Lattice energy-momentum tensor from the Yang-Mills gradient flow

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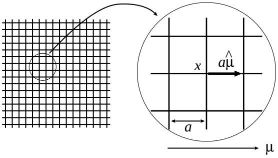
- H.S., Prog. Theor. Exp. Phys. (2013) 083B03 [arXiv:1304.0533 [hep-lat]].
- M. Asakawa, T. Hatsuda, E. Itou, M. Kitazawa, H.S. (FlowQCD Collaboration), arXiv:1312.7492 [hep-lat]
- Hiroki Makino and H.S., work in progress

Lattice field theory and the energy-momentum tensor (EMT)

- Lattice field theory



  is best successful non-perturbative formulation of QFT
- ... keeps internal gauge symmetries exact
- ... but quite incompatible with spacetime symmetries (translation, rotation, SUSY, conformal, ...)
- Ward–Takahashi (WT) relation associated with translational invariance ($T_{\mu\nu}(x)$: energy-momentum tensor (EMT))

$$\langle \partial_\mu T_{\mu\nu}(x) \mathcal{O}(y) \mathcal{O}(z) \cdots \rangle = -\delta(x-y) \langle \partial_\nu \mathcal{O}(y) \mathcal{O}(z) \cdots \rangle + \cdots$$

- Conservation law is a special case of this:

$$\langle \partial_\mu T_{\mu\nu}(x) \mathcal{O}(y) \mathcal{O}(z) \cdots \rangle = 0, \qquad \text{for } x \neq y,\ x \neq z,\ \ldots$$

- Can we construct lattice EMT which reproduces these relations in $a \to 0$?
- If this is possible, the application will be vast (thermodynamics, viscosities, conformal field theory, dilaton physics, vacuum energy, ...)

- Invent somehow a lattice formulation that is invariant under the desired symmetry (in the present case, translation) as the lattice chiral symmetry on the basis of the Ginsparg–Wilson relation
- This is certainly ideal, but seems formidable for spacetime symmetries . . . (eventually, SLAC derivative?)

Possible approaches

- A general argument (Caracciolo et al. (1989)) tells that (assuming the hypercubic symmetry) a linear combination of following dim. 4 operators

$$
\begin{aligned}
&T_{\mu\nu}(x) \\
&= C_1\left(\sum_{\rho} F^a_{\mu\rho}F^a_{\nu\rho} - \frac{1}{4}\delta_{\mu\nu}\sum_{\rho\sigma} F^a_{\rho\sigma}F^a_{\rho\sigma}\right) + C_2\delta_{\mu\nu}\sum_{\rho\sigma} F^a_{\rho\sigma}F^a_{\rho\sigma} + C_3\delta_{\mu\nu}\sum_{\rho} F^a_{\mu\rho}F^a_{\nu\rho}
\end{aligned}
$$

  is conserved for $a \to 0$
- We may determine ratios of these coefficients, C_2/C_1 and C_3/C_1 by imposing the conservation law
- Overall normalization C_1 should be fixed separately (expectation value in a one-particle state? current algebra?; matching to the free energy obtained by the other way?)
- No one yet studied whether this construction generates correct translations on composite operators (!)
- Approach on the basis of SUSY algebra and Ferrara–Zumino supermultiplet (H.S. (2012))

Our approach

- Use a UV finite quantity that can be related with EMT in a translational invariant regularization
- Any regularization (including lattice) will produce the same number for such a UV finite quantity
- To define this UV finite quantity, we employ the so-called Yang–Mills gradient flow
- Yang–Mills gradient flow (a diffusion equation wrt a fictitious time $t \in \mathbb{R}$)

$$\partial_t B_\mu(t,x) = -g_0^2 \frac{\delta S}{\delta B_\mu(t,x)} = D_\nu G_{\nu\mu}(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$

  where $G_{\mu\nu}$ is the field strength of the flowed gauge potential:

$$G_{\mu\nu}(t,x) = \partial_\mu B_\nu(t,x) - \partial_\nu B_\mu(t,x) + [B_\mu(t,x), B_\nu(t,x)], \qquad D_\mu = \partial_\mu + [B_\mu, \cdot]$$

