Fluctuations of Conserved Charges

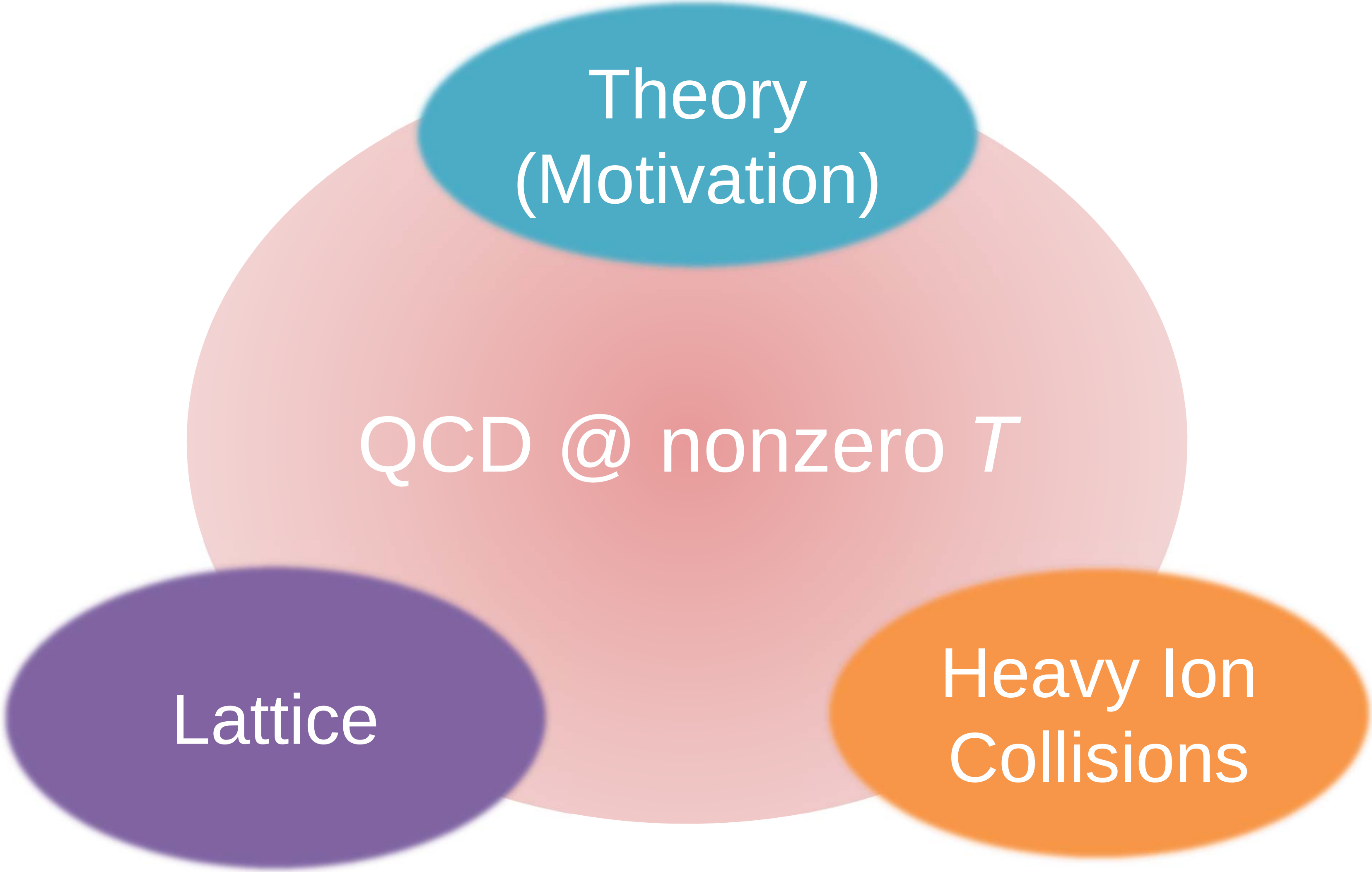
- Theory, Experiment, and Lattice -

Masakiyo Kitazawa

（Osaka U.）

KEK, 2014/Jan./20

QCD @ nonzero T



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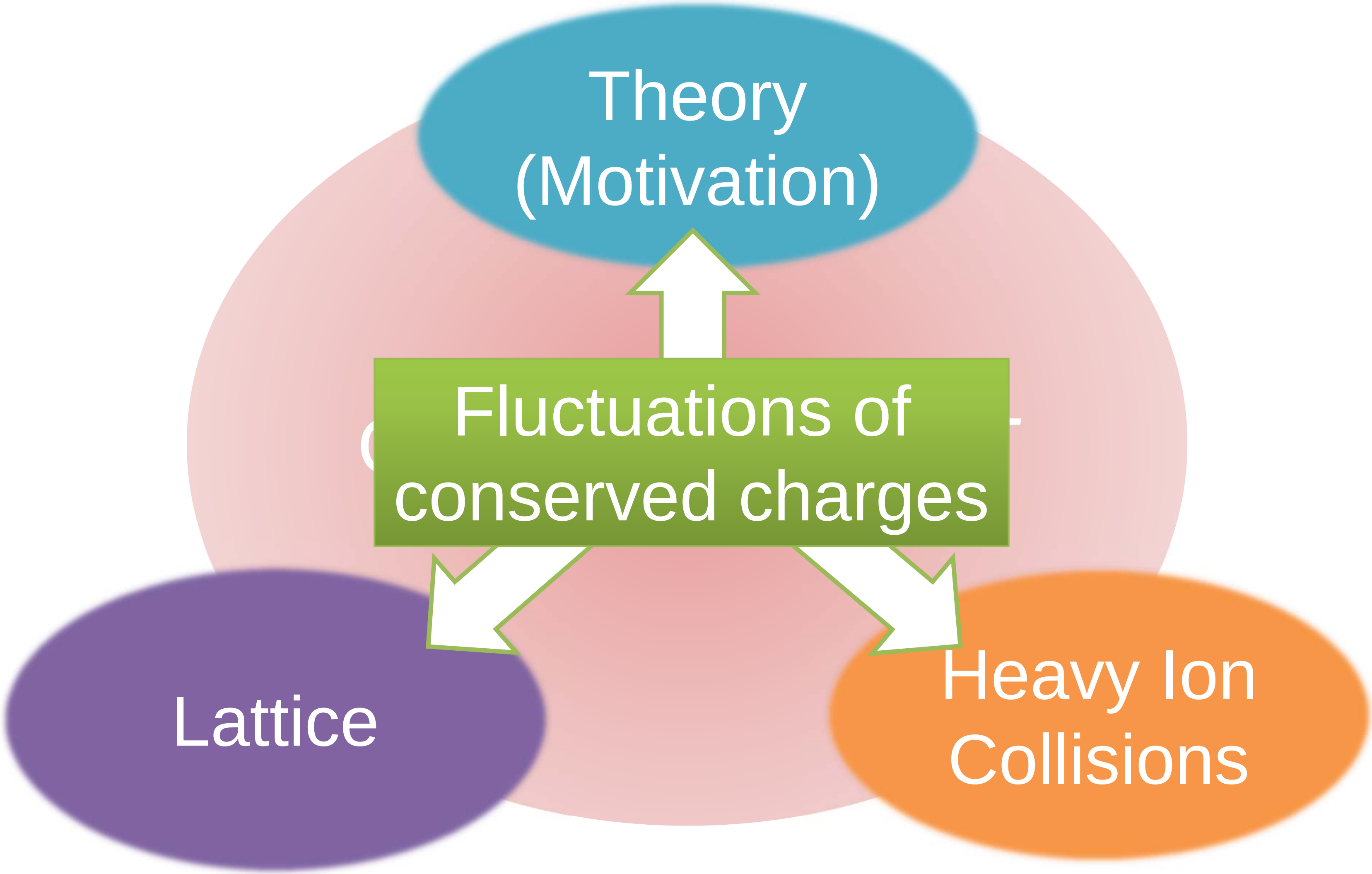

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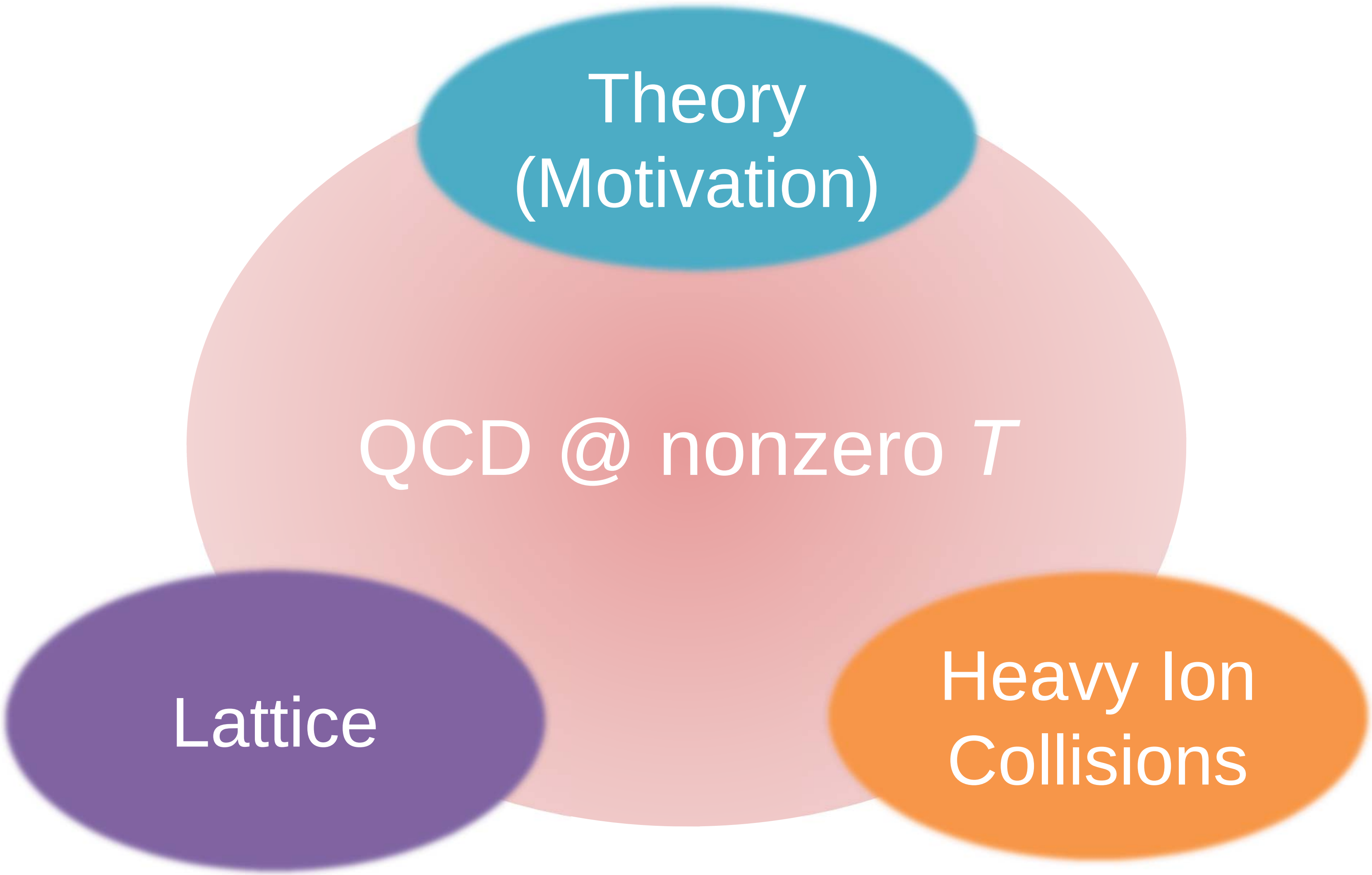
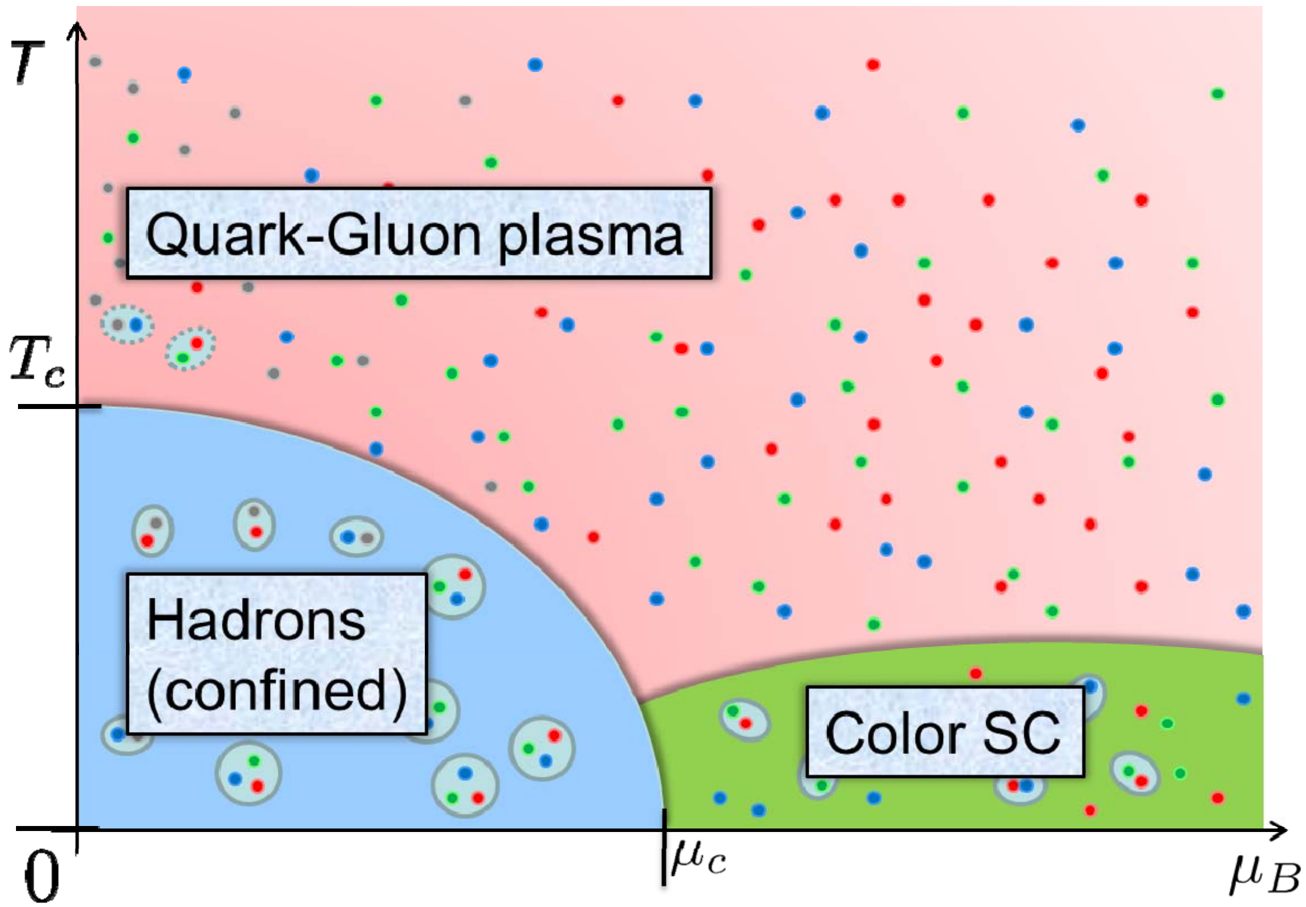


Why QCD @ nonzero T and μ?

- Form of the matter under extreme conditions
  - QCD Phase diagram
  - New many body properties



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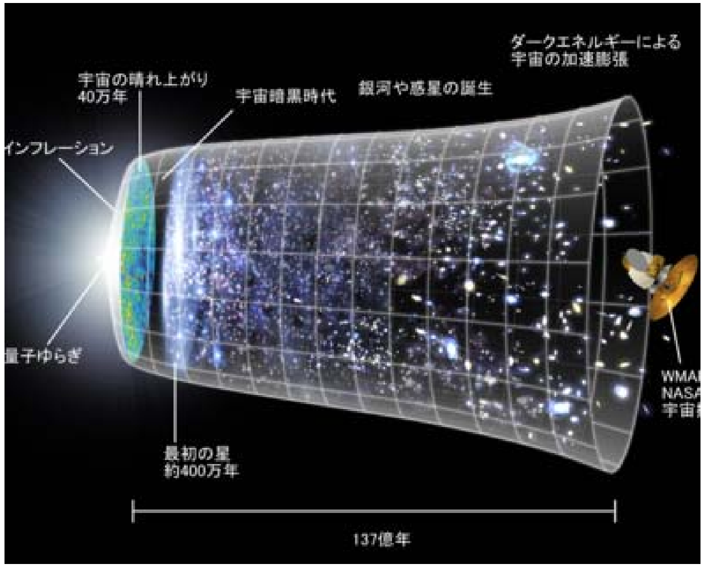
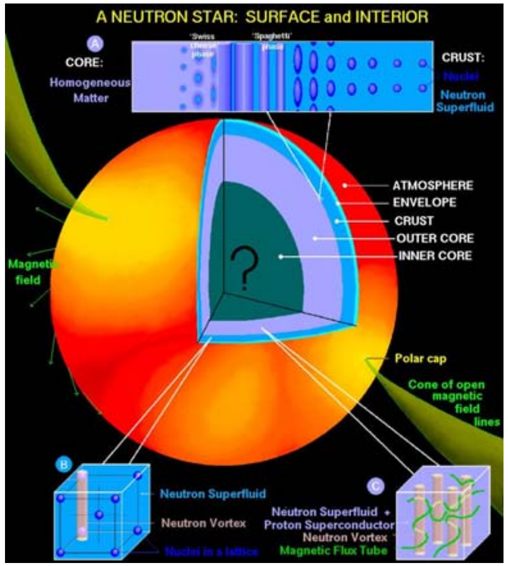
- State of the matter realized in
    - Early Universe
    - Compact stars





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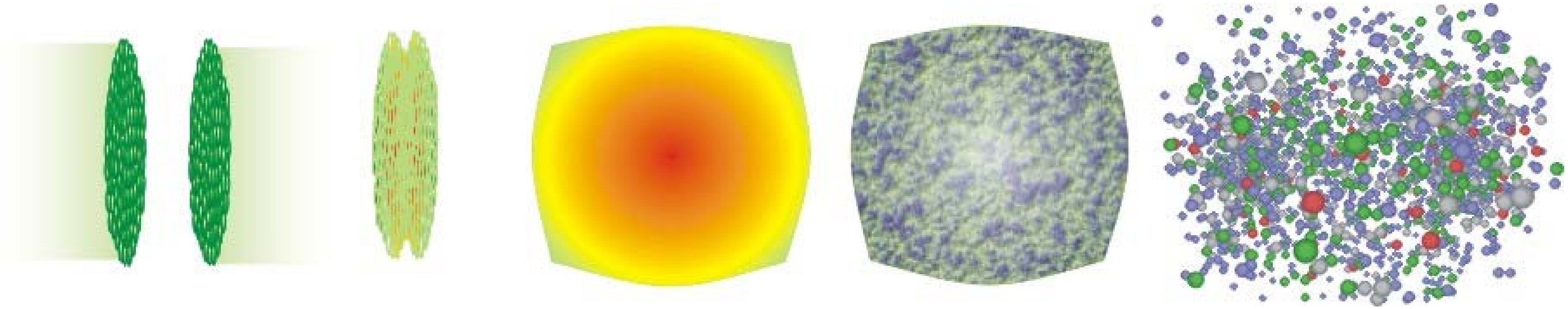
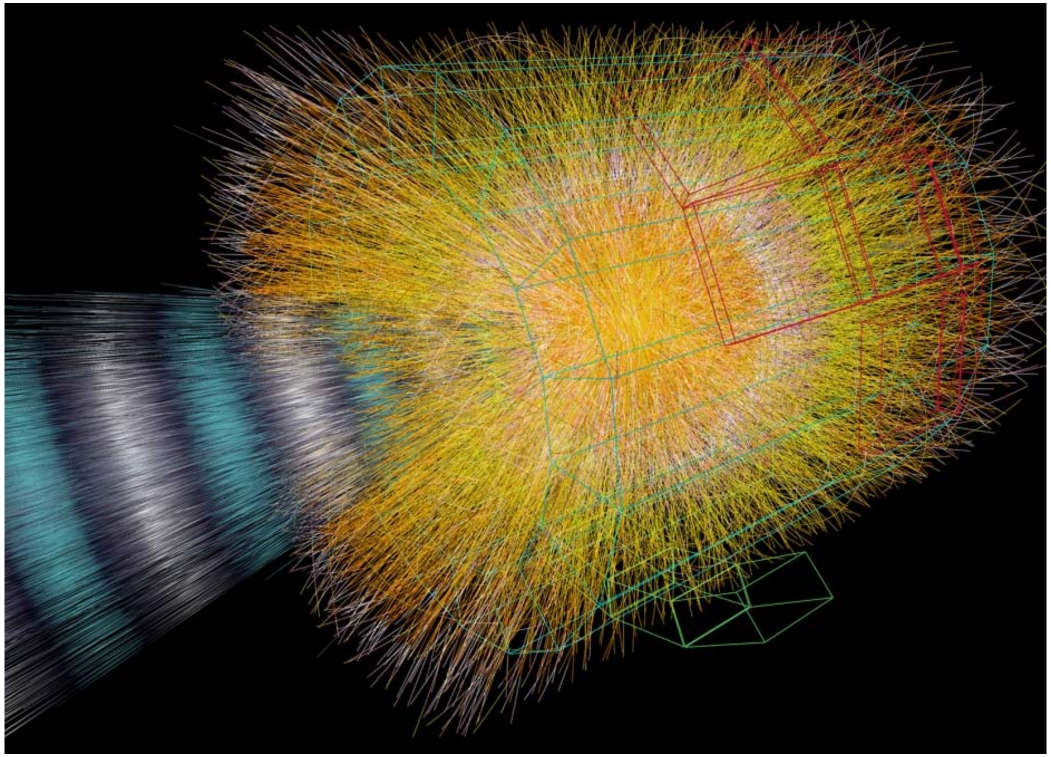
- Relativistic heavy ion collisions

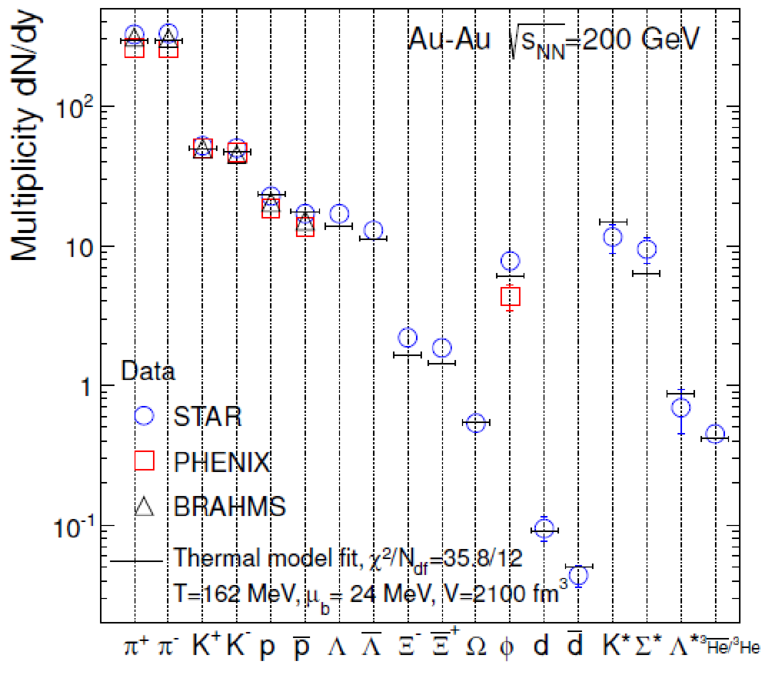
Relativistic Heavy Ion Collisions



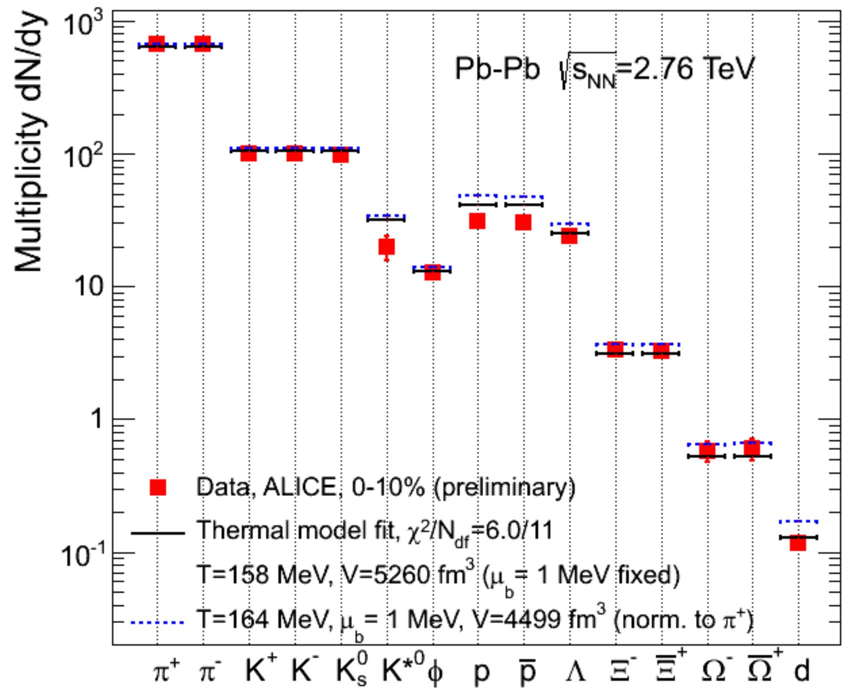


Chemical Freezeout

RHIC



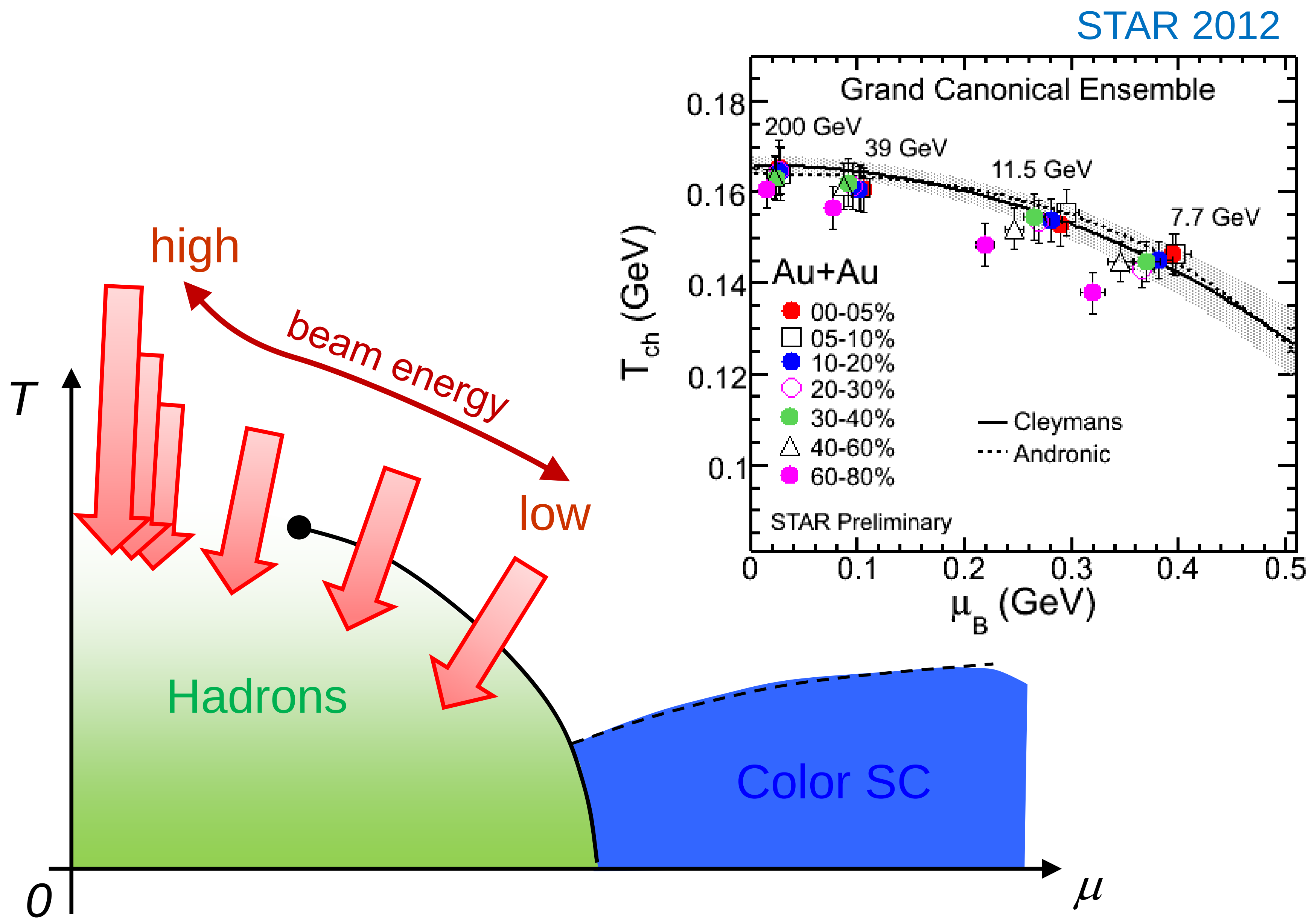
LHC



Particle yields can be well described only by T, μ_B!

chemical equilibration?

