09) 181602 [arXiv:0811.2081]

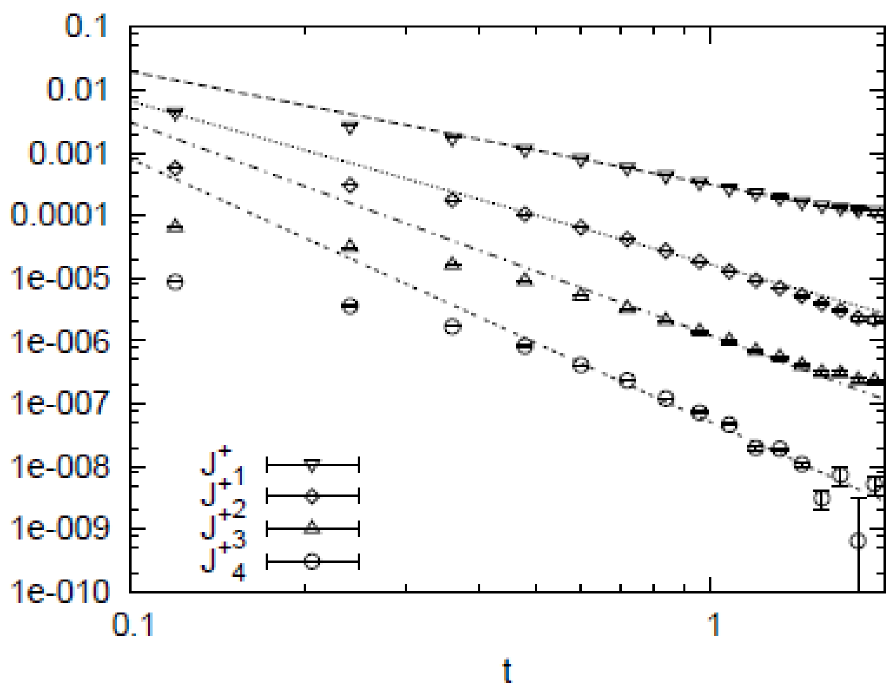


Correlation functions

Hanada-J.N.-Sekino-Yoneya,
PRL 104 ('10) 151601,
JHEP 12(2011) 020

$$\langle \mathcal{O}(t)\mathcal{O}(t')\rangle \propto \frac{1}{|t-t'|^{2\nu+1}}$$

predicted from dual geometry by Sekino-Yoneya ('99)
using Gubser-Klebanov-Polyakov-Witten relation ('98)



$$J^{+ij}_{l,i_1,\cdots,i_l} \equiv \frac{1}{N}\mathrm{Str}\left(F_{ij}X_{i_1}\cdots X_{i_l}\right)$$

$$F_{ij} \equiv -i[X_i, X_j]$$

$l=1$

$l=2$

$l=3$

$l=4$

$$\nu = \frac{2l}{5}$$

Predicted power law confirmed clearly.

$$N=3,\ \beta=4,\ \Lambda=16$$

2. 行列模型に基づく初期宇宙の研究

type IIB matrix model

Ishibashi-Kawai-Kitazawa-Tsuchiya ('96)

$$S_b = -\frac{1}{4g^2}\mathrm{tr}([A_\mu, A_\nu][A^\mu, A^\nu])$$
$$S_f = -\frac{1}{2g^2}\mathrm{tr}(\Psi_\alpha(\mathcal{C}\Gamma^\mu)_{\alpha\beta}[A_\mu, \Psi_\beta])$$

