1st or 2nd; the order of finite temperature phase transition of Nf=2 QCD from effective theory analysis

Yusuke Taniguchi (University of Tsukuba)
with
Sinya Aoki (Kyoto University)
Hidenori Fukaya (Osaka University)

Introduction

- Chiral symmetry in QCD
  - Broken in two different ways

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

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[Nambu 1961]

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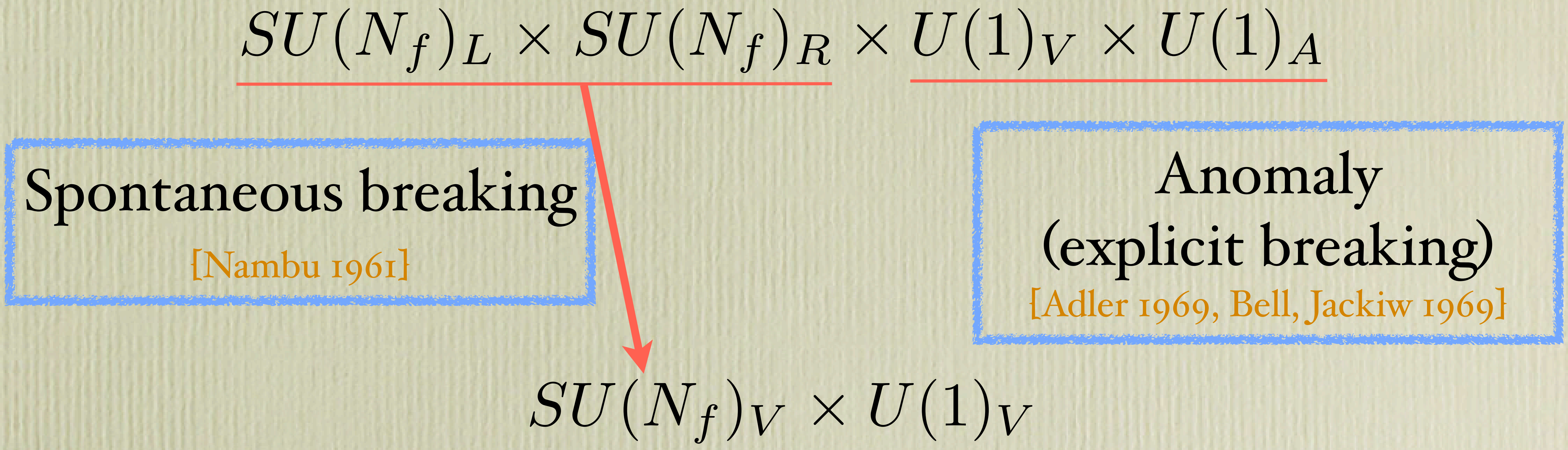

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Anomaly
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[Adler 1969, Bell, Jackiw 1969]

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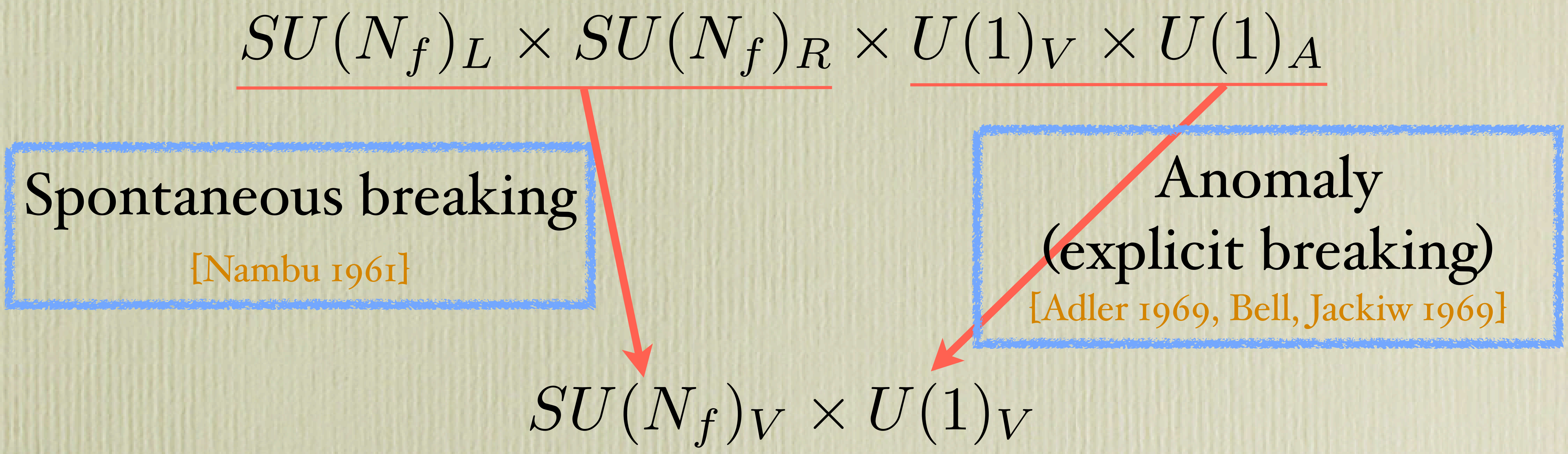

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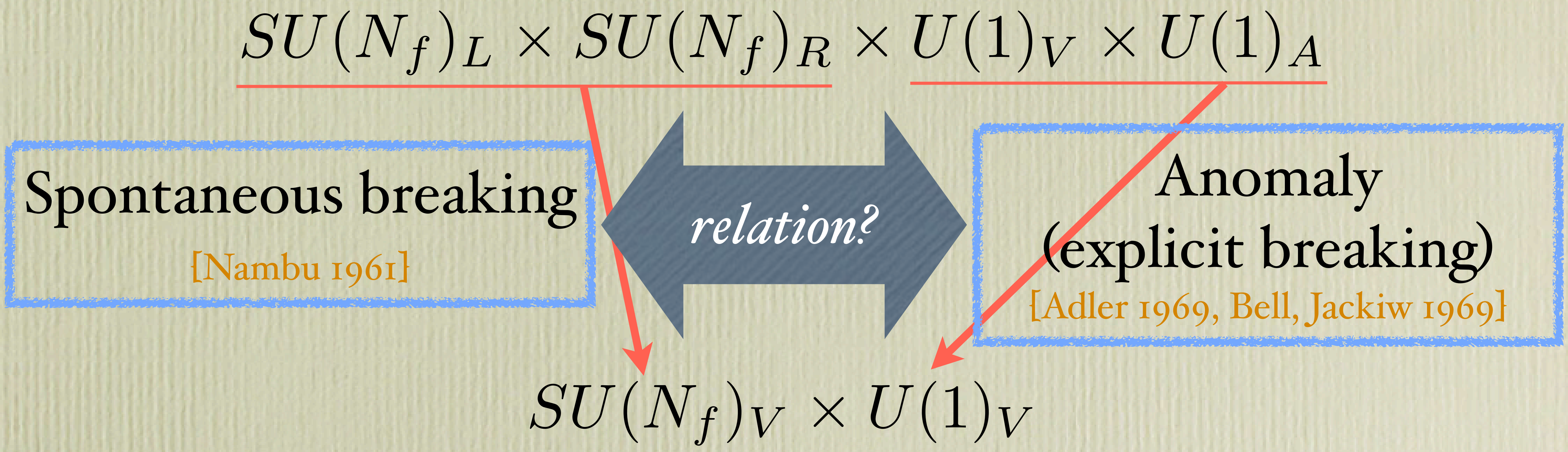

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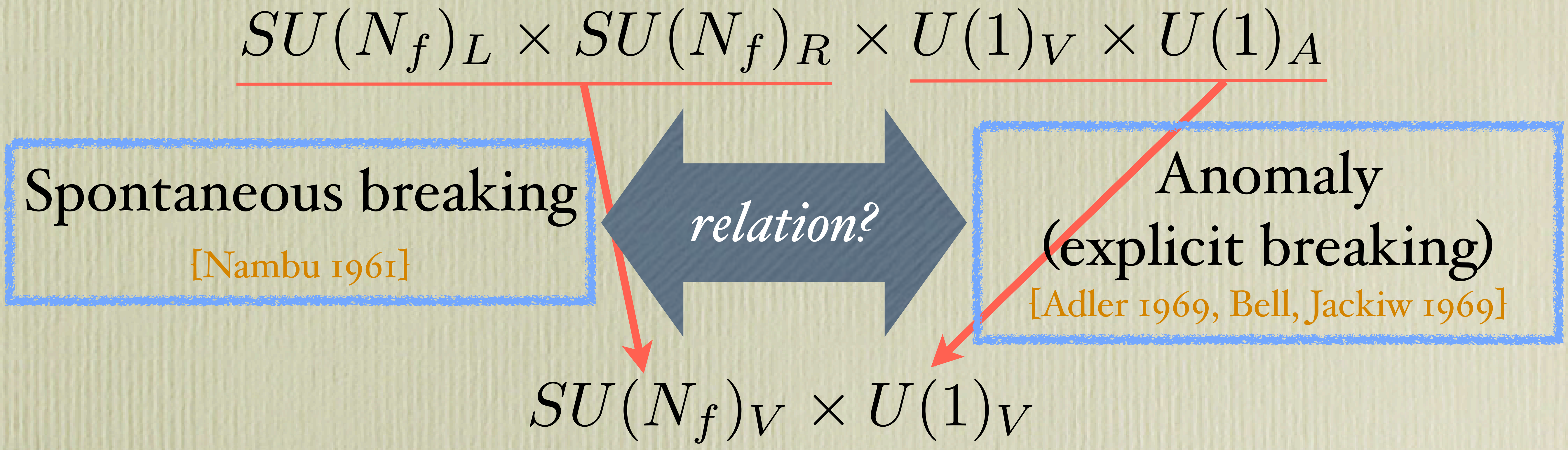

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We have shown a possibility of BOTH restoration at the SAME temperature.

(Aoki, Fukaya, Taniguchi; PRD 86, 114512)

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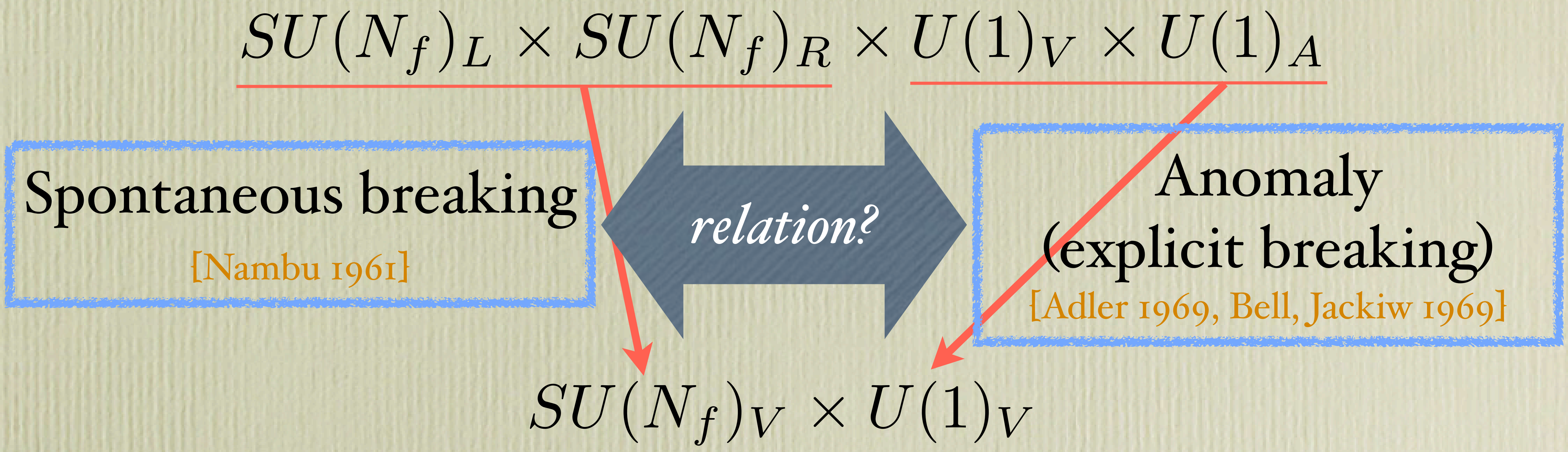

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Previous talk by Sinya Aoki

Restoration of U(1)$_A$ symmetry

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Order parameter of U(1)$_A$

Restoration of U(1)A symmetry

Order parameter of U(1)A $\langle \bar{\psi}\psi \rangle$

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Also the order parameter of SU(2)A

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Order parameter of U(1)A $\langle\bar{\psi}\psi\rangle$ =0 above Tc

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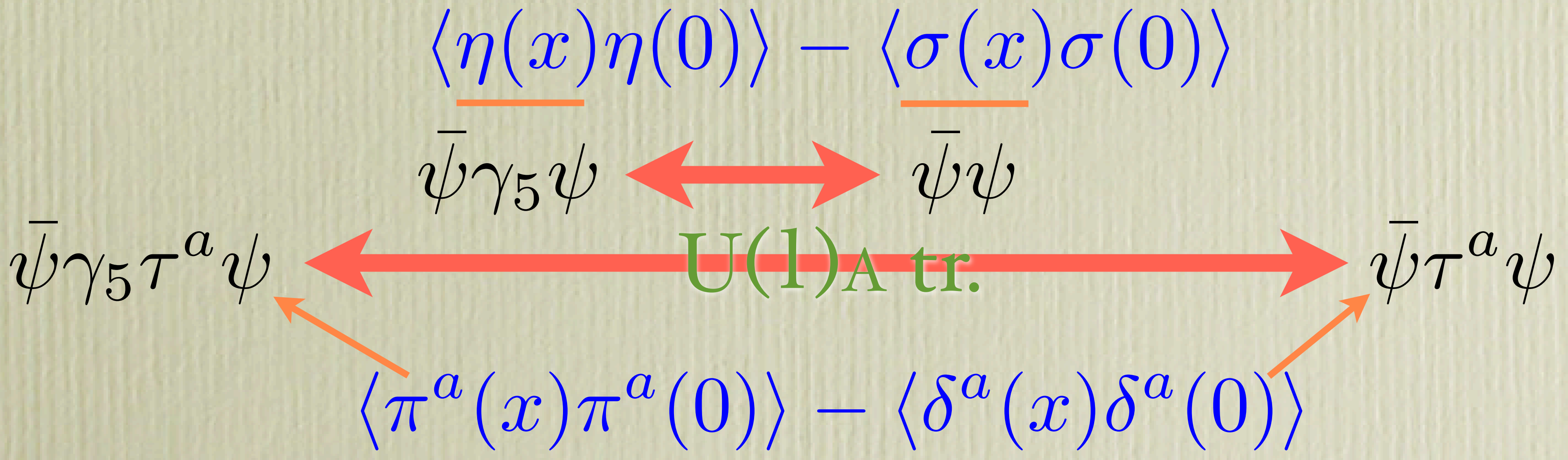

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Plan of the talk

✓ 1. Introduction

2. Previous works

3. Restoration U(1)A symmetry

4. Effective theory

5. Renormalization group analysis

6. Conclusion

Previous works on U(1)$_A$ restoration

There so many works.

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Cohen (1996) : YES.

- U(1)A is restored above Tc

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Lee & Hatsuda (1996) : NO.

Q=±1 instanton sector does break U(1)A.

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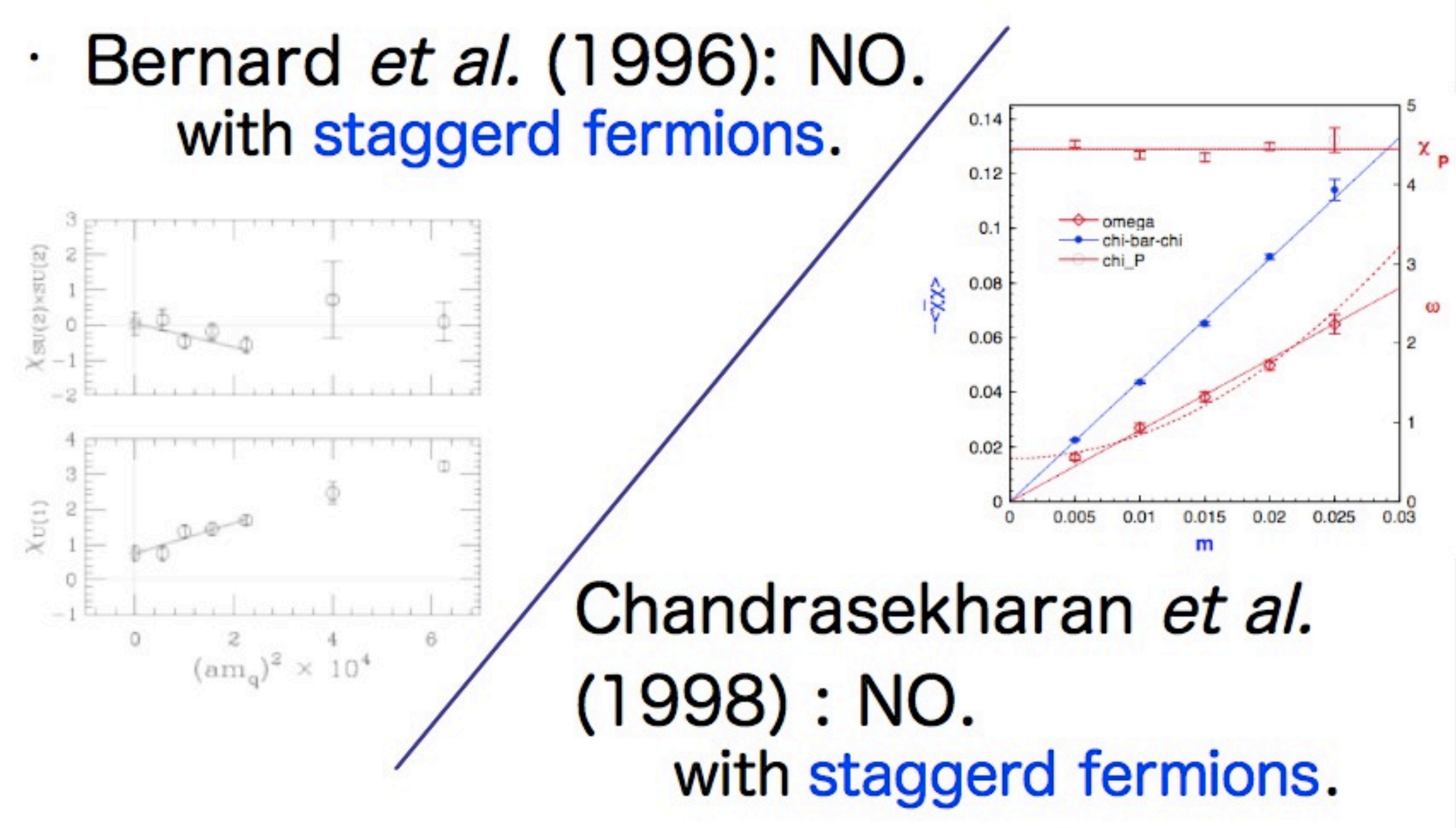


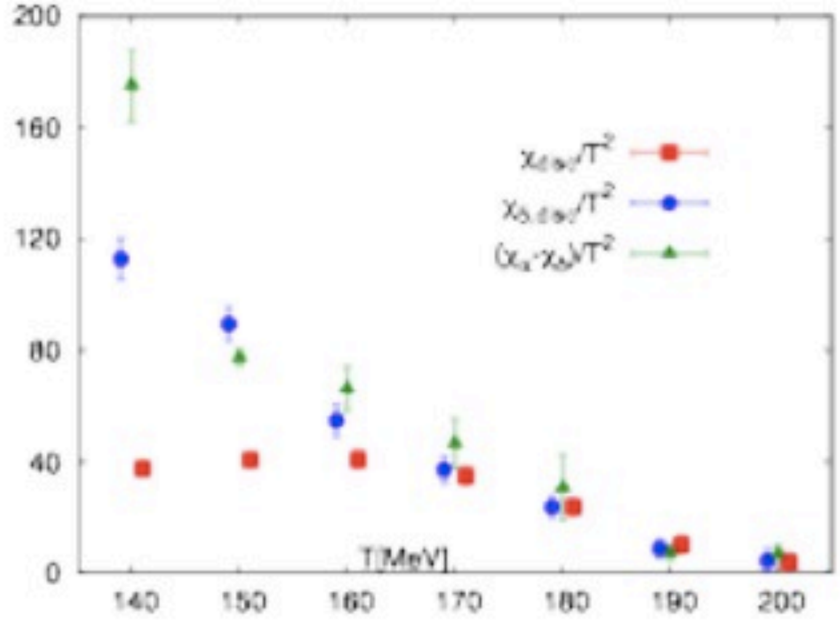
- Bernard *et al.* (1996): NO.
  with staggerd fermions.

Chandrasekharan *et al.* (1998) : NO.
with staggerd fermions.

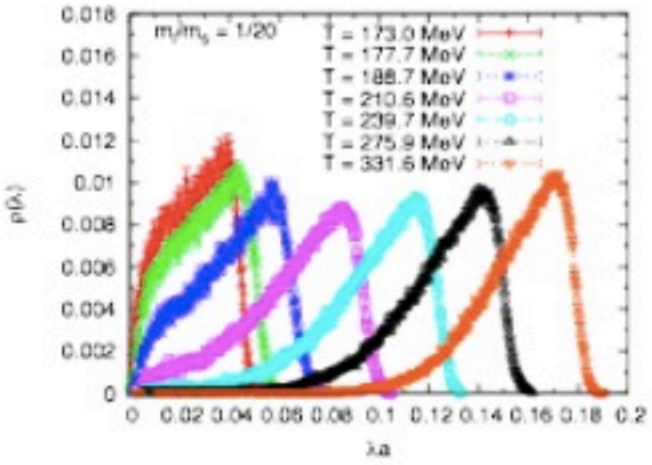
Previous works on U(1)A restoration

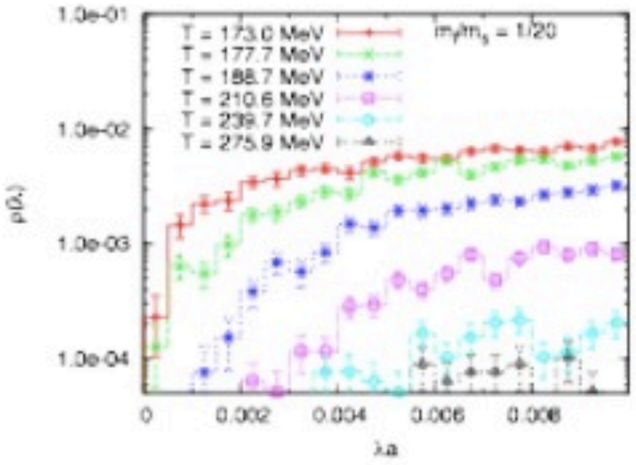
There so many works.

HotQCD (2011) : NO.
with domain-wall fermions.



Ohno *et al.* (2011) : NO.
with HISQ
(highly
improved
staggered)





Previous works on $U(1)_A$ restoration

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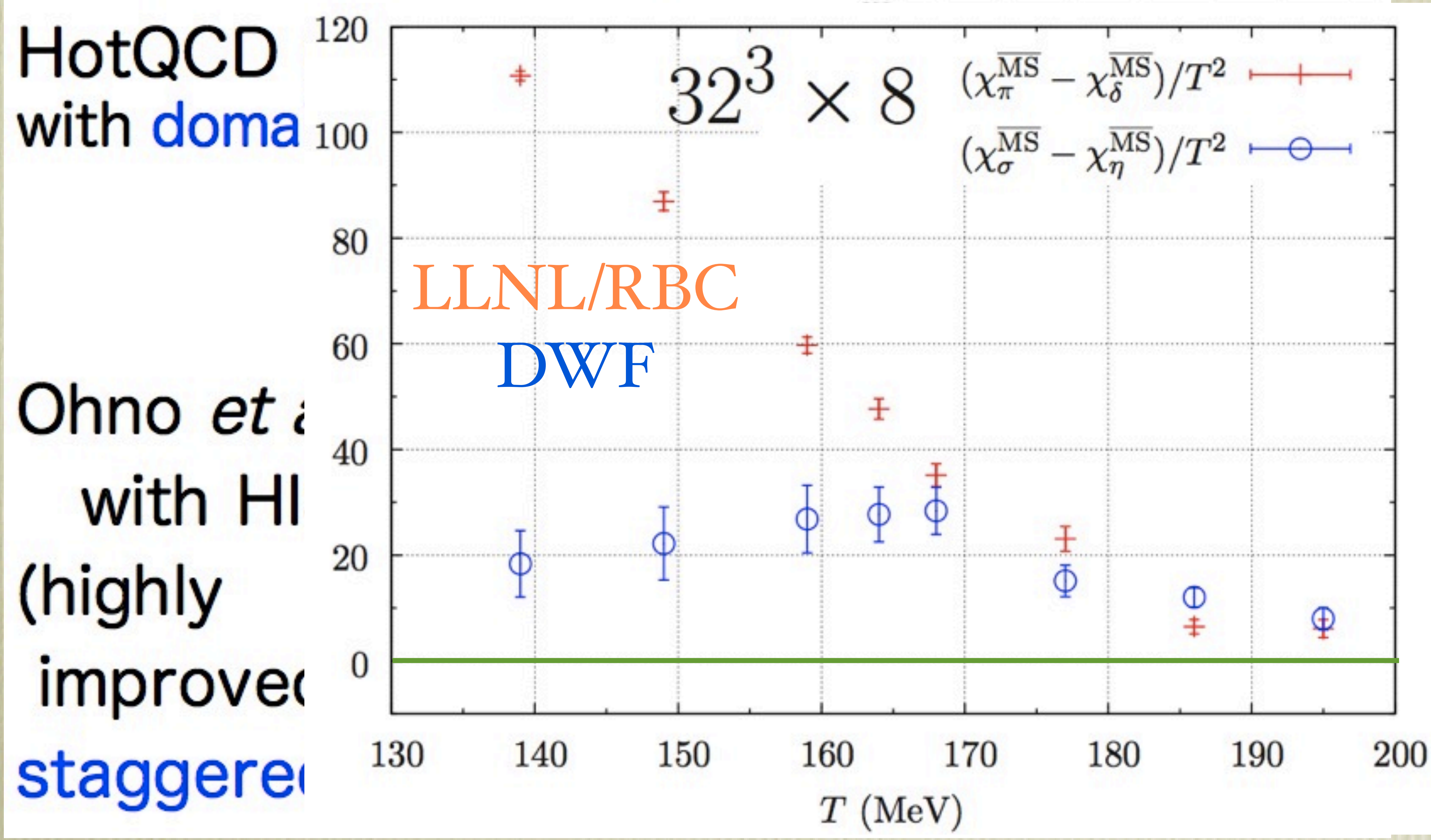

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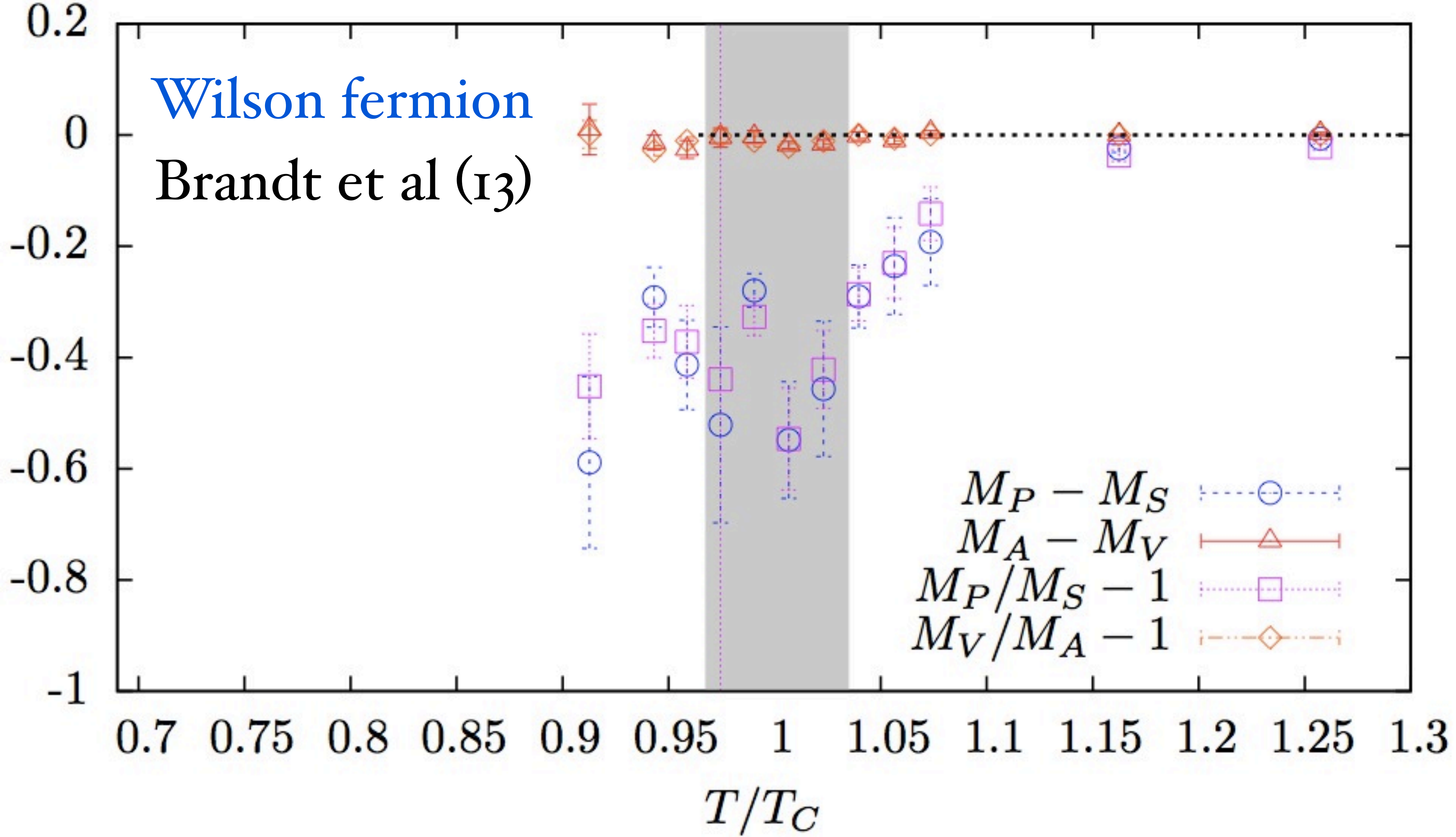

Wilson fermion

Brandt et al (13)

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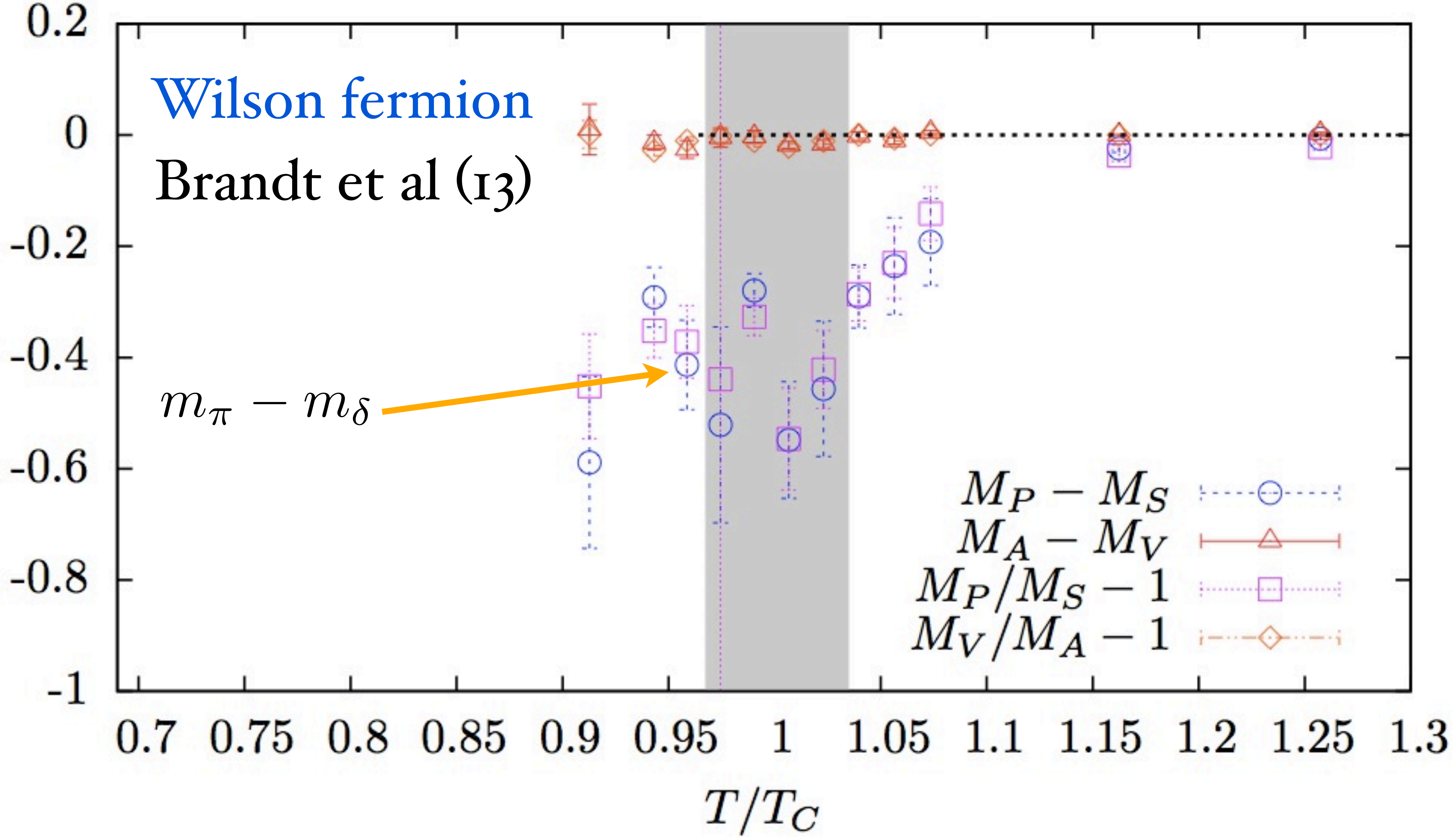

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$m_\pi - m_\delta$

Previous works on U(1)A restoration

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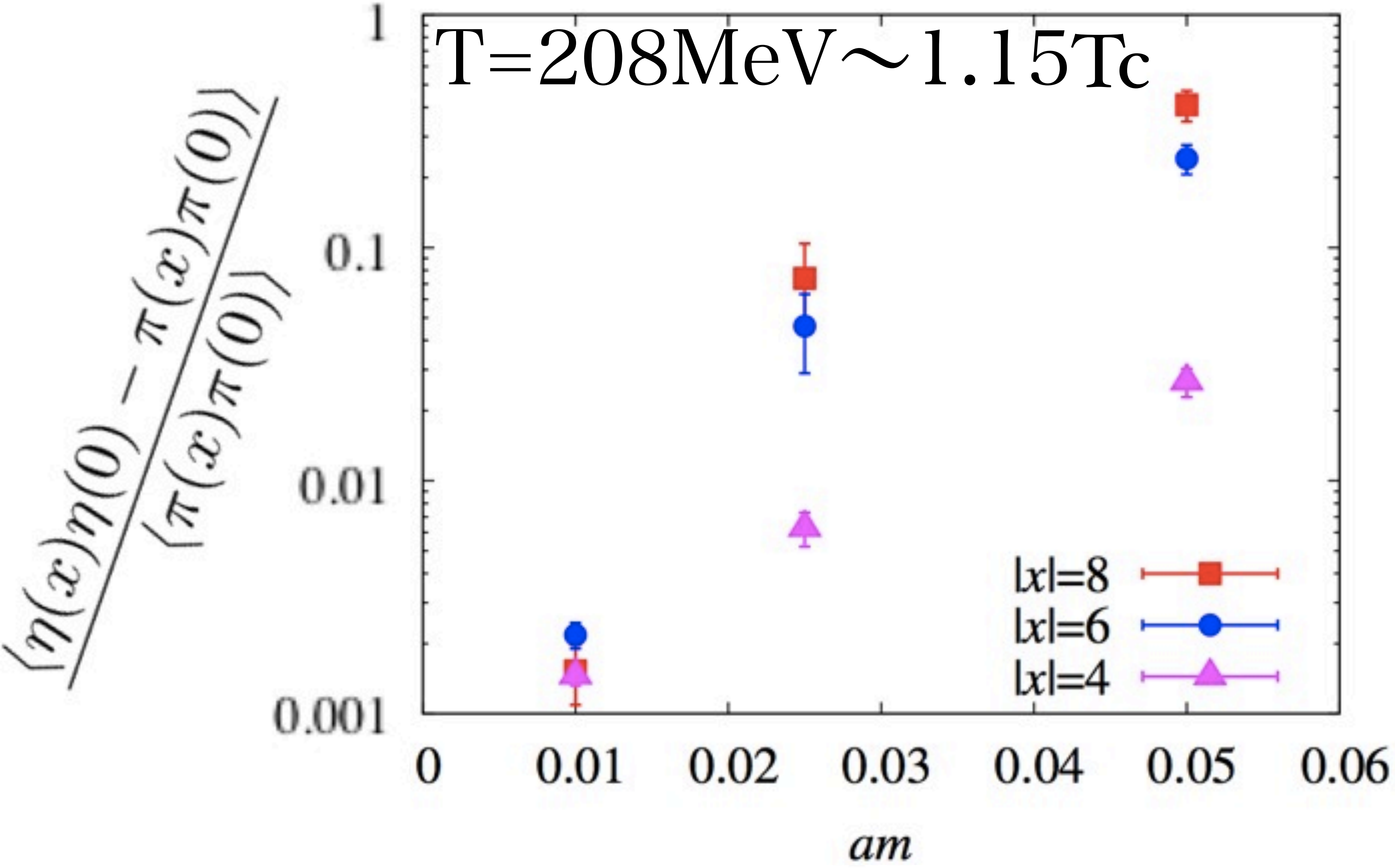
Cossu *et al.* [JLQCD] (2013) : YES.

with overlap fermions.



T=208MeV∼1.15Tc

Previous works on U(1)A restoration

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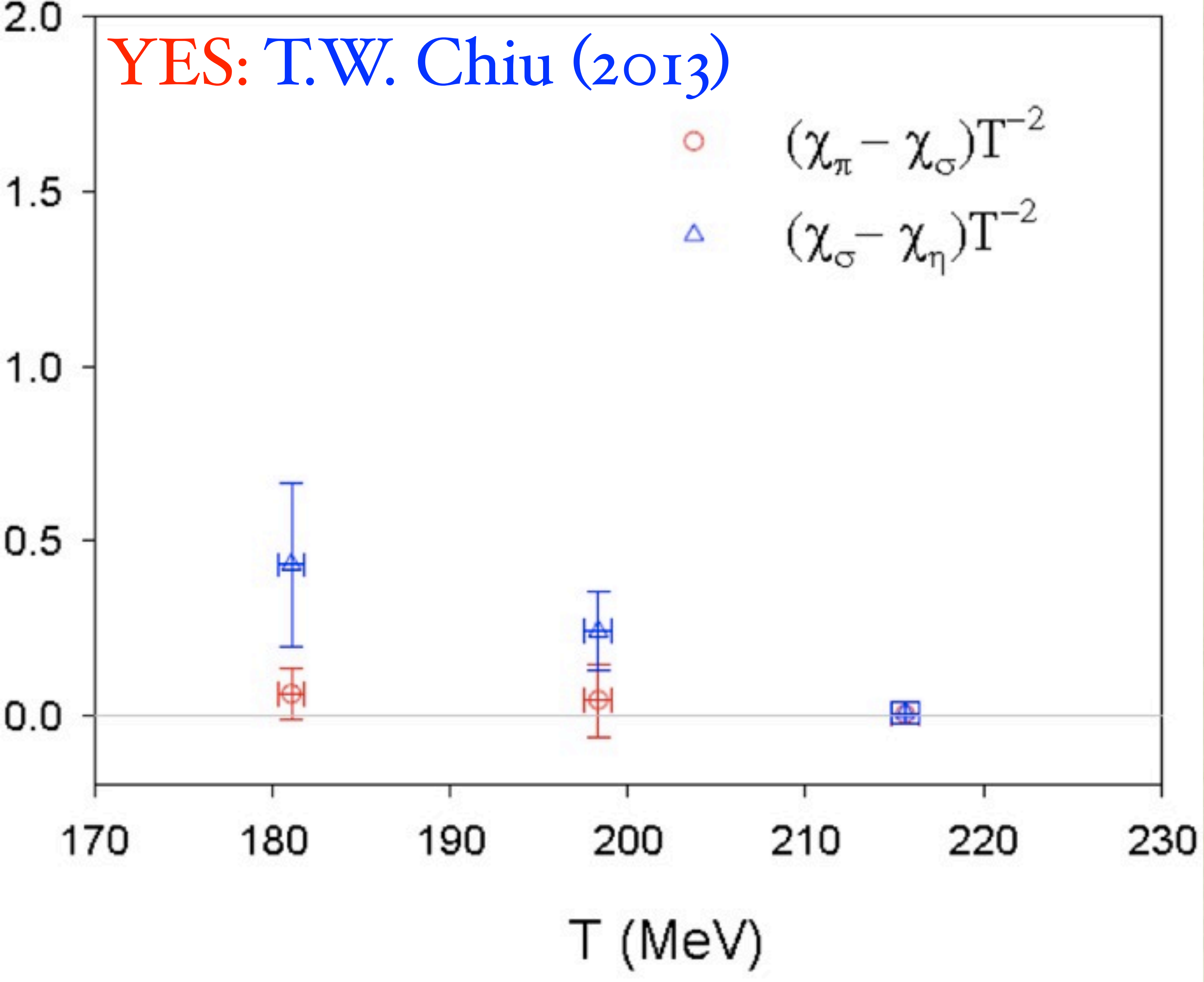

Previous wor

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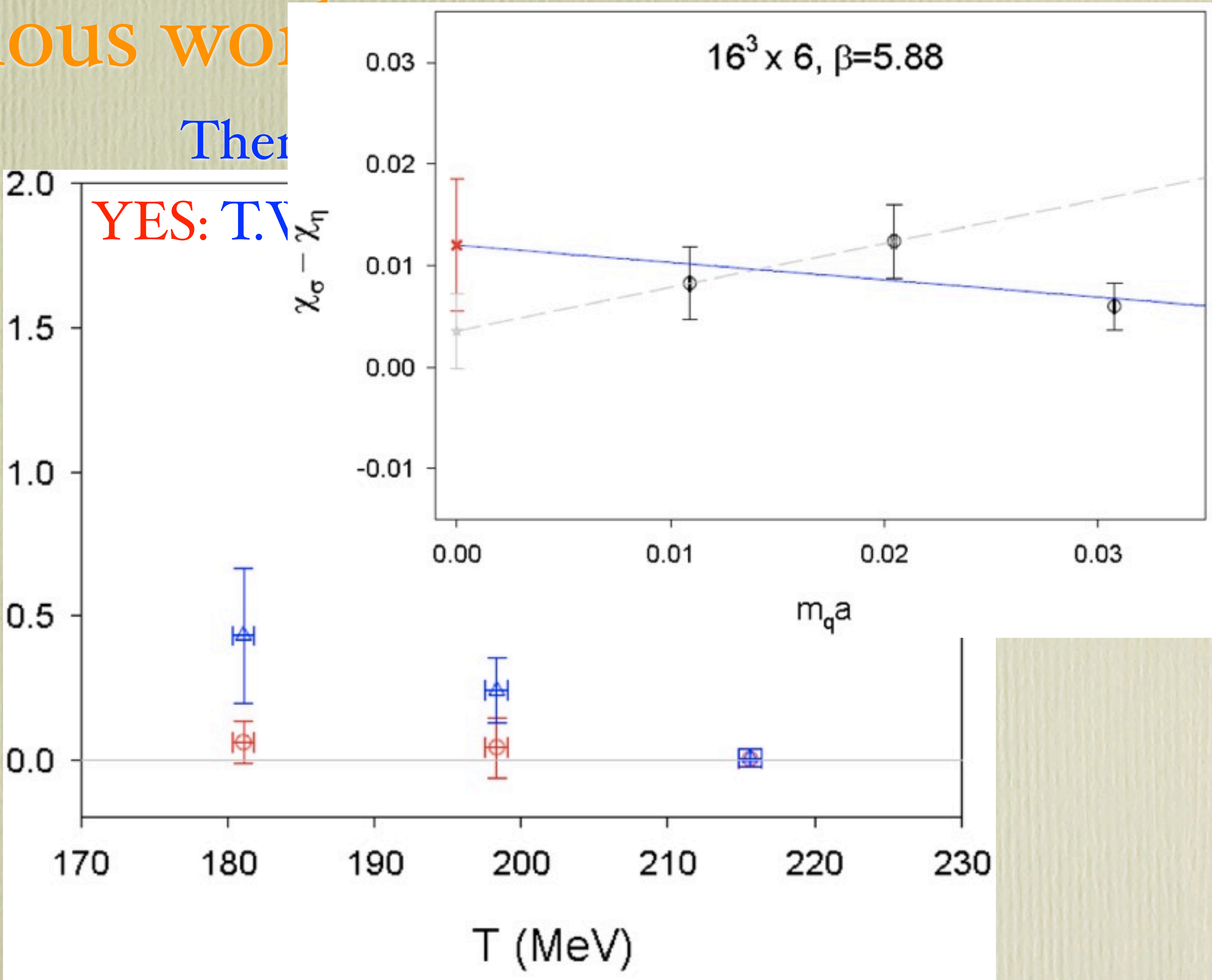

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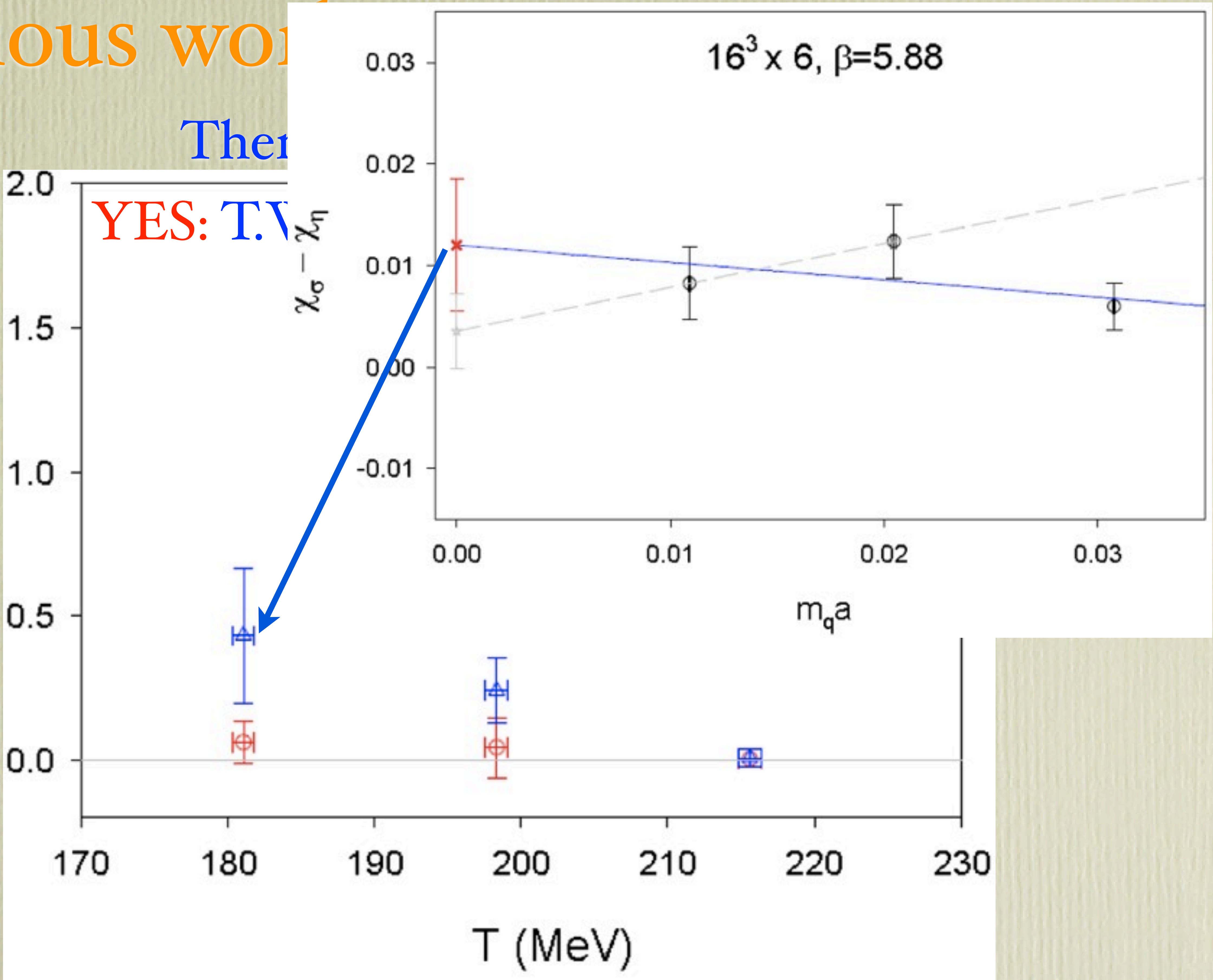

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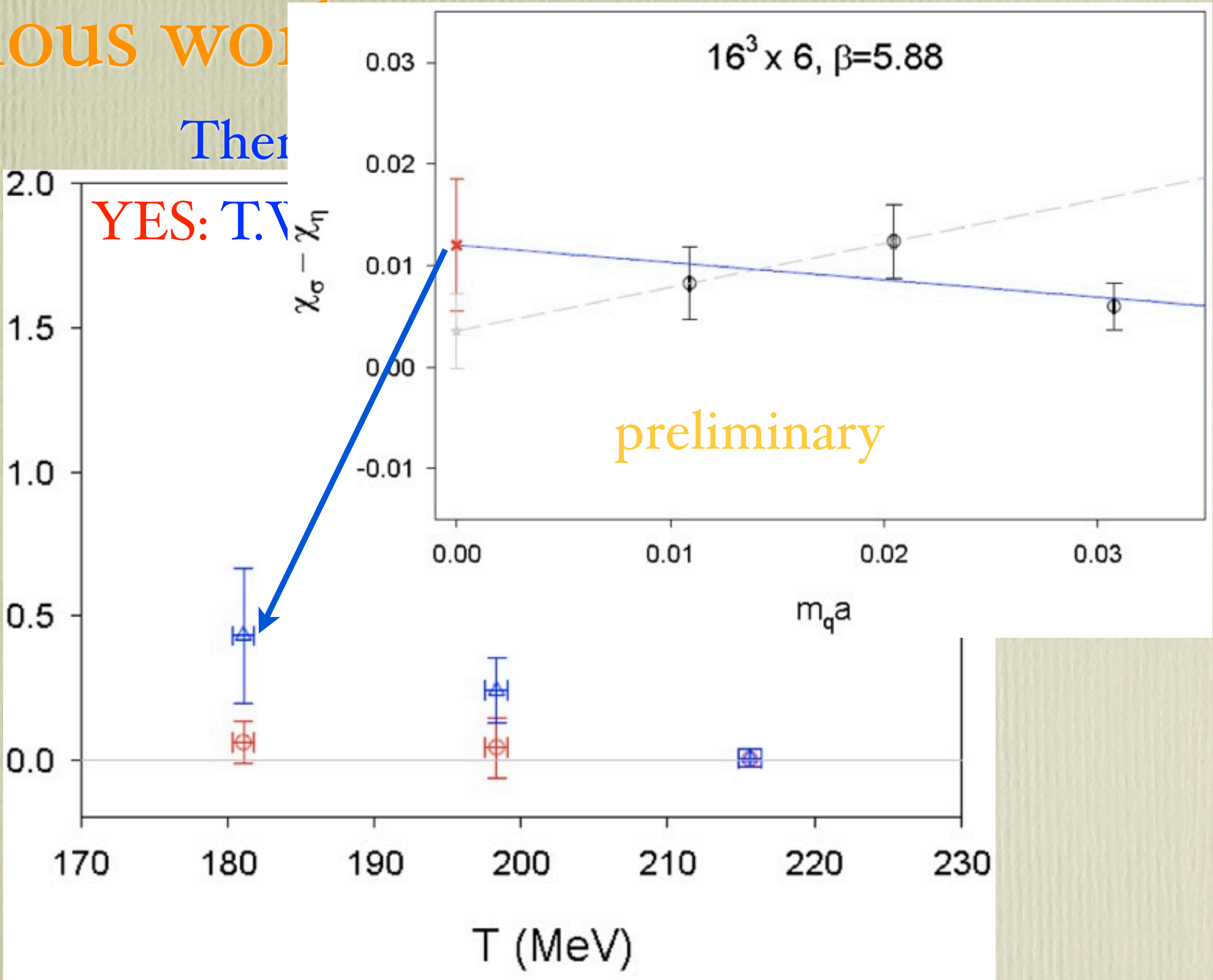

Previous works on U(1)A restoration

	U(1) restoration	instanton effect	exact chiral sym.	$V \to \infty$
Cohen	YES	×	○	○
Lee-Hatsuda	NO	○	○	×
staggered	NO	○	×	×
Wilson	NO	○	×	×
DFW	NO	○	△	×
overlap	YES	○	○	×
Our work	YES	○	○	○

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- The idea of Cohen [1996]:
  Eigenvalue spectrum of Dirac operator may link SU(2) SSB and U(1) anomaly

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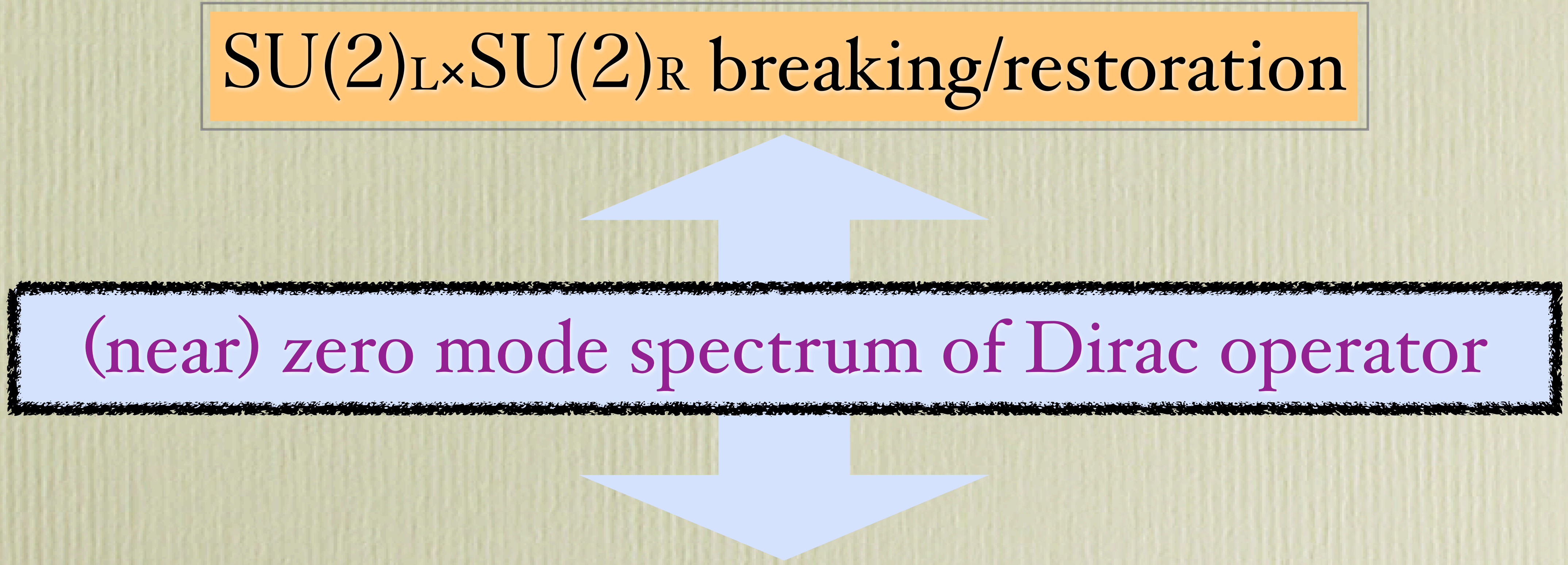