Beam-Energy Scan Program

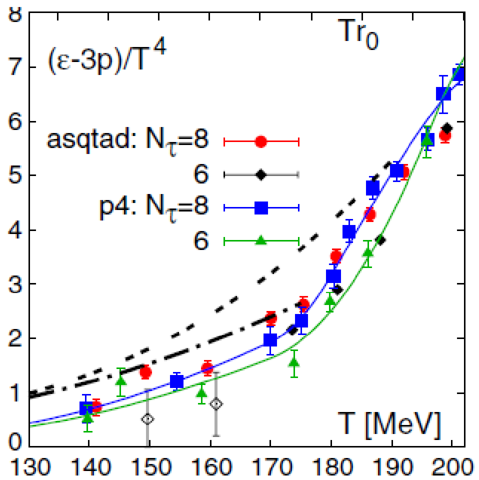


Hadron Resonance Gas (HRG) Model

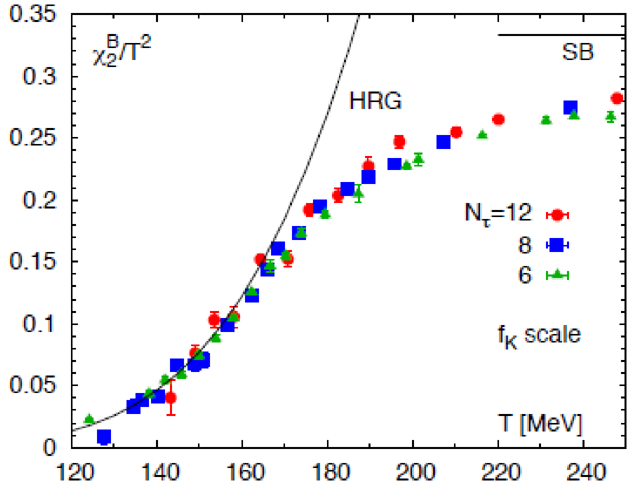
HRG model

free gas composed of known hadrons

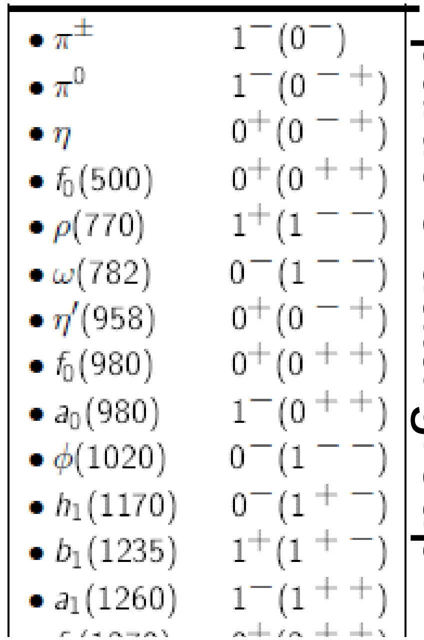
The HRG model well describes thermodynamics calculated on the lattice.



“Trace Anomaly”



Baryon # fluctuation

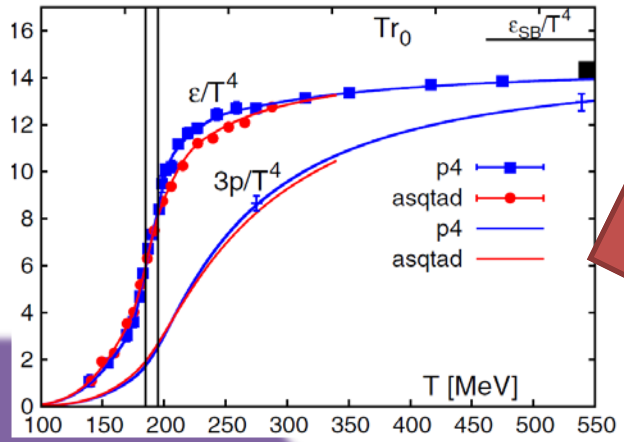


• $\pi^{\pm}$	$1^-(0^-)$
• π^0	$1^-(0^{-+})$
• η	$0^+(0^{-+})$
• $f_0(500)$	$0^+(0^{++})$
• $\rho(770)$	$1^+(1^{--})$
• $\omega(782)$	$0^-(1^{--})$
• $\eta'(958)$	$0^+(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$
• $a_0(980)$	$1^-(0^{++})$
• $\phi(1020)$	$0^-(1^{--})$
• $h_1(1170)$	$0^-(1^{+-})$
• $b_1(1235)$	$1^+(1^{+-})$
• $a_1(1260)$	$1^-(1^{++})$

particle data group

Lattice and HIC : EoS

Equation of states



Lattice

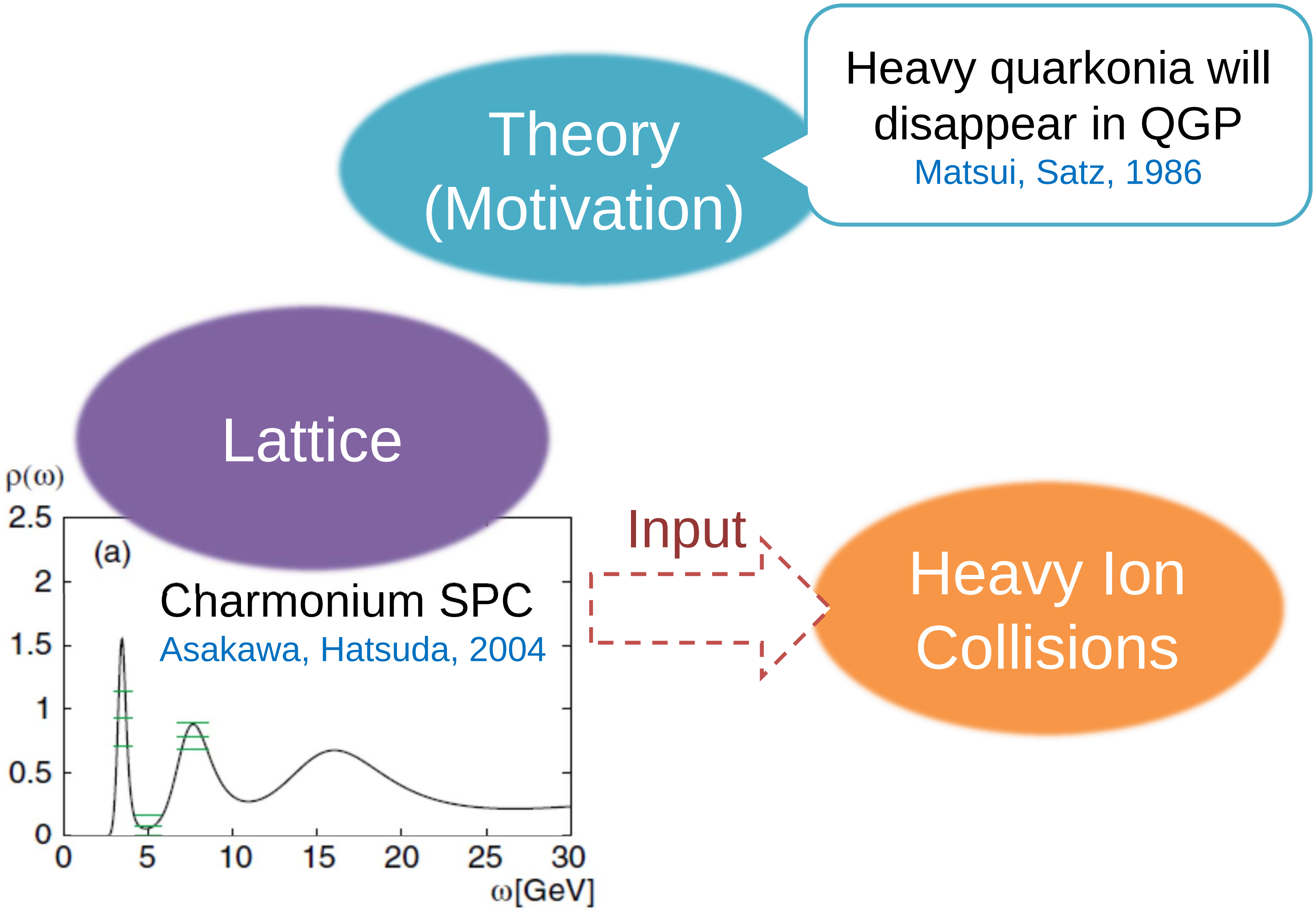
Input

Heavy Ion Collisions

Challenge!

- Robust modelling of space-time evolution
- Small shear viscosity

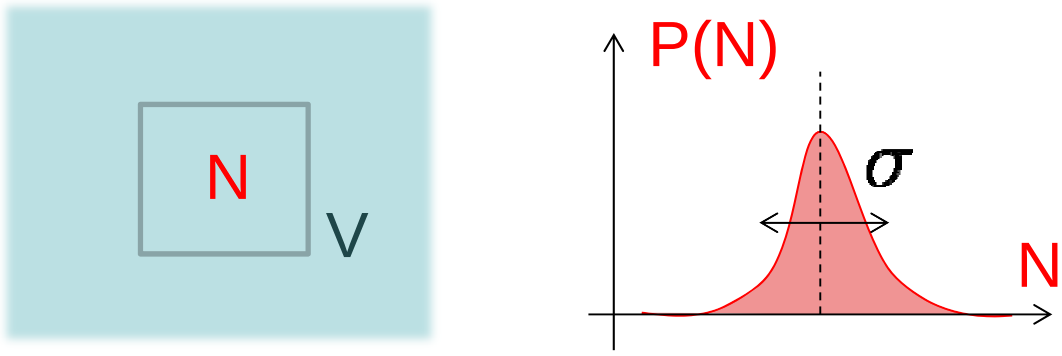
Lattice and HIC : Heavy Quarkonia



Fluctuations of Conserved Charges

Fluctuations

Observables in equilibrium are fluctuating.



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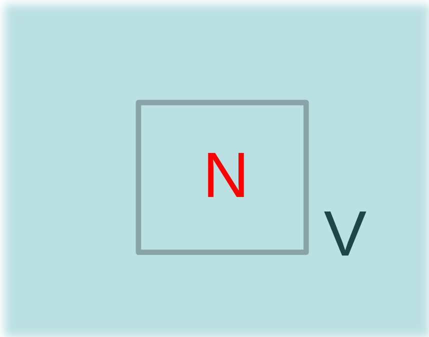
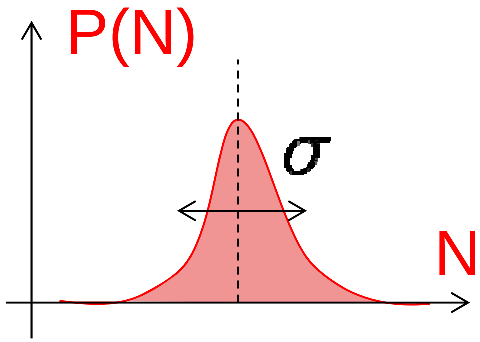




- Variance: $\langle \delta N^2 \rangle = V\chi_2 = \sigma^2$
- Skewness: $S = \dfrac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \dfrac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

$\delta N = N - \langle N \rangle$

Non-Gaussianity

Conserved Charge Fluctuations

- Definite definition of the operator $\mathcal{O}$

$$\rho = \frac{1}{Z} e^{-\beta H}$$

  - as a Noether current
  - Expectation value: $\langle \mathcal{O} \rangle = \mathrm{Tr}[\rho \mathcal{O}] = \int \mathcal{D}U \mathcal{O} e^{-S}$
  - Fluctuation: $\langle \delta \mathcal{O}^2 \rangle = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$

- Simple thermodynamic relation

$$\langle \delta \mathcal{O}^n \rangle_c = \frac{T^n}{V} \frac{\partial^n}{\partial \mu^n} \ln Z(\mu) \qquad Z(\mu) = \mathrm{Tr} e^{-\beta(H - \mu \mathcal{O})}$$

Taylor Expansion Method & Cumulants

$$P(T,\mu) = \frac{T}{V}\ln Z(\mu)$$

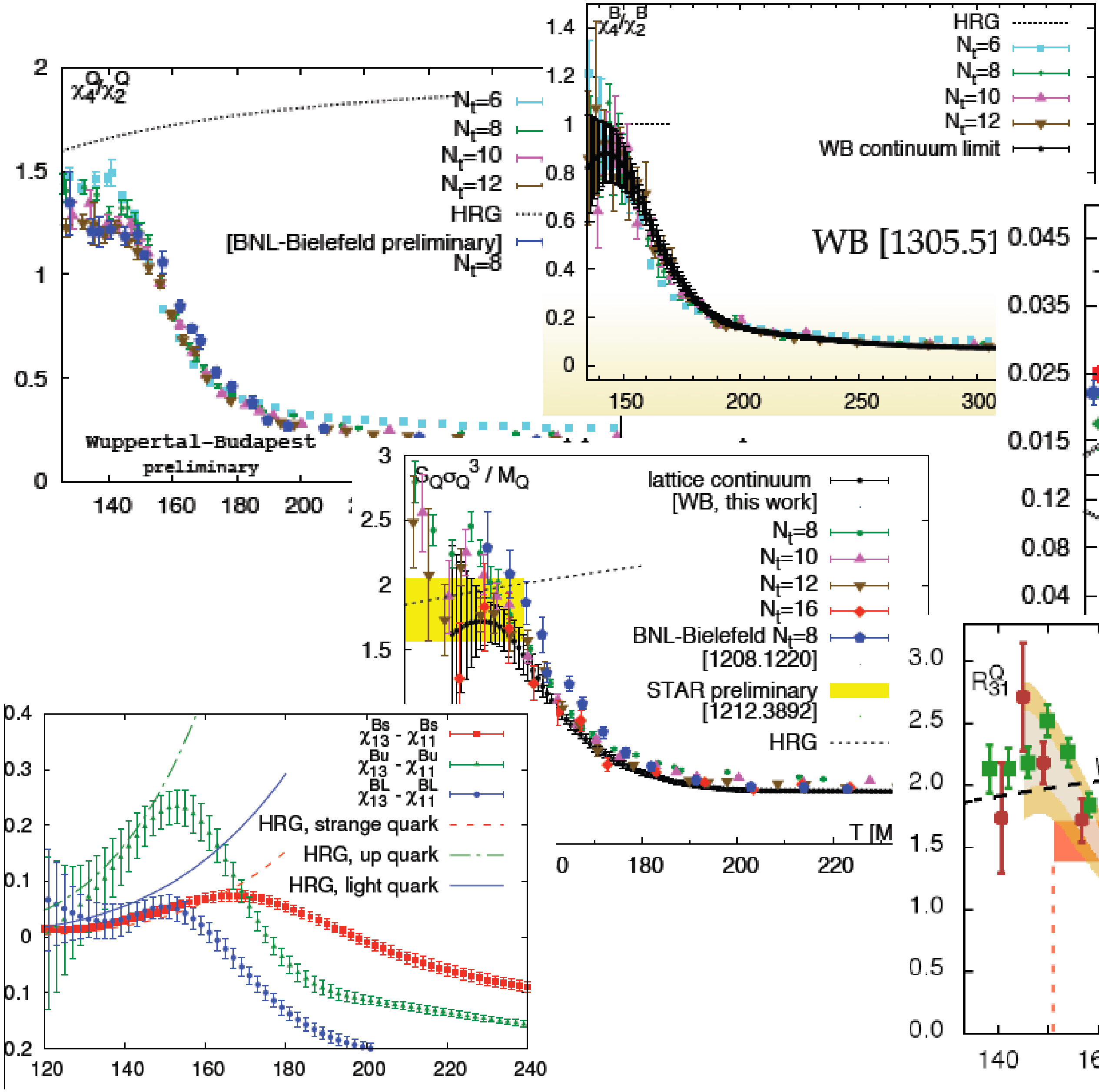
$$= P(T,0) + \frac{\mu}{T}\frac{\partial P(T,0)}{\partial(\mu/T)} + \frac{1}{2}\left(\frac{\mu}{T}\right)^2 \frac{\partial^2 P(T,0)}{\partial(\mu/T)^2} + \cdots$$

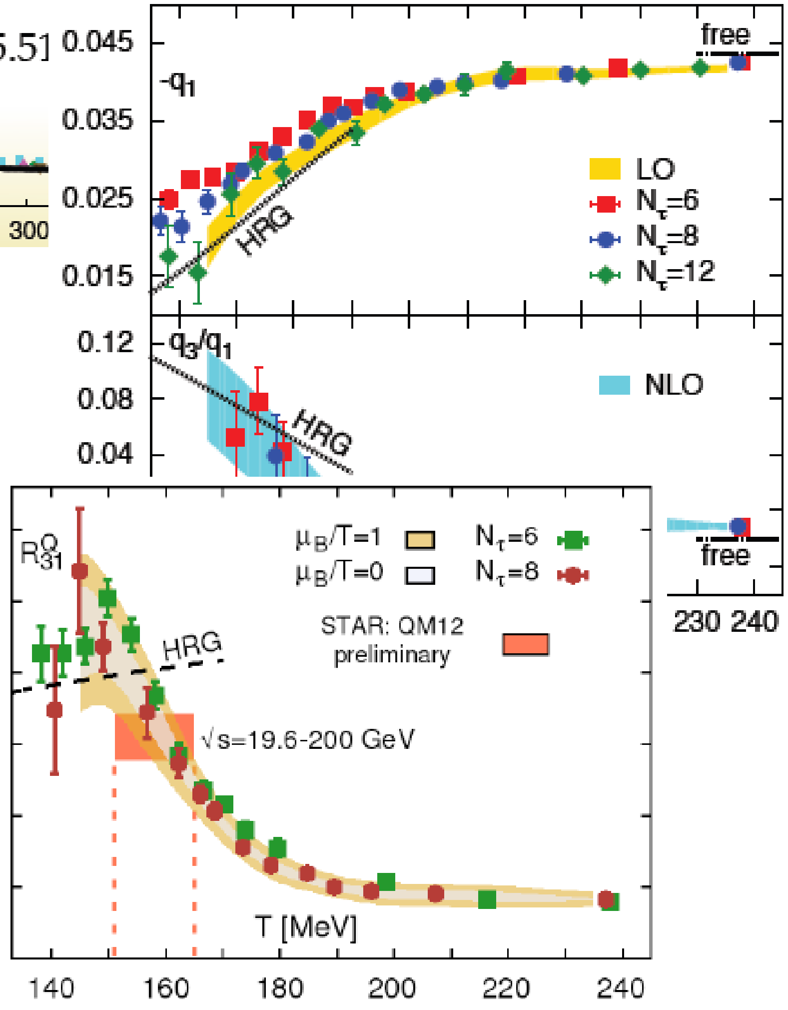
$\langle N \rangle$ $\qquad\qquad$ $\langle \delta N^2 \rangle_c$

Baryon number cumulants = Taylor expansion coeffs.

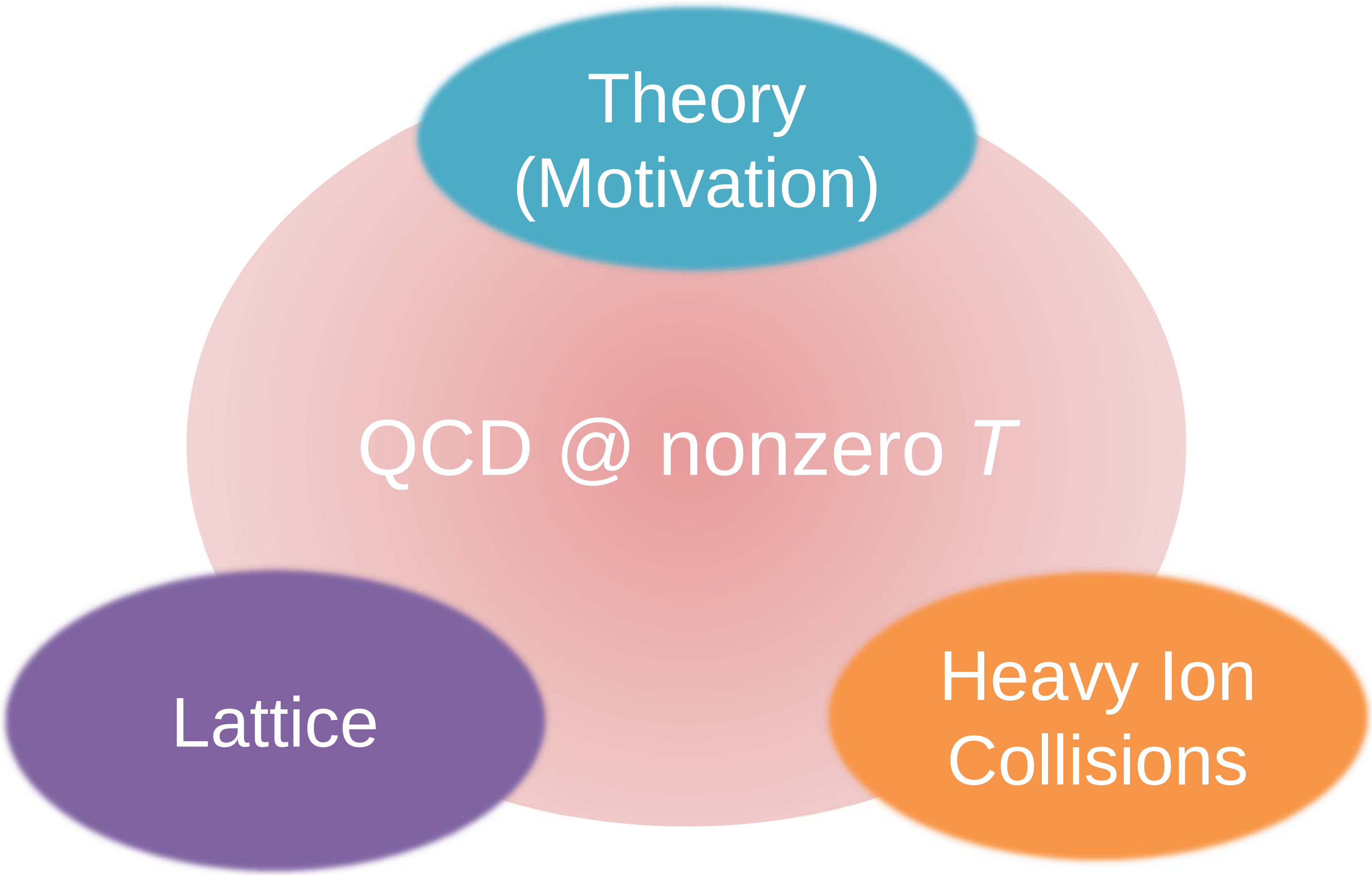
Recent Progress in Lattice Simulations

From LATTICE2013
presentations



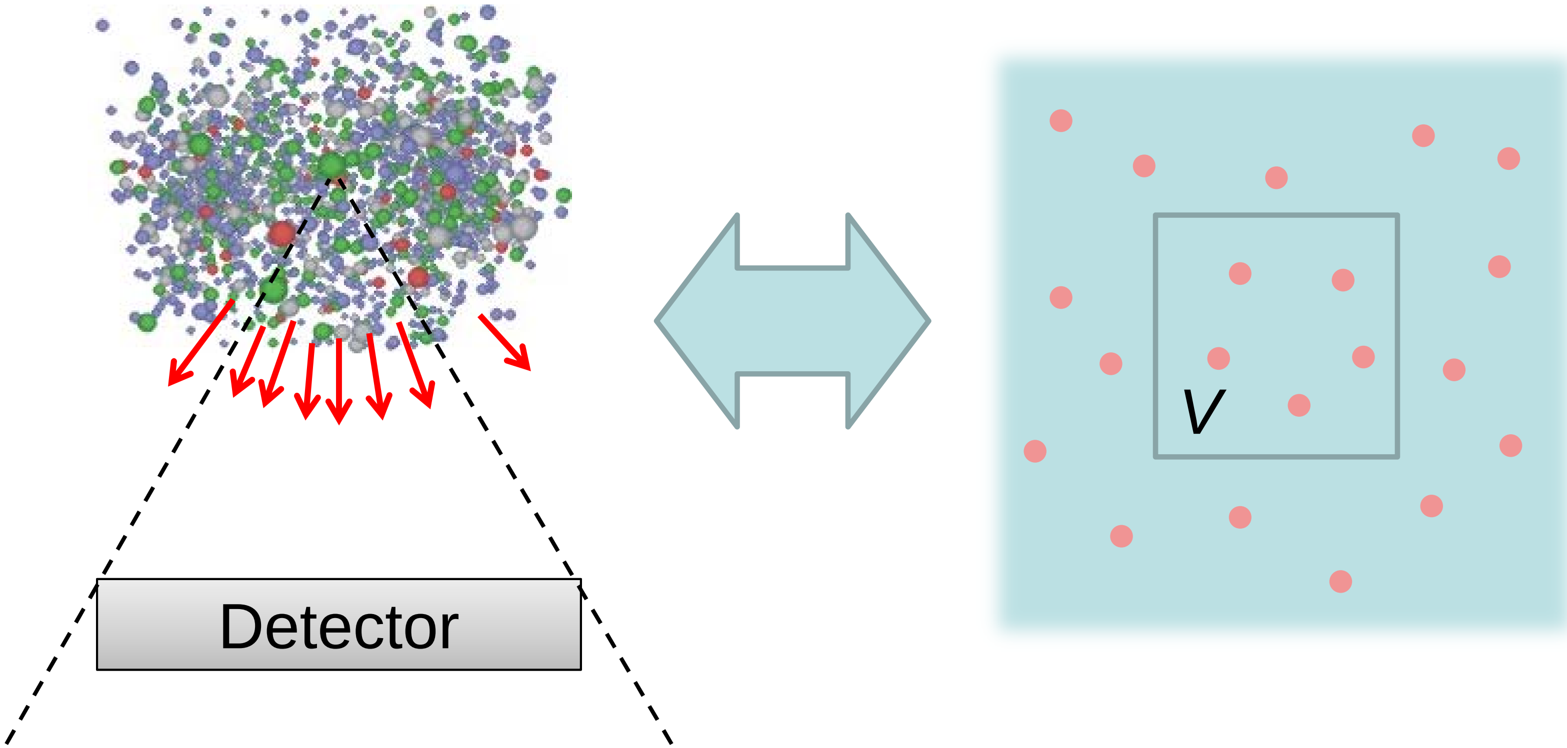


QCD @ nonzero T



Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.



