核力から出発した有効相互作用の構築とその応用

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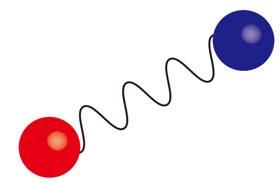
HPCI Field5: The origin of matter and the universe

QCD

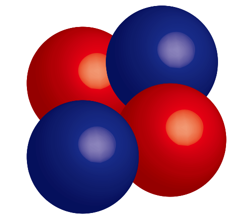
Lattice QCD
Effective Field Theory

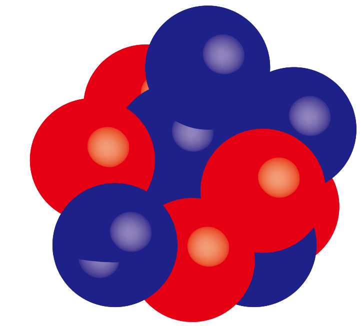
Nuclear force



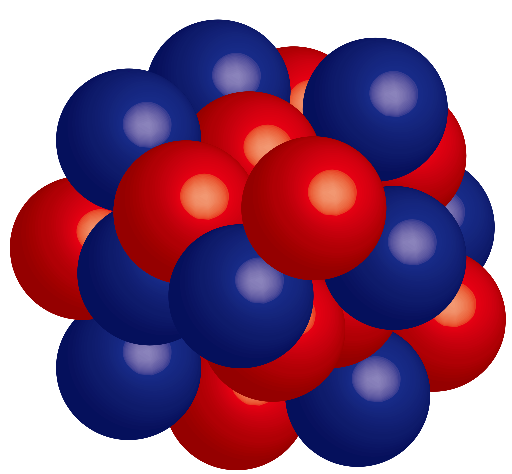
Few body techniques
No core shell model
and many others…