- Note: the mass dimension of t is -2

Yang–Mills gradient flow or the Wilson flow

- Yang–Mills gradient flow (continuum theory)

$$\partial_t B_\mu(t,x) = -g_0^2 \frac{\delta S}{\delta B_\mu(t,x)} = D_\nu G_{\nu\mu}(t,x) = \Delta B_\mu(t,x) + \cdots, \quad B_\mu(t=0,x) = A_\mu(x)$$

- Wilson flow (lattice theory)

$$\partial_t V(t,x,\mu) V(t,x,\mu)^{-1} = -g_0^2 \partial S_{\text{Wilson}}, \qquad V(t=0,x,\mu) = U(x,\mu)$$

- Application (Lüscher):
  - Definition of the topological charge
  - Scale setting (just like the Sommer scale r_0); taking for instance

$$E(t,x) \equiv \frac{1}{4} G_{\mu\nu}^a(t,x) G_{\mu\nu}^a(t,x)$$

  and set

$$t^2 \left\langle E(t,x) \right\rangle\Big|_{t=t_0} = 0.3, \qquad \text{and set (for instance) } \sqrt{8t_0} = 0.5\,\text{fm}$$

  - Computation of the chiral condensate
  - Define UV finite quantities ← Our usage here

Perturbative expansion of the gradient flow

- Yang–Mills gradient flow

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x), \qquad B_\mu(t=0,x) = A_\mu(x),$$

where the second term in RHS was introduced to suppress the gauge mode; it can be seen that gauge invariant quantities are independent of α_0. The formal solution is

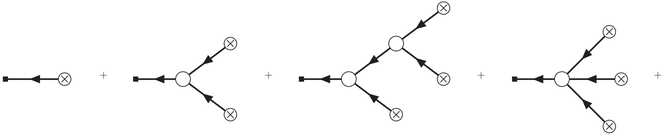
$$B_\mu(t,x) = \int d^D y \left[K_t(x-y)_{\mu\nu} A_\nu(y) + \int_0^t ds\, K_{t-s}(x-y)_{\mu\nu} R_\nu(s,y) \right],$$

where K is the heat kernel and R denotes non-linear terms

$$K_t(z)_{\mu\nu} = \int_p \frac{e^{ipz}}{p^2} \left[(\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-tp^2} + p_\mu p_\nu e^{-\alpha_0 t p^2} \right]$$

$$R_\mu = 2[B_\nu, \partial_\nu B_\mu] - [B_\nu, \partial_\mu B_\nu] + (\alpha_0 - 1)[B_\mu, \partial_\nu B_\nu] + [B_\nu, [B_\nu, B_\mu]]$$

Pictorially (cross: A_μ; open circle: flow vertex R),



Perturbative expansion of the gradient flow

- Quantum correlation function of the flowed gauge field

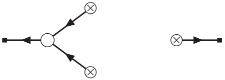
$$\langle B_{\mu_1}(t_1, x_1) \cdots B_{\mu_n}(t_n, x_n) \rangle ,$$

is obtained by taking the expectation value of the initial value $A_\mu(x)$. For example, the contraction of two A_μ's

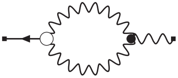
produces the propagator of the flowed field

$$\delta^{ab} g_0^2 \frac{1}{(p^2)^2} \left[(\delta_{\mu\nu} p^2 - p_\mu p_\nu) e^{-(t+s)p^2} + \frac{1}{\lambda_0} p_\mu p_\nu e^{-\alpha_0(t+s)p^2} \right] ,$$

(where t and s are flow times at the end points; λ_0 is the conventional gauge parameter). Similarly, for



considering the contraction with the usual Yang–Mills vertex (the full circle)



Gauge invariance of the gradient flow

- Under the infinitesimal gauge transformation,

$$B_\mu(t,x) \to B_\mu(t,x) + D_\mu\omega(t,x),$$

  the flow equation

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x)$$

  changes to

$$\partial_t B_\mu(t,x) = D_\nu G_{\nu\mu}(t,x) + \alpha_0 D_\mu \partial_\nu B_\nu(t,x) - D_\mu(\partial_t - \alpha_0 D_\nu \partial_\nu)\omega(t,x)$$