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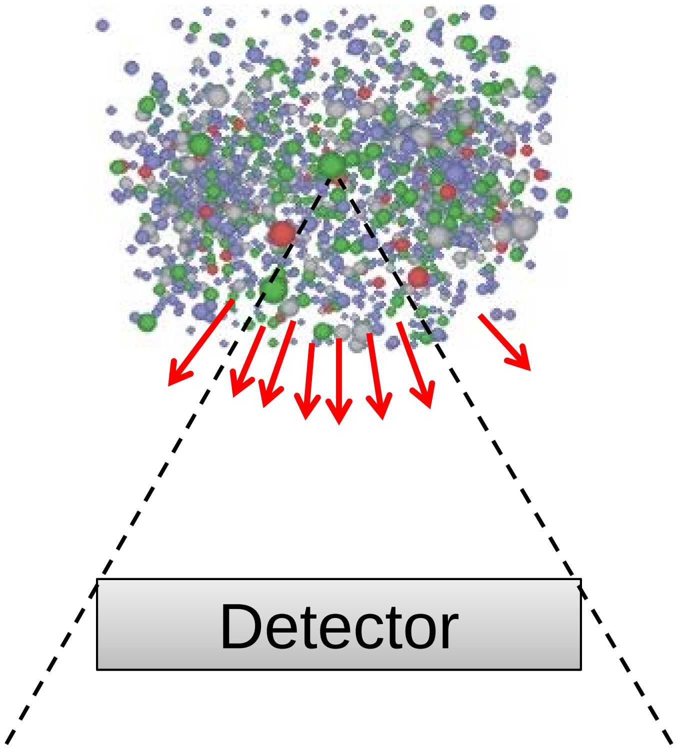
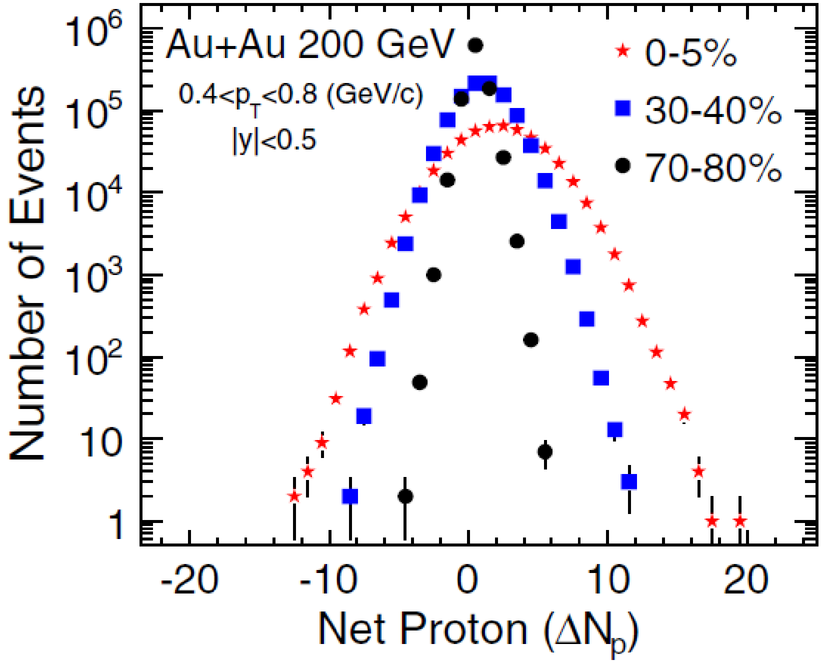
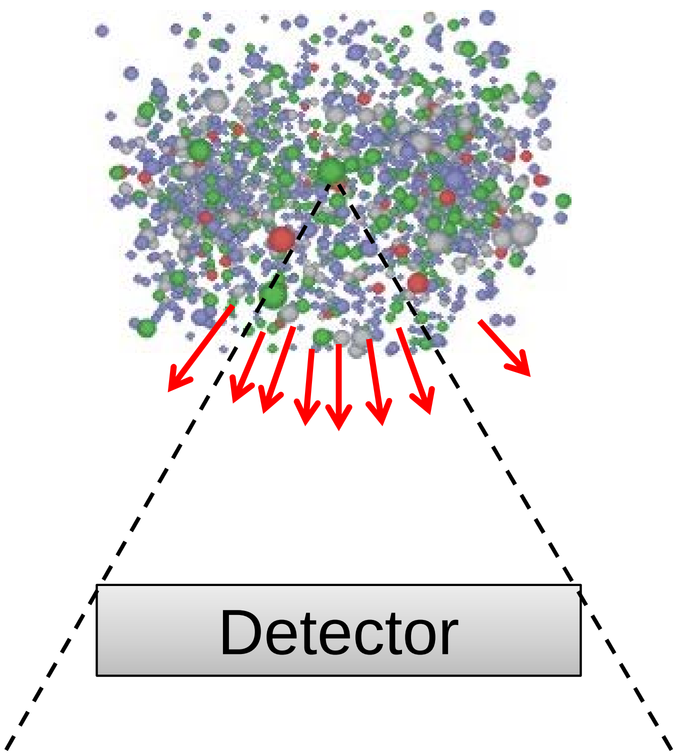


STAR, PRL**105** (2010)



$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$

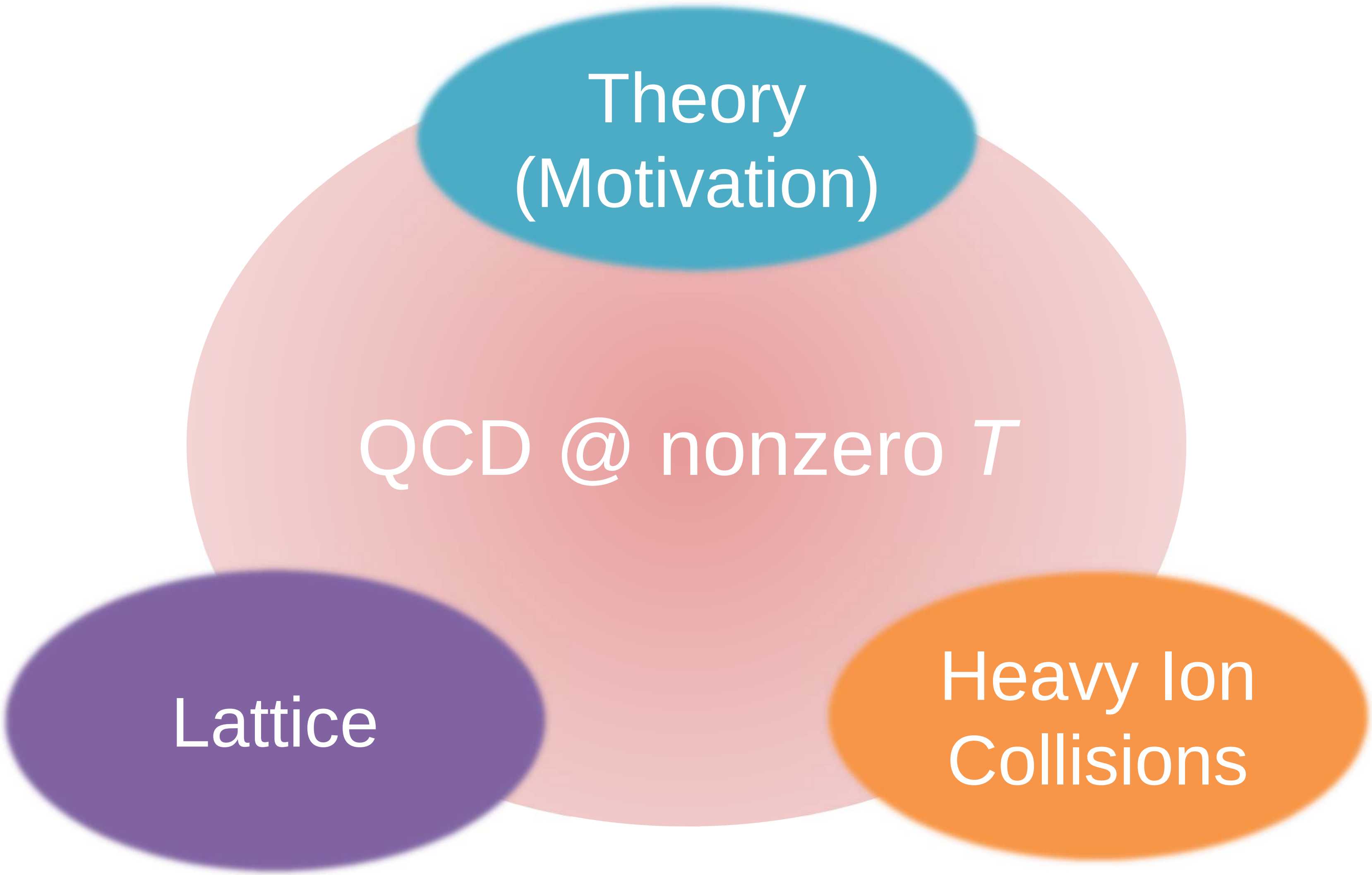
What are Fluctuations observed in HIC?



QUESTION:
When the experimentally-observed fluctuations are formed?

- at chemical freezeout?
- at kinetic freezeout?
- or, much earlier?

QCD @ nonzero *T*



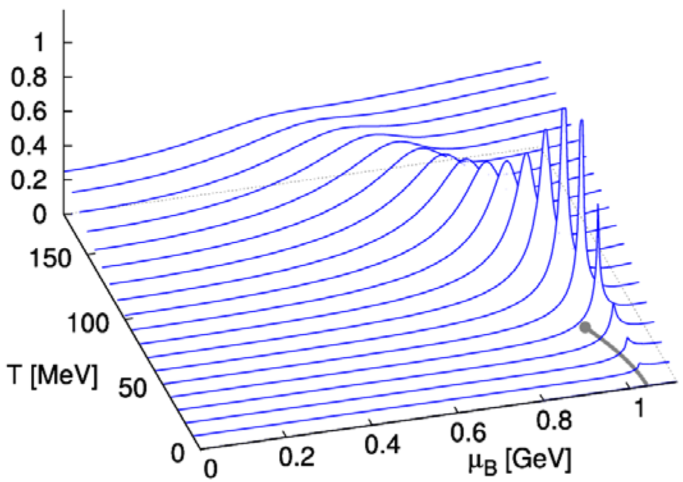
Fluctuations

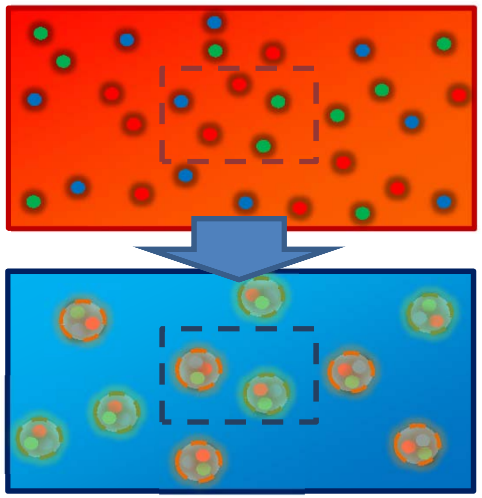
- Fluctuations reflect properties of matter.
  - Enhancement near the critical point

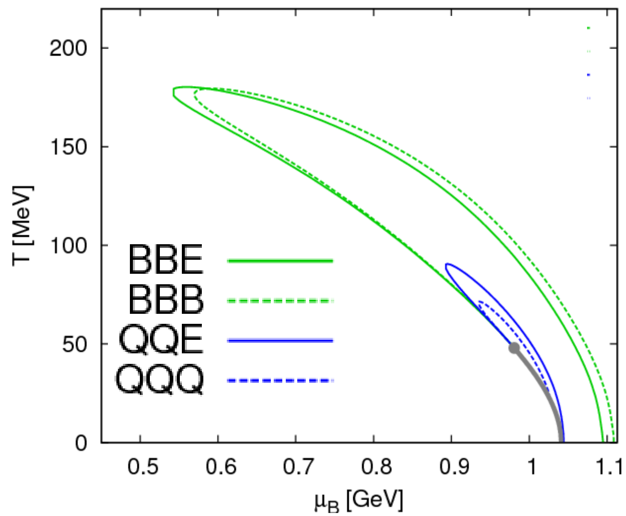
    Stephanov,Rajagopal,Shuryak('98); Hatta,Stephanov('02); Stephanov('09);…
  - Ratios between cumulants of conserved charges

    Asakawa,Heintz,Muller('00); Jeon, Koch('00); Ejiri,Karsch,Redlich('06)
  - Signs of higher order cumulants

    Asakawa,Ejiri,MK('09); Friman,et al.('11); Stephanov('11)



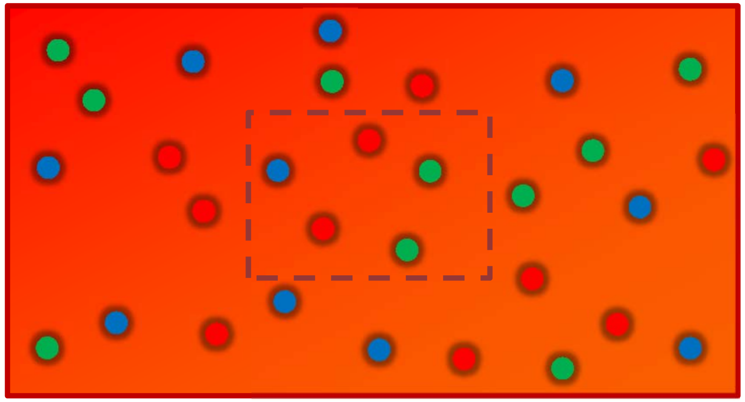




Fluctuations

Free Boltzmann → Poisson

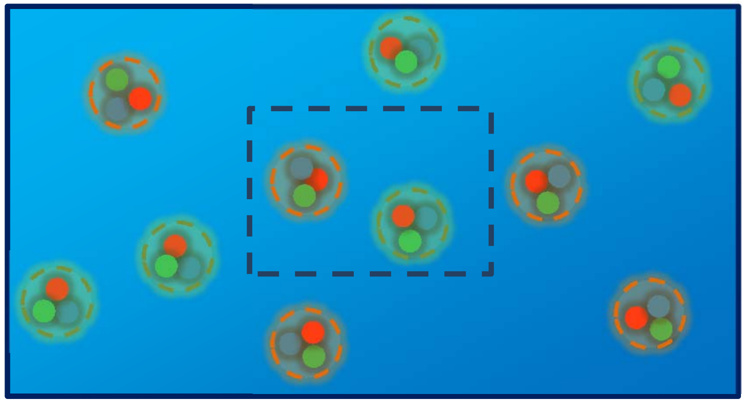
$$\langle \delta N^n \rangle_c = \langle N \rangle$$



$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\Rightarrow \langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$



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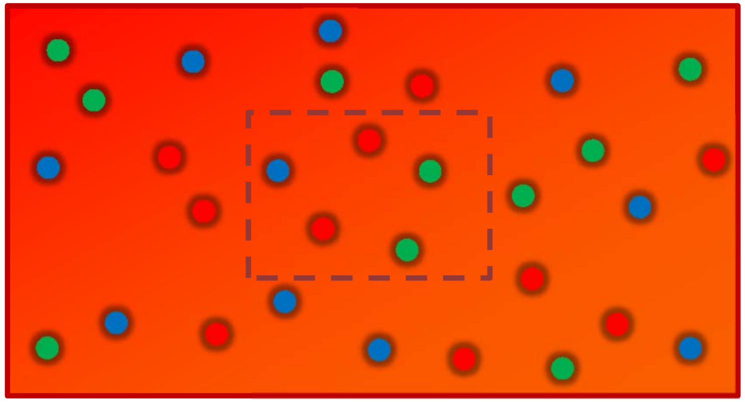

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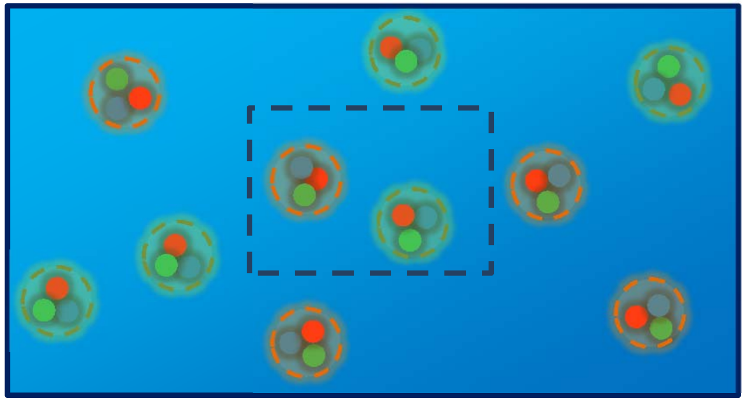

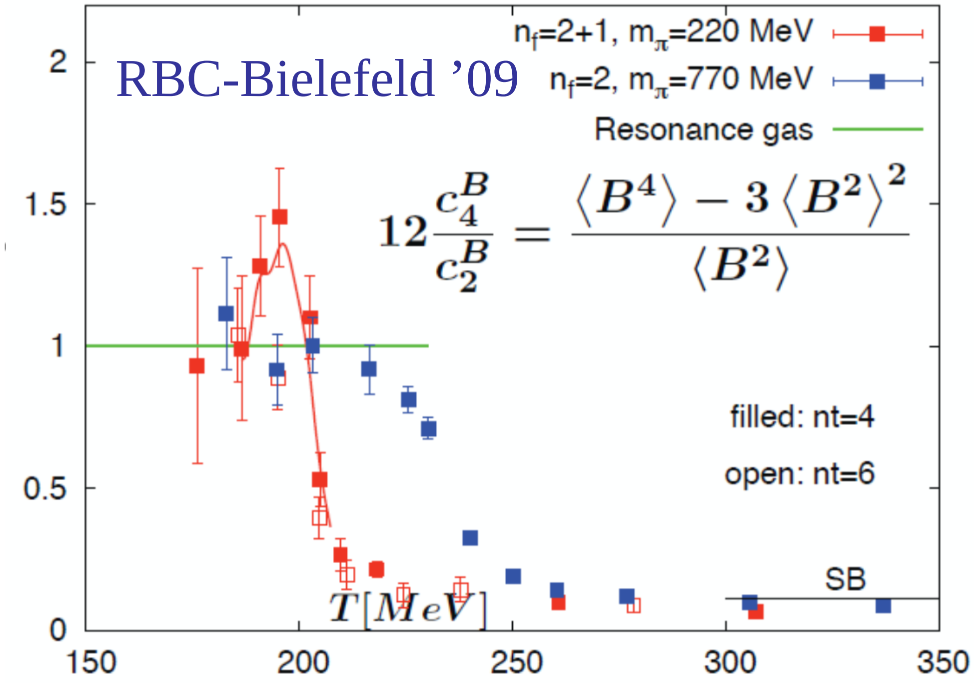

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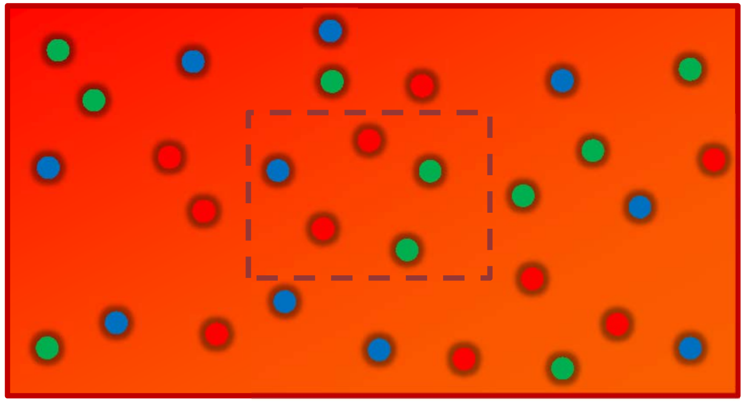

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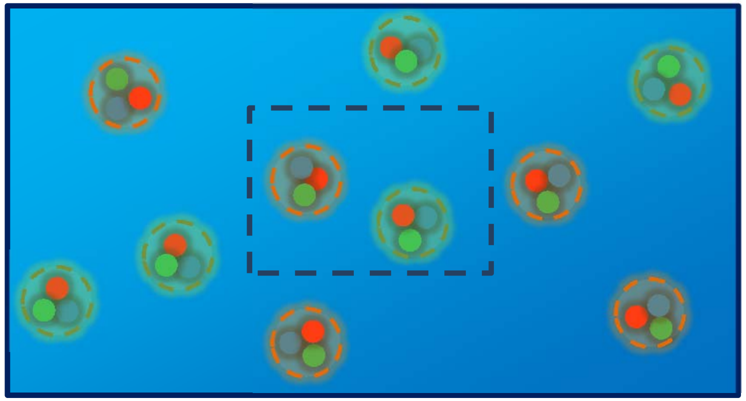
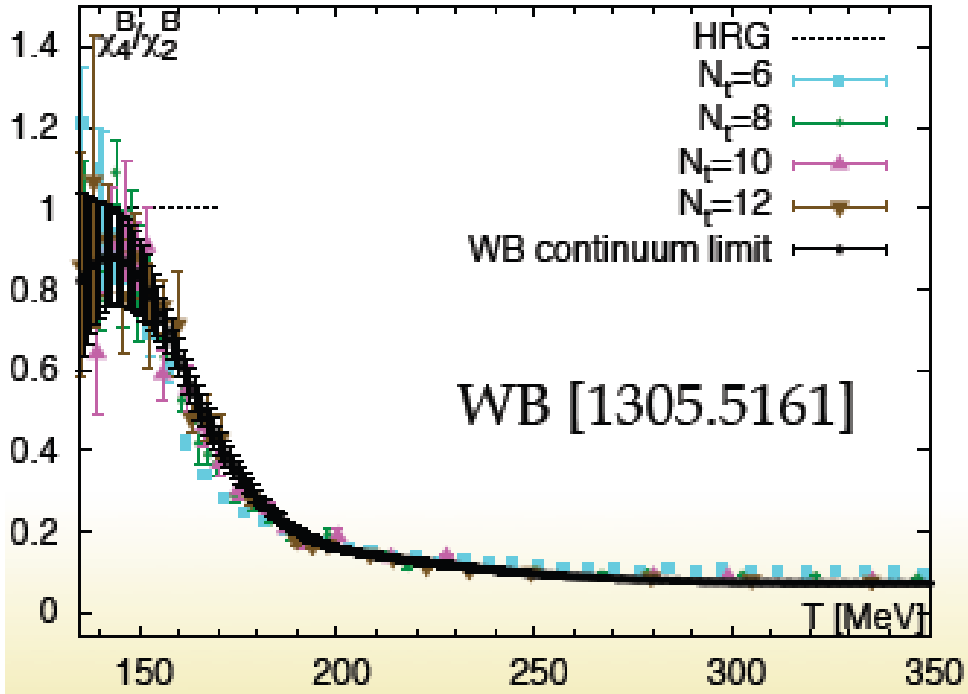




Skellam Distribution

- Poisson + Poisson = Poisson

$\langle N_1 \rangle$ $\langle N_2 \rangle$ $\langle \delta N^n \rangle_c = \langle N_1 + N_2 \rangle$

- Poisson — Poisson = **Skellam** distribution

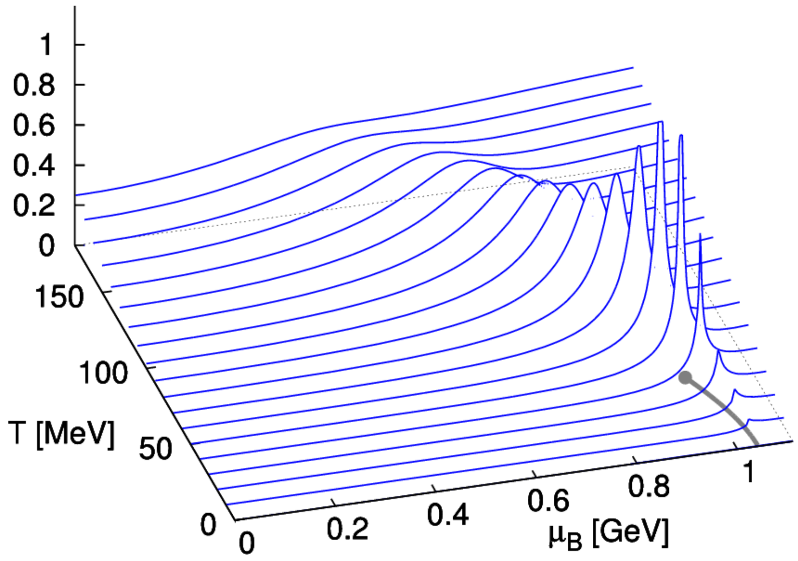
$\langle N_1 \rangle$ $\langle N_2 \rangle$ $\langle \delta N^n \rangle_c = \begin{cases} \langle N_1 + N_2 \rangle & \text{(n:even)} \\ \langle N_1 - N_2 \rangle & \text{(n:odd)} \end{cases}$

In the HRG model,
(Net-)baryon and electric charge fluctuations
are of Skellam distribution.

Search of QCD Critical Point

- Fluctuations diverge at the QCD critical point.

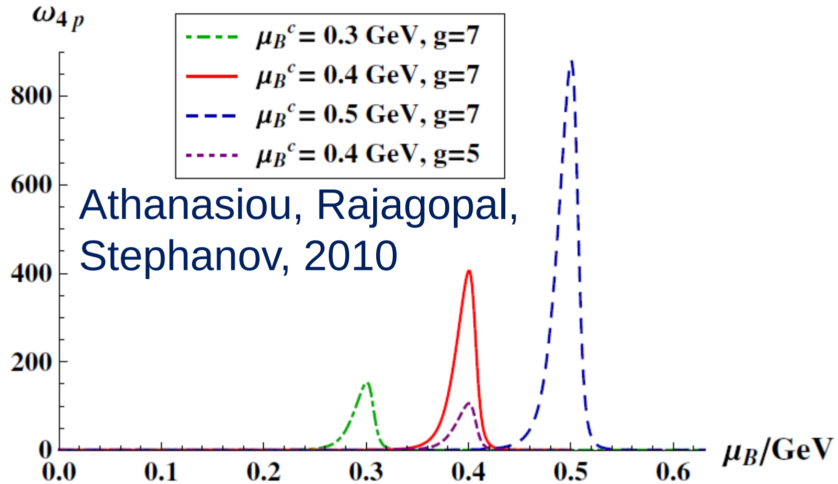
  Example: $\langle \delta N_B^2 \rangle$



- Higher order cumulants are more sensitive to correlation length

$$\begin{cases} \langle \delta N^2 \rangle \sim \xi^2 \\ \langle \delta N^3 \rangle \sim \xi^{4.5} \\ \langle \delta N^4 \rangle_c \sim \xi^7 \end{cases}$$

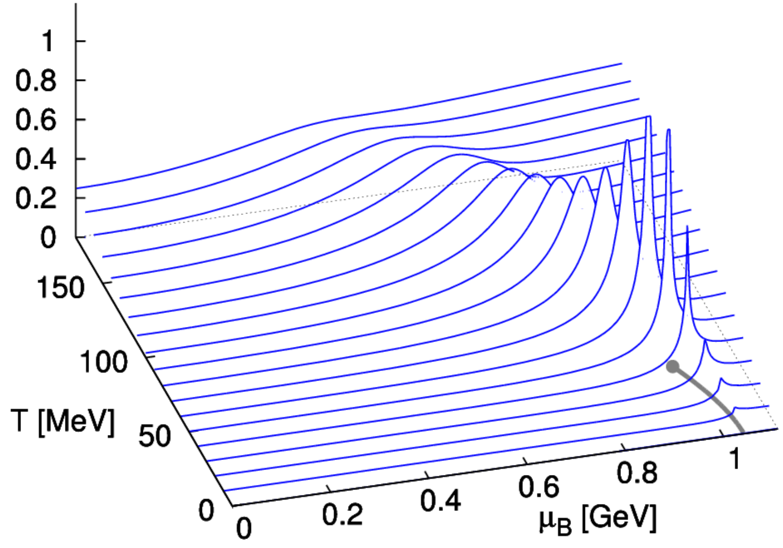
Stephanov, PRL, 2010



Athanasiou, Rajagopal, Stephanov, 2010

Sign of Higher Order Cumulants

Asakawa, Ejiri, MK, PRL,2009



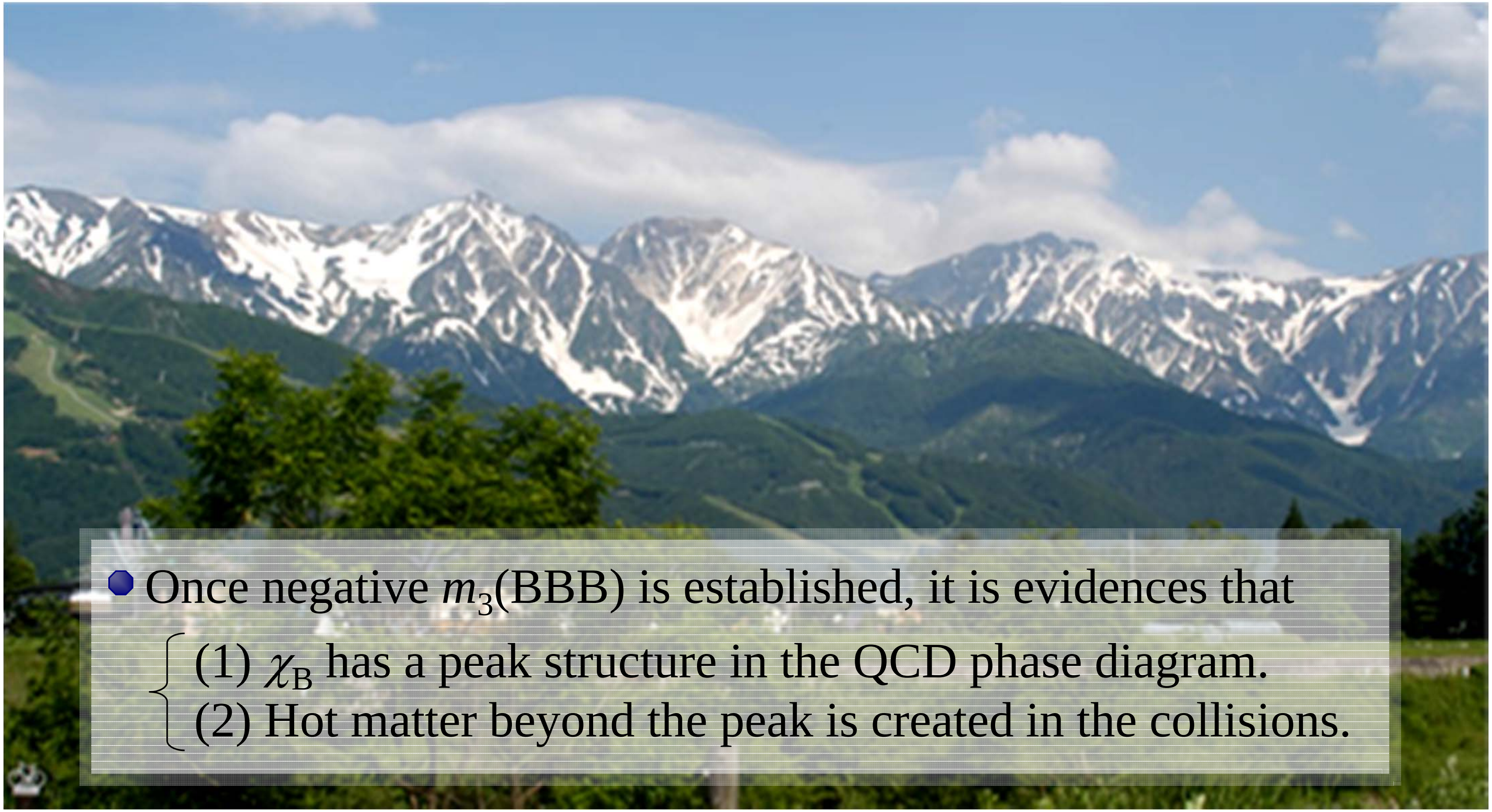
- χ_B has an edge along the phase boundary

↓

$\frac{\partial \chi_B}{\partial \mu_B}$ changes the sign at QCD phase boundary!