Light nuclei ~A≒10-20

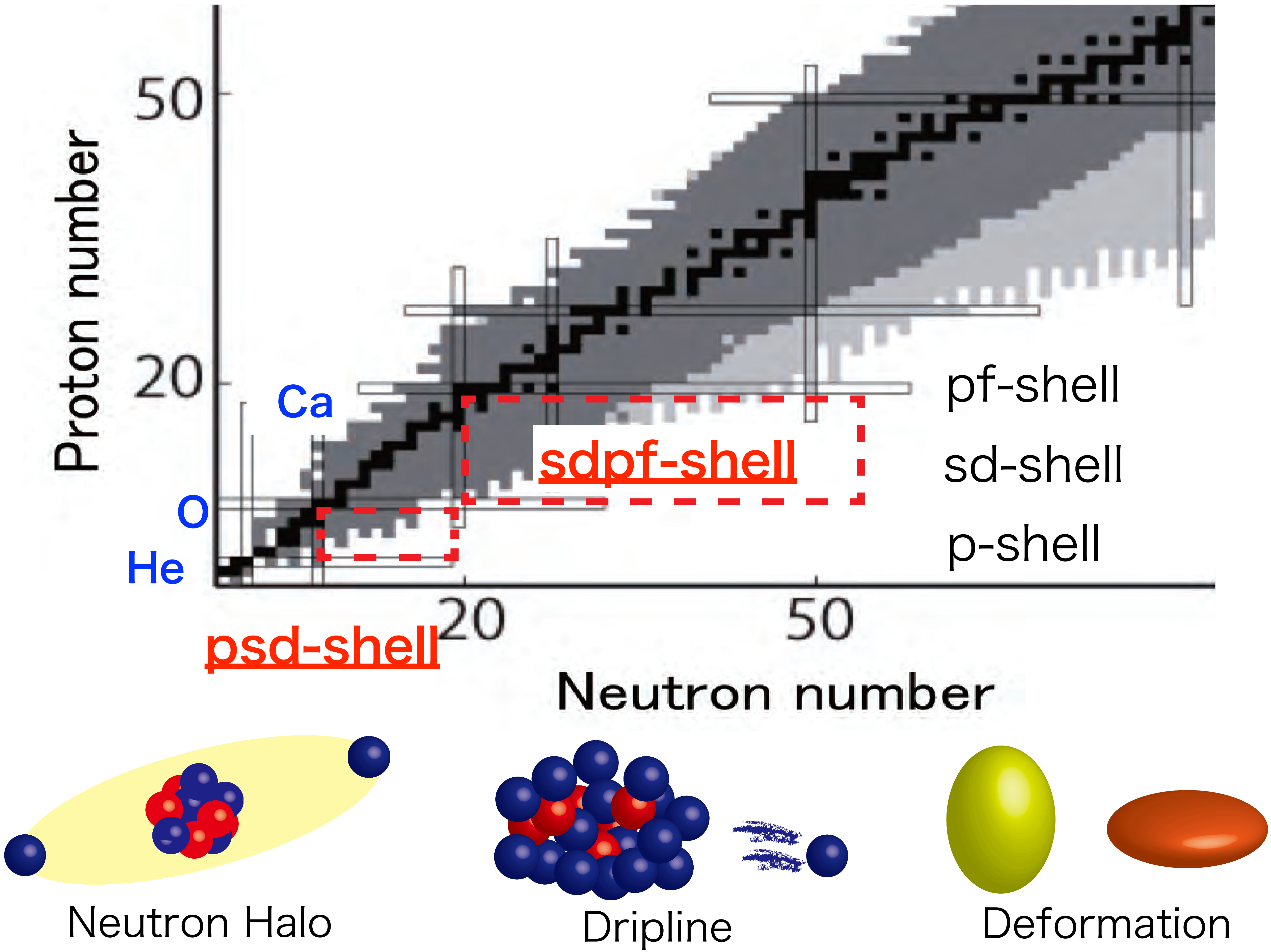




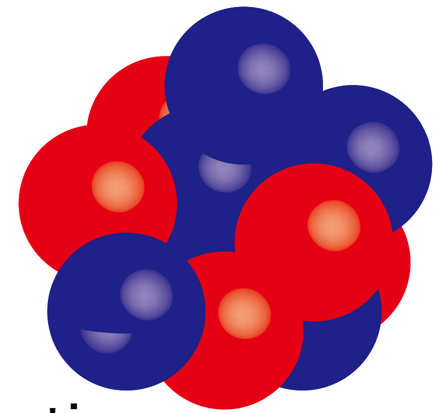
Medium mass nuclei~A≒20-100



shell model with core
via the **effective interaction**
derived from **nuclear force**



50
20
Proton number
Ca
O
He
pf-shell
sdpf-shell
sd-shell
p-shell
psd-shell
20
50
Neutron number
Neutron Halo
Dripline
Deformation



non-relativistic schrodinger equation

$$H|\Psi\rangle = E|\Psi\rangle$$

$|\Psi\rangle$: many-body wave function

Solve the Schrodinger equation with :

1. Nuclear force
2. Degrees of freedom which is appropriate for the problem we like to attack

What we have to overcome are:

A. Nuclear force has strong short range repulsion (singularity) -> Vlowk interaction (not today)
B. Unless we take whole Hilbert space, we need to fix **effective interaction** for selected model space

Nuclear force and Nuclear shell model

Shell model Hamiltonian

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k .$$

input

ε_i : single particle energies

$V_{ij,kl}$: two-body matrix elements

diagonalization →

output

Nuclear properties

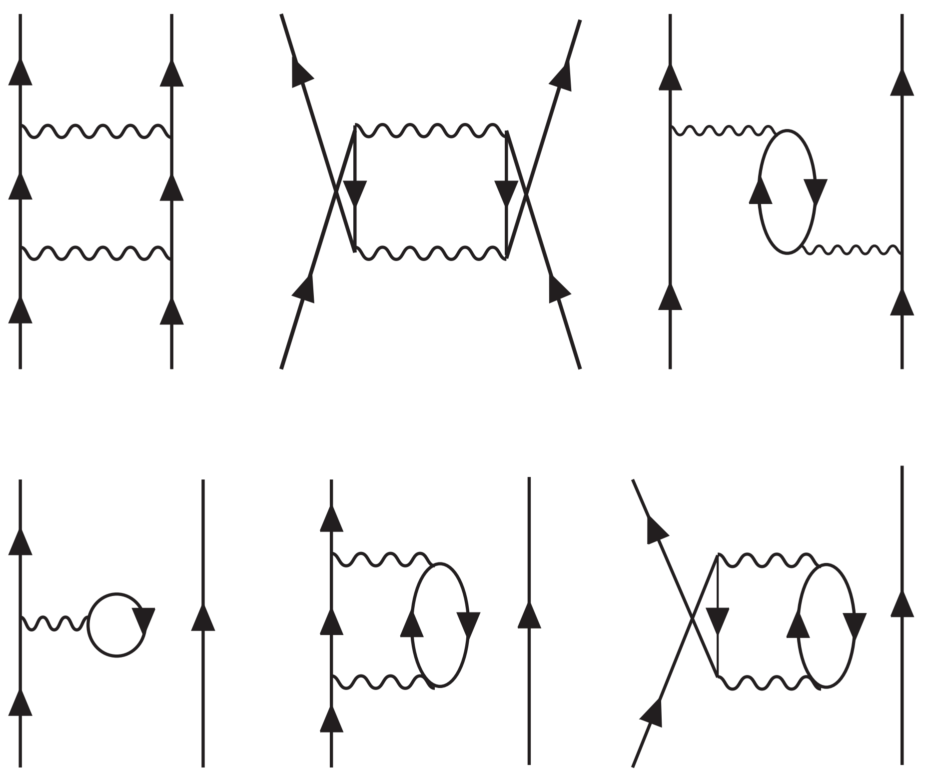
Binding energy, energy spectrum, transition probability , etc…

Derive Shell model Hamiltonian based on Nuclear force and many-body theories

Q-box expansion

Q-box is the ingredient of effective interaction and approximated by perturbation theory

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP$$
$$= PVP + PVQ\frac{1}{E - QH_0Q}QVP + PVQ\frac{1}{E - QH_0Q}QVQ\frac{1}{E - QH_0Q}QVP + \cdots$$



Diagrams appearing in 2nd order Q-box

beyond perturbation contribution

$$V_{\rm eff}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\rm eff}^{(n-1)}\}^k.$$

