

エキゾチックハドロン系の精密科学

根村英克

筑波大学数理物質科学研究科計算科学研究センター

Plan of research



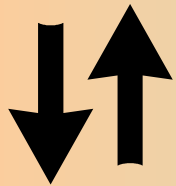
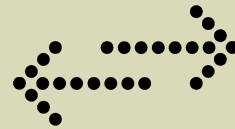
J-PARC
hyperon-nucleon (YN)
scattering



QCD



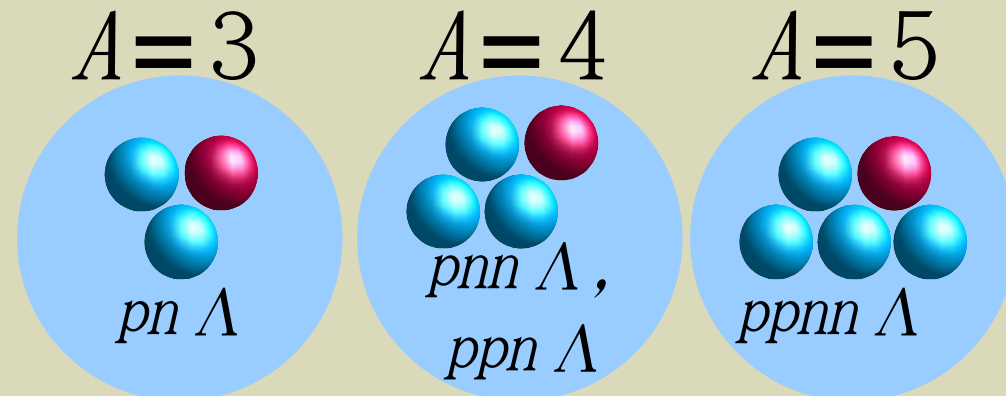
Baryon interaction

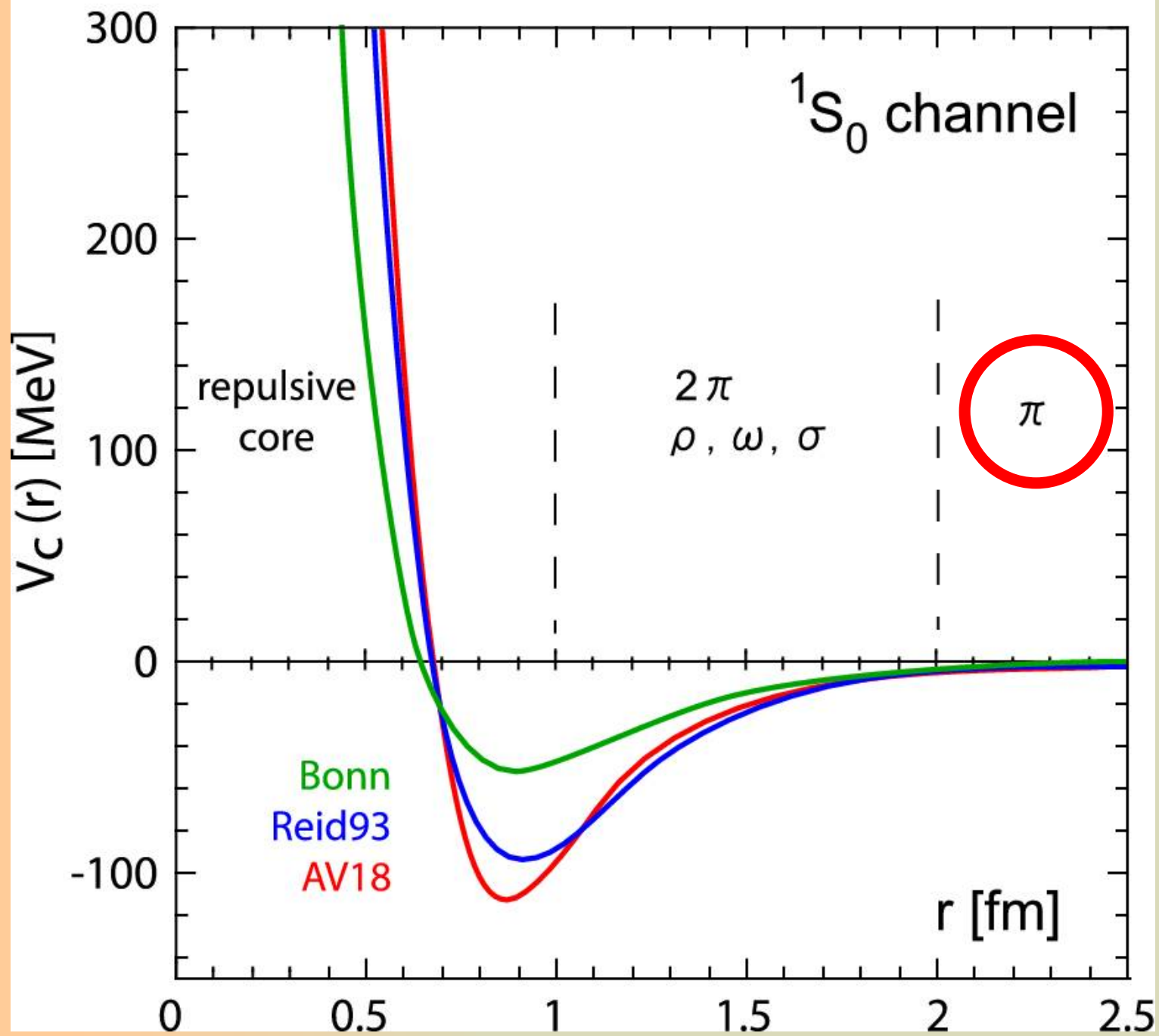


Structure and reaction of
(hyper)nuclei

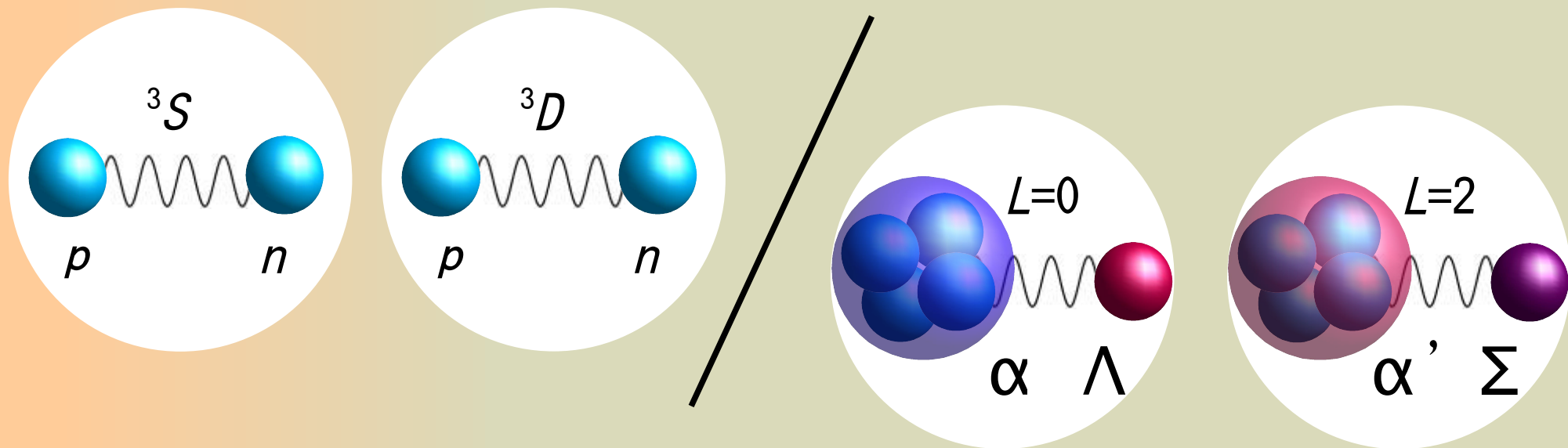
Equation of State (EoS)
of nuclear matter

Neutron star and
supernova





Comparison between $d=p+n$ and $\text{core}+Y$



	$\langle T_S \rangle$ (MeV)	$\langle T_D \rangle$ (MeV)	$\langle V_{NN}(\text{central}) \rangle$ (MeV)	$\langle V_{NN}(\text{tensor}) \rangle$ (MeV)	$\langle V_{NN}(\text{LS}) \rangle$ (MeV)
AV8	8.57	11.31	-4.46	-16.64	-1.02
G3RS	10.84	5.64	-7.29	-11.46	0.00
	$\langle T_{Y-c} \rangle_{\Lambda}$	$\langle T_{Y-c} \rangle_{\Sigma} + \Delta \langle H_c \rangle$	$\langle V_{YN}(\text{のこり}) \rangle$	$2\langle V_{\Lambda N-\Sigma N}(\text{tensor}) \rangle$	
${}^{\Lambda}_5\text{He}$	9.11	3.88+4.68	-0.86	-19.51	
${}^{\Lambda}_4\text{H}^*$	5.30	2.43+2.02	0.01	-10.67	
${}^{\Lambda}_4\text{H}$	7.12	2.94+2.16	-5.05	-9.22	

Introduction:

⊗ Tensor ΛN - ΣN force plays a key role for the light hypernuclei:

⊗ An example: $pn\Lambda + NN\Sigma$ three-body system.