$SU(2)_L \times SU(2)_R$ breaking/restoration

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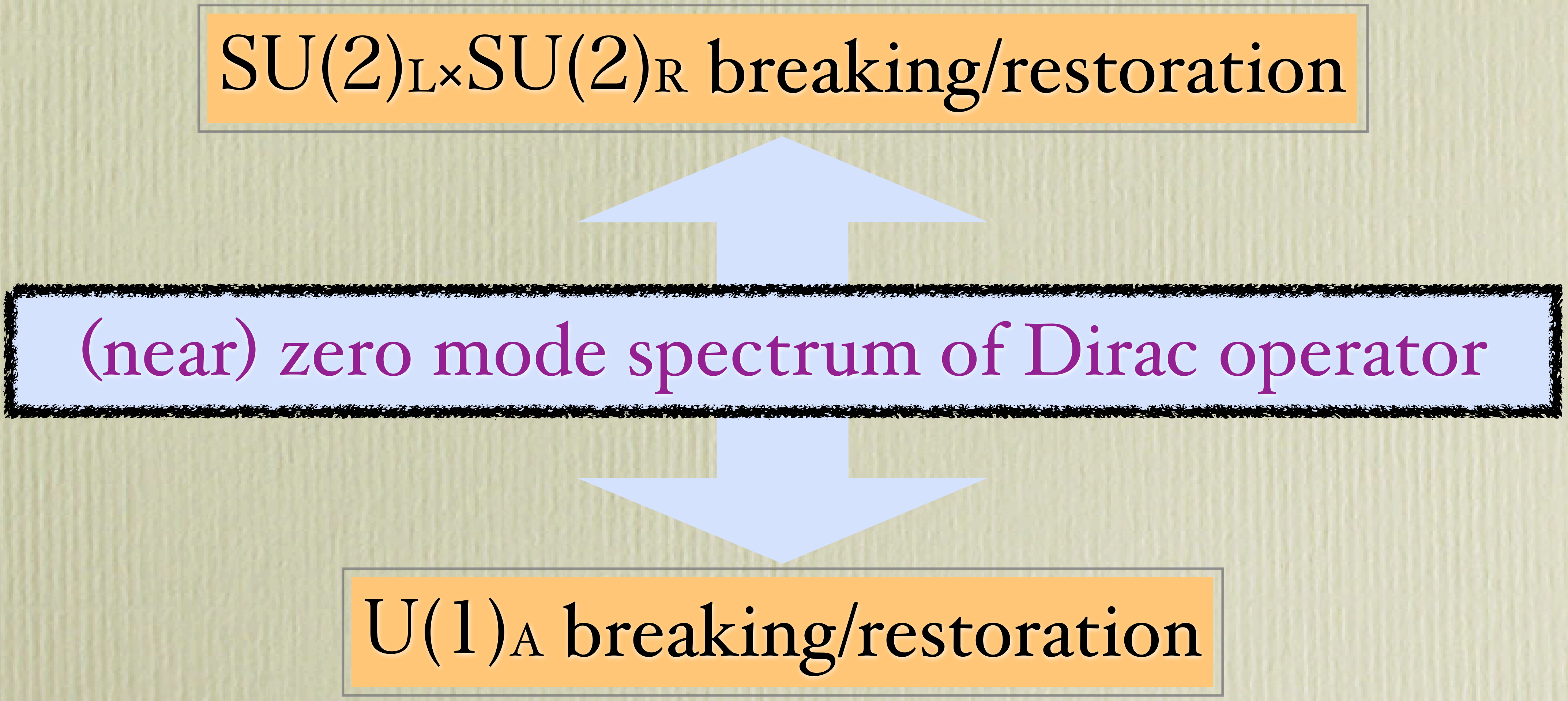

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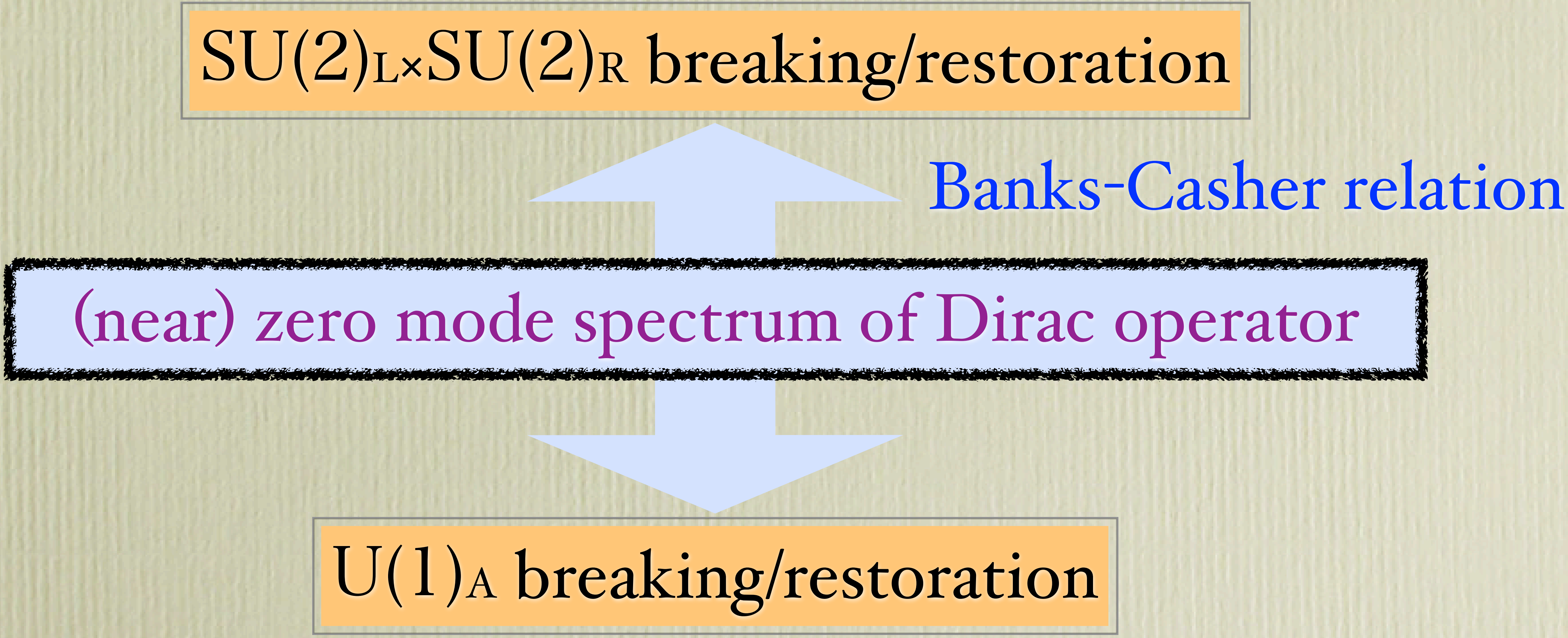

SU(2)$_L$×SU(2)$_R$ breaking/restoration

Banks-Casher relation

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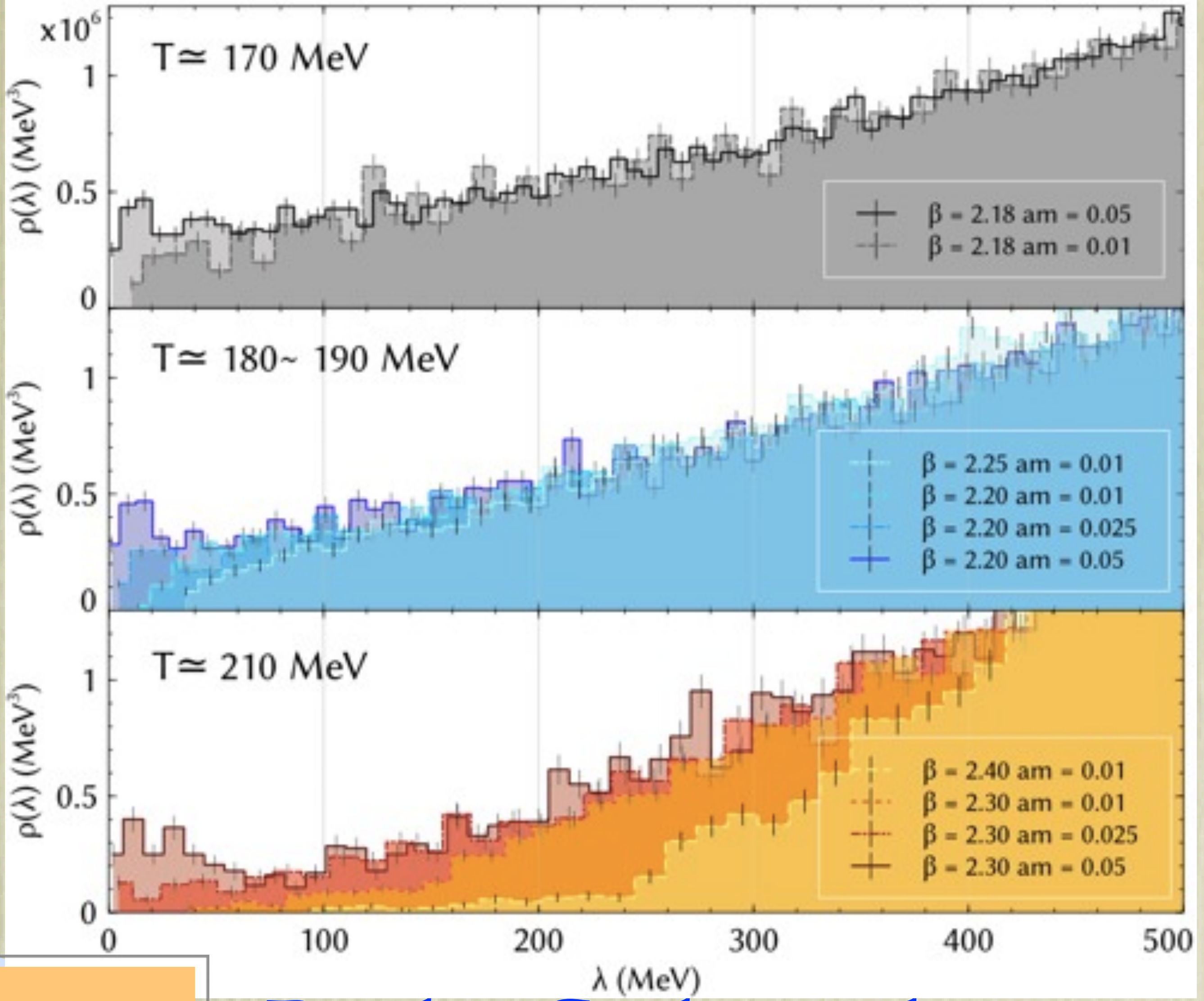

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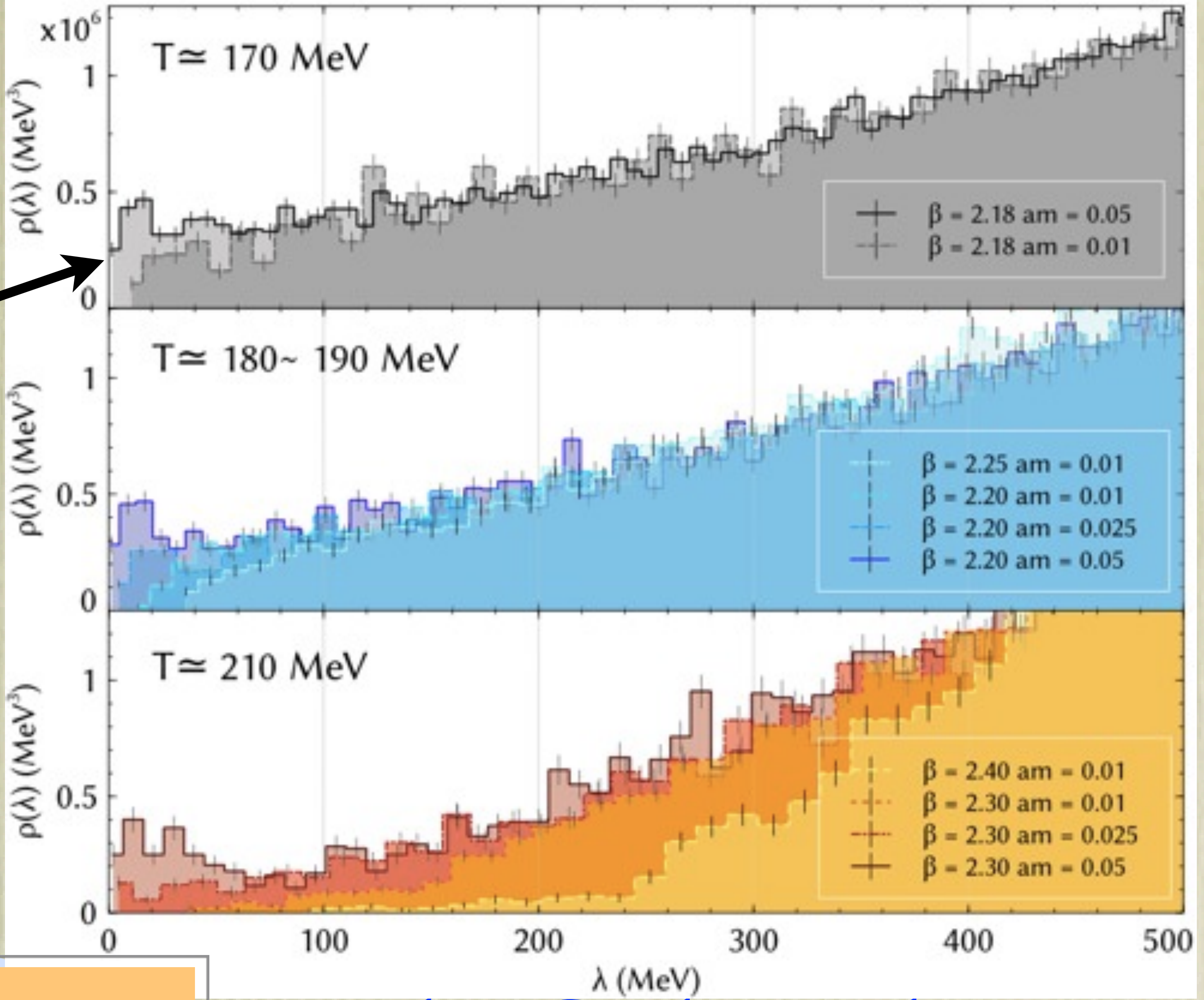
intercept

$\langle\bar{\psi}\psi\rangle$



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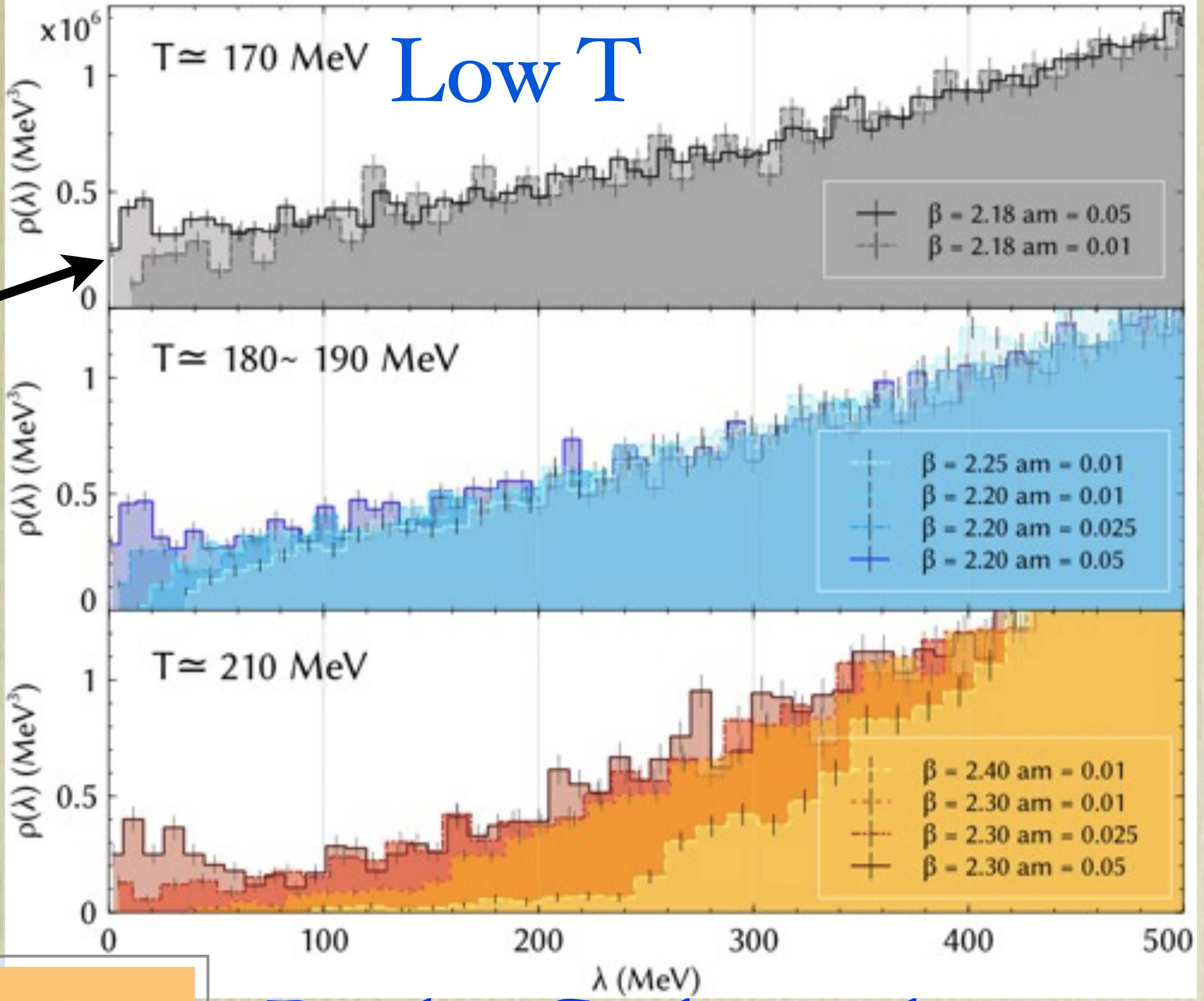

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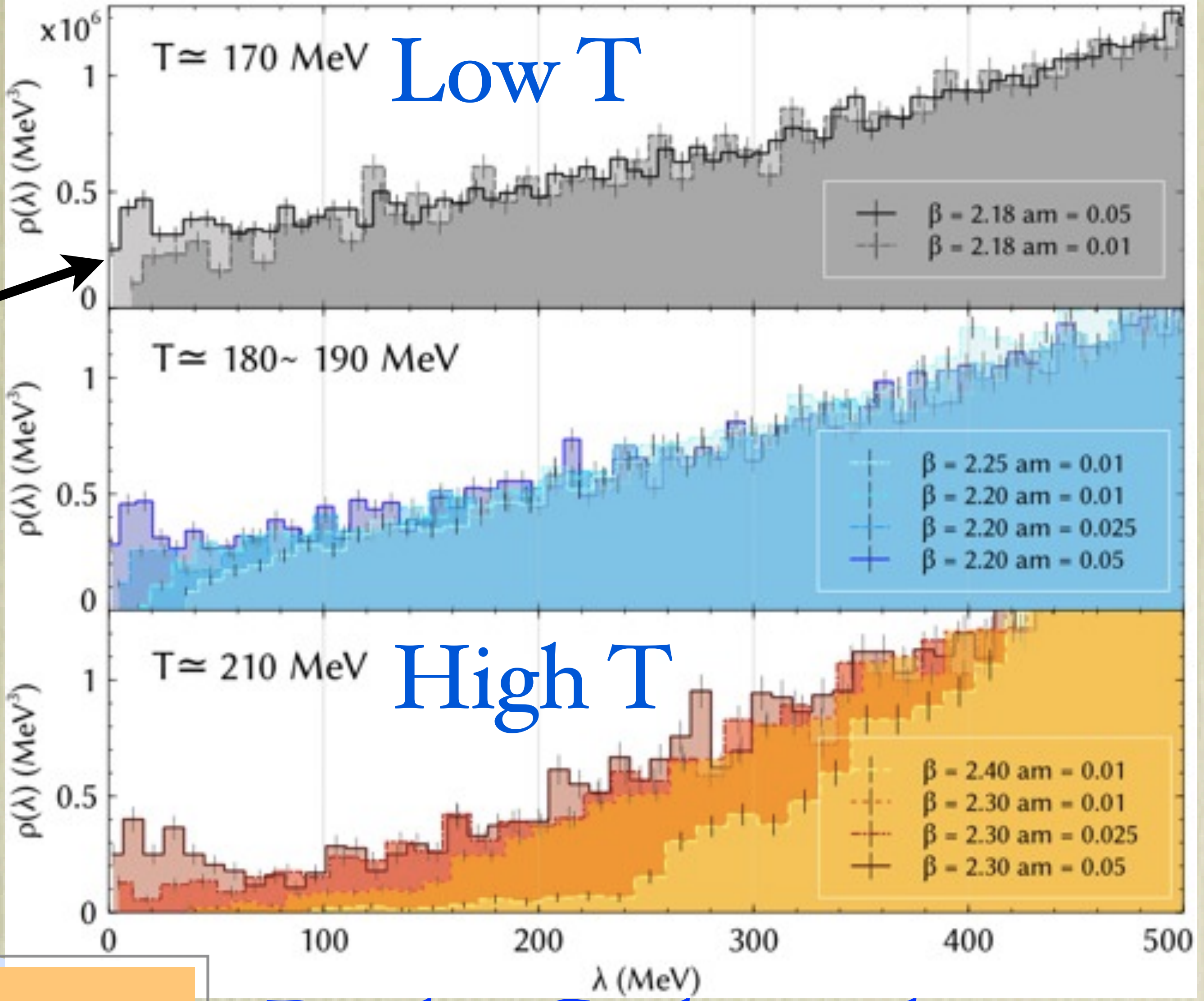

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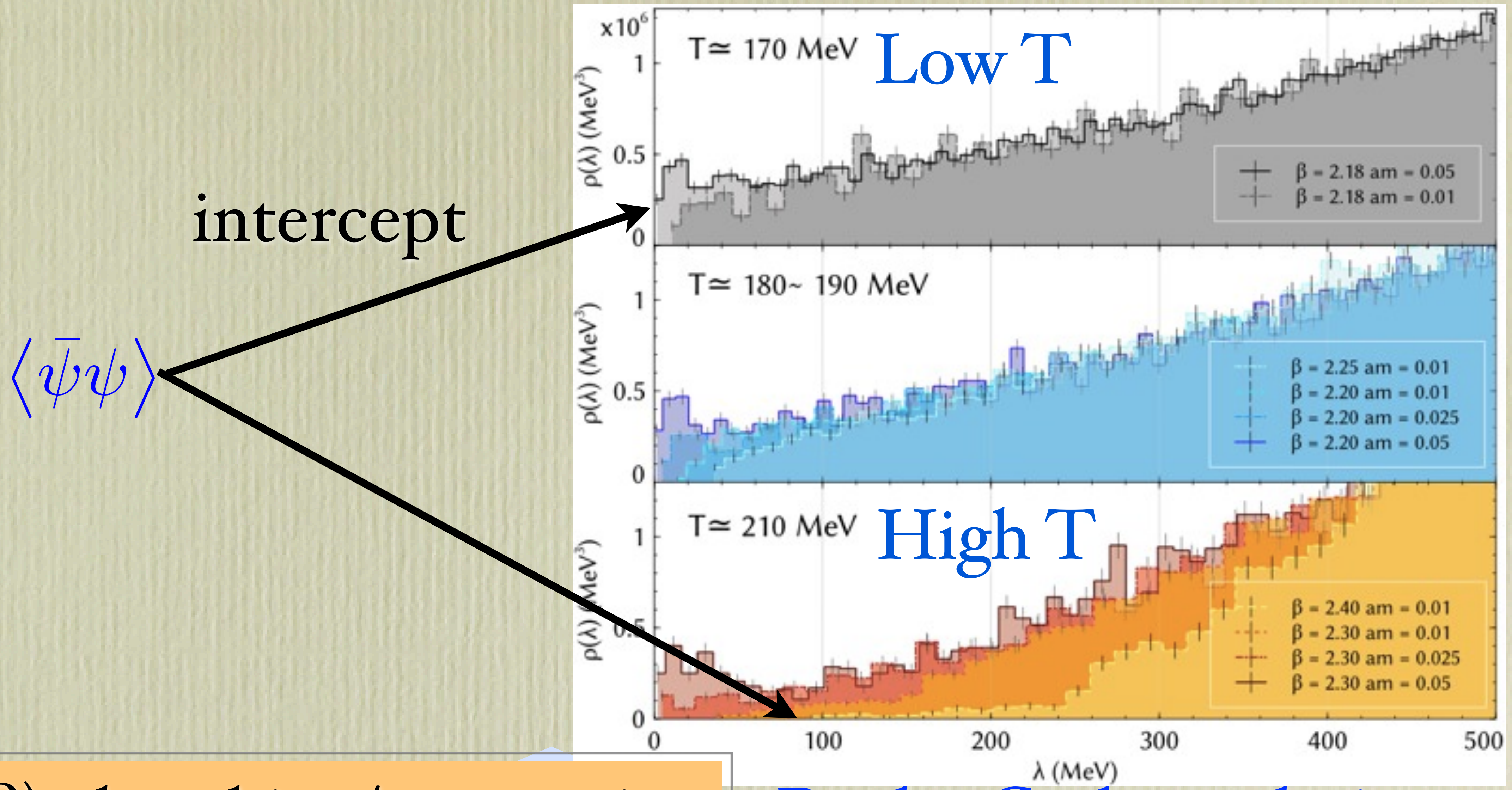

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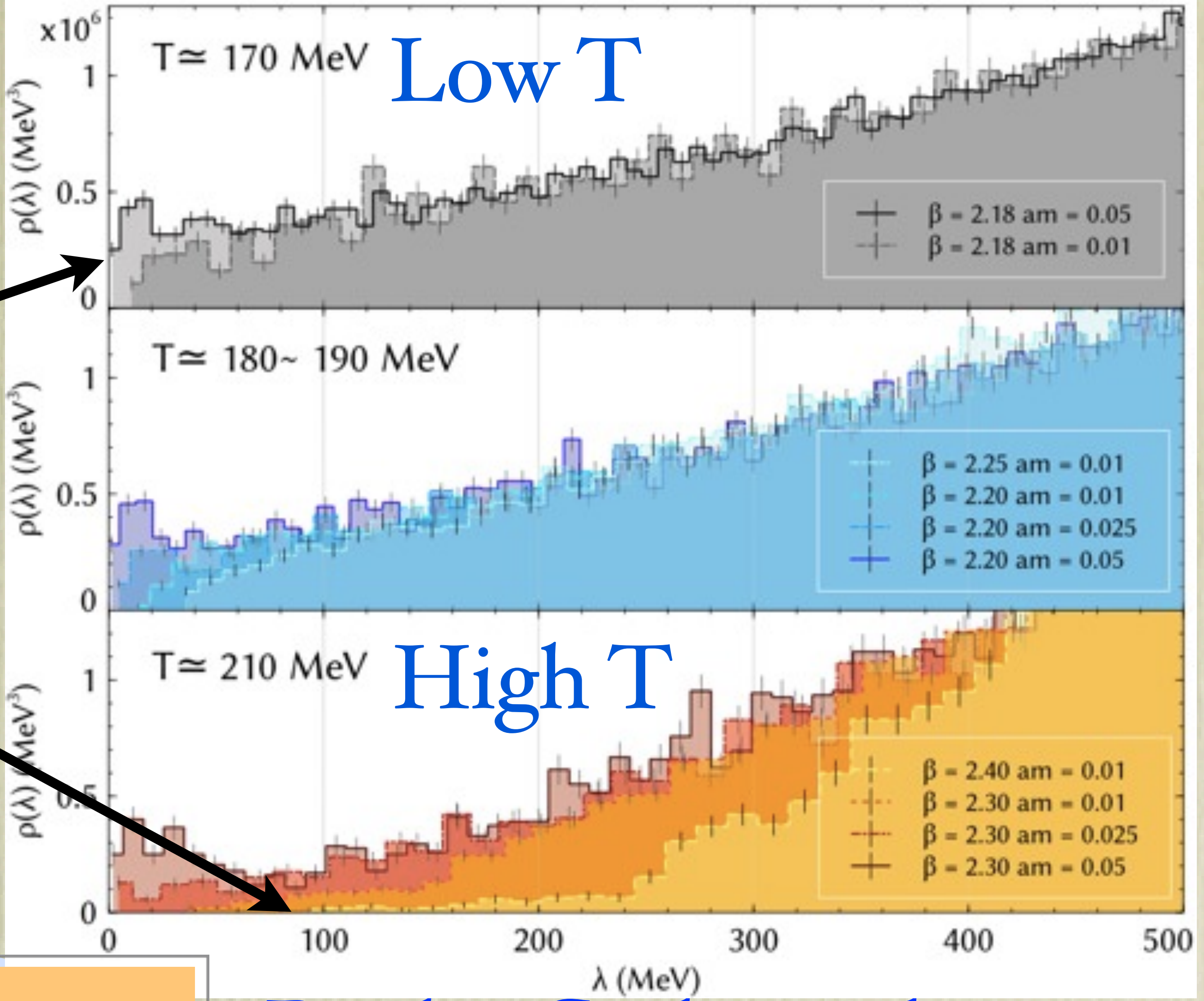
intercept



SU(2)L×SU(2)R breaking/restoration

Banks-Casher relation

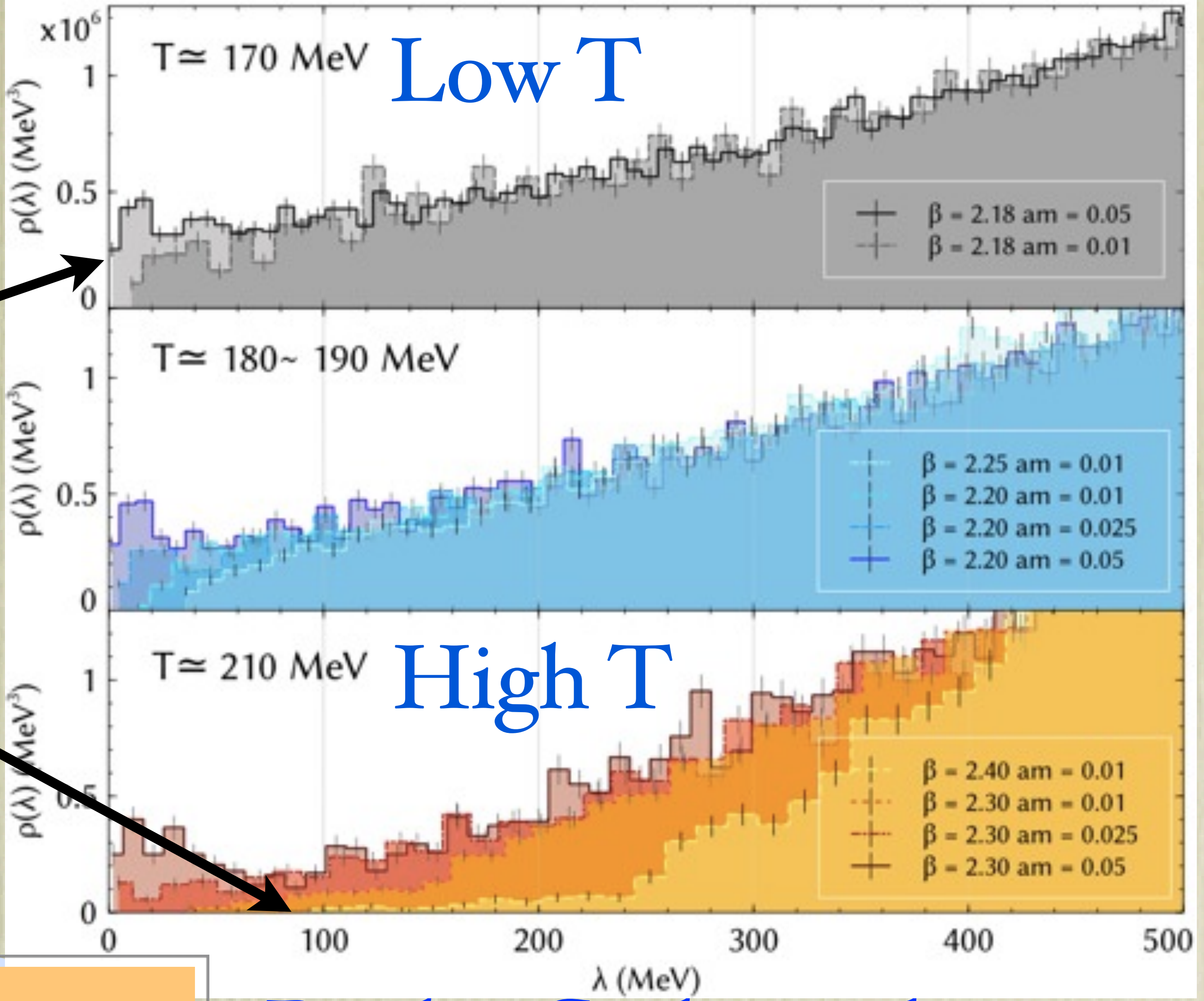
(near) zero mode spectrum of Dirac operator

index theorem

U(1)A breaking/restoration

intercept

$\langle\bar{\psi}\psi\rangle$



SU(2)L×SU(2)R breaking/restoration

Banks-Casher relation

(near) zero mode spectrum of Dirac operator

index theorem

U(1)A breaking/restoration

anomaly$\sim N_+ - N_-$

Restoration of $U(1)_A$ symmetry

We assume: $SU(2)_L \times SU(2)_R$ is fully recovered above Tc

Restoration of U(1)A symmetry

We assume: SU(2)L×SU(2)R is fully recovered above Tc

Order parameters of SU(2)L×SU(2)R

$$\frac{1}{V^k}\langle\delta^a\mathcal{O}_{n_1,n_2,n_3,n_4}\rangle=0$$

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δ^a :

SU(2)A tr.

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non-singlet, parity odd operator

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

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a=1,2,3: SU(2) triplet

(sum is not taken on a)

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$$P^{(a/0)} = \int d^4x\bar{q}\gamma_5\tau^{(a/0)}q(x)$$

S: scalar

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Order parameters of SU(2)L×SU(2)R

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k: minimum number to make the VEV finite

non-singlet, parity odd operator

$$\mathcal{O}_{n_1,n_2,n_3,n_4} = (P^a)^{n_1}(S^a)^{n_2}(P^0)^{n_3}(S^0)^{n_4}$$

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constraint ↓

Restoration of U(1)$_A$ symmetry

We assume: SU(2)$_L$×SU(2)$_R$ is fully recovered above Tc

Order parameters of SU(2)$_L$×SU(2)$_R$

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constraint

- Eigenvalue density of Dirac operator

$$\rho^A(\lambda) = \lim_{V\to\infty}\frac{1}{V}\sum_n \delta\left(\lambda - \sqrt{\bar{\lambda}_n^A \lambda_n^A}\right) = \sum_{n=0}^{\infty} \rho_n^A \frac{\lambda^n}{n!}$$

Restoration of U(1)A symmetry

We assume: SU(2)L×SU(2)R is fully recovered above Tc

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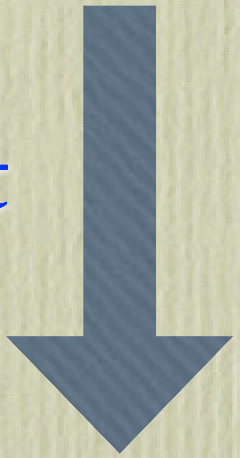

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$$\rho_0^A,\ \rho_1^A,\ \rho_2^A \to 0$$

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- Number of zero mode N$_{R+L}$(A), topological charge Q(A)

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- Number of zero mode NR+L(A), topological charge Q(A)

→ 0

Restoration of U(1)A symmetry

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Order parameters of SU(2)L×SU(2)R

$$\frac{1}{V^k}\langle \delta^a \mathcal{O}_{n_1,n_2,n_3,n_4} \rangle = 0$$ as an input

constraint $\Downarrow$

- Eigenvalue density of Dirac operator

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- Number of zero mode $N_{R+L}(A)$, topological charge $Q(A)$

$\to 0$ $\qquad$ $\to 0$

Restoration of U(1)$_A$ symmetry

We assume: SU(2)$_L$×SU(2)$_R$ is fully recovered above Tc

Order parameters of SU(2)$_L$×SU(2)$_R$

$$\frac{1}{V^k}\langle \delta^a \mathcal{O}_{n_1,n_2,n_3,n_4}\rangle = 0$$

as an input

constraint

we have shown

U(1)$_A$ order parameter

$$\frac{1}{V^{k'}}\langle \delta^0 \mathcal{O}'_{n_1,n_2,n_3,n_4}\rangle = 0$$

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- Number of zero mode N$_{R+L}$(A), topological charge Q(A)

$\rightarrow 0$ $\rightarrow 0$

Restoration of SU(2)A symmetry

Restoration of SU(2)$_A$ symmetry

Order parameters of SU(2)$_L$×SU(2)$_R$

Restoration of SU(2)A symmetry

Order parameters of SU(2)L×SU(2)R

← decomposition by the Dirac eigen-mode

Restoration of $SU(2)_A$ symmetry

Order parameters of $SU(2)_L \times SU(2)_R$

← decomposition by the Dirac eigen-mode

Overlap lattice fermion (Nf=2) [Neuberger 1998]

Restoration of $SU(2)_A$ symmetry

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Overlap lattice fermion (Nf=2) [Neuberger 1998]

$$S = \bar{\psi}\left[D + mF(D)\right]\psi, \quad F(D) = 1 - \frac{aR}{2}D$$

Restoration of SU(2)A symmetry

Order parameters of SU(2)L×SU(2)R

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Ginsparg-Wilson relation [1982]

$$D\gamma_5 + \gamma_5 D = aRD\gamma_5 D$$

Restoration of SU(2)A symmetry

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Exact chiral symmetry on lattice:

Restoration of SU(2)A symmetry

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$$\psi \to e^{i\gamma_5\alpha(1-aD)}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\gamma_5\alpha}$$

Restoration of SU(2)A symmetry

Order parameters of SU(2)L×SU(2)R

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Exact chiral symmetry on lattice: well defined zero modes

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Restoration of SU(2)$_A$ symmetry

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Dirac eigen-modes appear by pair

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Exact chiral symmetry on lattice: well defined zero modes

Dirac eigen-modes appear by pair

$$D\phi_n = \lambda_n \phi_n, \quad D\gamma_5\phi_n = \lambda_n^\dagger \gamma_5 \phi_n$$

Restoration of SU(2)A symmetry

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Restoration of $SU(2)_A$ symmetry

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Exact chiral symmetry on lattice: well defined zero modes

Dirac eigen-modes appear by pair $\gamma_5 D \gamma_5 = D^\dagger$

$$D\phi_n = \lambda_n \phi_n, \quad D\gamma_5\phi_n = \lambda_n^\dagger \gamma_5 \phi_n$$

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Ginsparg-Wilson relation [1982]

$$D\gamma_5 + \gamma_5 D = aRD\gamma_5 D \longrightarrow DD^\dagger = D^\dagger D$$

Exact chiral symmetry on lattice: well defined zero modes

Dirac eigen-modes appear by pair $\quad \gamma_5 D\gamma_5 = D^\dagger$

$$D\phi_n = \lambda_n\phi_n, \quad D\gamma_5\phi_n = \lambda_n^\dagger\gamma_5\phi_n$$

Restoration of $SU(2)_A$ symmetry

Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

$$S(x,y) = \sum_n \left[\frac{\phi_n(x)\phi_n^\dagger(y)}{f_m\lambda_n + m} + \frac{\gamma_5\phi_n(x)\phi_n^\dagger(y)\gamma_5}{f_m\lambda_n^\dagger + m} \right] + \sum_{k=1}^{N_{R+L}} \frac{1}{m}\phi_k(x)\phi_k^\dagger(y) + \sum_{K=1}^{N_D} \frac{aR}{2}\phi_K(x)\phi_K^\dagger(y)$$

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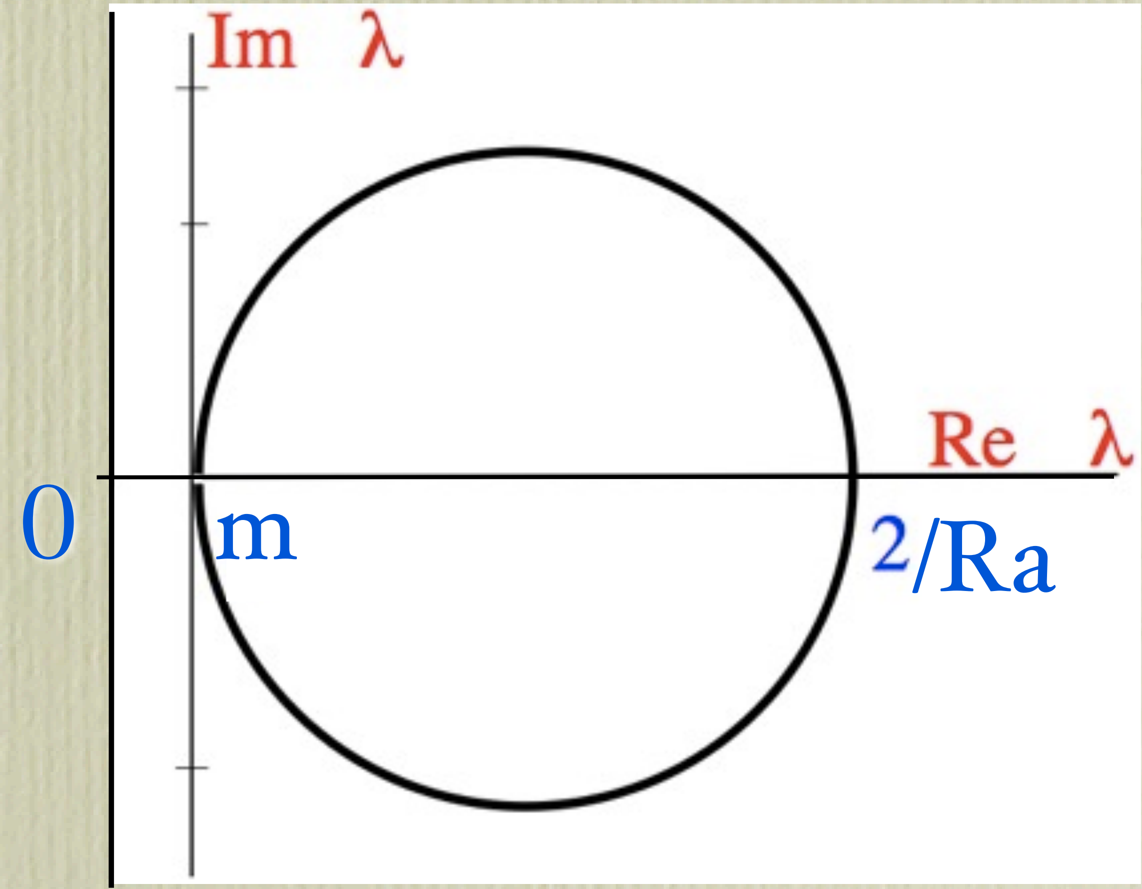

Restoration of SU(2)A symmetry

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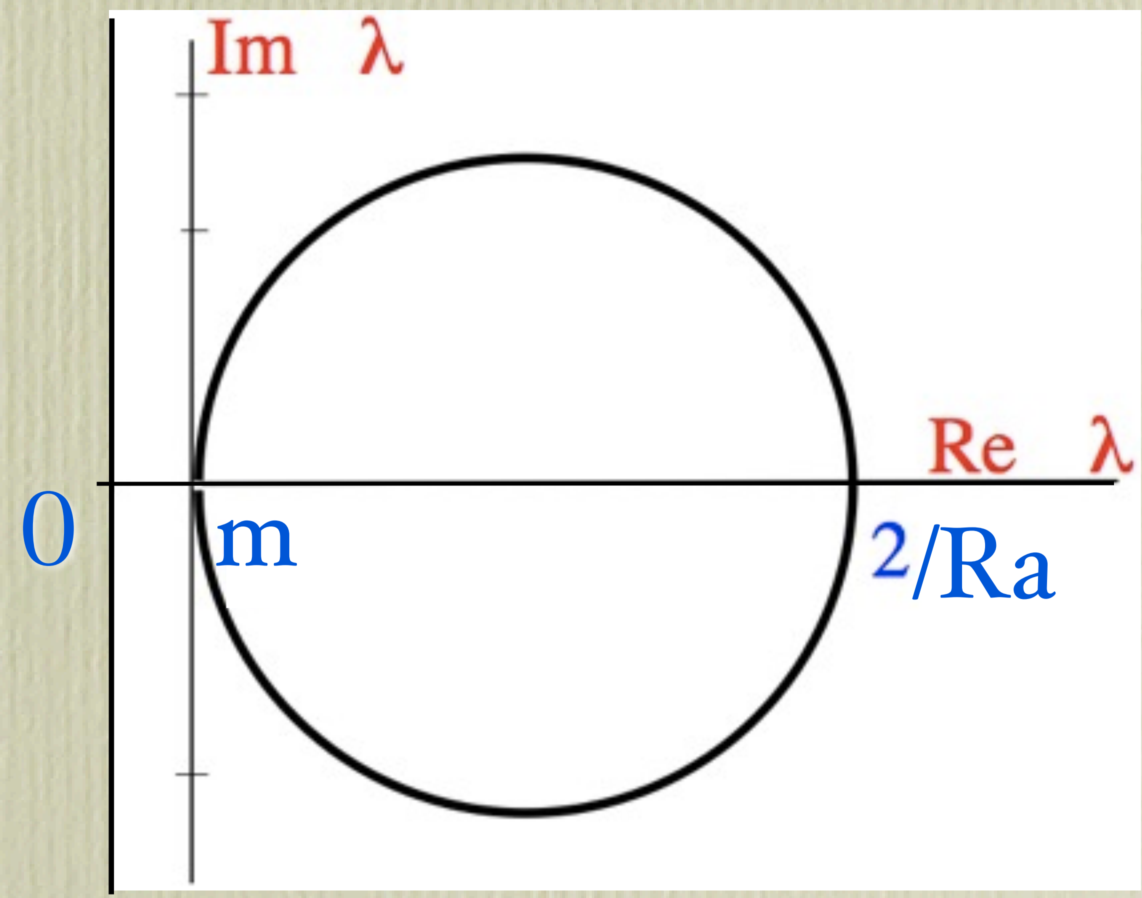
bulk modes



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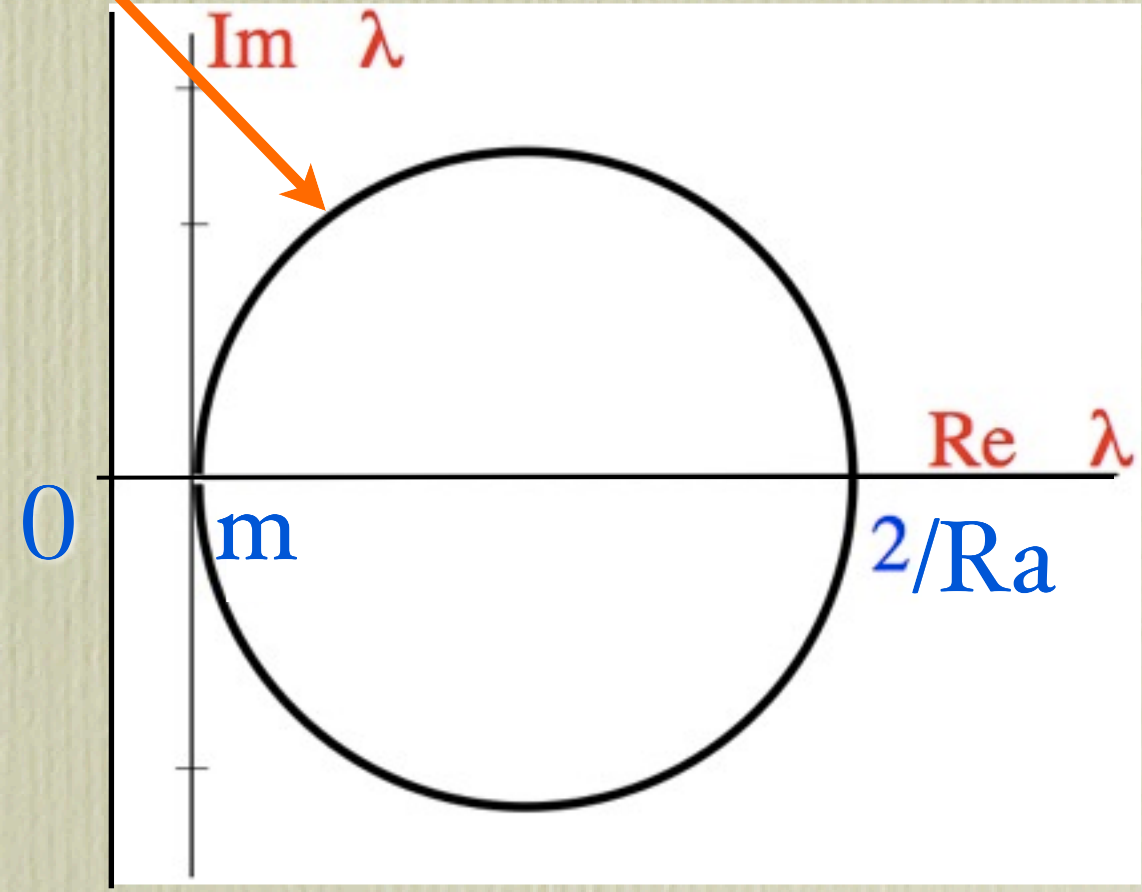

Restoration of SU(2)A symmetry

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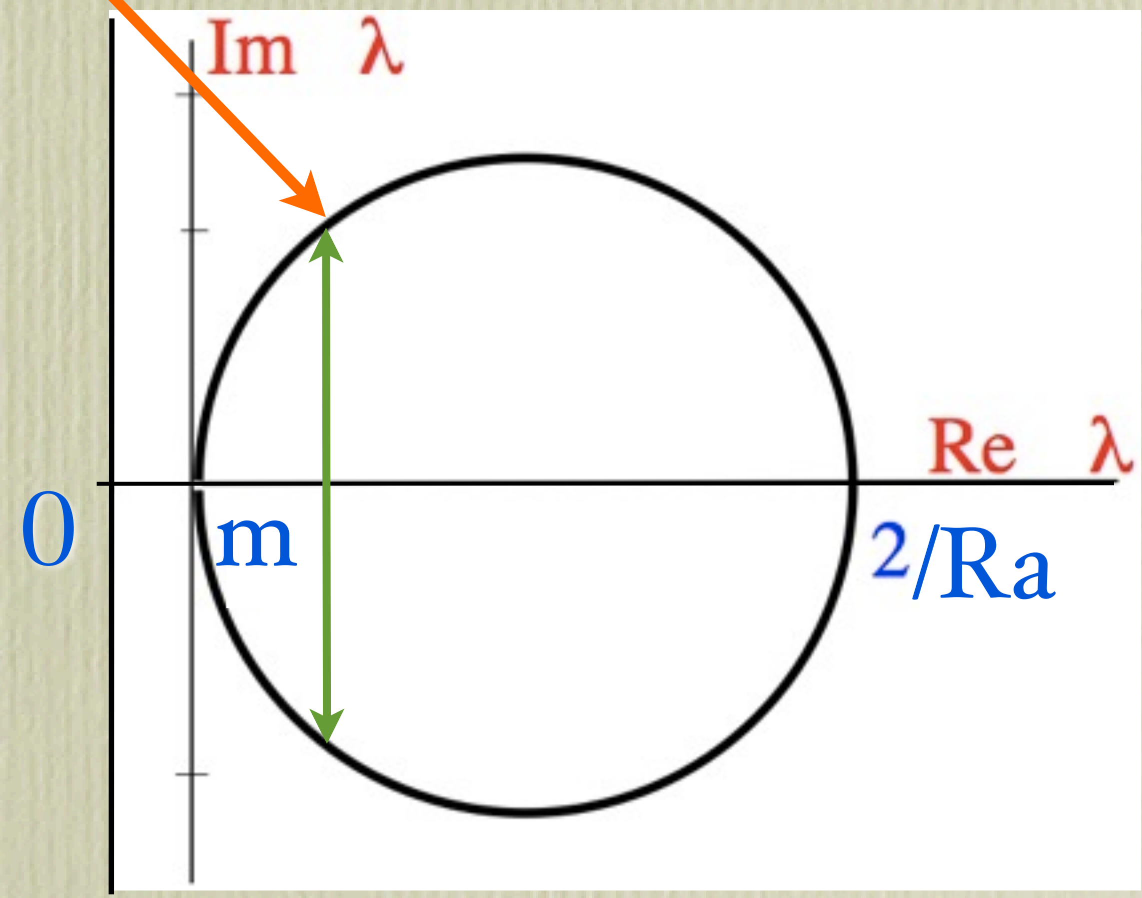

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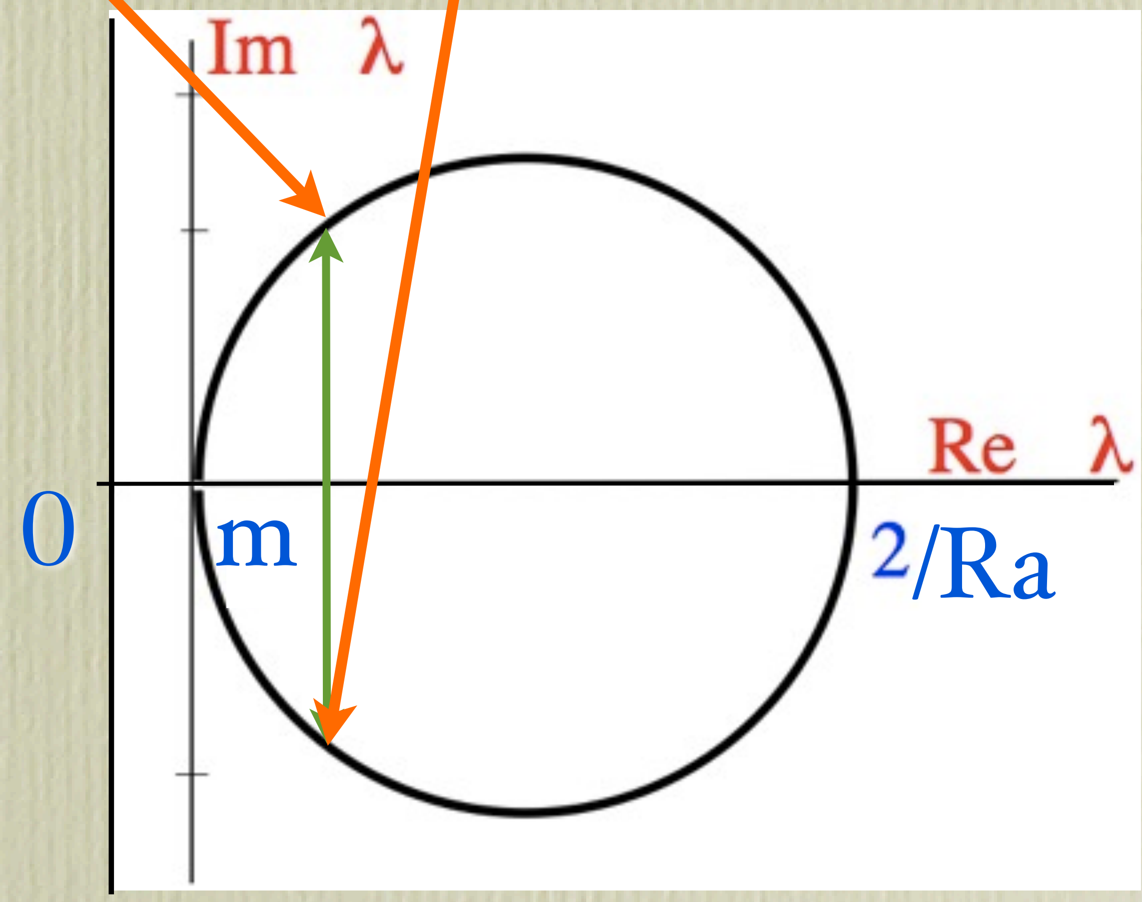