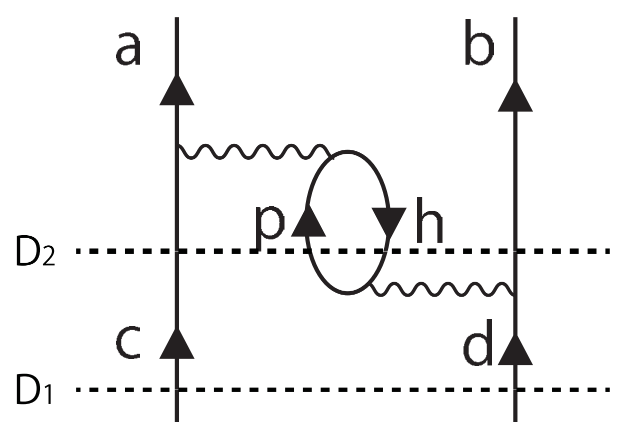
$$\hat{Q}_k(E) = \frac{1}{k!}\frac{\mathrm{d}^k\hat{Q}(E)}{\mathrm{d}E^k}$$

Divergent problem of Q-box in non-degenerate model space

(A)Folded diagram theory requires assumption that the model space is **degenerate**

(B)Naive perturbation theory leads a **divergence** in non-degenerate model space

Example



$$\frac{V_{ah,cp}V_{pb,hd}}{(\epsilon_c+\epsilon_d)-\epsilon_c-\epsilon_p+\epsilon_h-\epsilon_b}$$

→ **Energy denominator is zero when $\varepsilon_d - \varepsilon_b = \varepsilon_p - \varepsilon_h$**

We need a theory which satisfies

(a) The assumption of degenerate model space is **removed**

(b) **Avoid** the divergence appearing in Q-box diagrams

→ **EKK method as a re-summation scheme of KK method**

Extended KK method as a re-summation of the perturbative series

EKK method is derived with the following re-interpretation of the Hamiltonian

$$H = H_0' + V' = \begin{pmatrix} E & 0 \\ 0 & QH_0Q \end{pmatrix} + \begin{pmatrix} P\tilde{H}P & PVQ \\ QVP & QVQ \end{pmatrix},$$

New parameter E (arbitrary parameter)

Change PH_0P part of the unperturbed Hamiltonian

KK method

$$\hat{Q}(E) = PVP + PVQ\frac{1}{E - QHQ}QVP$$

$$V_{\rm eff}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\rm eff}^{(n-1)}\}^k.$$

EKK method

$$H_{\rm BH}(E) = PHP + PVQ\frac{1}{E - QHQ}QVP.$$

$$\tilde{H}_{\rm eff}^{(n)} = \tilde{H}_{\rm BH}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\rm eff}^{(n-1)}\}^k.$$

- One can take E so as to avoid the divergence !
- Final result does not depends on E.

Extended KK method as an analogy of Taylor series

KK method

$$V_{\rm eff}^{(n)} = \hat{Q}(\epsilon_0) + \sum_{k=1}^{\infty} \hat{Q}_k(\epsilon_0)\{V_{\rm eff}^{(n-1)}\}^k.$$

$$e^x = 1 + \sum_{k=1}^{\infty} \frac{1}{k!} x^k$$

Taylor expansion

around x=0

EKK method

$$\tilde{H}_{\rm eff}^{(n)} = \tilde{H}_{\rm BH}(E) + \sum_{k=1}^{\infty} \hat{Q}_k(E)\{\tilde{H}_{\rm eff}^{(n-1)}\}^k.$$

$$e^x = e^E + \sum_{k=1}^{\infty} \frac{e^E}{k!} (x-E)^k$$

Taylor expansion

around x=E

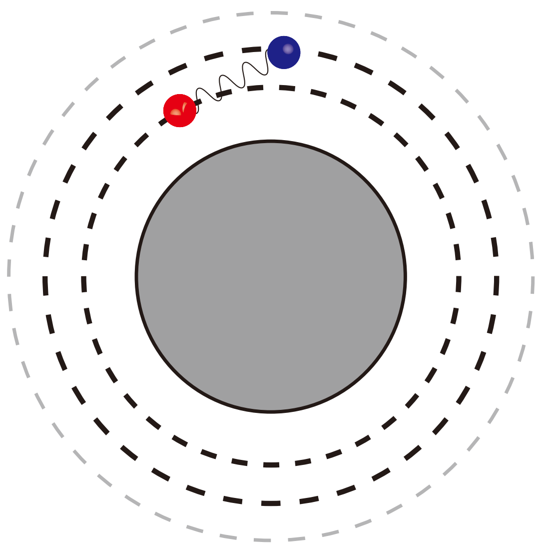
→ Result does not depend on E

Nuclear force and Nuclear shell model

Shell model Hamiltonian

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{ijkl} V_{ij,kl} a_i^\dagger a_j^\dagger a_l a_k.$$