Miyagawa, *et al.*, PRC 51,

TABLE II. The various kinetic and potential energy contributions of Eqs. (6) in the hypertriton, using the Nijmegen YN and Nijmegen 93 NN interactions. The potential energy of the hyperon-nucleon interaction is broken up further into its contribution from the states 1S_0 and 3S_1 - 3D_1 . All numbers are in units of MeV.

Partial wave	$\langle V_{\Lambda N, \Lambda N} \rangle$	$\langle V_{\Lambda N, \Sigma N} \rangle$	$\langle V_{\Sigma N, \Lambda N} \rangle$	$\langle V_{\Sigma N, \Sigma N} \rangle$	$\langle V_{YN} \rangle$
1S_0	-1.60	-0.19	-0.19	0.03	-1.95
3S_1 - 3D_1	0.02	-0.77	-0.77	-0.06	-1.57
all	-1.58	-0.97	-0.97	-0.02	-3.54
	$\langle V_{NN} \rangle_\Lambda$	$\langle V_{NN} \rangle_\Sigma$			$\langle V_{NN} \rangle$
all	-22.22	-0.03			-22.25
	$\langle T_{NN} \rangle_\Lambda$	$\langle T_{NN} \rangle_\Sigma$			$\langle T_{NN} \rangle$
all	20.25	0.23			20.48
	$\langle T_{\Lambda-NN} \rangle$	$\langle T_{\Sigma-NN} \rangle$			$\langle T_{Y-NN} \rangle$
all	2.18	0.79			2.97

Tensor

ΛN - ΣN

FY calculation with and w/o 3NF

⊗ Three nucleon force does not change the B_Λ so much.

⊗ A. Nogga, *et al.*, PRL88, 172501 (2002).

TABLE II. NN and $3N$ interaction dependence of the ${}^4_\Lambda\text{He}$ SE's E_{sep}^Λ and the 0^+-1^+ splitting Δ . We show results for different combinations of YN , NN , and $3N$ forces (YNF , NNF , and $3NF$). All energies are given in MeV.

YNF	NNF	$3NF$	$E_{\text{sep}}^\Lambda(0^+)$	$E_{\text{sep}}^\Lambda(1^+)$	Δ
SC97e	Bonn B	...	1.66	0.80	0.84
SC97e	Nijm 93	...	1.54	0.72	0.79
SC97e	Nijm 93	TM	1.56	0.70	0.82
SC89	Bonn B	...	2.25
SC89	Nijm 93	...	2.14	0.02	2.06
SC89	Nijm 93	TM	2.19

ハイペロンポテンシャルは、NNと切り離して決めることはできない。

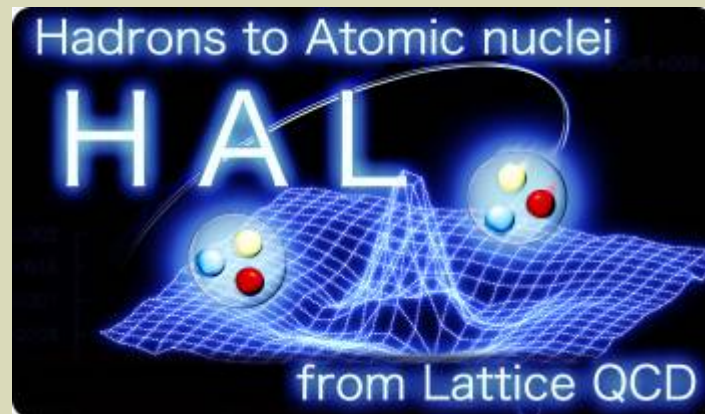
Lattice QCD calculation

Baryon-baryon potentials from lattice QCD

H. Nemura¹,

for HAL QCD Collaboration

S. Aoki², B. Charron³, T. Doi⁴, F. Etminan¹,
T. Hatsuda⁴, Y. Ikeda⁴, T. Inoue⁵, N. Ishii¹,
K. Murano², K. Sasaki¹, and M. Yamada¹,



¹*Center for Computational Science, University of Tsukuba, Japan*

²*Yukawa Institute for Theoretical Physics, Kyoto University, Japan*

³*Department of Physics, University of Tokyo, Japan*

⁴*Theoretical Research Division, Nishina Center RIKEN, Japan*

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⁶*Strangeness Nuclear Physics, Nishina Center RIKEN, Japan*

Outline

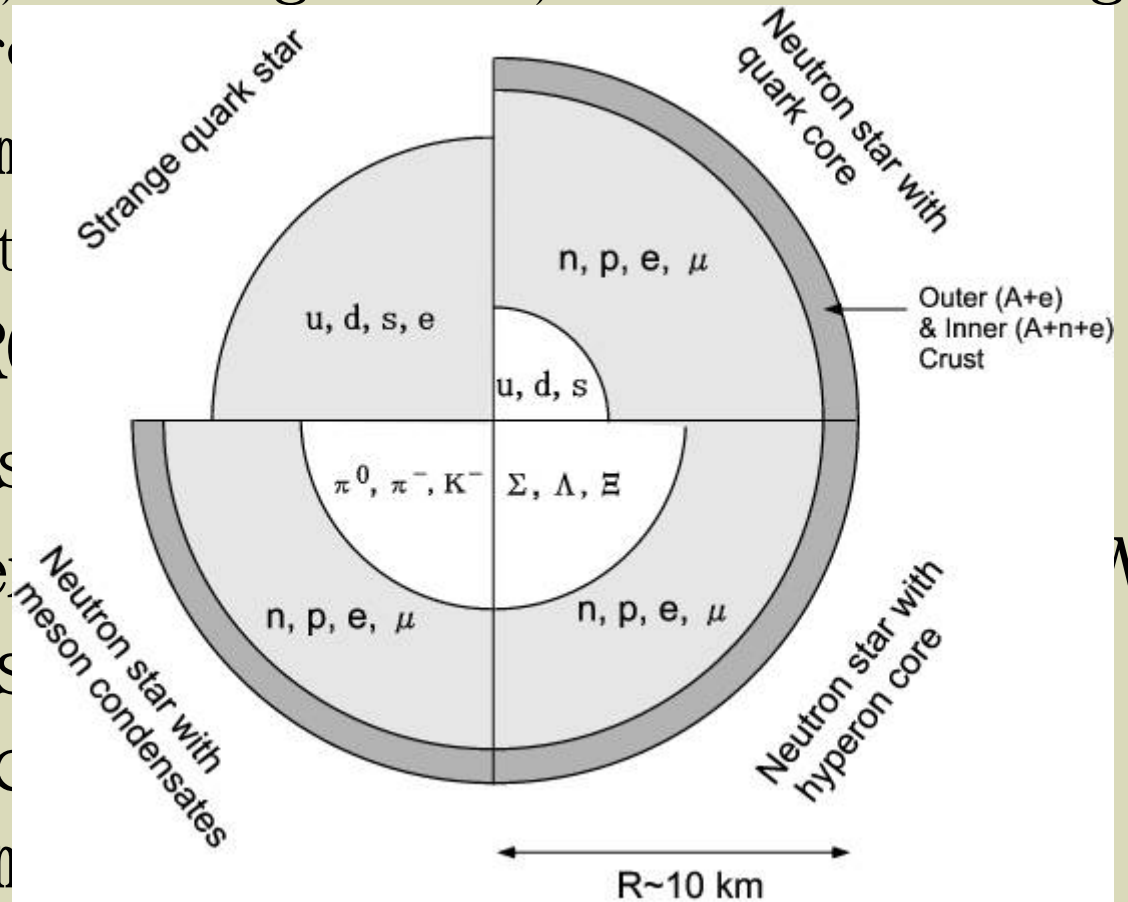
- ⊗ Introduction
- ⊗ Formulation --- potential (central + tensor)
- ⊗ Numerical results:
 - ⊗ Λ force ($V_C + V_T$)
 - ⊗ $\Lambda\Sigma$ (I=3/2) force ($V_C + V_T$)
- ⊗ Recent work on lattice QCD
- ⊗ Stochastic variational calculation of 4He with using a lattice potential
- ⊗ Summary and outlook

Introduction:

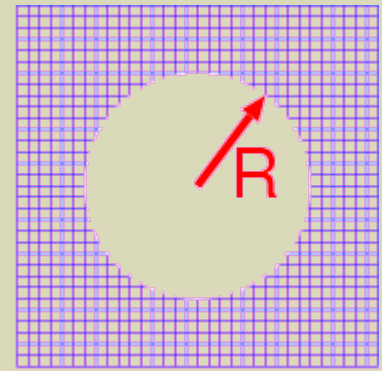
- ⊗ Study of hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions is one of the important subjects in the nuclear physics.
 - ⊗ Structure of the neutron-star core,
 - ⊗ Hyperon mixing, softening of EOS, inevitable strong repulsive force,
 - ⊗ H-dibaryon problem,
 - ⊗ To be, or not to be,
- ⊗ The project at J-PARC:
 - ⊗ Explore the multistrange world,
- ⊗ However, the phenomenological description of YN and YY interactions has large uncertainties, which is in sharp contrast to the nice description of phenomenological NN potential.