- Therefore, by choosing $\omega(t,x)$ as the solution of

$$(\partial_t - \alpha_0 D_\nu \partial_\nu)\omega(t,x) = -\delta\alpha_0 \partial_\nu B_\nu(t,x), \qquad \omega(t=0,x) = 0,$$

  α_0 can be changed as

$$\alpha_0 \to \alpha_0 + \delta\alpha_0$$

  That is, $B_\mu(t,x)$'s corresponding to different α_0's are related by a gauge transformation

- Also, by choosing $\omega(t,x)$ as the solution of

$$(\partial_t - \alpha_0 D_\nu \partial_\nu)\omega(t,x) = 0, \qquad \omega(t=0,x) = \omega(x),$$

  the D dimensional gauge transformation $\omega(x)$ can be extended to a $D+1$ dimensional gauge transformation $\omega(t,x)$ that leaves the flow equation unchanged

UV finiteness of the gradient flow I

- Correlation function of the flowed gauge field

$$\langle B_{\mu_1}(t_1,x_1)\cdots B_{\mu_n}(t_n,x_n)\rangle\,,\qquad t_1>0,\ldots,t_n>0,$$

  when expressed in terms of renormalized parameters, is UV finite without the wave function renormalization

- Tree-level two-point function

$$\left\langle \tilde{B}^a_\mu(t,p)\tilde{B}^b_\nu(s,q)\right\rangle \sim \delta^{ab}g_0^2\frac{1}{(p^2)^2}\left[(\delta_{\mu\nu}p^2-p_\mu p_\nu)e^{-(t+s)p^2}+\frac{1}{\lambda_0}p_\mu p_\nu e^{-\alpha_0(t+s)p^2}\right]$$

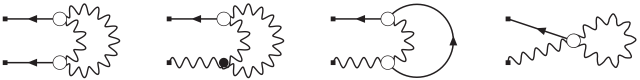
- 1-loop two point function (those containing only Yang–Mills vertices)

- The last counter term comes from rewriting to renormalized parameters as

$$g_0^2=\mu^{2\epsilon}g^2Z,\qquad \lambda_0=\lambda Z_3^{-1}$$

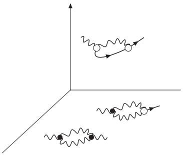
- Usually, this becomes UV finite only by taking the wave function renormalization factor into account ...

- … here, we have also diagrams containing flow vertices



which give rise to the precisely same effect as the wave function renormalization factor

- All order proof (Lüscher–Weisz (2011))



- When a loop contains a vertex in the bulk ($t > 0$), the loop integral contains the flow-time evolution factor

$$\sim e^{-t\ell^2}$$

which makes the loop integral finite; no bulk counterterm is necessary

- By using a BRS symmetry, it can be shown that all boundary ($t = 0$) counterterms are those of the Yang–Mills theory

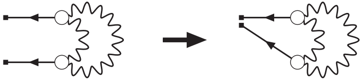
UV finiteness of the gradient flow II

- Correlation function of the flow gauge field

$$\langle B_{\mu_1}(t_1, x_1) B_{\mu_2}(t_2, x_2) \cdots B_{\mu_n}(t_n, x_n) \rangle , \qquad t_1 > 0, \ldots, t_n > 0,$$

remains finite even for the equal-point product

$$t_1 \to t_2, \qquad x_1 \to x_2,$$



- The new loop always contains the flow-time evolution factor $\sim e^{-t\ell^2}$ and this makes integral finite; no new UV divergence arises
- This is an extremely powerful property!

$$B_\mu(t,x) B_\nu(t,x)|_{\text{Dimensional Regularization}} = B_\mu(t,x) B_\nu(t,x)|_{\text{Lattice}}$$

- On the other hand, the difficulty in the present problem comes from

$$(A_R)_\mu(x)(A_R)_\nu(x)|_{\text{Dimensional Regularization}} \neq (A_R)_\mu(x)(A_R)_\nu(x)|_{\text{lattice}}$$

- Using this property of the gradient flow, we relate a certain quantity defined by the gradient flow and EMT in the dimensional regularization