Effective interaction in two-body space



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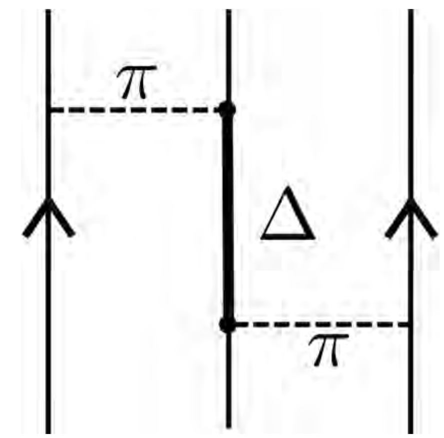
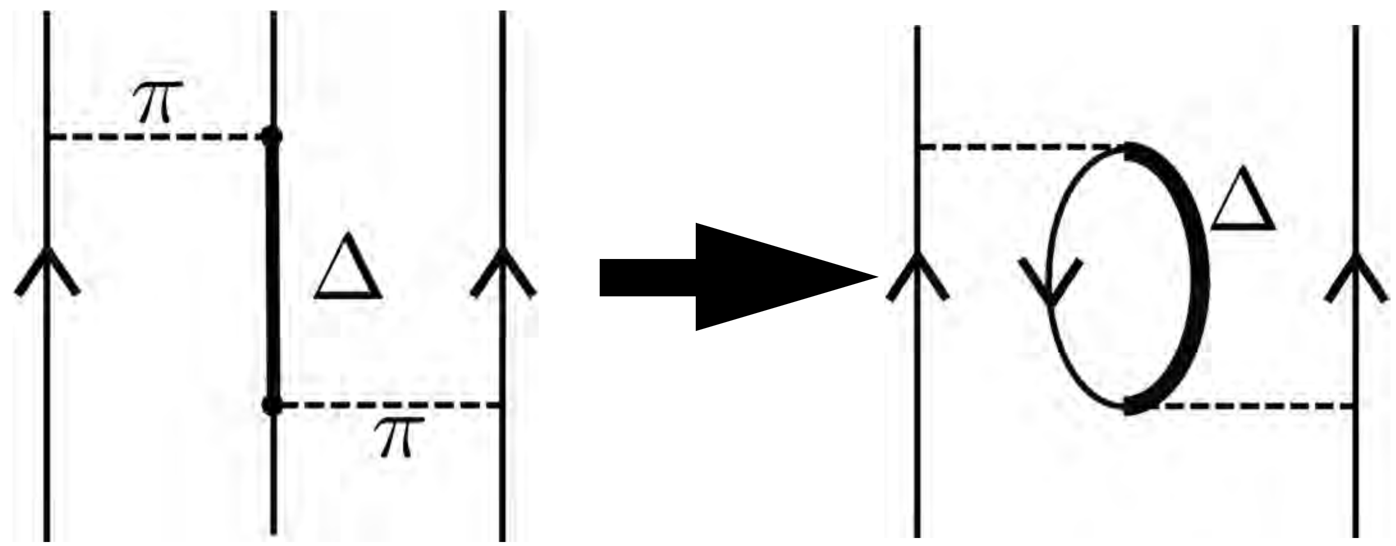
Three-body force



Reduce interaction to the model space perturbatively
Many-body perturbation theory