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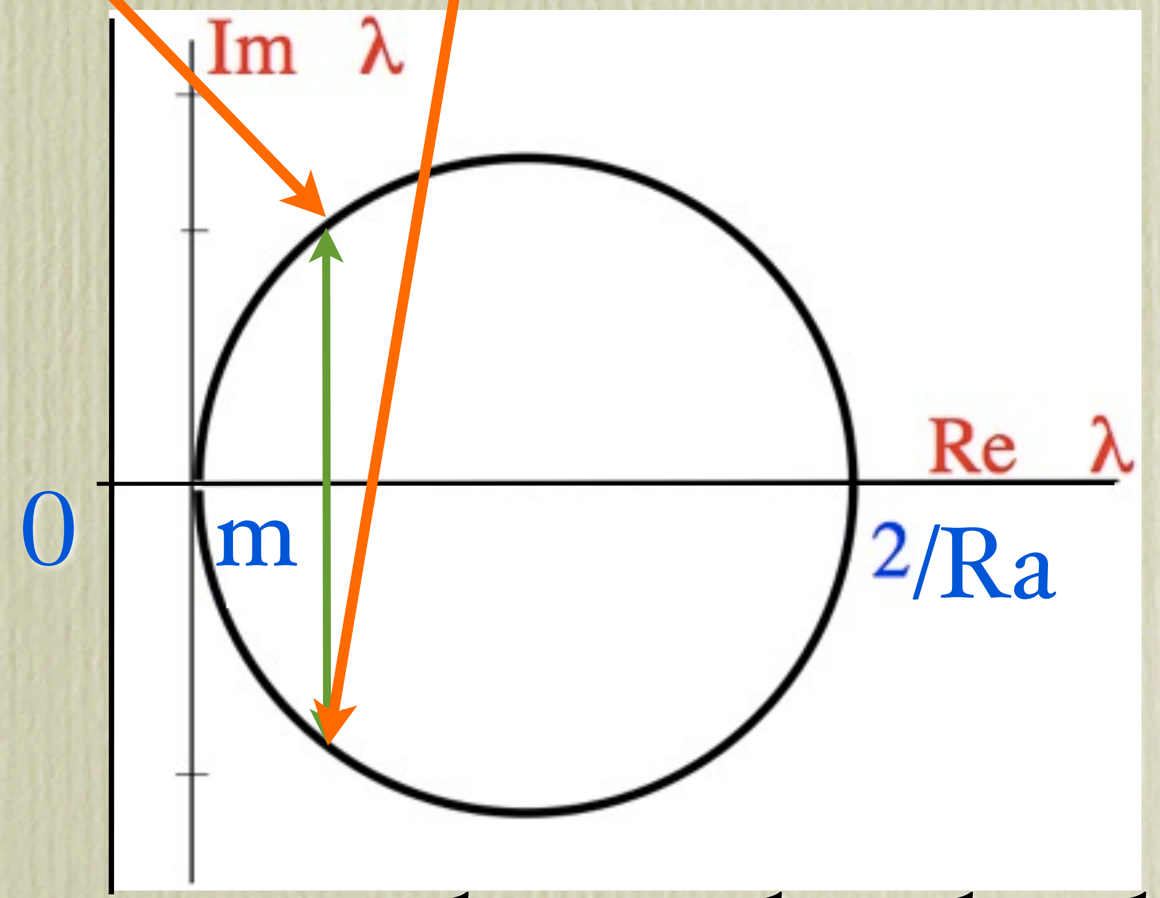

orthogonal each other: $\int \phi_n^\dagger \gamma_5 \phi_n = 0$

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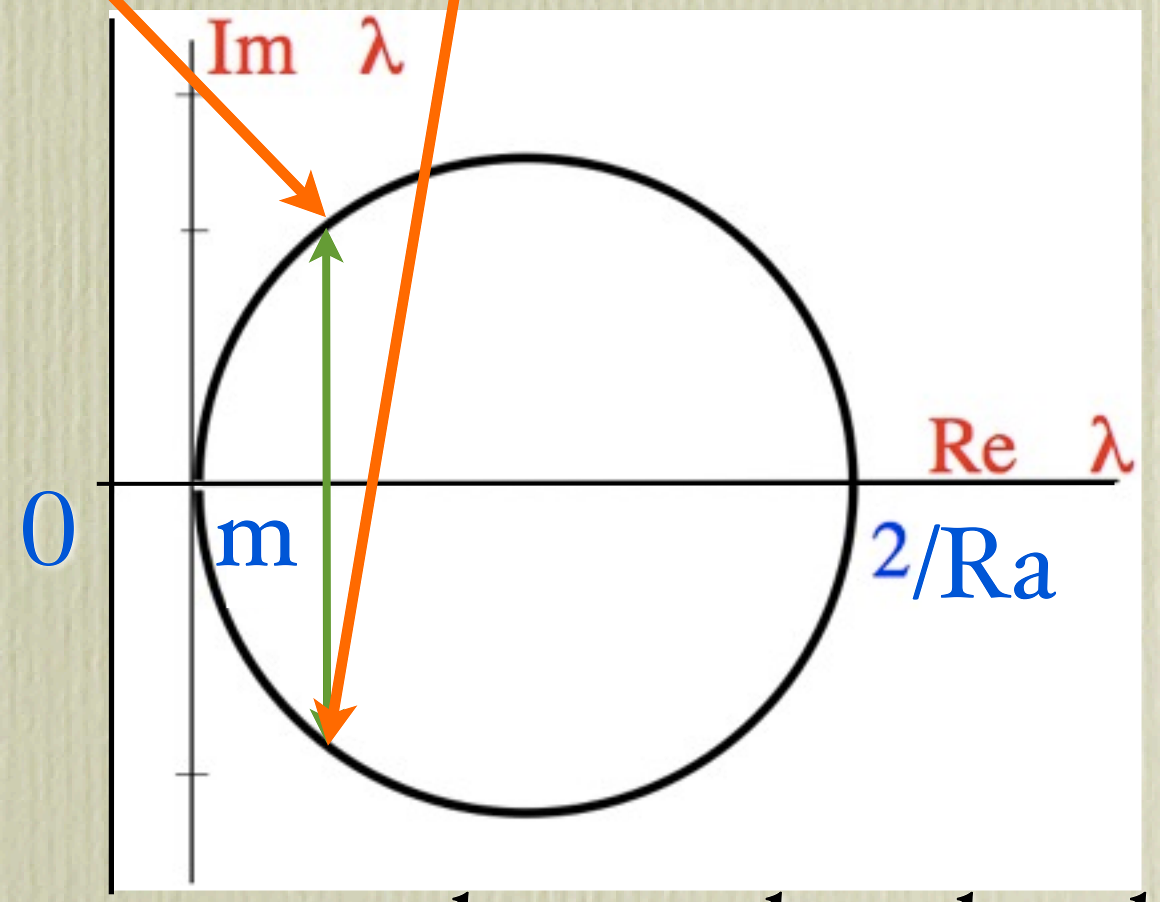

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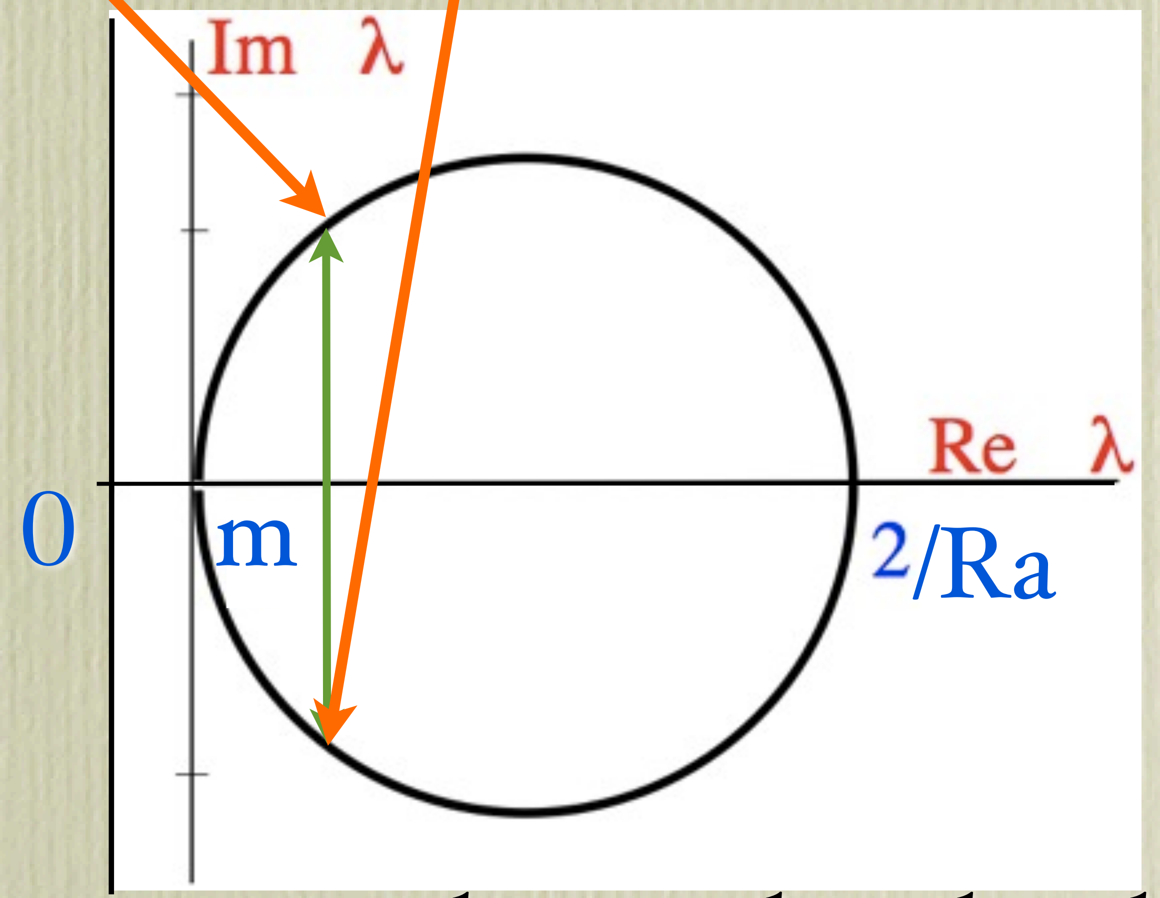
bulk modes

$$f_m = 1 - \frac{Rma}{2}$$



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Restoration of SU(2)$_A$ symmetry

Quark propagator in Dirac eigen-mode expansion

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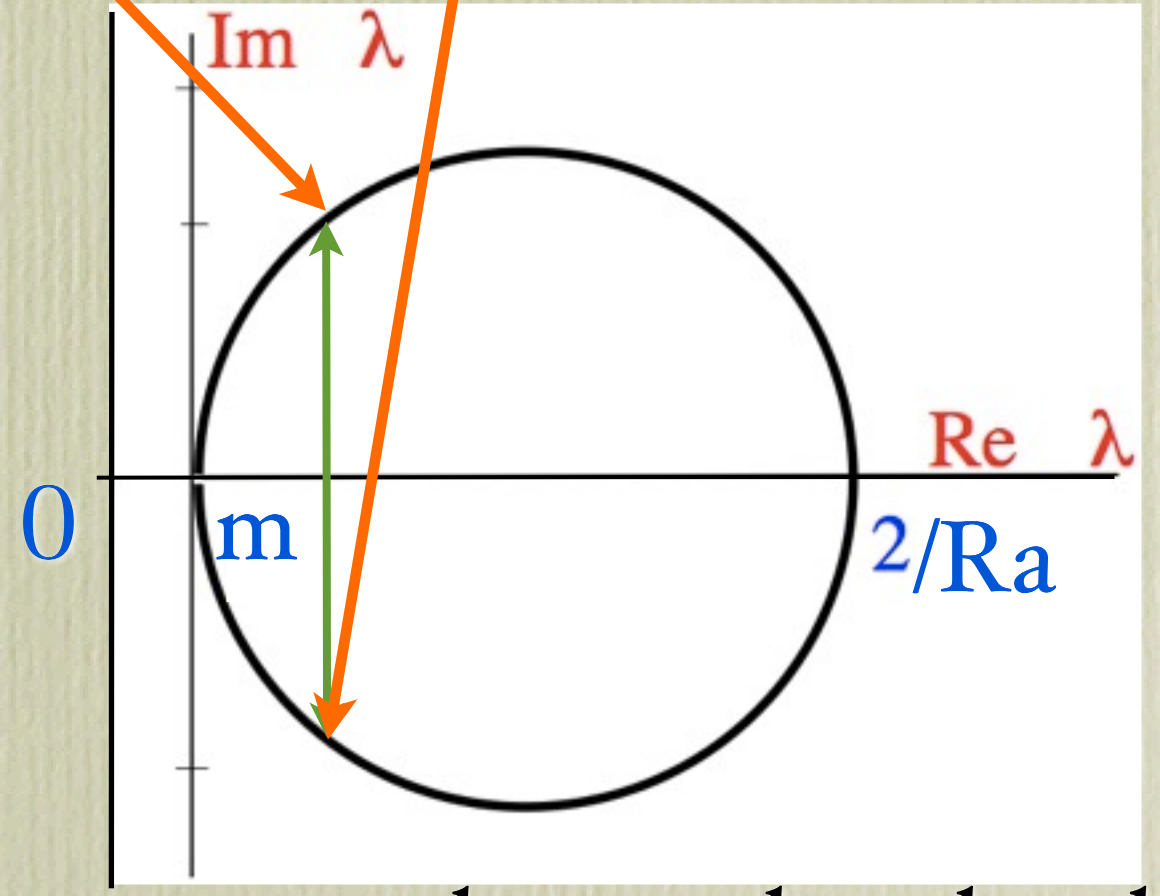
bulk modes

$$f_m = 1 - \frac{Rma}{2}$$

mass term:

$$m\left(1 - \frac{Ra}{2}D\right)$$



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Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

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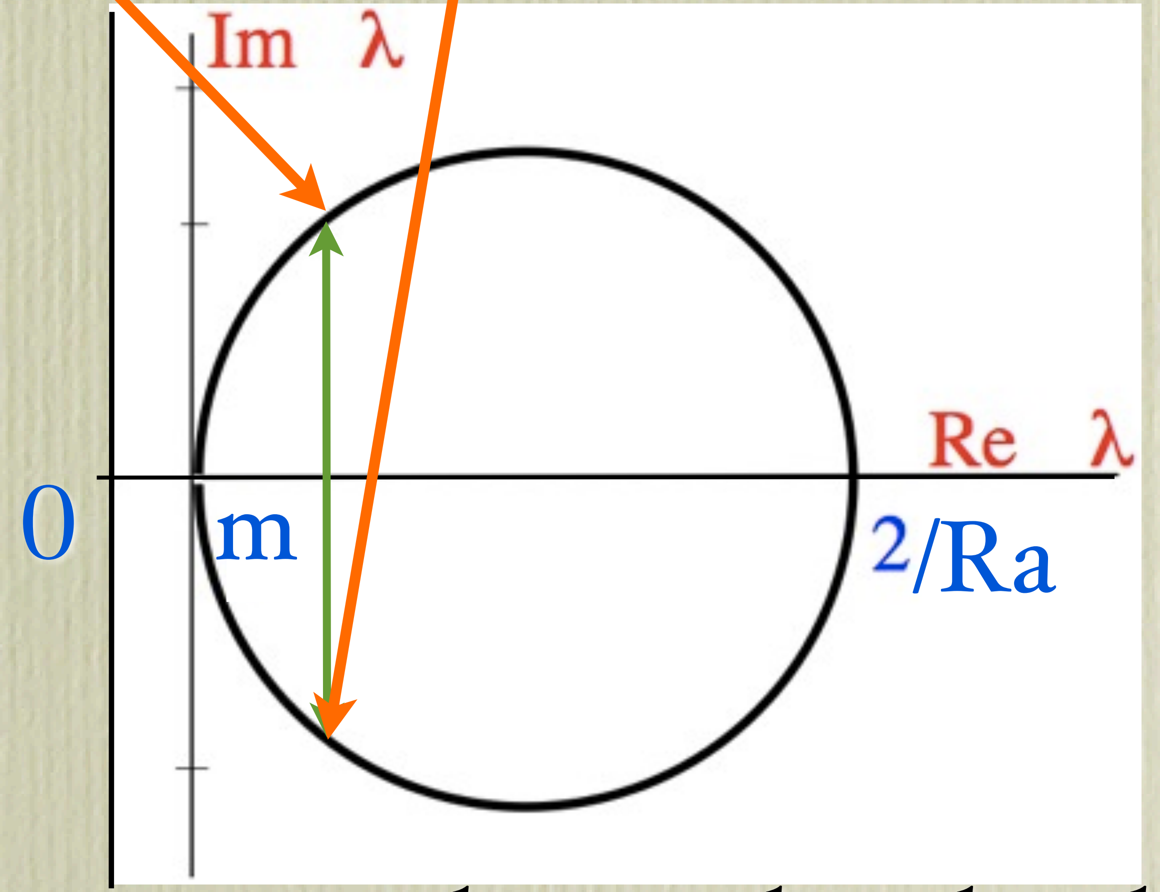
bulk modes

zero modes

$$f_m = 1 - \frac{Rma}{2}$$

mass term:

$$m\left(1-\frac{Ra}{2}D\right)$$



orthogonal each other: $\int \phi_n^\dagger \gamma_5 \phi_n = 0$

Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

$$S(x,y)=\sum_n\left[\frac{\phi_n(x)\phi_n^\dagger(y)}{f_m\lambda_n+m}+\frac{\gamma_5\phi_n(x)\phi_n^\dagger(y)\gamma_5}{f_m\lambda_n^\dagger+m}\right]+\sum_{k=1}^{N_{R+L}}\frac{1}{m}\phi_k(x)\phi_k^\dagger(y)+\sum_{K=1}^{N_D}\frac{aR}{2}\phi_K(x)\phi_K^\dagger(y)$$

bulk modes

zero modes

$$f_m=1-\frac{Rma}{2}$$

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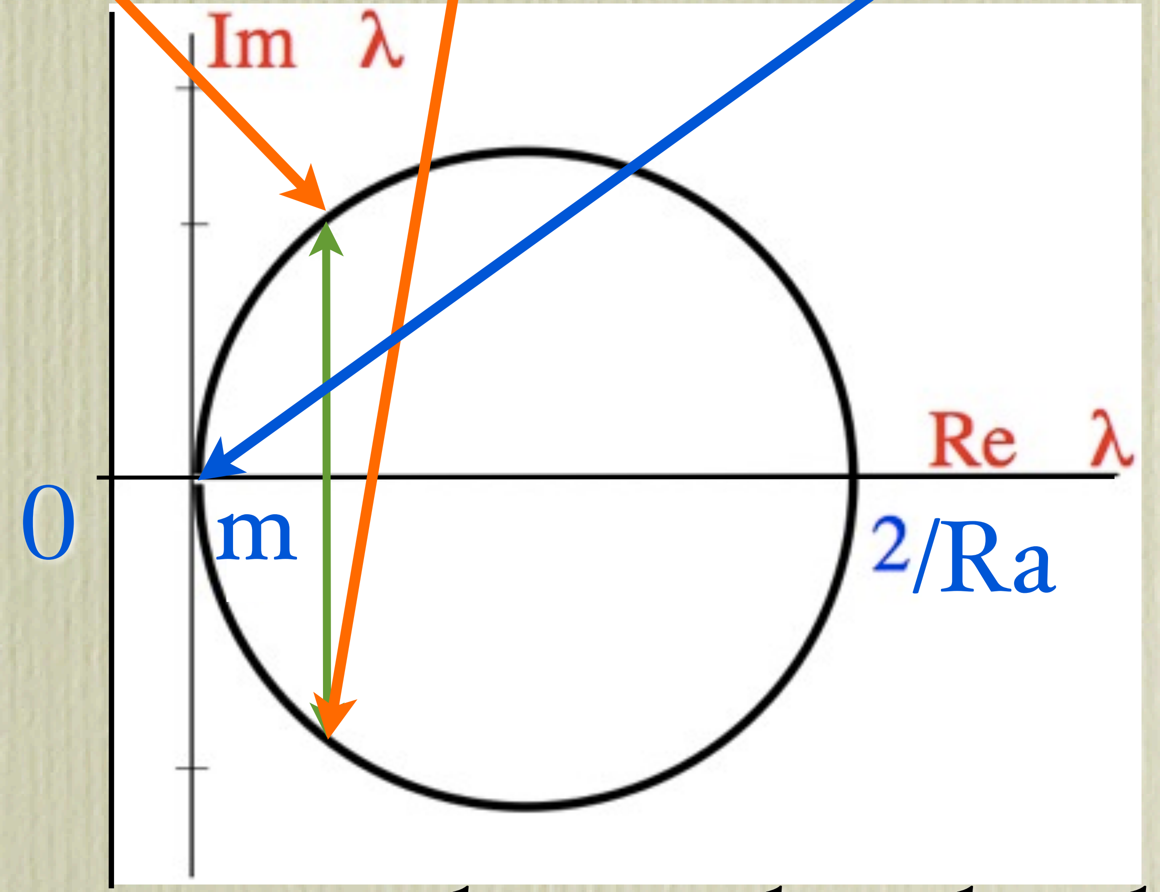

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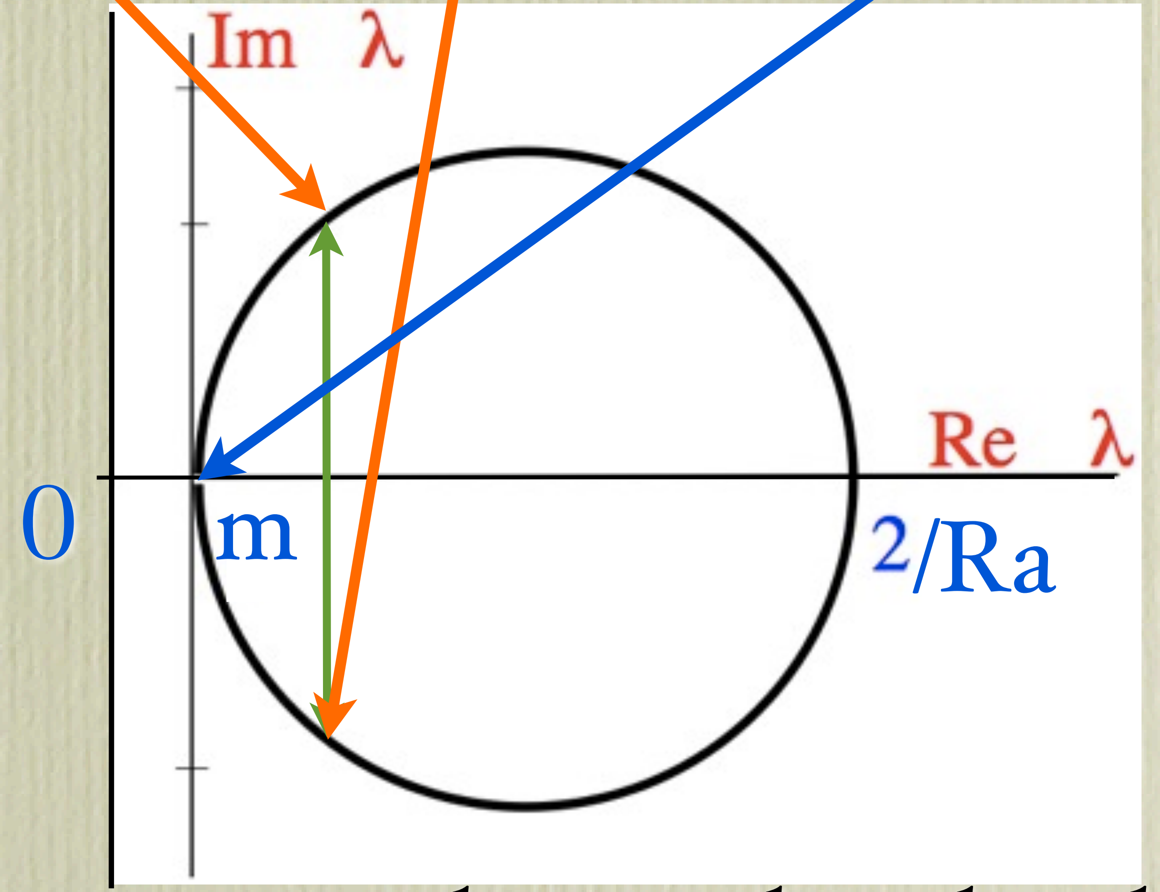

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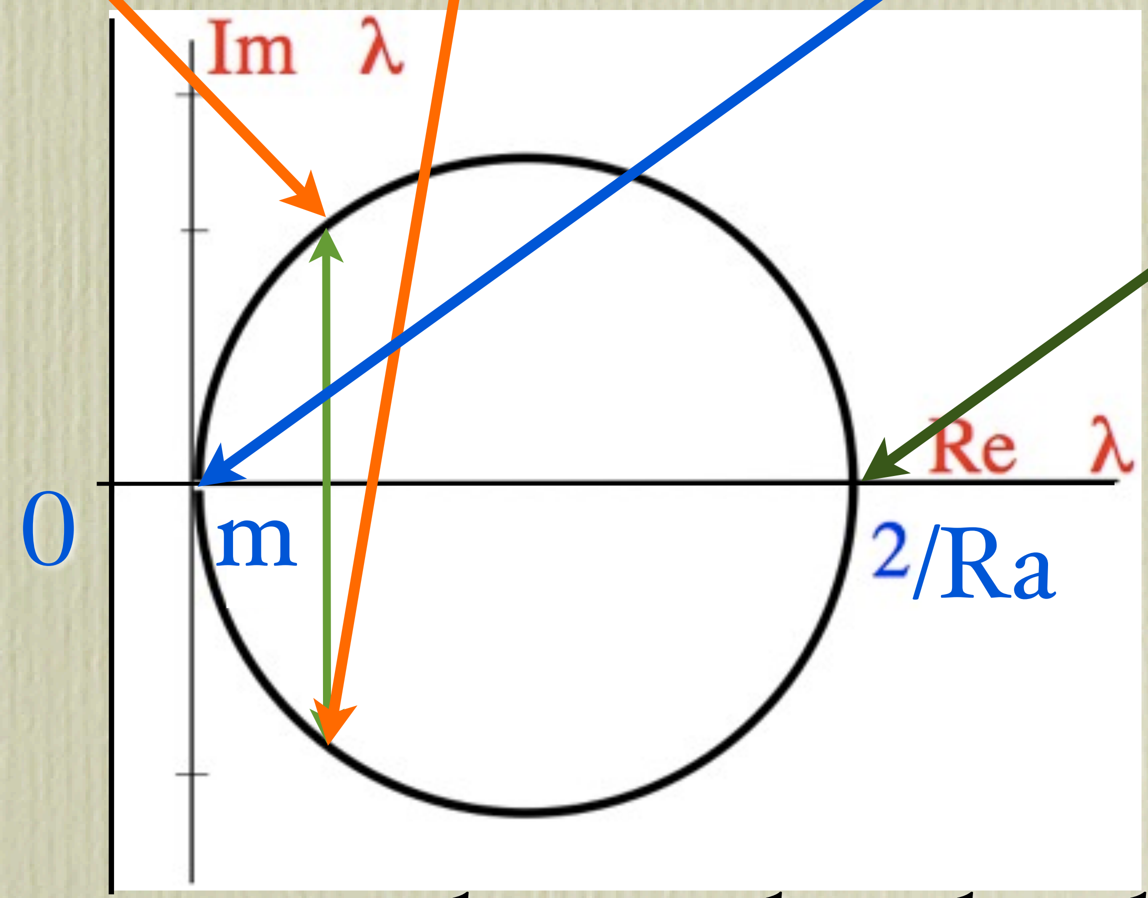

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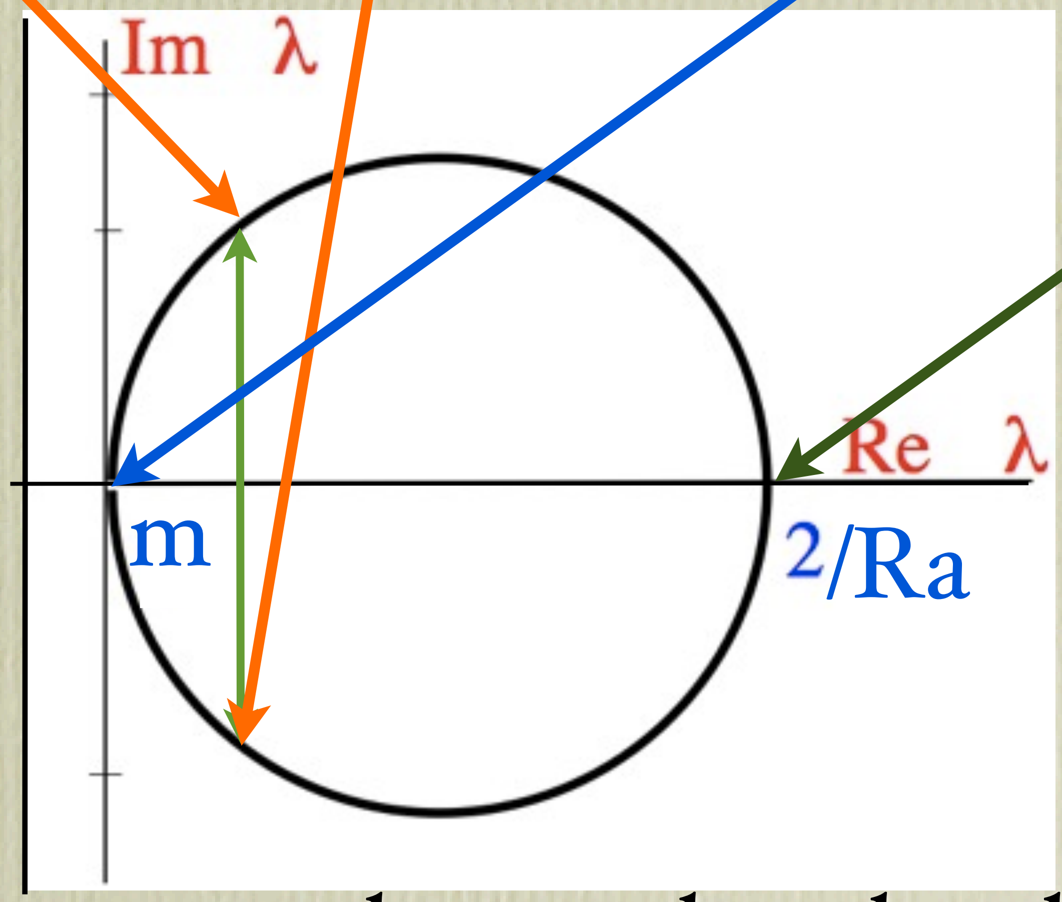

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Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

total number of zero modes

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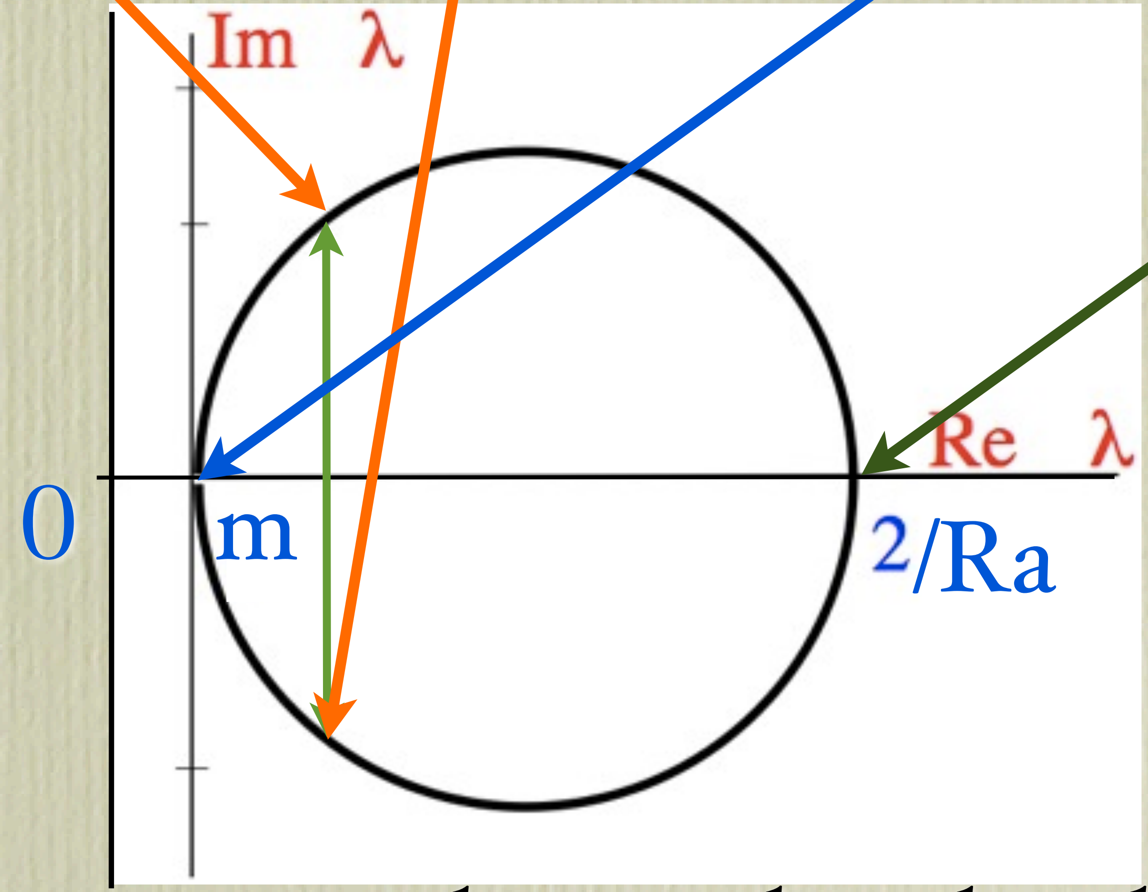

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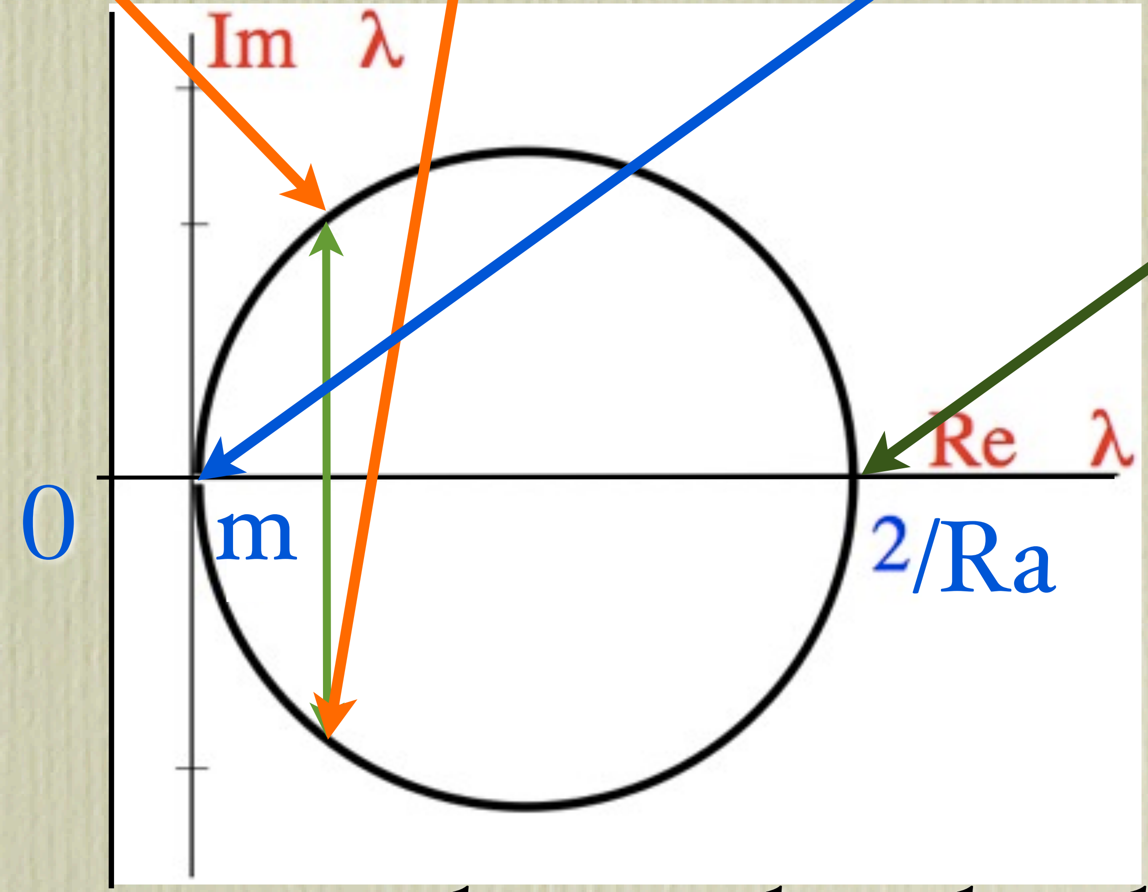

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Restoration of SU(2)$_A$ symmetry

Quark propagator in Dirac eigen-mode expansion

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bulk modes zero modes doublers

Eigenvalue density for a given configuration A

Restoration of SU(2)A symmetry

Quark propagator in Dirac eigen-mode expansion

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bulk modes zero modes doublers

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$$\rho^A(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\lambda_n^{\dagger A}\lambda_n^A}\right)$$

Restoration of SU(2)A symmetry

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$$\rho^A(\lambda) = \lim_{V\to\infty} \frac{1}{V} \sum_n \delta\left(\lambda - \sqrt{\lambda_n^{\dagger A}\lambda_n^A}\right)$$

expected to be a smooth function in V→∞ limit

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle$$

←

$$-\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

Restoration of SU(2)$_A$ symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\underset{\downarrow}{\underline{\mathrm{Tr}S(x,x)}}\right\rangle \longleftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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$$\Big\downarrow$$

$$\sum_n\int \mathrm{tr}\left[\frac{\phi_n(x)\phi_n^\dagger(x)}{f_m\lambda_n+m}+\frac{\gamma_5\phi_n(x)\phi_n^\dagger(x)\gamma_5}{f_m\lambda_n^\dagger+m}\right]+\sum_{k=1}^{N_{R+L}}\frac{1}{m}\int\phi_k^\dagger(x)\phi_k(x)+\sum_{K=1}^{N_D}\frac{aR}{2}\int\phi_K^\dagger(x)\phi_K(x)$$

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$$= \lim_{V\to\infty} \frac{1}{V} \left\langle \frac{N_{R+L}^A}{m} + \sum_{\mathrm{Im}\lambda_n^A>0} \frac{2m}{\lambda_n^{\dagger A}\lambda_n^A + m^2} \right\rangle + \mathcal{O}(a)$$

Restoration of SU(2)A symmetry

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$$-\langle \bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\underline{\mathrm{Tr}S(x,x)}\right\rangle \longleftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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gauge dependent

$$=\lim_{V\to\infty}\frac{1}{V}\left\langle\frac{N^A_{R+L}}{m}+\sum_{\mathrm{Im}\lambda_n^A>0}\frac{2m}{\lambda_n^{\dagger A}\lambda_n^A+m^2}\right\rangle+\mathcal{O}(a)$$

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numerator=1

$$\underline{\sum_n \int \mathrm{tr}\left[\frac{\phi_n(x)\phi_n^\dagger(x)}{f_m\lambda_n+m}+\frac{\gamma_5\phi_n(x)\phi_n^\dagger(x)\gamma_5}{f_m\lambda_n^\dagger+m}\right]} + \sum_{k=1}^{N_{R+L}}\frac{1}{m}\underbrace{\int \phi_k^\dagger(x)\phi_k(x)}_{1} + \sum_{K=1}^{N_D}\frac{aR}{2}\int \phi_K^\dagger(x)\phi_K(x)$$

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Restoration of SU(2)$_A$ symmetry

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numerator=1

0

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eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

$$= \lim_{V\to\infty} \frac{1}{V} \left\langle \frac{N^A_{R+L}}{m} + \sum_{\mathrm{Im}\lambda^A_n>0} \frac{2m}{\lambda^{\dagger A}_n \lambda^A_n + m^2} \right\rangle + \mathcal{O}(a)$$

Restoration of SU(2)$_A$ symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

$$= \lim_{V\to\infty}\frac{1}{V}\left\langle \frac{N^A_{R+L}}{m} + \sum_{\mathrm{Im}\lambda^A_n>0}\frac{2m}{\lambda^{\dagger A}_n\lambda^A_n+m^2}\right\rangle + \mathcal{O}(a)$$

$$= \lim_{V\to\infty}\left[\frac{1}{mV}\langle N^A_{R+L}\rangle_m + \int_0^{2/Ra} d\lambda\,\langle\rho^A(\lambda)\rangle_m\,\frac{2m}{\lambda^2+m^2}\right]$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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$$= \lim_{V\to\infty} \left[\frac{1}{mV} \langle N^A_{R+L} \rangle_m + \int_0^{2/Ra} d\lambda \, \langle \underline{\rho^A(\lambda)} \rangle_m \frac{2m}{\lambda^2 + m^2} \right]$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

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Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

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Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

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m independent operators

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \longleftarrow -\left\langle\bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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m independent operators

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

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m independent operators

due to Boltzmann factor

$\mathrm{Det}\,D(m)^2$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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In the m→0 limit, Banks-Casher relation [1980] appears

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle=\lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle\bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle=\lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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In the m→0 limit, Banks-Casher relation [1980] appears

$$\lim_{m\to0}\frac{2m}{\lambda^2+m^2}=\pi\delta(\lambda)$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar q q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar q\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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$$= \lim_{V\to\infty}\left[\frac{1}{mV}\langle N^A_{R+L}\rangle_m + \int_0^{\underline{2/Ra}} d\lambda\,\langle\rho^A(\lambda)\rangle_m\frac{2m}{\lambda^2+m^2}\right]$$

In the m→0 limit, Banks-Casher relation [1980] appears

$$\lim_{m\to 0}\frac{2m}{\lambda^2+m^2} = \pi\delta(\lambda)$$

$$\lim_{m\to 0}\langle -\bar q q\rangle = \pi\left\langle\rho^A(0)\right\rangle_m + \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{mV}\left\langle N^A_{R+L}\right\rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$$\lim_{m\to 0}\langle -\bar{q}q\rangle = \pi\left\langle\rho^A(0)\right\rangle_m + \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{mV}\left\langle N^A_{R+L}\right\rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$\uparrow$

$$\lim_{m\to 0} \langle -\bar{q}q \rangle = \pi \langle \rho^A(0) \rangle_m + \lim_{m\to 0} \lim_{V\to\infty} \frac{1}{mV} \langle N^A_{R+L} \rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle \longleftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$$\overset{\overset{0}{\uparrow}}{\lim_{m\to 0} \langle -\bar{q}q \rangle} = \pi \langle \rho^A(0) \rangle_m + \lim_{m\to 0} \lim_{V\to\infty} \frac{1}{mV} \langle N^A_{R+L} \rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$$\lim_{m\to0}\overset{\overset{0}{\uparrow}}{\langle-\bar{q}q\rangle} = \pi\left\langle\overset{\uparrow}{\rho^A}(0)\right\rangle_m + \lim_{m\to0}\lim_{V\to\infty}\frac{1}{mV}\left\langle N^A_{R+L}\right\rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle\bar{q}q\rangle = \lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \longleftarrow -\left\langle\bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle\frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$$\lim_{m\to0}\langle-\bar{q}q\rangle = \pi\left\langle\rho^A(0)\right\rangle_m + \overset{\overset{0}{\uparrow}}{\lim_{m\to0}}\;\lim_{V\to\infty}\frac{1}{mV}\left\langle N^A_{R+L}\right\rangle_m$$

$$\overset{\overset{\mathcal{O}(m^2)}{\uparrow}}{\pi\left\langle\rho^A(0)\right\rangle_m}$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q \rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \right\rangle \leftarrow -\left\langle \bar{q} \left(1 - \frac{Ra}{2} D\right) q \right\rangle = \lim_{V\to\infty} \left\langle \frac{1}{V} \mathrm{Tr} S(x,x) \left(1 - \frac{Ra}{2} D\right) \right\rangle$$

$$= \lim_{V\to\infty} \frac{1}{V} \left\langle \frac{N^A_{R+L}}{m} + \sum_{\mathrm{Im}\lambda^A_n > 0} \frac{2m}{\lambda^{\dagger A}_n \lambda^A_n + m^2} \right\rangle + \mathcal{O}(a)$$

$$= \lim_{V\to\infty} \left[\frac{1}{mV} \langle N^A_{R+L} \rangle_m + \int_0^{\underline{2/Ra}} d\lambda \, \langle \rho^A(\lambda) \rangle_m \frac{2m}{\lambda^2 + m^2} \right]$$

Restoration of SU(2)A

$$\lim_{m\to 0} \overset{\overset{0}{\uparrow}}{\langle -\bar{q}q \rangle} = \pi \overset{\overset{\mathcal{O}(m^2)}{\uparrow}}{\langle \rho^A(0) \rangle_m} + \lim_{m\to 0} \lim_{V\to\infty} \frac{1}{mV} \langle \overset{\uparrow}{N^A_{R+L}} \rangle_m$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: scalar density

$$-\langle \bar{q}q\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\right\rangle \leftarrow -\left\langle \bar{q}\left(1-\frac{Ra}{2}D\right)q\right\rangle = \lim_{V\to\infty}\left\langle \frac{1}{V}\mathrm{Tr}S(x,x)\left(1-\frac{Ra}{2}D\right)\right\rangle$$

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Restoration of SU(2)A

$$\overset{\overset{0}{\uparrow}}{\lim_{m\to 0}\langle -\bar{q}q\rangle} = \pi\,\overset{\overset{\mathcal{O}(m^2)}{\uparrow}}{\langle\rho^A(0)\rangle_m} + \lim_{m\to 0}\lim_{V\to\infty}\frac{1}{mV}\overset{\overset{\mathcal{O}(m^2)}{\uparrow}}{\langle N^A_{R+L}\rangle_m}$$

Restoration of SU(2)$_A$ symmetry

eigenvalue decomposition: U(1)$_A$ WT identity

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\langle \partial_\mu A_\mu(x)\eta(y)\rangle = 2m_q \langle \eta(x)\eta(y)\rangle + 2\delta(x-y)\langle \sigma(x)\rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y)\right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\langle \partial_\mu A_\mu(x) \eta(y) \rangle = 2m_q \langle \eta(x)\eta(y) \rangle + 2\delta(x-y) \langle \sigma(x) \rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y) \right\rangle$$

↓

$$\bar{q} i \gamma_5 q$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\langle \partial_\mu A_\mu(x) \underset{\substack{\big\downarrow \\ \bar{q} i \gamma_5 q}}{\eta(y)} \rangle = 2m_q \langle \eta(x)\eta(y) \rangle + 2\delta(x-y) \langle \underset{\substack{\big\downarrow \\ \bar{q} q}}{\sigma(x)} \rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y) \right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\langle \partial_\mu A_\mu(x)\eta(y)\rangle = 2m_q \langle \eta(x)\eta(y)\rangle + 2\delta(x-y)\langle \sigma(x)\rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y)\right\rangle$$

$\bar{q}i\gamma_5 q$ $\bar{q}q$ anomaly

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\int_{\mathrm{PBC}} \langle \partial_\mu A_\mu(x)\eta(y)\rangle = 2m_q \langle \eta(x)\eta(y)\rangle + 2\delta(x-y)\langle\sigma(x)\rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y)\right\rangle$$

$\downarrow$ $\bar{q}i\gamma_5 q$ $\downarrow$ $\bar{q}q$ $\uparrow$ anomaly

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\int_{\mathrm{PBC}} \underset{\substack{\downarrow \\ 0}}{\langle \partial_\mu A_\mu(x) \eta(y) \rangle} = 2m_q \left\langle \eta(x) \underset{\substack{\downarrow \\ \bar{q} i \gamma_5 q}}{\eta(y)} \right\rangle + 2\delta(x-y) \left\langle \underset{\substack{\downarrow \\ \bar{q} q}}{\sigma(x)} \right\rangle - 2iCg^2 \left\langle \underset{\substack{\uparrow \\ \text{anomaly}}}{F\tilde{F}}(x) \eta(y) \right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\underset{\substack{\downarrow\\ 0}}{\int_{\text{PBC}}} \langle \partial_\mu A_\mu(x)\underset{\substack{\downarrow\\ \bar{q} i\gamma_5 q}}{\eta}(y)\rangle = 2m_q \underline{\langle \eta(x)\eta(y)\rangle} + 2\delta(x-y)\langle \underset{\substack{\downarrow\\ \bar{q}q}}{\sigma}(x)\rangle - 2iCg^2 \left\langle F\underset{\substack{\uparrow\\ \text{anomaly}}}{\tilde{F}}(x)\eta(y)\right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\int_{\mathrm{PBC}} \langle \partial_\mu A_\mu(x)\eta(y)\rangle = 2m_q \underline{\langle \eta(x)\eta(y)\rangle} + 2\delta(x-y)\langle\sigma(x)\rangle - 2iCg^2\left\langle F\tilde{F}(x)\eta(y)\right\rangle$$

$\bar{q}i\gamma_5 q$

massive

$\bar{q}q$

anomaly

0

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\underset{\substack{\downarrow\\ 0}}{\int_{\mathrm{PBC}}} \langle \partial_\mu \underset{\substack{\downarrow\\ \bar{q}i\gamma_5 q}}{A_\mu}(x)\eta(y)\rangle = 2\underset{\substack{\downarrow\\ 0}}{m_q}\underbrace{\langle \eta(x)\eta(y)\rangle}_{\text{massive}} + 2\delta(x-y)\langle \underset{\substack{\downarrow\\ \bar{q}q}}{\sigma}(x)\rangle - 2iCg^2\left\langle F\underset{\substack{\uparrow\\ \text{anomaly}}}{\tilde{F}}(x)\eta(y)\right\rangle$$

Restoration of SU(2)A symmetry

eigenvalue decomposition: U(1)A WT identity

$$\int_{\mathrm{PBC}} \langle \partial_\mu A_\mu(x)\eta(y)\rangle = 2m_q \langle \eta(x)\eta(y)\rangle + 2\delta(x-y)\langle \sigma(x)\rangle - 2iCg^2 \left\langle F\tilde{F}(x)\eta(y)\right\rangle$$

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$$\|$$

$$\left\langle \frac{Q^2}{mV}\right\rangle$$

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$$\begin{tikzcd}
\frac{1}{V}\int\langle\sigma(y)\rangle \arrow[d, phantom, "\|"] \arrow[dr] & = i\frac{1}{V}\int\int dxCg^2\left\langle F\tilde{F}(x)\eta(y)\right\rangle \\
\left\langle\frac{Q^2}{mV}\right\rangle \arrow[dr] & 0 \\
& \left\langle\frac{Q^2(A)}{V}\right\rangle = \mathcal{O}(m^2)
\end{tikzcd}$$

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$$||$$

$$\left\langle \frac{Q^2}{mV}\right\rangle$$

SU(2)A restoration

$$\rightarrow \left\langle \frac{Q^2(A)}{V}\right\rangle = \mathcal{O}(m^2)$$

Typical term in chiral susceptibility

Restoration of SU(2)$_A$ symmetry

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$$\Vert$$

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SU(2)A restoration

$$\left\langle \frac{Q^2}{mV}\right\rangle \searrow \left\langle \frac{Q^2(A)}{V}\right\rangle = \mathcal{O}(m^2)$$

we used assumptions here.

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Our Assumptions

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due to Boltzmann factor

Det D(m)/Z(m) : even function in m

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analytic at m=0 above Tc since there is no massless modes

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Restoration of SU(2)A symmetry

SU(2)A symmetry restoration at 2 point fn.

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$$\chi^{\pi-\sigma} = -4\left\langle\left(\frac{N^A_{R+L}}{mV} + \pi\rho^A_0\right)^2\right\rangle + \mathcal{O}(1/V) + \mathcal{O}(m)$$

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$$\longrightarrow \langle \rho_1^A \rangle = \frac{\langle Q(A)^2 \rangle}{m^2 V}$$

Restoration of SU(2)A symmetry

Restoration of SU(2)A symmetry

The same discussion continues (let me skip details)

Restoration of $SU(2)_A$ symmetry

The same discussion continues (let me skip details)

N=3 point function order parameter

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Larger N point function order parameter

Restoration of SU(2)$_A$ symmetry

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$$\rightarrow \quad \lim_{V\to\infty} \left\langle \frac{N_{R+L}^A}{V} \right\rangle = \mathcal{O}(m^N) = 0$$

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U(1)A anomaly is invisible for these set of operators

Plan of the talk

- ✓ 1. Introduction
- ✓ 2. Previous works
- ✓ 3. Restoration U(1)A symmetry
- 4. Effective theory
- 5. Renormalization group analysis
- 6. Conclusion

Restoration of U(1)A symmetry

As was discussed by Pisarski and Wilczek; PRD 29 (1984) 338

Restoration of U(1)A symmetry

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Restoration of U(1)A

Restoration of $U(1)_A$ symmetry

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Restoration of $U(1)_A$ ⟷

Restoration of U(1)$_A$ symmetry

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Restoration of U(1)$_A$ ↔ Order of phase transition

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Restoration of $U(1)_A$ ⟷ Order of phase transition

- For Nf=2 QCD
- Phase transition is first order if $U(1)_A$ is restored

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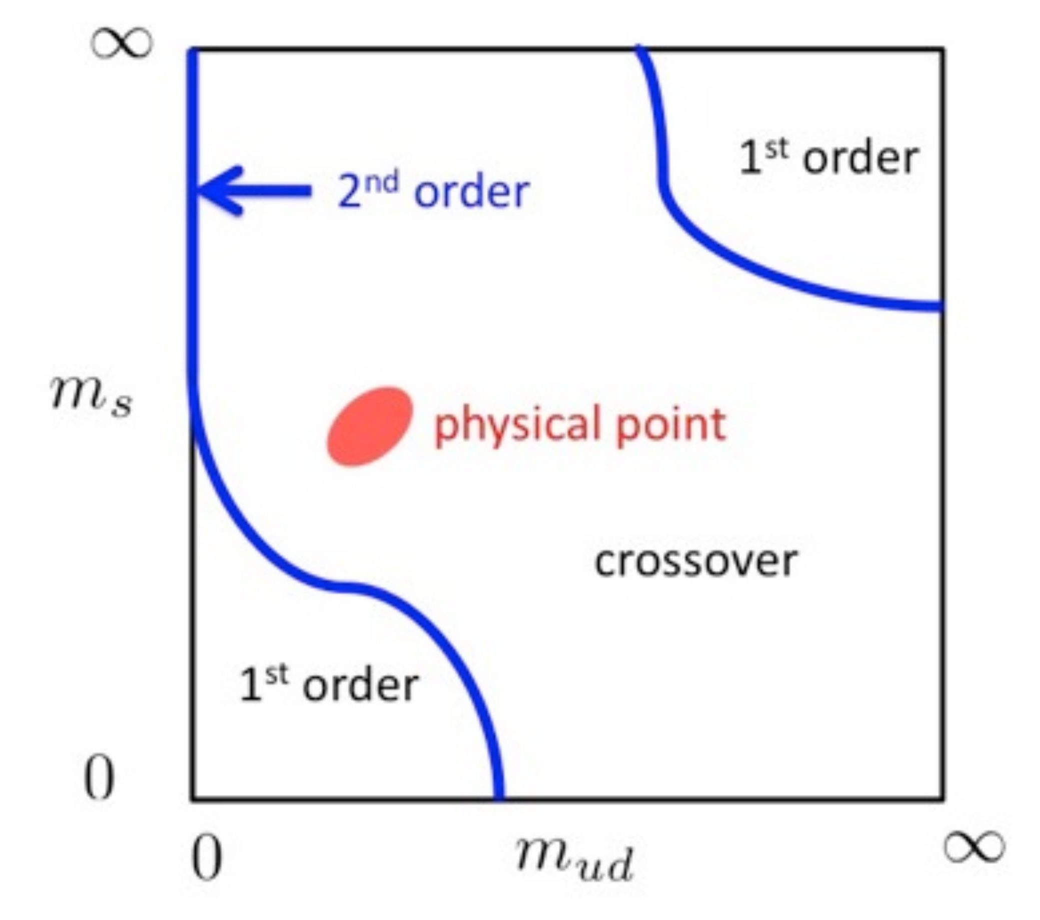

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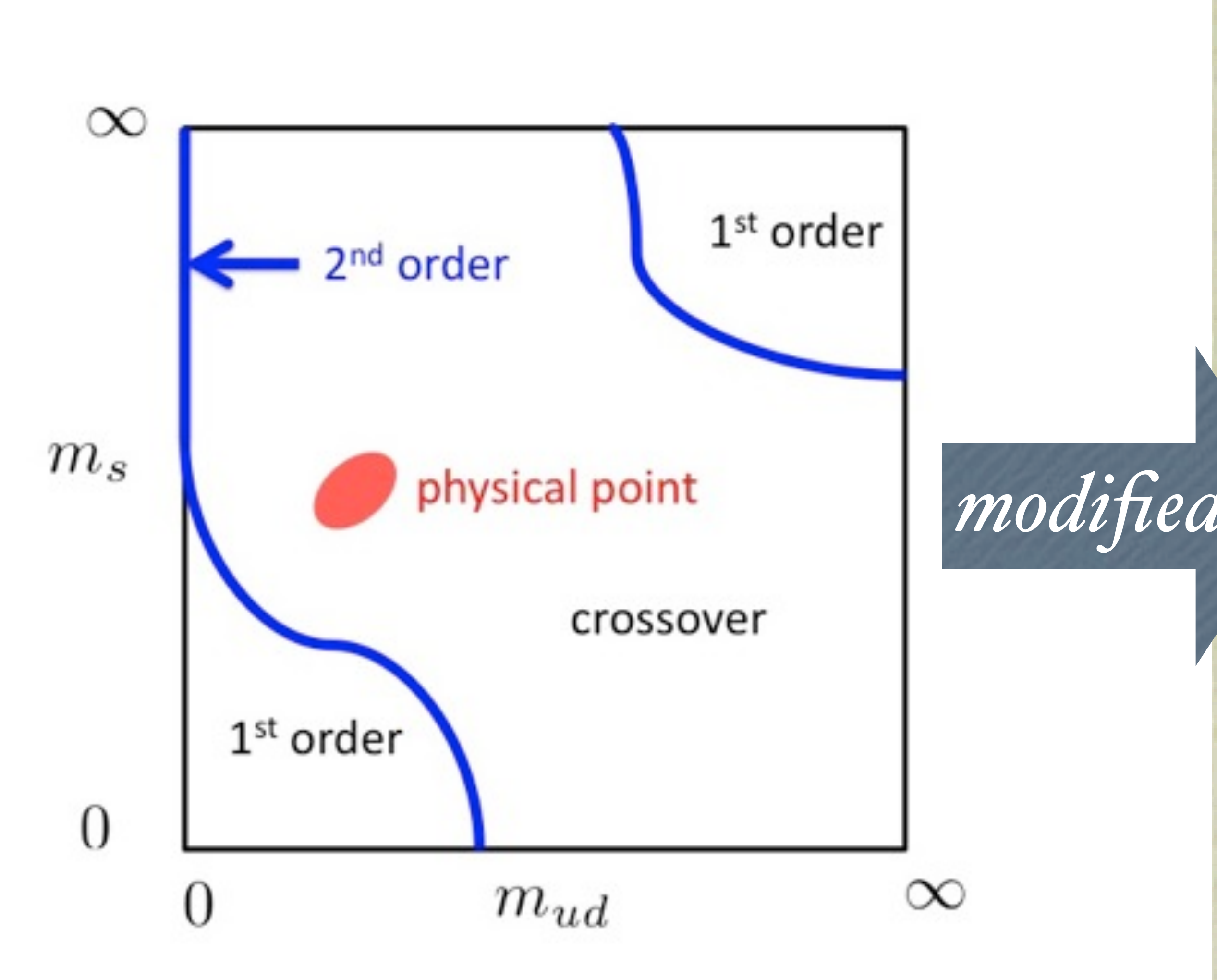

modified?