EMT in the dimensional regularization

- $SU(N)$ Yang–Mills theory in $D = 4 - 2\epsilon$ dimensions

$$S = \frac{1}{4g_0^2} \int d^D x \, F_{\mu\nu}^a(x) F_{\mu\nu}^a(x)$$

- Assuming the dimensional regularization, since it preserves the translational invariance, the naive expression

$$T_{\mu\nu}(x) = \frac{1}{g_0^2} \left[F_{\mu\rho}^a(x) F_{\nu\rho}^a(x) - \frac{1}{4} \delta_{\mu\nu} F_{\rho\sigma}^a(x) F_{\rho\sigma}^a(x) \right]$$

  fulfills the correct WT relation

$$\langle \partial_\mu T_{\mu\nu}(x) \mathcal{O}(y) \mathcal{O}(z) \cdots \rangle = -\delta(x-y) \langle \partial_\nu \mathcal{O}(y) \mathcal{O}(z) \cdots \rangle + \cdots$$

- It follows from this that $T_{\mu\nu}(x)$ does not receive the multiplicative renormalization
- So, we define a renormalized (finite) EMT by subtracting VEV, with dimensional regularization,

$$\{T_{\mu\nu}\}_R(x) = T_{\mu\nu}(x) - \langle T_{\mu\nu}(x) \rangle$$

Local product in finite flow time and EMT

- Now, we consider the following dim. 4 gauge invariant combinations of the flowed field;

$$U_{\mu\nu}(t,x) \equiv G^a_{\mu\rho}(t,x)G^a_{\nu\rho}(t,x) - \frac{1}{4}\delta_{\mu\nu}G^a_{\rho\sigma}(t,x)G^a_{\rho\sigma}(t,x)$$

$$E(t,x) \equiv \frac{1}{4}G^a_{\mu\nu}(t,x)G^a_{\mu\nu}(t,x)$$

- The flow equation is a diffusion equation whose diffusion length is $\sim \sqrt{8t}$. So, in $t \to 0$ limit, $U_{\mu\nu}(t,x)$ and $E(t,x)$ can be regarded as local operators in D dimensional x space
- Moreover, from the UV finiteness of the gradient flow, these are UV finite
- From these facts, for $t \to 0$, above local products can be expressed by an asymptotic series of D dimensional renormalized operators (coefficients will be finite too):

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

Here, we have used the fact that $U_{\mu\nu}(x)$ is traceless for $D=4$. $O(t)$ is the contribution of operators with dim. 6 or higher

Local product in finite flow time and EMT

- From the above expansions,

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t),$$

by eliminating the trace part $\{T_{\rho\rho}\}_R(x)$, we have

$$\{T_{\mu\nu}\}_R(x) = \frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{1}{4\alpha_E(t)}\delta_{\mu\nu}\left[E(t,x) - \langle E(t,x)\rangle\right] + O(t)$$

- Therefore, if we know the $t \to 0$ behavior of the coefficients $\alpha_U(t)$ and $\alpha_E(t)$, the EMT can be obtained by $t \to 0$ limit of the combination in RHS
- Now we show that $\alpha_U(t)$ and $\alpha_E(t)$ for $t \to 0$ can be determined by perturbation theory

Renormalization group argument

- We apply

$$\left(\mu\frac{\partial}{\partial\mu}\right)_0, \qquad \mu\text{: renormalization scale, 0: bare quantities fixed}$$

to both sides of

$$U_{\mu\nu}(t,x) = \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] + O(t),$$

$$E(t,x) = \langle E(t,x)\rangle + \alpha_E(t)\{T_{\rho\rho}\}_R(x) + O(t)$$

- Expressed in terms of bare quantities, LHS does not contain μ. So,

$$\left(\mu\frac{\partial}{\partial\mu}\right)_0 \alpha_U(t)\left[\{T_{\mu\nu}\}_R(x) - \frac{1}{4}\delta_{\mu\nu}\{T_{\rho\rho}\}_R(x)\right] = 0,$$

$$\left(\mu\frac{\partial}{\partial\mu}\right)_0 \alpha_E(t)\{T_{\rho\rho}\}_R(x) = 0$$