Introduction:

- Study of hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions is one of the important subjects in the nuclear physics.
 - Structure of the neutron-star core,
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- The project at J-PARC
 - Explore the multi-species hypernuclei
- However, the phenomenology of YN and YY interactions which is in sharp contrast with the description of phenomena



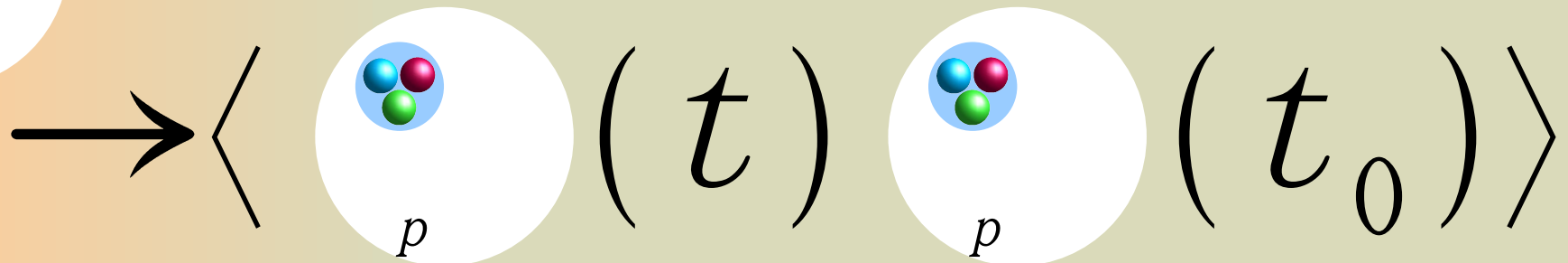
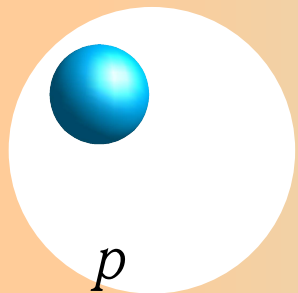
Formulation



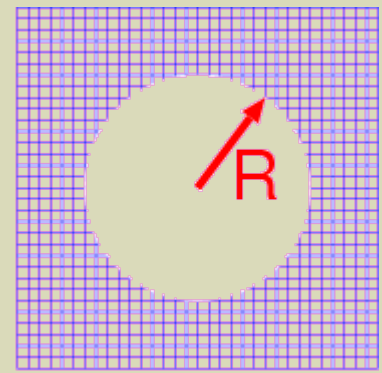
Lattice QCD simulation

$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$

$$\begin{aligned} \langle O(\bar{q}, q, U) \rangle &= \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U) \\ &= \int dU \det D(U) e^{-S_v(U)} O(D^{-1}(U)) \\ &= \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N O(D^{-1}(U_i)) \end{aligned}$$



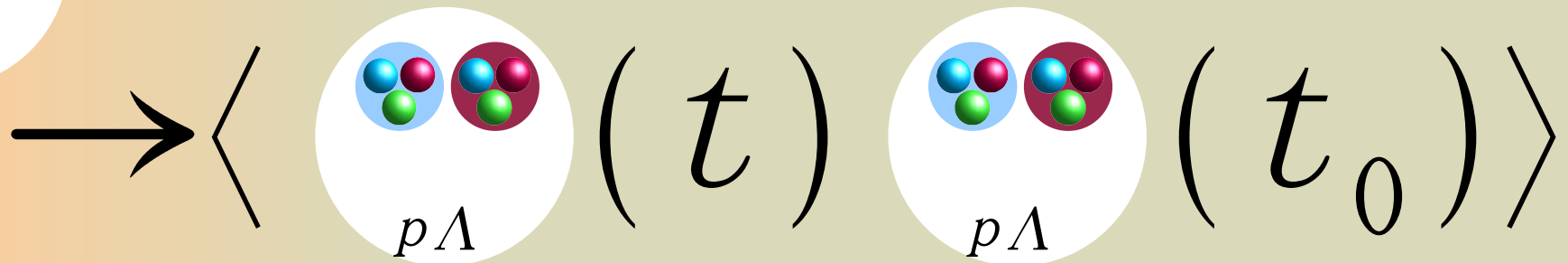
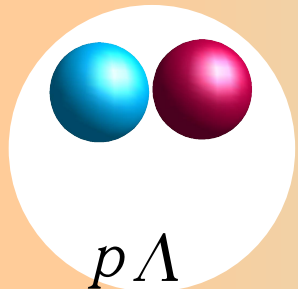
Formulation



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Formulation

i) basic procedure:

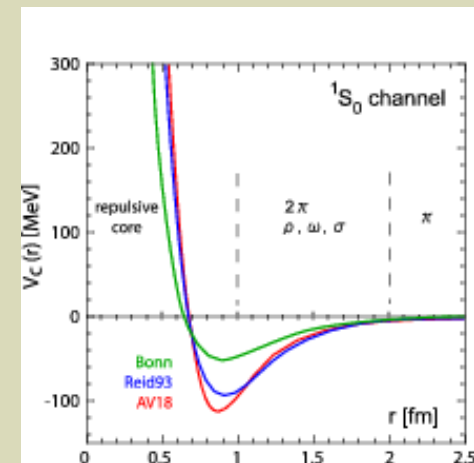
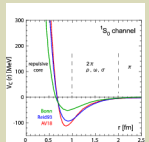
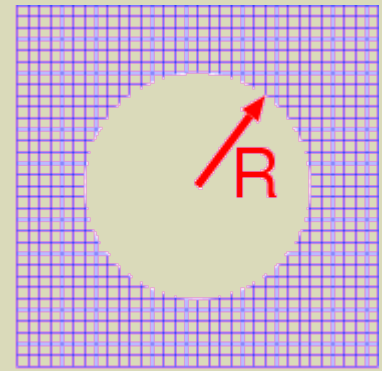
asymptotic region

→ phase shift

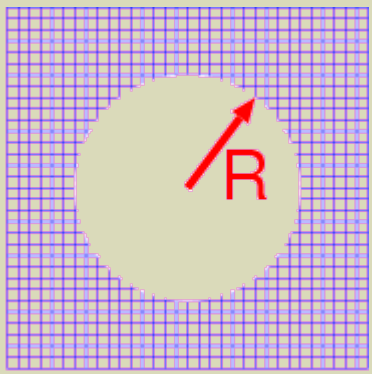
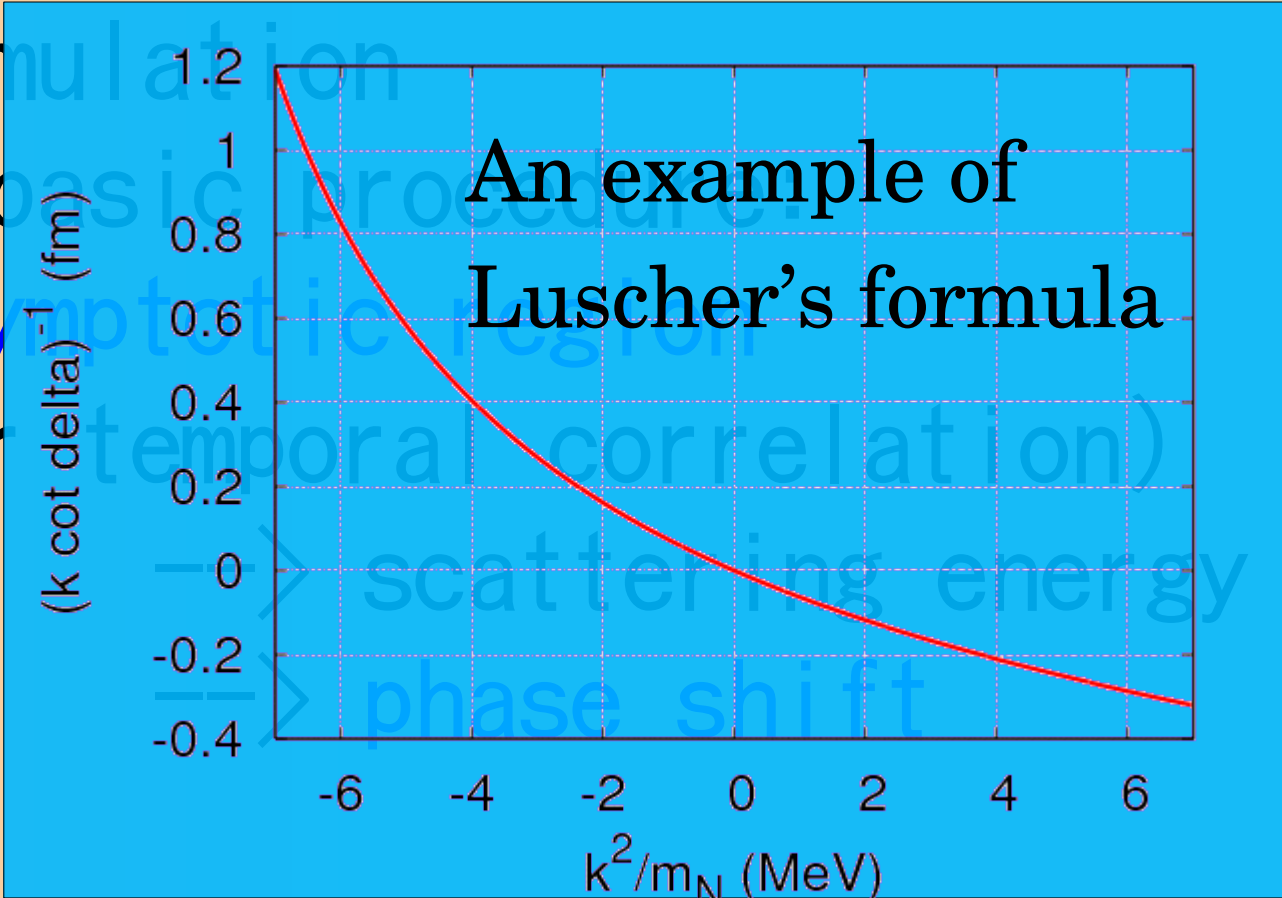
ii) advanced (HAL's) pro-