a nonperturbative formulation of superstring theory

$N \times N$ Hermitian matrices

$A_\mu \quad (\mu = 0, \cdots, 9)$ Lorentz vector

$\Psi_\alpha \quad (\alpha = 1, \cdots, 16)$ Majorana-Weyl spinor

raised and lowered by the metric

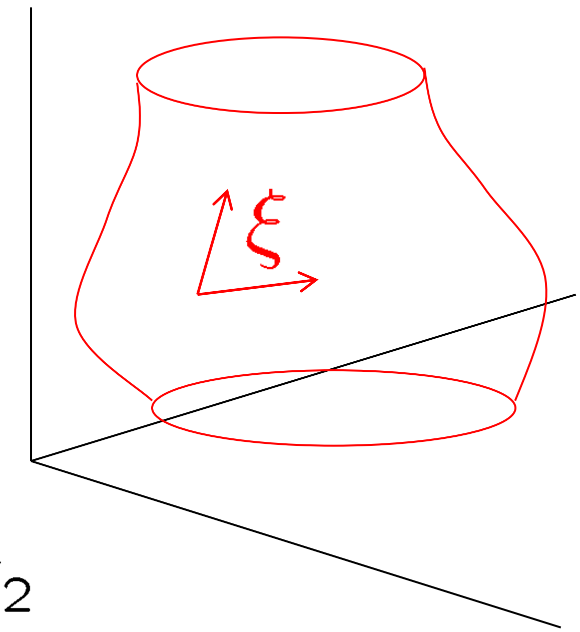
$$\eta = \mathrm{diag}(-1, 1, \cdots, 1)$$

Wick rotation $(A_0 = -iA_{10}\ , \quad \Gamma^0 = i\Gamma_{10})$

⇒ Euclidean model with SO(10) symmetry

Connection to the worldsheet formulation

$X^\mu(\xi), \ \Psi(\xi)$



- worldsheet action

$$S = \int d^2\xi \sqrt{g} \left(\frac{1}{4}\{X^\mu, X^\nu\}^2 + \frac{1}{2}\bar{\Psi}\gamma^\mu\{X^\mu, \Psi\} \right)$$

$$\{X, Y\} \equiv \frac{1}{\sqrt{g}} \epsilon^{ab} \frac{\partial X}{\partial \xi^a} \frac{\partial Y}{\partial \xi^b}$$

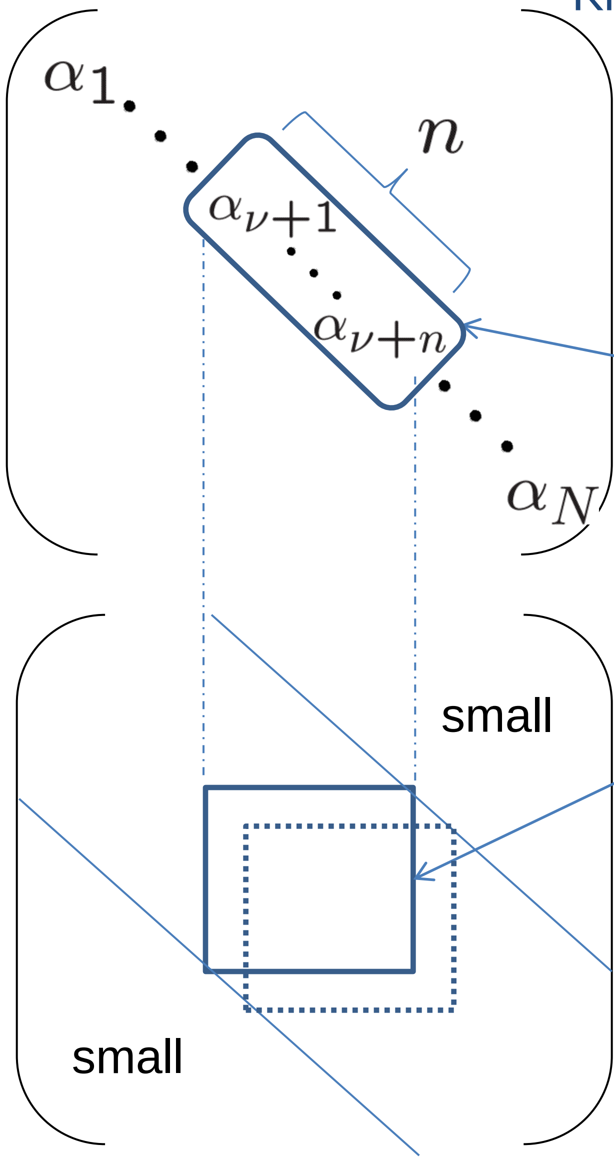
Poisson bracket (regarding ξ_1 and ξ_2 as p and q in Hamilton dynamics)