- Further, since the EMT is not renormalized (it is a bare quantity),

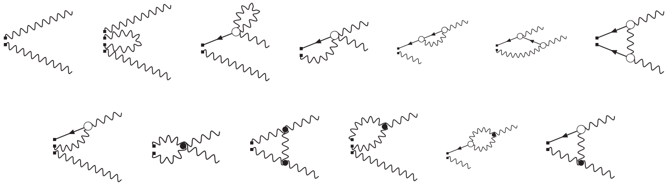
$$\left(\mu\frac{\partial}{\partial\mu}\right)_0 \alpha_{U,E}(t) = 0$$

Renormalization group argument

- Introducing the β function,
$$\beta \equiv \left(\mu \frac{\partial}{\partial \mu}\right)_0 g$$
the above relation becomes
$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g}\right) \alpha_{U,E}(t)(g;\mu) = 0$$
- This implies that in terms of the running coupling $\bar{g}$ defined by
$$q \frac{d\bar{g}(q)}{dq} = \beta\left(\bar{g}(q)\right), \qquad \bar{g}(q=\mu) = g,$$
the coefficients do not depend on the renormalization scale:
$$\alpha_{U,E}(t)(\bar{g}(q);q) = \alpha_{U,E}(t)(\bar{g}(q');q').$$
- So, we may set
$$q = \mu, \qquad q' = \frac{1}{\sqrt{8t}}$$
and thus
$$\alpha_{U,E}(t)(g;\mu) = \alpha_{U,E}(t)(\bar{g}(1/\sqrt{8t});1/\sqrt{8t})$$
- Because of the asymptotic freedom, $\bar{g}(1/\sqrt{8t}) \to 0$ for $t \to 0$. The coefficients can evaluated by the perturbation theory (a sort of factorization)

Perturbative calculation of coefficients

- To find the coefficients in next to the leading order, we need to evaluate following flow-line Feynman diagrams



- In terms of the renormalized gauge coupling in the $\overline{\text{MS}}$ scheme,

$$\alpha_U(t)(g;\mu) = g^2 \left\{1 + 2b_0 \left[\ln(\sqrt{8t}\mu) + \bar{s}_1\right] g^2 + O(g^4)\right\},$$

$$\alpha_E(t)(g;\mu) = \frac{1}{2b_0} \left\{1 + 2b_0 \bar{s}_2 g^2 + O(g^4)\right\},$$

where

$$\bar{s}_1 = \frac{7}{16} + \frac{1}{2}\gamma_E - \ln 2, \qquad \bar{s}_2 = \frac{109}{176} - \frac{b_1}{2b_0^2} = \frac{383}{1936},$$

and $b_0 = 11N/(48\pi^2)$ and $b_1 = 17N^2/(384\pi^4)$ are the first two coefficients of the β function; we see that $\alpha_{U,E}(t)$ are actually UV finite

Master formula

- By the RG argument,

$$\alpha_U(t) = \bar{g}(1/\sqrt{8t})^2 \left\{1 + 2b_0\bar{s}_1\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right\},$$

$$\alpha_E(t) = \frac{1}{2b_0}\left\{1 + 2b_0\bar{s}_2\bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4)\right\},$$

where

$$\bar{g}(q)^2 = \frac{1}{b_0\ln(q^2/\Lambda^2)} - \frac{b_1\ln[\ln(q^2/\Lambda^2)]}{b_0^3\ln^2(q^2/\Lambda^2)} + O\left(\frac{\ln^2[\ln(q^2/\Lambda^2)]}{\ln^3(q^2/\Lambda^2)}\right)$$

- Gathering all the above arguments,

$$\{T_{\mu\nu}\}_R(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{1}{4\alpha_E(t)}\delta_{\mu\nu}\left[E(t,x) - \langle E(t,x)\rangle\right]\right\}$$

- This extracts a correctly normalized conserved EMT from local products defined by the gradient flow
- Correlation functions of the quantities in RHS can (in principle) be computed non-perturbatively by using lattice regularization

Master formula

- The master formula for EMT

$$\{T_{\mu\nu}\}_R(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{1}{4\alpha_E(t)}\delta_{\mu\nu}\left[E(t,x) - \langle E(t,x)\rangle\right]\right\}$$

- Correlation functions of the quantities in RHS can (in principle) be computed non-perturbatively by using lattice regularization
- The ordering of the limits is very important: first $a \to 0$ (continuum Yang–Mills) and then $t \to 0$
- Practically, we cannot simply take $a \to 0$ so we should take t as small as possible in the window

$$a \ll \sqrt{8t} \ll \frac{1}{\Lambda}$$

and the applicability is not obvious a priori...