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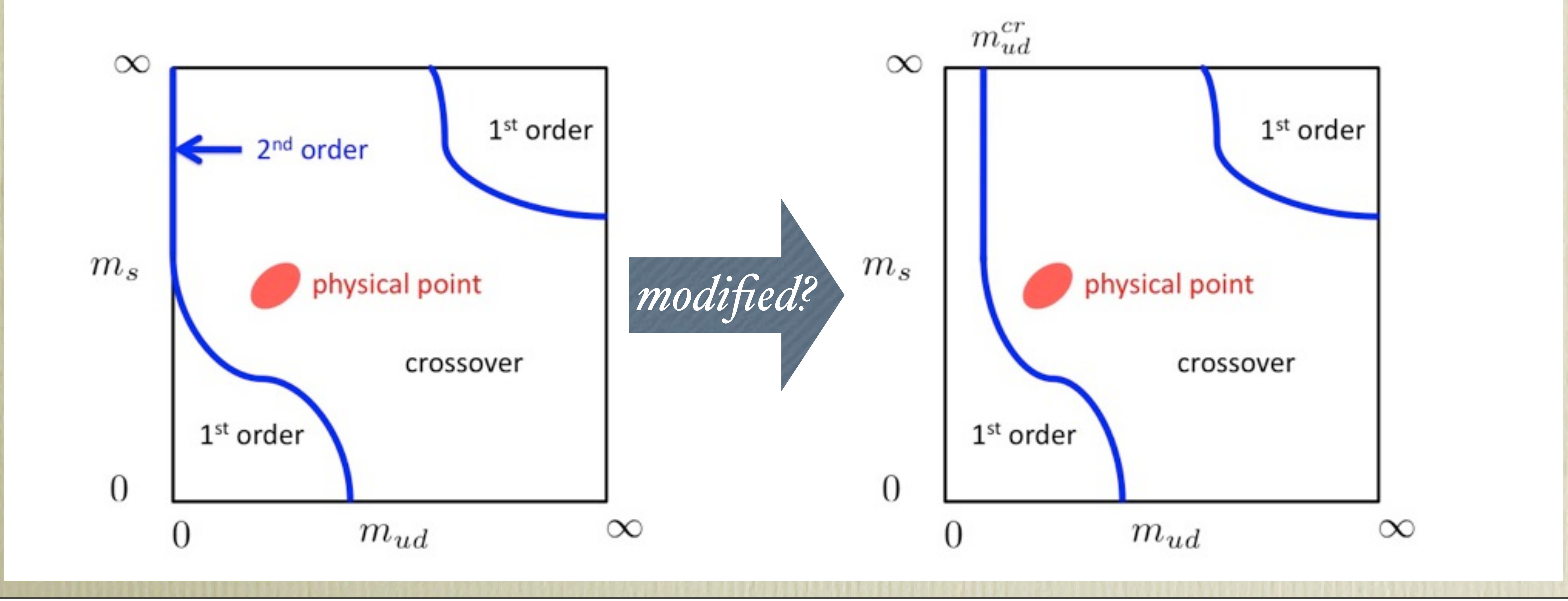

Restoration of $U(1)_A$ symmetry

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  - Use the low energy effective theory of QCD

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Effective theory of Nf=2 QCD

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$$\psi \to e^{i\frac{\alpha}{2}\gamma_5}\psi, \quad \bar{\psi} \to \bar{\psi}e^{i\frac{\alpha}{2}\gamma_5}$$

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$$\begin{aligned}\mathcal{L}_{\mathrm{U}(1)_A} = c' \left(\det \Phi + \det \Phi^\dagger\right) &+ \frac{x}{4}\left(\mathrm{tr}\Phi^\dagger \Phi\right)\left(\det \Phi + \det \Phi^\dagger\right) \\ &+ \frac{y}{4}\left(\left(\det \Phi\right)^2 + \left(\det \Phi^\dagger\right)^2\right)\end{aligned}$$

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- Symmetry of U(1)$_A$ breaking Lagrangian

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- U(1)A breaking parameters: c’, x, y

Constraint on c’, x, y

Constraint on c', x, y

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$$\chi^{\pi-\delta} = \frac{1}{2V}\langle \delta^0 \left(P^a S^a\right)\rangle = \frac{1}{V}\langle P^a P^a - S^a S^a \rangle$$

$$\chi^{\pi-\delta} = \langle \pi^a \pi^a (p=0)\rangle - \langle \delta^a \delta^a (p=0)\rangle$$

different masses

$$c' \left(\det \Phi + \det \Phi^\dagger\right) = \frac{c'}{2}\left(\pi^a \pi^a - \delta^a \delta^a + \sigma^2 - \eta^2\right)$$

$\chi^{\pi-\delta} = 0$ *constraint* $c' = 0$

Constraint on c'

- One loop correction

$$\langle \pi^a \pi^a (p=0) \rangle = \frac{1}{m_\Phi^2 + \delta m_\Phi^2 + c' + \delta c'}$$

$$\langle \delta^a \delta^a (p=0) \rangle = \frac{1}{m_\Phi^2 + \delta m_\Phi^2 - (c' + \delta c')}$$

$$\chi^{\pi-\xi} = 0 \xrightarrow{\text{constraint}} c'_{\mathrm{R}} = c' + \delta c' = 0$$

fine tuning of bare parameter c'

Constraint on c', x, y

- For general $n_1 + n_2 + n_3 + n_4 =$ even case

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$$\left(\frac{1}{V}\langle P^a P^a\rangle\right)^{k_1}\left(\frac{1}{V}\langle S^a S^a\rangle\right)^{k_2}\left(\frac{1}{V}\langle P^0 P^0\rangle\right)^{k_3}\left(\frac{1}{V}\langle P^a S^a P^0\rangle\right)^{k_4}$$

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Effective theory of Nf=2 QCD

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- $SU(2)_L \times SU(2)_R \times U(1)_A$ symmetric Lagrangian

Effective theory of Nf=2 QCD

- SU(2)L×SU(2)R×U(1)A symmetric Lagrangian

$$\mathcal{L}_0 = \mathrm{tr}\left(\partial_\mu \Phi^\dagger \partial_\mu \Phi\right) + m_\Phi^2 \mathrm{tr}\Phi^\dagger\Phi + \frac{\lambda_1}{2}\left(\mathrm{tr}\Phi^\dagger\Phi\right)^2 + \frac{\lambda_2}{2}\mathrm{tr}\left(\Phi^\dagger\Phi\right)^2$$

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- U(1)A breaking Lagrangian

$$\mathcal{L}_{\mathrm{U}(1)_A} = \frac{x}{4}\left(\mathrm{tr}\Phi^\dagger\Phi\right)\left(\det\Phi + \det\Phi^\dagger\right) + \frac{y}{4}\left(\left(\det\Phi\right)^2 + \left(\det\Phi^\dagger\right)^2\right)$$

Effective theory of Nf=2 QCD

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- No mass splitting between π and η
  - all mesons contribute to the running
- The interaction has U(1)A breaking effect

Plan of the talk

- ✓ 1. Introduction
- ✓ 2. Previous works
- ✓ 3. Restoration U(1)A symmetry
- ✓ 4. Effective theory
- 5. Renormalization group analysis
- 6. Conclusion

Renormalization group analysis

Wilson and Kogut, Zinn-Justin, Pisarski and Wilczek

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- Instead of 3D we adopt ε -expansion analysis

Renormalization group analysis

4 couplings $g_i = \{\lambda_1, \lambda_2, x, y\}$

β function $\beta_{g_i} = \mu \frac{\partial}{\partial \mu} g_i$

Dimensional regularization d=4- ε and MS scheme

$$g_i^B = \mu^{\epsilon} \sum_j Z_{ij} g_j$$

β function $\beta_{g_i} = \mu \frac{\partial}{\partial \mu} g_i$ ← $\mu \frac{d g_i^B}{d\mu} = 0$

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$$\beta_{\lambda_1} = -\epsilon \lambda_1 + \frac{1}{8\pi^2}\left(8\lambda_1^2 + 8\lambda_1\lambda_2 + 3\lambda_2^2 + \frac{3}{2}x^2 + \frac{5}{4}y^2\right)$$

$$\beta_{\lambda_2} = -\epsilon \lambda_2 + \frac{1}{8\pi^2}\left(4\lambda_2^2 + 6\lambda_1\lambda_2 - \frac{3}{4}x^2 - y^2\right)$$

$$\beta_x = -\epsilon x + \frac{1}{8\pi^2} x \left(12\lambda_1 + 6\lambda_2 + 3y\right)$$

$$\beta_y = -\epsilon y + \frac{1}{8\pi^2}\left(6\lambda_1 y + \frac{3}{2}x^2\right)$$

Renormalization group analysis

- Fixed points of β-function $(\lambda_1, \lambda_2, x, y)$

Fixed points	*Property*
(0,0,0,0)	UV FP
$\epsilon\pi^2$(1,0,0,0)	saddle point
$\epsilon\pi^2/3$(4,-2,0,-4)	saddle point
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Phase transition is FIRST order!

Conclusion

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U(1)A order parameter vanishes $$\frac{1}{V^k}\langle \delta^0 \mathcal{O}_{n_1,n_2,n_3,n_4}\rangle = 0$$

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π - η splitting mass=0 but $U(1)_A$ breaking int. remains

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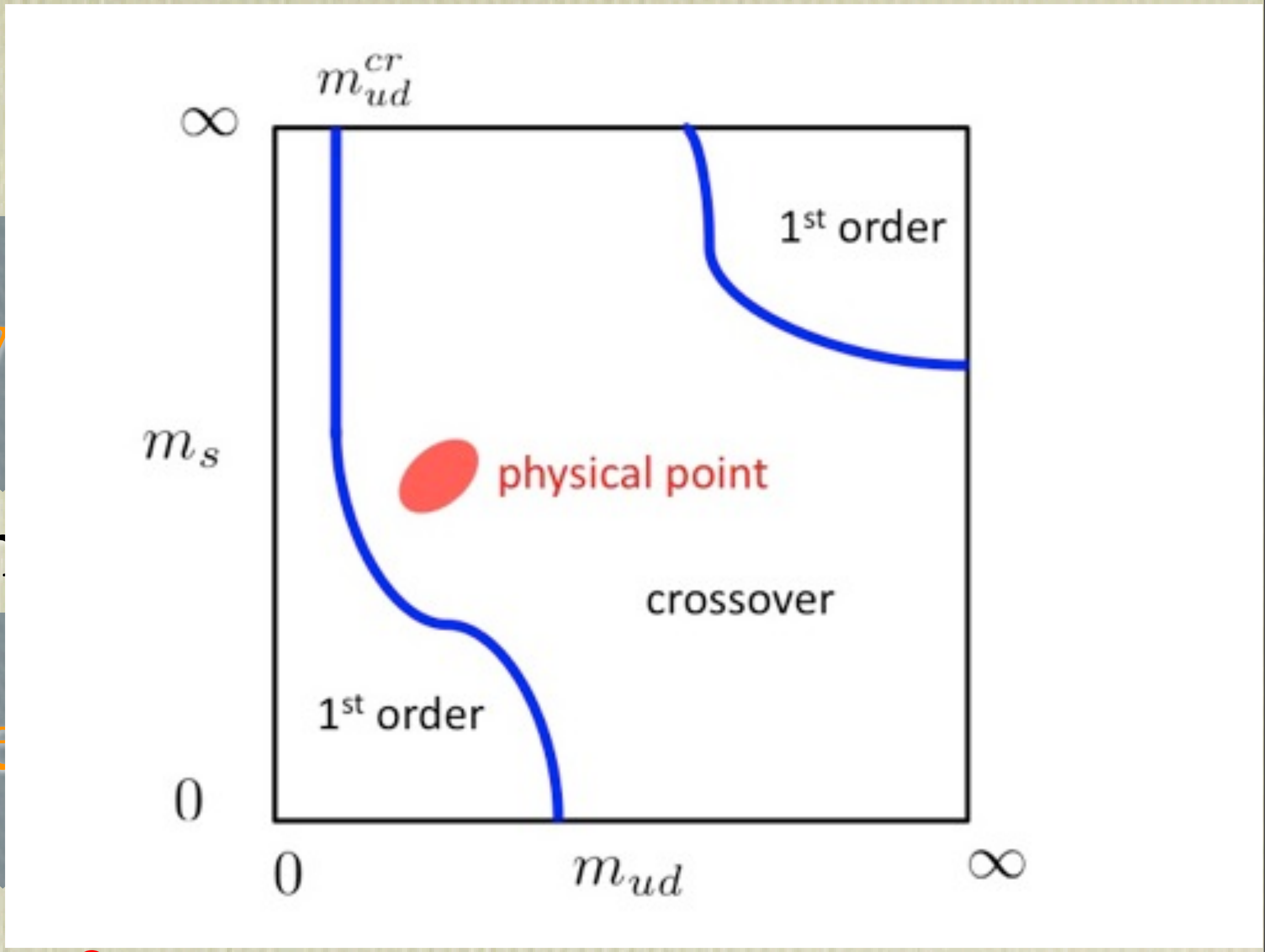
Conclusion



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$\varepsilon \rightarrow 1$

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$\varepsilon \to 1$ ⟶

Non-perturbative scenario?

$\varepsilon \to 1$ ⟶ Non-perturbative β-function

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1. No stable IR FP is found → first order

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Plan of the talk

✓ 1. Introduction

✓ 2. Effective theory

✓ 3. Renormalization group analysis

✓ 4. Conclusion

5. Discussions

Objection!

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O(4) scaling!

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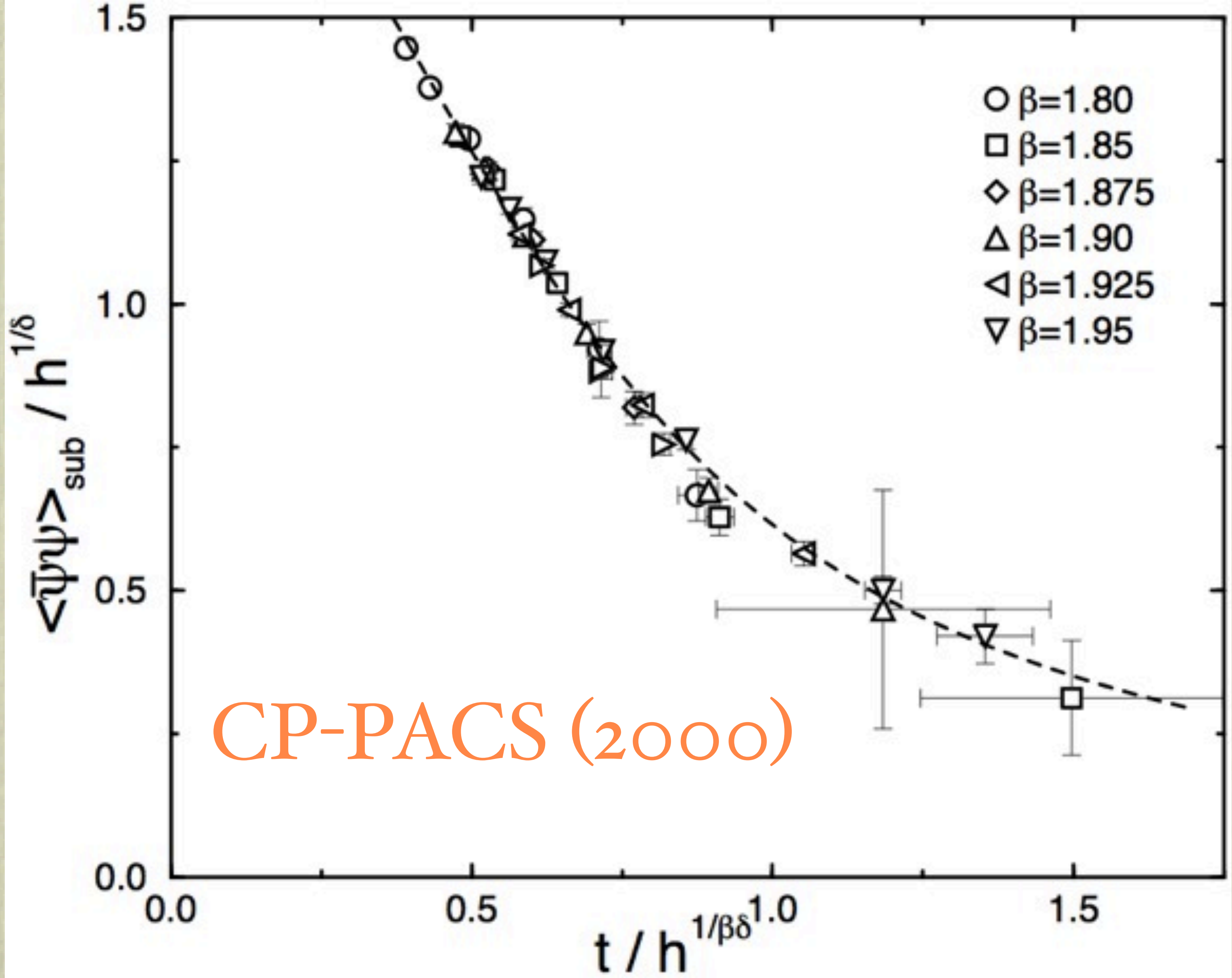
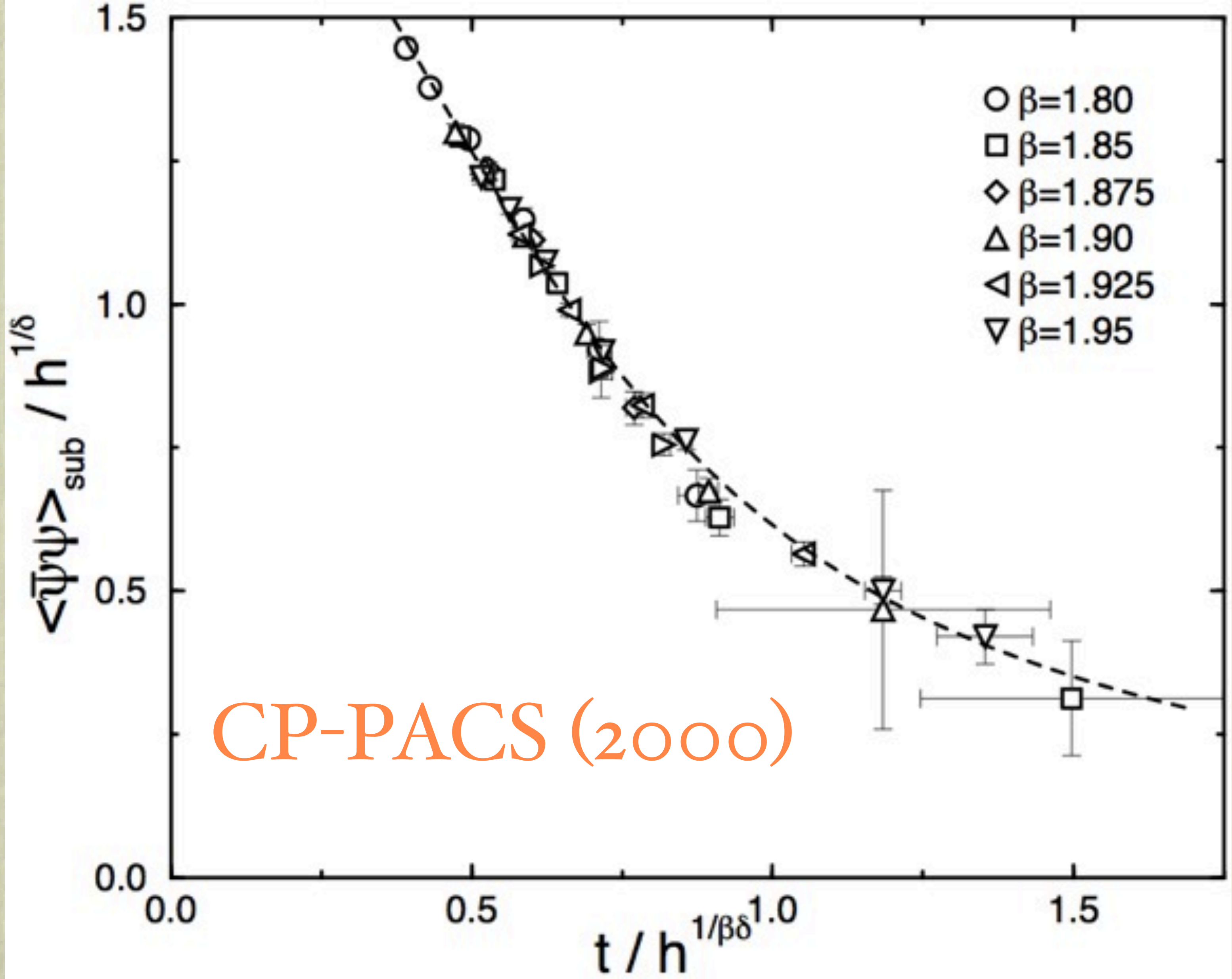
O(4) scaling!



CP-PACS (2000)

Objection!

O(4) scaling! → second order phase transition!



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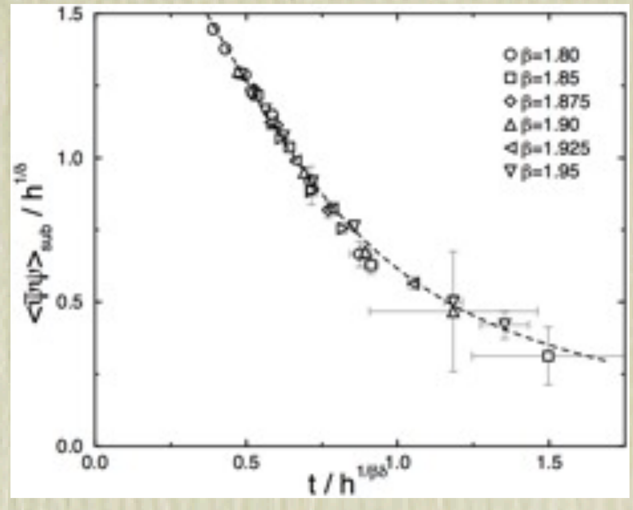

- Consistent scenarios

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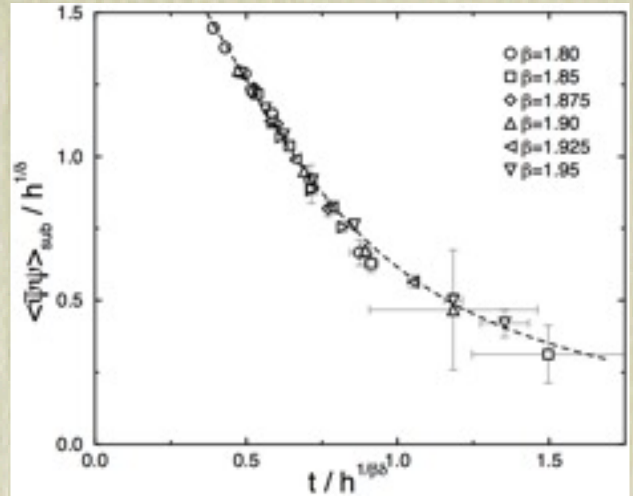

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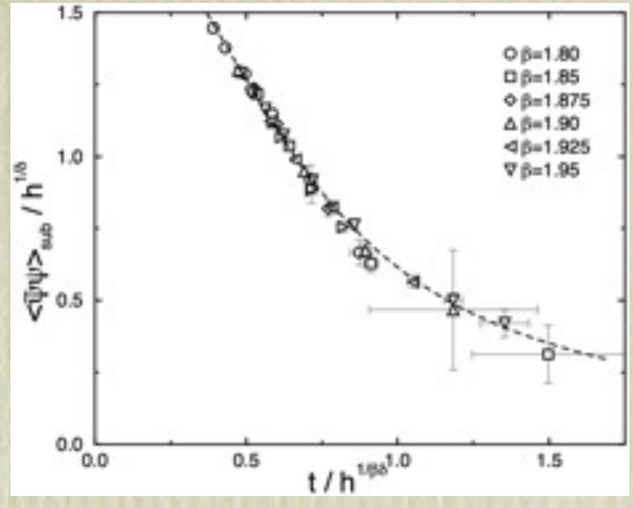

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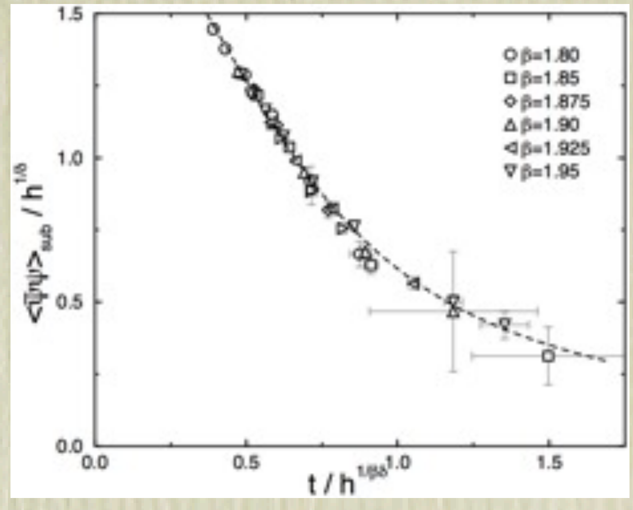

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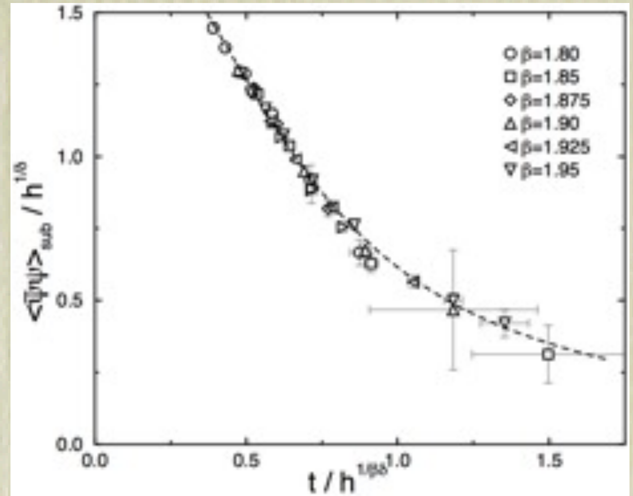

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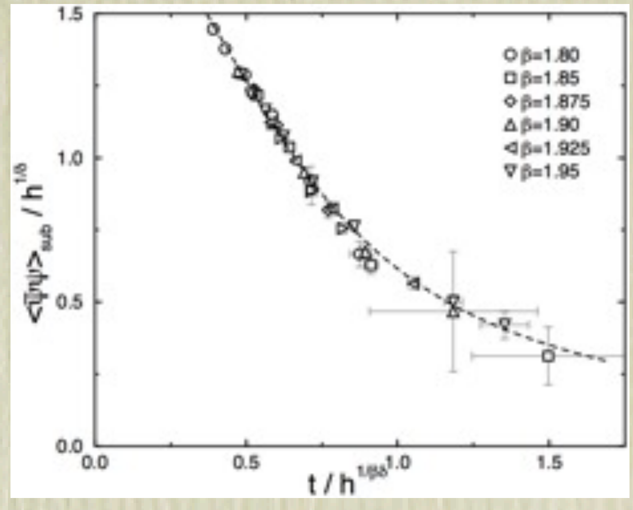

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critical exponent may be very similar

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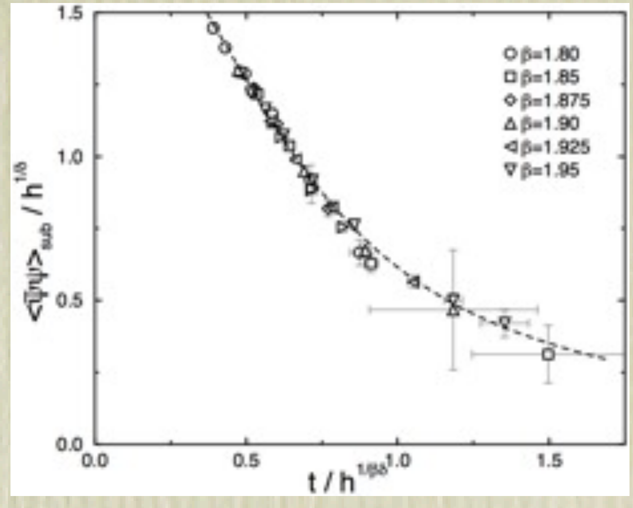

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★Second order and O(4) universality class

- Stable IR FP is found at non-zero x

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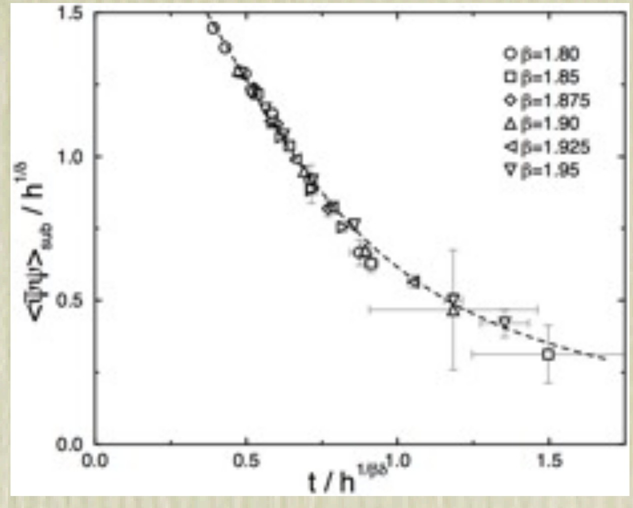

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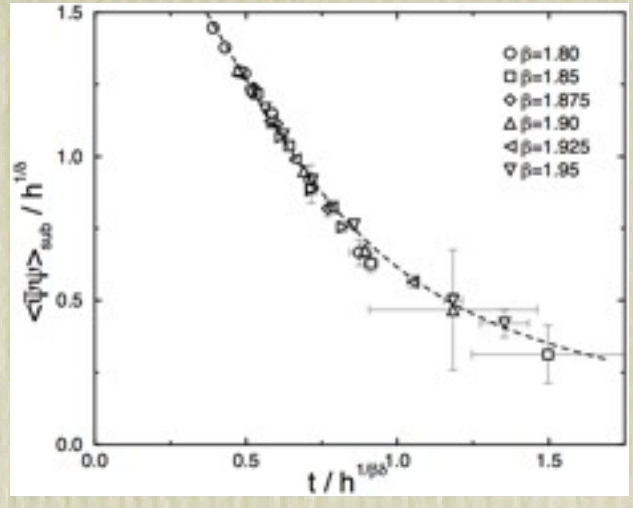

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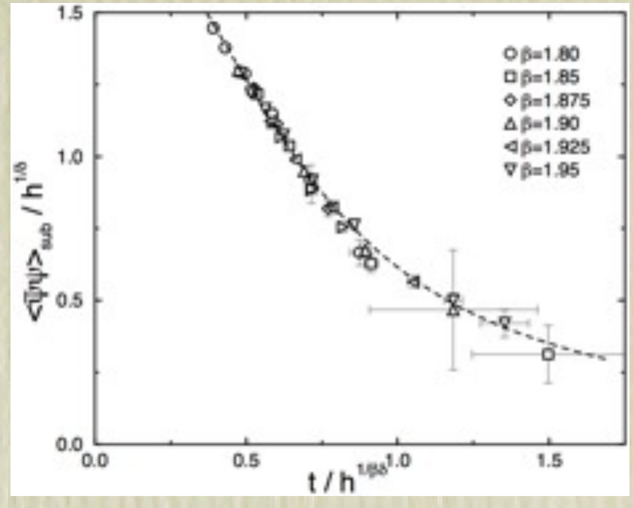

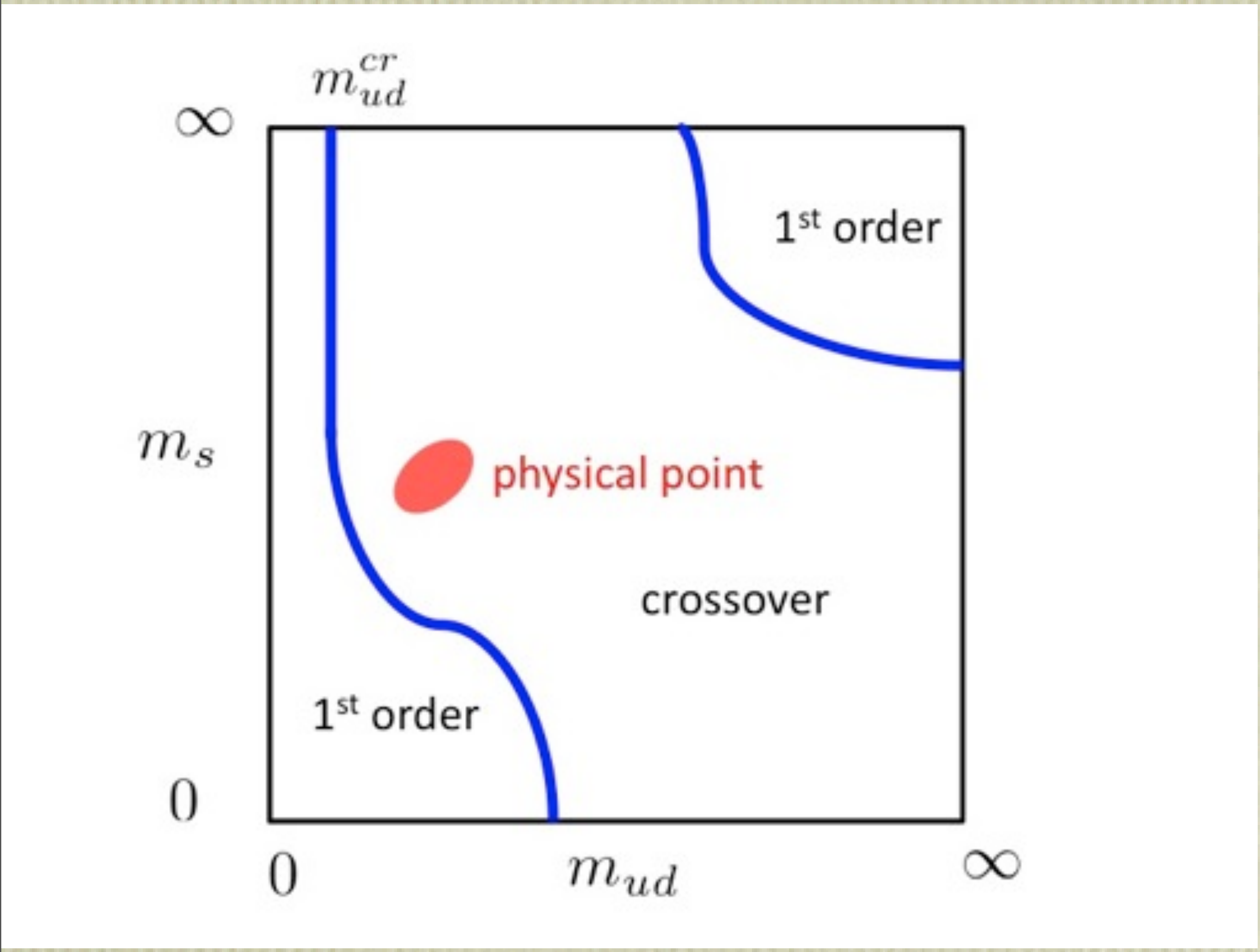

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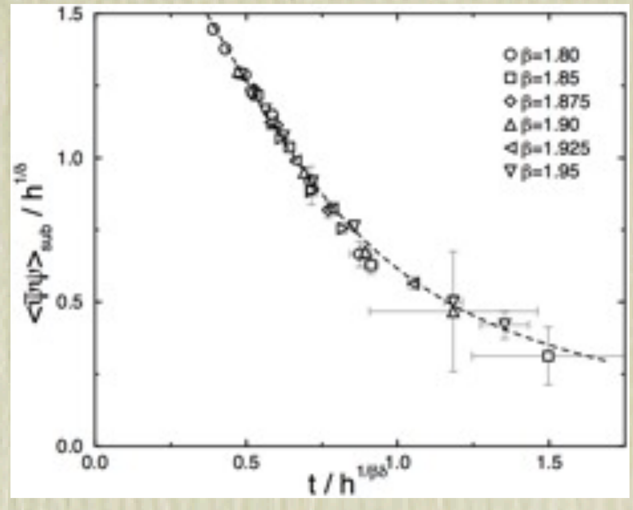

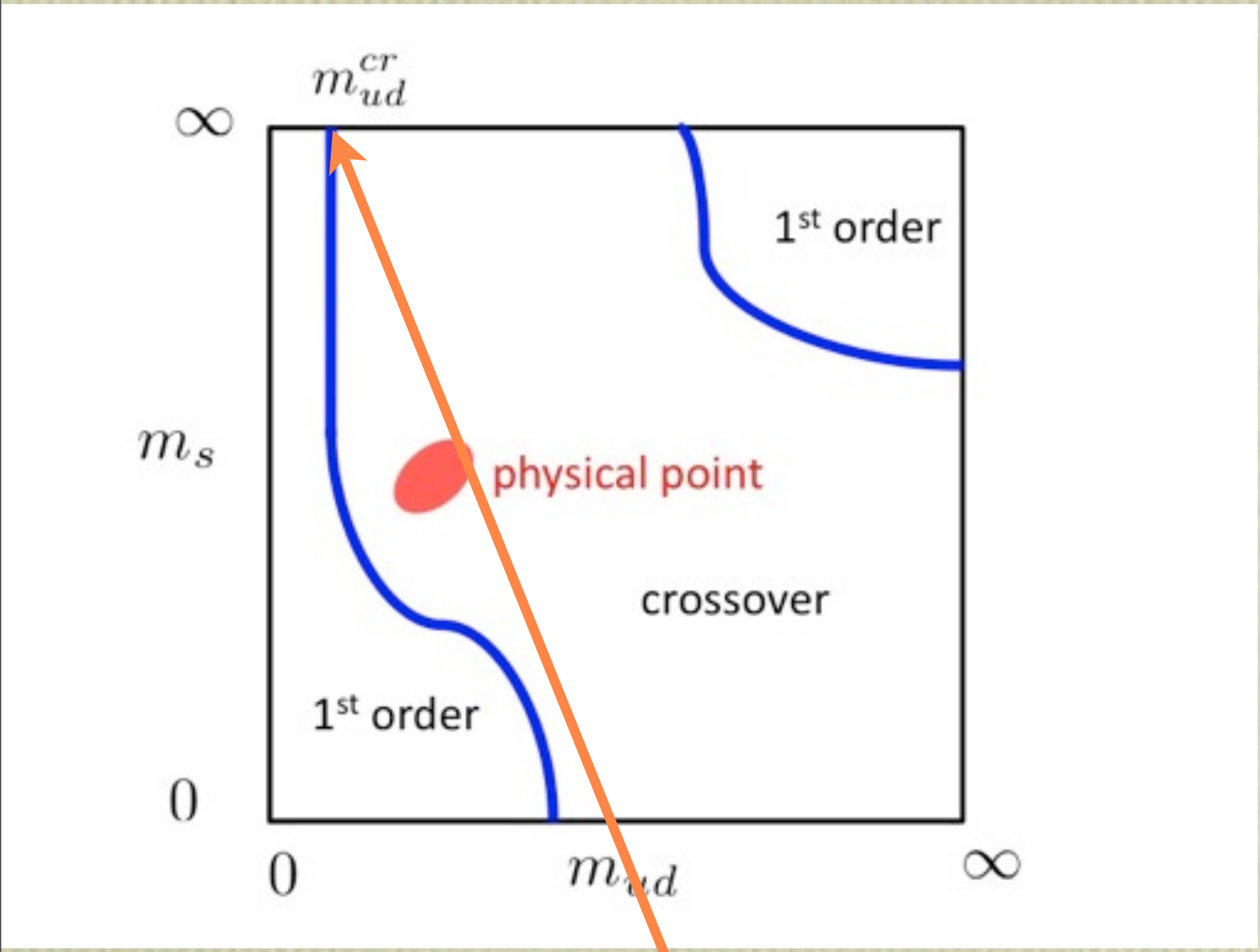

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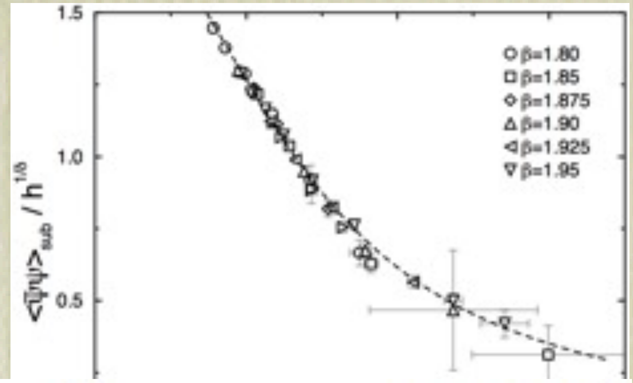
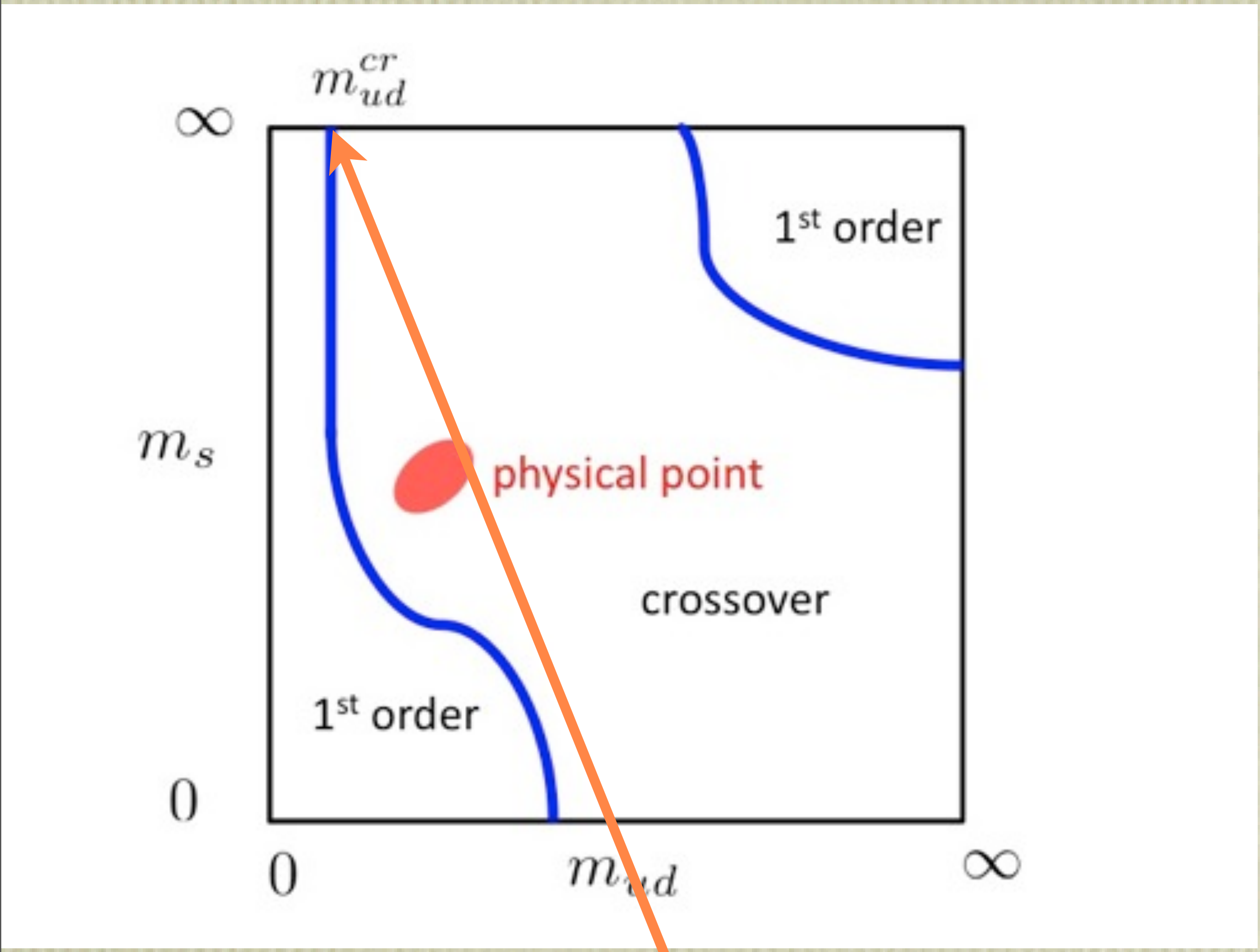


Brandt et al

$$T_C(m_{ud}) = T_C(0)\ \left[1 + C\, m_{ud}^{1/(\delta\beta)}\right]$$



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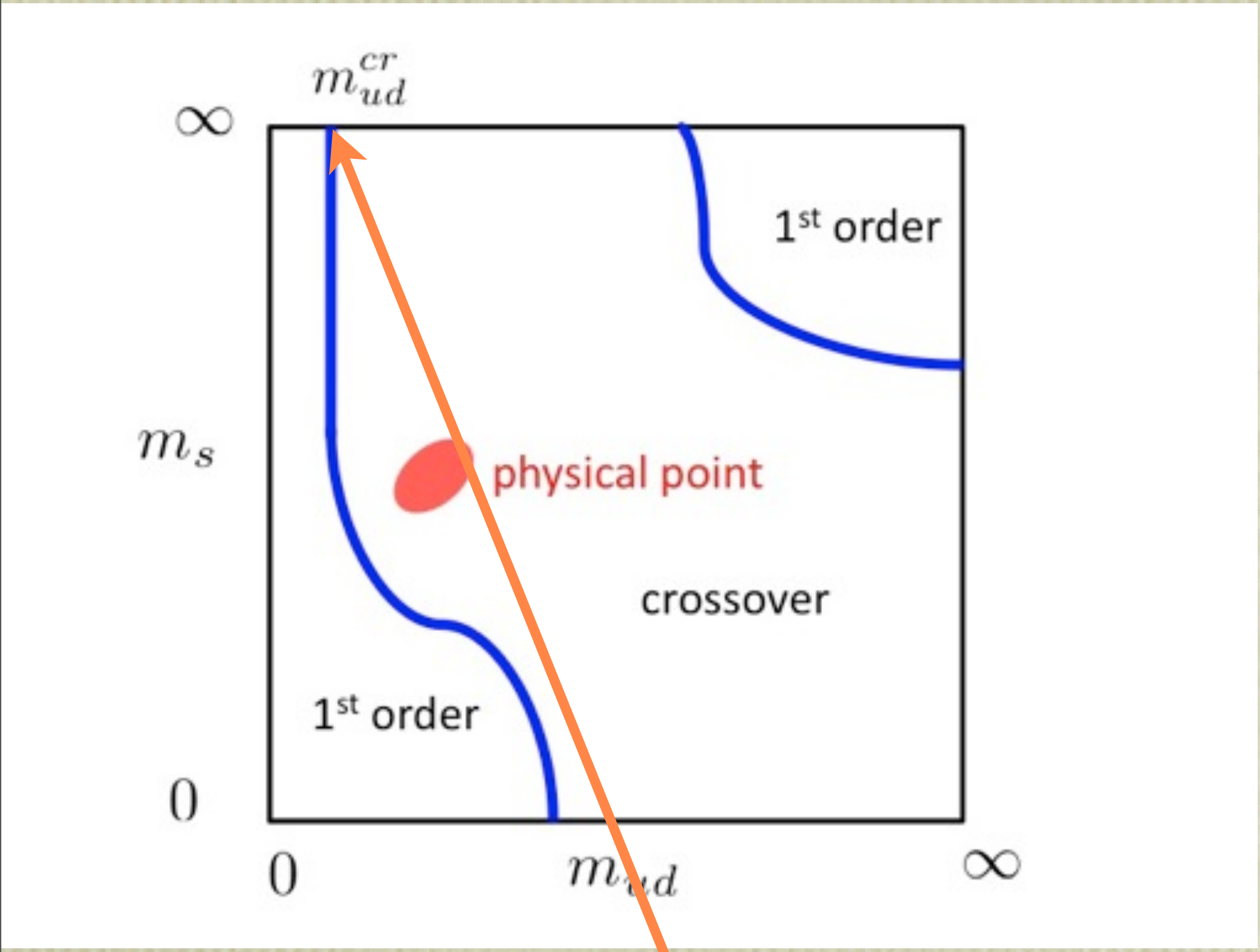

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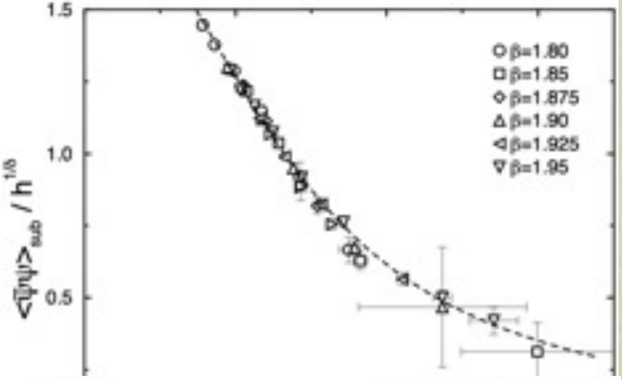

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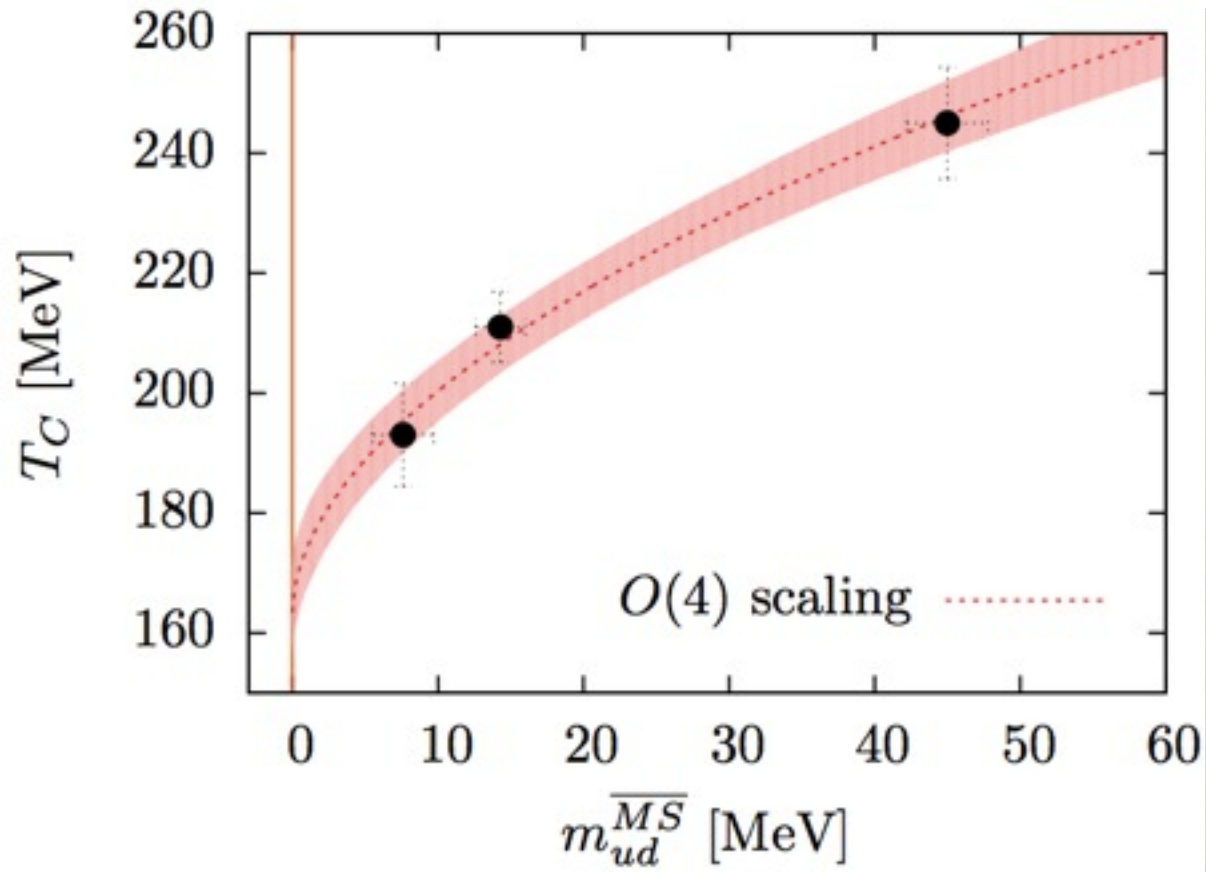