Fujita-Miyazawa interaction

3N interaction



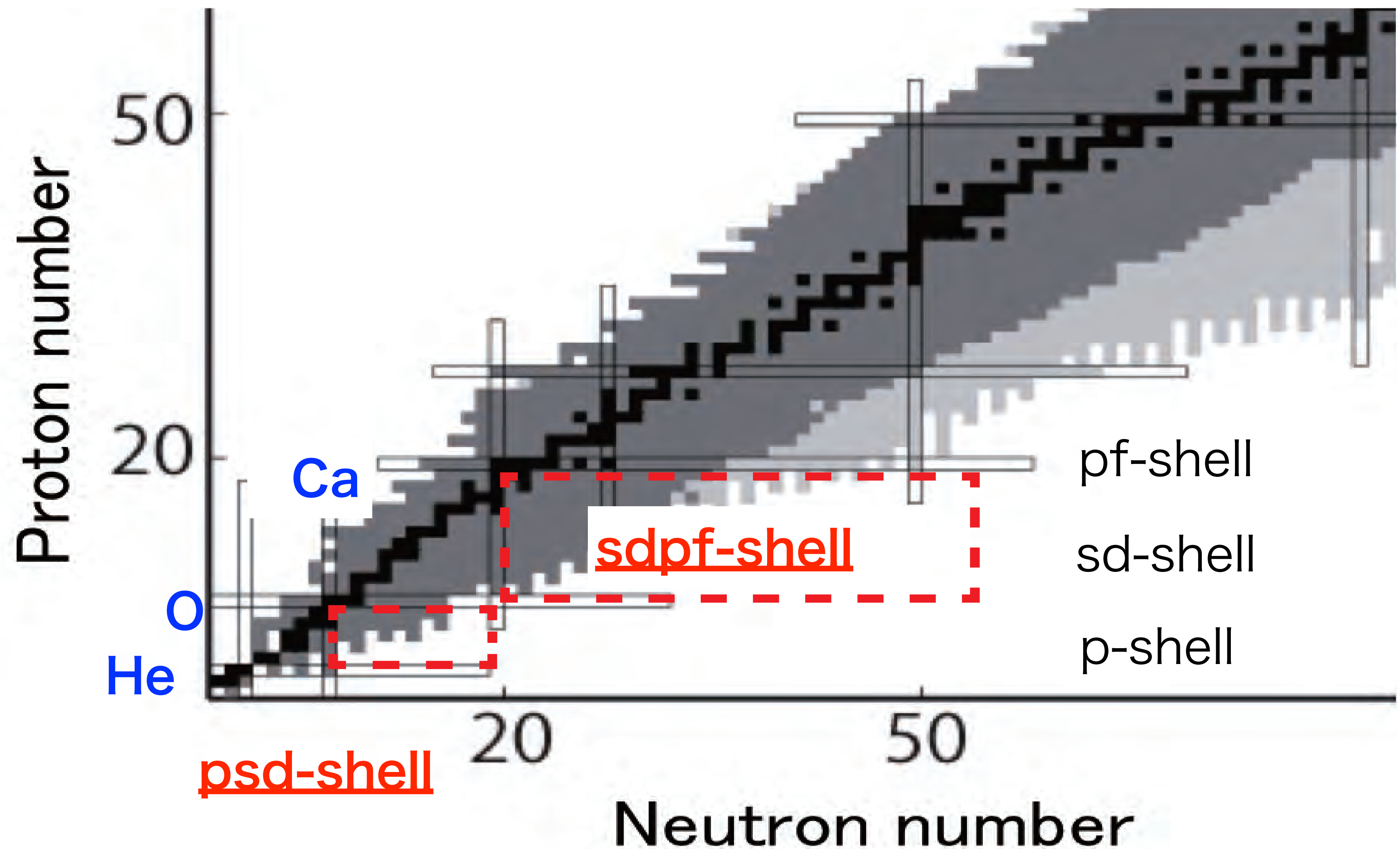
summation with hole

Fujita-Miyazawa type
3N interaction

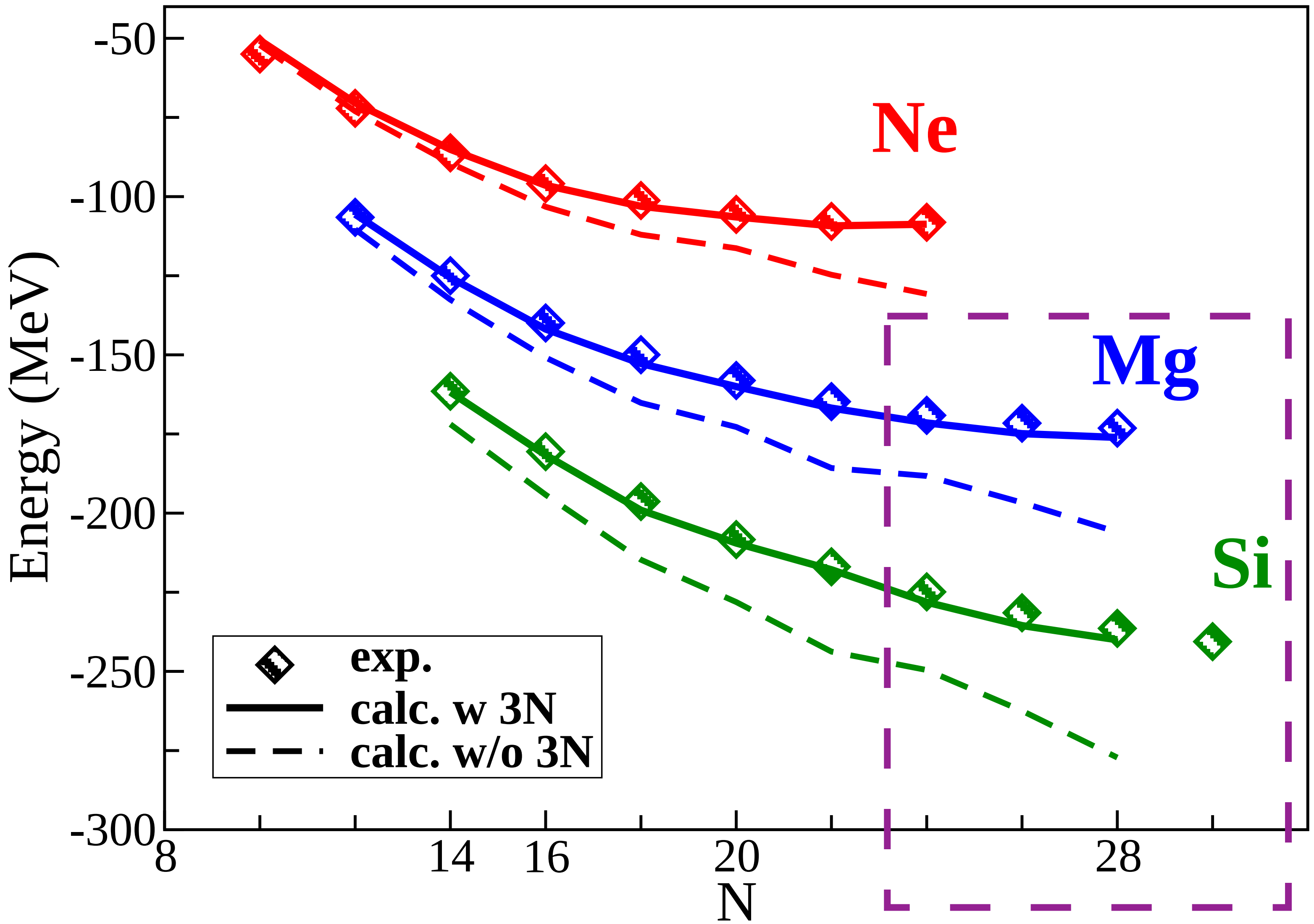
Effective
2N interaction

- Adding up effective 2N interaction derived from 3N interaction to EKK 2N effective interaction
- This is one of the lowest order interaction from 3N force and for higher order we are working on…