quantization $\Longrightarrow$ type IIB matrix model $\quad (\hbar \sim \frac{1}{N})$

$$\{X^\mu(\xi), X^\nu(\xi)\} \mapsto -i[A^\mu, A^\nu]$$

How to extract time-evolution

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601



diagonalize A_0

$\alpha_1 < \cdots < \alpha_N$

SU(N) transformation

definition of time "t"

$$t = \frac{1}{n} \sum_{i=1}^{n} \alpha_{\nu+i}$$

The state of the universe $\bar{A}_i(t)$ at time t

A_i has a band diagonal structure

non-trivial dynamical property

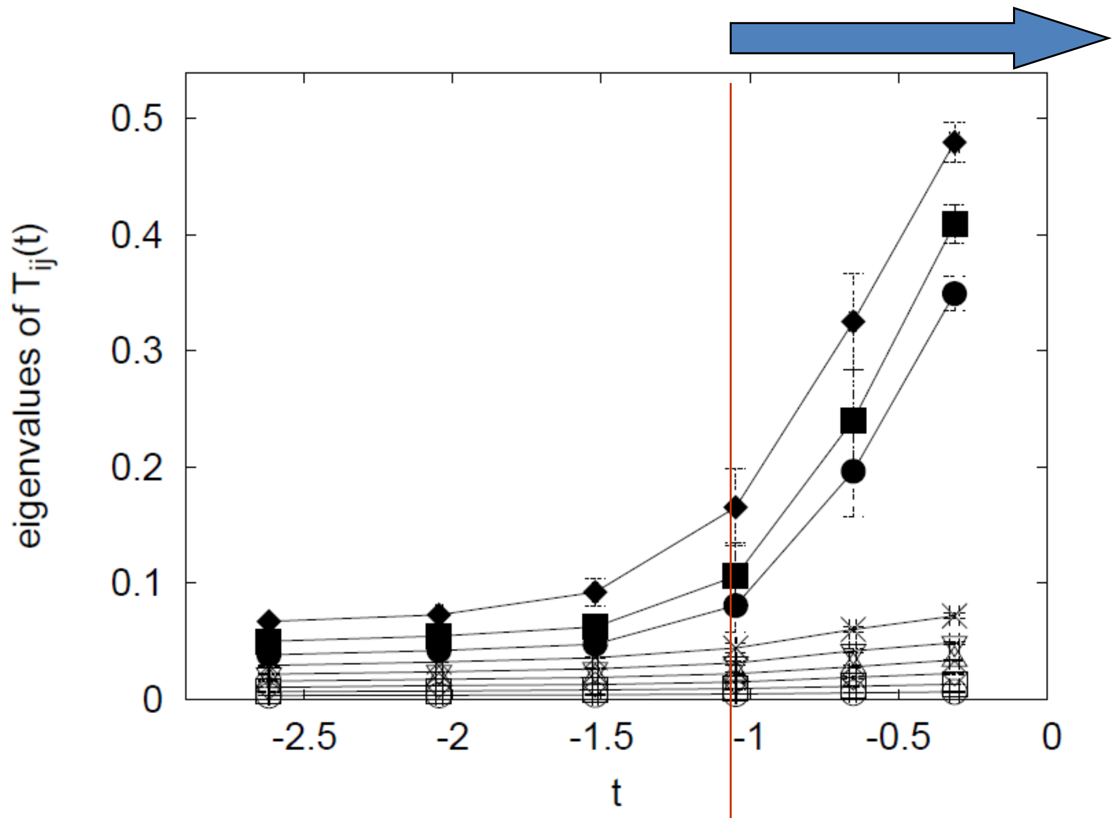
Spontaneous breaking of SO(9)