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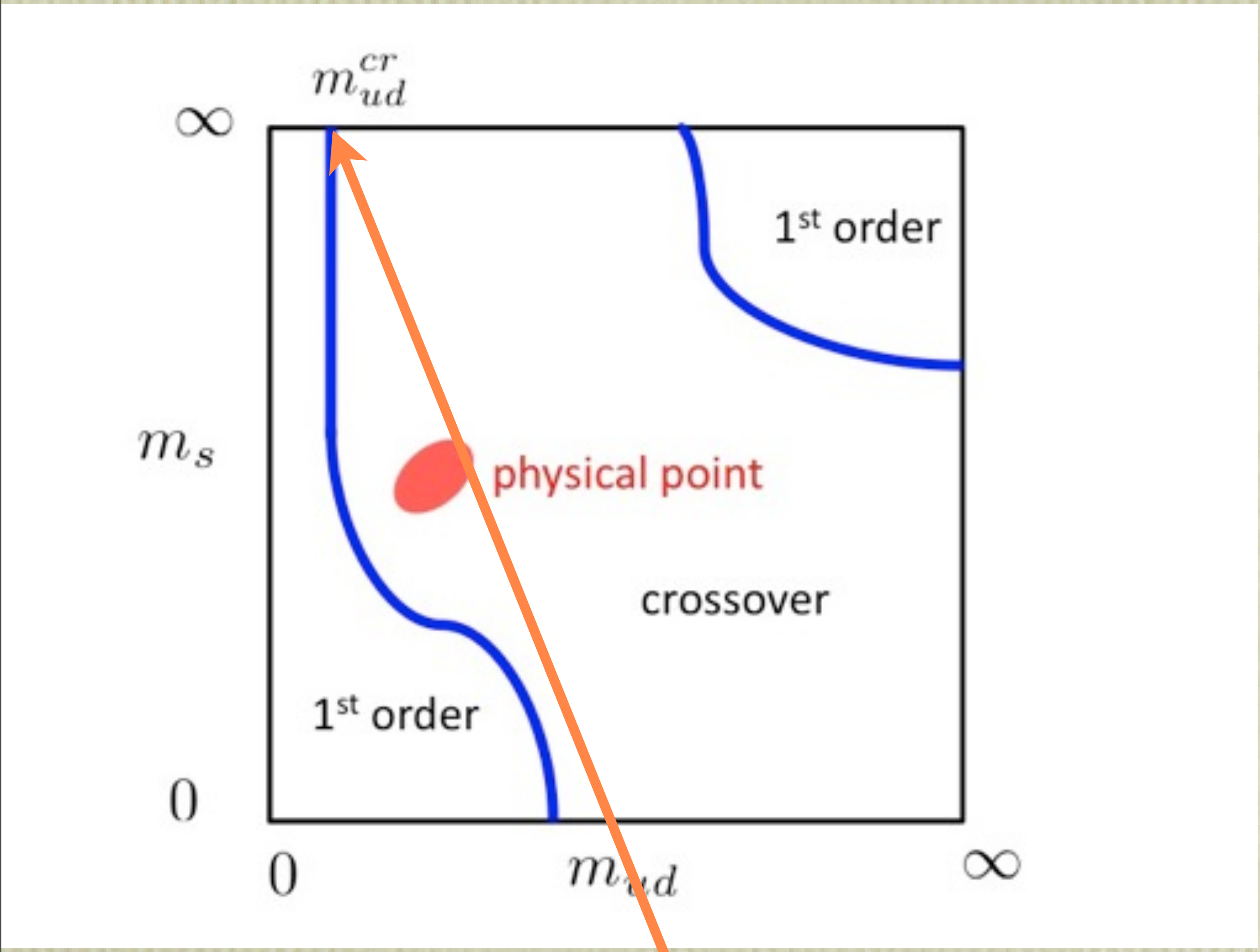

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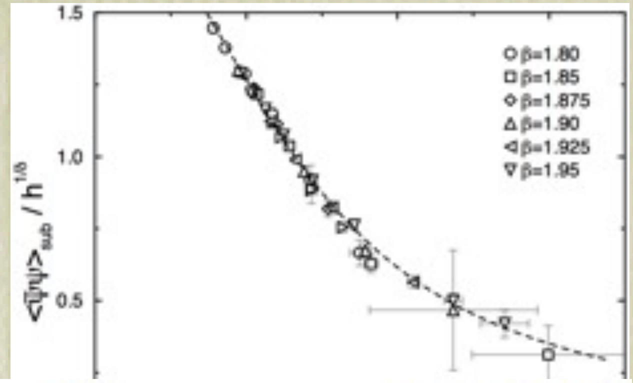

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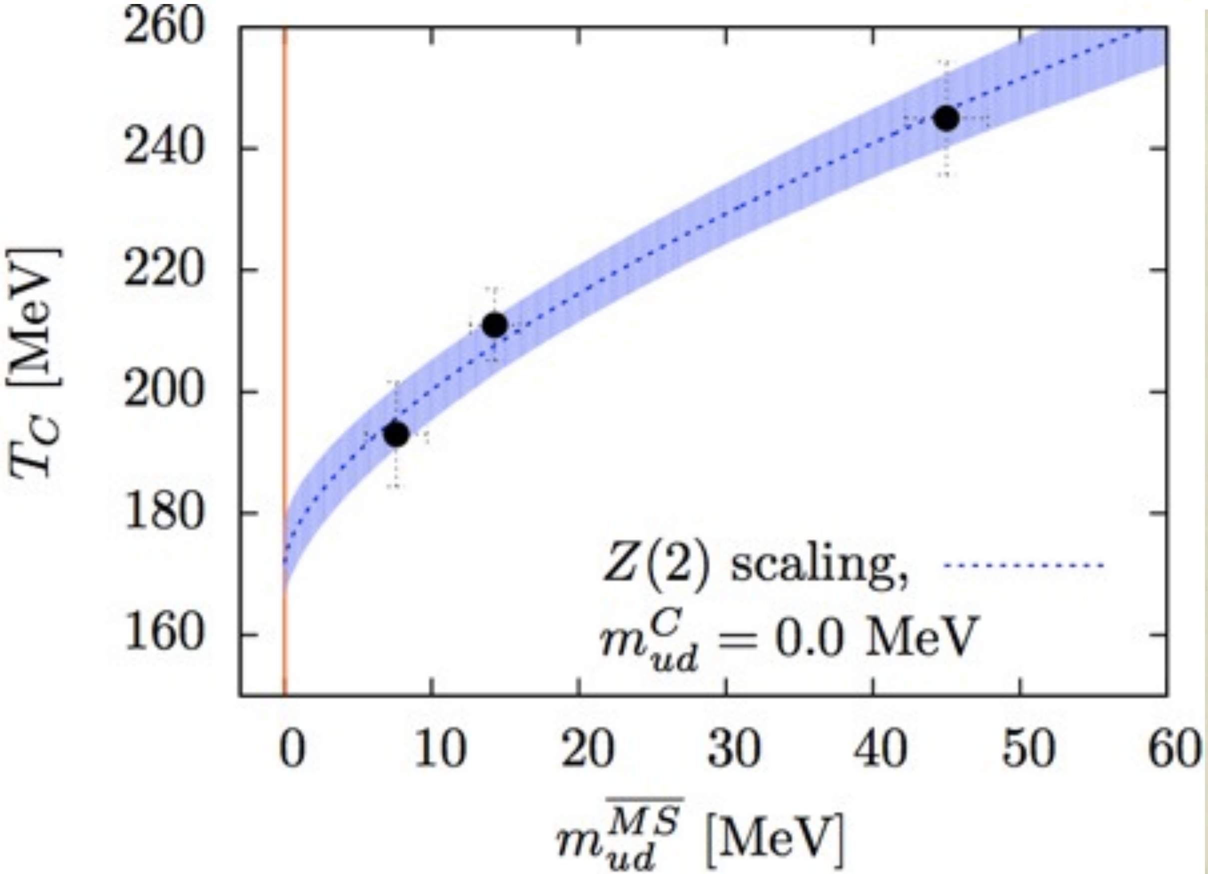

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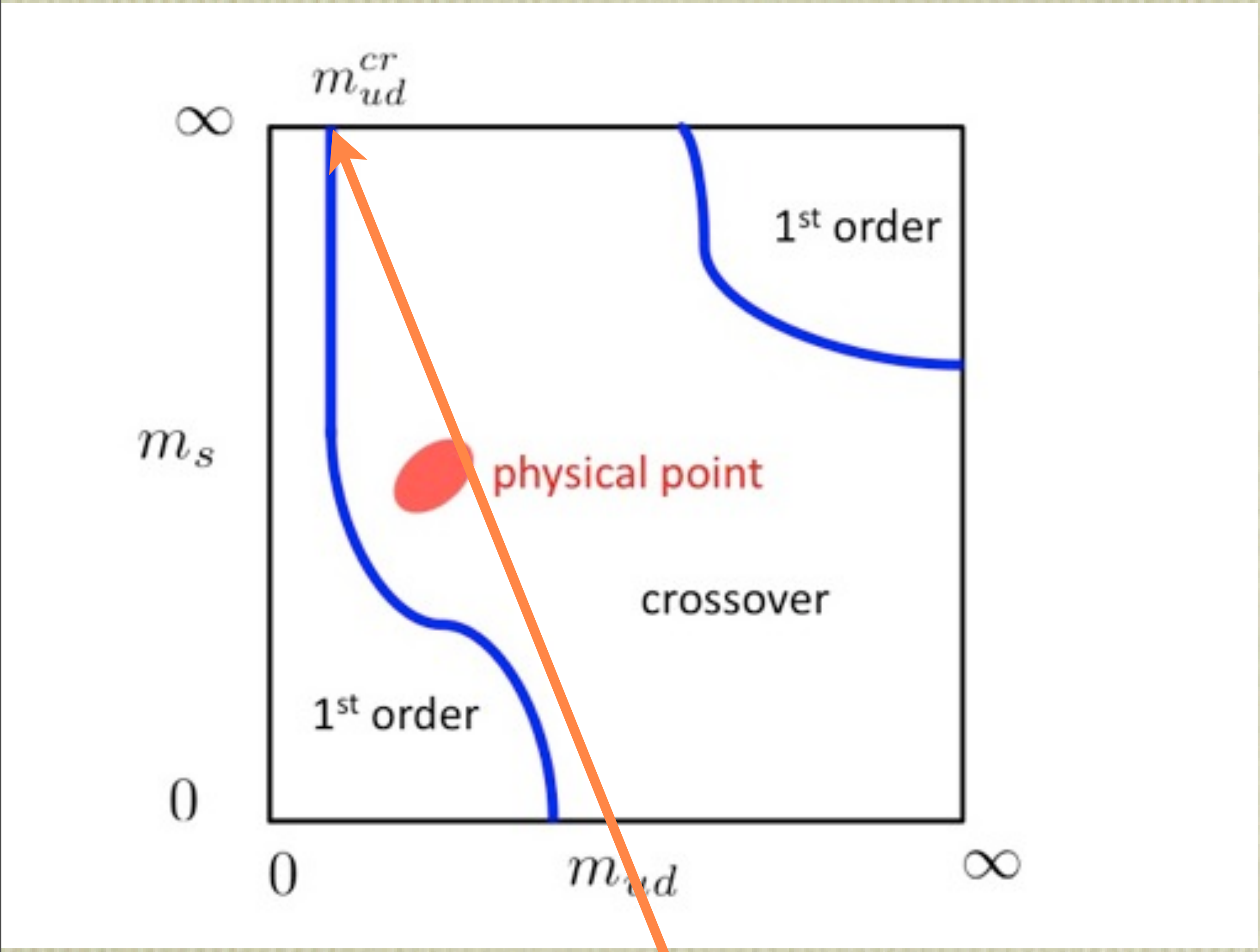

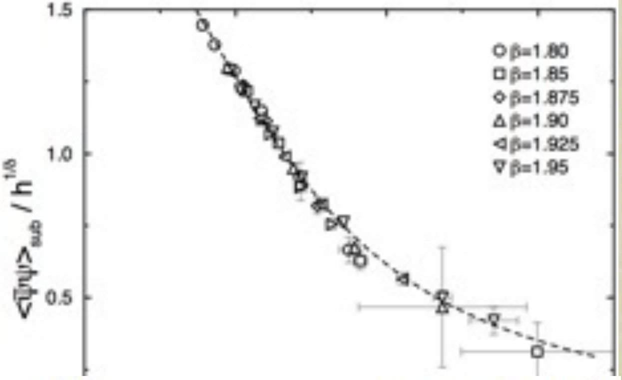

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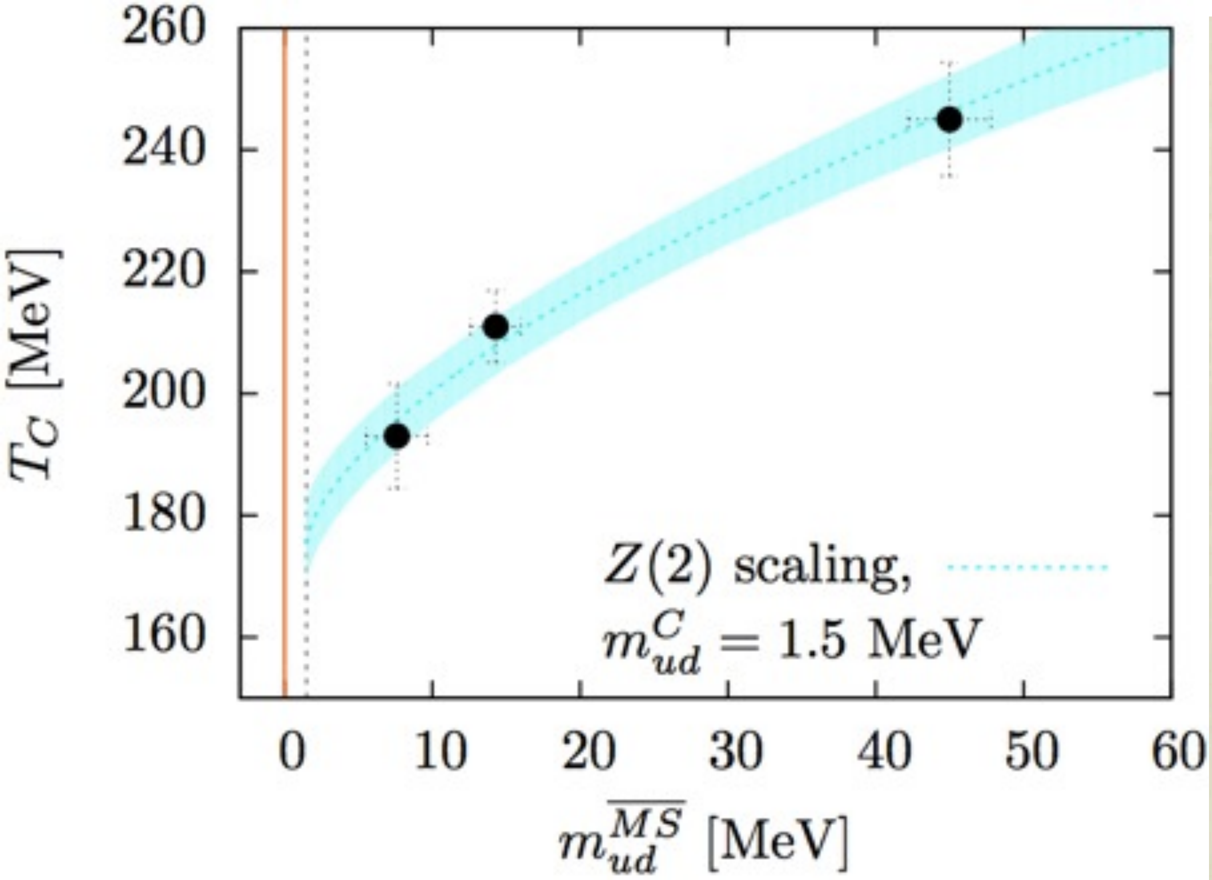

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So many numerical results for U(1)$_A$ breaking at Tc

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LLNL/RBC collaboration arXiv:1309.4149

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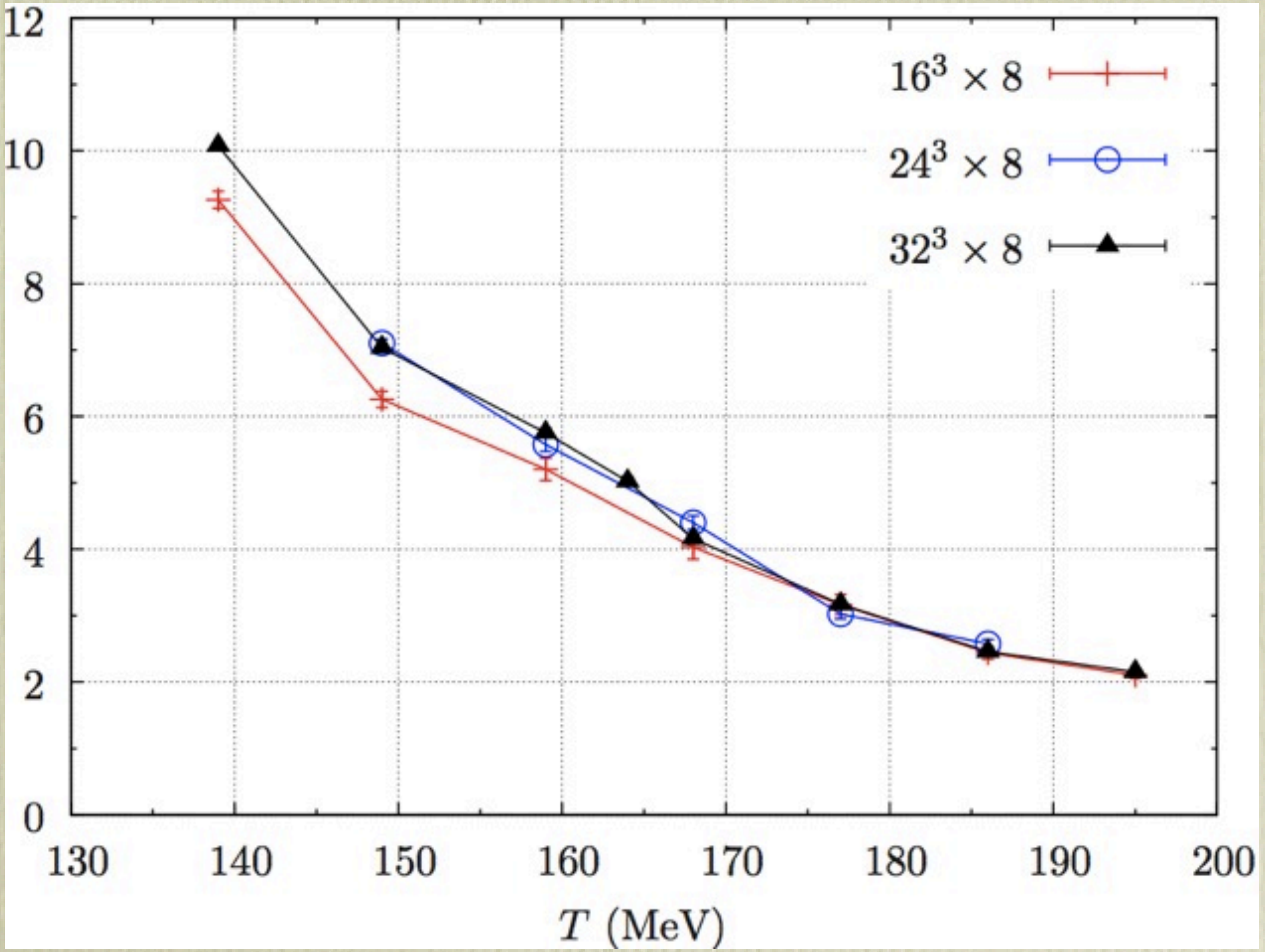
$$\langle\bar{\psi}\psi\rangle$$



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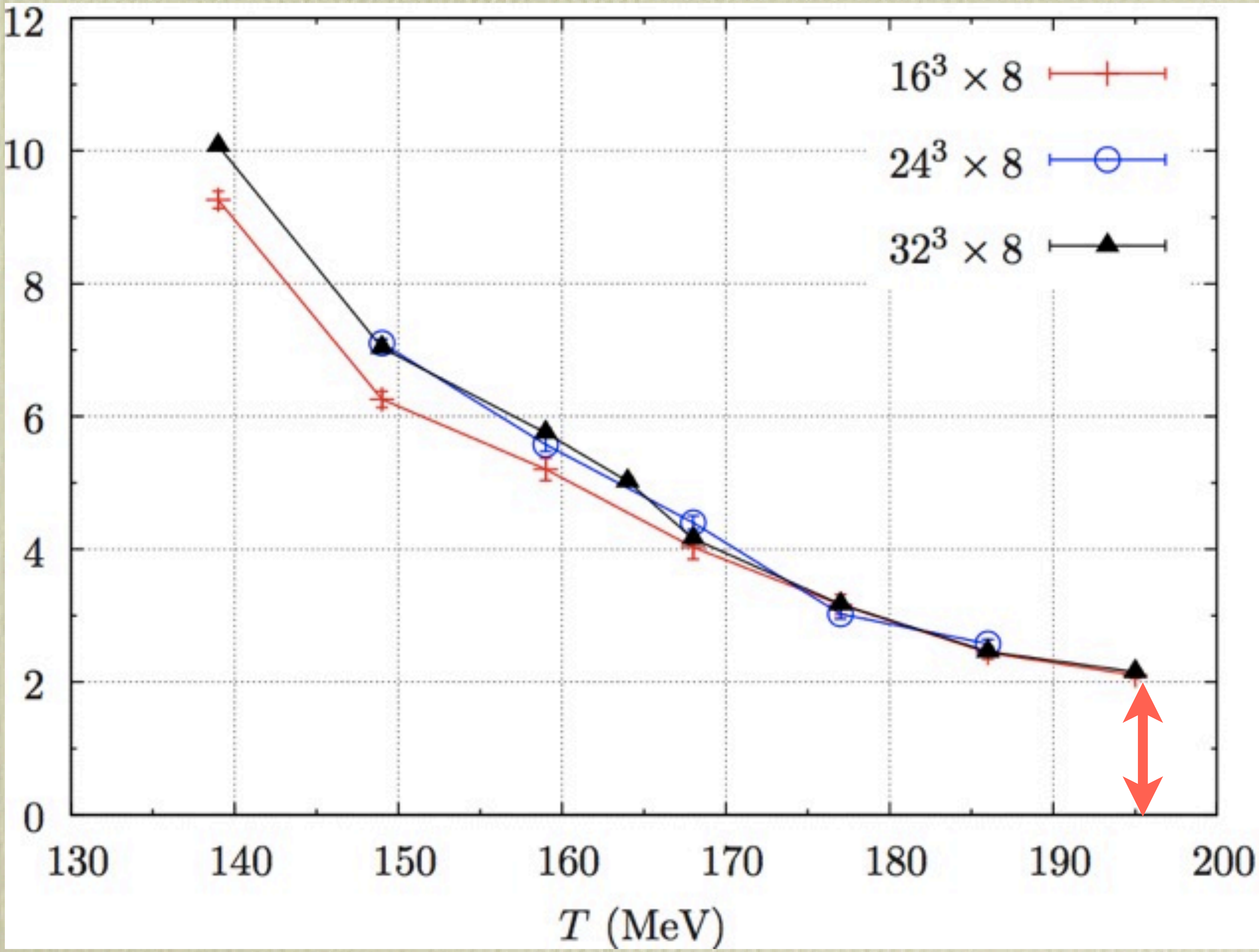

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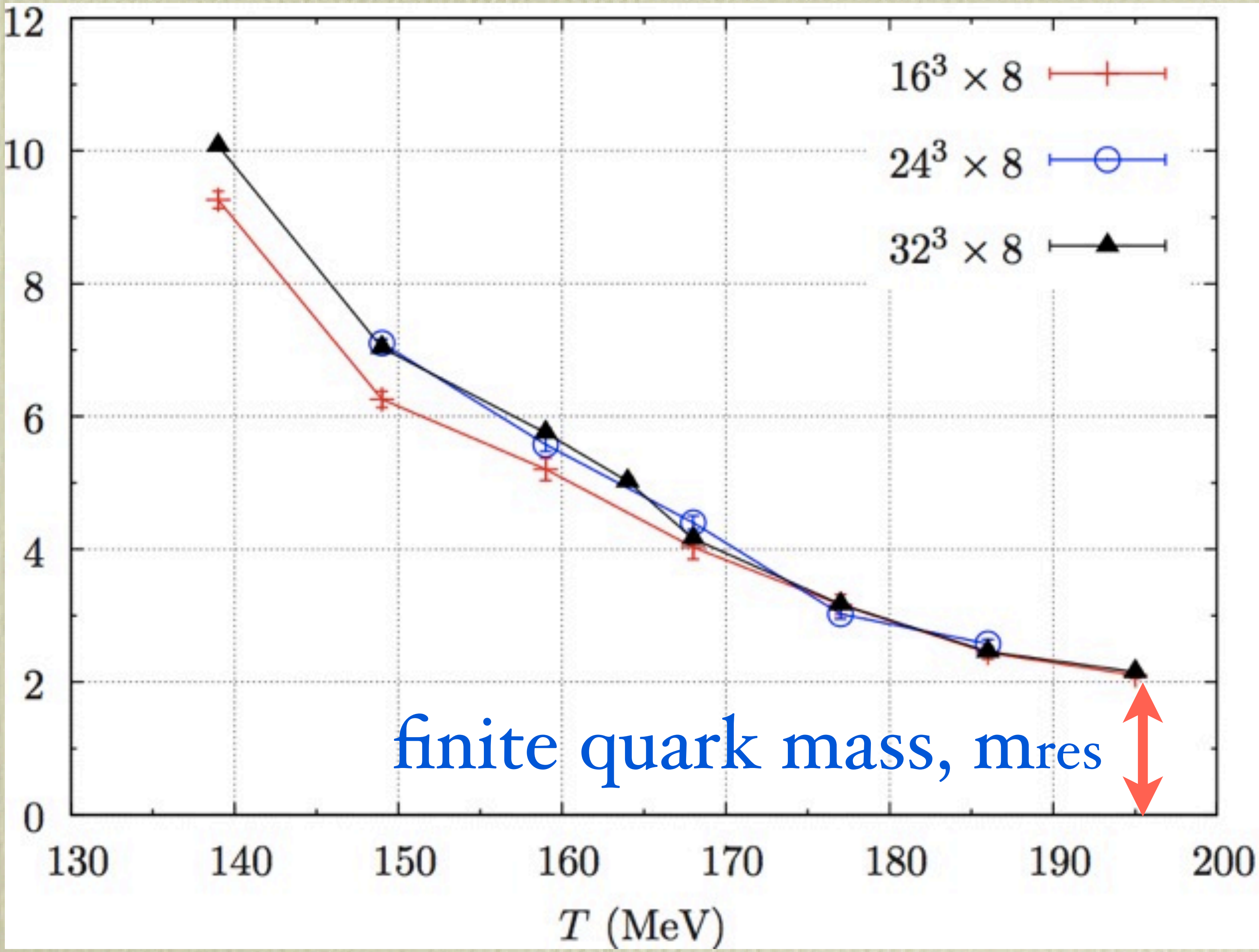

finite quark mass, m_{res}

Objection!

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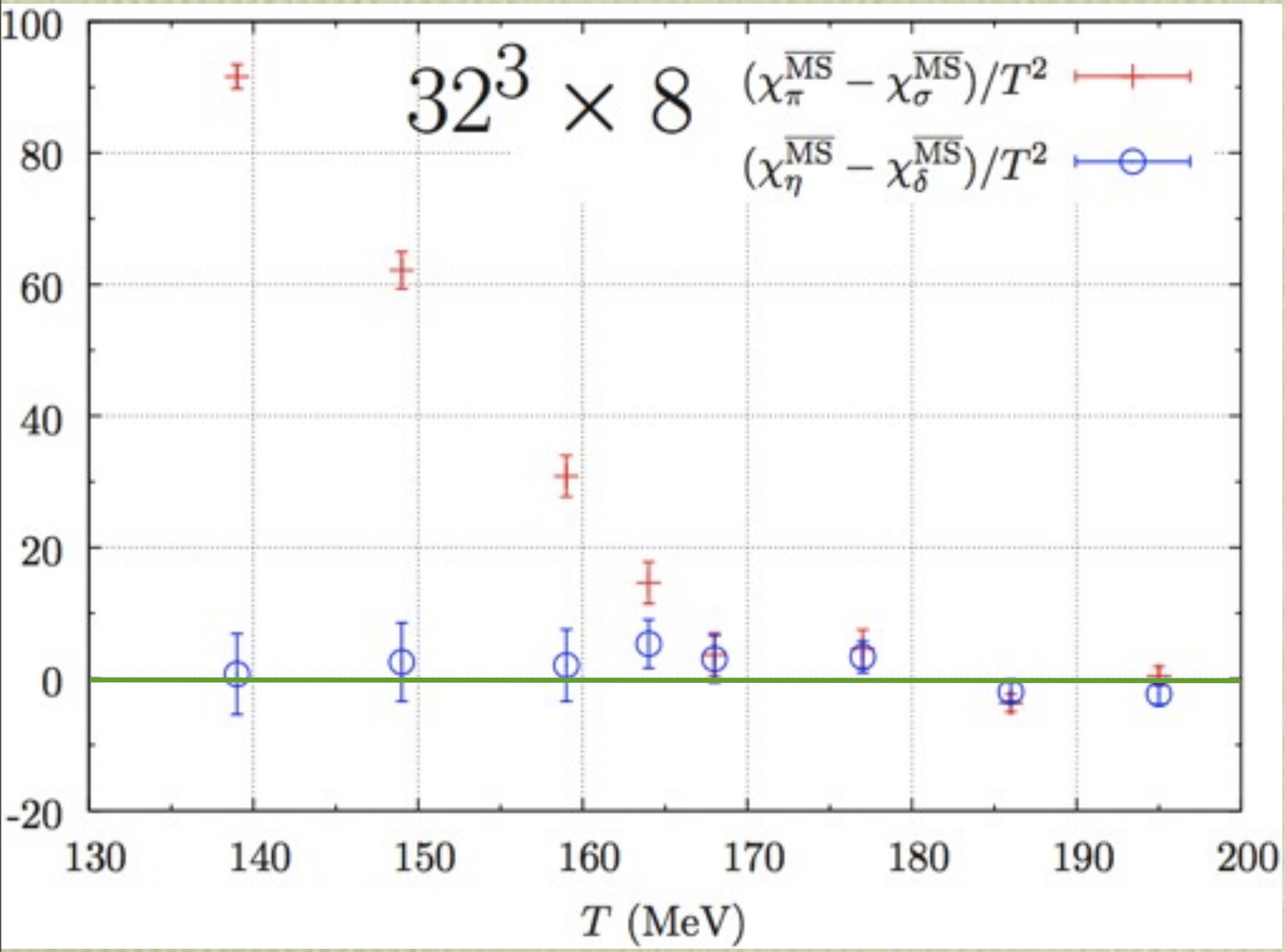
2pt. SU(2)A order parameter



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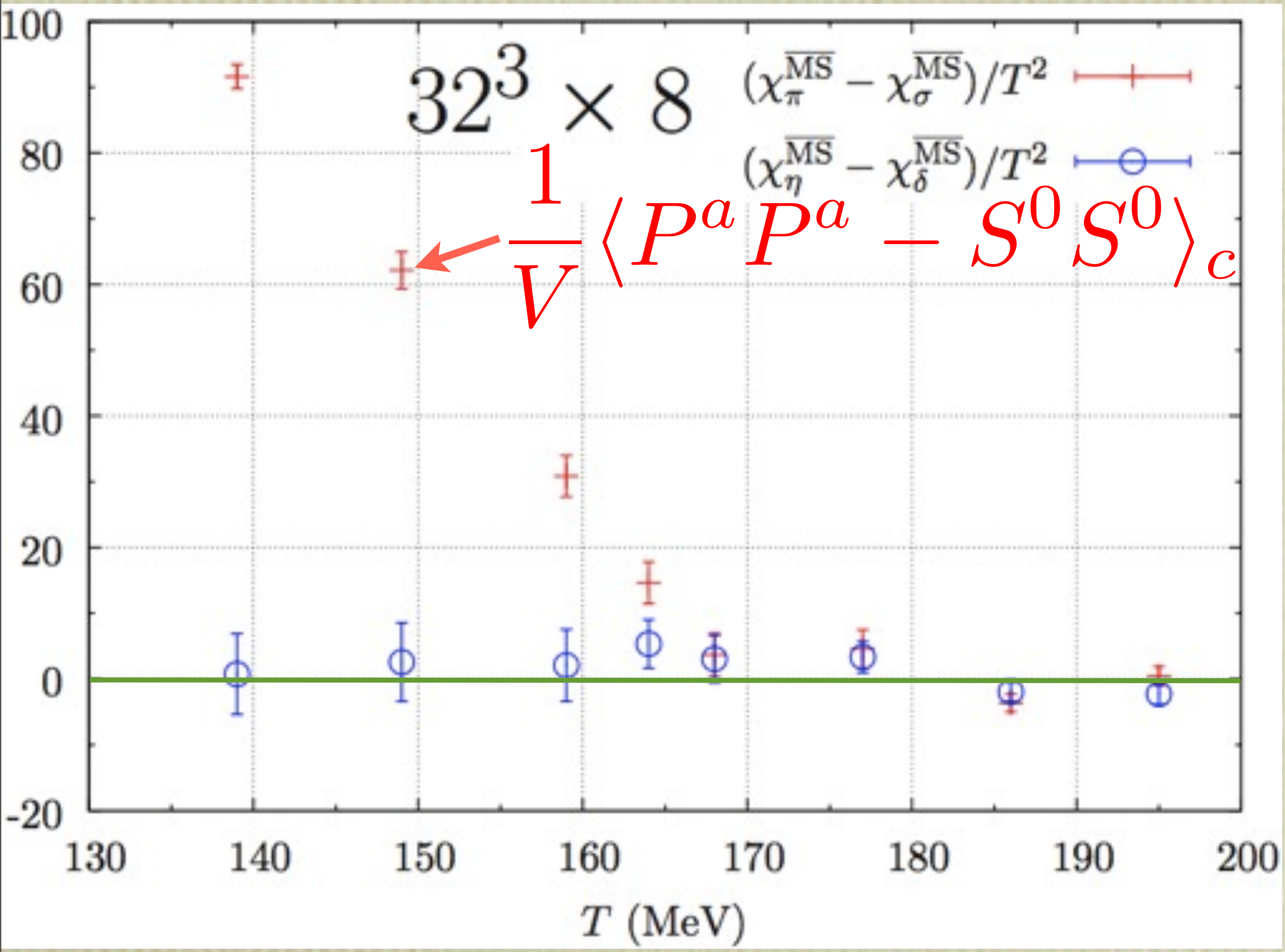

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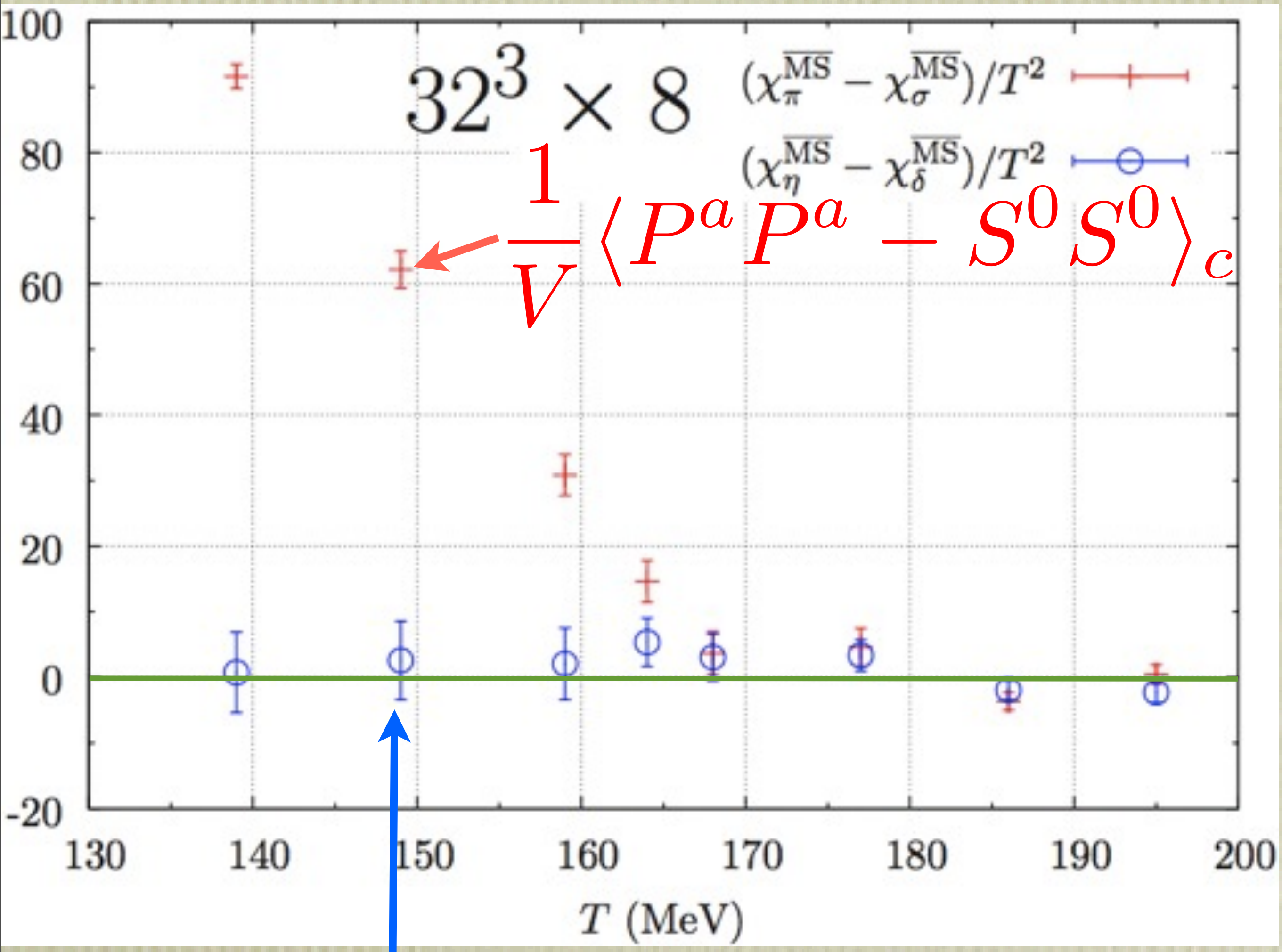

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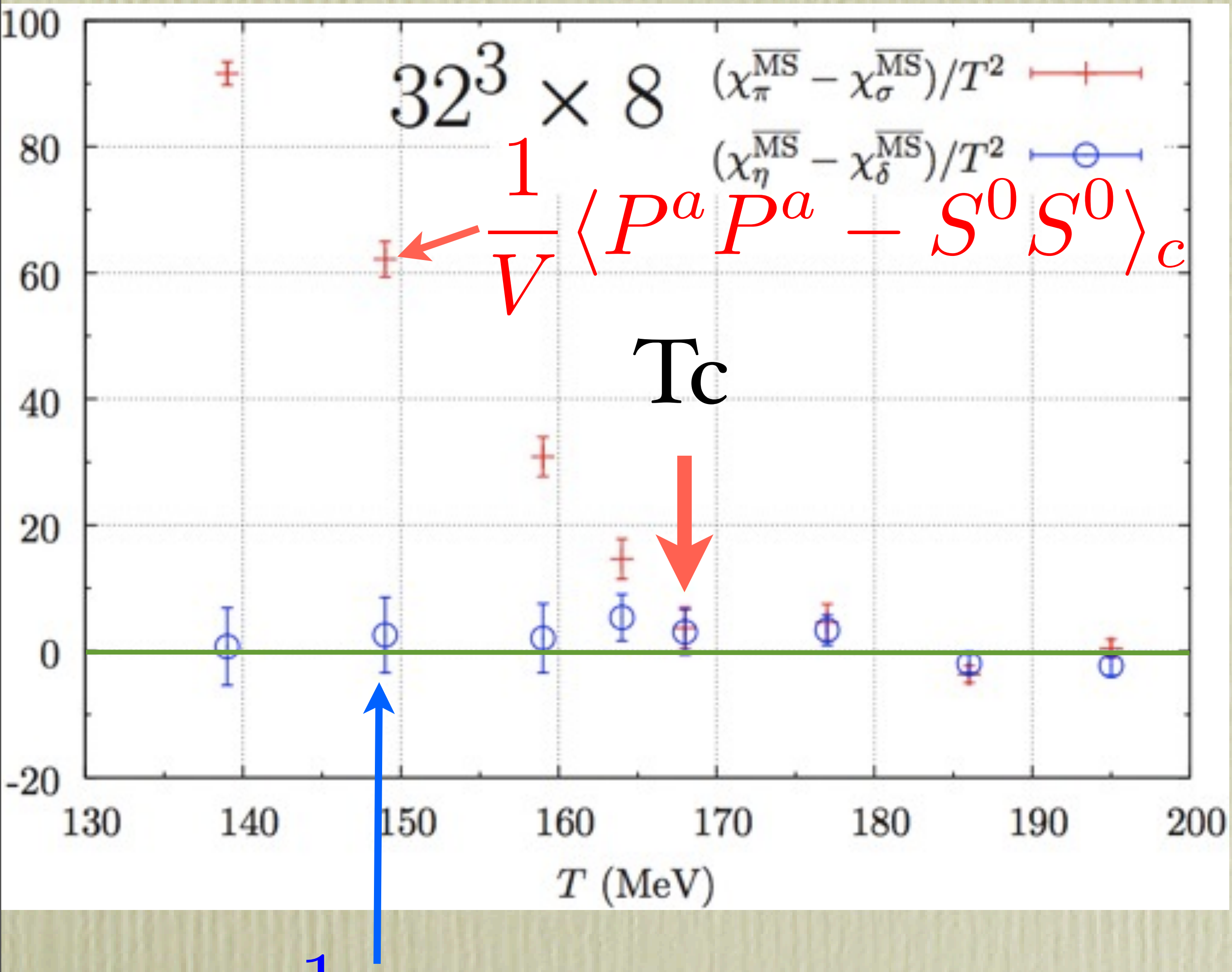

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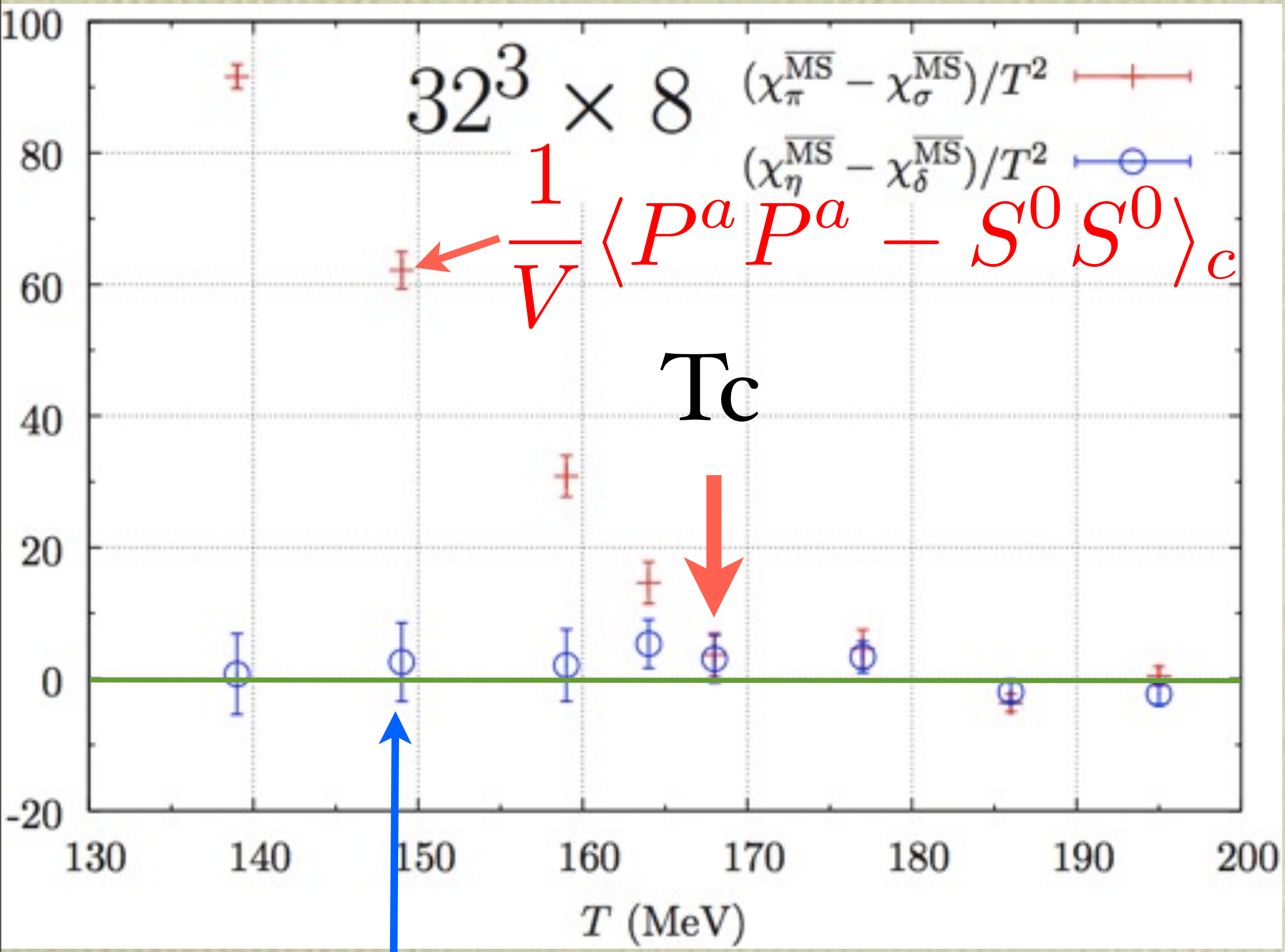

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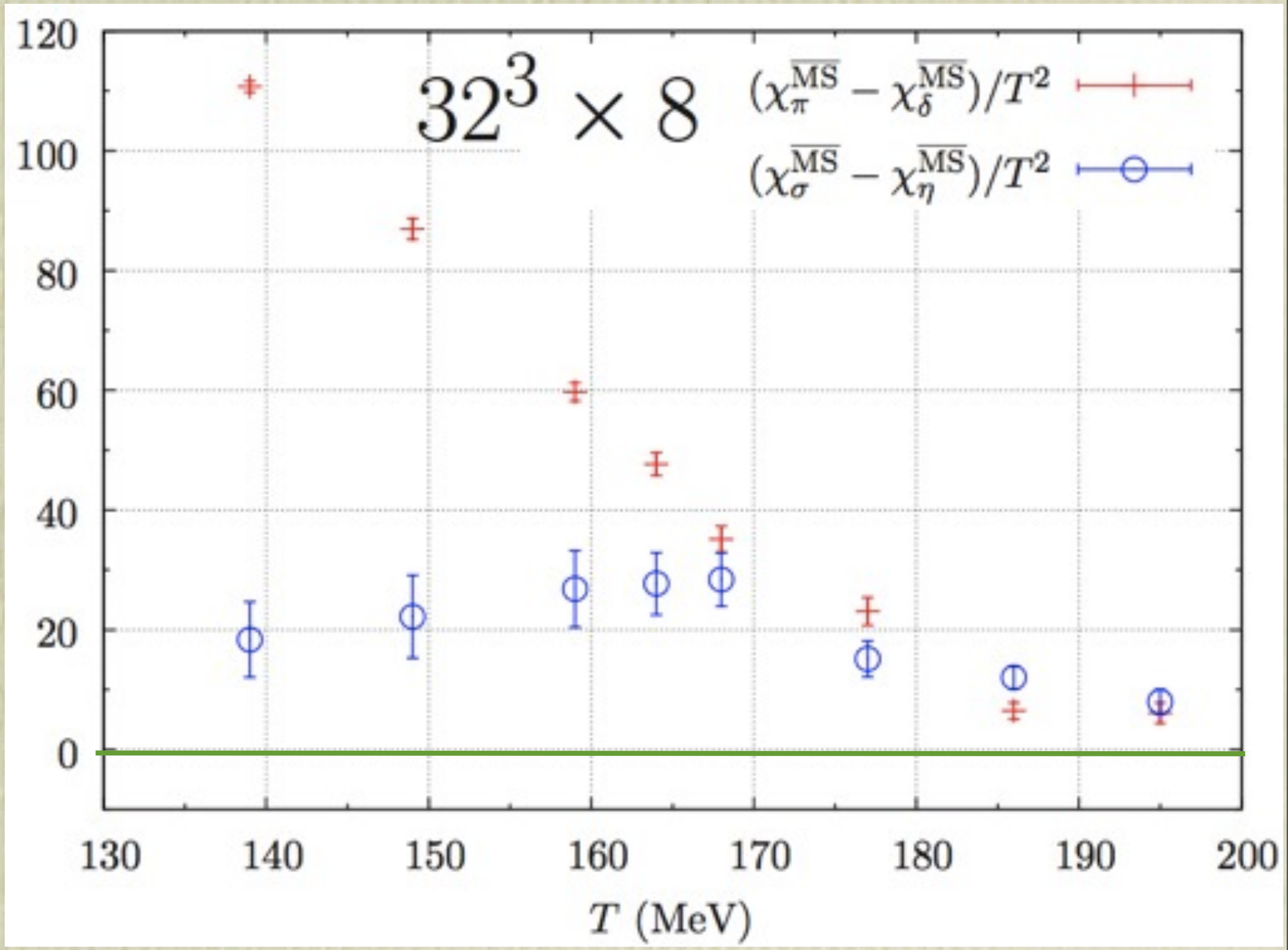

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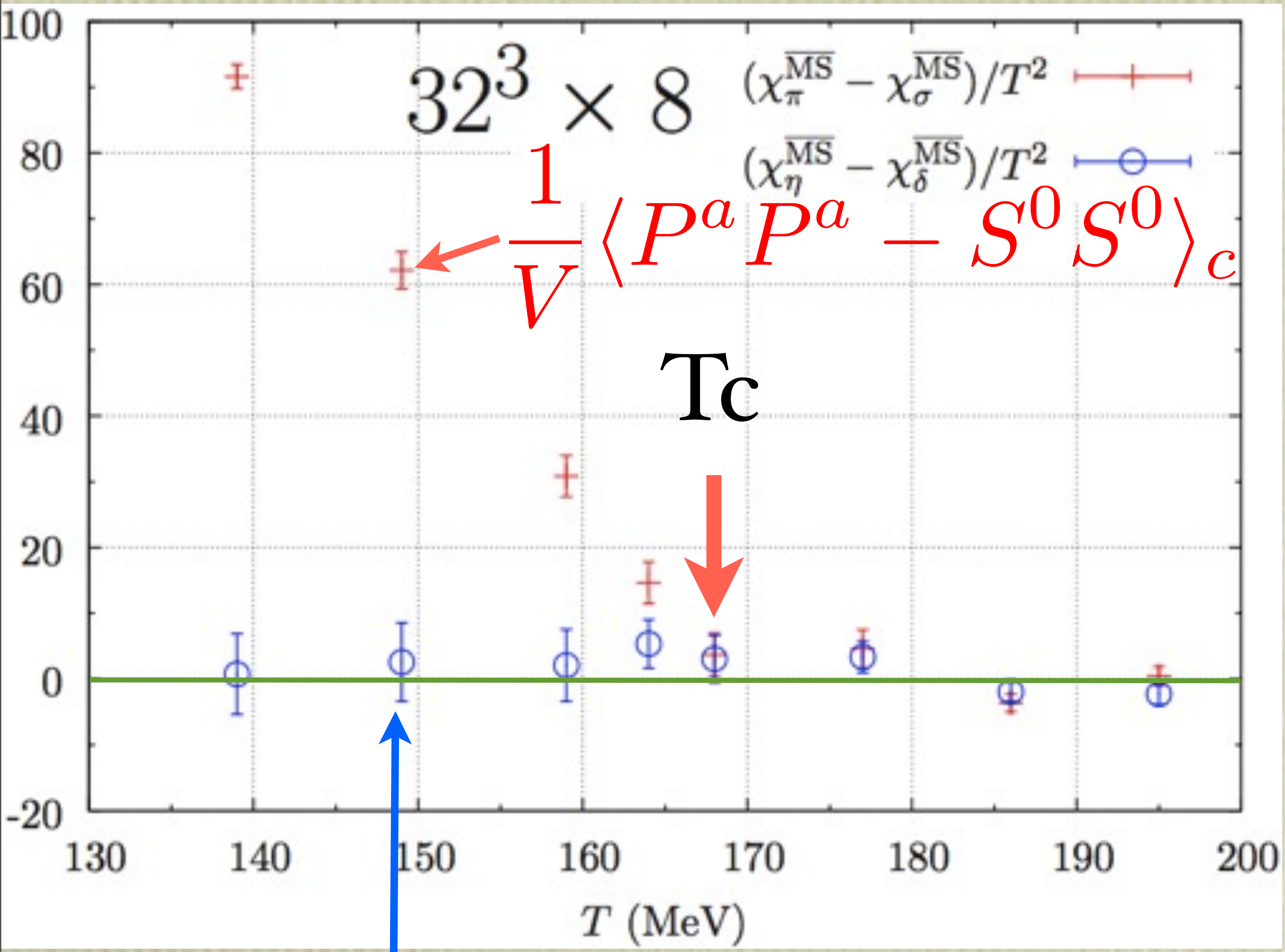

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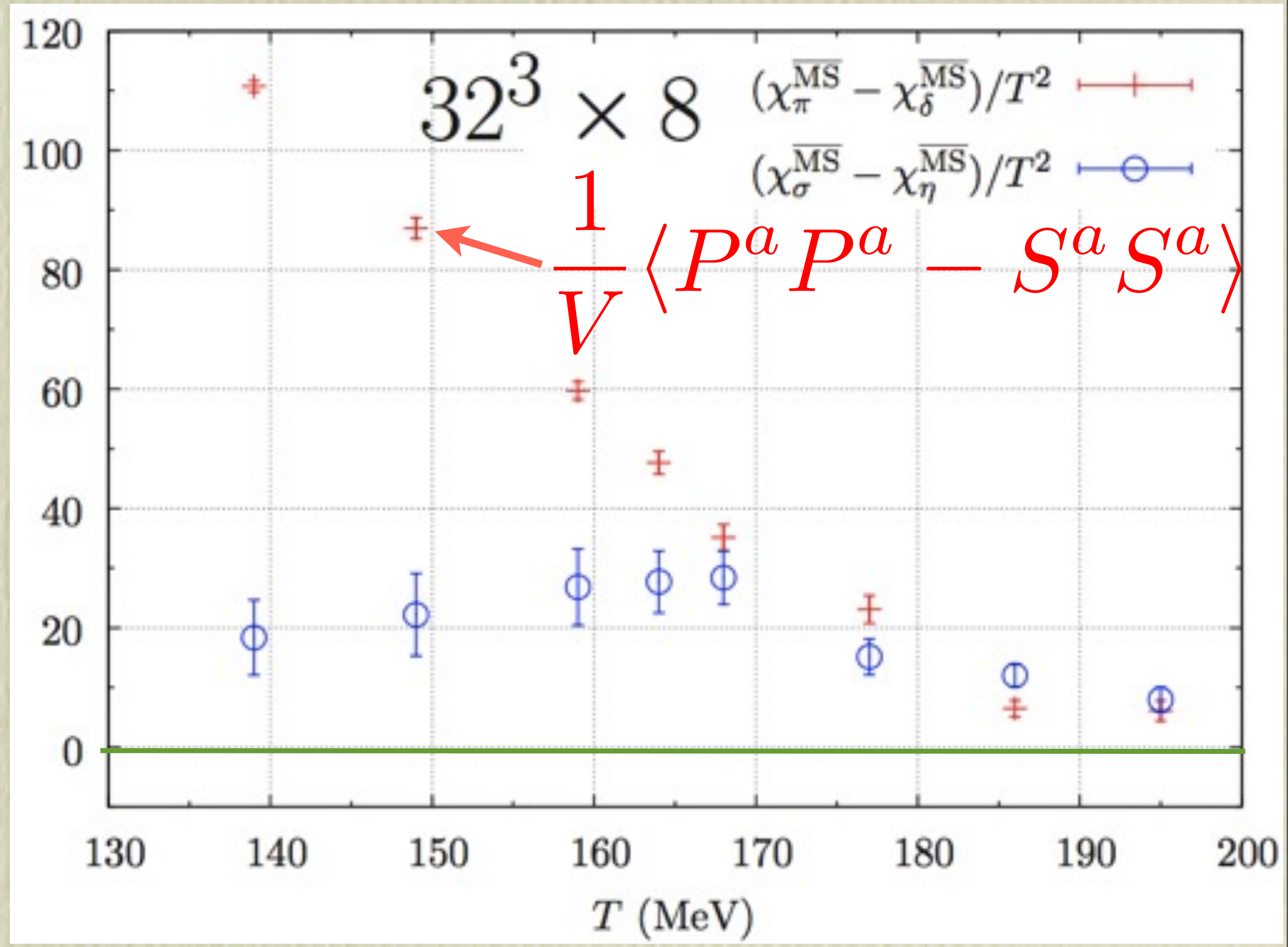