- $$\begin{cases} \chi_B = -\frac{1}{V}\frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\langle (\delta N_B)^2 \rangle}{VT} \\ \frac{\partial \chi_B}{\partial \mu_B} = -\frac{1}{V}\frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\langle (\delta N_B)^3 \rangle}{VT^2} \end{cases}$$

Impact of Negative Third Moments

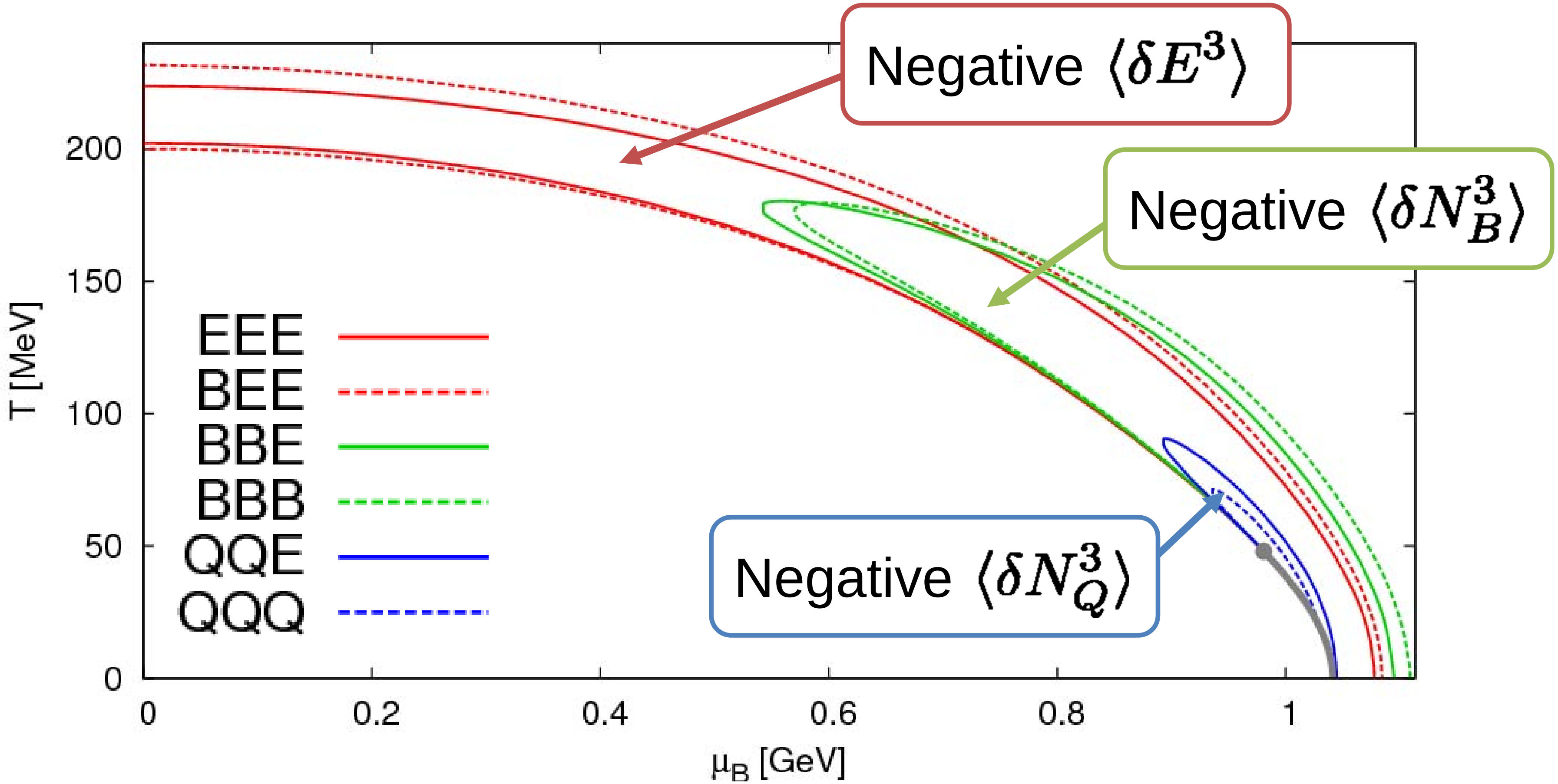


- Once negative m_3(BBB) is established, it is evidences that
  - (1) χ_B has a peak structure in the QCD phase diagram.
  - (2) Hot matter beyond the peak is created in the collisions.

-
  - **No** dependence on any specific models.
  - Just the sign! **No** normalization (such as by N_{ch}).

Various Third Moments

Asakawa, Ejiri, MK, PRL,2009

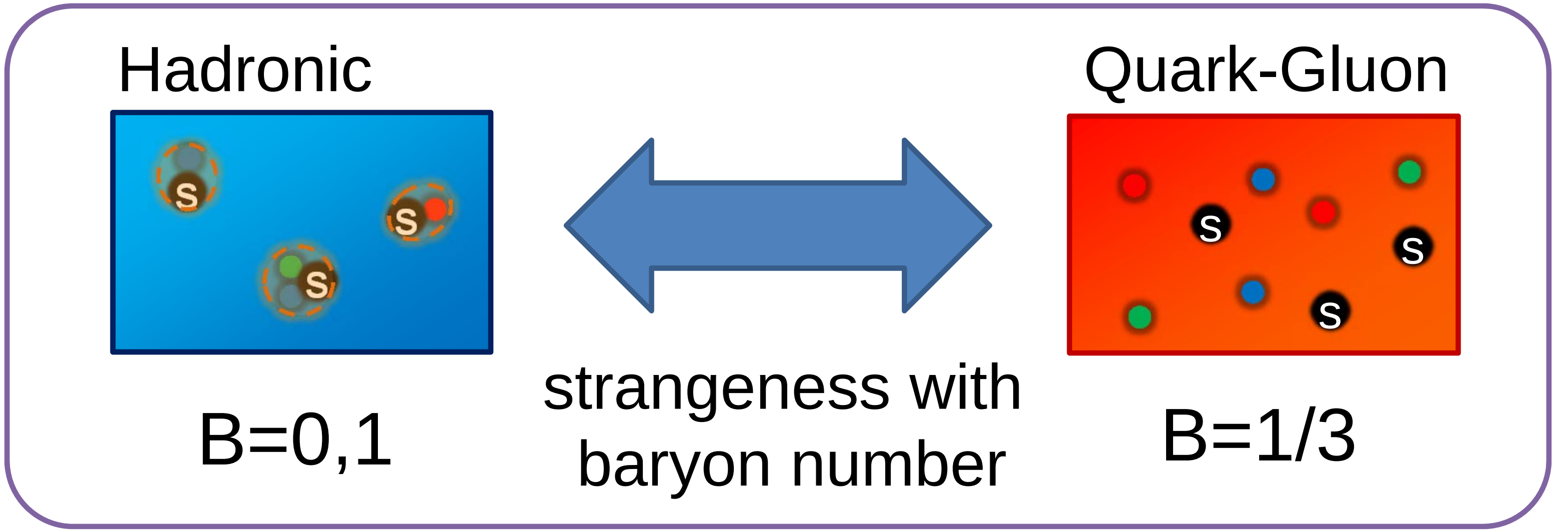


- Various third moments, $\langle \delta N_B^3 \rangle$, $\langle \delta N_Q^3 \rangle$, $\langle \delta E^3 \rangle$ become negative near the phase boundary.

➡ The behaviors can be checked by lattice and HIC!

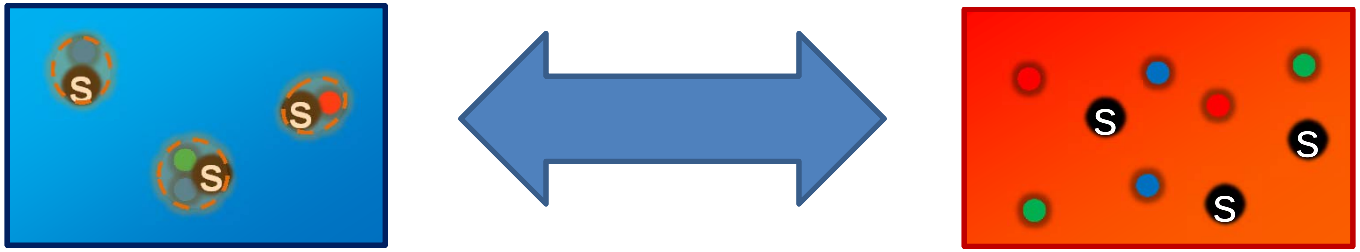
See also, Friman,et al.('11); Stephanov('11)

Exploring Medium Properties



Exploring Medium Properties

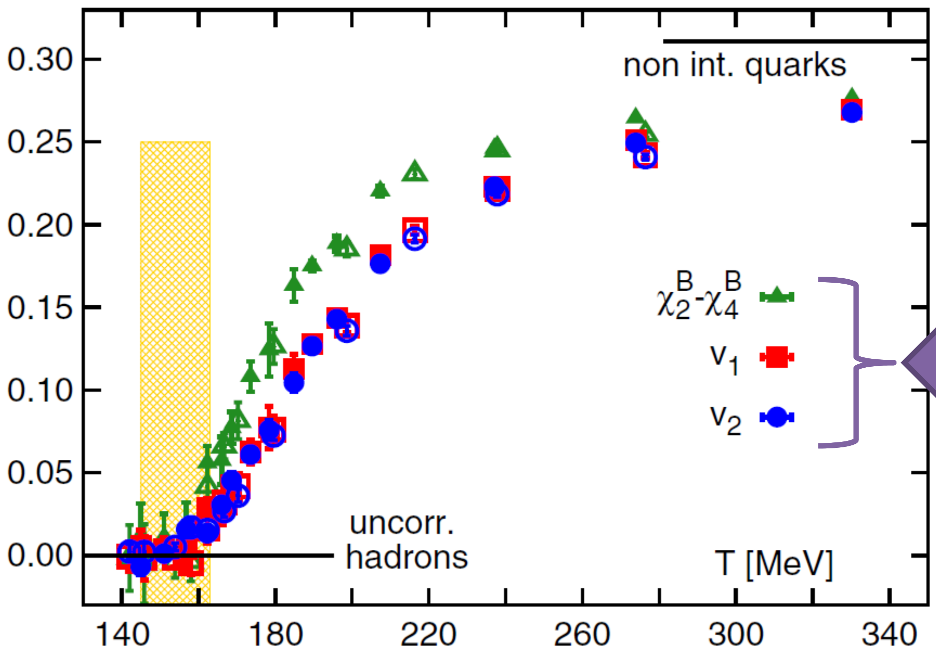
Hadronic | Quark-Gluon



B=0,1

strangeness with baryon number

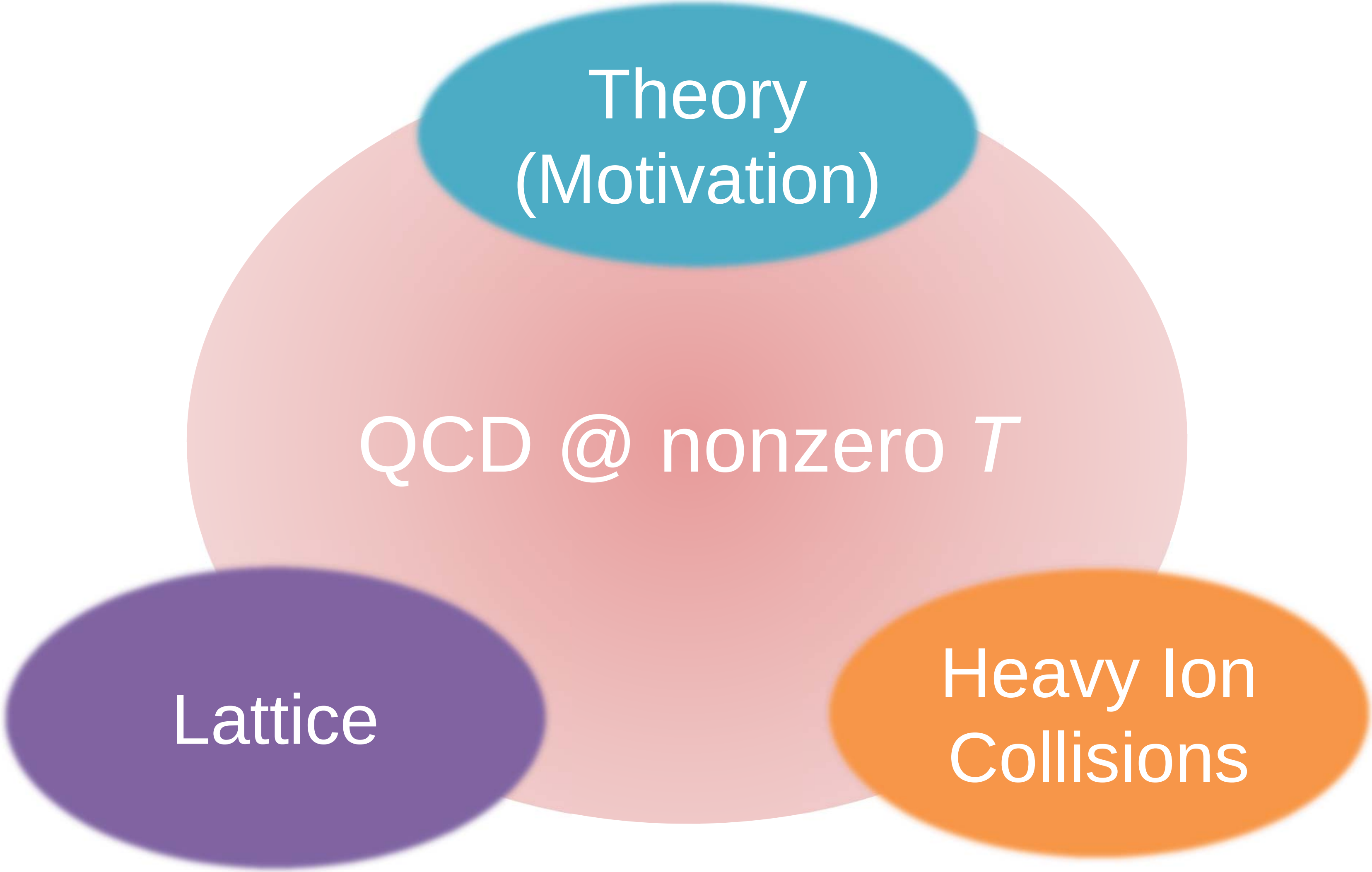
B=1/3



BNL-Bielefeld, PRL 2013

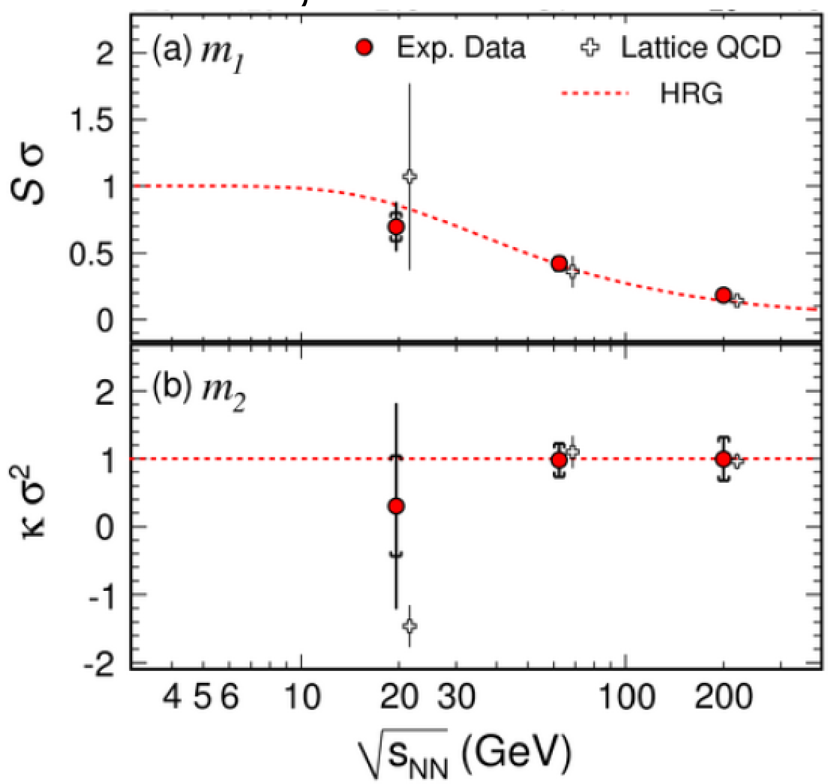
Combinations of cumulants which vanish in the HRG model

QCD @ nonzero *T*

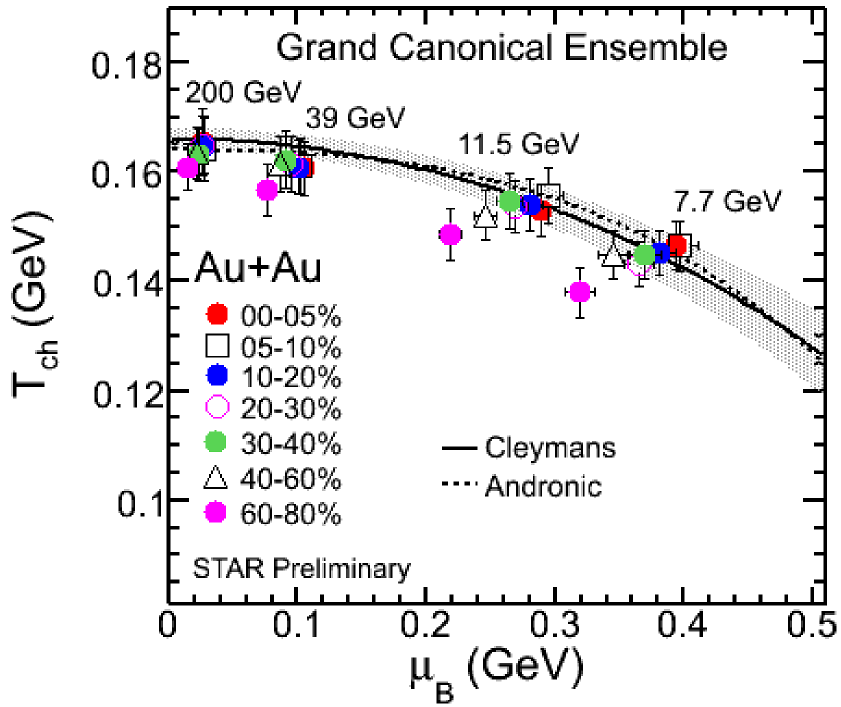


Proton # Fluctuations @ STAR-BES

STAR, PRL2010



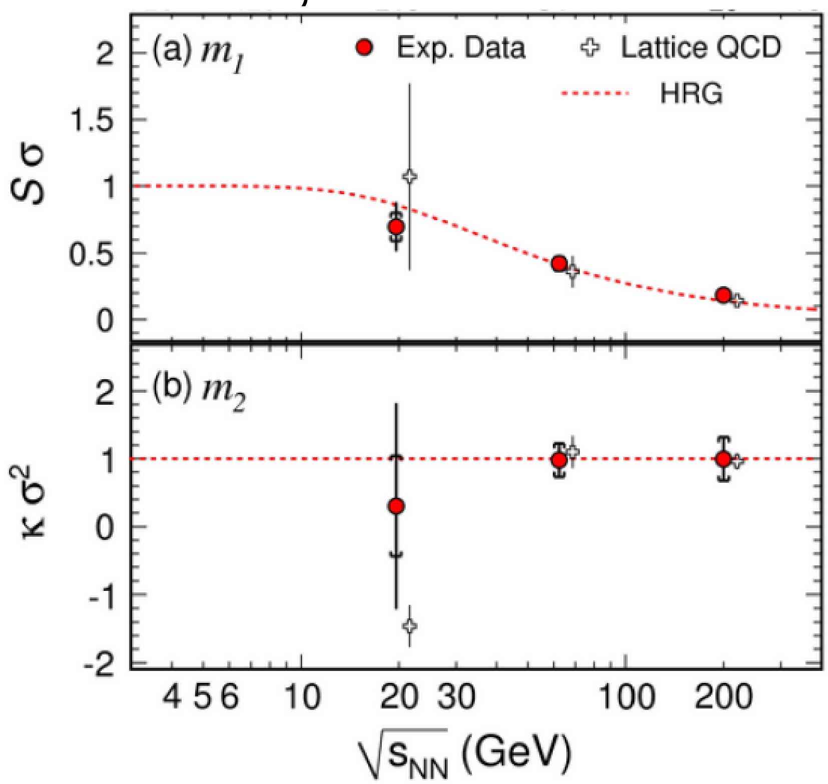
STAR 2012



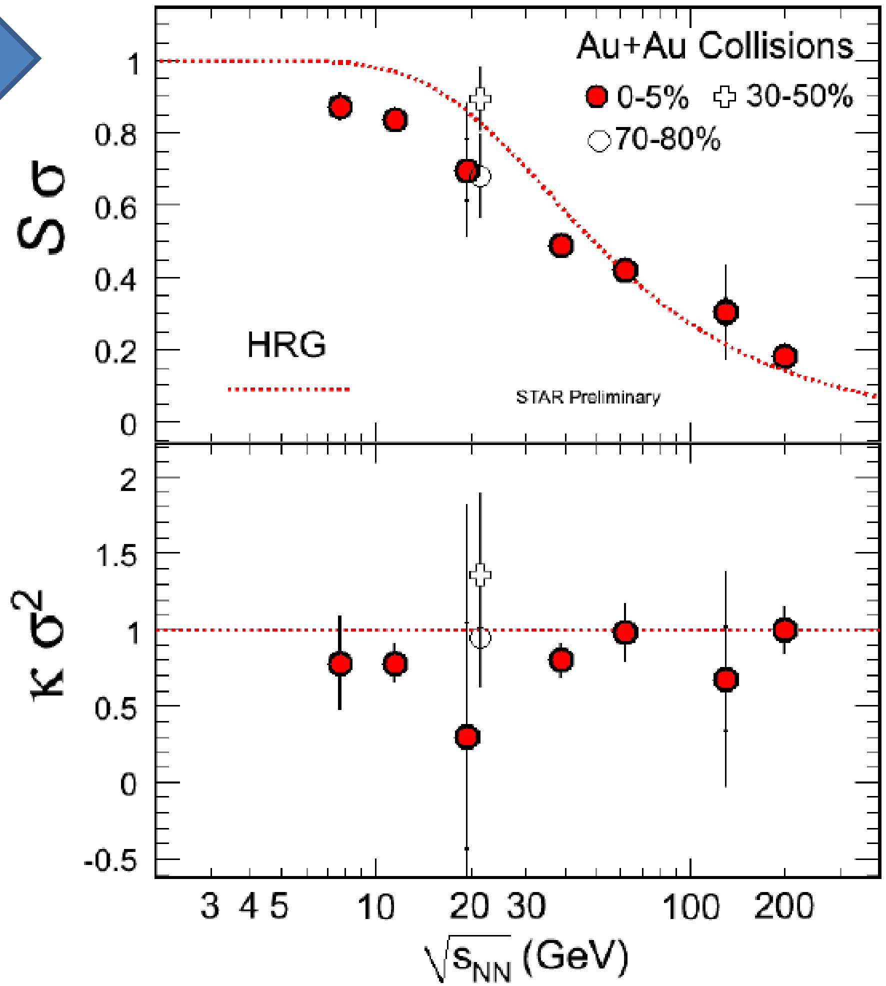
$$S\sigma = \frac{\langle (\delta N_p^{(\mathrm{net})})^3 \rangle}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}, \quad \kappa\sigma^2 = \frac{\langle (\delta N_p^{(\mathrm{net})})^4 \rangle_c}{\langle (\delta N_p^{(\mathrm{net})})^2 \rangle}$$

Proton # Fluctuations @ STAR-BES

STAR, PRL2010



STAR, 2011



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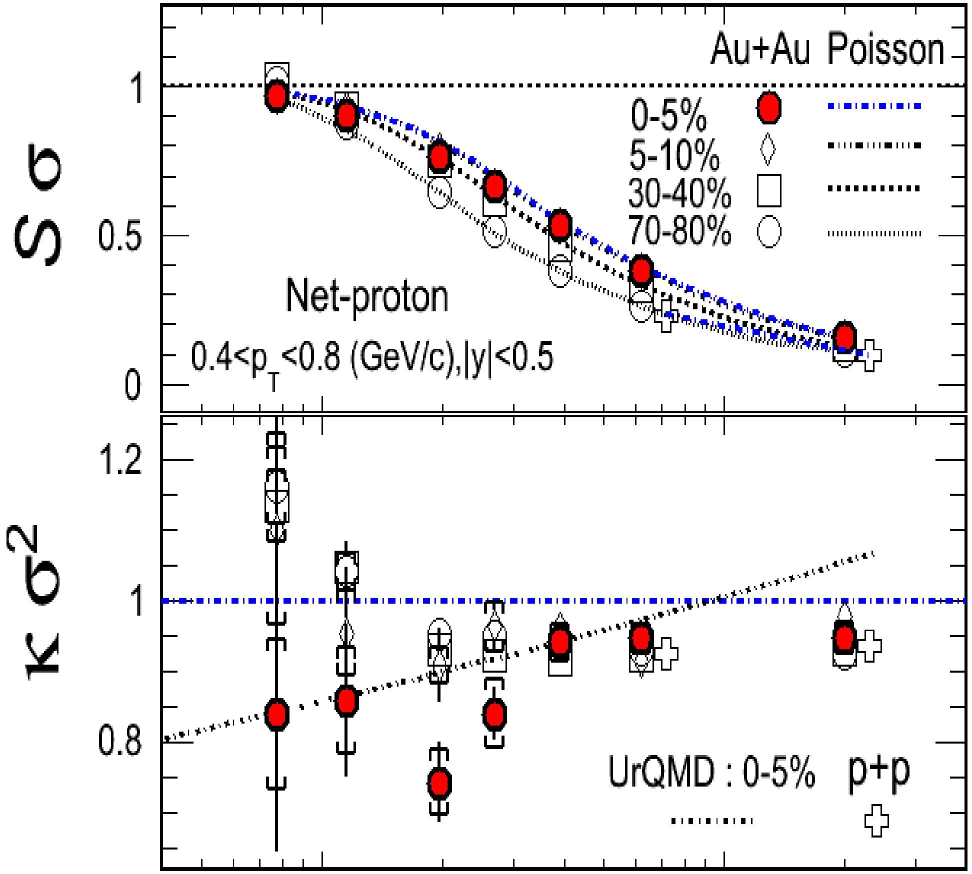
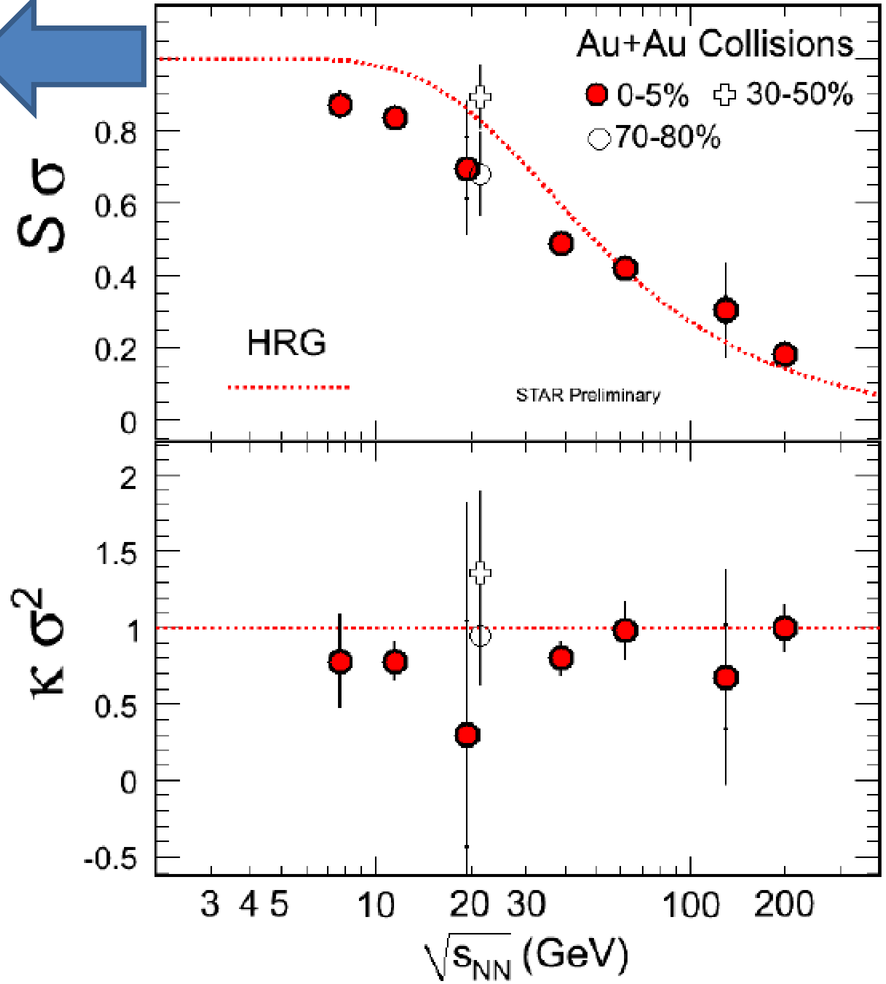
high μ ← → low μ