Matching with the perturbation theory

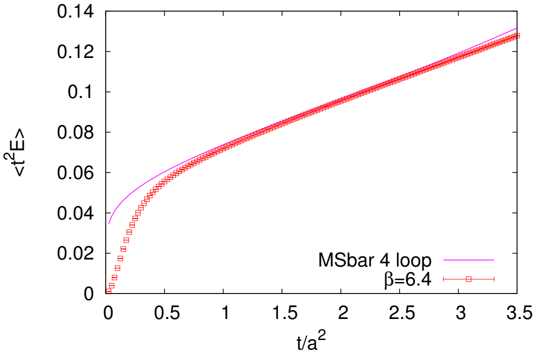
- Example: 1-point function of the "energy" operator (Lüscher (2010))

$$t^2 \langle E(t,x) \rangle = \frac{3(N^2-1)}{128\pi^2} \bar{g}(1/\sqrt{8t})^2 \left[1 + 2b_0 \bar{g}(1/\sqrt{8t})^2 + O(\bar{g}^4) \right],$$

where, in the $\overline{\text{MS}}$ scheme,

$$c \equiv \frac{\gamma_E}{2} + \frac{26}{33} - \frac{9}{22} \ln 3$$

- With the 4-loop running coupling



Application to the bulk thermodynamics of $SU(3)$ YM

- First physical application! (Asakawa–Hatsuda–Itou–Kitazawa–H.S. (FlowQCD Collaboration))
- The objective of this workshop
- Bulk thermodynamical quantities are obtained by the expectation value of EMT just at that temperature (no integration wrt the temperature)
- "Trace anomaly", or the interaction measure,

$$\langle \varepsilon - 3p \rangle_T = - \left\langle \{T_{\mu\mu}\}_R(x) \right\rangle_T ,$$

- Entropy density

$$\langle \varepsilon + p \rangle_T = - \left\langle \{T_{00}\}_R(x) \right\rangle_T + \frac{1}{3} \sum_{i=1,2,3} \left\langle \{T_{ii}\}_R(x) \right\rangle_T ,$$

- We do not need to determine the overall normalization (or the non-perturbative β function) separately!; the normalization is already fixed in the master formula

Application to the bulk thermodynamics of $SU(3)$ YM

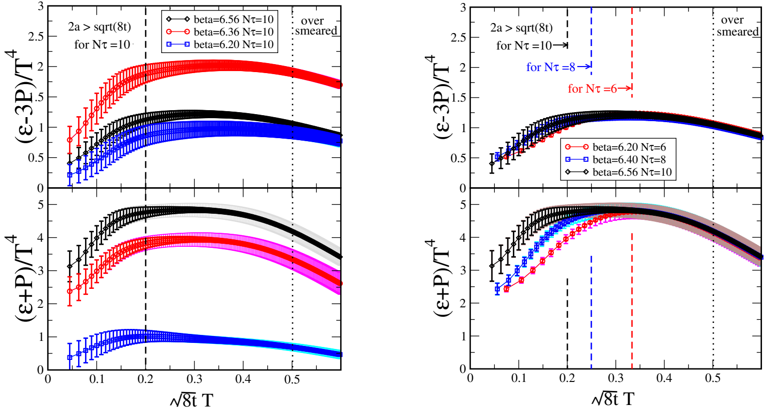
- Wilson plaquette action, 1 pseudo-heatbath sweep and 5 over-relaxations
- $N_s^3 \times N_\tau = 32^3 \times (6, 8, 10, 32)$

N_τ	6	8	10	T/T_c
	6.20	6.40	6.56	1.65
$\beta = 6/g_0^2$	6.02	6.20	6.36	1.24
	5.89	6.06	6.20	0.99

- For each parameter set, 300 configurations separated by 200–500 sweeps
- Wilson flow: 4th order Runge–Kutta with $\epsilon/a^2 = 0.025$
- Scale setting: $\beta \leftrightarrow a\Lambda$ from ALPHA Collaboration), aT_c at $\beta = 6.20$ from Boyd et al.
- 4-loop running coupling in $\overline{\text{MS}}$ scheme used in the coefficients
- Clover-type field strength $G_{\mu\nu}^a(x)$