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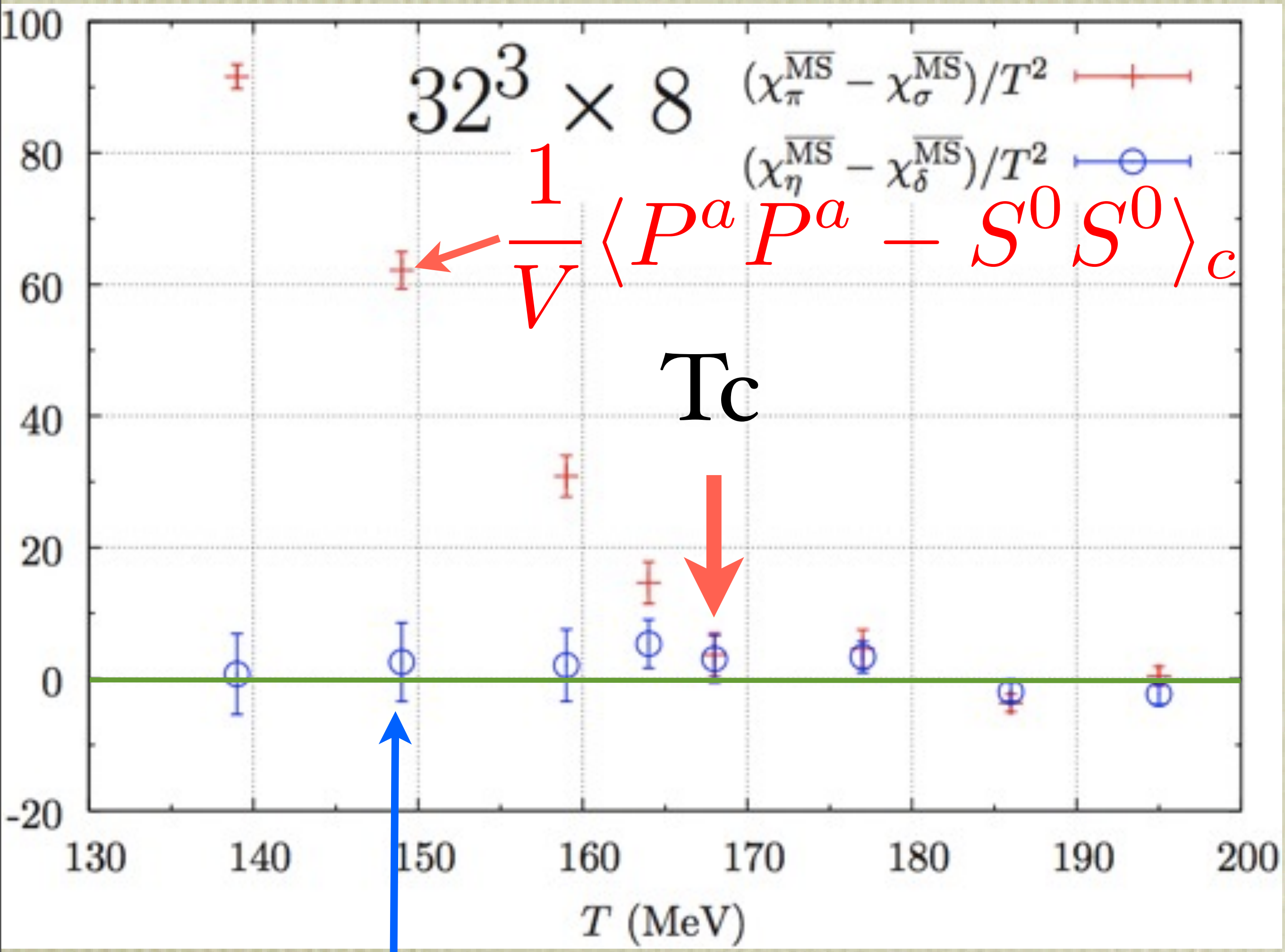

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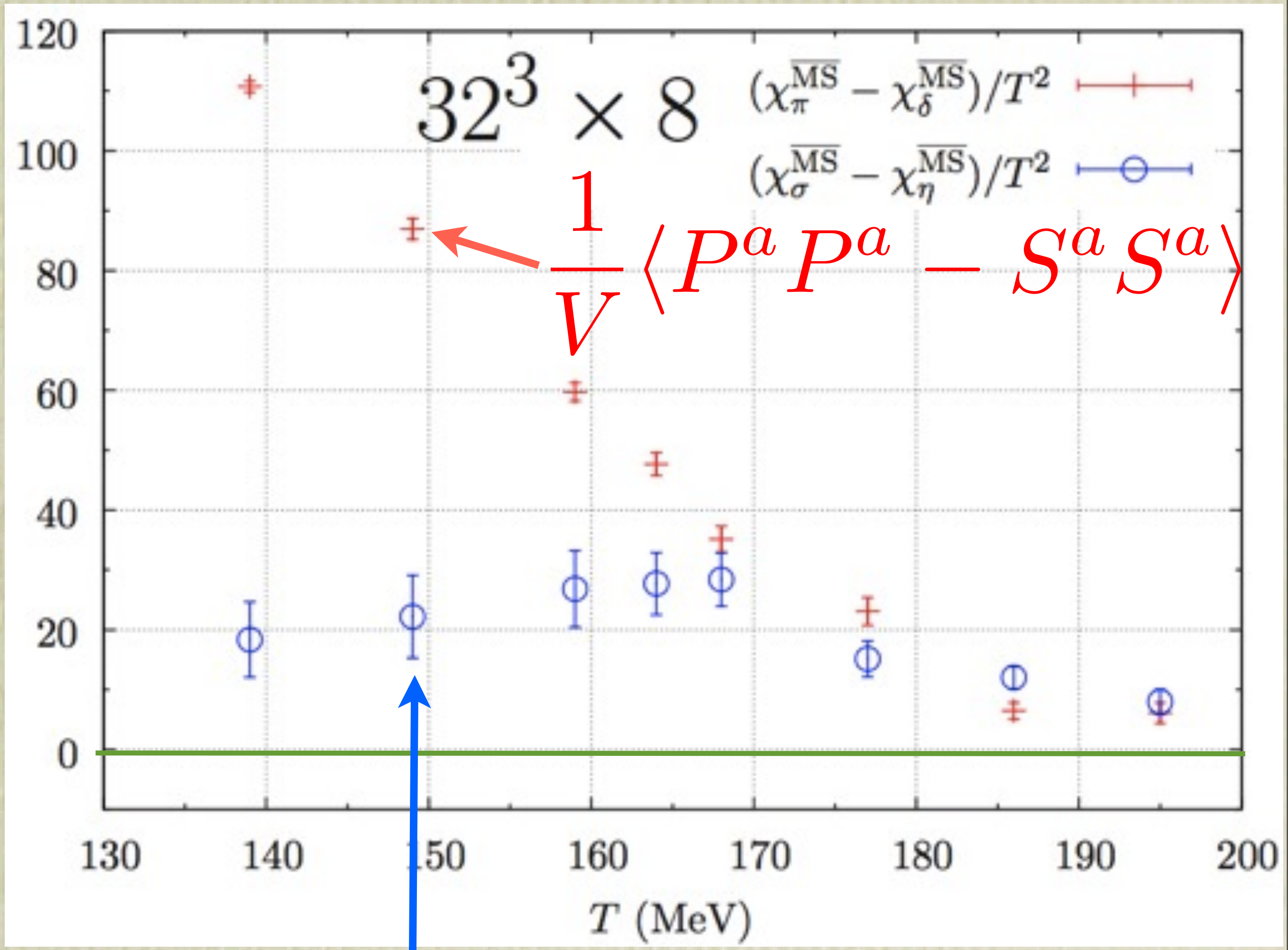

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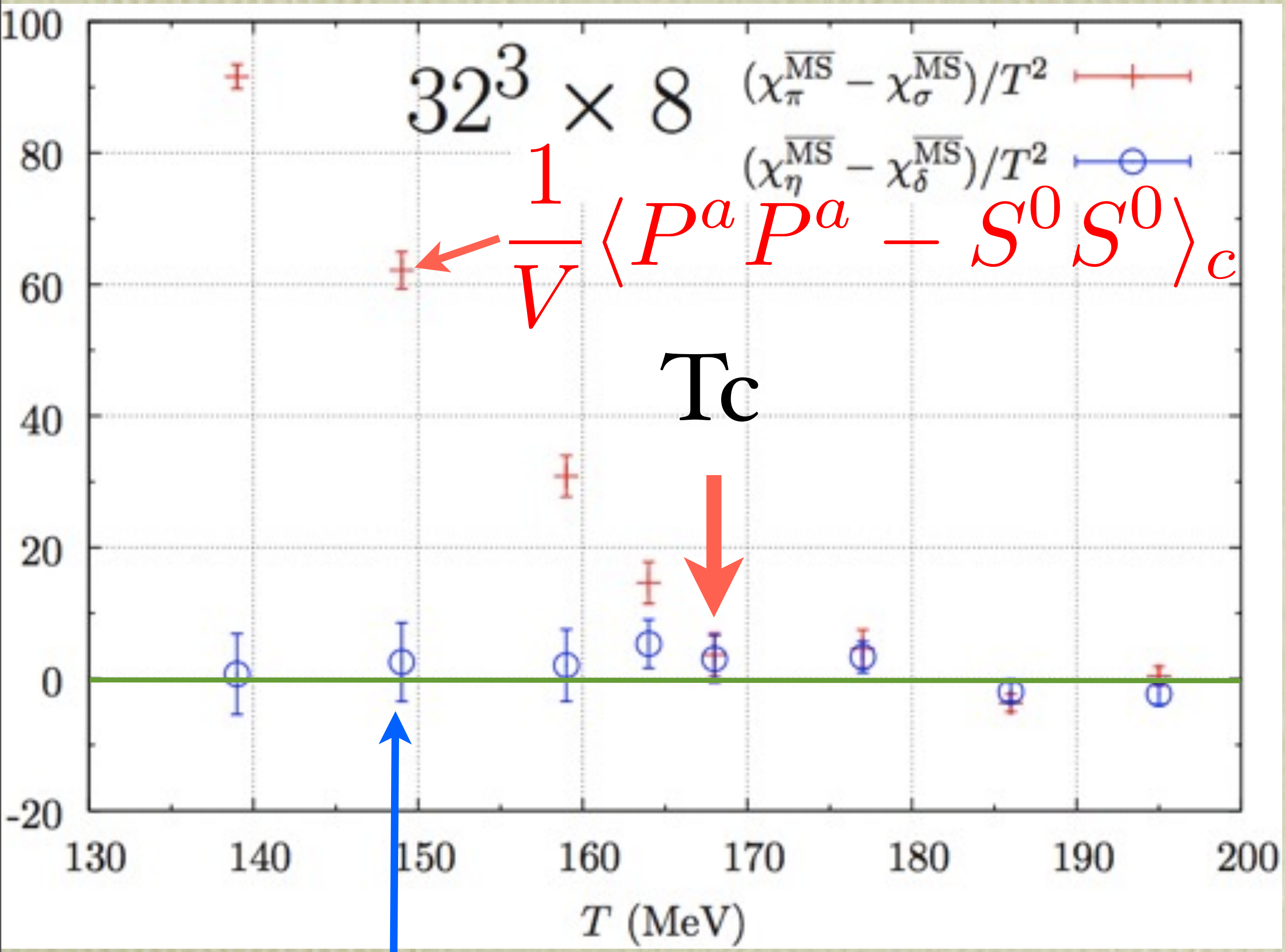

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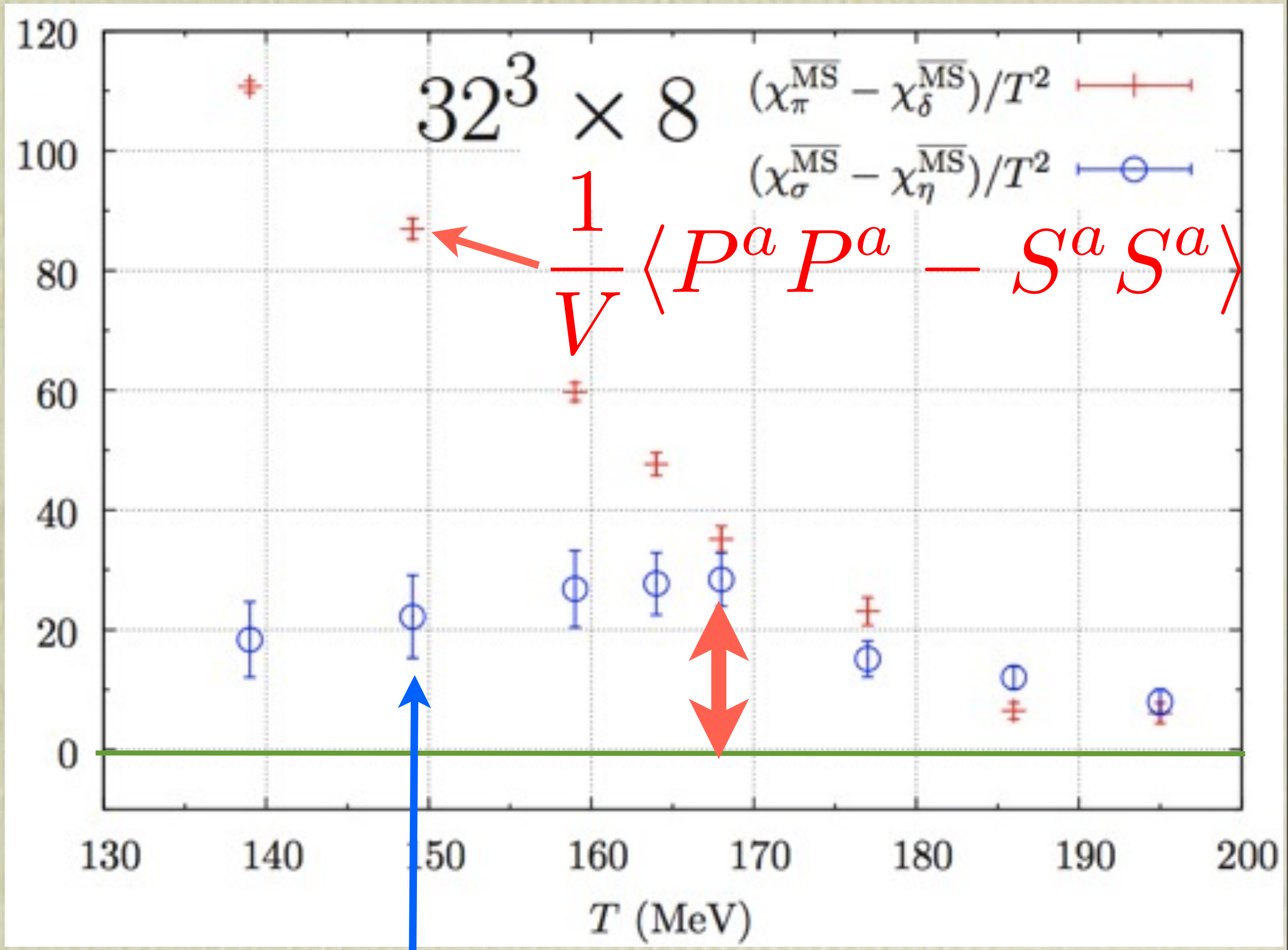

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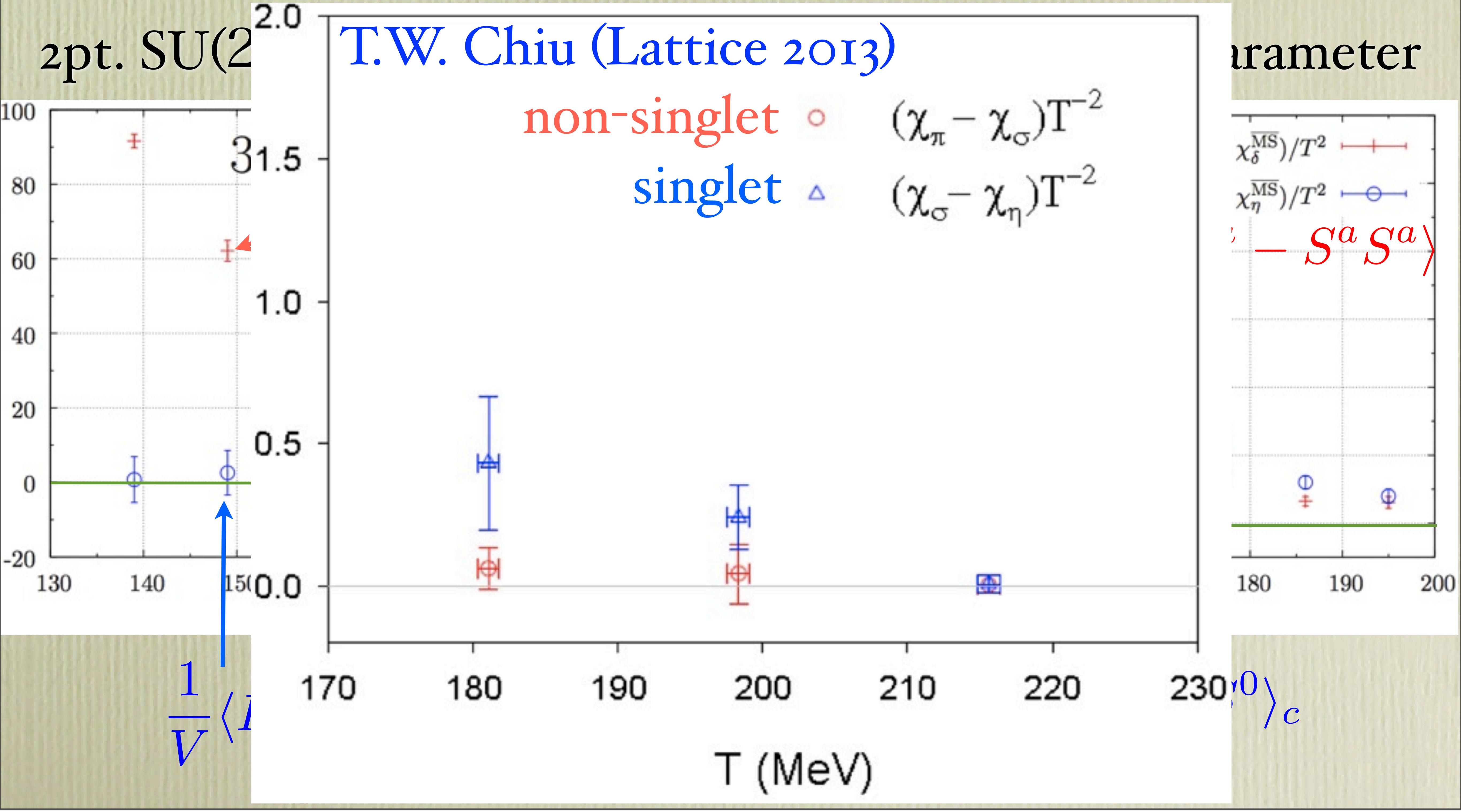


T.W. Chiu (Lattice 2013)

non-singlet ○ $(\chi_\pi - \chi_\sigma)T^{-2}$

singlet △ $(\chi_\sigma - \chi_\eta)T^{-2}$

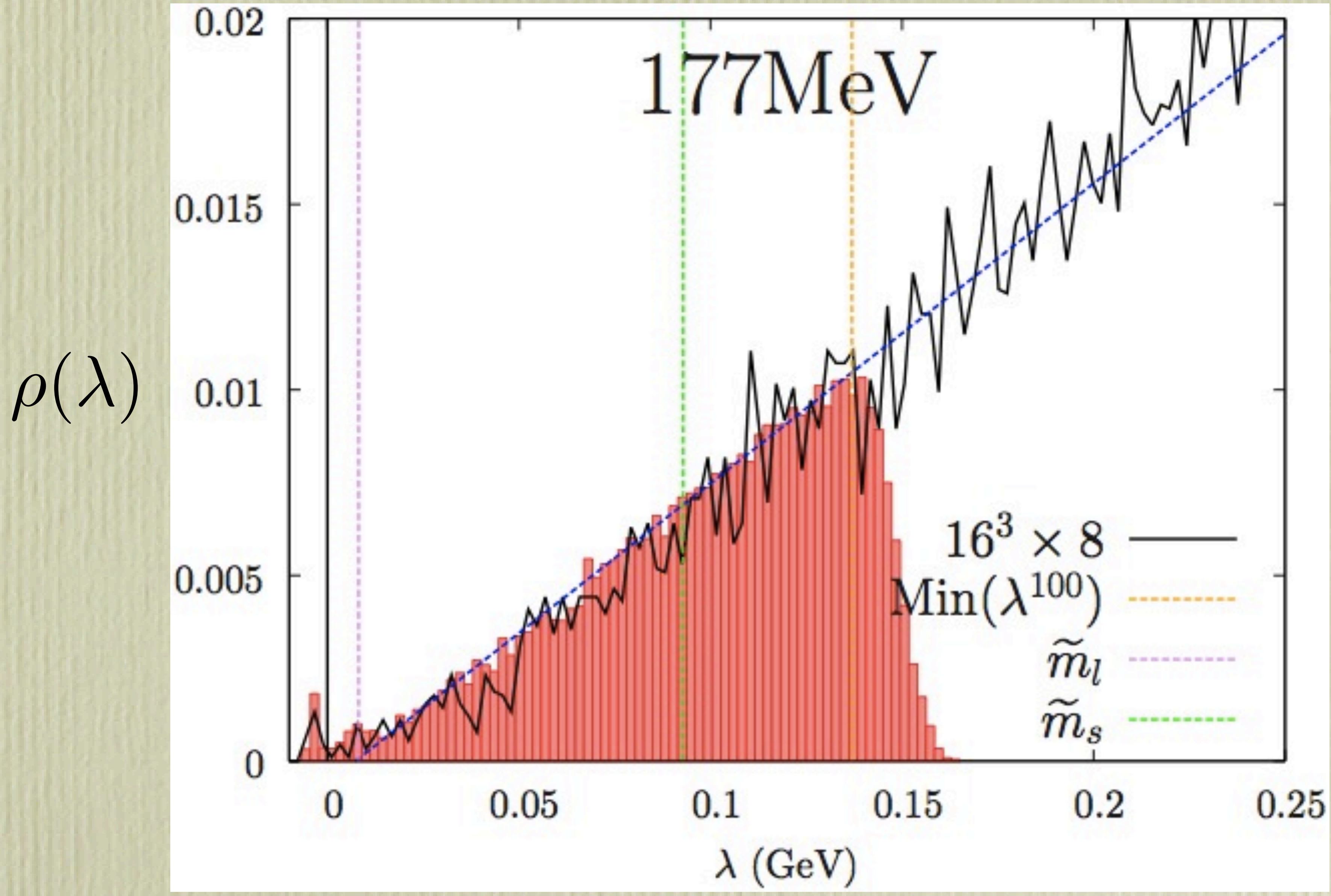
$- S^a S^a\rangle$

$\frac{1}{V}\langle I$ … $^0\rangle_c$

Lattice artifact?

Lattice artifact?

Eigenvalue distribution function



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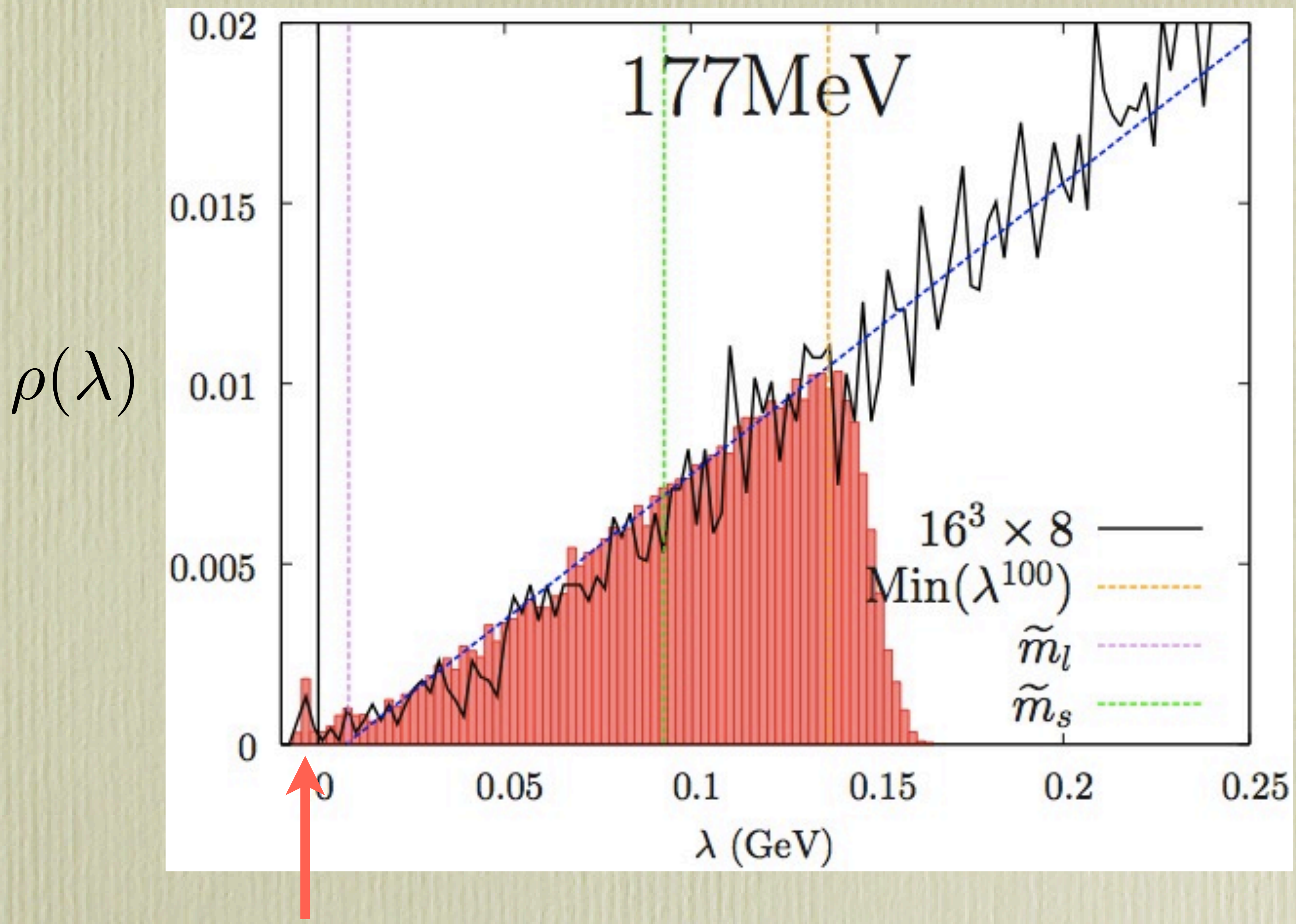

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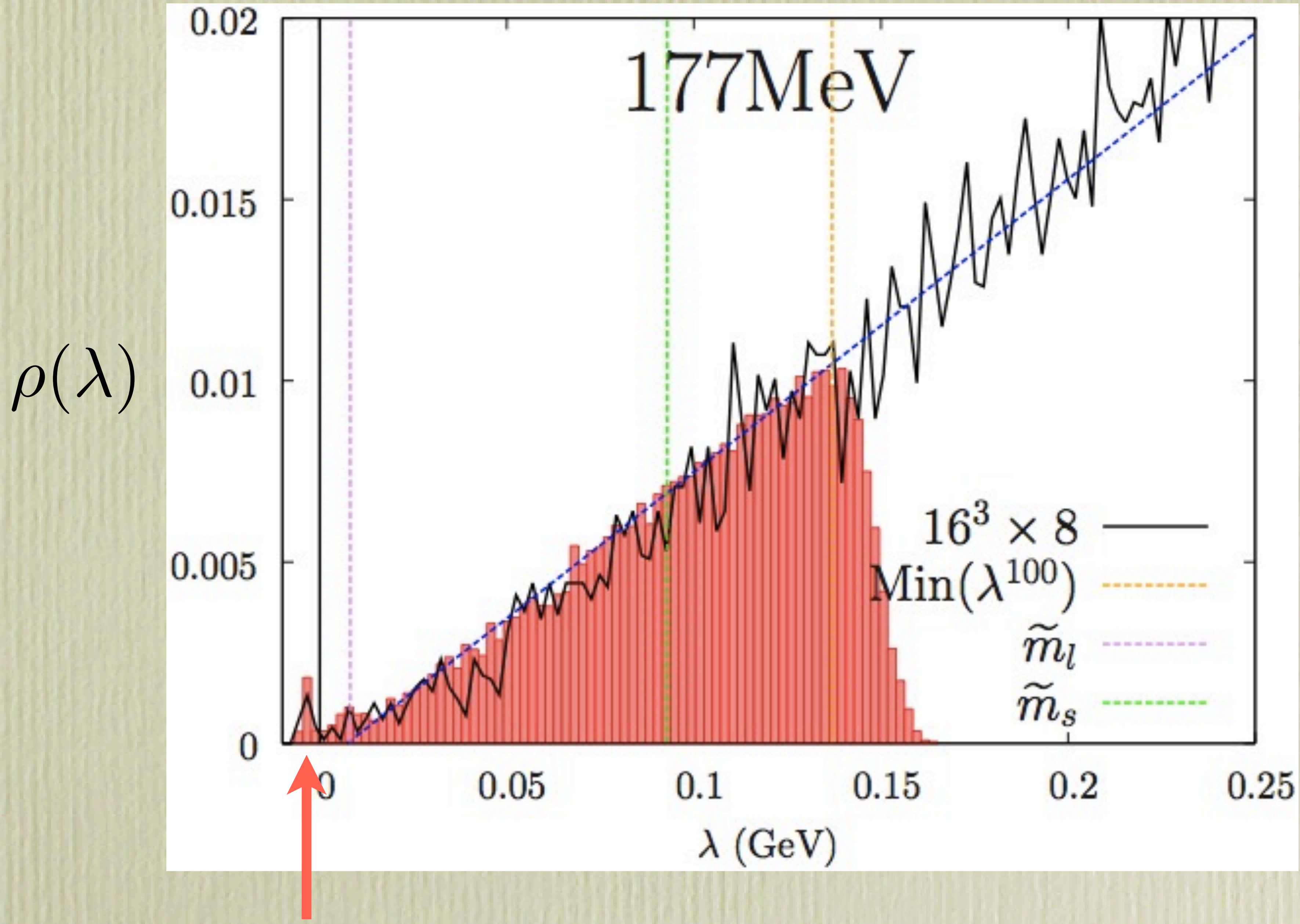

$\rho(\lambda)$

$m^2\delta(\lambda)$

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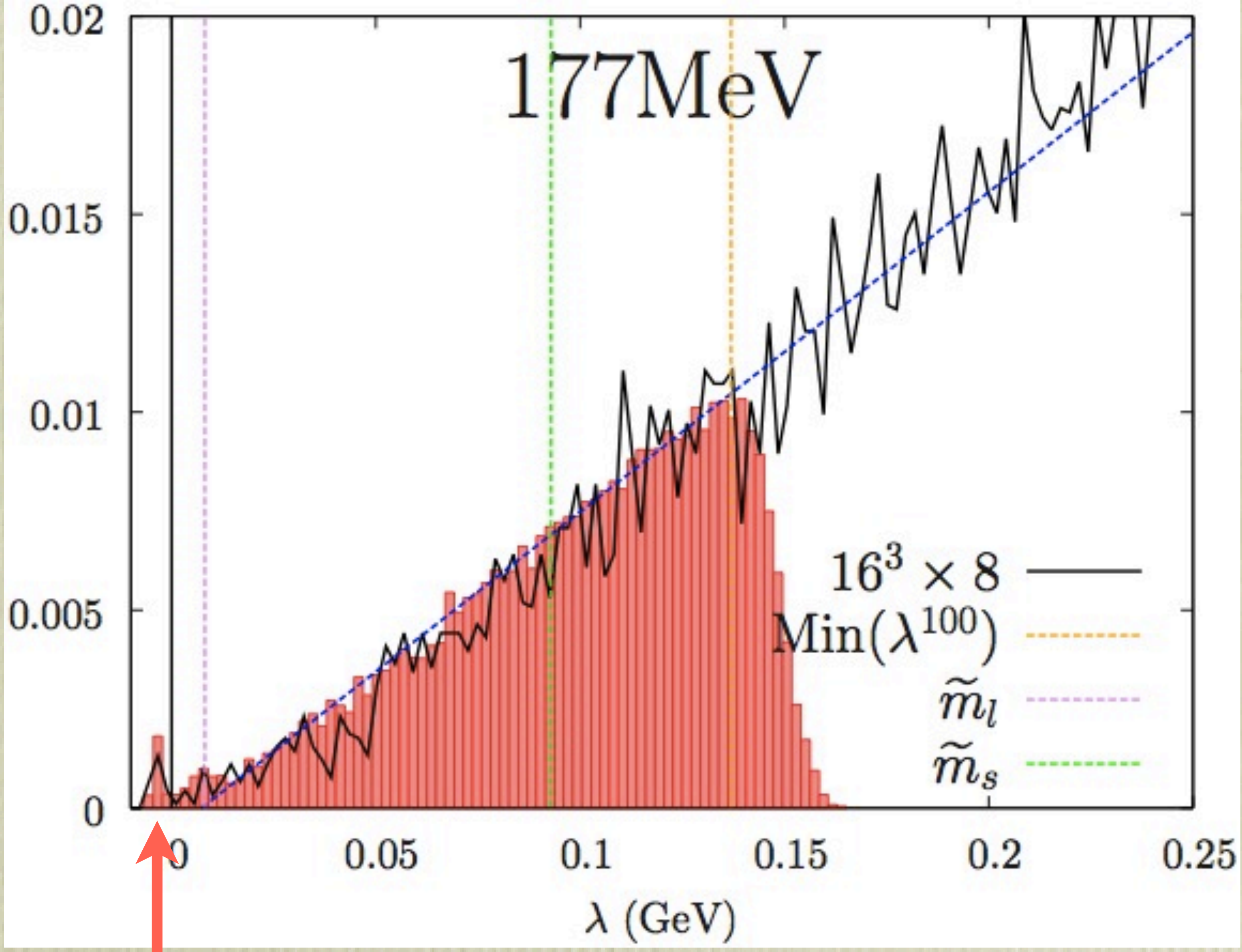

$m^2\delta(\lambda)$: dilute instanton gas

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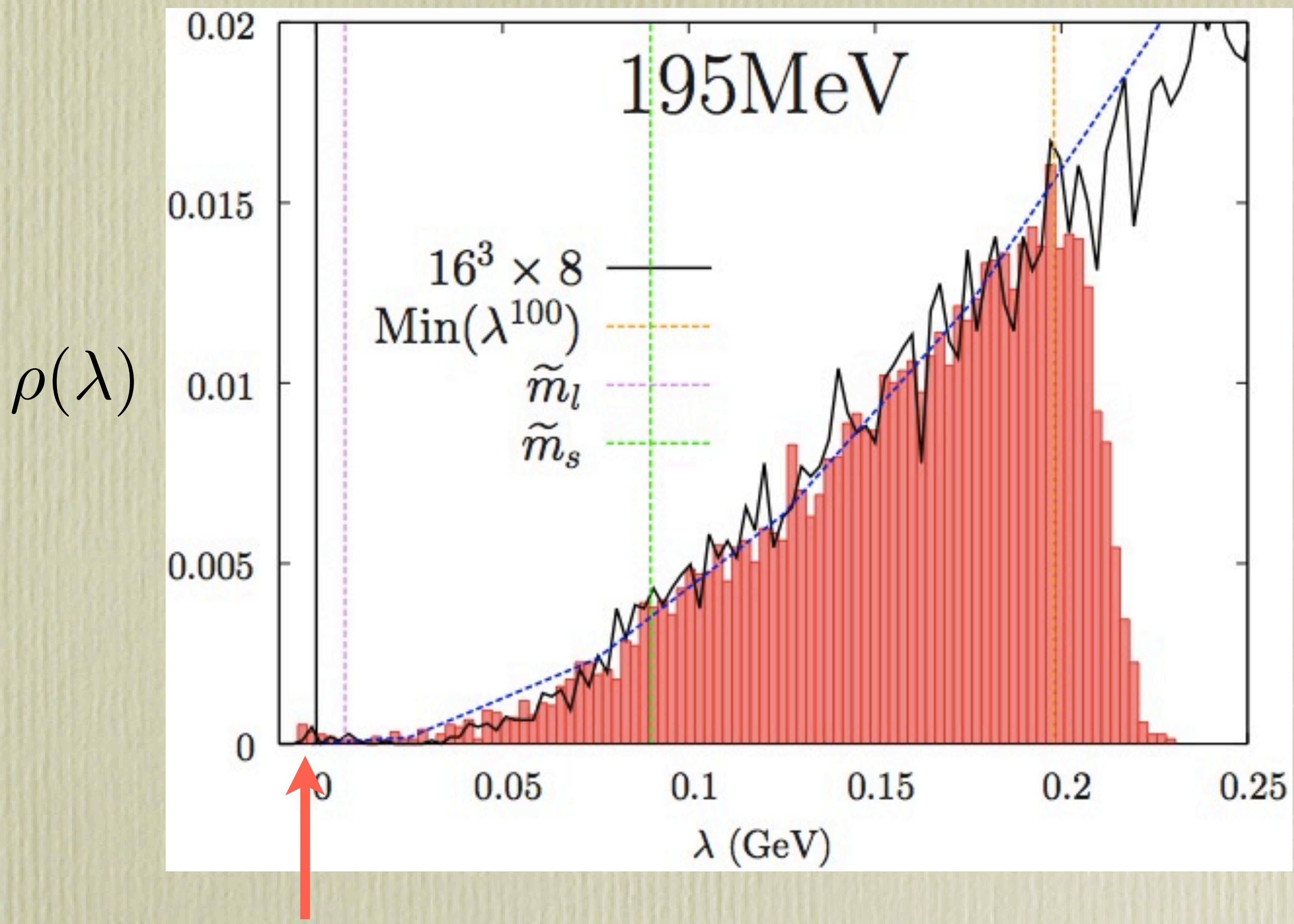

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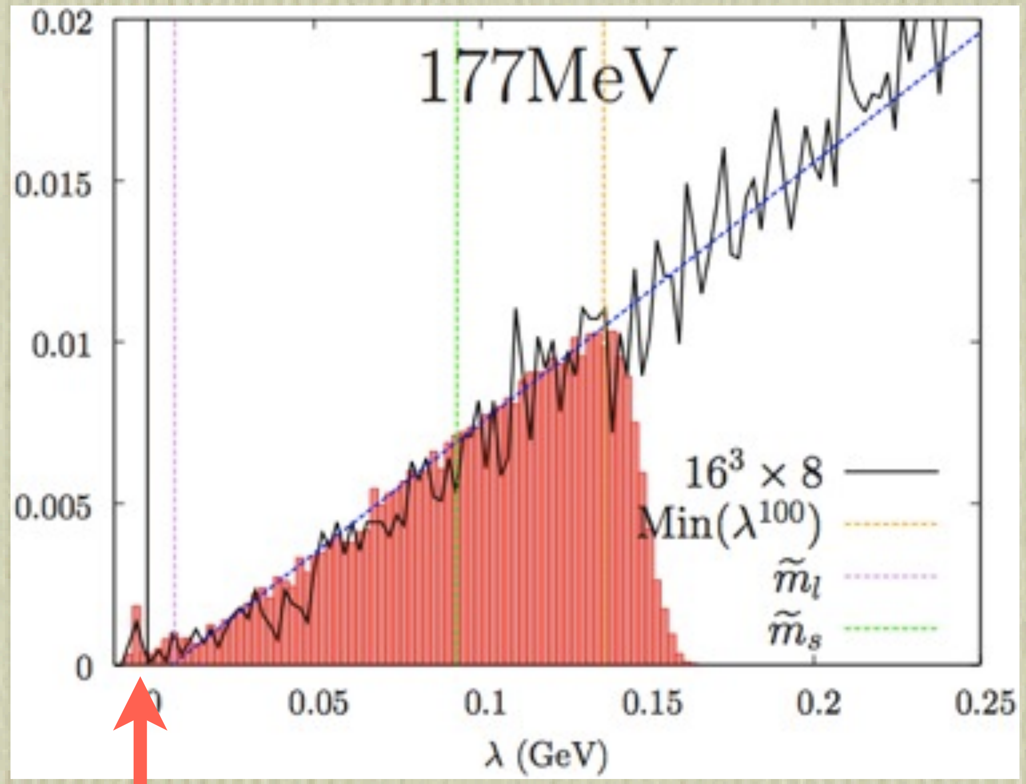

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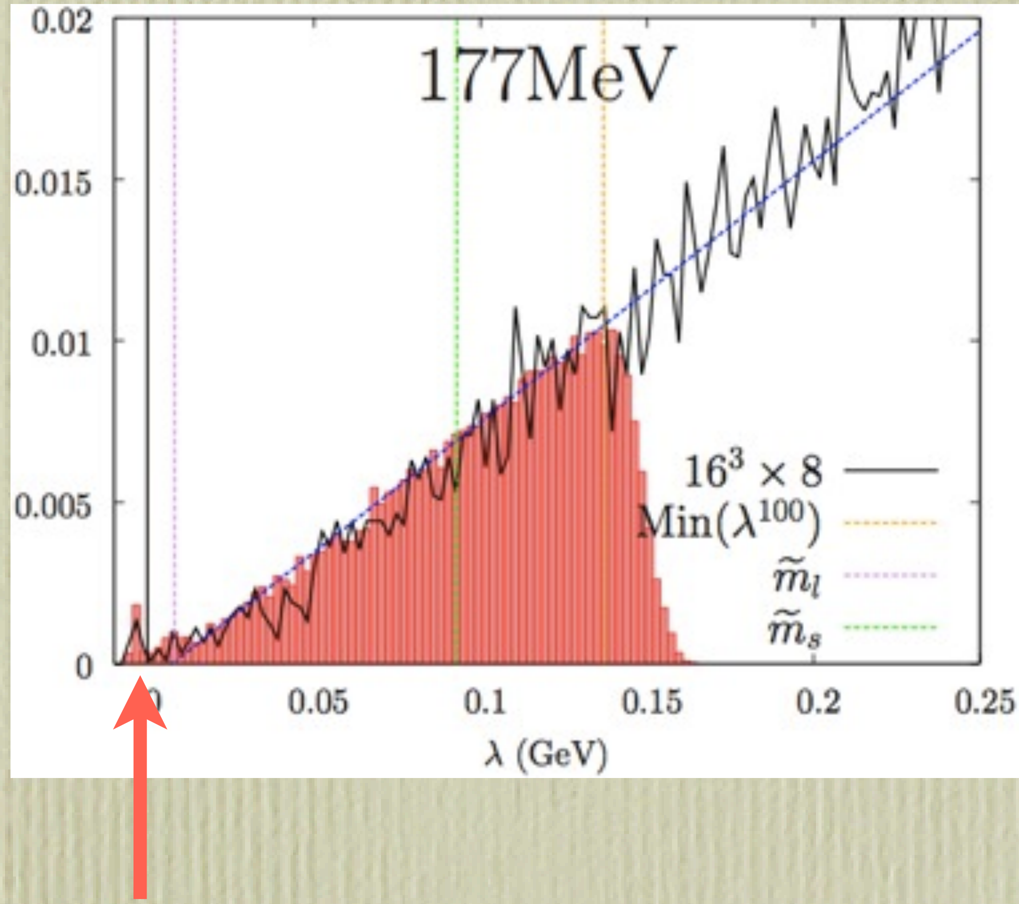

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$$\langle\bar{\psi}\psi\rangle \sim \int d\lambda \, \langle\rho(\lambda)\rangle \, \frac{2m}{\lambda^2 + m^2} \to 0$$

Lattice artifact?

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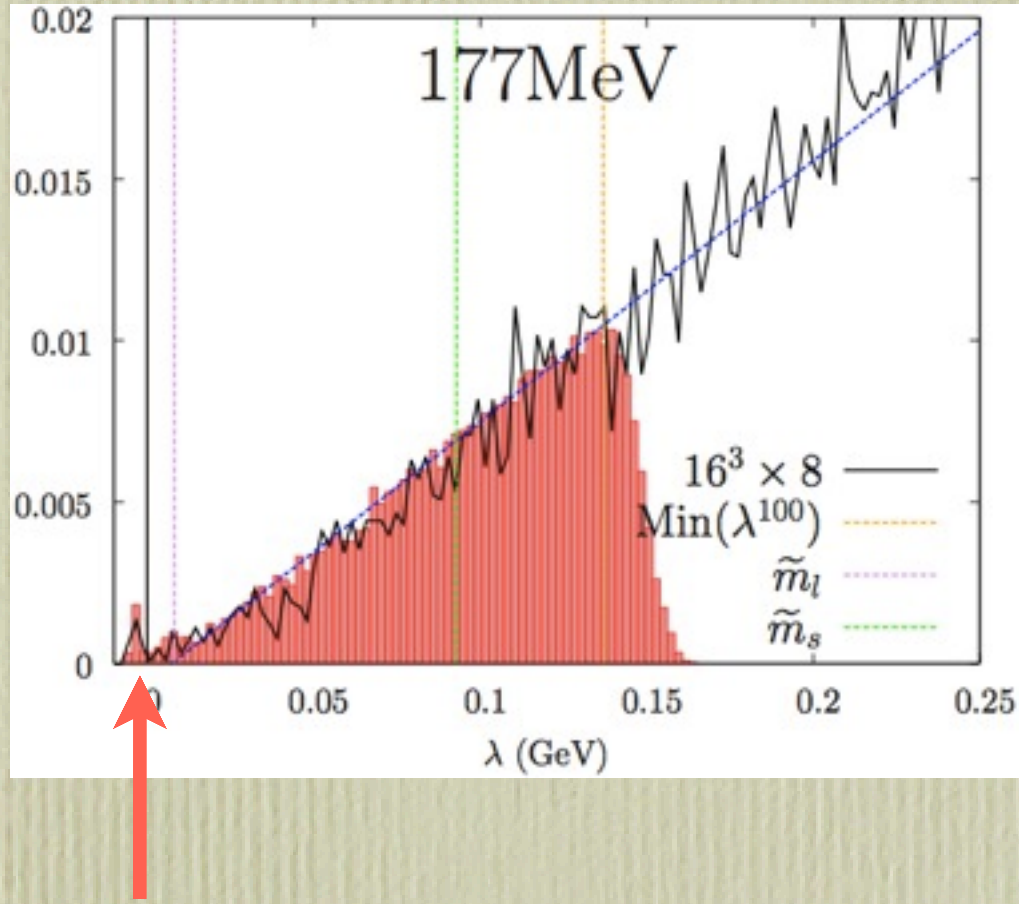

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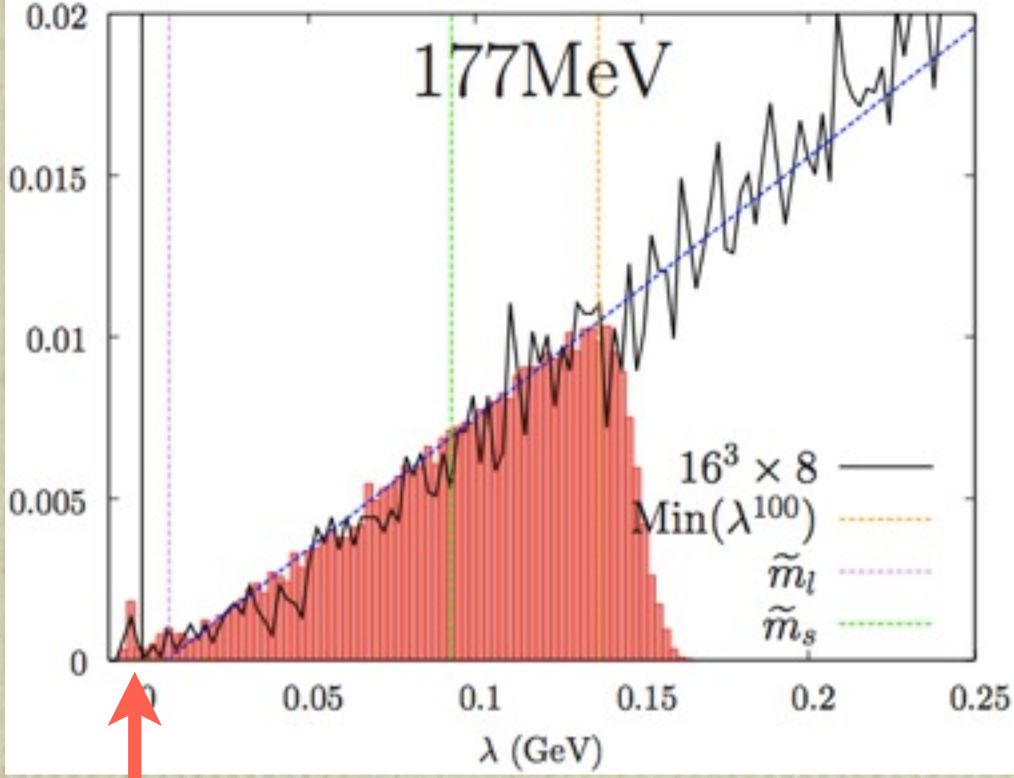

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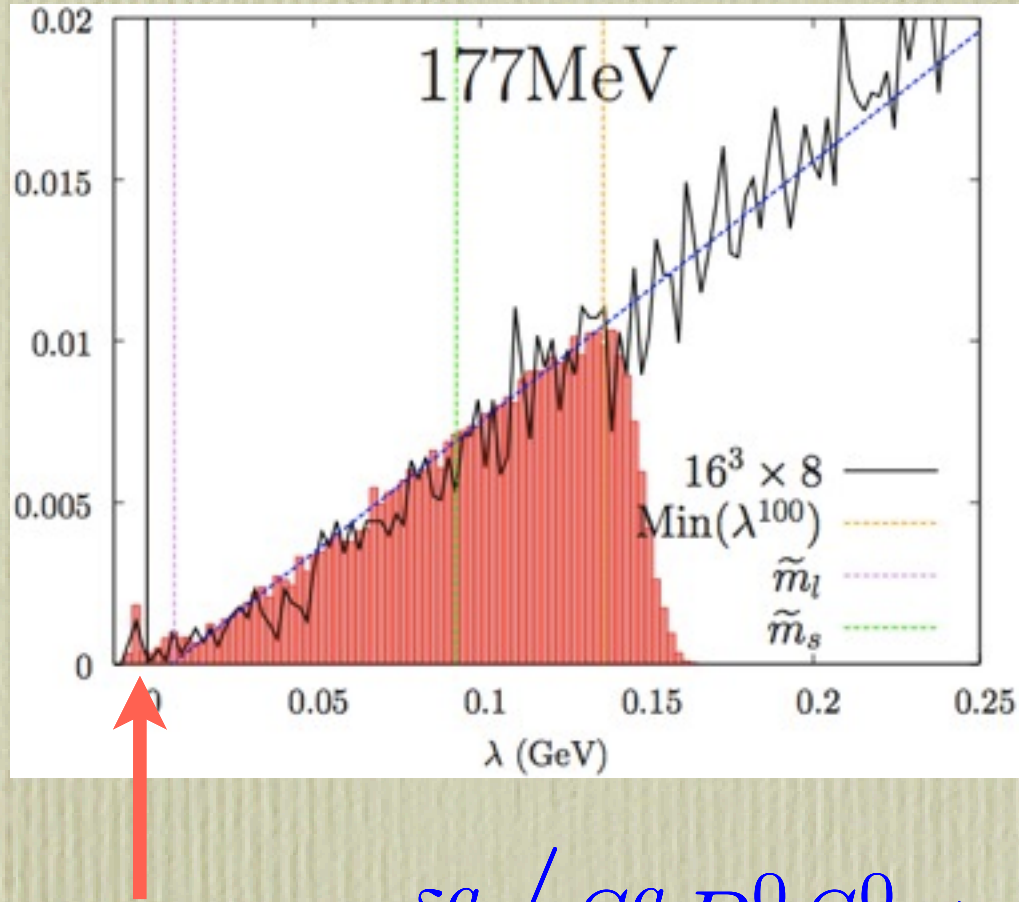

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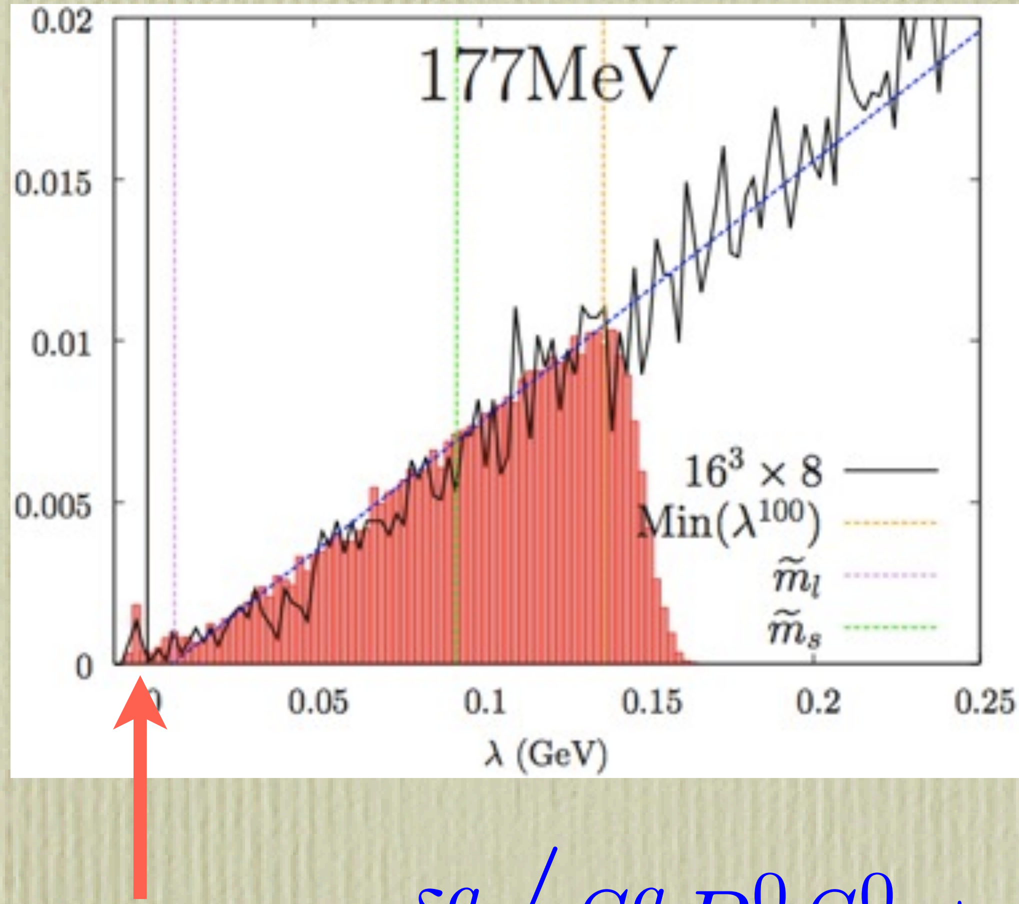

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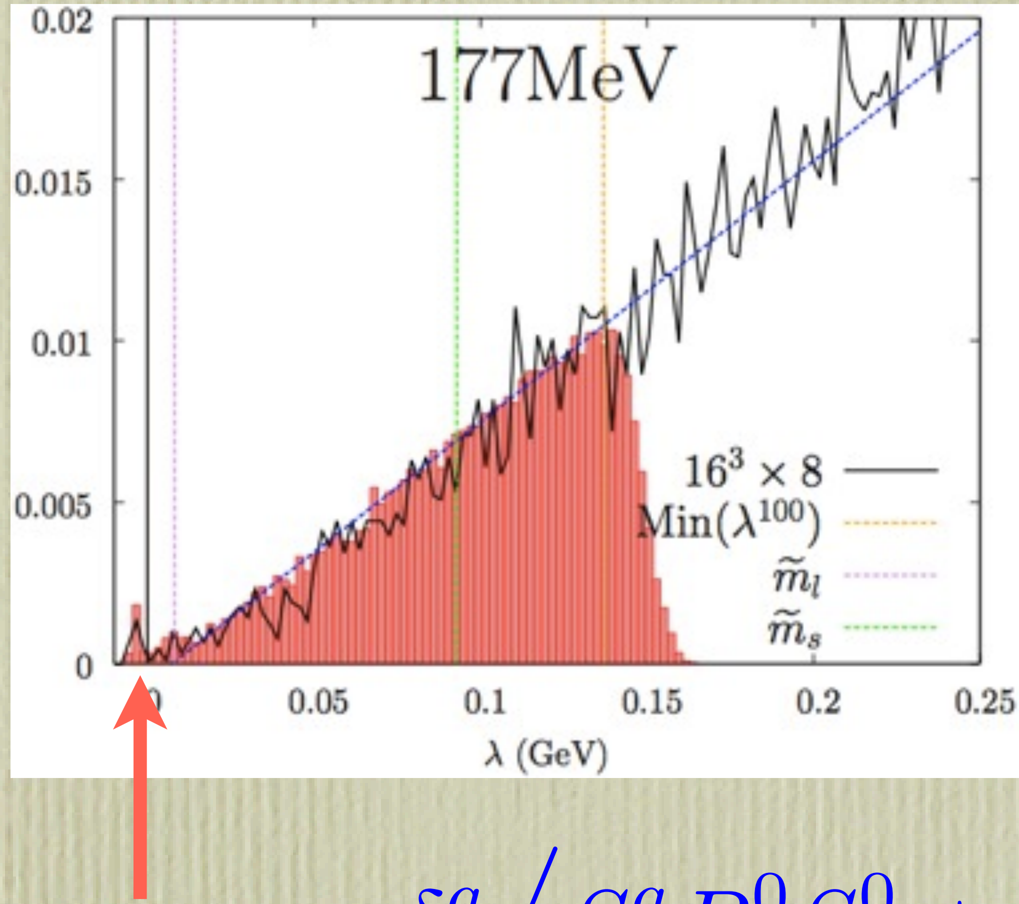

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canceled with $\left\langle \frac{Q^2}{m^3 V} \right\rangle$

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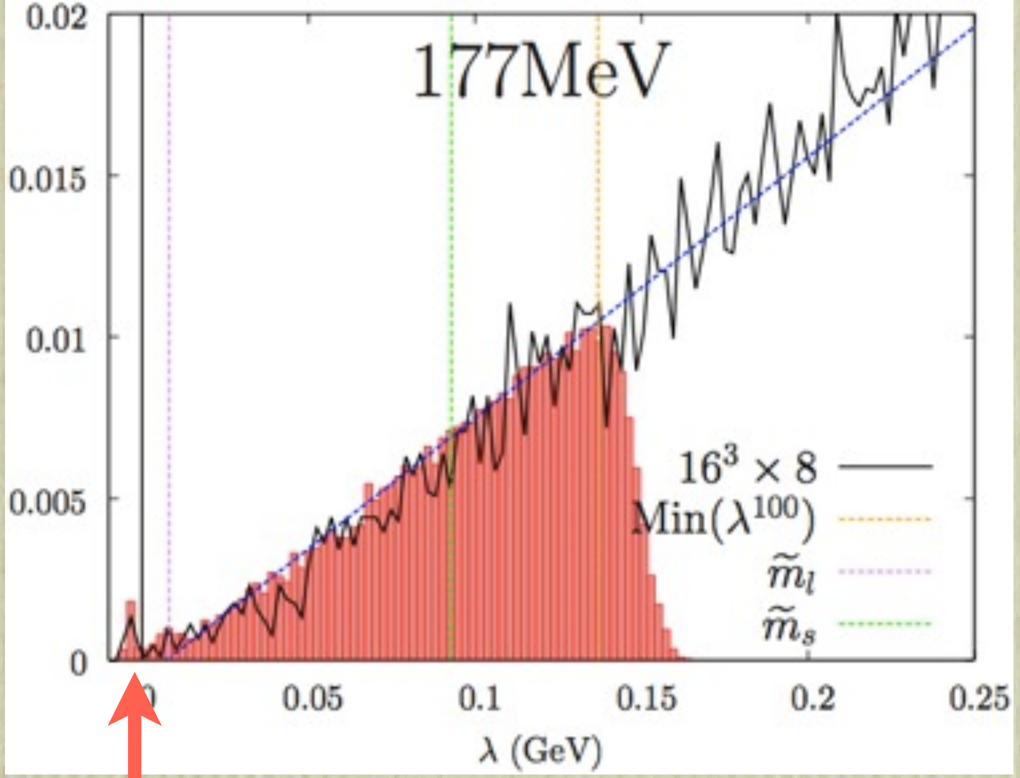

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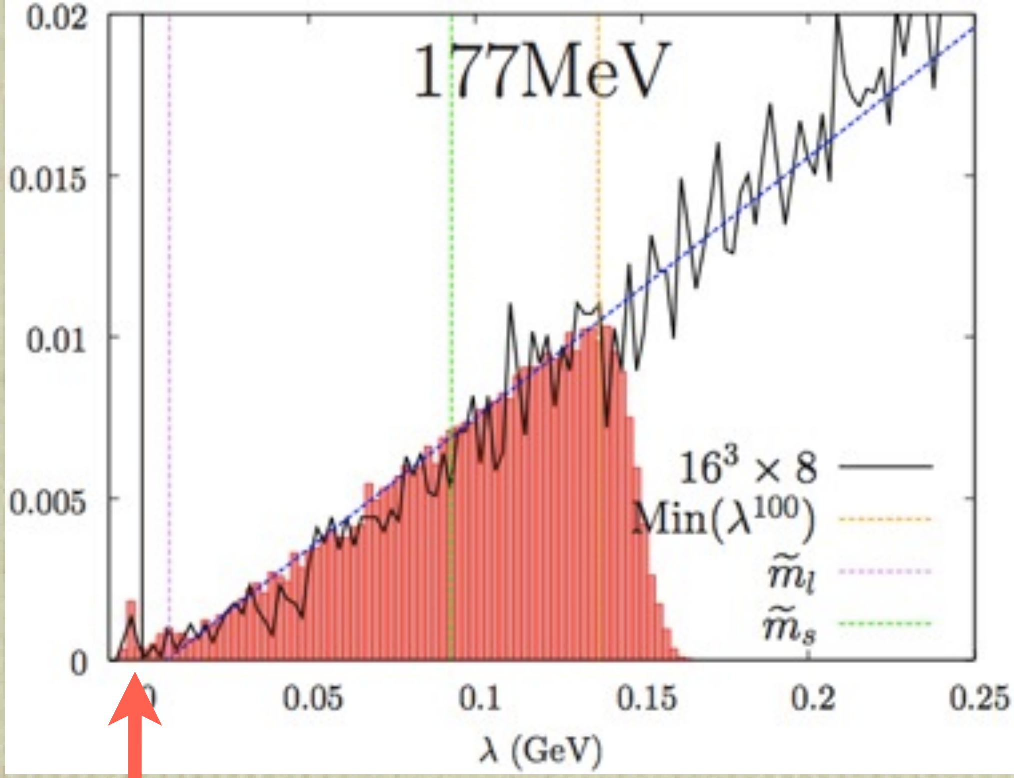
may be
a lattice artifact

canceled with $\left\langle \dfrac{Q^2}{m^3 V} \right\rangle$

Lattice artifact?