Proton # Fluctuations @ STAR-BES

STAR, 2012 (Quark Matter)



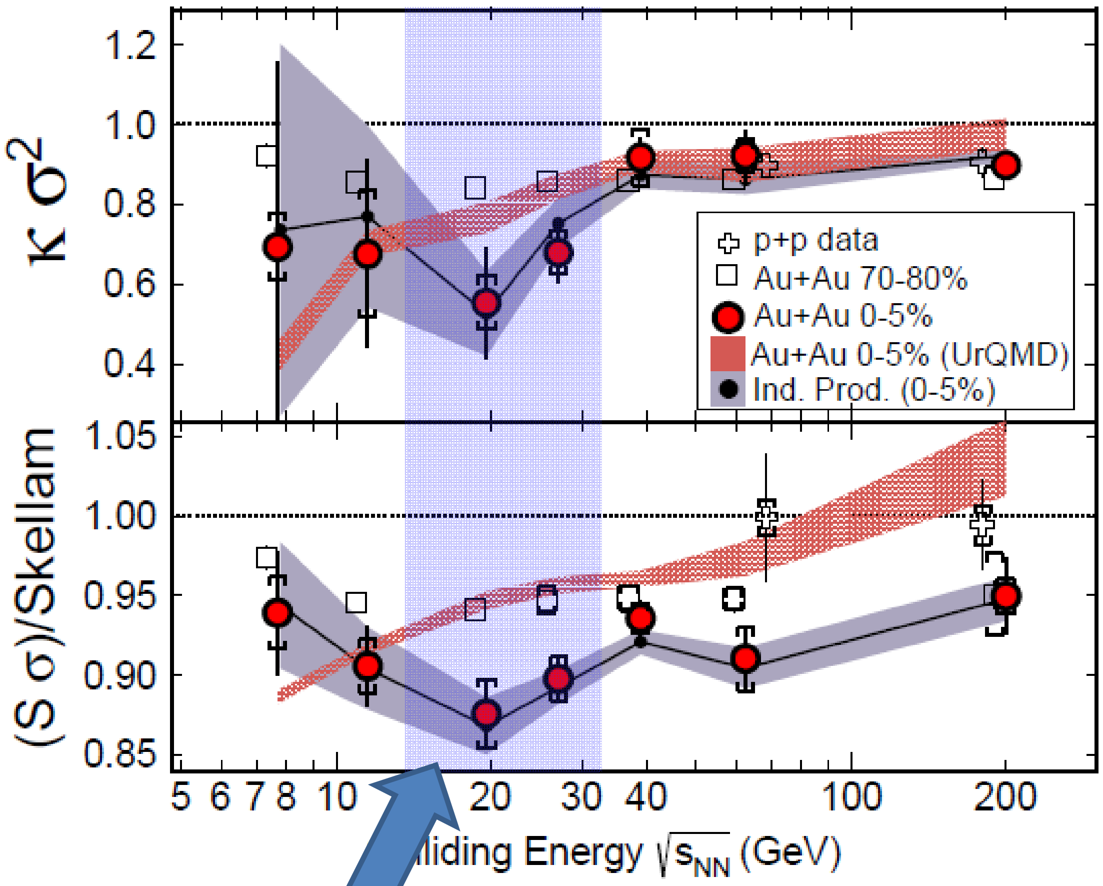
STAR, 2011



high μ

low μ

Proton # Cumulants @ STAR-BES



STAR,1309.5681

$$\frac{C_4}{C_2}$$

$$\frac{C_3}{C_1} = \frac{C_3/C_2}{\text{Poissonian}}$$

Something interesting??

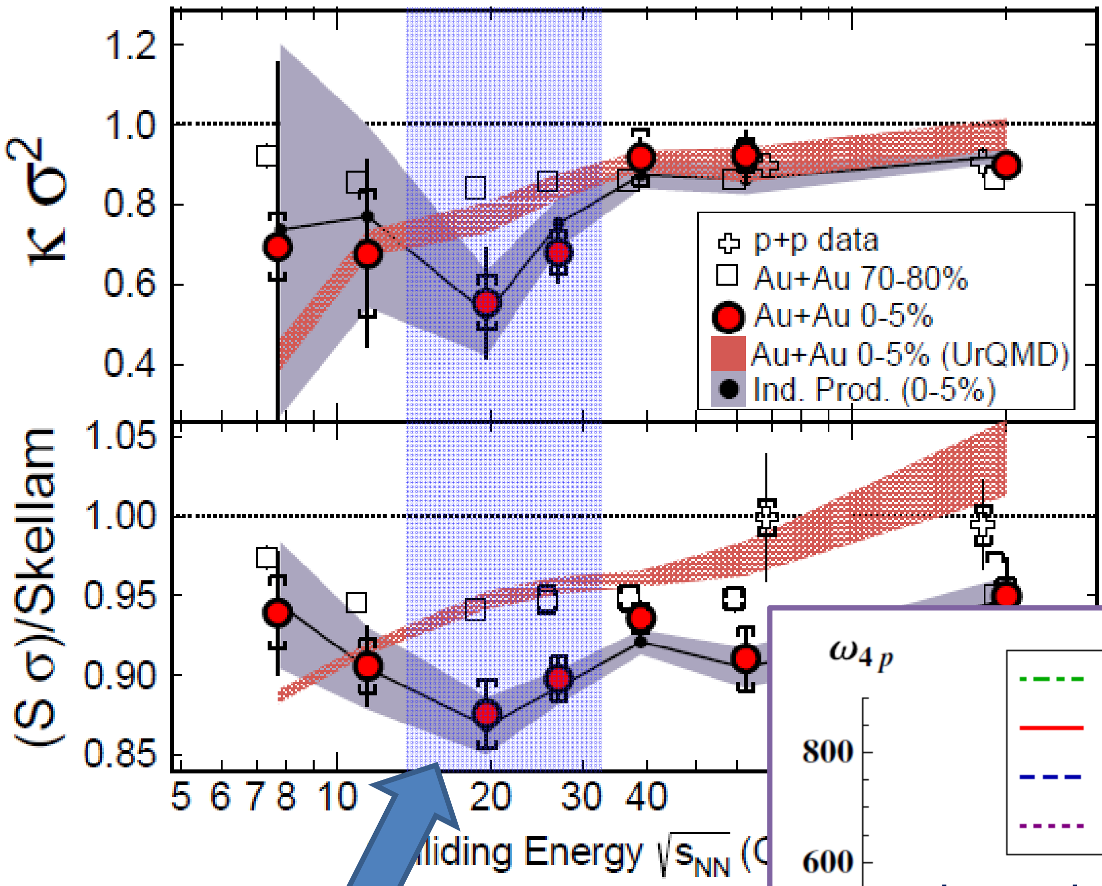
CAUTION!

proton number $\neq$ baryon number

MK, Asakawa, 2011;2012

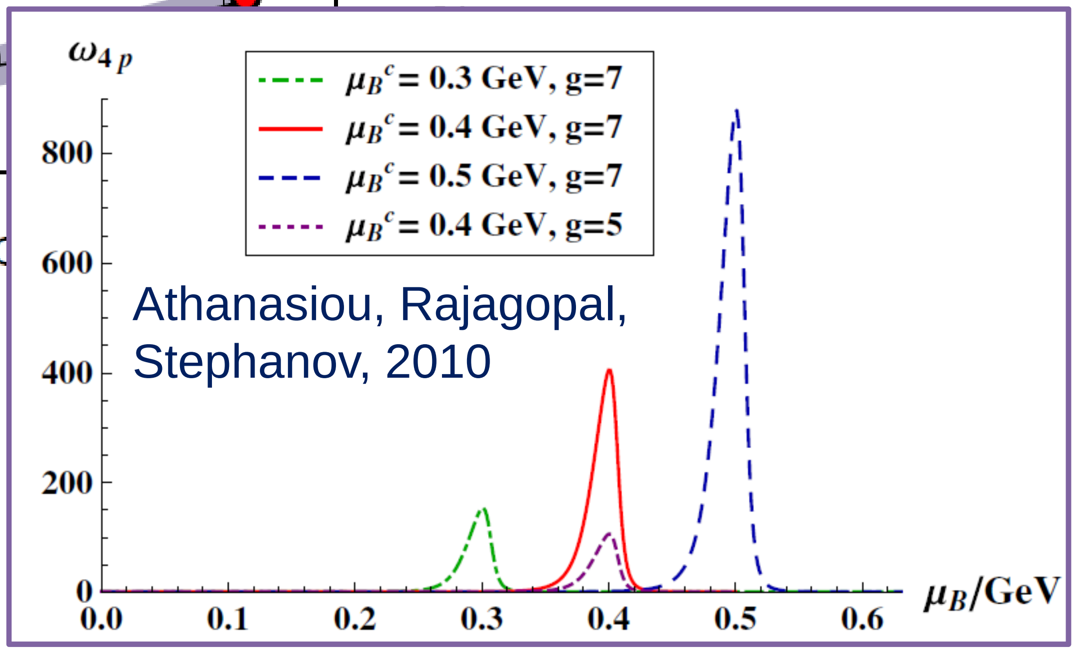
Proton # Cumulants @ STAR-BES

STAR,1309.5681



$$\frac{C_4}{C_2}$$

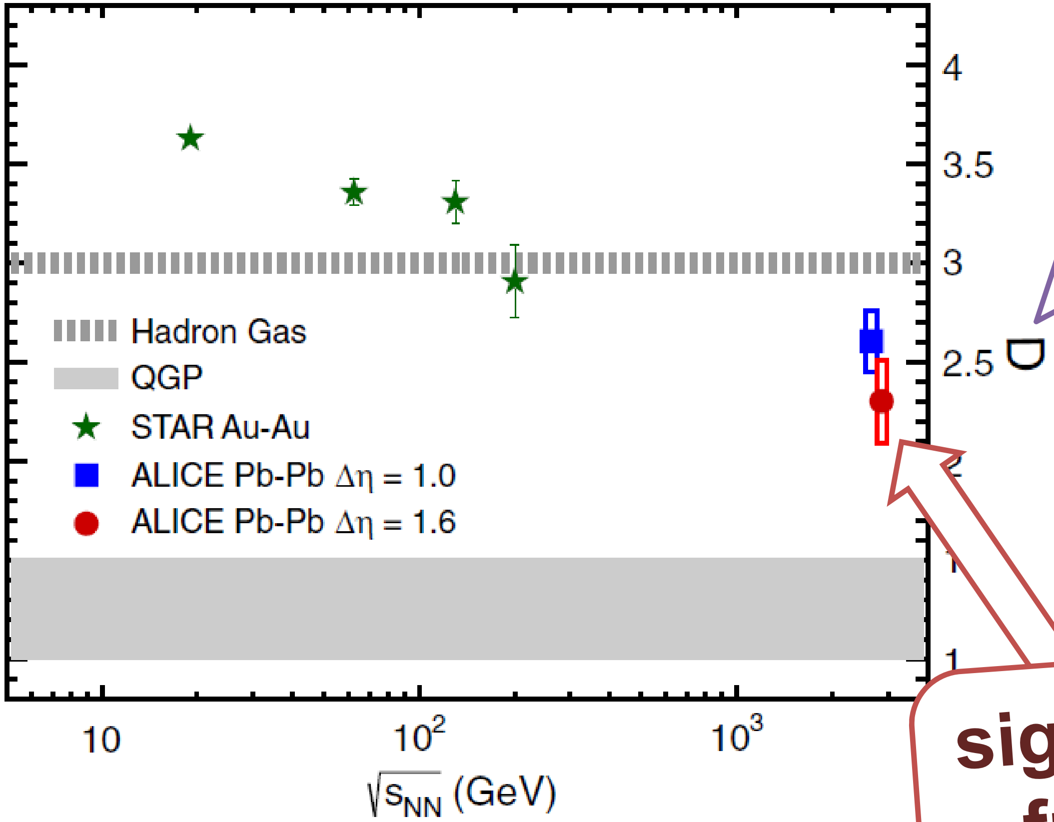
$$\frac{C_3}{} = \frac{C_3/C_2}{}$$



Athanasiou, Rajagopal, Stephanov, 2010

Something interesting??

Electric Charge Fluctuation @ LHC



ALICE, PRL110,152301(2013)

D-measure

$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

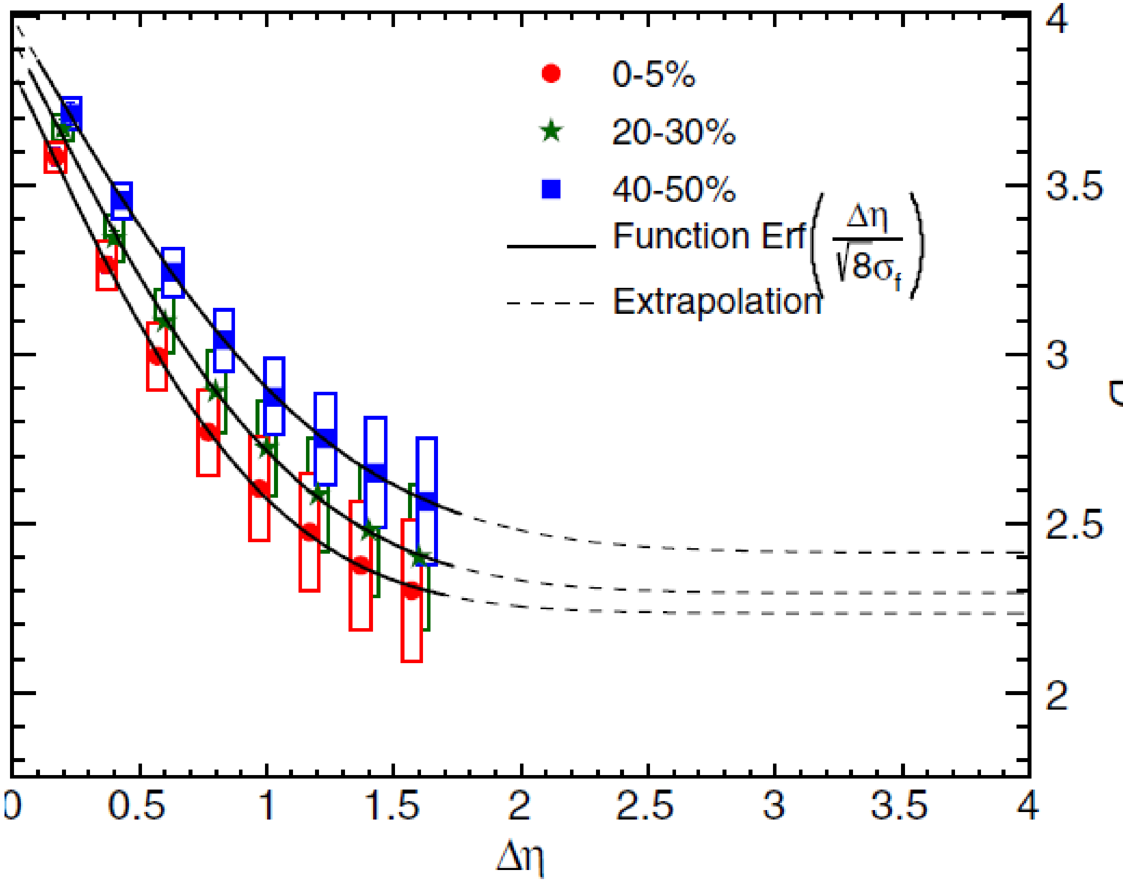
- D ~ 3-4 Hadronic
- D ~ 1-1.5 Quark

significant suppression from hadronic value at LHC energy!

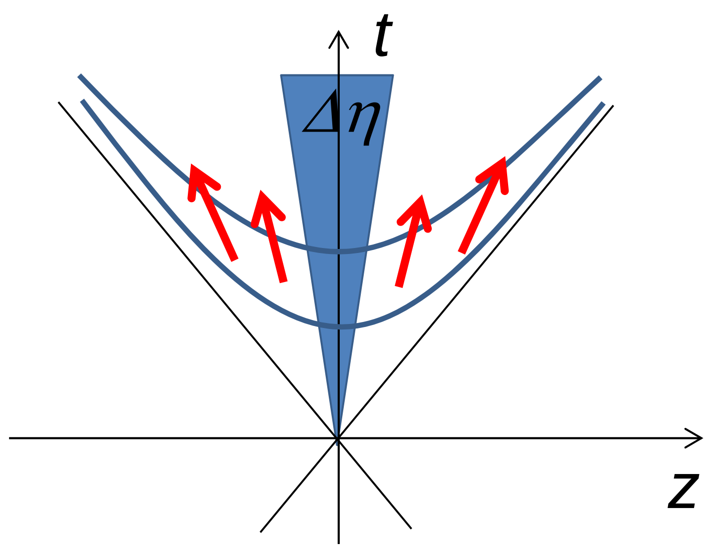
$\langle \delta N_Q^2 \rangle$ is not equilibrated at freeze-out at LHC energy!

Δη Dependence @ ALICE

ALICE
PRL 2013

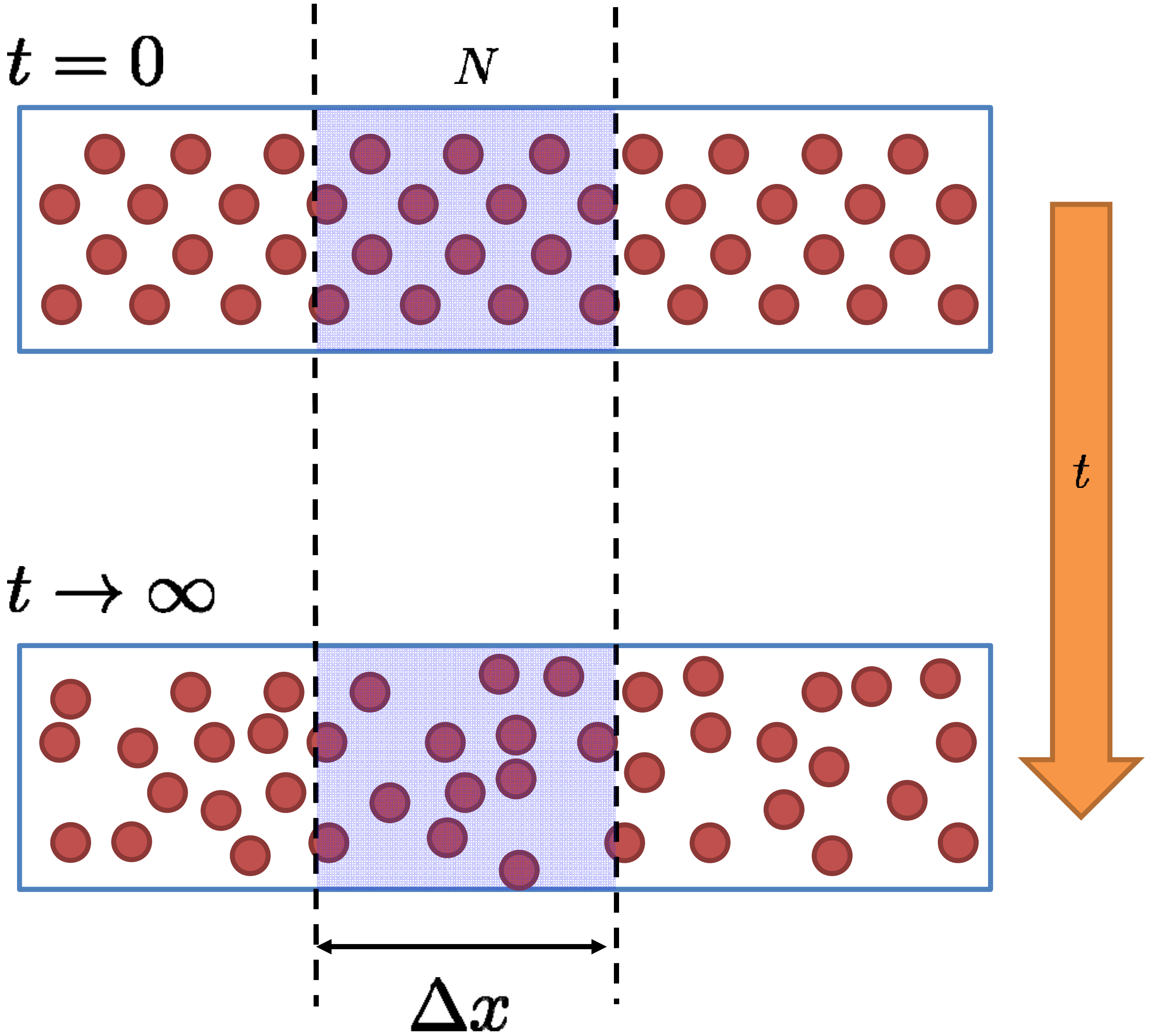


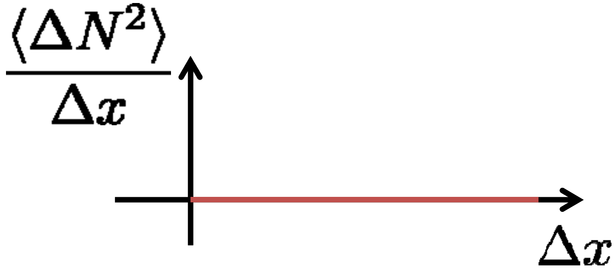
rapidity window



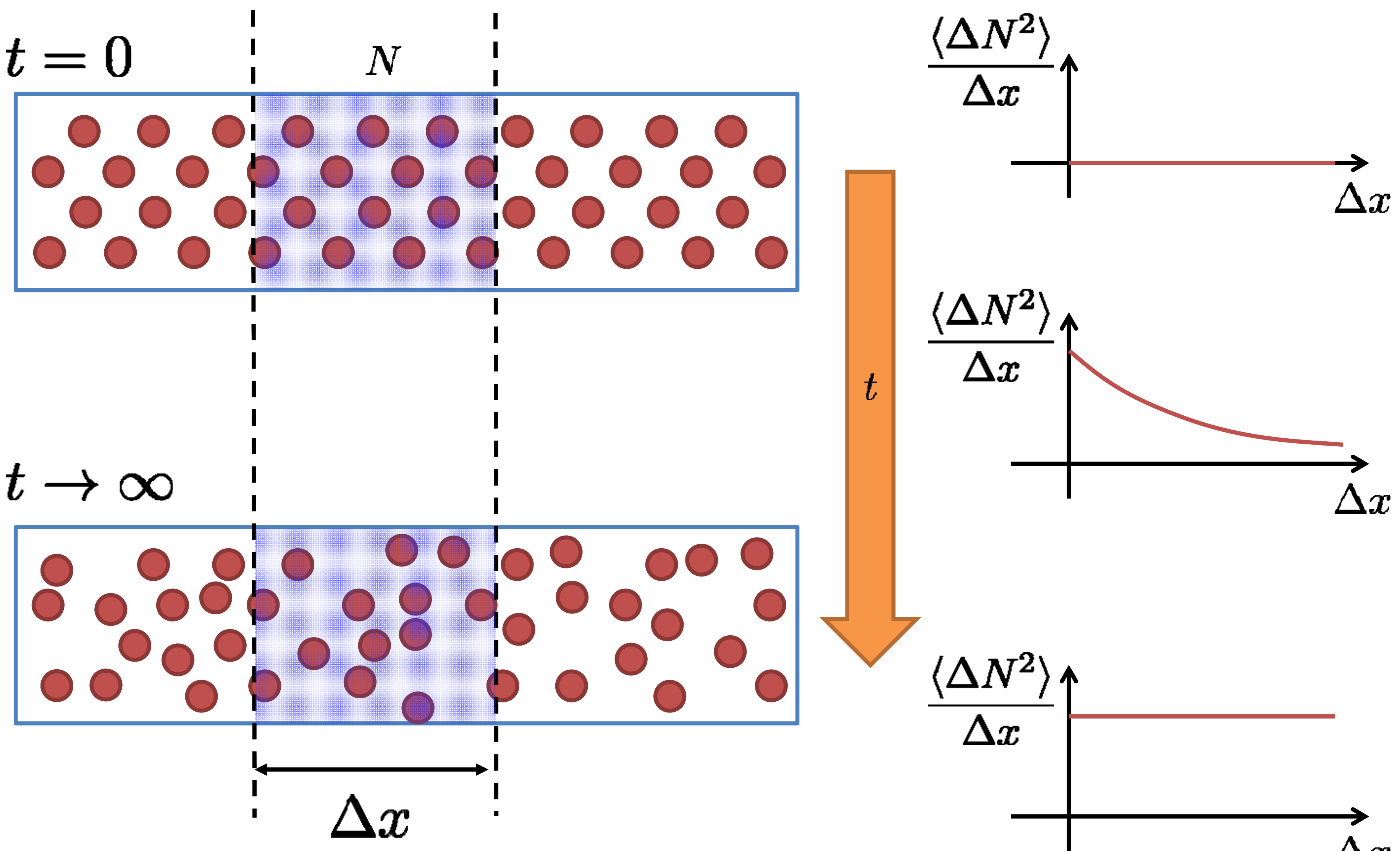
$$D = 4\frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