Application to the bulk thermodynamics of $SU(3)$ YM

- Thermal expectation values versus the flow time $\sqrt{8t}$



- We observe plateau behavior for $2a < \sqrt{8t}$ (and for $\sqrt{8t} < 1/(2T)$) which may be regarded as would-be $t \to 0$ limit

Application to the bulk thermodynamics of $SU(3)$ YM

- Continuum limit of the values at $\sqrt{8t}T = 0.40$

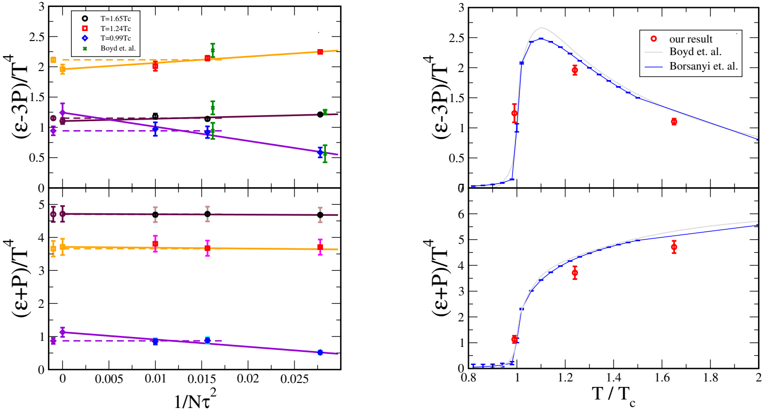


Figure: cf. Boyd et al. (1996); Borsanyi et al. (2012)

- Consistent within 2σ; quite encouraging! A much comprehensive study using finer and larger lattices is carrying out

Summary and prospects

- We developed a formula that relates a correctly-normalized conserved EMT and quantities defined by the Yang–Mills gradient flow:

$$\{T_{\mu\nu}\}_R(x) = \lim_{t\to 0}\left\{\frac{1}{\alpha_U(t)}U_{\mu\nu}(t,x) + \frac{1}{4\alpha_E(t)}\delta_{\mu\nu}\left[E(t,x) - \langle E(t,x)\rangle\right]\right\}$$

- Correlation functions of RHS can be computed by lattice Monte Carlo simulations
- Possible obstacle would be

$$a \ll \sqrt{8t}$$

- The measurement of one-point functions in the finite temperature indicates that our reasoning is correct and the present approach is promising
- The conservation law of EMT is still needed to be demonstrated by Monte Carlo simulations

Summary and prospects

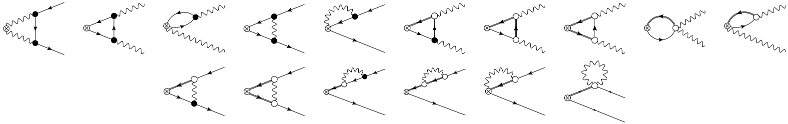
- Inclusion of matter fields: flowed matter field requires the wave function renormalization (Lüscher (2013))

$$\chi(t,x)=Z_\chi^{-1/2}\chi_R(t,x),\qquad \bar{\chi}(t,x)=Z_\chi^{-1/2}\bar{\chi}_R(t,x)$$

  To avoid the matching of Z_χ between the continuum and lattice theories, we may use fields normalized by their "condensation" as, for example,

$$\tilde{\chi}(t,x)\equiv\sqrt{\frac{-2\dim(R)N_fm}{(4\pi)^2t\langle\bar{\chi}(t,x)\chi(t,x)\rangle}}\,\chi(t,x)$$

- Composite operators of the tilded field are UV finite and our argument applies
- To find mixing coefficients in the next to leading order, requires



- $\sim$ 90% of required calculation was over (but we still have some inconsistency. . .)

Summary and prospects

- Further physical applications: Viscosities, conformal field theory, dilaton physics, vacuum energy, . . .
- By a similar idea using the Yang–Mills gradient flow, can we construct other Noether currents, such as the chiral current or the SUSY current, on the lattice???