cedure: interacting region

→ potential



Formulation
 i) basic
 asymptotic region
 (or scattering energy
 phase shift)



$$E = \frac{k^2}{2\mu}$$

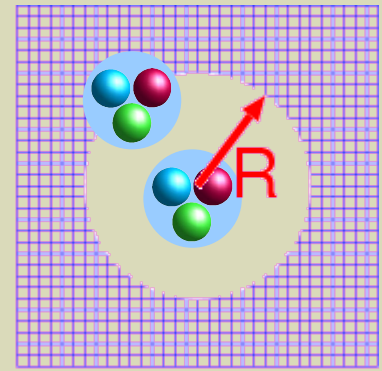
$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} \frac{1}{(n^2 - q^2)^s} \quad \Re s > \frac{3}{2}$$

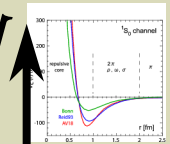
Luscher, NPB354, 531 (1991).
 Aoki, et al., PRD71, 094504 (2005).

Formulation

Lattice QCD simulation

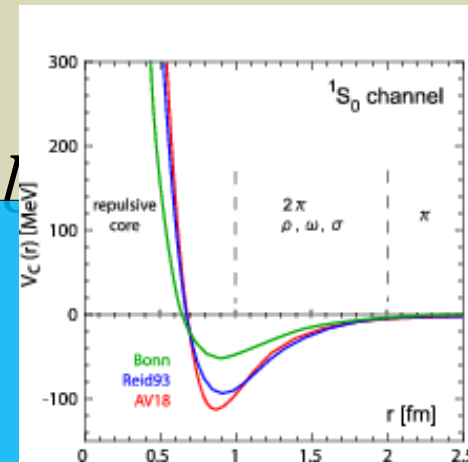


$$L = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \bar{q} \gamma^\mu (i \partial_\mu - g t^a A_\mu^a) q - m \bar{q} q$$



$$\langle O(\bar{q}, q, U) \rangle = \int dU d\bar{q} dq e^{-S(\bar{q}, q, U)} O(\bar{q}, q)$$

$$= \int dU \det D(U) e^{-S_U(U)} O(D^{-1}(U))$$



$$F_{\alpha\beta}^{(JM)}(\vec{r}, t - t_0)$$

$$\rightarrow \left\langle \left(\text{p}\Lambda \right) (\vec{r}, t) \left(\text{p}\Lambda \right) (t_0) \right\rangle$$

Calculate the scattering state

HAL formulation

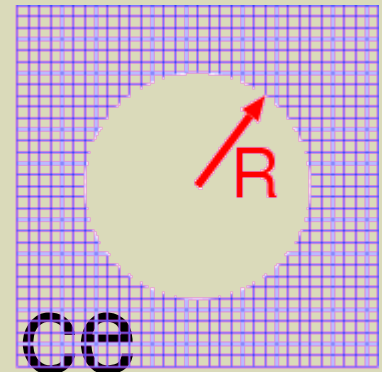
ii) advanced procedure:

make better use of the lattice

output ! (wave function)

interacting region

→ potential



Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., arXiv:0805.2462[hep-ph].

NOTE:

- > Potential is not a direct experimental observable.
- > Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

HAL formulation

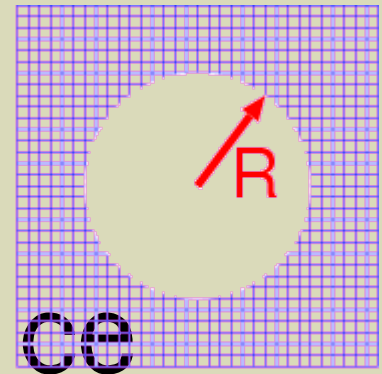
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⇒

- > Phase shift
- > Nuclear many-body problems

Numerical results

Full QCD calculations by using $N_F=2+1$ PACS-CS gauge configurations:

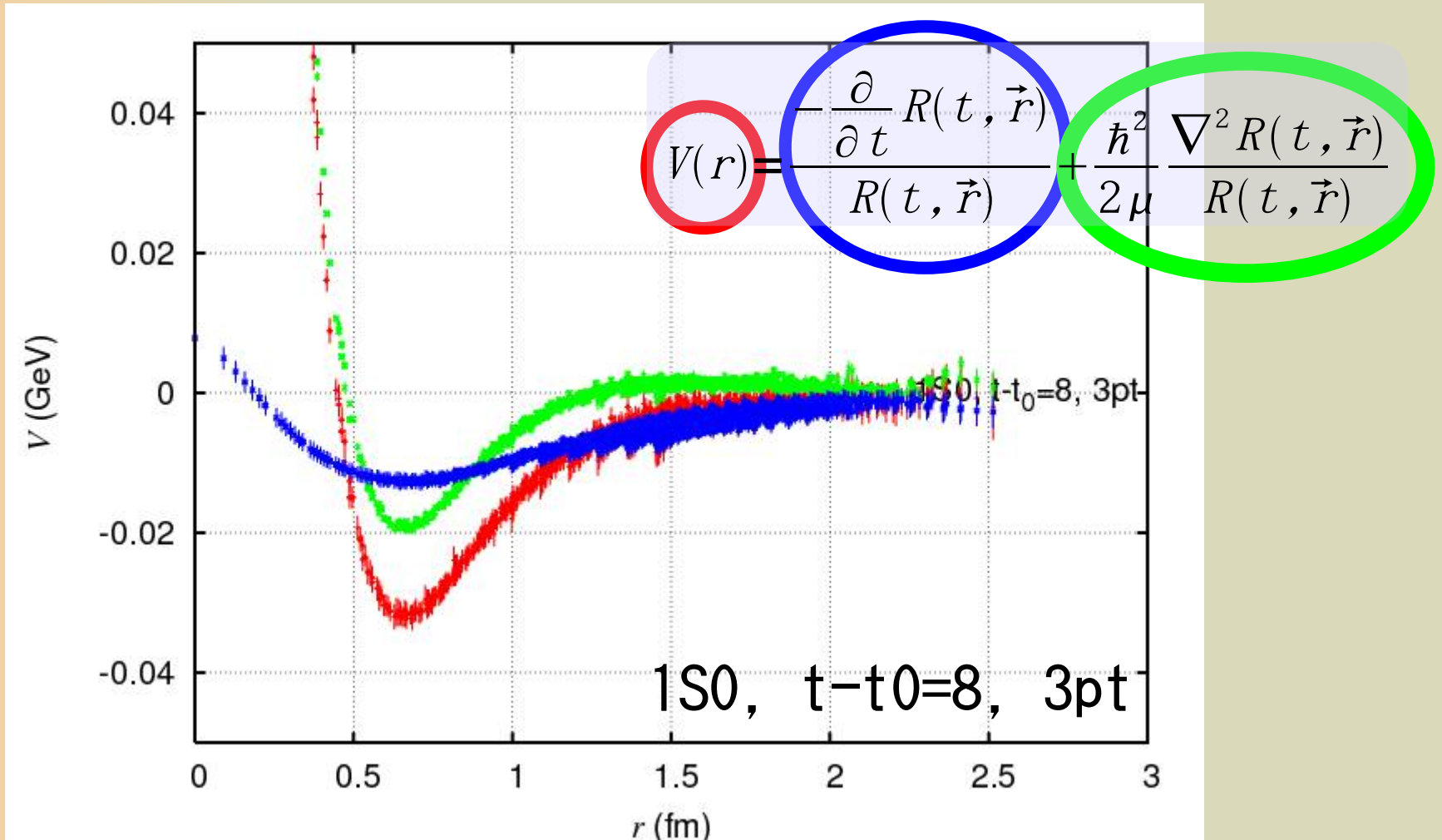
- ⊗ S. Aoki, et al., (PACS-CS Collaboration), PRD79, 034503 (2009), arXiv:0807.1661 [hep-lat].
- ⊗ Iwasaki gauge action at $\beta=1.90$ on $32^3 \times 64$ lattice
- ⊗ $O(a)$ improved Wilson quark action
- ⊗ $1/a = 2.17$ GeV ($a = 0.0907$ fm)