Dissipation of a Conserved Charge

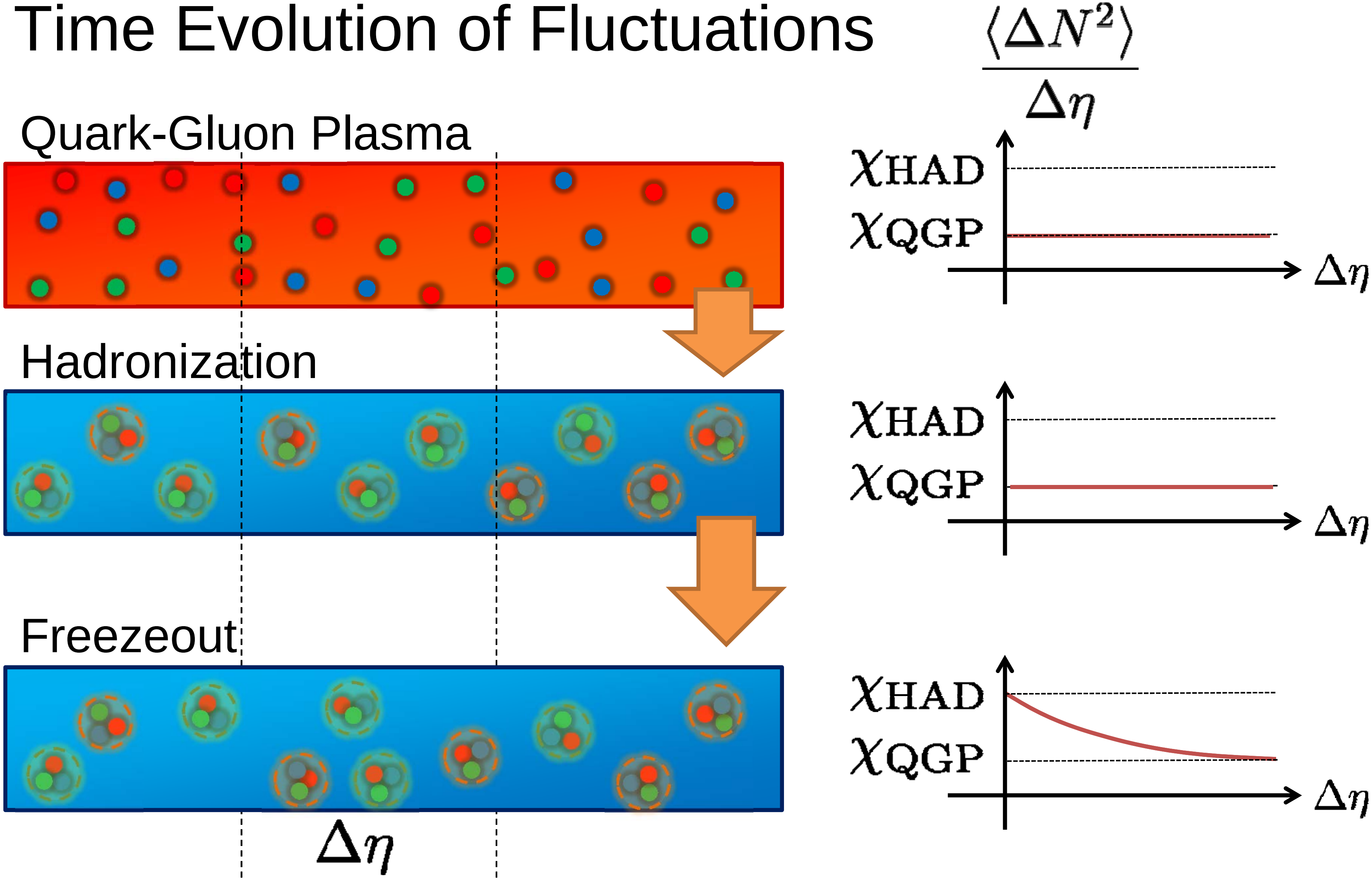




Dissipation of a Conserved Charge



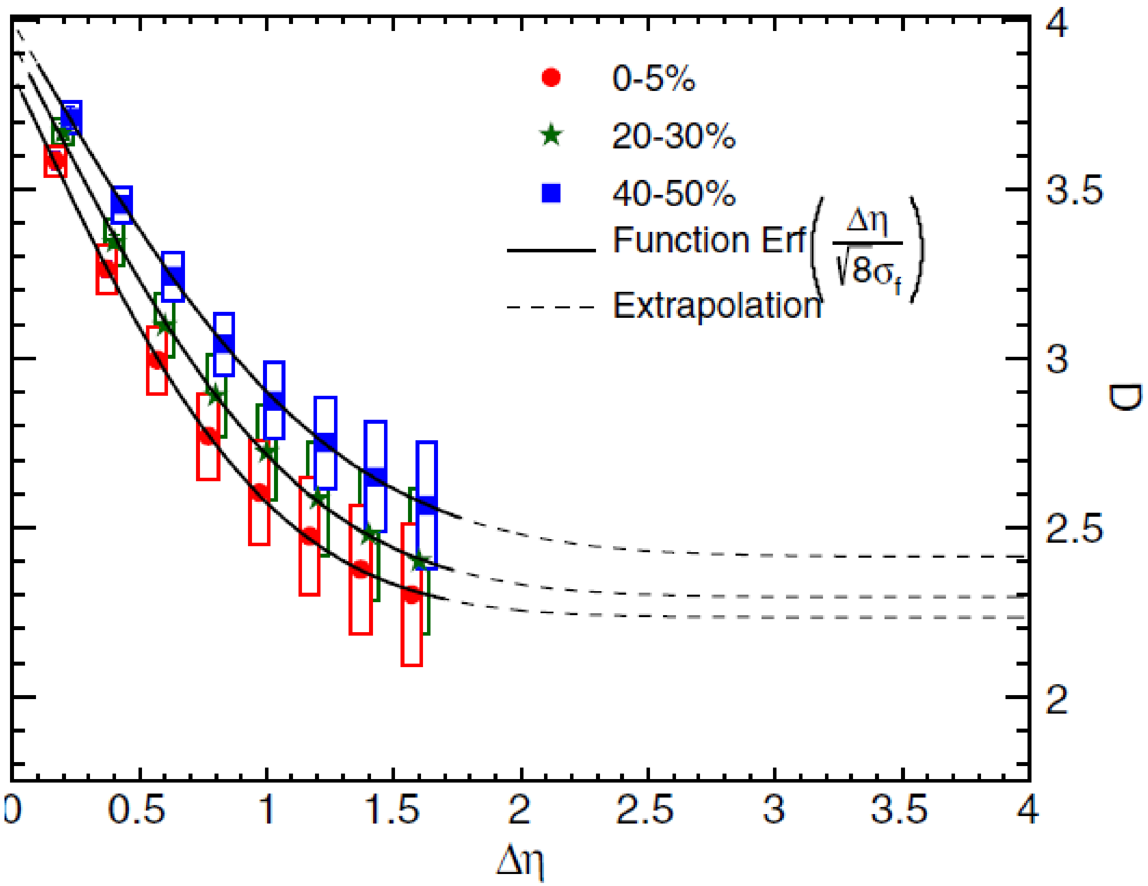
Time Evolution of Fluctuations

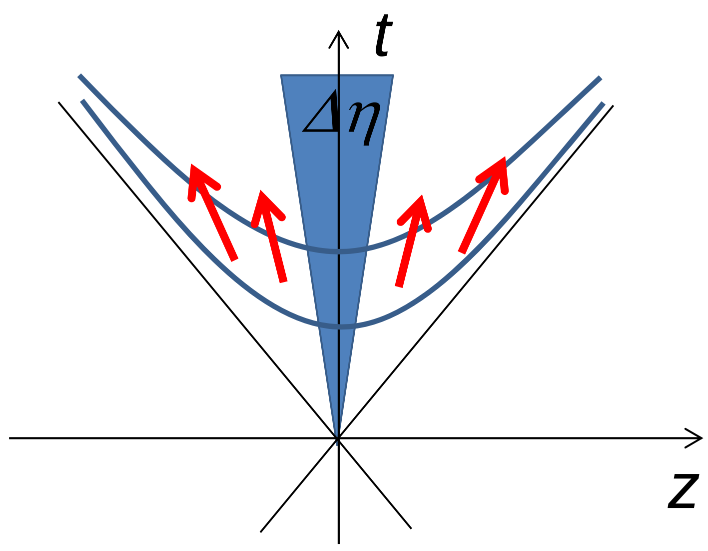


Variation of a conserved charge is achieved only through diffusion. ⇨ The larger $\Delta\eta$, the slower diffusion

Δη Dependence @ ALICE

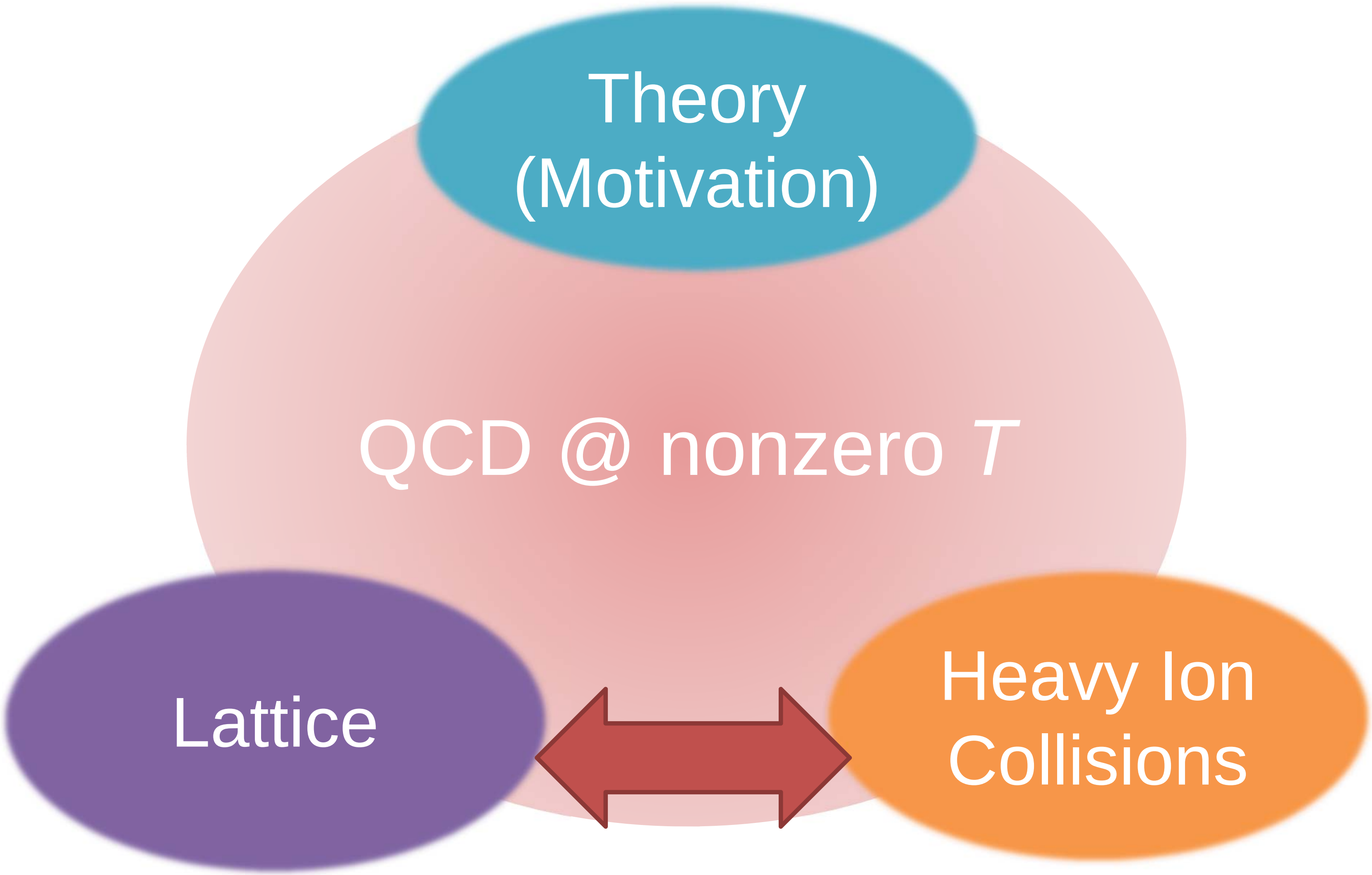
ALICE
PRL 2013



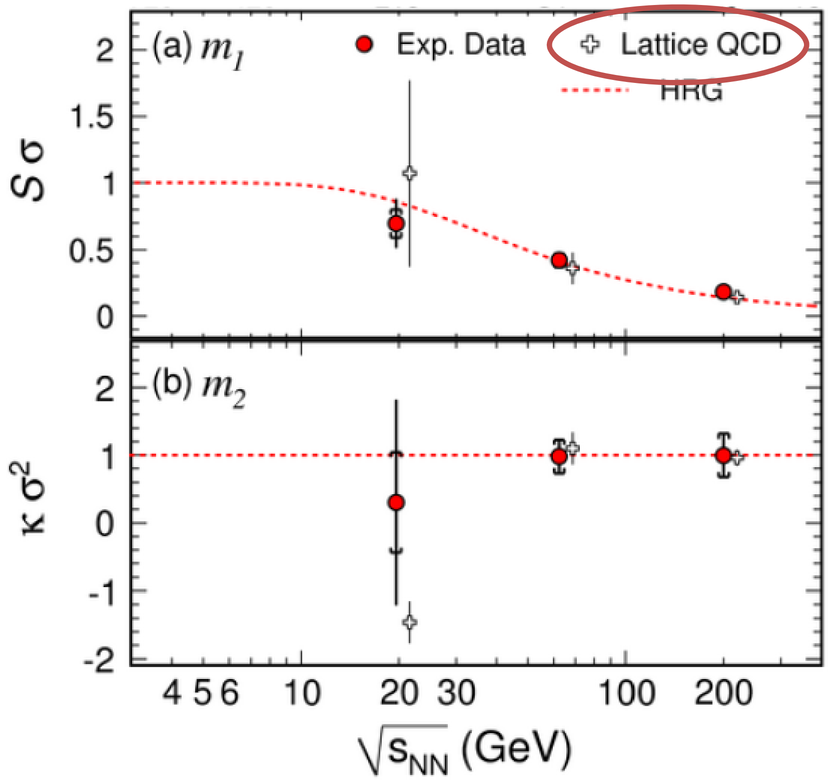


Δη dependences of conserved charge fluctuations encode history of dynamical evolution

QCD @ nonzero *T*



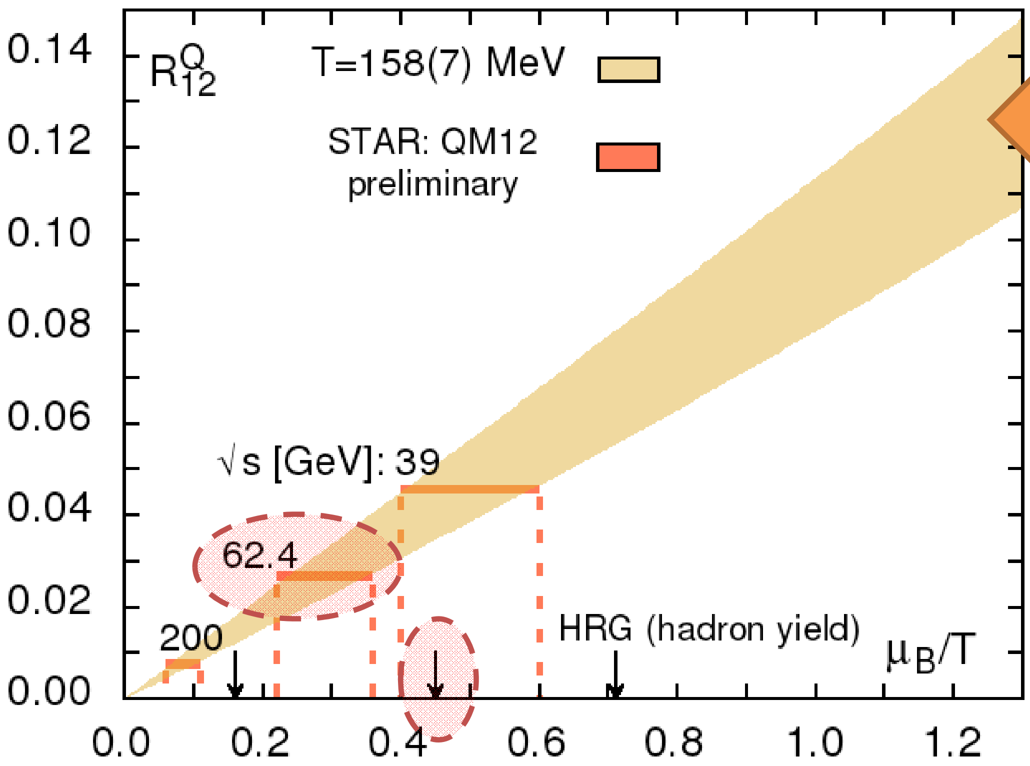
Comparison b/w Lattice & HIC



Gupta, Xu, et al., Science, 2009

- Taylor expansion method
- Chemical freezeout T,μ
- Pade approx.

Cumulants : HIC@RHIC vs Lattice



parameter window constrained by lattice

BNL-Bielefeld, LATTICE2013

fluctuations "exp + lattice" ⟷ μ/T ⟷ particle abundance (chem. freezeout T)

discrepancy

Many Things to Do

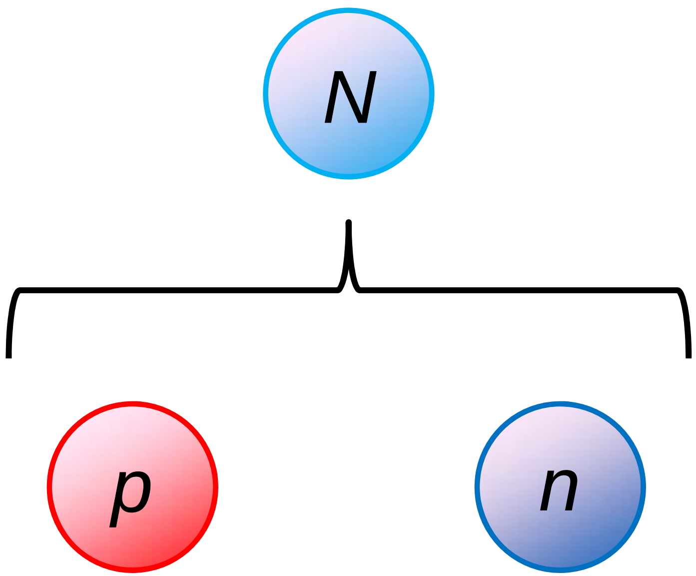
- Proton vs baryon number cumunants
- Are fluctuations generated with fixed *T*?
- Experimental environments
  - Acceptance, efficiency
  - Particle missid
  - Global charge conservation

Baryon vs Proton Number Fluctuations

MK, Asakawa, PRC85,021901C(2012); PRC86, 024904(2012)

- $$\frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c}$$
- $\langle \delta N_B^n \rangle_c$ are experimentally observable

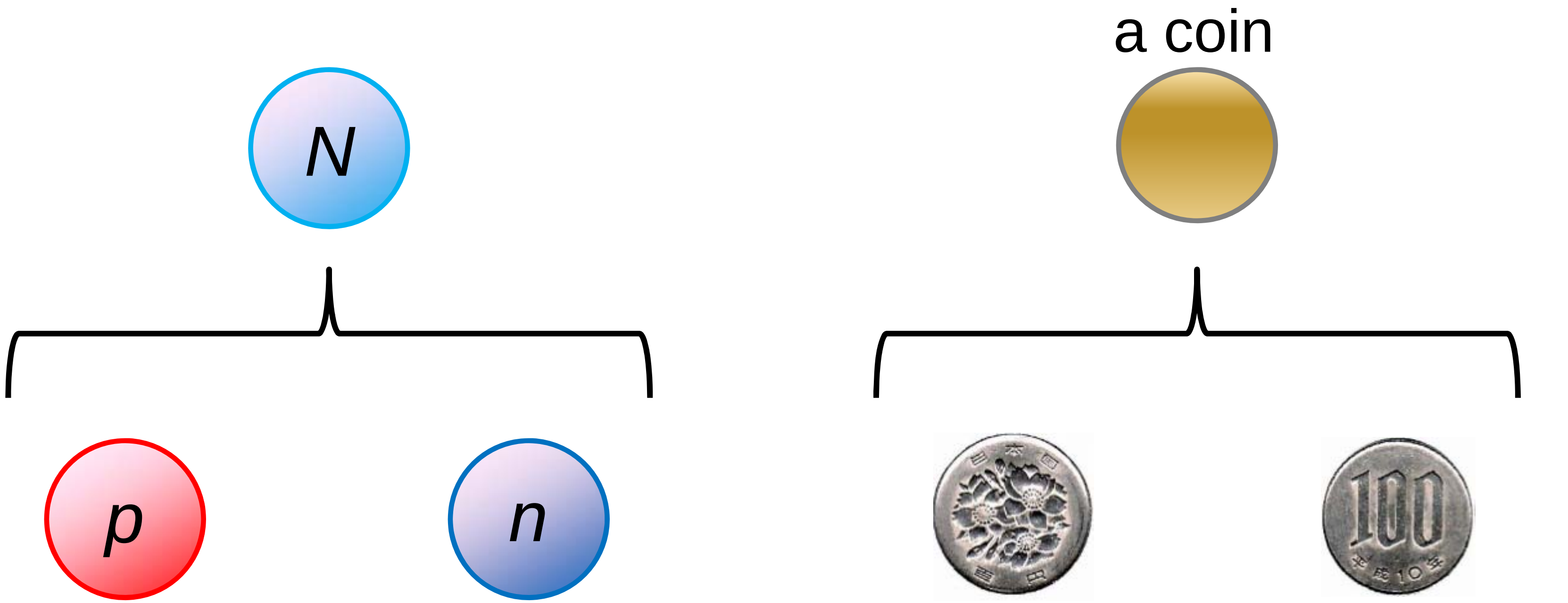
Nucleon Isospin as Two Sides of a Coin



Nucleons have
two isospin states.

MK, Asakawa,2012

Nucleon Isospin as Two Sides of a Coin

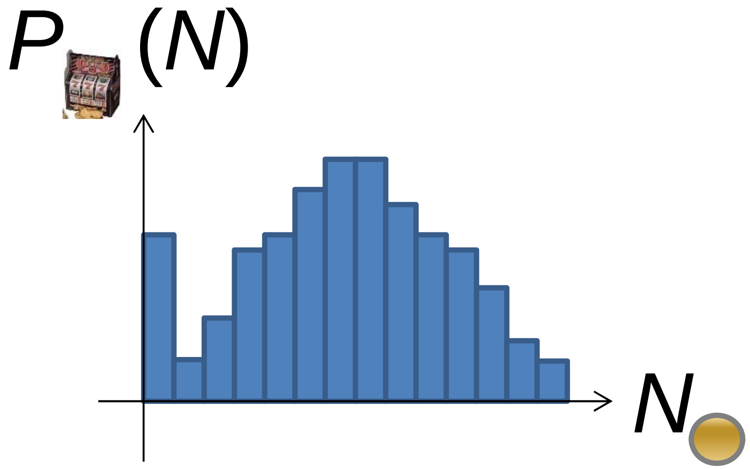


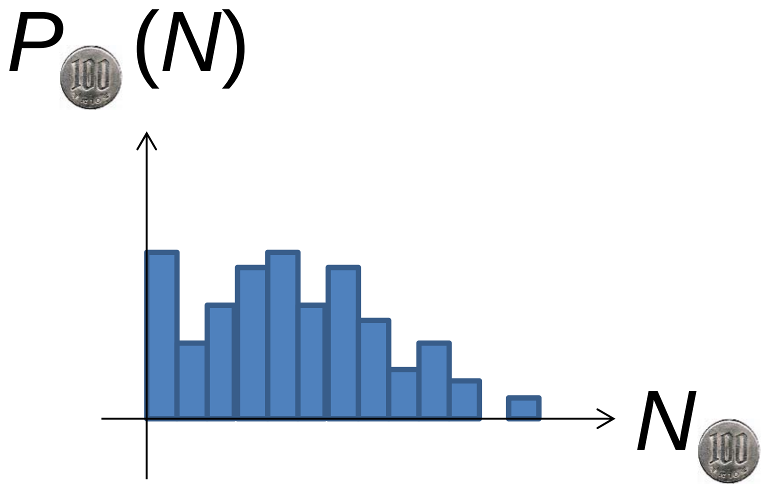
Nucleons have
two isospin states.

Coins have two sides.

MK, Asakawa,2012

Slot Machine Analogy

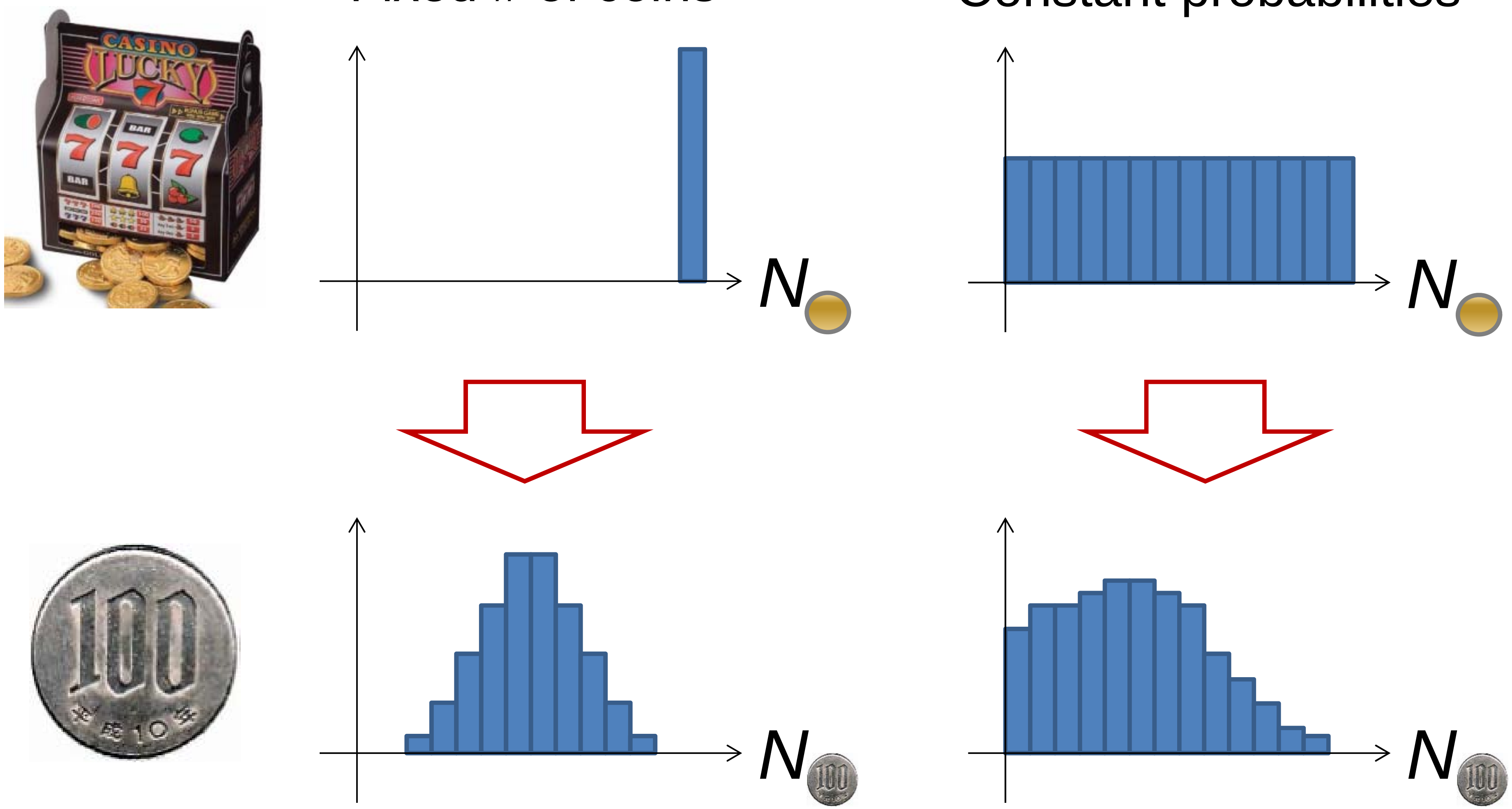




Extreme Examples

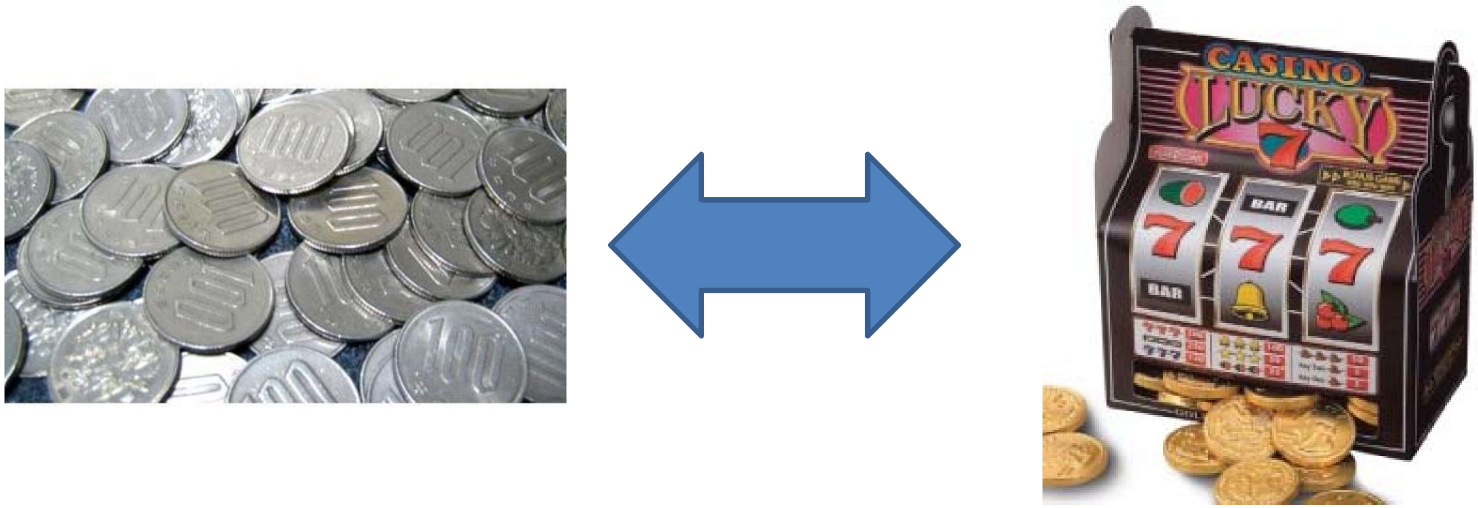
Fixed # of coins

Constant probabilities



Reconstructing Total Coin Number

$$P_{100}(N_{100}) = \sum_{\bullet} P_{\text{slot}}(N_{\bullet}) B_{1/2}(N_{100}; N_{\bullet})$$



$$B_p(k; N) = p^k(1-p)^{N-k}\ {}_kC_N$$:binomial distr. func.