Kim-J.N.-Tsuchiya, PRL 108 (2012) 011601

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}\left\{\bar{A}_i(t)\bar{A}_j(t)\right\}$$

$$SO(9) \xrightarrow{\text{SSB}} SO(3)$$

$N = 16$, $\kappa = 4.0$

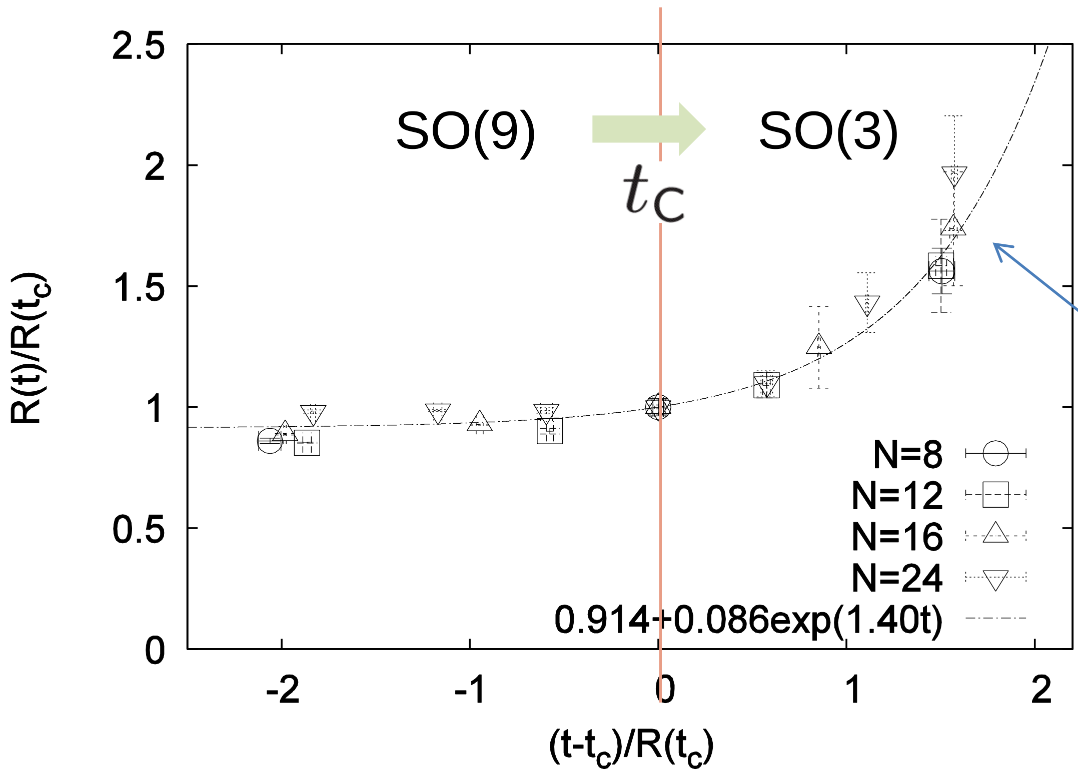


"critical time"

Exponential expansion

Ito-Kim-J.N.-Tsuchiya,
arXiv:1311.5579 [hep-lat]

$$R(t)^2 \equiv \frac{1}{n}\mathrm{tr}\,\bar{A}_i(t)^2$$



fitted well to

$$f(x) = a + (1-a)e^{bx}$$

Exponential expansion

Inflation

Effects of fermionic action

$$S_f = \mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \Psi_\beta])$$
$$= \mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^0)_{\alpha\beta} [A_0, \Psi_\beta]) + \mathrm{tr}(\bar{\Psi}_\alpha (\Gamma^i)_{\alpha\beta} [A_i, \Psi_\beta])$$

dominant term
at early times

dominant term
at late times

keep only the first term

simplified model at early times

$$\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j} (\alpha_i - \alpha_j)^{2(d-1)}$$