$(\kappa_{ud})_{N_{\text{conf}}}$	m_π	m_ρ	m_K	m_{K^*}	m_N	m_Λ	m_Σ	m_E
2+1 flavor QCD by PACS-CS with $\kappa_s = 0.13640$ @ present calc (Dirichlet BC along T)								
(0.13700)	700.0(4)	1108(3)	785.8(3)	1159(2)	1573(4)	1632(4)	1650(5)	1700(4)
(0.13754)	415(1)	903(5)	639.7(8)	1024(4)	1232(10)	1354(6)	1415(7)	1512(4)
Exp.	135	770	494	892	940	1116	1190	1320



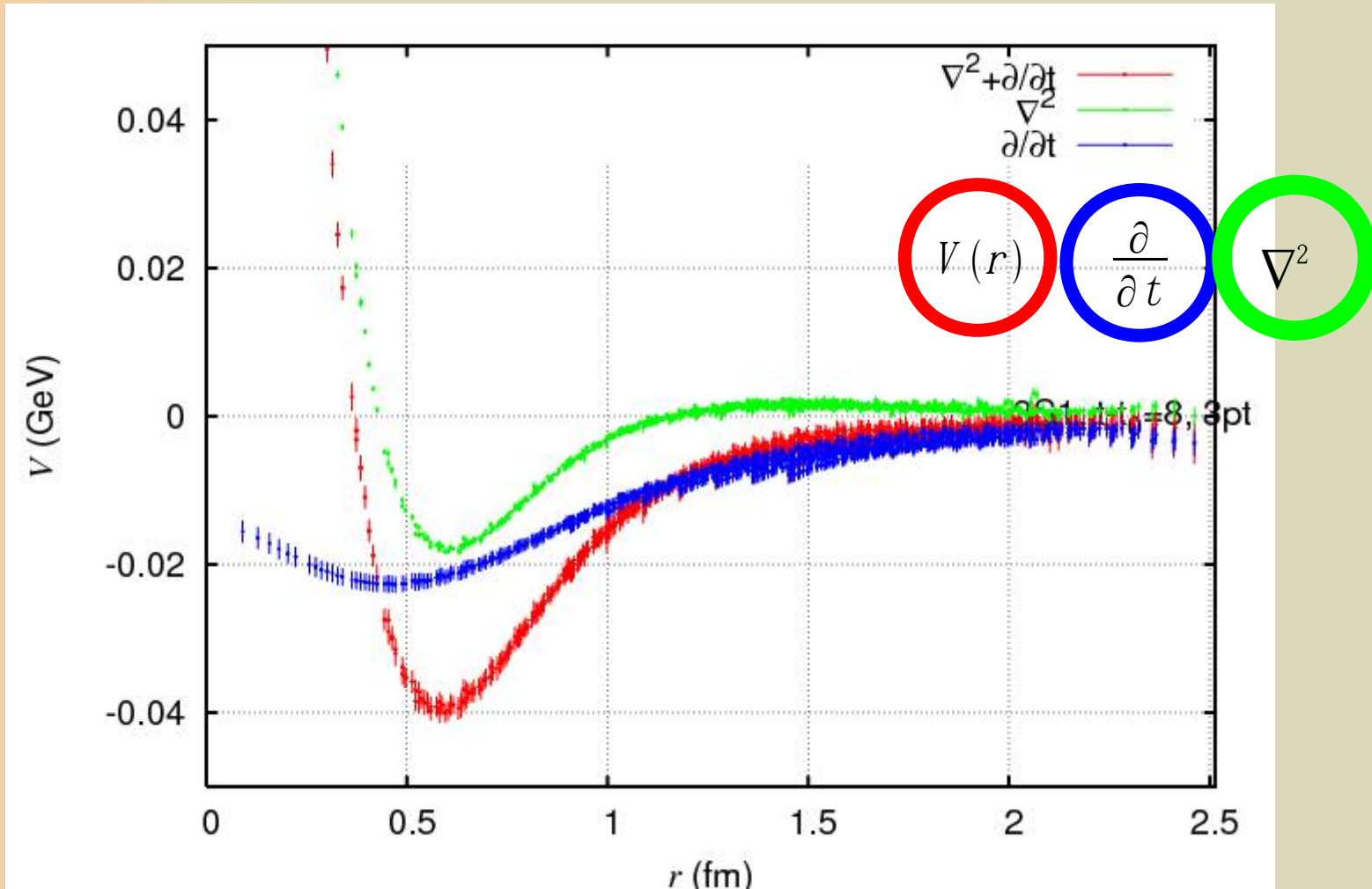
ΛN potential

$V_c(\Lambda N; 1S0)$



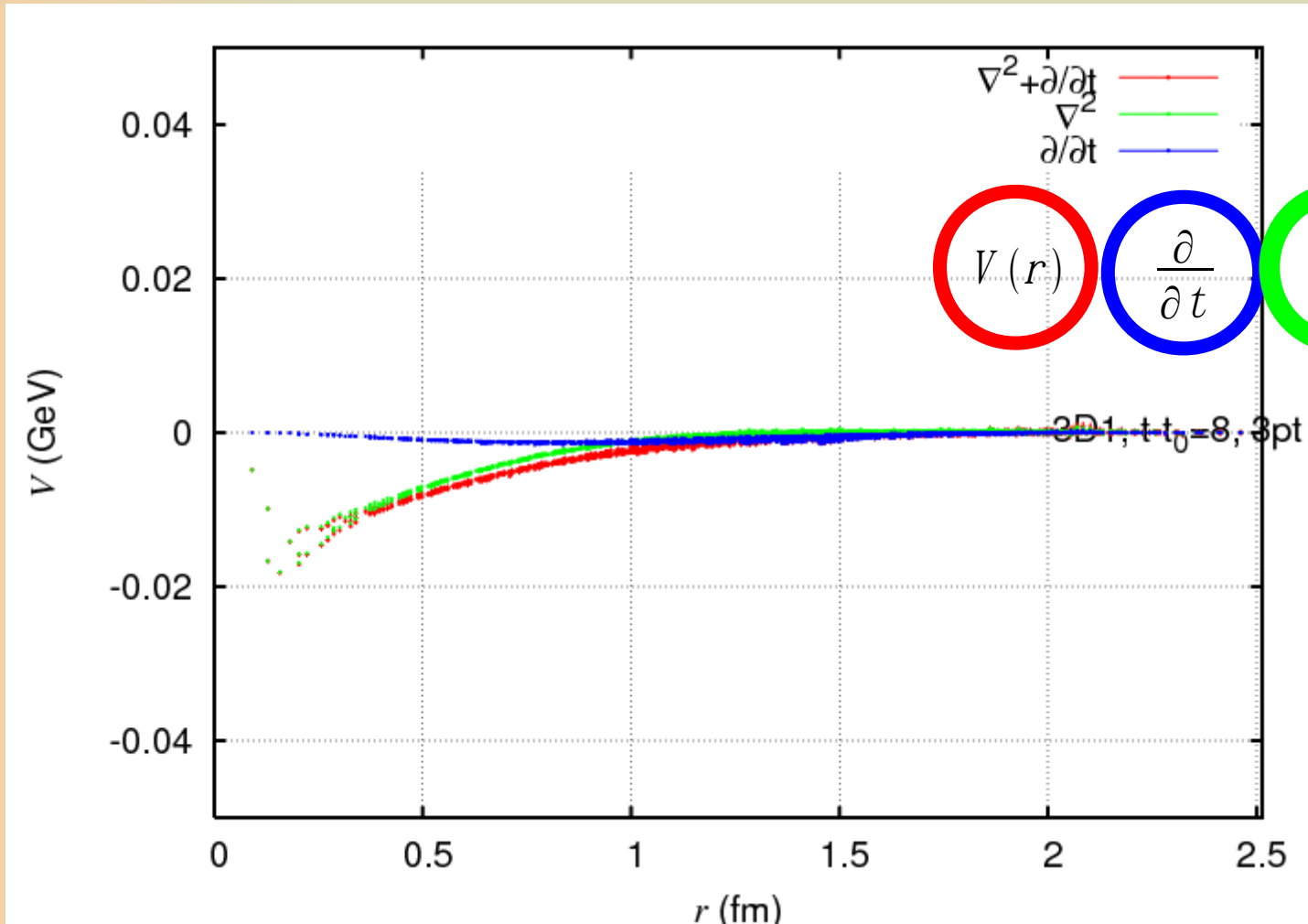
- $\{27\} + \{8s\}$
- Similar to NN (1S0)
- Sizable contribution from time-derivative part

$V_C(\Lambda N; 3S1-3D1)$



- $\{10^*\} + \{8a\}$
- Sizable attractive contribution from time-derivative part