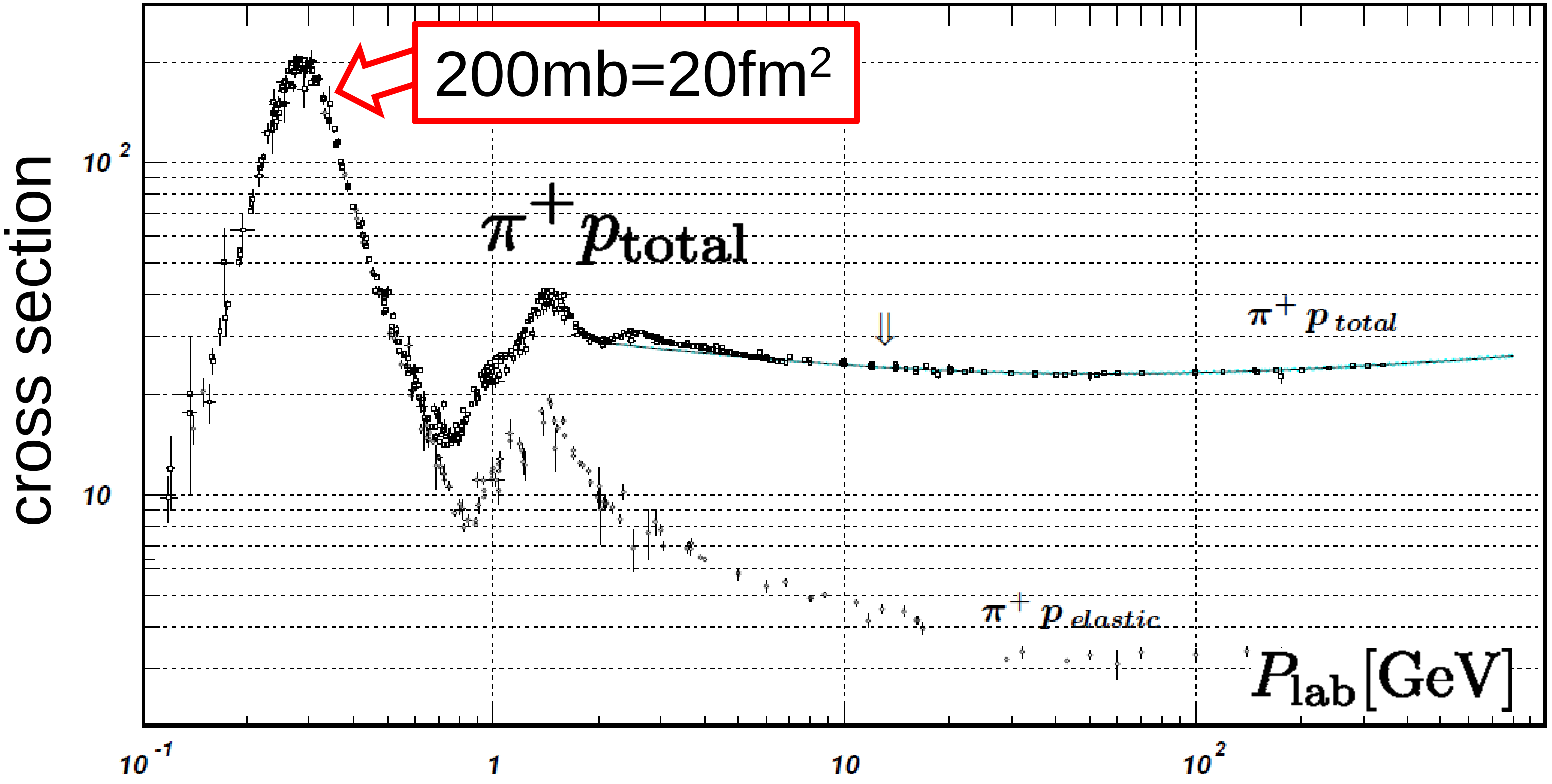
Nucleon Isospin in Hadronic Medium

- Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by Δ(1232):

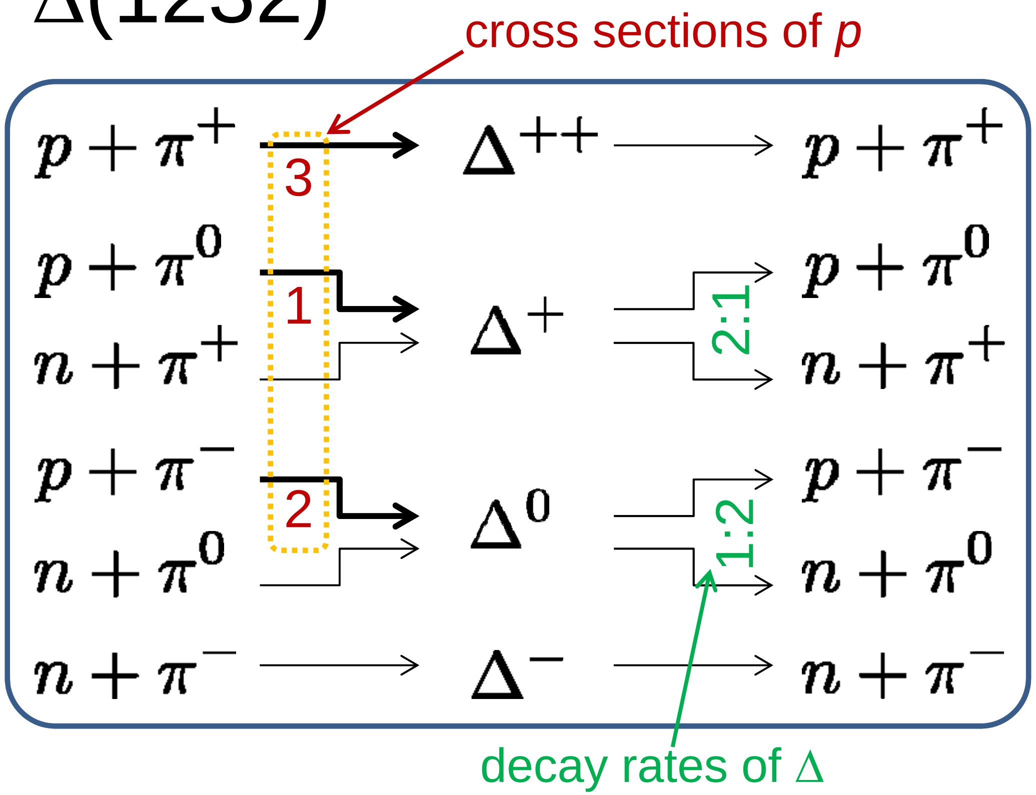
$$p, n \; \pi \rightarrow \Delta(1232) \rightarrow p, n \; \pi$$

$$I = 3/2$$

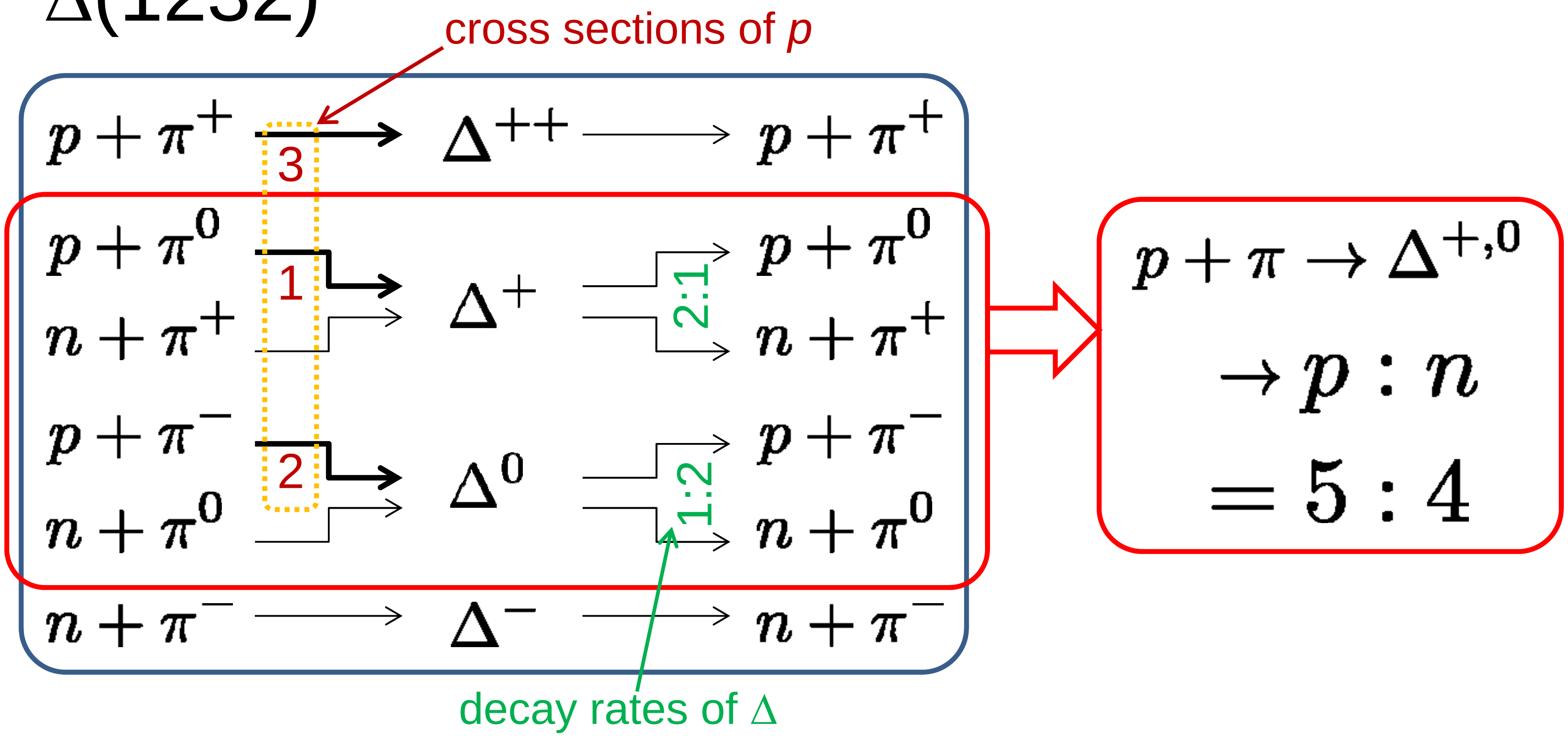
$$\Gamma \simeq 1.8 \; [\mathrm{fm}]$$



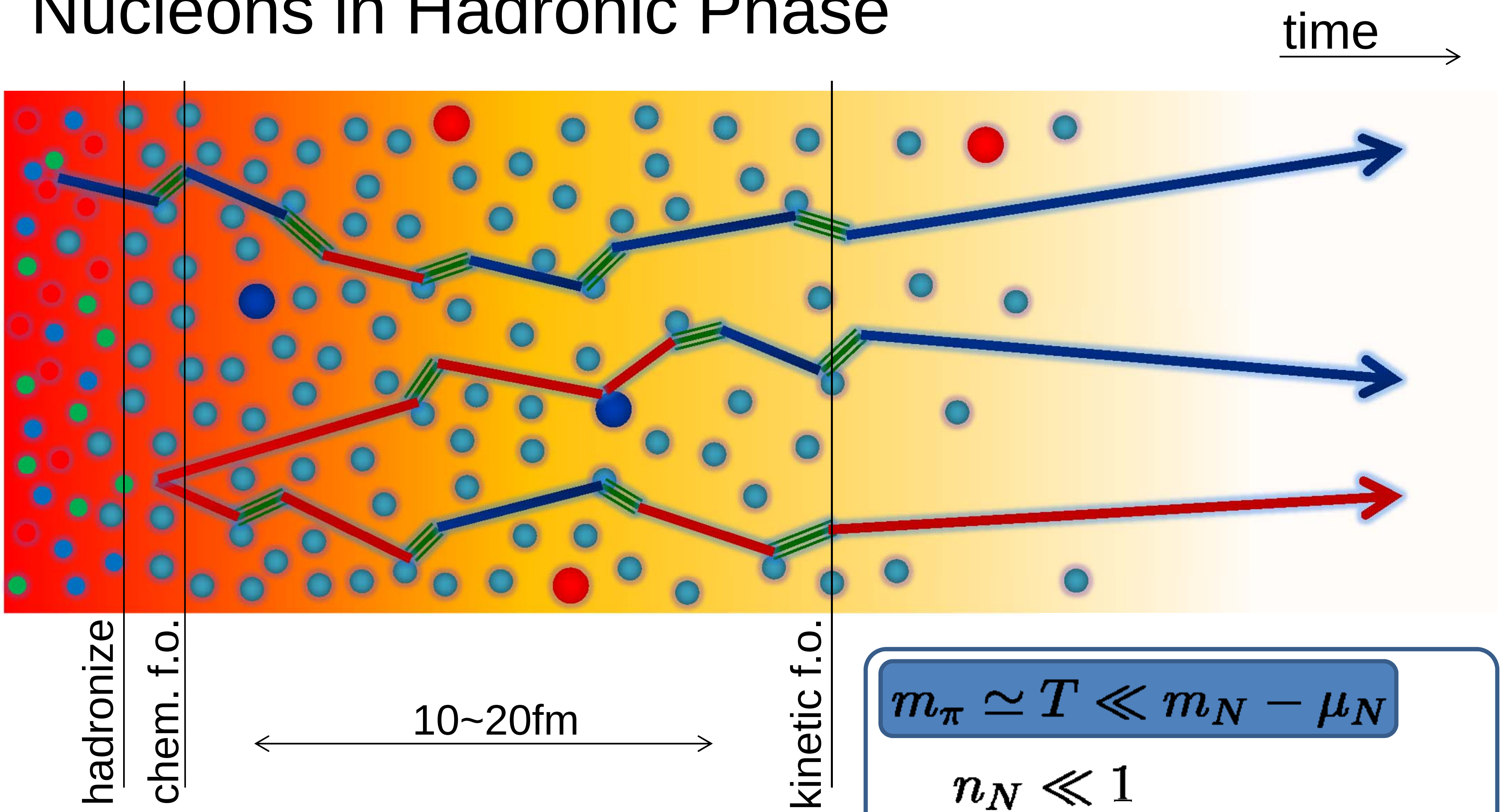
Δ(1232)

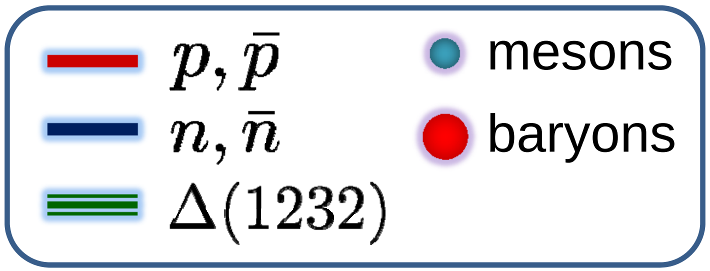

cross sections of *p*
$p+\pi^+$
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
3
1
2
Δ^{++}
Δ^+
Δ^0
Δ^-
2:1
1:2
$p+\pi^+$
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
decay rates of Δ

Δ(1232)


cross sections of p
$p+\pi^+$
3
Δ^{++}
$p+\pi^+$
$p+\pi^0$
1
$n+\pi^+$
Δ^+
2:1
$p+\pi^0$
$n+\pi^+$
$p+\pi^-$
2
$n+\pi^0$
Δ^0
1:2
$p+\pi^-$
$n+\pi^0$
$n+\pi^-$
Δ^-
$n+\pi^-$
decay rates of Δ
$p+\pi \rightarrow \Delta^{+,0}$
$\rightarrow p : n$
$= 5 : 4$

Nucleons in Hadronic Phase





$m_\pi \simeq T \ll m_N - \mu_N$

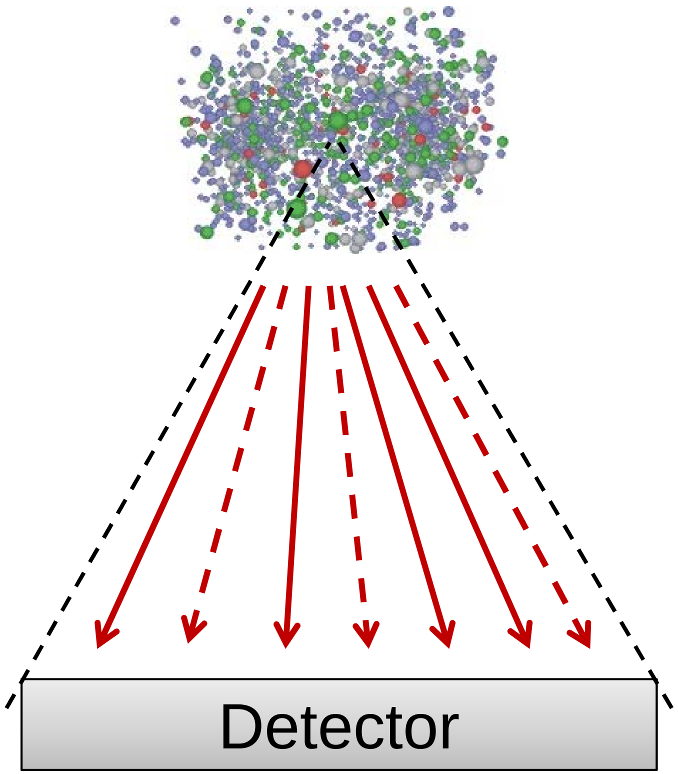
$n_N \ll 1$

- rare NN collisions
- no quantum corr.

$n_N \ll n_\pi$

- many pions

Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$



- $\begin{cases} N_N \text{ nucleons} \\ N_{\bar{N}} \text{ anti-nucleons} \end{cases}$

  ⇒ $F(N_N, N_{\bar{N}})$

- $N_N \begin{cases} N_p \text{ protons} \\ N_n \text{ neutrons} \end{cases}$

  ⇒ $B(N_p; N_N)$ — binomial distribution func.

$$\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$$
$$= F(N_N, N_{\bar{N}}) B(N_p; N_N) B(N_{\bar{p}}; N_{\bar{N}})$$

➢ for any phase space in the final state.

Difference btw Baryon and Proton Numbers

(1) $N_B^{(\mathrm{net})} = N_B - N_{\bar{B}}$ deviates from the equilibrium value.

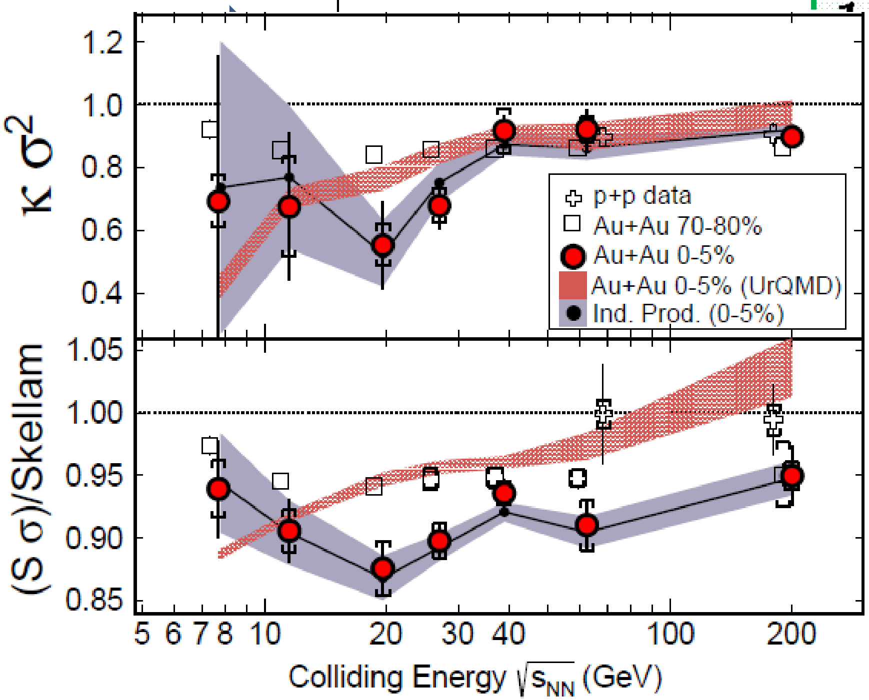
(2) Boltzmann (Poisson) distribution for $N_B, N_{\bar{B}}$.

$$2\langle(\delta N_p^{(\mathrm{net})})^2\rangle = \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle + \frac{1}{2}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^2\rangle_{\mathrm{free}}$$

$$\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle + \frac{3}{4}\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^3\rangle_{\mathrm{free}}$$

$$\langle(\delta N_{\mathrm{B}}^{(\mathrm{net})})^4\rangle_c + \cdots$$

enuine info. | noise



For free gas

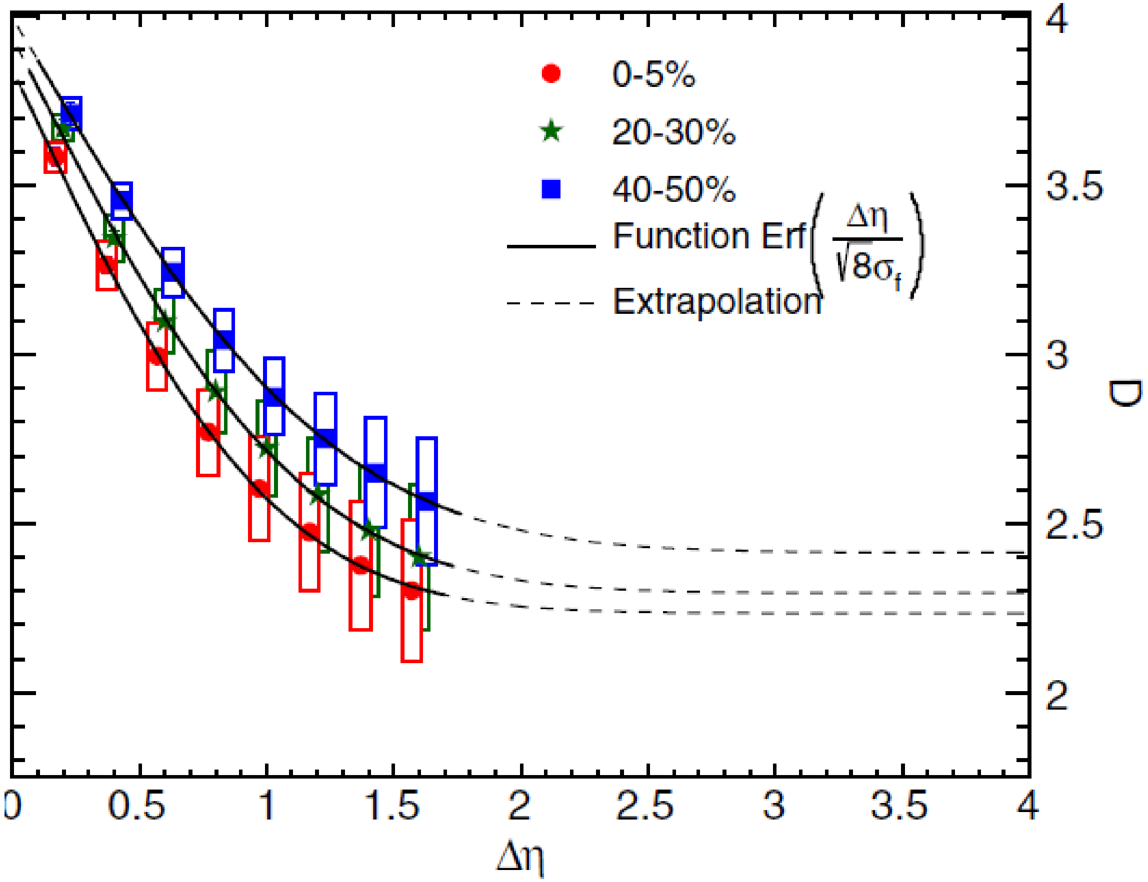
$$2\langle(\delta N_p^{(\mathrm{net})})^n\rangle_c = \langle(\delta N_{\mathrm{N}}^{(\mathrm{net})})^n\rangle_c$$

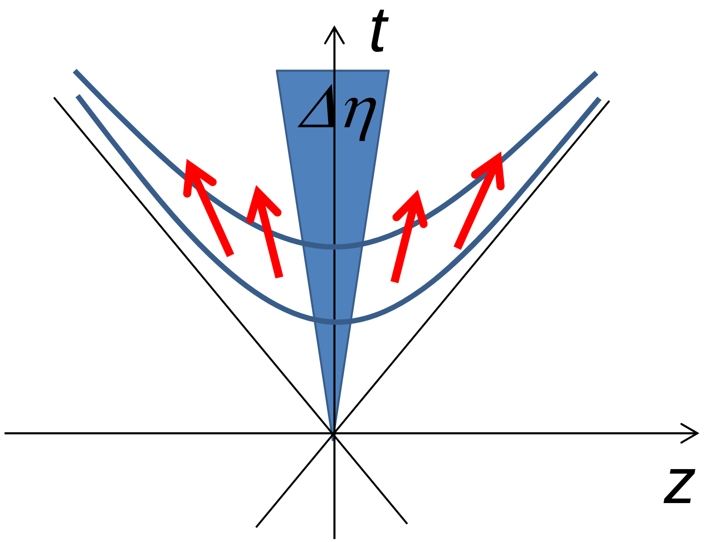
Time Evolution of
Higher Order Cumulants