repulsive force between eigenvalues of A_0

simplified model at late time

$$\mathrm{Pf}\mathcal{M}(A) \simeq 1$$

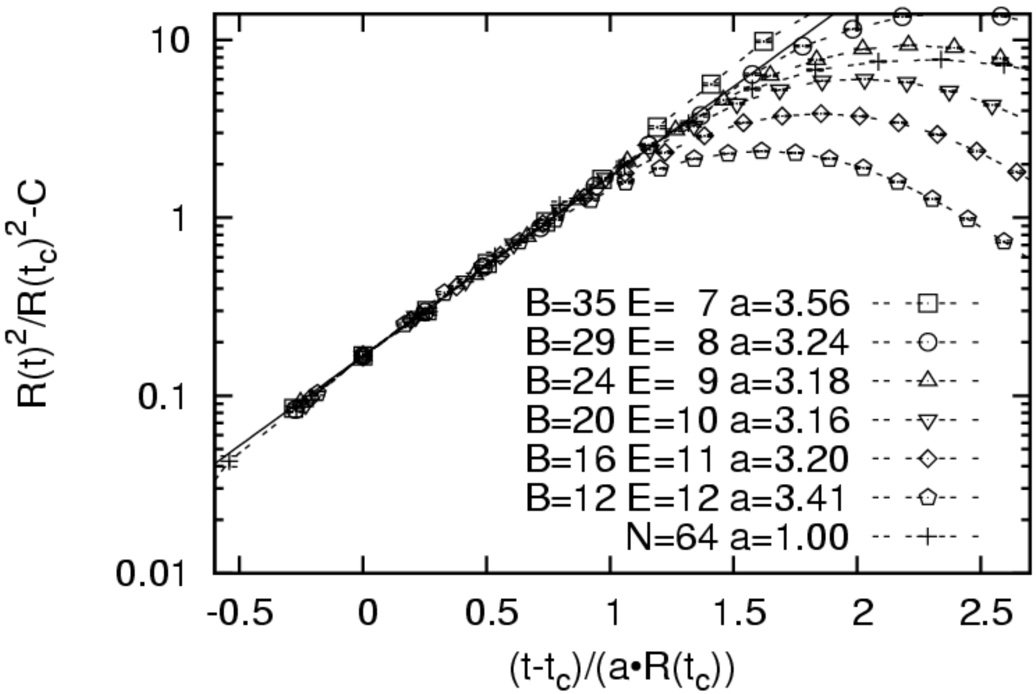
quench fermions

Exponential expansion at early times

- simplified model at early times

Ito-Kim-Koizuka-J.N.-Tsuchiya,
arXiv:1312.5415 [hep-th]
PTEP (2014) 083B01

$$\mathrm{Pf}\mathcal{M}(A) \simeq \Delta^{d-1} = \prod_{i<j} (\alpha_i - \alpha_j)^{2(d-1)}$$