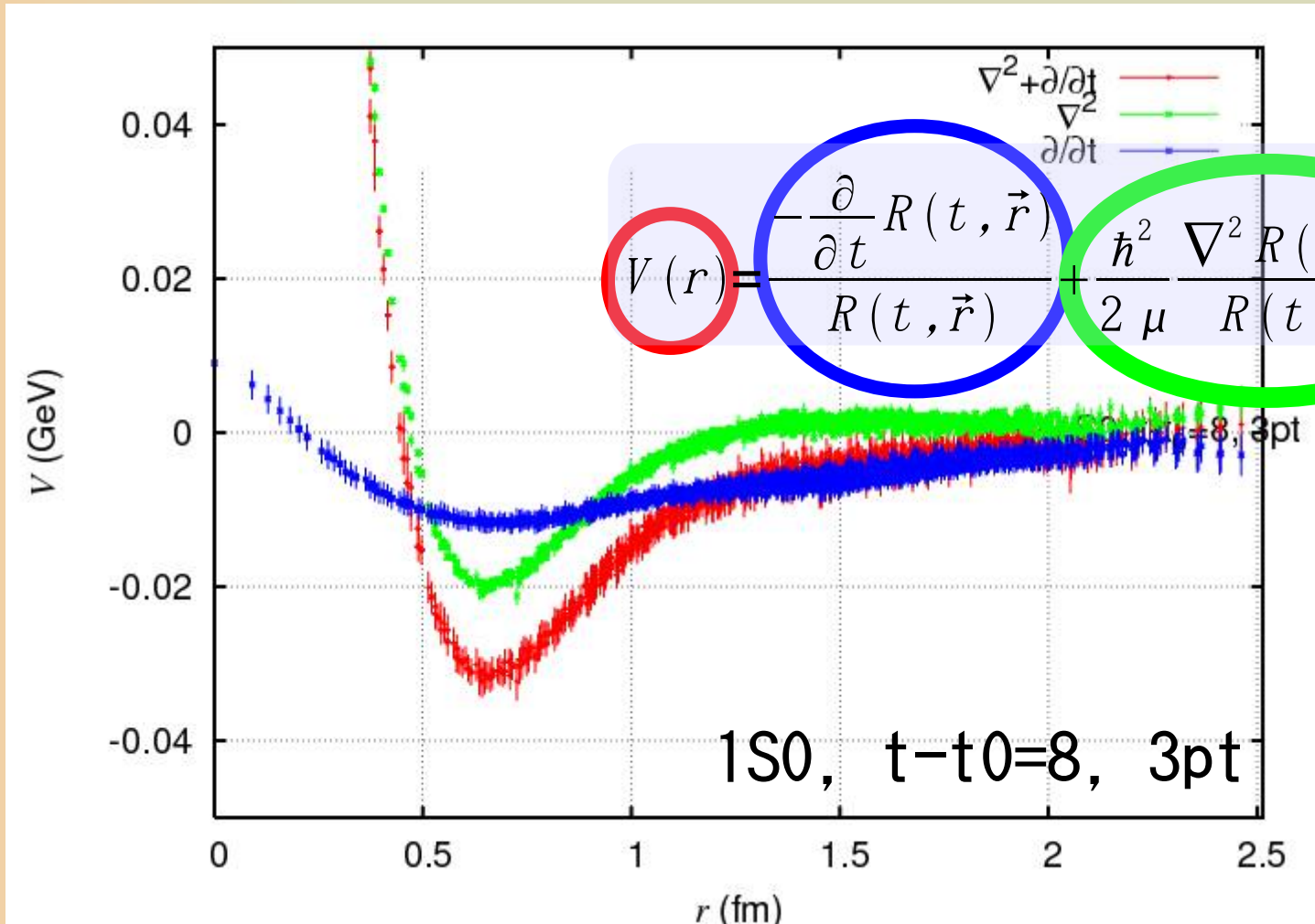
$V_T(\Lambda N; 3S1-3D1)$



- Weaker tensor force than NN
- Small contribution from time-derivative part

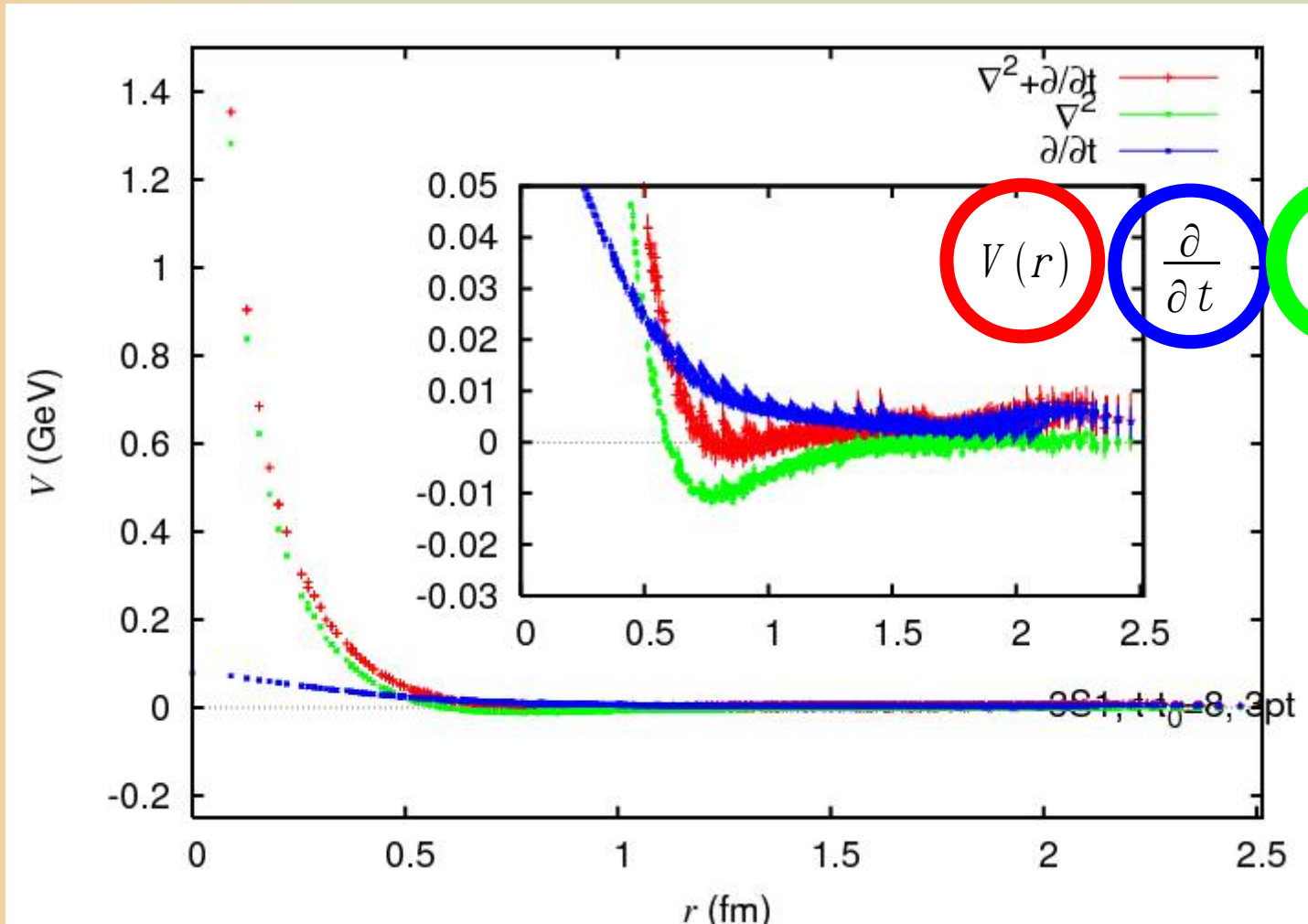
$\Sigma N(l=3/2)$ potential

$V_C(\Sigma N(I=3/2); 1S0)$



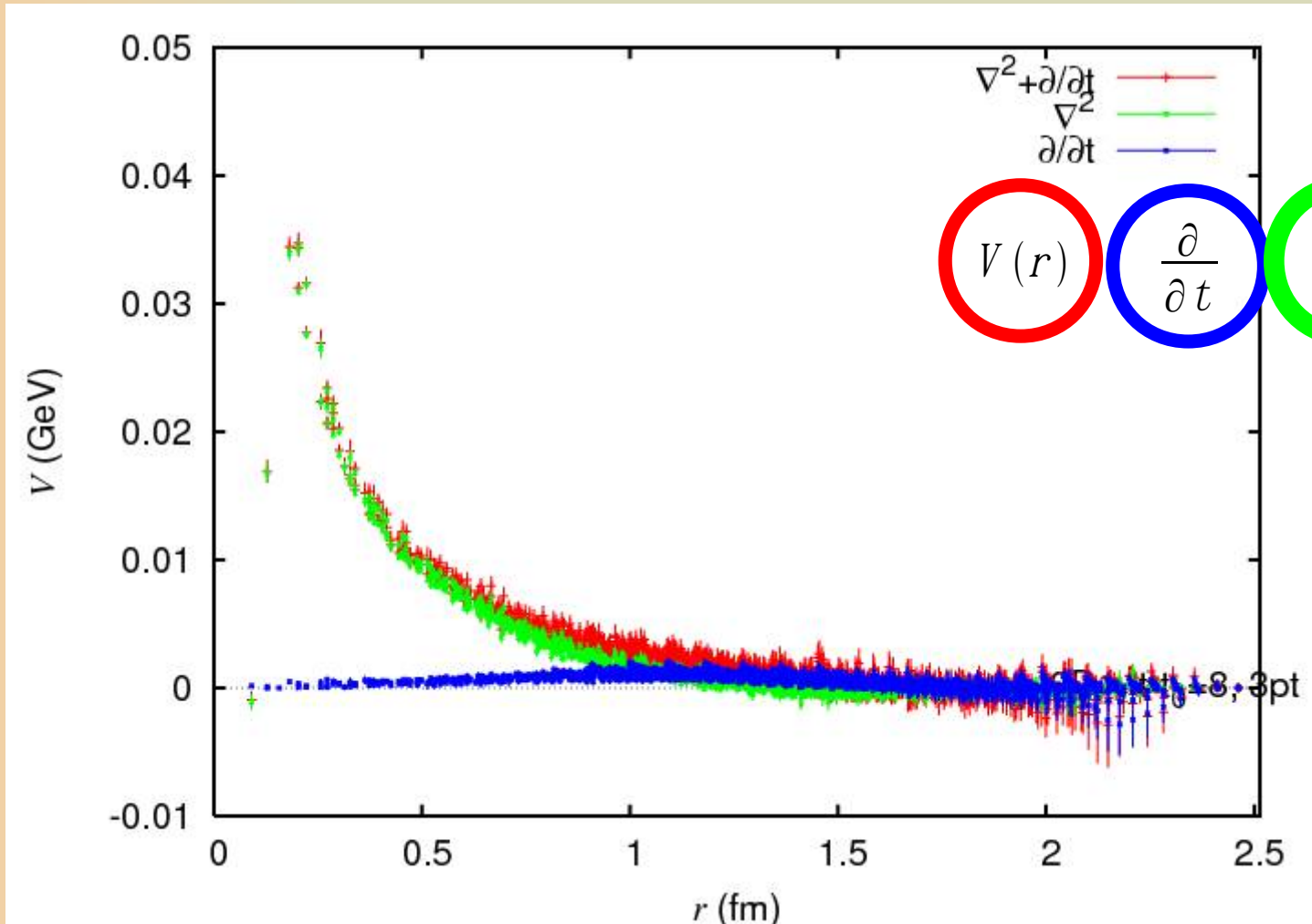
- {27}
- Similar to NN (1S0) (as well as Lambda-N (1S0))
- Sizable contribution from time-derivative part

$V_C(\Sigma N(I=3/2); 3S1-3D1)$



- $\{10\}$
- Repulsive potential (consistent with quark model)
- sizable repulsive contribution from time-derivative part

$V_T(\Sigma N(I=3/2); 3S1-3D1)$



- Weak tensor force
- Small contribution from time-derivative part

Scattering phase shifts

Proton-Lambda scattering (preliminary)

**Parametrized
potential**



Phase shift

(Hyper-)Nuclear few-body problem

Stochastic variational calculation of ^4He with using a lattice potential

- ⊗ For NN potential, we use Inoue-san's SU(3) potential at the lightest quark mass ($m_{ps} = 469 \text{ MeV}$), which has been reported to have a $4N$ bound state (about 5.1 MeV) within a tensor-included effective central potential; NPA881, 28-43 (2011).

Stochastic variational calculation of 4He with using a lattice potential

The wave function of A -body system is described by a linear combination of basis functions as

$$\Psi = \sum_{k=1}^K c_k \varphi_k, \quad \text{with} \quad \varphi_k = \mathcal{A}\{G(\mathbf{x}; A_k)[\theta_{(LL')_k}(\mathbf{x}; (uu')_k), \chi_{S_k}]_{JM\eta_{kIM_I}}\}, \quad (11)$$

where c_k is the linear variational parameter determined by the variational principle, \mathcal{A} is antisymmetrizer for identical particles. χ_{S_k} (η_{kIM_I}) is the spin (isospin) function of the system. $G(\mathbf{x}; A_k)$ is the correlated Gaussian function which is given by

$$G(\mathbf{x}; A_k) = \exp \left\{ -\frac{1}{2} \sum_{i < j}^A \alpha_{kij} (\mathbf{r}_i - \mathbf{r}_j)^2 \right\} = \exp \left\{ -\frac{1}{2} \sum_{i,j=1}^{A-1} A_{kij} \mathbf{x}_i \cdot \mathbf{x}_j \right\}. \quad (12)$$