Eigenvalue distribution function



$$\langle\bar{\psi}\psi\rangle \sim \int d\lambda \, \langle\rho(\lambda)\rangle \, \frac{2m}{\lambda^2 + m^2} \to 0$$

$$\chi_{\pi-\sigma} \sim \int d\lambda \, \langle\rho(\lambda)\rangle \, \frac{2m^2}{\left(\lambda^2 + m^2\right)^2}$$

finite

$m^2\delta(\lambda)$

$$\delta^a \left\langle S^a P^0 S^0 + P^a \left(P^0\right)^2 - P^a \left(S^a\right)^2 \right\rangle$$

$$= \langle P^a S^a P^0 \rangle \sim \int d\lambda \, \langle\rho(\lambda)\rangle \, \frac{2m}{\left(\lambda^2 + m^2\right)^2} \to \infty$$

may be
a lattice artifact

canceled with $\left\langle \frac{Q^2}{m^3 V} \right\rangle$

$$\left\langle \frac{Q^2(A)}{V} \right\rangle = \mathcal{O}(m^2)$$

Lattice artifact?

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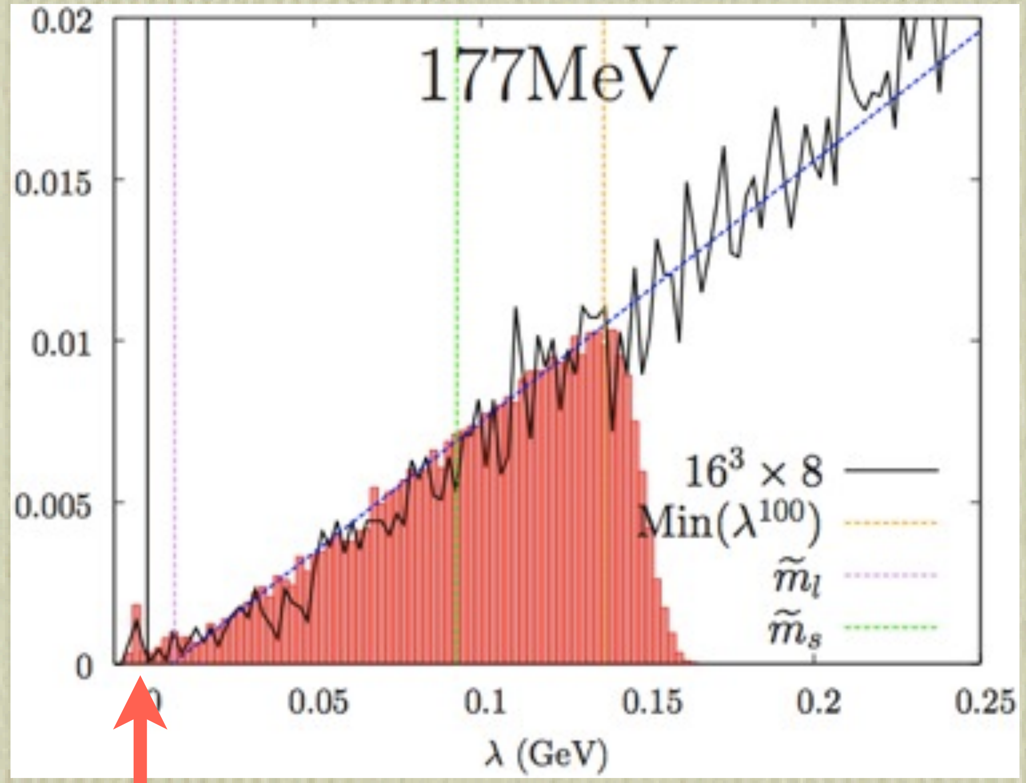

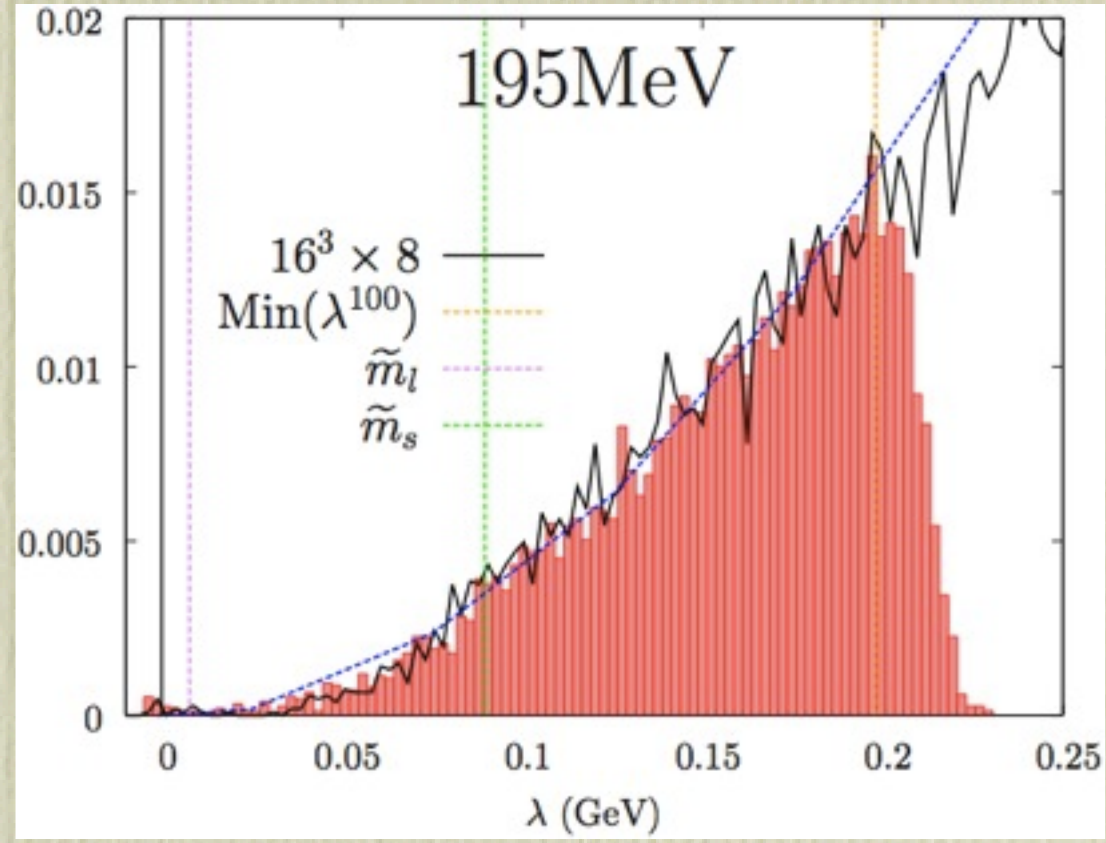

$m^2\delta(\lambda)$

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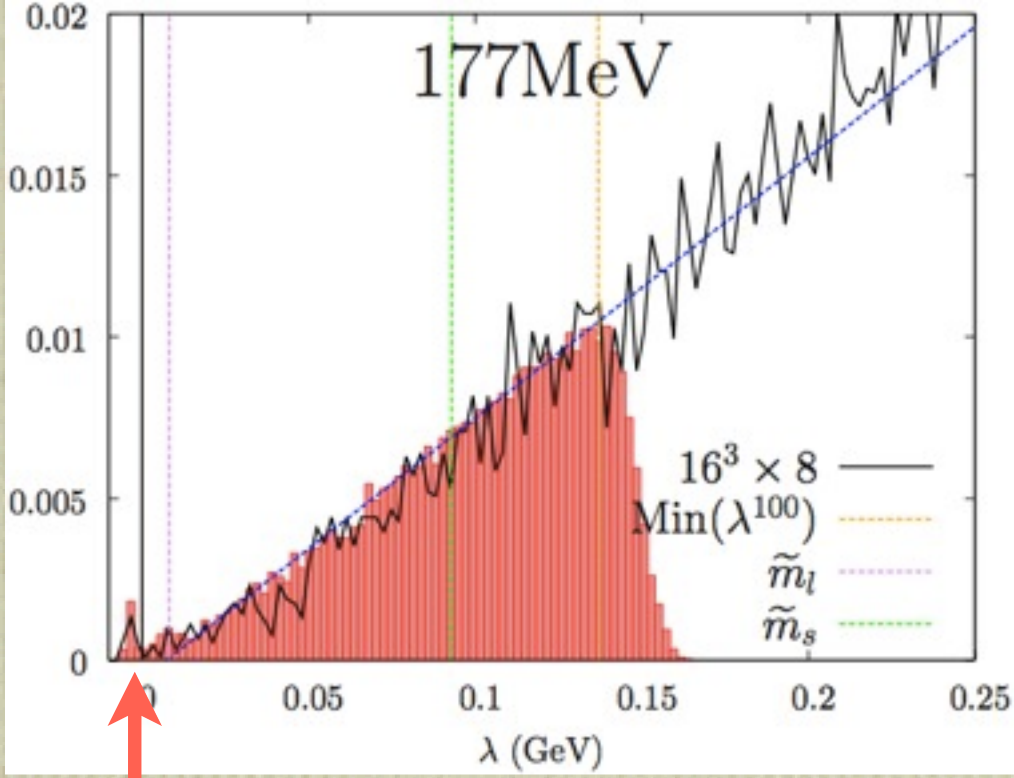
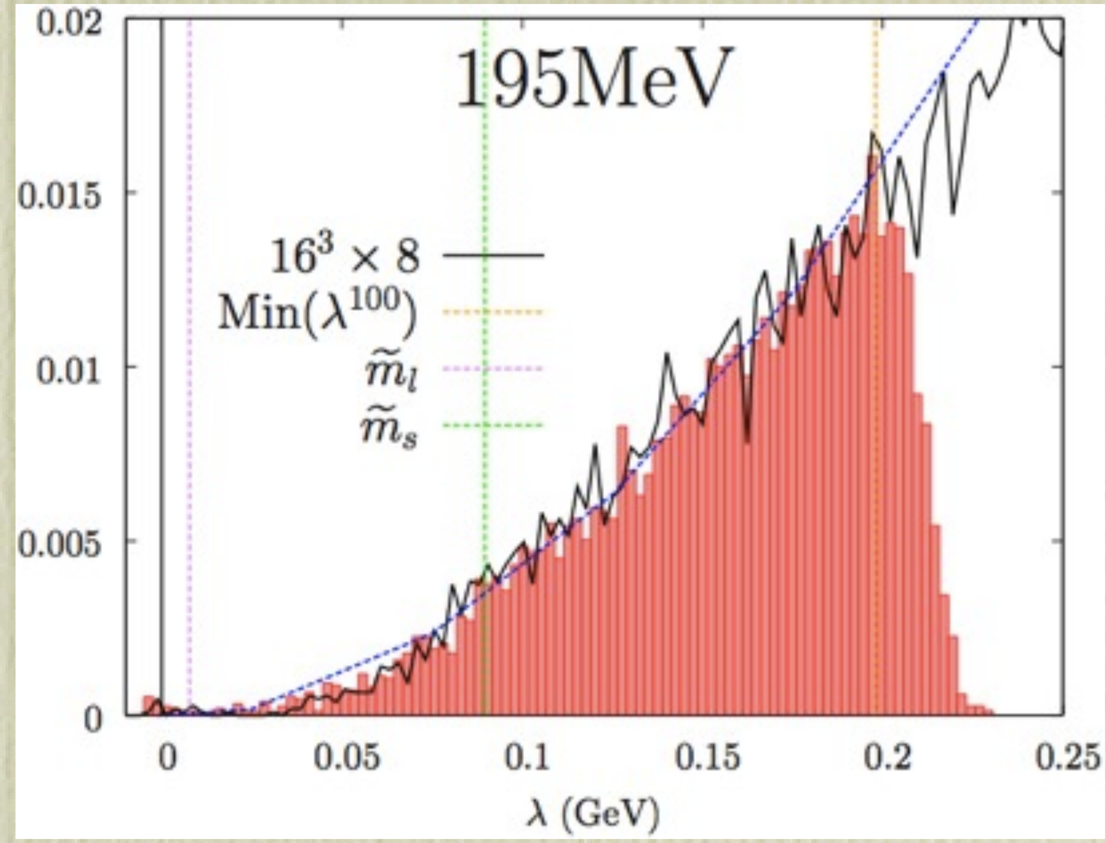
Lattice artifact?

Eigenvalue distribution function





$m^2\delta(\lambda)$

may be
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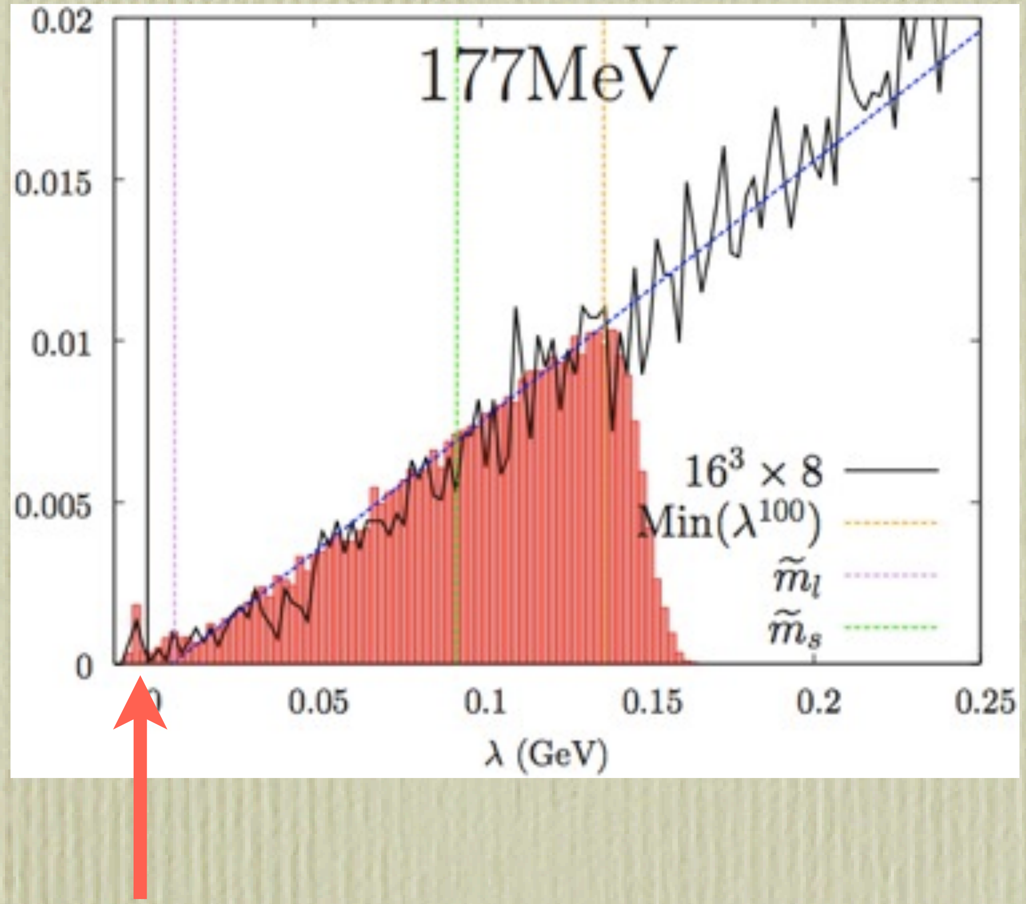
Nt: fixed
β→large

Lattice artifact?

Eigenvalue distribution function



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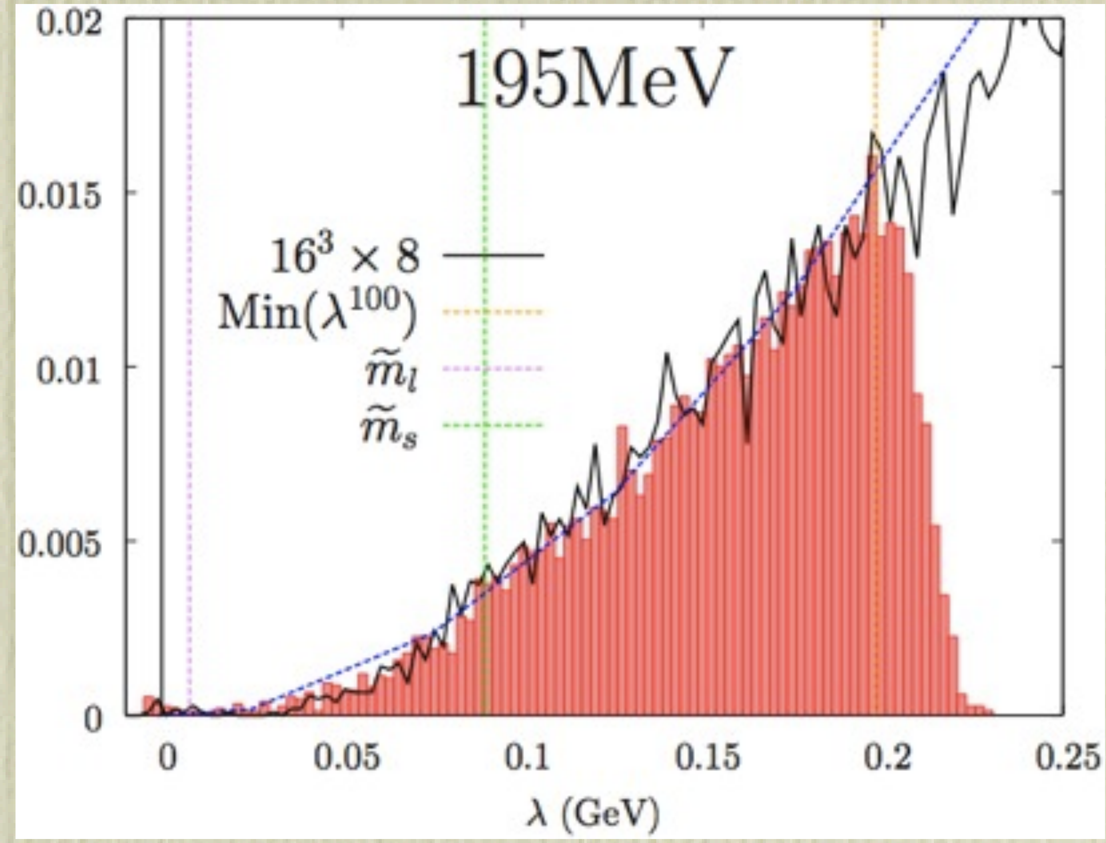

Nt: fixed

β→large

may be
a lattice artifact

cannot distinguish a high T effect
from a lattice artifact

Is this consistent with our result?

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SU(2)xSU(2) restoration at Tc,
but U(1)A breaking at Tc

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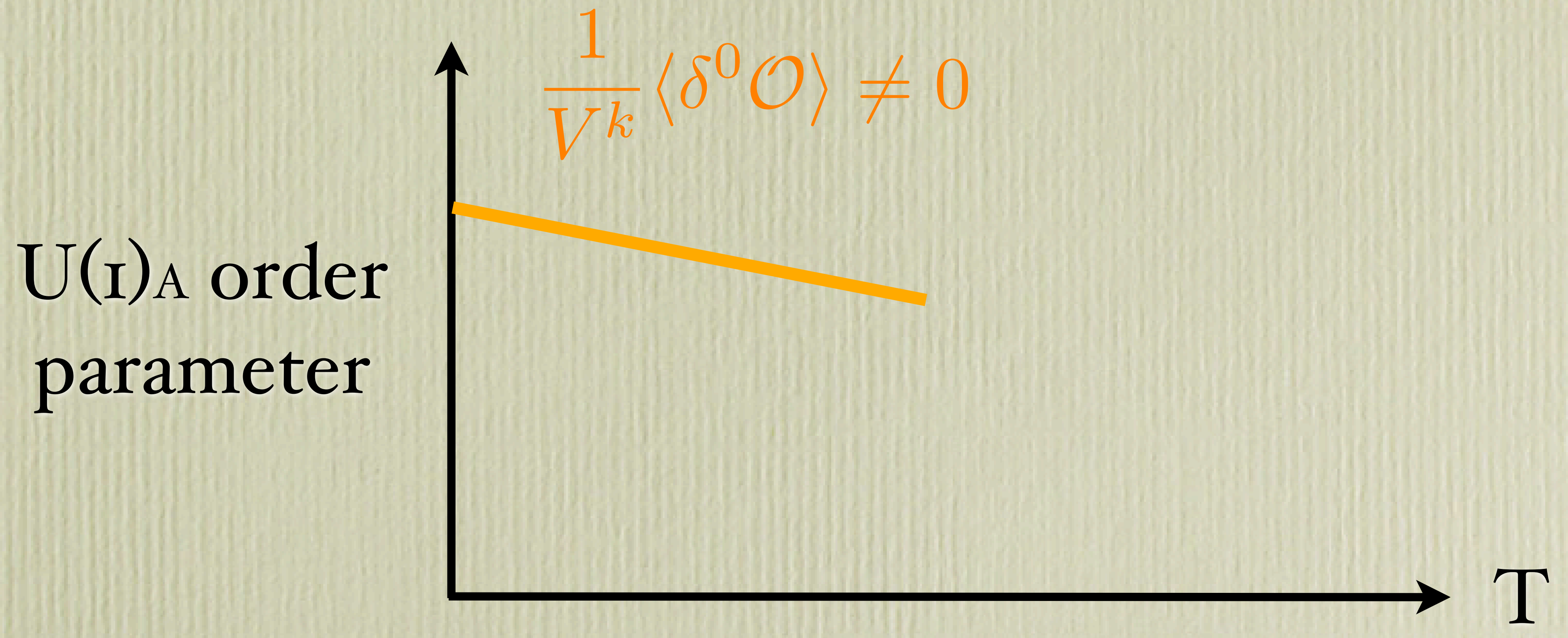

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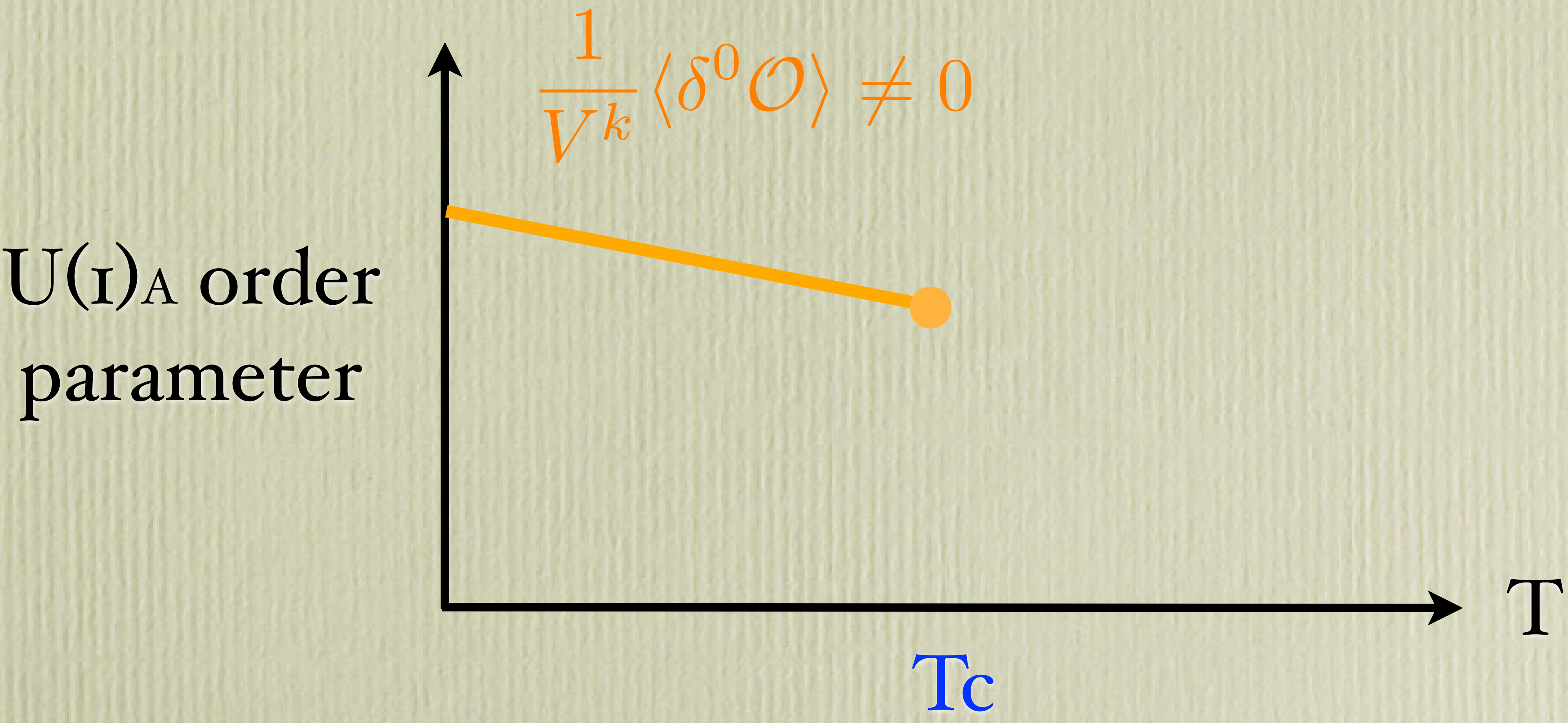

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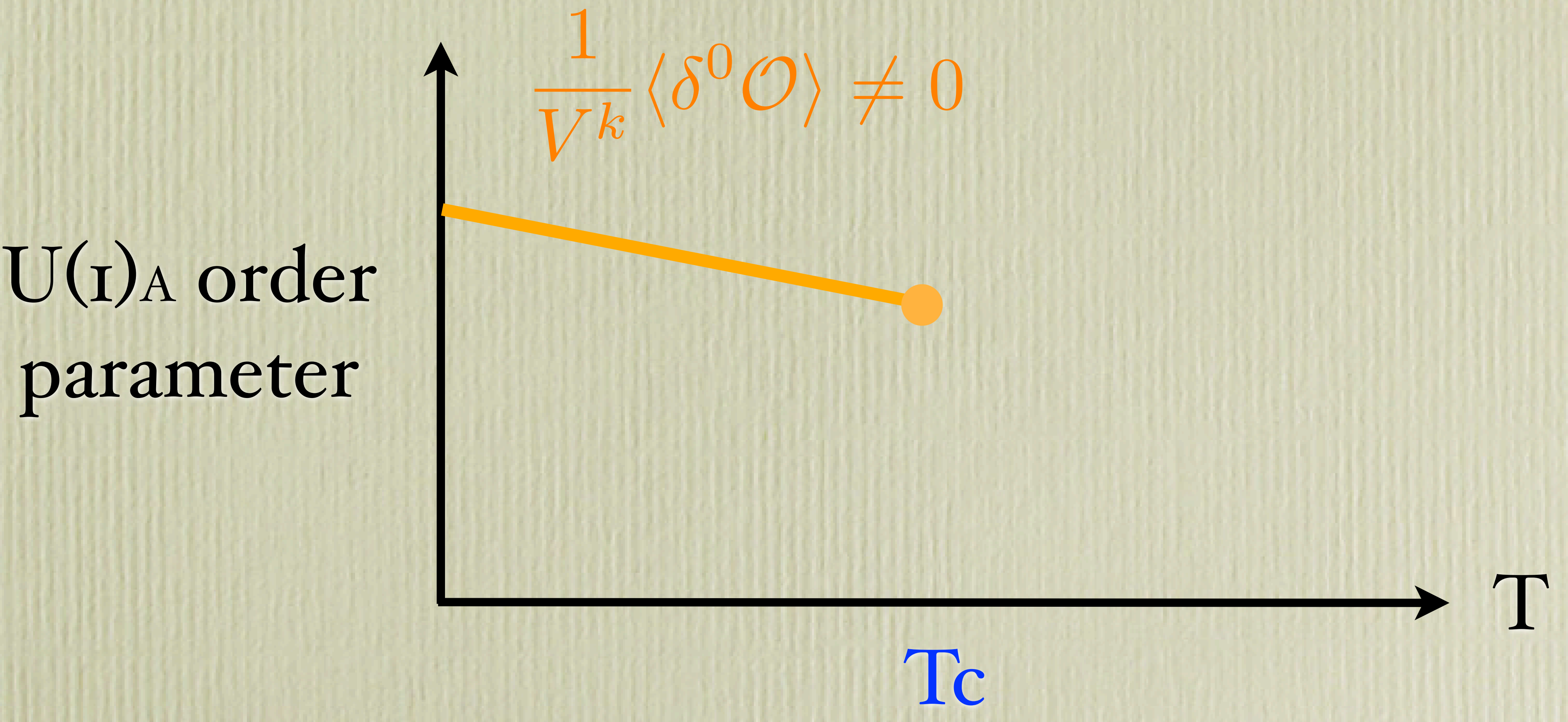

$$\frac{1}{V^k}\langle\delta^0\mathcal{O}\rangle \neq 0$$

Our requirement:

Is this consistent with our result?

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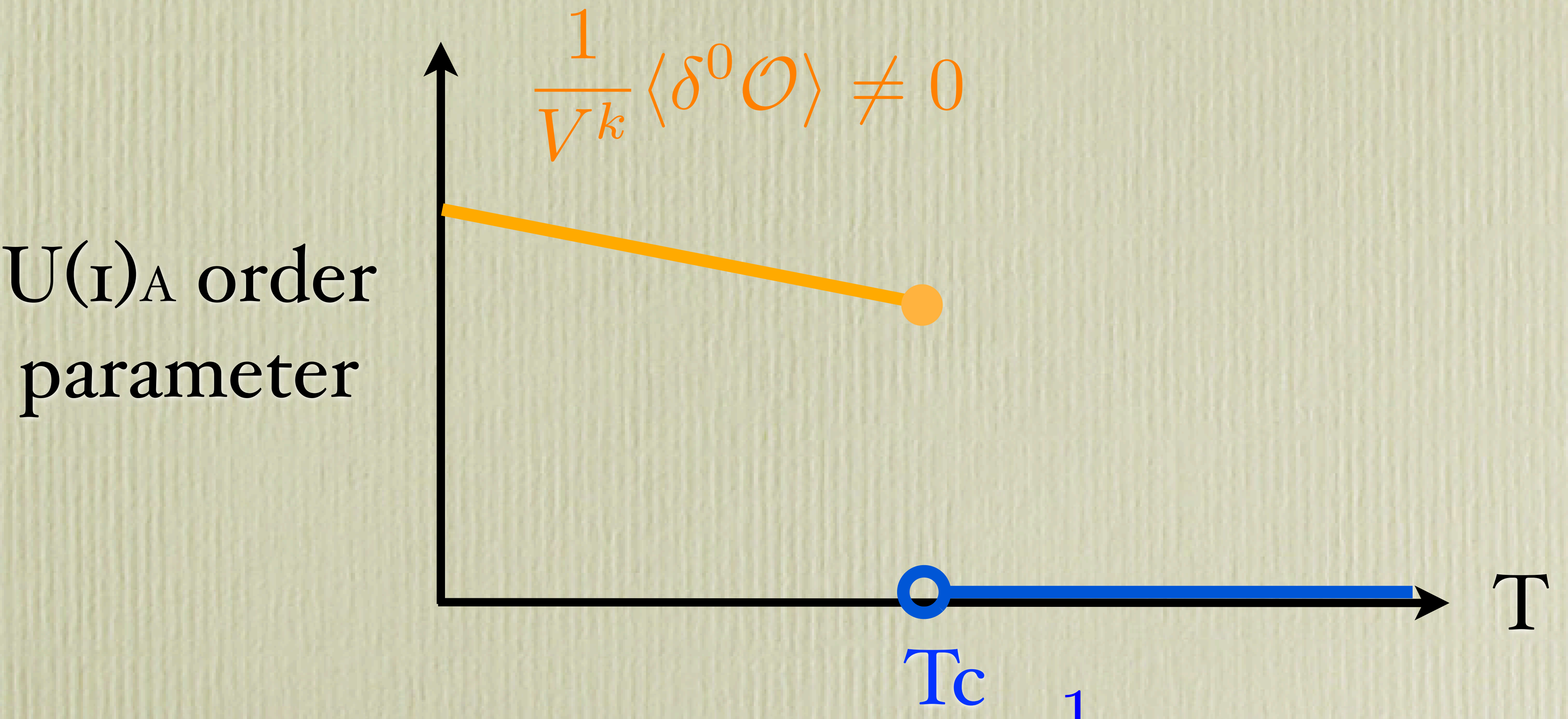

Our requirement: $\frac{1}{V^k}\langle\delta^0\mathcal{O}\rangle = 0$ just above Tc

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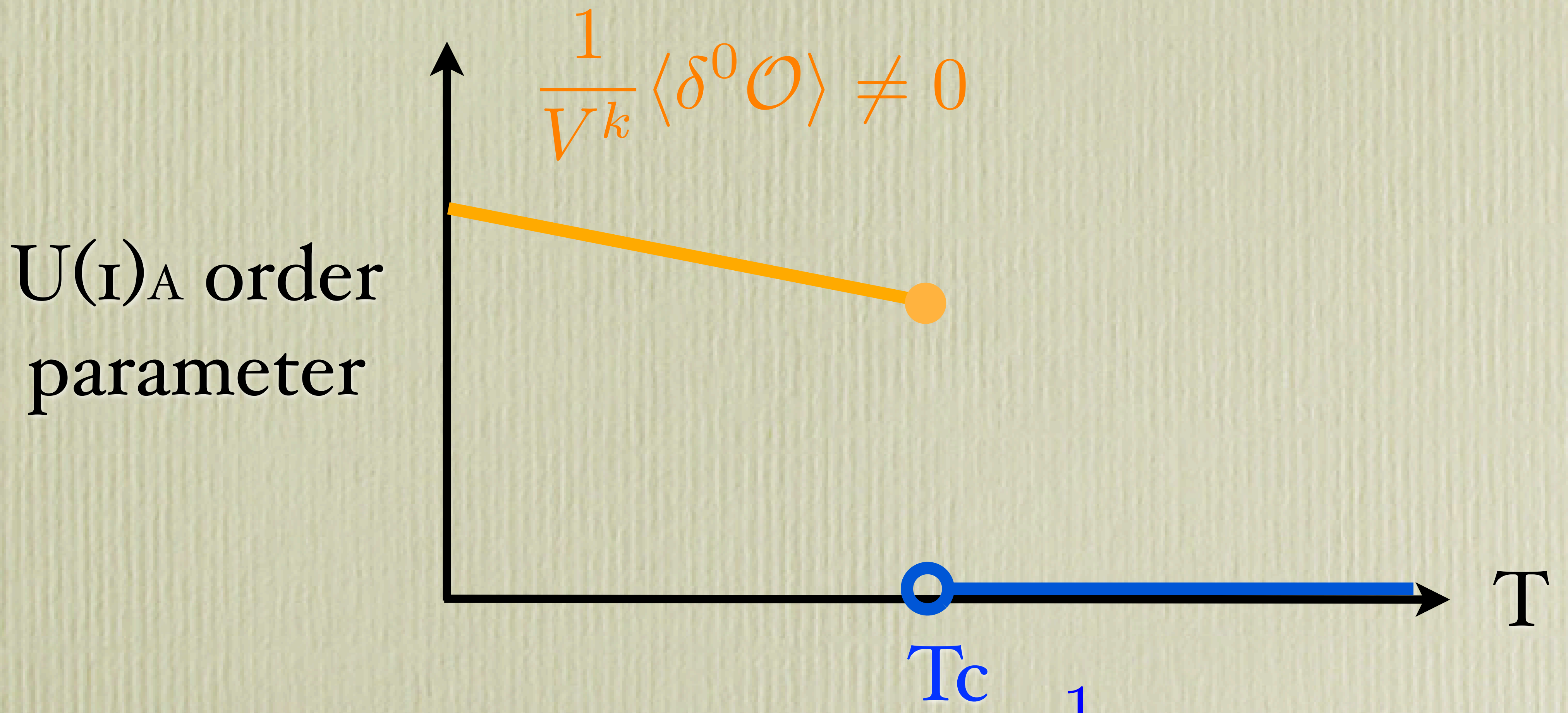

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We can compromise just on Tc

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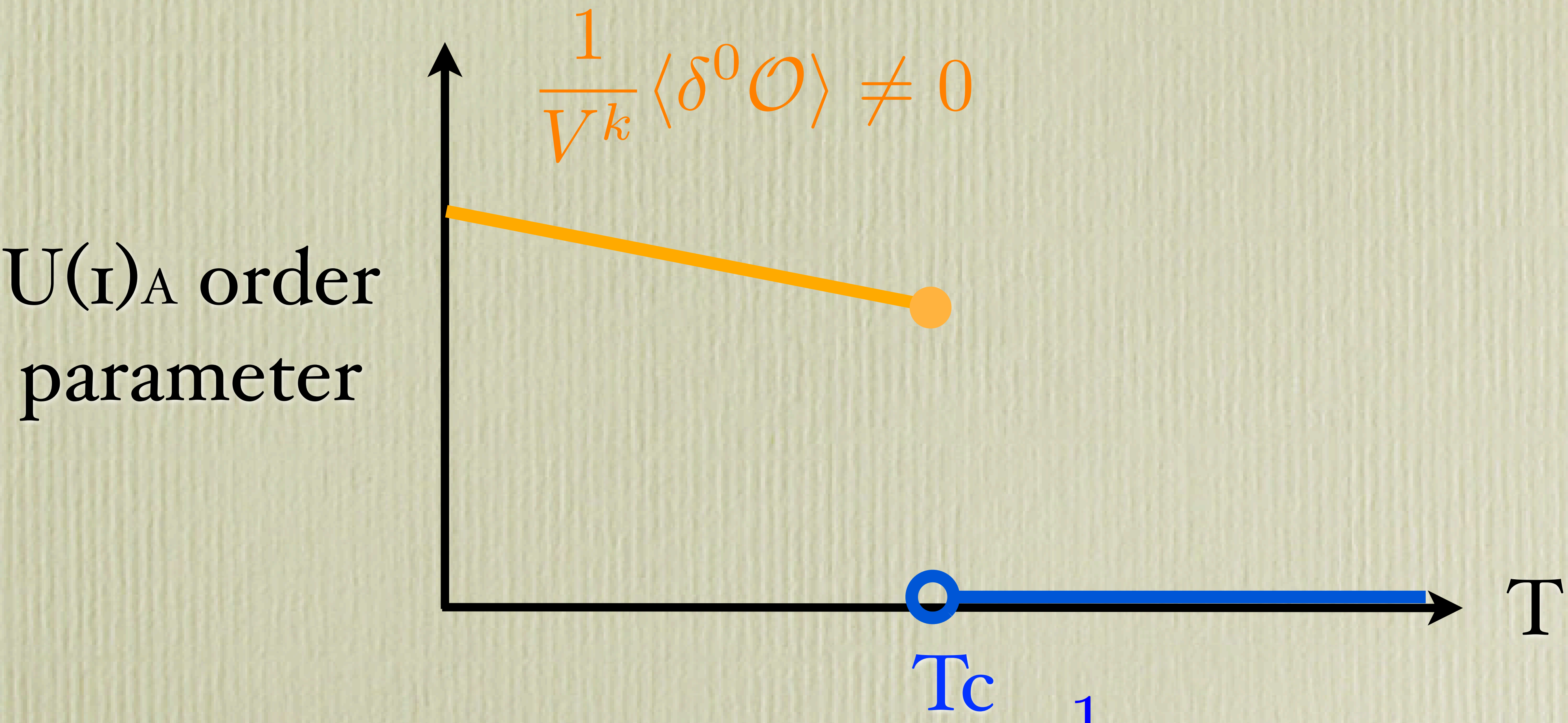

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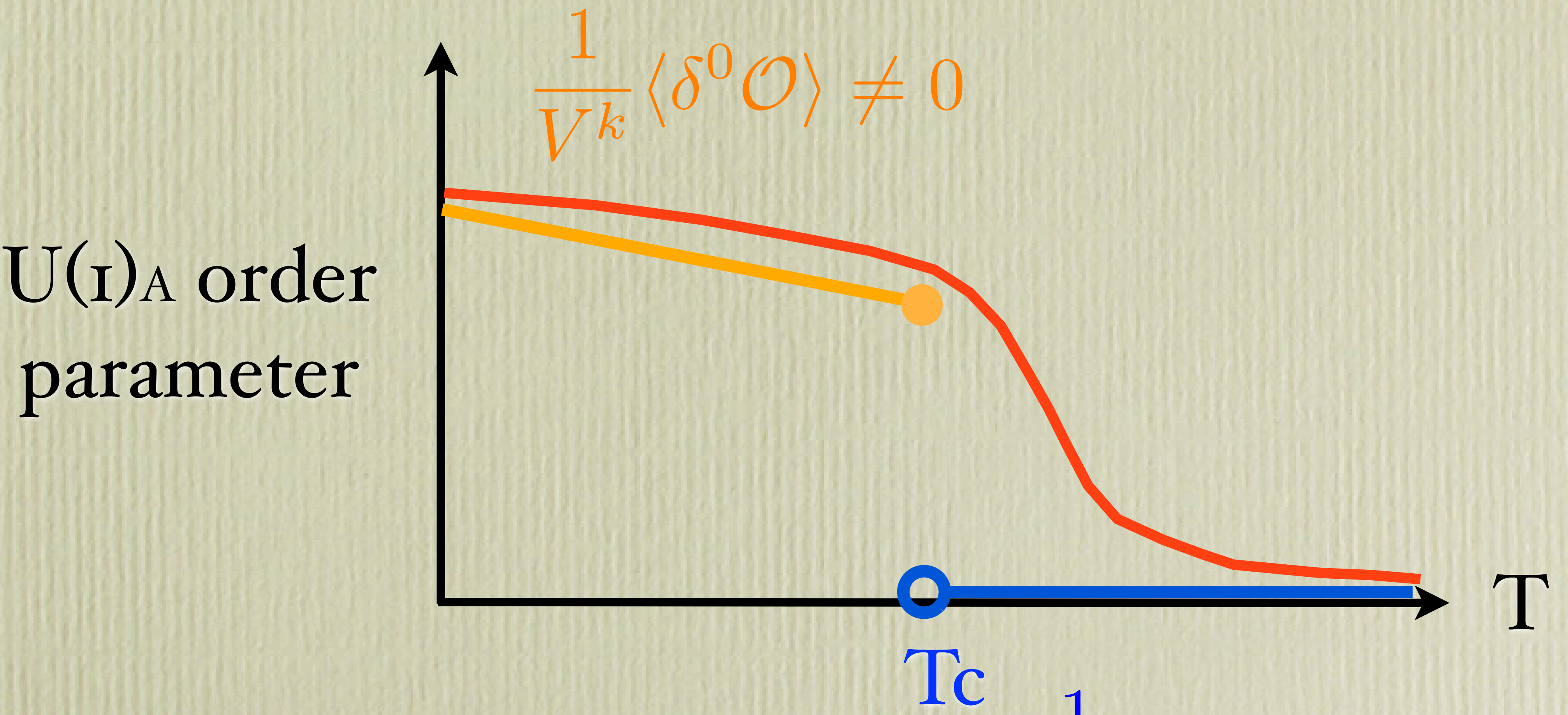

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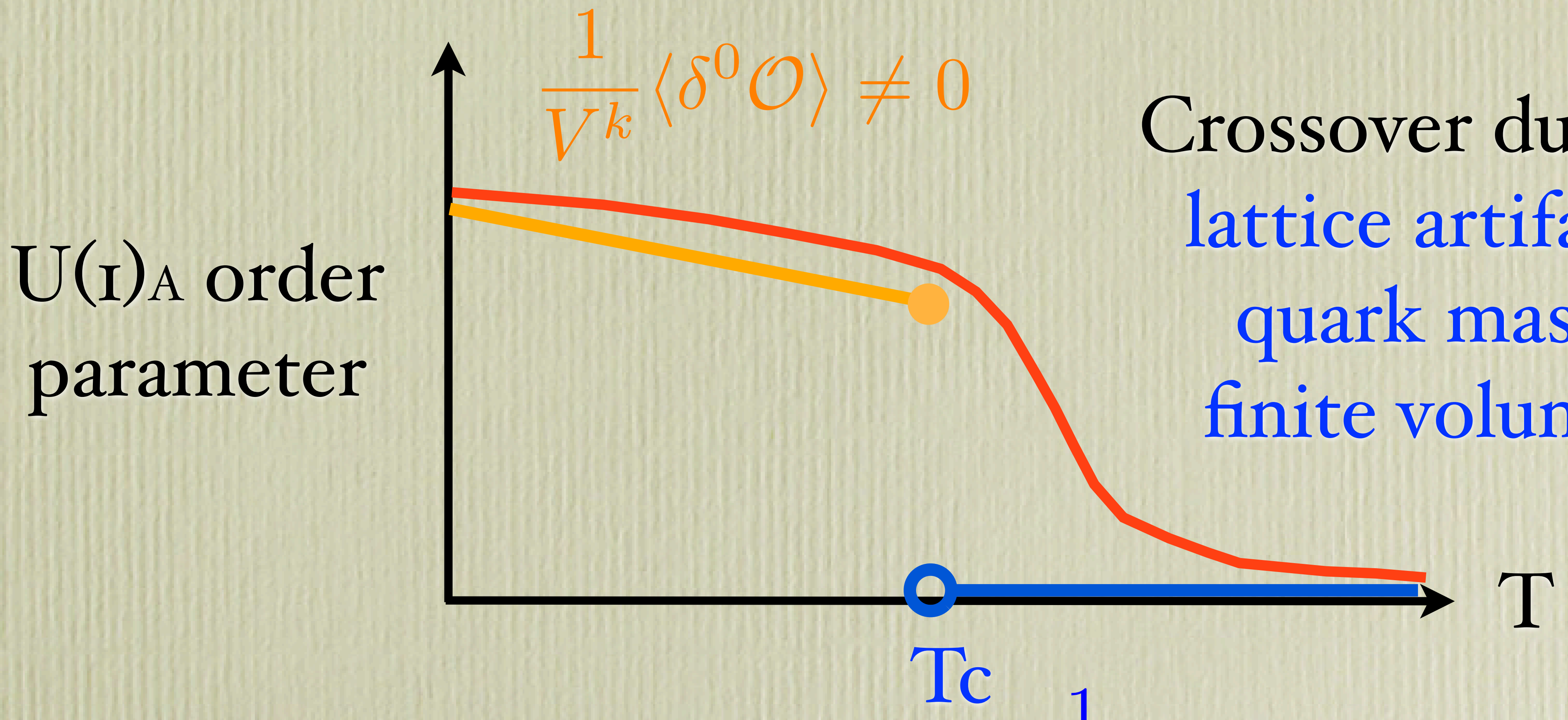