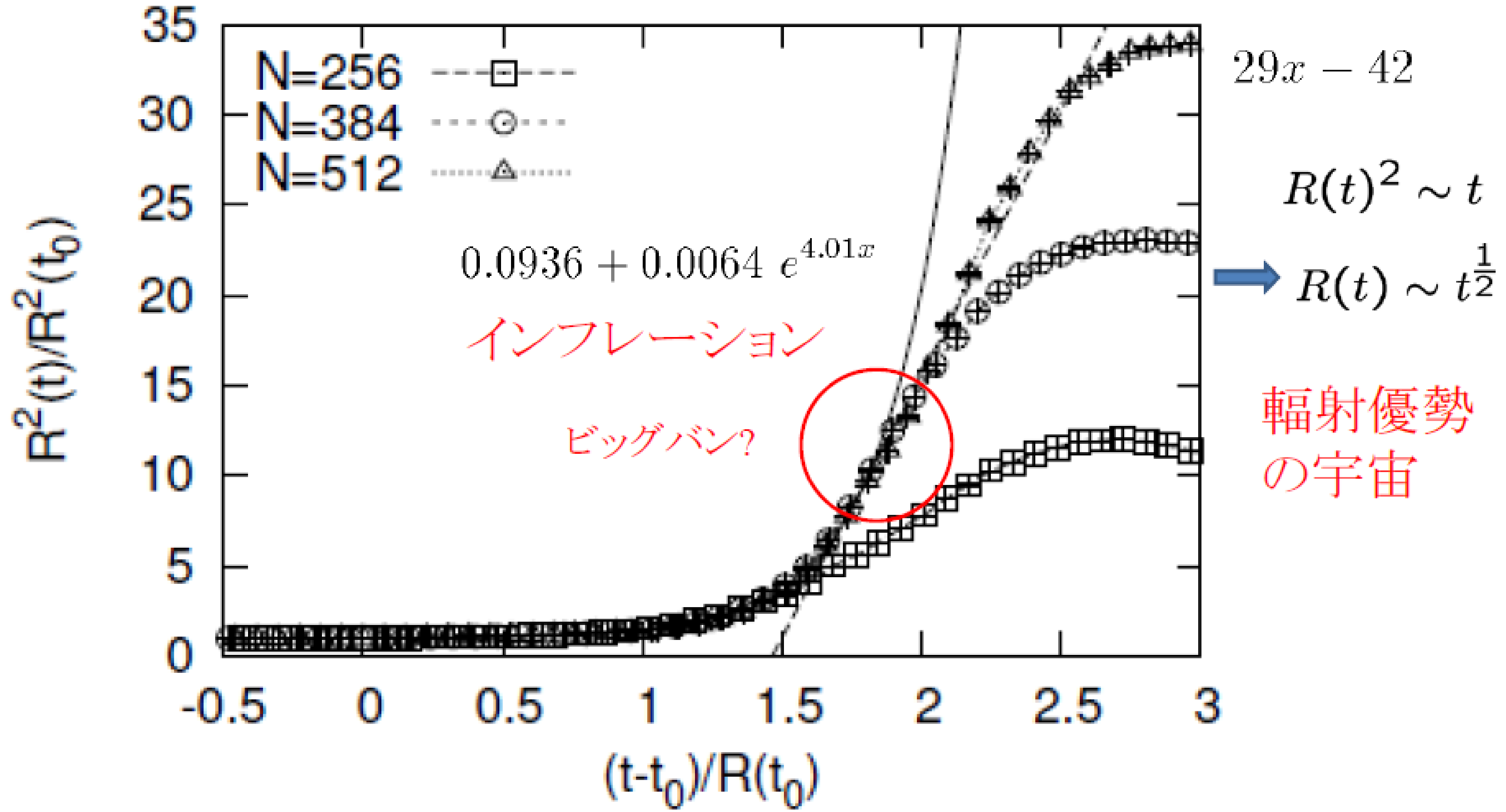
exponential expansion

The first term is important for exponential expansion.