Stochastic variational calculation of 4He with using a lattice potential

A set of relative coordinates $\{\mathbf{x}_1, \dots, \mathbf{x}_{A-1}\}$ and the center-of-mass coordinate \mathbf{x}_A are given by a linear transformation of single particle coordinates $\{\mathbf{r}_1, \dots, \mathbf{r}_A\}$ such as

$$\mathbf{x}_i = \sum_{j=1}^A U_{ij} \mathbf{r}_j, \quad (i = 1, \dots, A). \quad (13)$$

In order to obtain the accurate solution of the four-nucleon bound state with explicitly utilizing the the tensor potential, we consider nonzero orbital angular momentum states $(L, S)J^\pi = (1, 1)0^+$ and $(2, 2)0^+$ in addition to the $(0, 0)0^+$ configuration. We employ the global vector representation[11] for these nonzero orbital angular momentum states. Therefore, the angular part of the basis function is given by

$$\theta_{(LL')_k}(\mathbf{x}; (uu')_k) = v_k^{L_k} v_k'^{L'_k} [Y_{L_k}(\hat{\mathbf{v}}_k) \times Y_{L'_k}(\hat{\mathbf{v}}'_k)]_{L_k}, \quad \begin{pmatrix} \mathbf{v} \\ \mathbf{v}' \end{pmatrix}_k = \sum_{i=1}^{A-1} \mathbf{x}_i \begin{pmatrix} u \\ u' \end{pmatrix}_{ki}. \quad (14)$$

The validity of the present choice of basis function is examined for several realistic NN potentials[11]. The A_{kij} and $(u, u')_{ki}$ are the nonlinear variational parameters which are determined by the stochastic variational method[12].

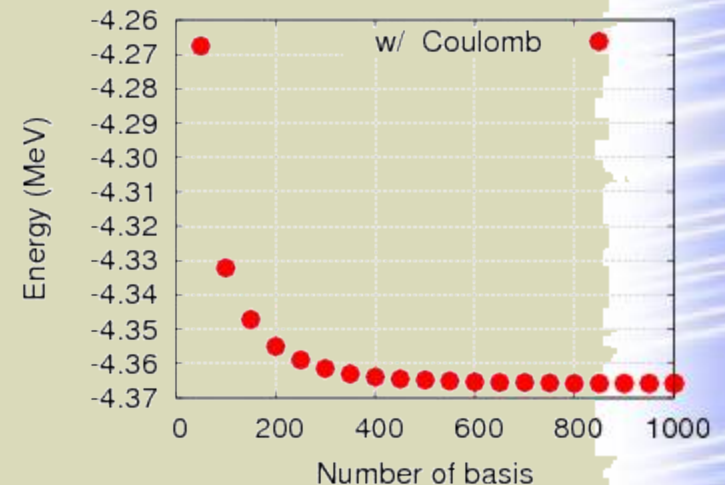
Results of few-body calculation

★ Inputs:

- $m=1161.0$ MeV,
- $\hbar c = 197.3269602$ MeV fm
- $\hbar c/e^2 = 137.03599976$
- V_{NN} is treated as a Serber-type potential.

★ Results:

- $B(4\text{He})=4.37$ MeV (w/ Coulomb)
 - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
 - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
- $B(4\text{He})=5.09$ MeV (w/o Coulomb)
 - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.4%)



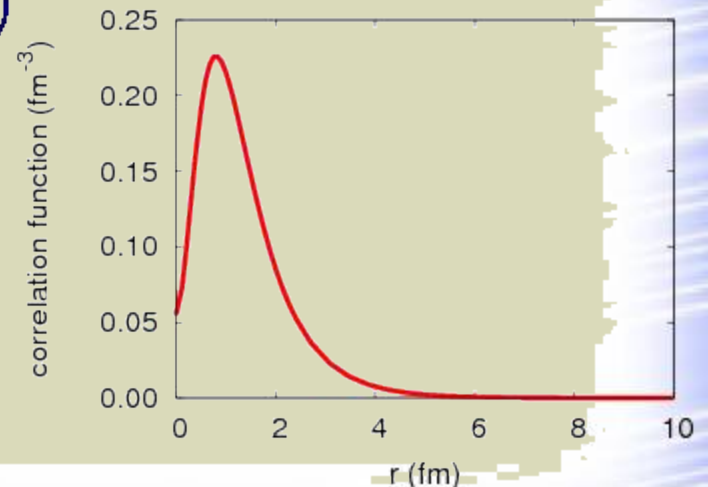
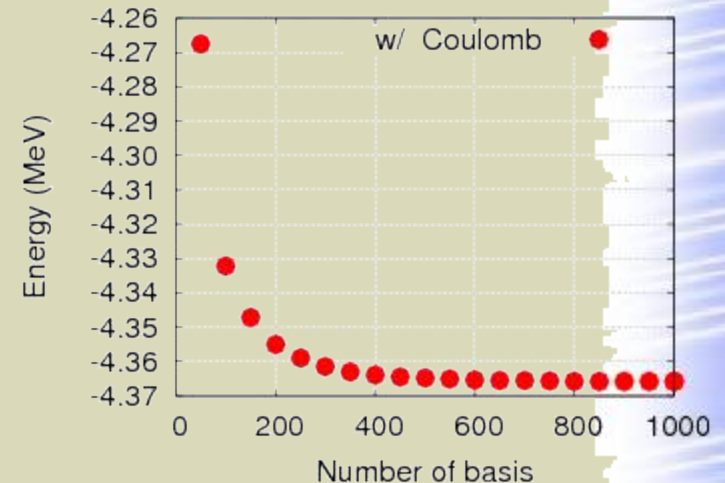
Results of few-body calculation

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- V_{NN} is treated as a Serber-type potential.