Application to sdpf-shell



Ground state energies



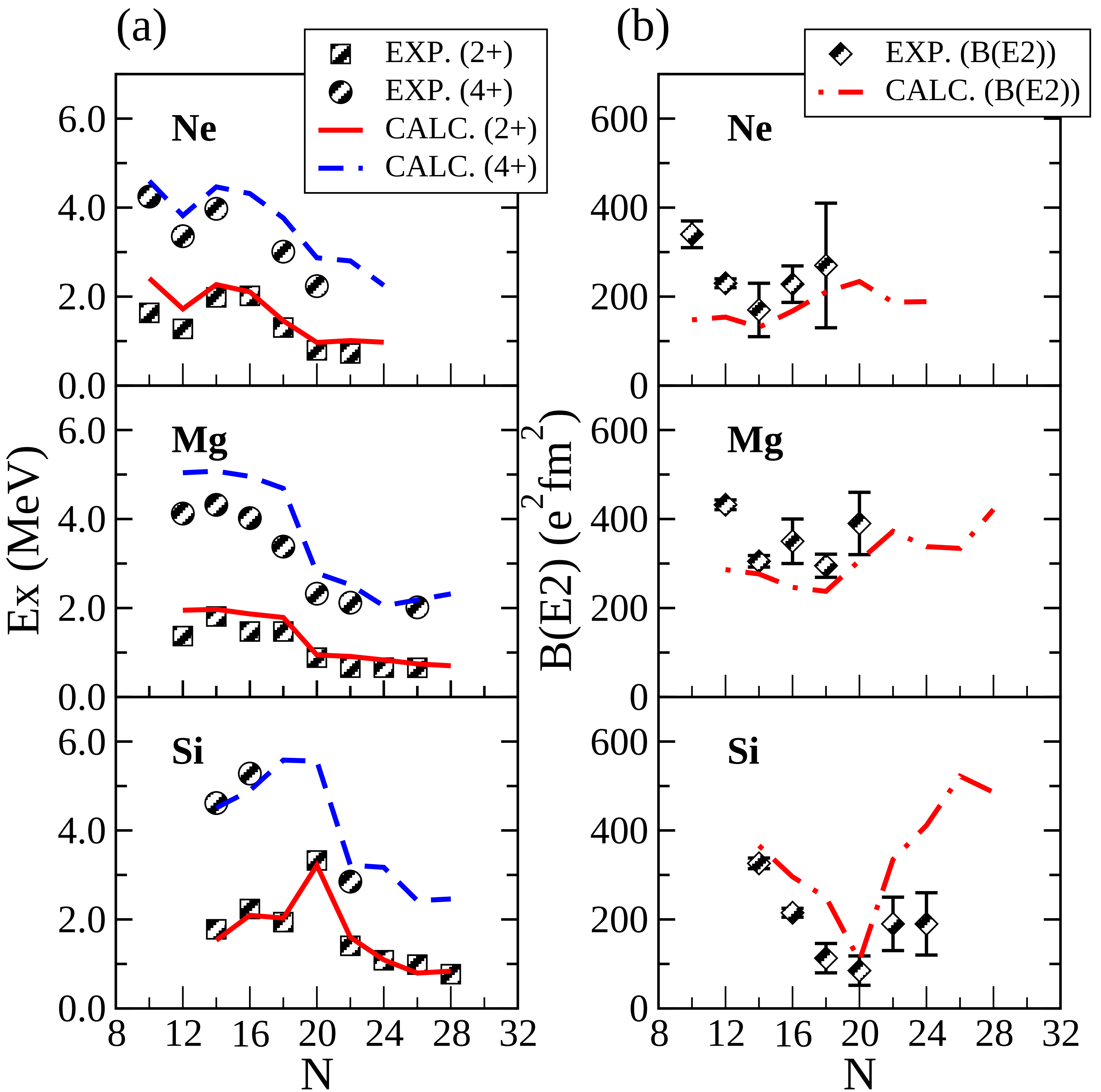
Typical dimension to diagonalize (4hw)

^{32}Ne = 1.78x10^8
^{34}Mg = 2.26x10^9
^{36}Si = 8.10x10^9
^{38}Si = 4.53x10^11

Monte Carlo Shell model

PC	workstation	super comp.	**k-comp.**	**MCSM**	
10^6	10^8	10^9	10^11		dim.