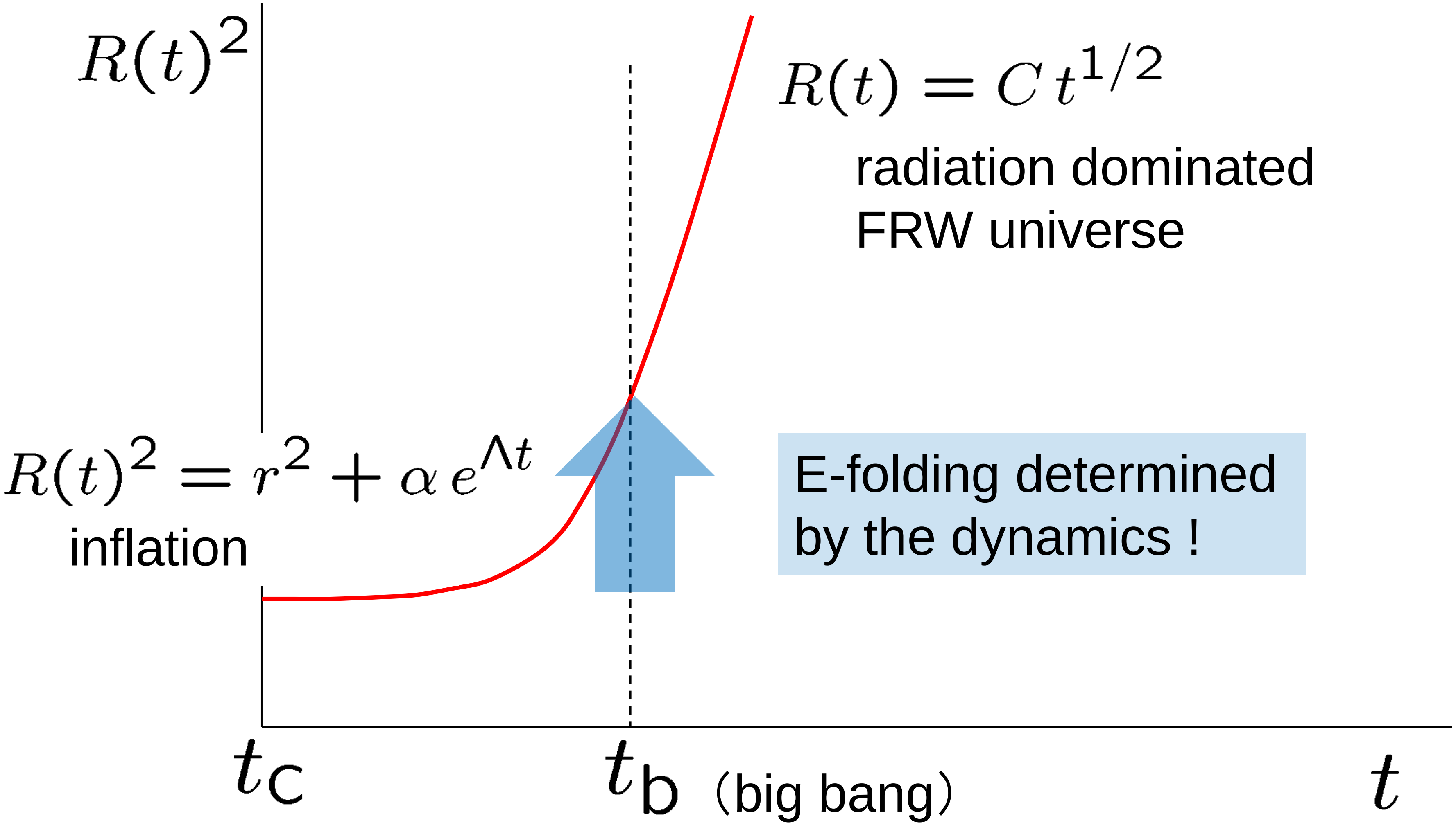
Late timesにおいて簡単化した模型

京コンピュータでの計算

Ito-Nishimura-A.T., in preparation



Expected scenario for the full Lorentzian IIB matrix model



4. まとめと展望

Large-*N* as a key to the Planck scale physics

- 1d SYM with 16 supercharges
  - Black hole thermodynamics
    gauge/gravity duality holds including
    α' corrections and string loop corrections

- Lorentzian type IIB matrix model
  - Expanding behavior of the early Universe
    (SSB from 9d to 3d, inflation + graceful exit)
    from nonperturbative dynamics of superstring theory

Future prospects

- D0-brane system : gauge/gravity duality
  lower temperature, larger matrices

- 1d SYM as a nonperturbative definition of M theory
  (BFSS conjecture) Banks-Fischler-Shenker-Susskind ('96)

- plane-wave matrix model
  the dual geometry is given by bubbling geometry
  description of 5-brane
  (Lin-Maldacena '06)

- Supersymmetric gauge theories in 2(,3,4) dimensions

  So far, only 2d SYM with 4 supercharges has been studied.
  Technical issues related to flat directions should be clarified.
  16 supercharges : Matrix String Theory (Dijkgraav-Verlinde-Verlinde '97)
  Improvement of the method: Matsuura-Sugino, **JHEP 1404 (2014) 088**