★ Results:

- $B(4\text{He})=4.37$ MeV (w/ Coulomb)
 - Probabilities of (S, P, D) waves = (98.6%, 0.003%, 1.3%)
 - I also calculate the correlation function.



Results when we cut off the tensor potential

★ Inputs:

- $m=1161.0$ MeV,
- $\hbar c = 197.3269602$ MeV fm
- $\hbar c/e^2 = 137.03599976$
- V_{NN} is treated as a Serber-type potential **with just cutting off the tensor part.**

★ Results:

- $B(4\text{He})=1.61$ MeV (w/ Coulomb)
 - Probabilities of (S, P, D) waves = (100%, 0%, 0%)
 - cf. roughly speaking (S,P,D)~(<90%, <0.1%, >10%) for a realistic NN force
- $B(4\text{He})=2.25$ MeV (w/o Coulomb)
 - (Probability of each component is almost same as the case including Coulomb)

Summary

Summary

(1) Lattice QCD calculation for hyperon potentials toward the physical point calculation.

Lambda-N, Sigma-N: central, tensor

(2) (hyper-)nuclear few-body calculation of stochastic variational method

(3) recent misc work

(萌芽的研究プロジェクトに関連していそうなその他の報告)

萌芽的研究プロジェクト

分野5の計算科学技術推進体制構築における萌芽的研究プロジェクト支援は**将来の主要な研究開発課題になるべきプロジェクトを開拓すること**を目的として行っているものです。アイデアの豊富な若手研究者に自由な発想で研究する機会を与え、分野全体で**新しい研究を育てる**ことを目指しています。このため研究支援チームの皆さんには、ユーザからのアルゴリズム・コーディング等の支援要請への対応・共通コード作成と同時に、**新しい発想に基づく**萌芽的研究課題に取り組むことを推奨してきました。萌芽的研究プロジェクトの一覧は以下の通りです。

Recent work

- (1) Porting the C++ program to **Bridge++**, which can calculate the four-point correlation function of Lambda-Nucleon system. The C++ program also has been used to **study other baryon-baryon potential for student**.
- (2) Improve the computational performance by implementing the **hybrid parallel** program with **MPI** and **OpenMP**
- (3) Generalize the target system to various baryon-baryon Channels (e.g., **52 channels** would be required to study the complete set of baryon-baryon potentials on **2+1 QCD calculation**)
- (4) In this approach, the number of iterations to obtain the four-point correlation function is remarkably smaller than the numbers given in the unified contraction algorithm[2]

[1] H.N. Pos(LAT2013)426;(LAT2008)156;(LAT2009)152;(LAT2011)167.

[2] Doi and Endres, Comput. Phys. Commun. 184, 117 (2013).

Effective block algorithm to calculate the 52 channels of 4pt correlator

$$\langle p n \overline{p n} \rangle, \tag{4.1}$$

$$\langle p \Lambda \overline{p \Lambda} \rangle, \langle p \Lambda \overline{\Sigma^+ n} \rangle, \langle p \Lambda \overline{\Sigma^0 p} \rangle, \tag{4.2}$$

$$\langle \Sigma^+ n \overline{p \Lambda} \rangle, \langle \Sigma^+ n \overline{\Sigma^+ n} \rangle, \langle \Sigma^+ n \overline{\Sigma^0 p} \rangle,$$

$$\langle \Sigma^0 p \overline{p \Lambda} \rangle, \langle \Sigma^0 p \overline{\Sigma^+ n} \rangle, \langle \Sigma^0 p \overline{\Sigma^0 p} \rangle,$$

$$\langle \Lambda \Lambda \overline{\Lambda \Lambda} \rangle, \langle \Lambda \Lambda \overline{p \Sigma^-} \rangle, \langle \Lambda \Lambda \overline{n \Sigma^0} \rangle, \langle \Lambda \Lambda \overline{\Sigma^+ \Sigma^-} \rangle, \langle \Lambda \Lambda \overline{\Sigma^0 \Sigma^0} \rangle, \tag{4.3}$$

$$\langle p \Sigma^- \overline{\Lambda \Lambda} \rangle, \langle p \Sigma^- \overline{p \Sigma^-} \rangle, \langle p \Sigma^- \overline{n \Sigma^0} \rangle, \langle p \Sigma^- \overline{\Sigma^+ \Sigma^-} \rangle, \langle p \Sigma^- \overline{\Sigma^0 \Sigma^0} \rangle, \langle p \Sigma^- \overline{\Sigma^0 \Lambda} \rangle,$$

$$\langle n \Sigma^0 \overline{\Lambda \Lambda} \rangle, \langle n \Sigma^0 \overline{p \Sigma^-} \rangle, \langle n \Sigma^0 \overline{n \Sigma^0} \rangle, \langle n \Sigma^0 \overline{\Sigma^+ \Sigma^-} \rangle, \langle n \Sigma^0 \overline{\Sigma^0 \Sigma^0} \rangle, \langle n \Sigma^0 \overline{\Sigma^0 \Lambda} \rangle,$$

$$\langle \Sigma^+ \Sigma^- \overline{\Lambda \Lambda} \rangle, \langle \Sigma^+ \Sigma^- \overline{p \Sigma^-} \rangle, \langle \Sigma^+ \Sigma^- \overline{n \Sigma^0} \rangle, \langle \Sigma^+ \Sigma^- \overline{\Sigma^+ \Sigma^-} \rangle, \langle \Sigma^+ \Sigma^- \overline{\Sigma^0 \Sigma^0} \rangle, \langle \Sigma^+ \Sigma^- \overline{\Sigma^0 \Lambda} \rangle,$$

$$\langle \Sigma^0 \Sigma^0 \overline{\Lambda \Lambda} \rangle, \langle \Sigma^0 \Sigma^0 \overline{p \Sigma^-} \rangle, \langle \Sigma^0 \Sigma^0 \overline{n \Sigma^0} \rangle, \langle \Sigma^0 \Sigma^0 \overline{\Sigma^+ \Sigma^-} \rangle, \langle \Sigma^0 \Sigma^0 \overline{\Sigma^0 \Sigma^0} \rangle,$$

$$\langle \Sigma^0 \Lambda \overline{p \Sigma^-} \rangle, \langle \Sigma^0 \Lambda \overline{n \Sigma^0} \rangle, \langle \Sigma^0 \Lambda \overline{\Sigma^+ \Sigma^-} \rangle, \langle \Sigma^0 \Lambda \overline{\Sigma^0 \Lambda} \rangle,$$

$$\langle \Sigma^- \Lambda \overline{\Sigma^- \Lambda} \rangle, \langle \Sigma^- \Lambda \overline{\Sigma^- \Sigma^0} \rangle, \langle \Sigma^- \Lambda \overline{\Sigma^0 \Sigma^-} \rangle, \tag{4.4}$$

$$\langle \Sigma^- \Sigma^0 \overline{\Sigma^- \Lambda} \rangle, \langle \Sigma^- \Sigma^0 \overline{\Sigma^- \Sigma^0} \rangle, \langle \Sigma^- \Sigma^0 \overline{\Sigma^0 \Sigma^-} \rangle,$$

$$\langle \Sigma^0 \Sigma^- \overline{\Sigma^- \Lambda} \rangle, \langle \Sigma^0 \Sigma^- \overline{\Sigma^- \Sigma^0} \rangle, \langle \Sigma^0 \Sigma^- \overline{\Sigma^0 \Sigma^-} \rangle,$$

$$\langle \Sigma^- \Sigma^0 \overline{\Sigma^- \Sigma^0} \rangle. \tag{4.5}$$

★ Elapse times to calculate the 52 matrix correlators (MPI+OpenMP)

★ [tasks_per_node] x [OMP_NUM_THREADS]

★ 64x1 32x2 16x4 8x4 4x8 2x16 1x32

★ Step-1 0:14 0:16 0:09 0:09 0:07 0:06 0:06

★ Step-2 0:10 0:11 0:12 0:12 0:12 0:13 0:14