Crossover due to
lattice artifact
quark mass
finite volume

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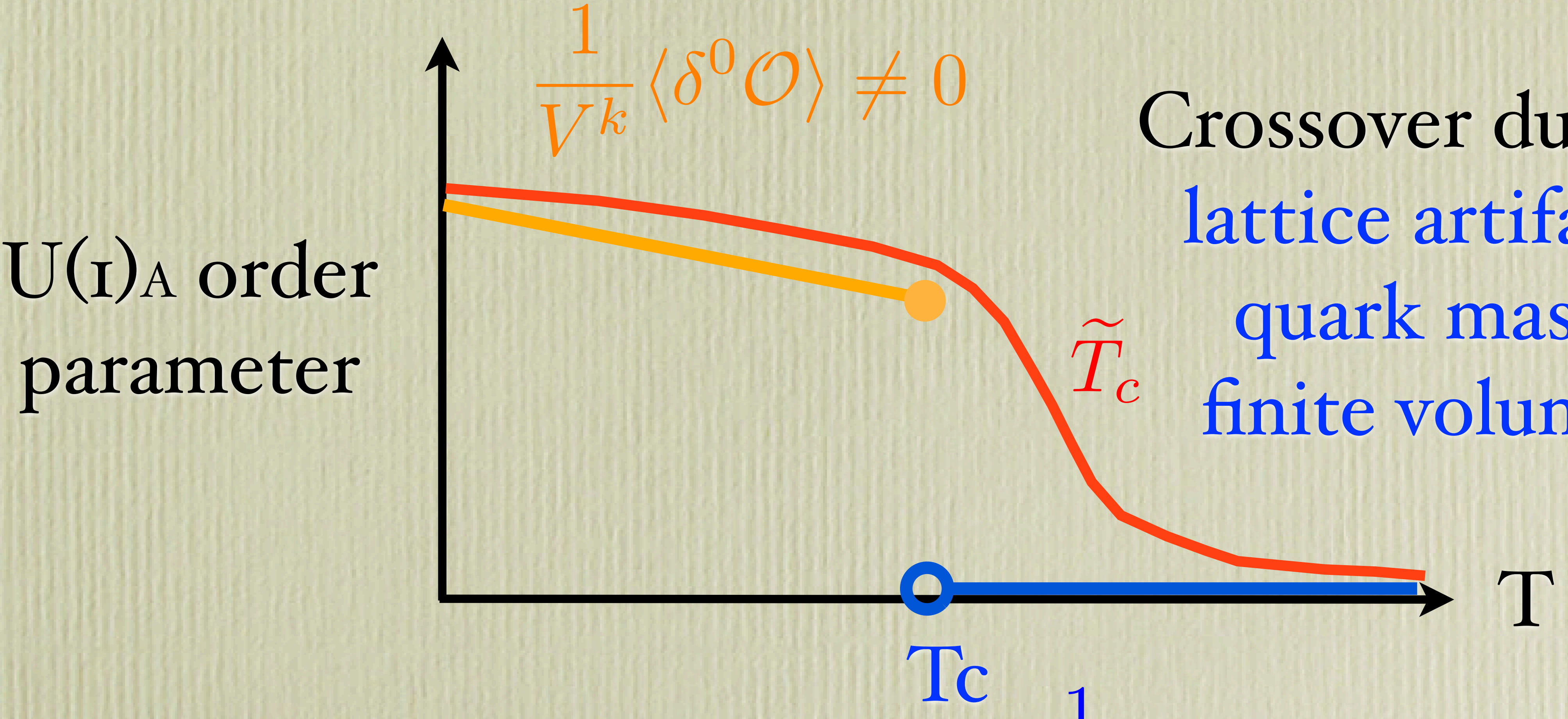

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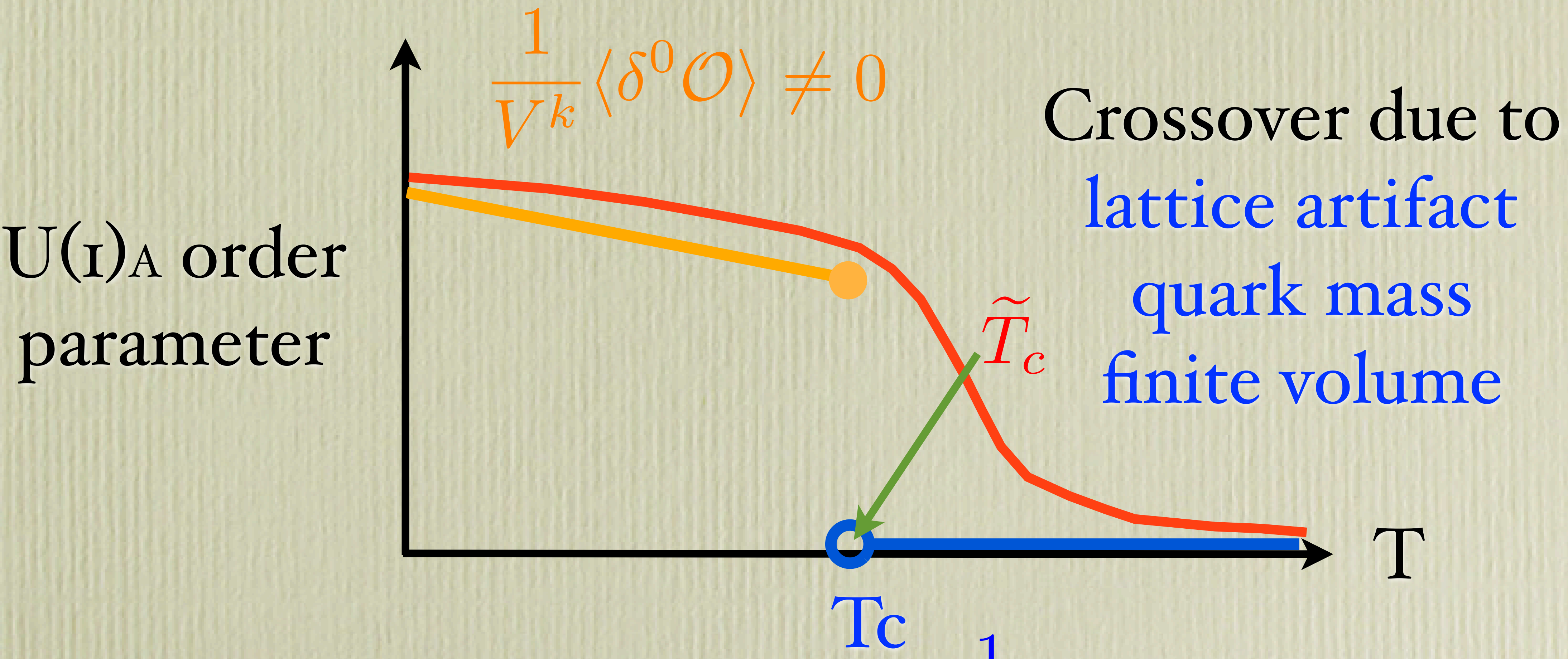

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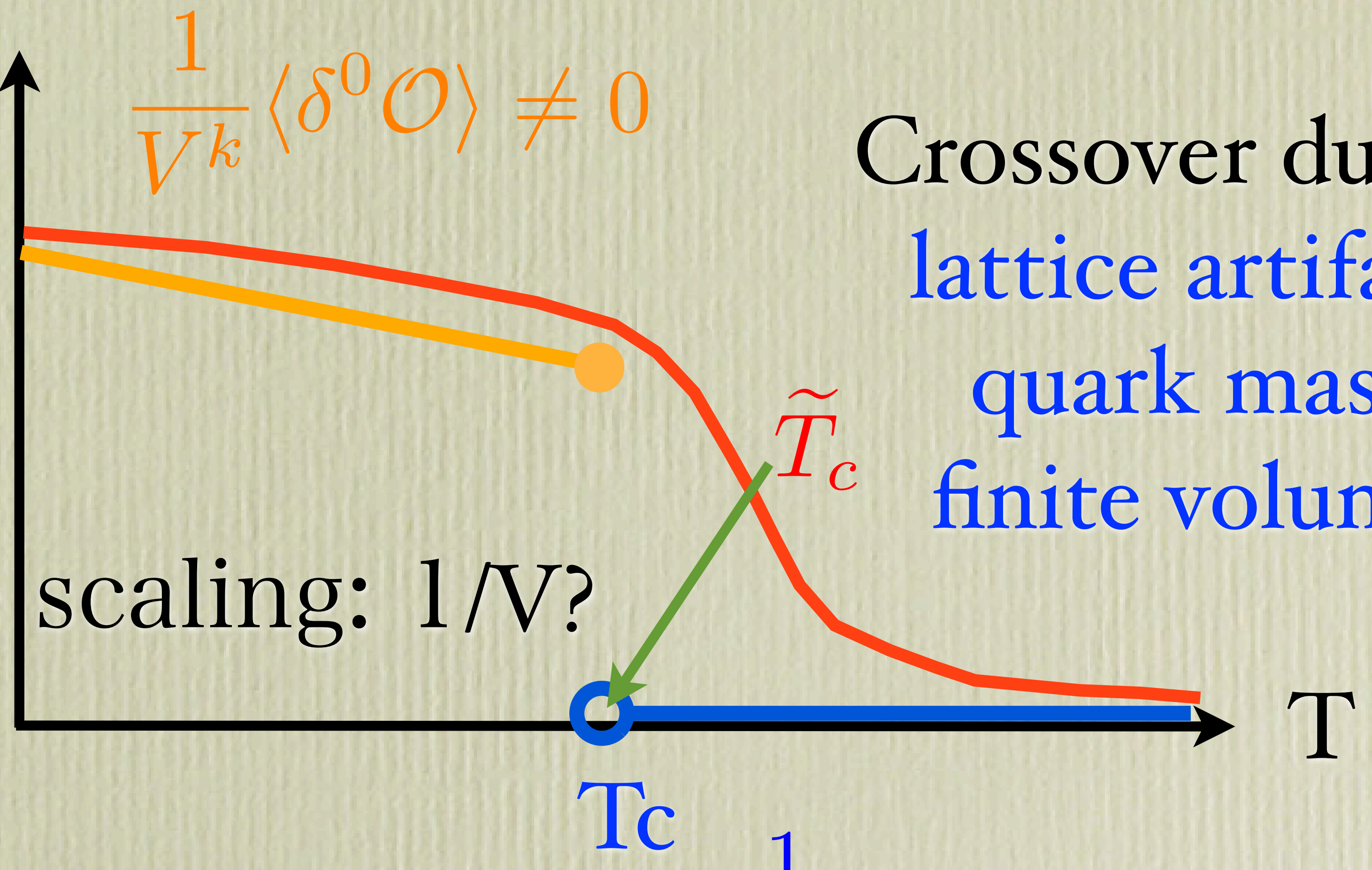

$\frac{1}{V^k}\langle\delta^0\mathcal{O}\rangle \neq 0$

U(1)A order parameter

Crossover due to
lattice artifact
quark mass
finite volume

finite size scaling: 1/V?

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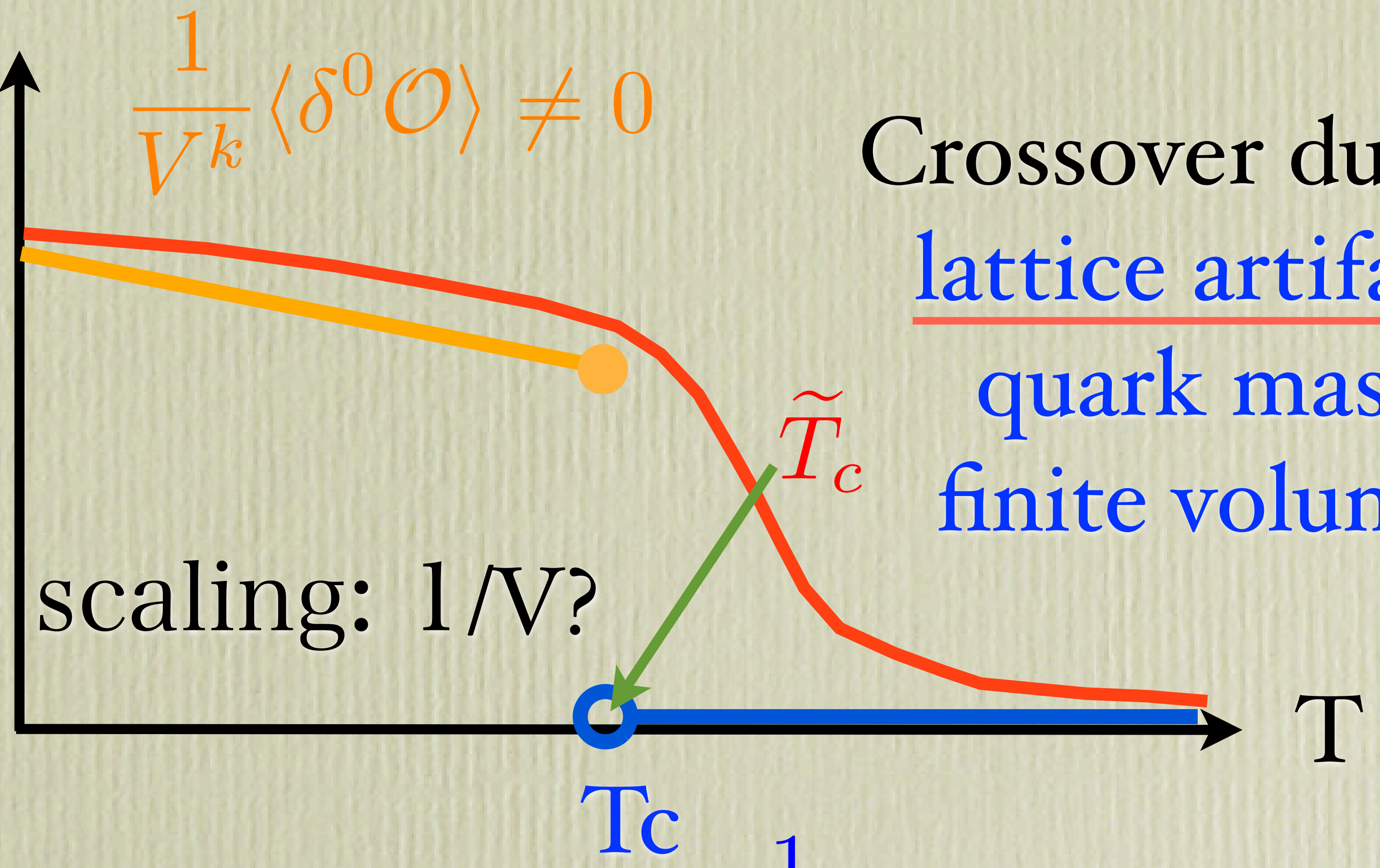

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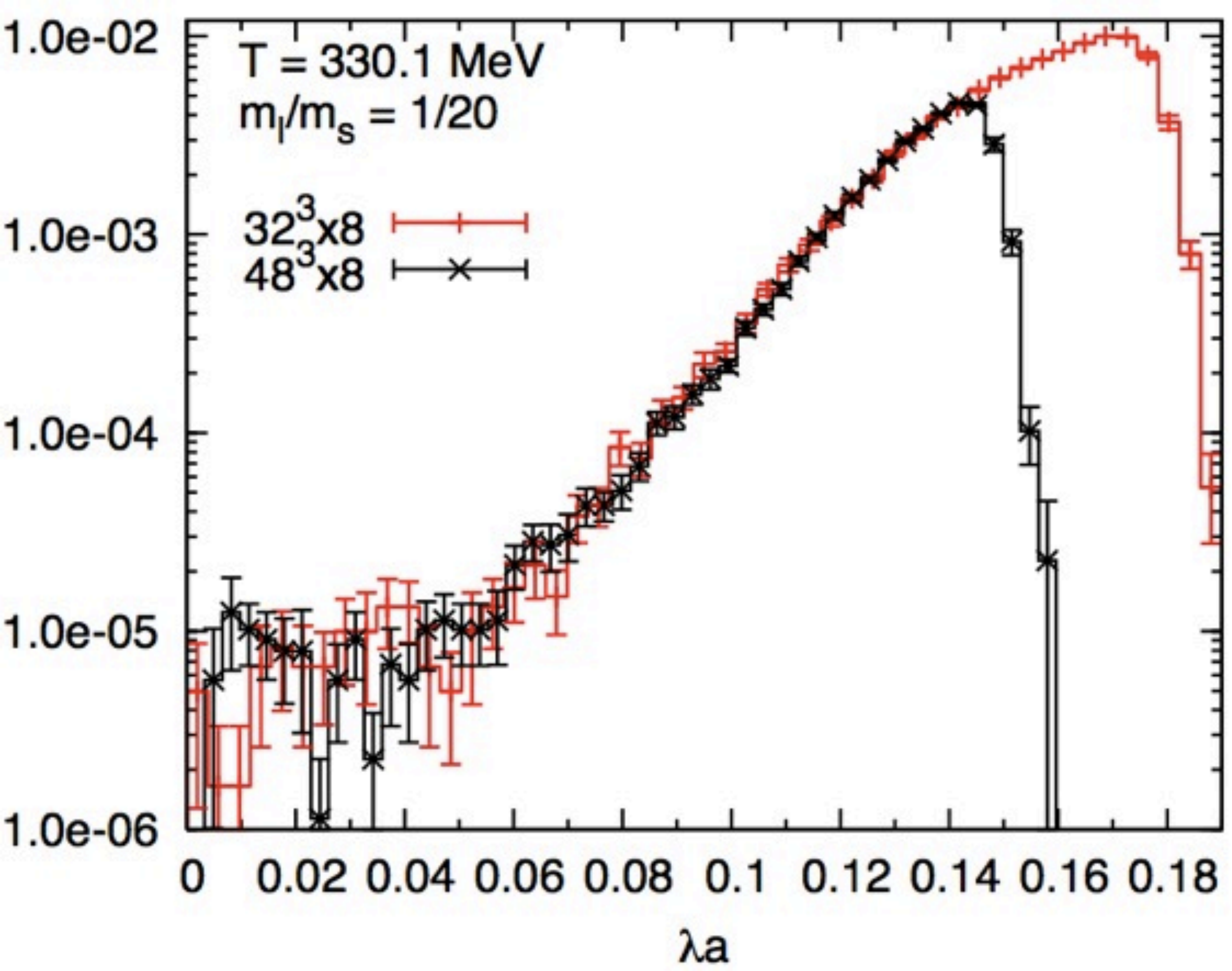
Lattice artifact?

Lattice artifact?

Ohno et. al. arXiv:1211.2591

$\rho(\lambda)$



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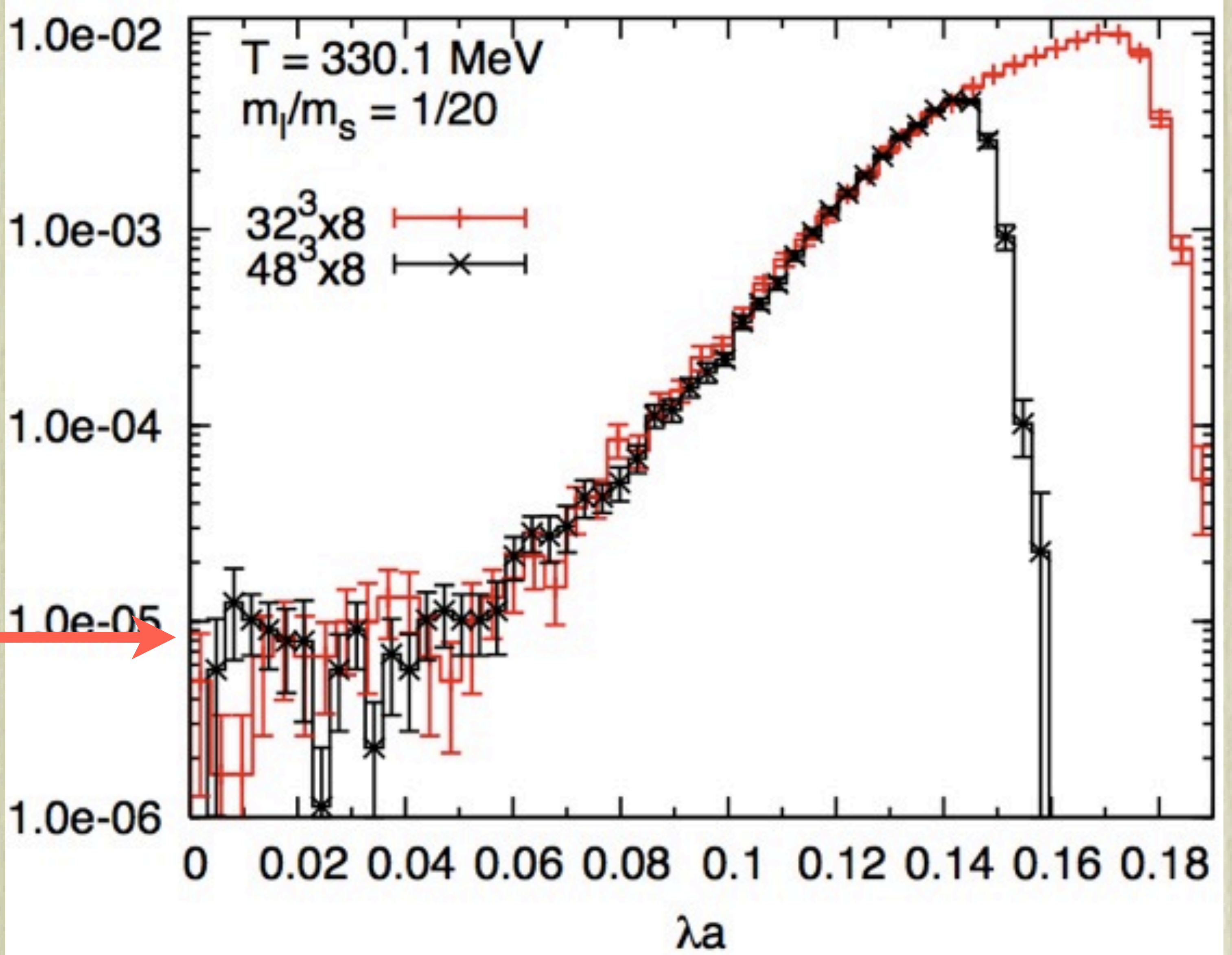

$\langle\rho(0)\rangle \propto m$ →

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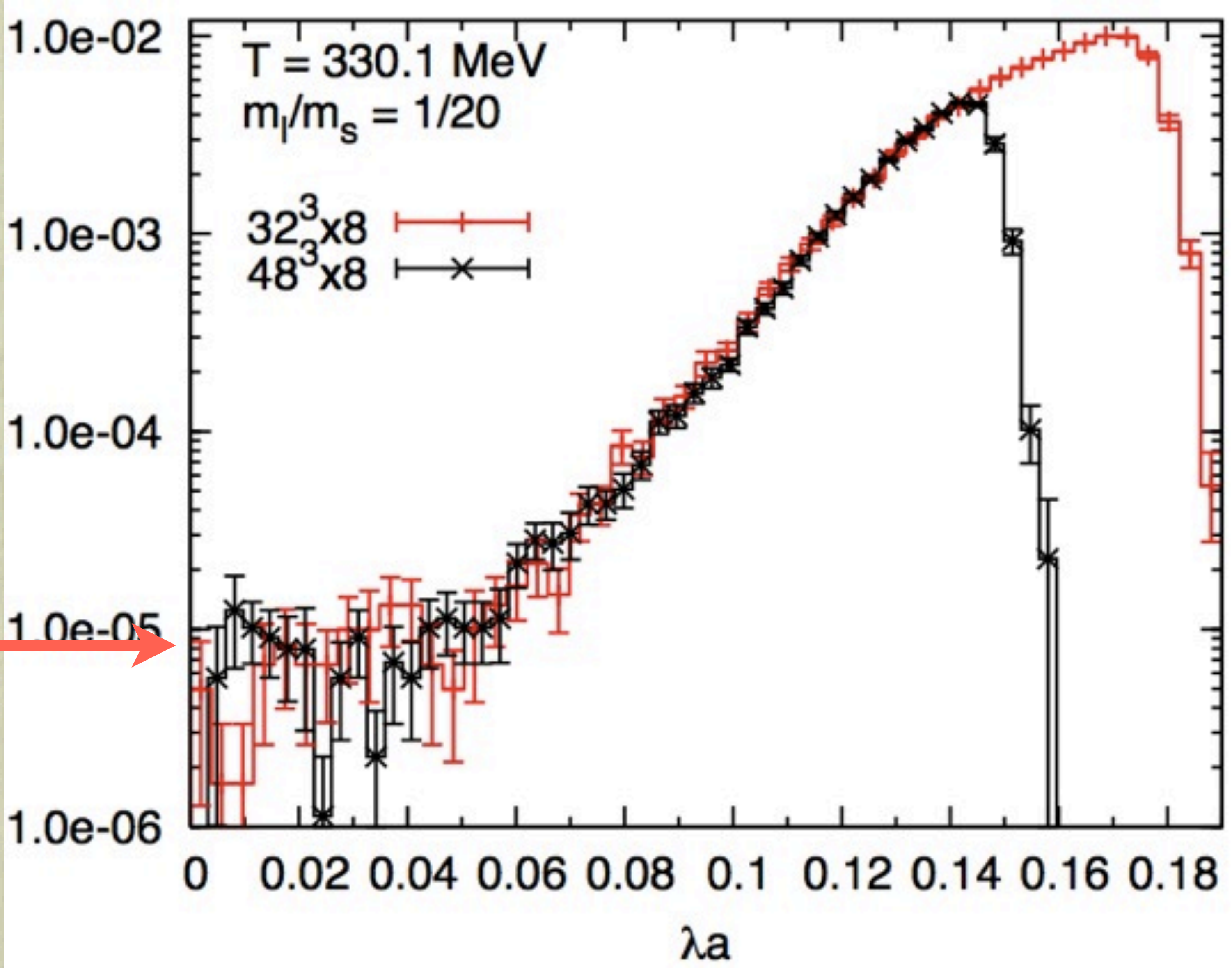

$\langle\rho(0)\rangle \propto m$

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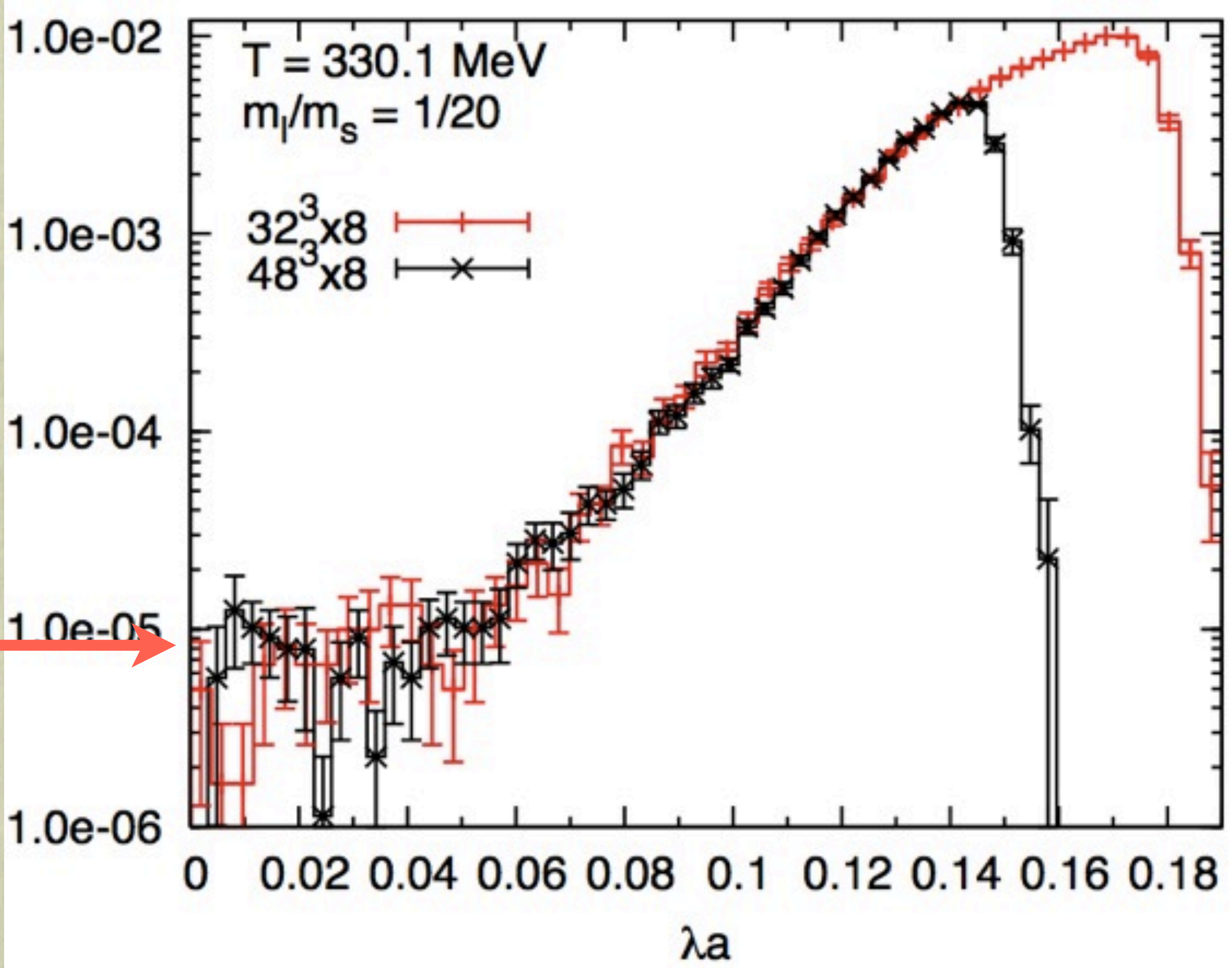

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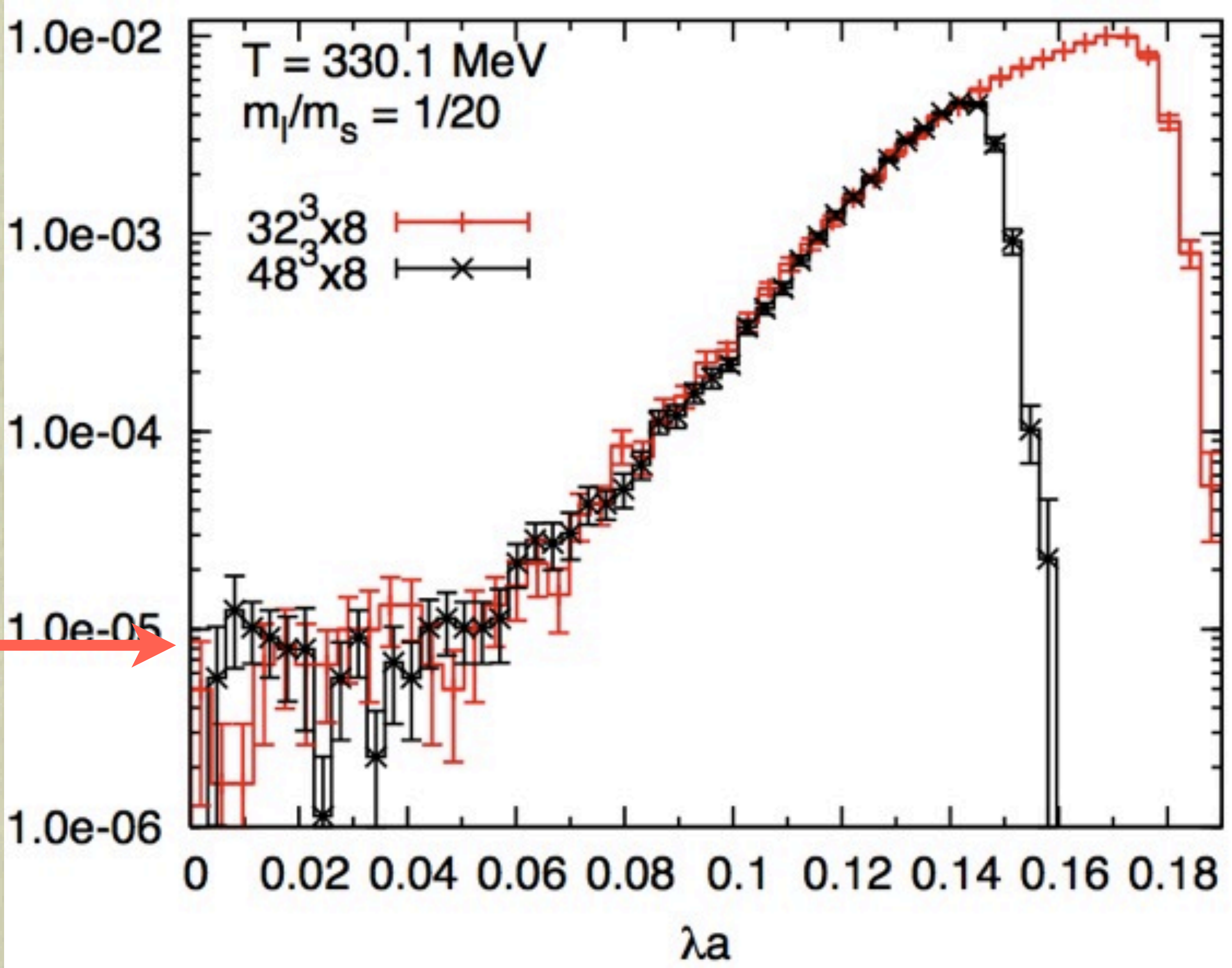

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$$\langle \bar{\psi}\psi \rangle \sim \langle \rho(0) \rangle \to 0 \qquad \chi_{\eta-\sigma} \sim \frac{1}{m} \langle \rho(0) \rangle = \text{finite}$$

$$\langle P^a S^a P^0 \rangle \sim \langle \rho(0) \rangle \frac{2}{m^3} \to \infty$$

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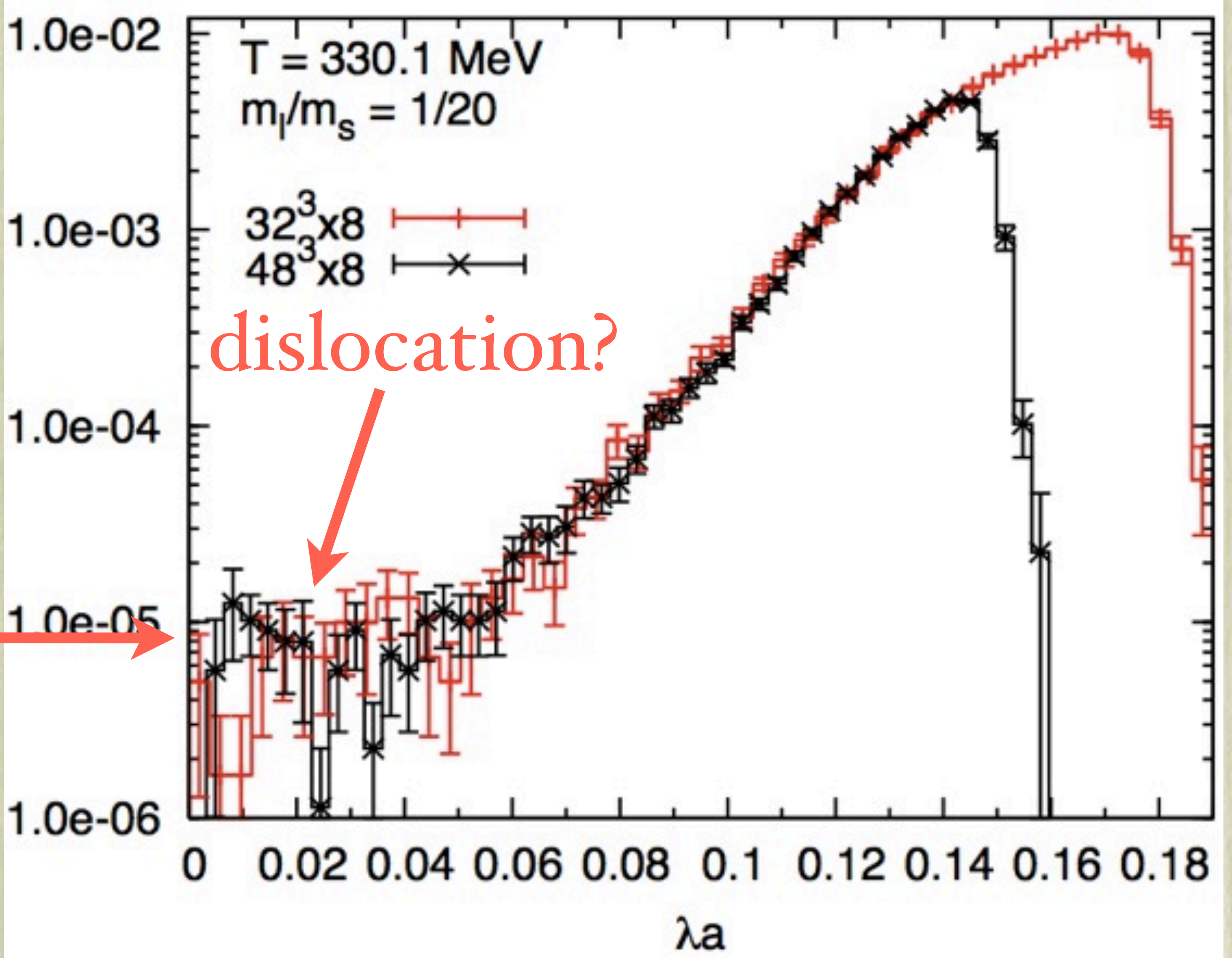

dislocation?

$$\langle\bar{\psi}\psi\rangle \sim \langle\rho(0)\rangle \to 0$$

$$\chi_{\eta-\sigma} \sim \frac{1}{m}\langle\rho(0)\rangle = \text{finite}$$

$$\langle P^a S^a P^0\rangle \sim \langle\rho(0)\rangle \frac{2}{m^3} \to \infty$$

Lattice artifact?

Ohno et. al. arXiv:1211.2591

$\rho(\lambda)$

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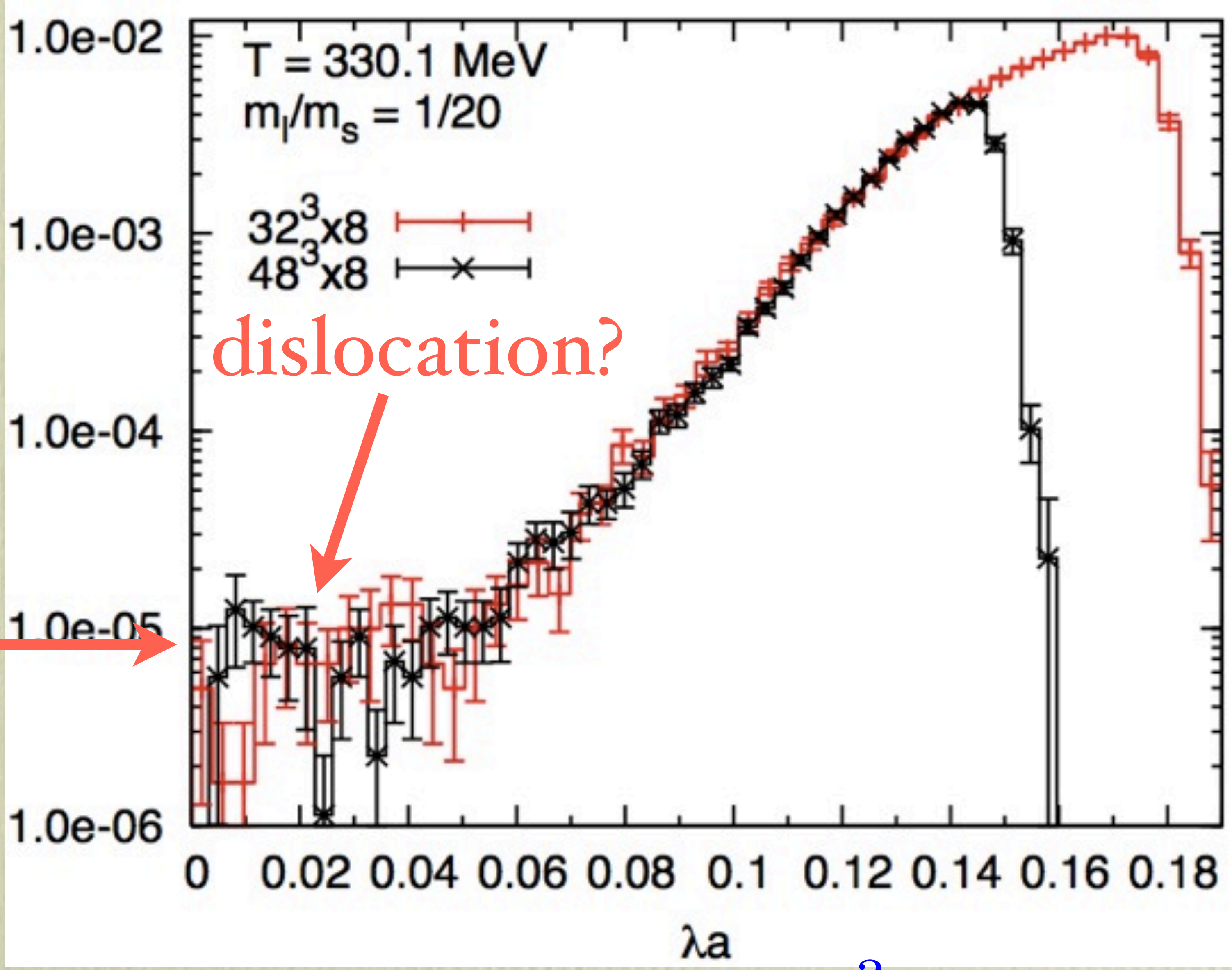
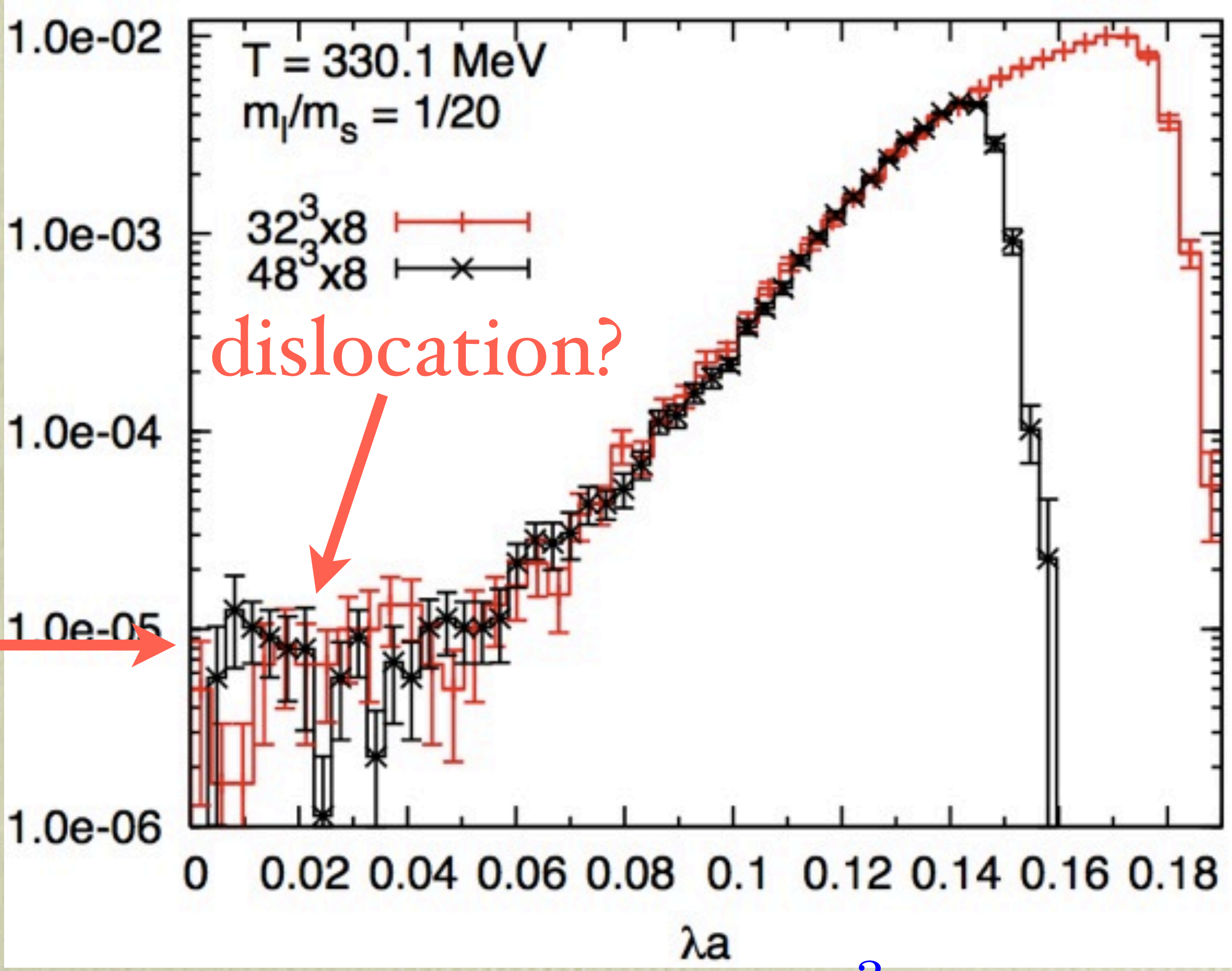


$$\langle\rho^A(\lambda)\rangle_m = \langle\rho_3^A\rangle_0 \frac{\lambda^3}{3!} + \cdots$$

Lattice artifact?

Ohno et. al. arXiv:1211.2591



dislocation?

$\rho(\lambda)$

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chiral sym. breaking

Lattice artifact?

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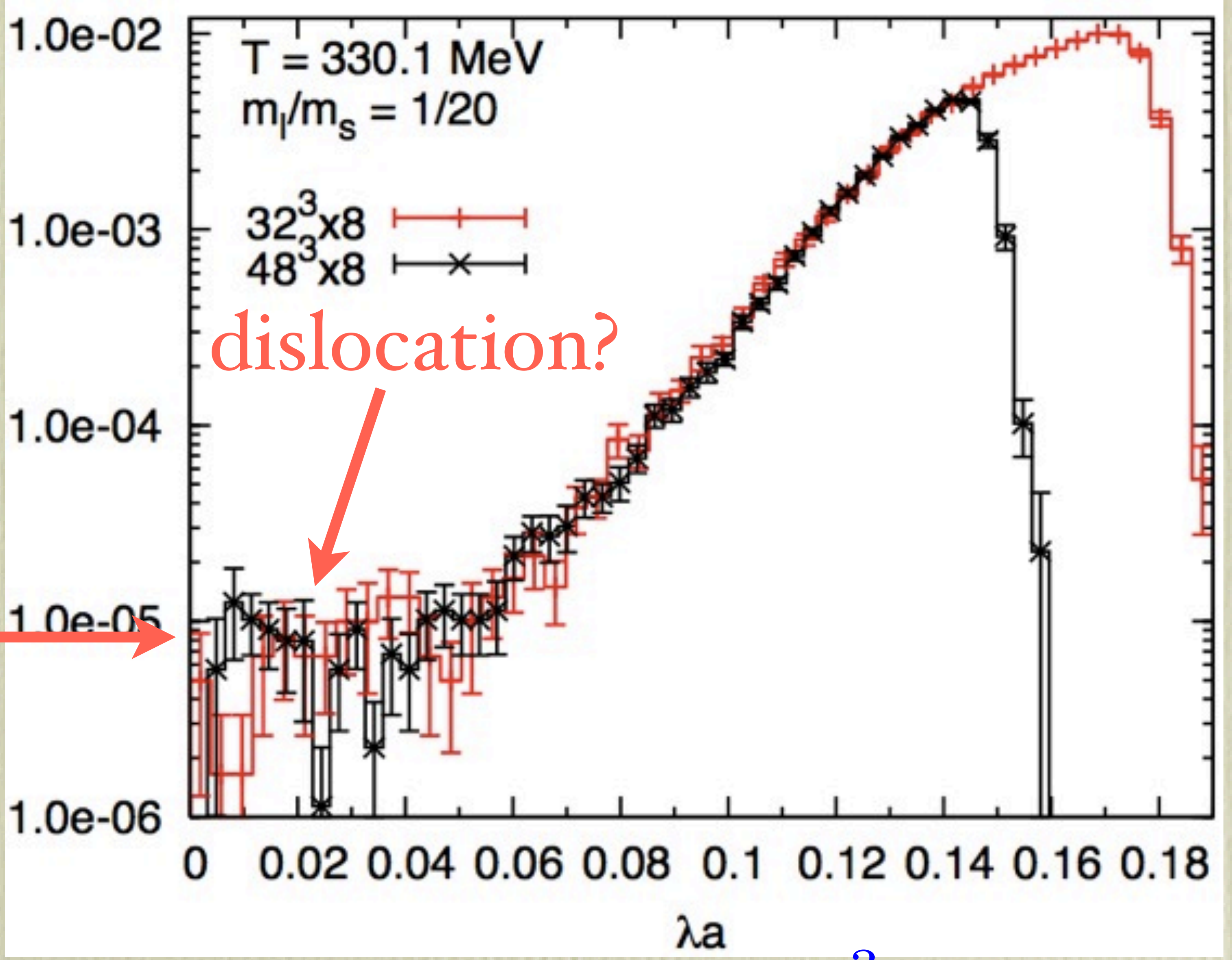

$\rho(\lambda)$

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chiral sym. breaking

$$\langle \rho^A(\lambda) \rangle_m = \alpha m_{\mathrm{break}} \Lambda_{\mathrm{QCD}} \lambda + \cdots$$

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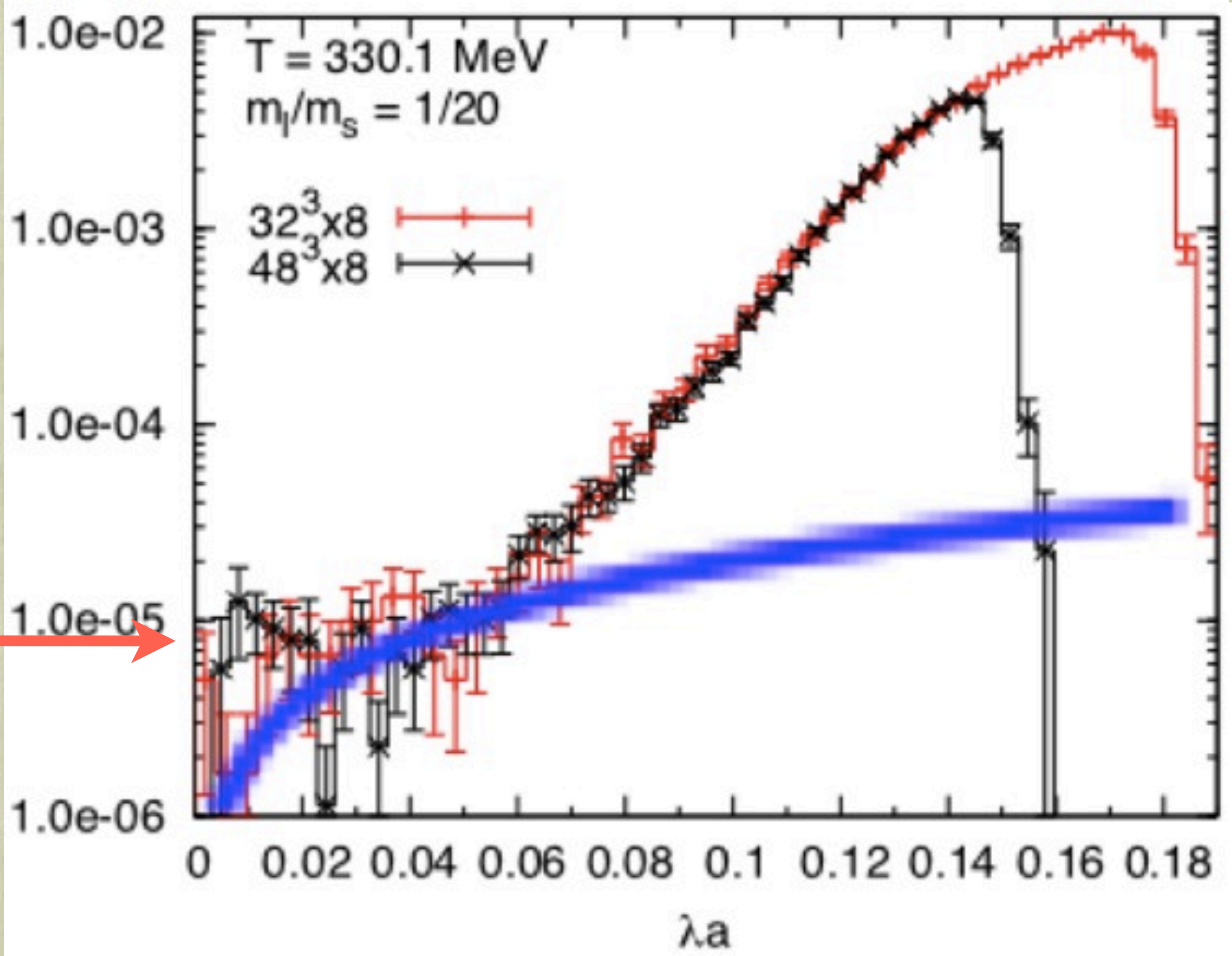

$\rho(\lambda)$

$\langle \rho(0) \rangle \propto m$

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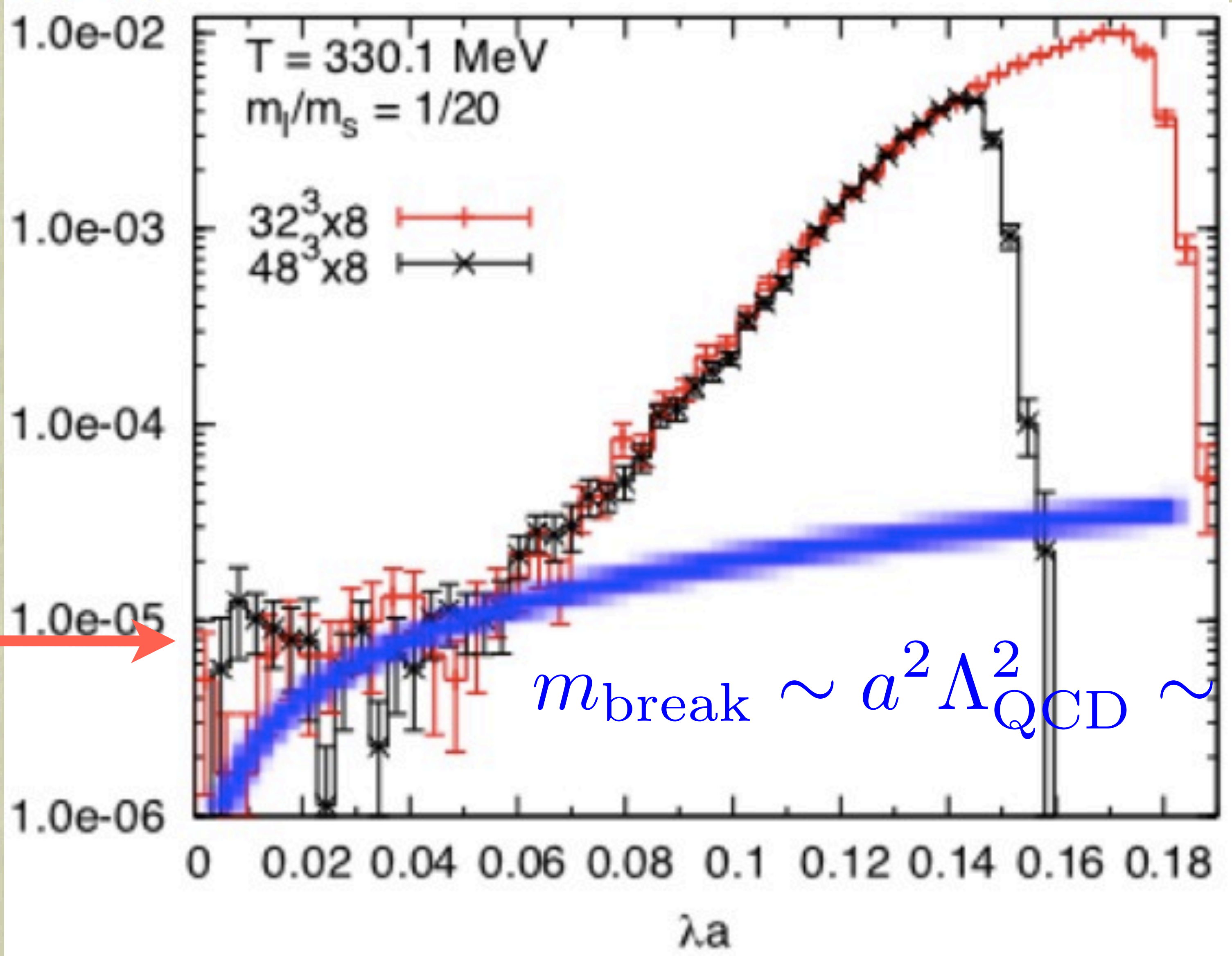

$\rho(\lambda)$

$\langle\rho(0)\rangle \propto m$

$m_{\rm break} \sim a^2\Lambda_{\rm QCD}^2 \sim 4$ MeV

$$\langle\rho^A(\lambda)\rangle_m = \langle\rho_3^A\rangle_0 \frac{\lambda^3}{3!} + \cdots$$

chiral sym. breaking

$$\langle\rho^A(\lambda)\rangle_m = \alpha m_{\rm break}\Lambda_{\rm QCD}\lambda + \cdots$$

If you can read this
I am on the wrong page.