shell structure ~ island of inversion

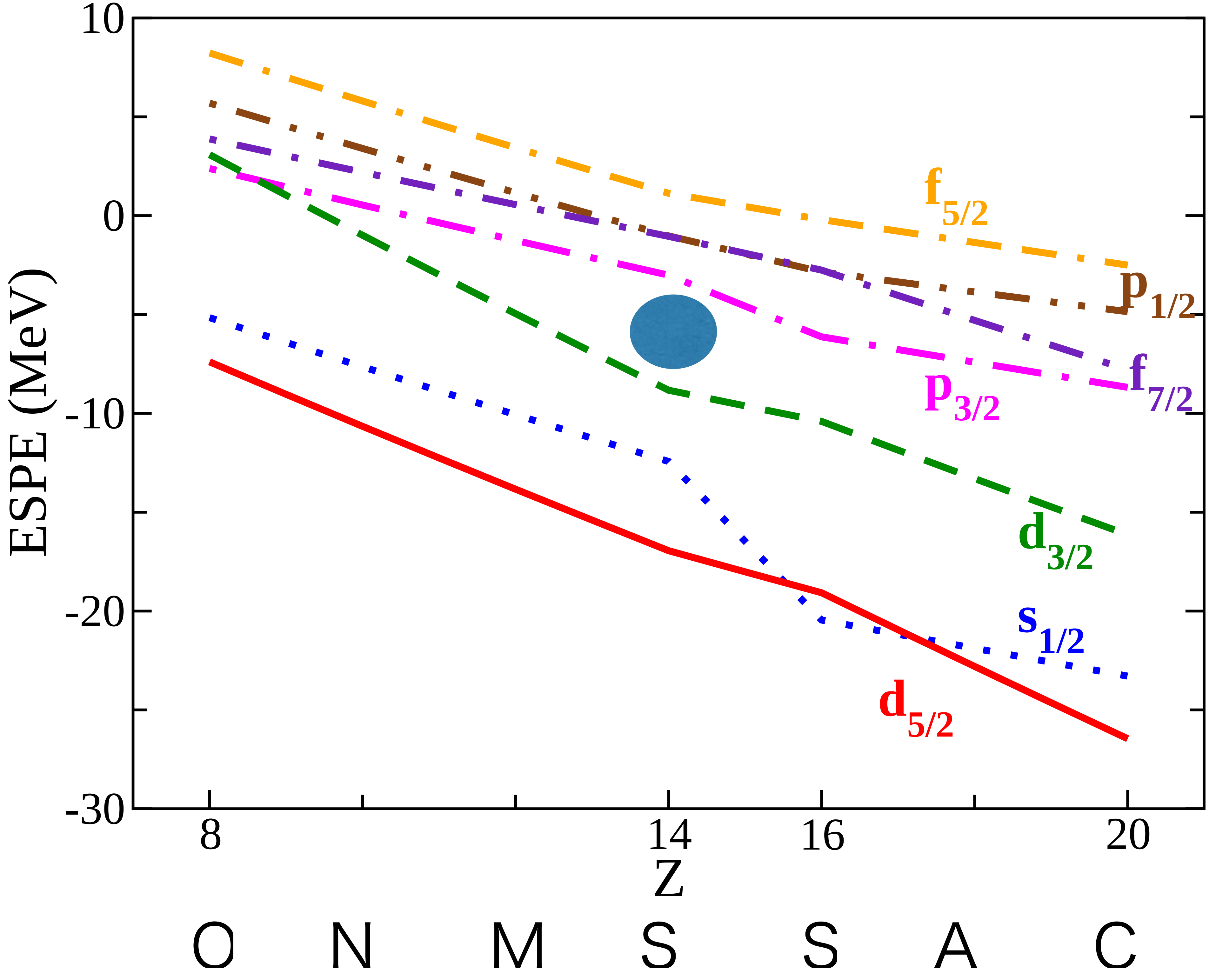


Ne, Mg: N=20 gap Vanishes

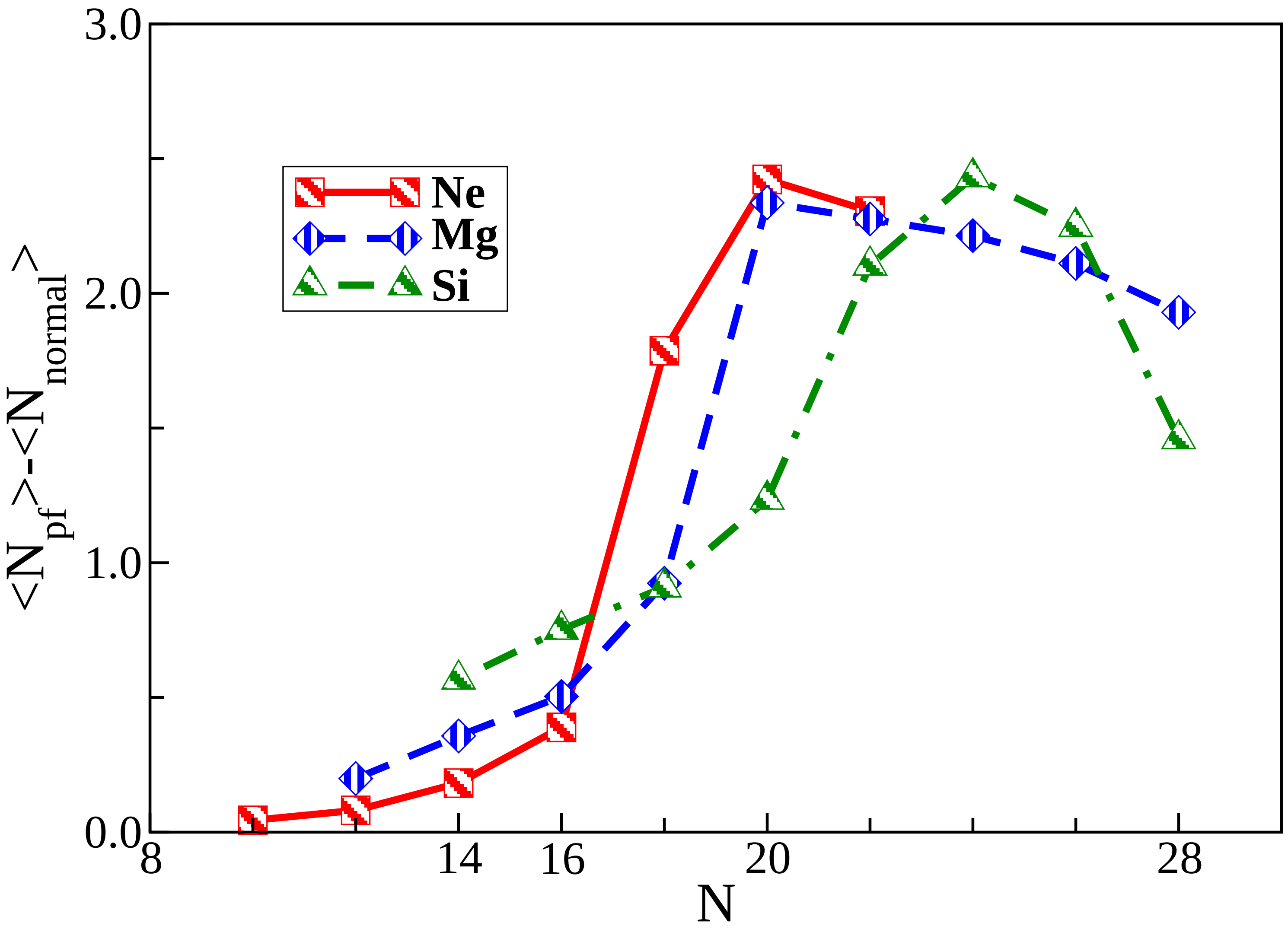
vs

Si: N=20 gap appears

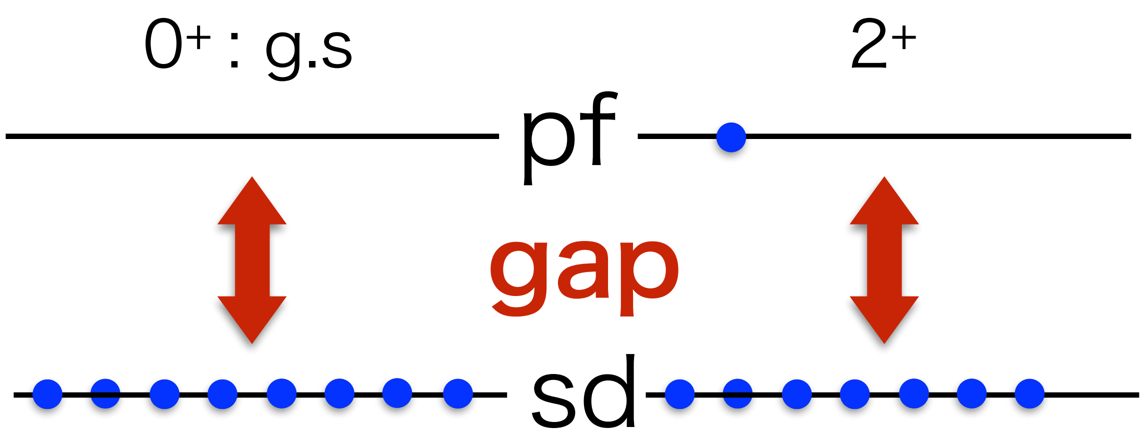
Effective single particle energy



Island of inversion

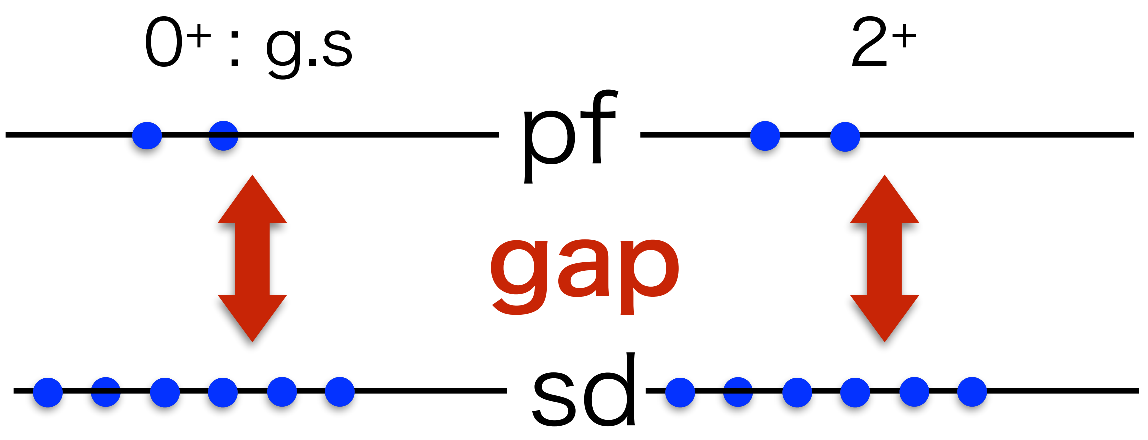


Normal config.



large 2+ energy

Intruder config.



small 2+ energy

Summary and conclusion

- To describe medium mass nuclei starting from nuclear force, we need some method to derive effective interaction.
- EKK method is developed to derive effective interaction for multi-shell, which has an energy parameter E that we can estimate the accuracy of the approximation via E-dependence of the final results.
- As the application of EKK method Ne, Mg, Si isotopes are discussed.
  - Physics in island of inversion is well described with EKK+3N framework

Collaborators

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