MK, Asakawa, Ono, PL**B728**, 386, 2014

Δη Dependence @ ALICE

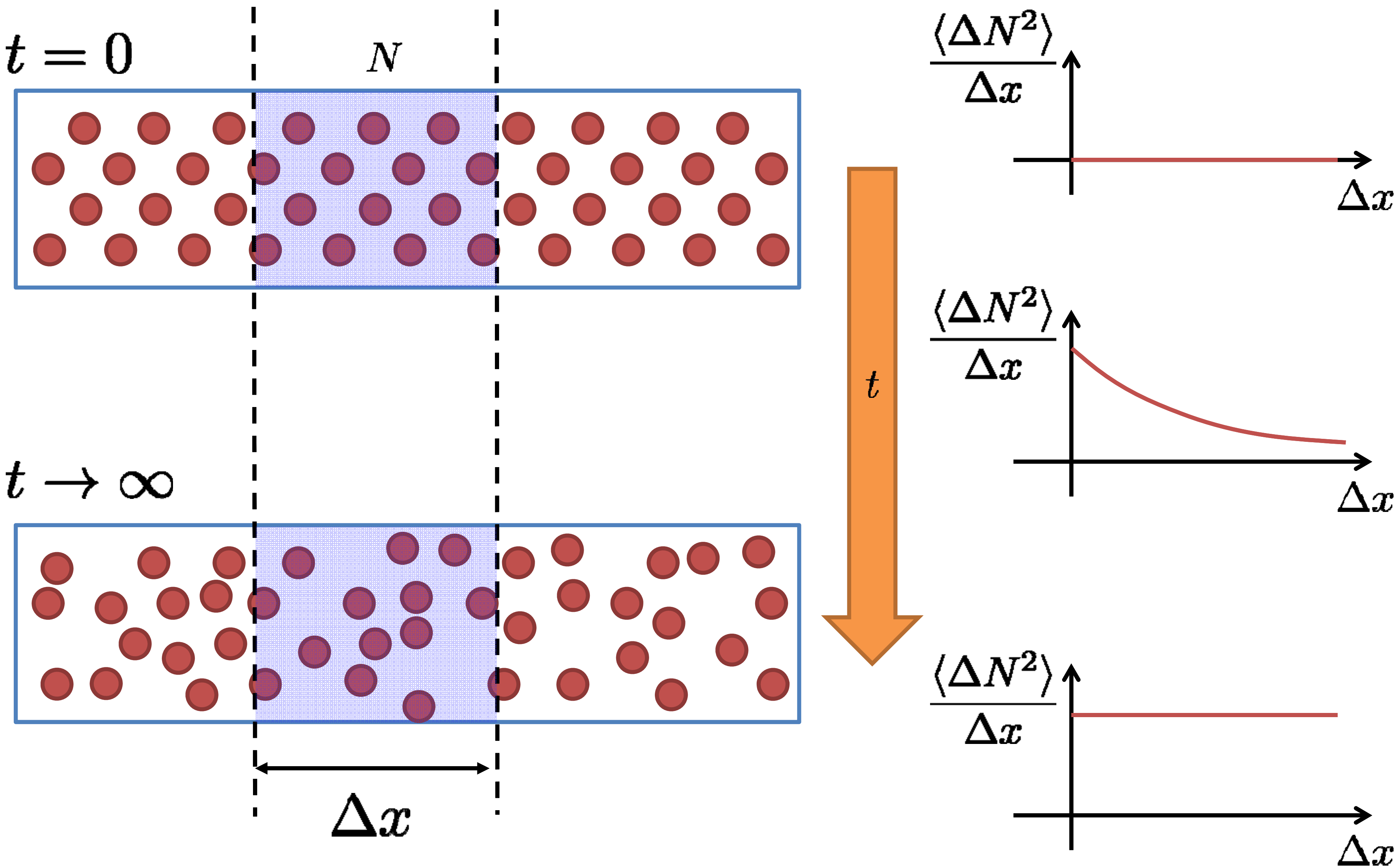
ALICE
PRL 2013





rapidity window

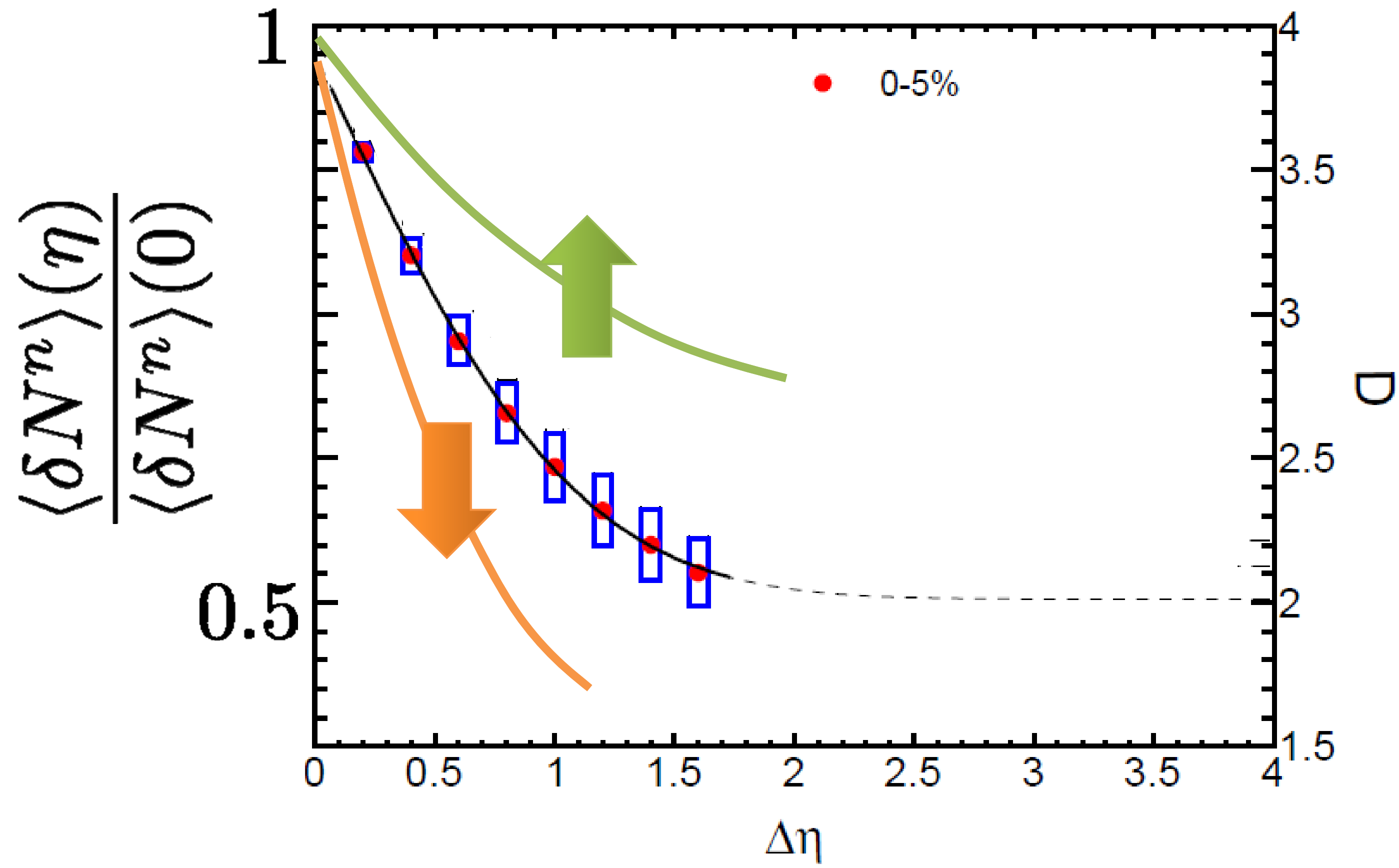
Dissipation of a Conserved Charge



$<\delta N_Q^4>$ @ LHC ?

How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of Δη?

suppression or enhancement



Hydrodynamic Fluctuations

Landau, Lifshitz, Statistical Mechaniqs II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

Markov (white noise)
+
continuity

Gaussian noise

Fluctuation of *n* is
Gaussian in equilibrium

cf) Gardiner, “Stochastic Methods”

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta,\tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n*

How to Introduce Non-Gaussianity?

Stochastic diffusion equation

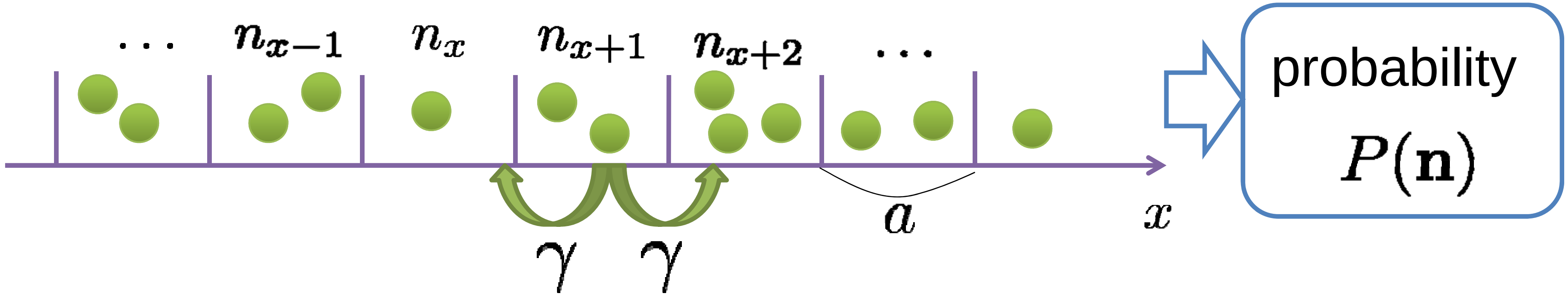
$$\partial_\tau n = D\partial_\eta^2 n + \partial_\eta \xi(\eta, \tau)$$

- Choices to introduce non-Gaussianity in equil.:
  - *n* dependence of diffusion constant *D*(*n*)
  - colored noise
  - discretization of *n* ⇐ **our choice**

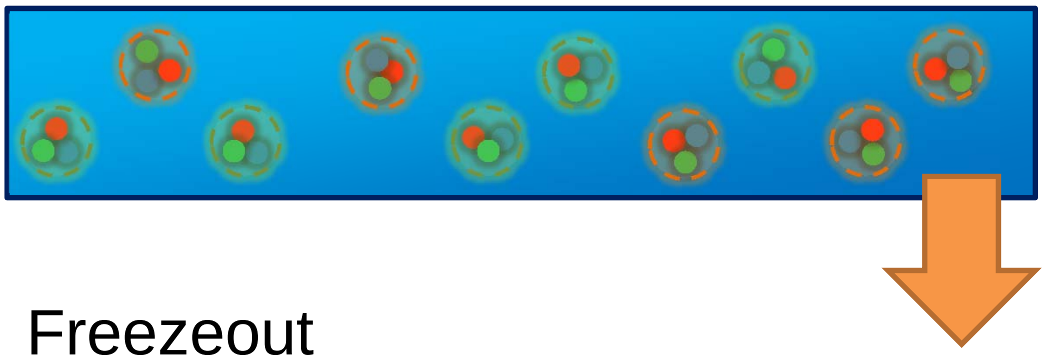
REMARK: Fluctuations measured in HIC are almost Poissonian.

Diffusion Master Equation

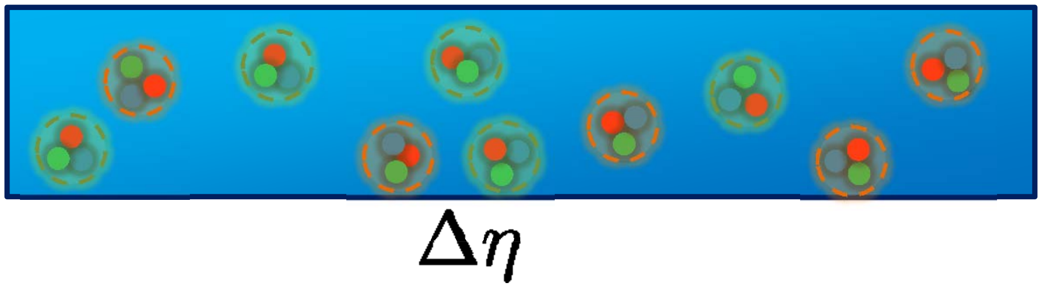
Divide spatial coordinate into discrete cells



Hadronization

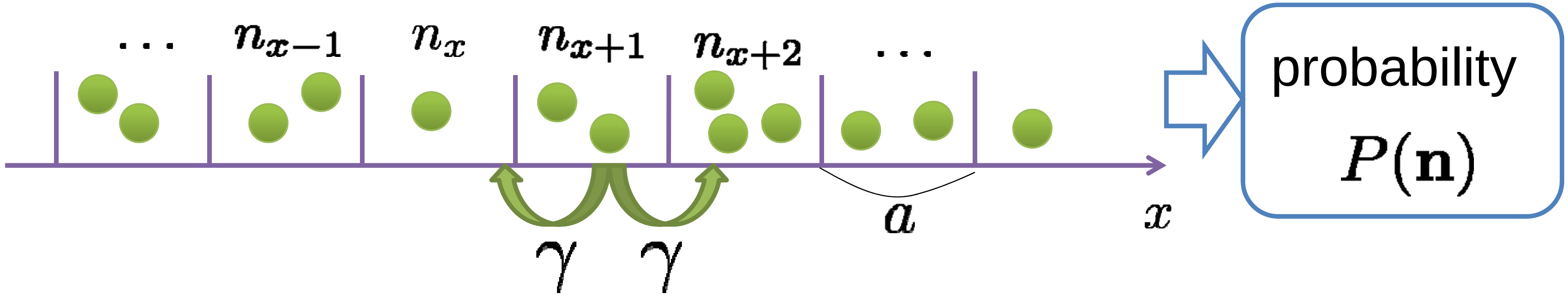


Freezeout



Diffusion Master Equation

Divide spatial coordinate into discrete cells



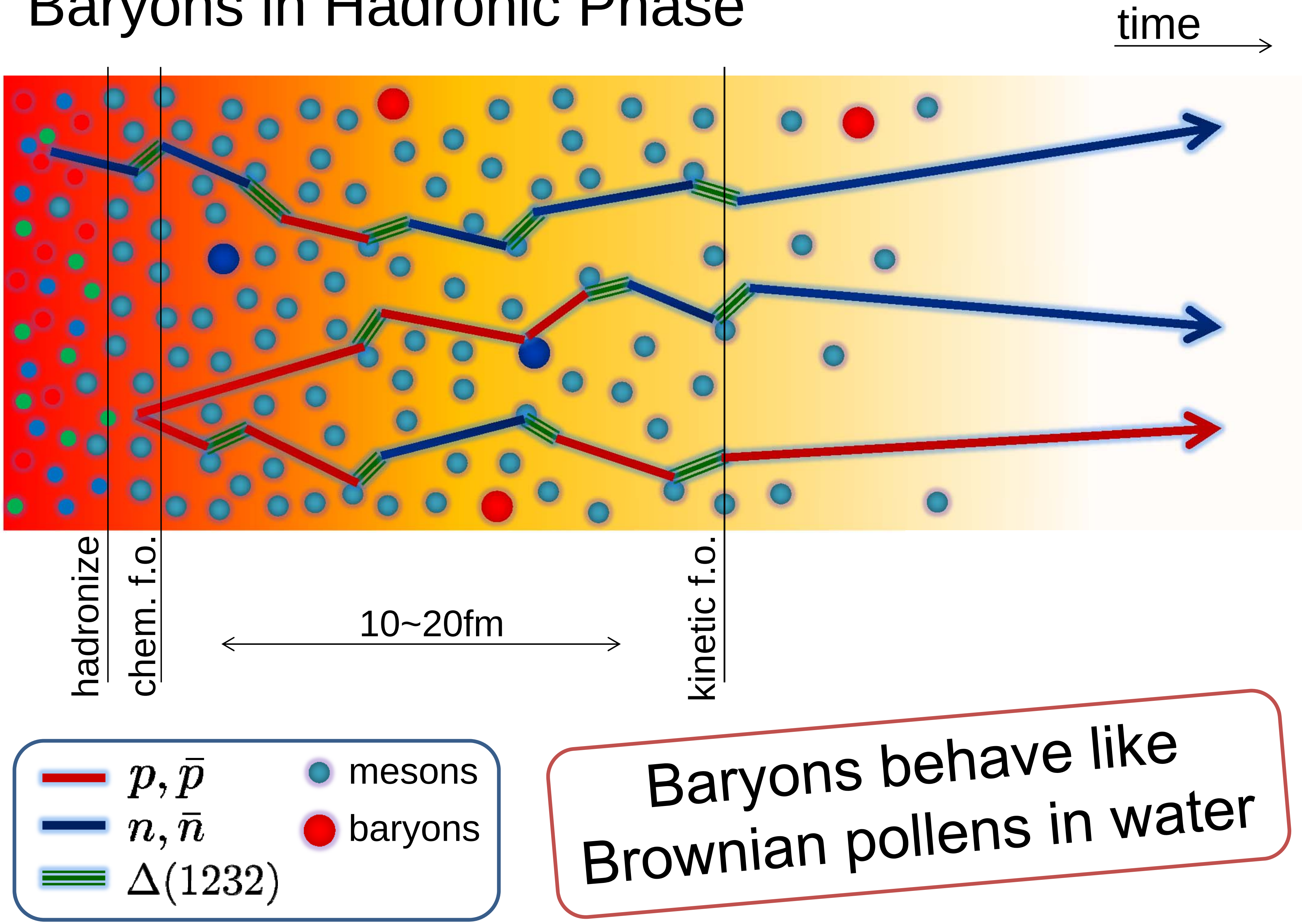
Master Equation for *P*(*n*)

$$\frac{\partial}{\partial t}P(\mathbf{n}) = \gamma \sum_x [(n_x + 1)\{P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x+1}) + P(\mathbf{n} + \mathbf{e}_x - \mathbf{e}_{x-1})\} - 2n_x P(\mathbf{n})]$$

Solve the DME **exactly**, and take *a*→0 limit

No approx., ex. van Kampen's system size expansion

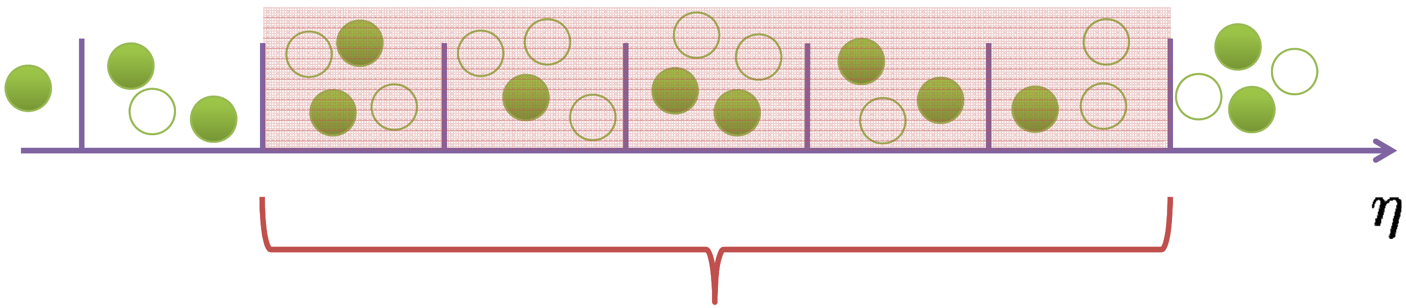
Baryons in Hadronic Phase



Baryons behave like
Brownian pollens in water

Net Charge Number

Prepare 2 species of (non-interacting) particles



$$\bar{Q}(\tau) = \int_0^{\Delta\eta} d\eta \, (n_1(\eta,\tau) - n_2(\eta,\tau))$$

Let us investigate

$\langle \bar{Q}^2 \rangle_c$ $\langle \bar{Q}^4 \rangle_c$ at freezeout time t

Solution of DME in a→0 Limit

1st order (deterministic) $\langle n \rangle$

- consistent with diffusion equation with $D=\gamma a^2$

  ⇨ Continuum limit with fixed $D=\gamma a^2$

2nd order $\langle \delta n^2 \rangle$

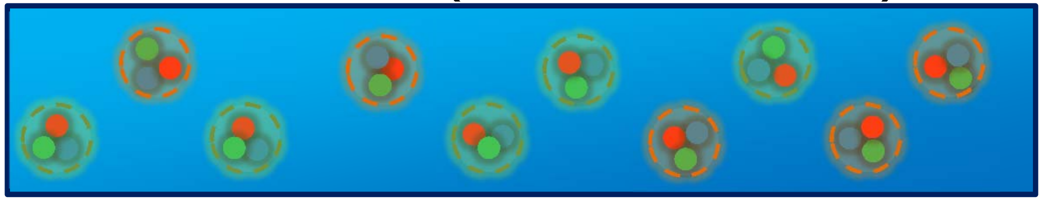
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Shuryak, Stephanov, 2001

Nontrivial results for non-Gaussian fluctuations

Time Evolution in Hadronic Phase

Hadronization (initial condition)



- Boost invariance / infinitely long system
- Local equilibration / local correlation

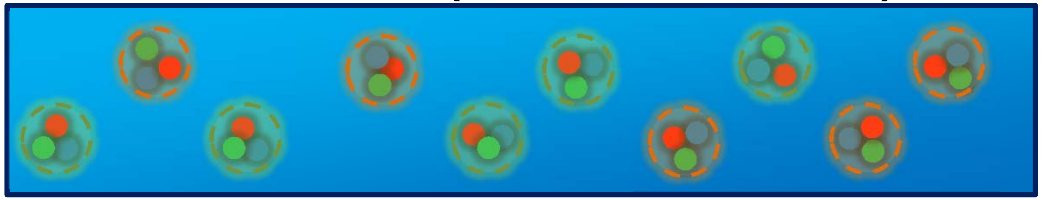
$\langle \bar{Q}^2 \rangle_c$ $\langle \bar{Q}^4 \rangle_c$ $\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$ $\langle Q^2_{(\mathrm{tot})} \rangle_c$

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Time Evolution in Hadronic Phase

Hadronization (initial condition)



Time evolution via DME

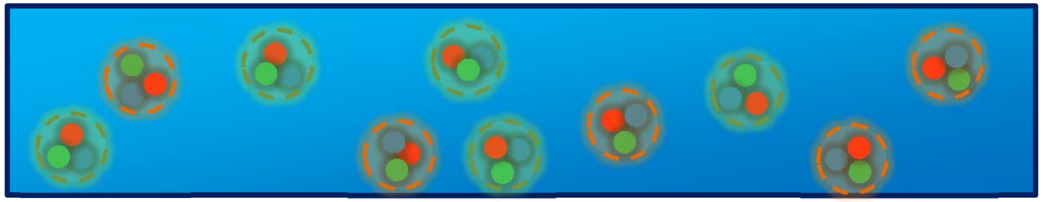
- Boost invariance / infinitely long system
- Local equilibration / local correlation

$\langle \bar{Q}^2 \rangle_c$ $\langle \bar{Q}^4 \rangle_c$ — suppression owing to local charge conservation

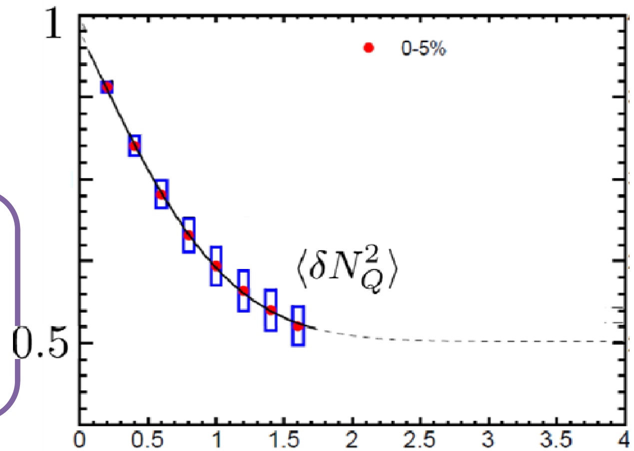
$\langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c$

$\langle Q^2_{(\mathrm{tot})} \rangle_c$ — strongly dependent on hadronization mechanism

Freezeout

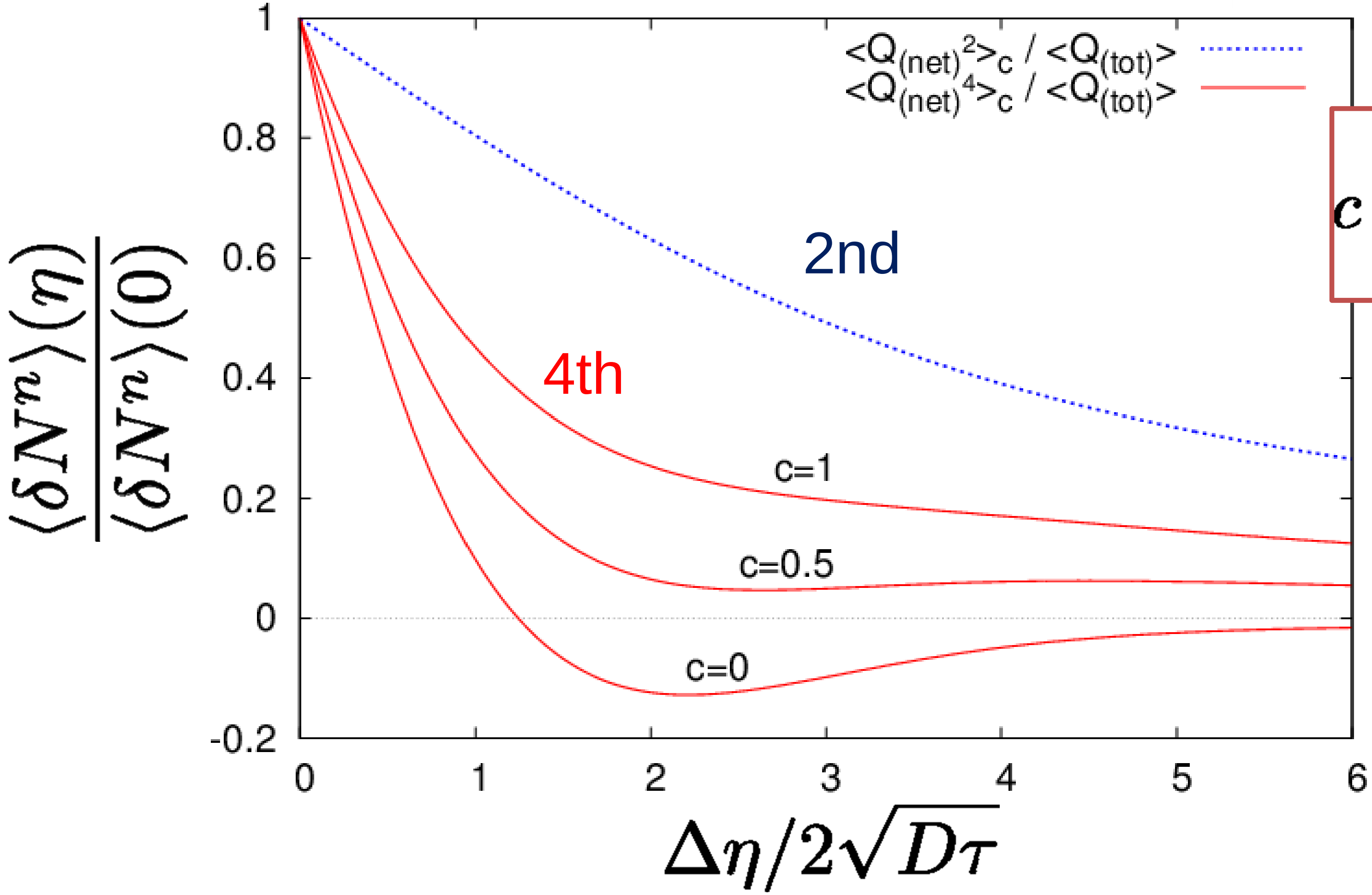


$\Delta\eta$ Dependence at Freezeout



Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(\mathrm{tot})} \rangle_c = 0$$

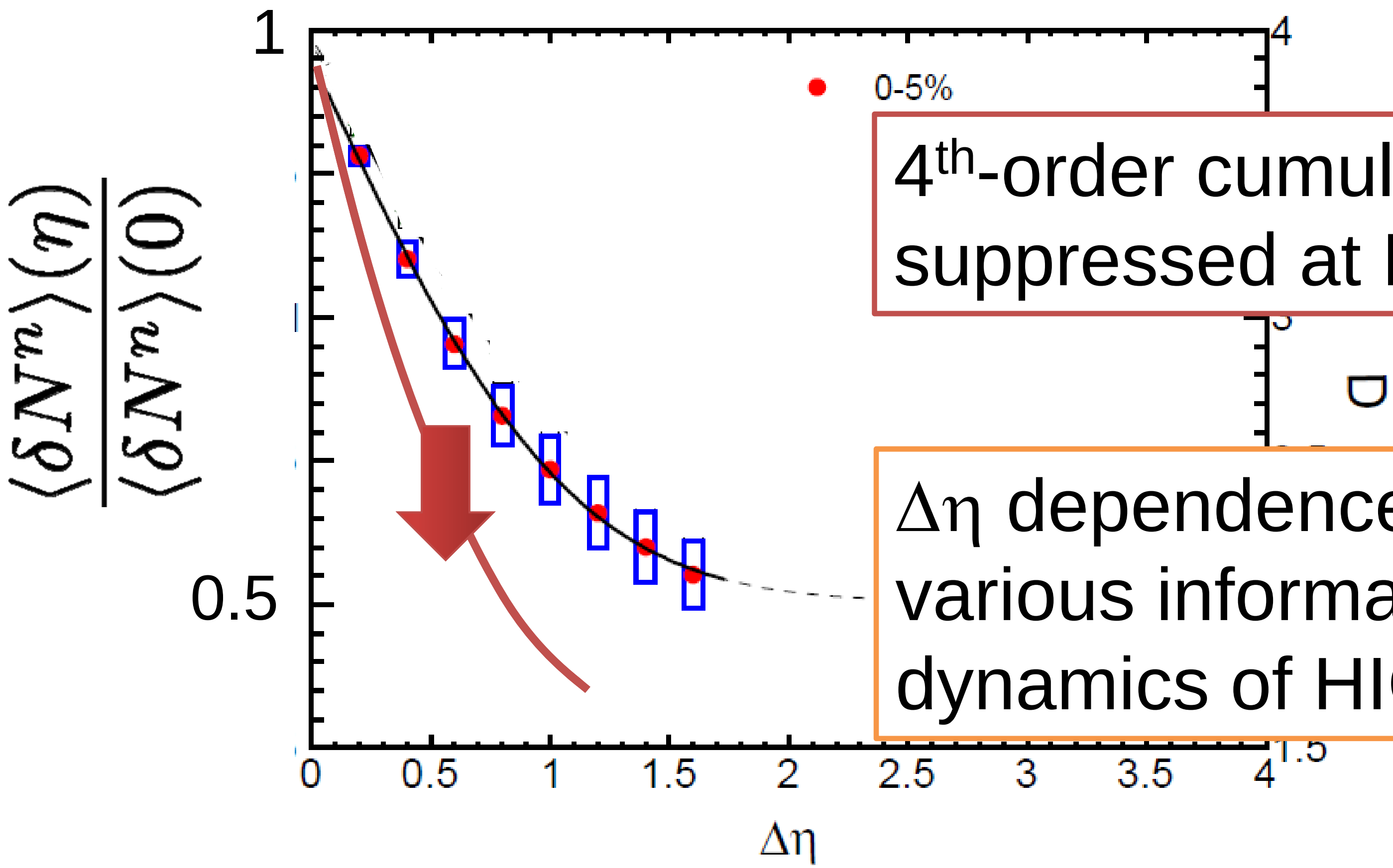


$$c = \frac{\langle Q_{(\mathrm{tot})}^2 \rangle_c}{\langle Q_{(\mathrm{tot})} \rangle}$$

parameter sensitive to hadronization

$\langle \delta N_Q^4 \rangle$ @ LHC

Assumptions
- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage



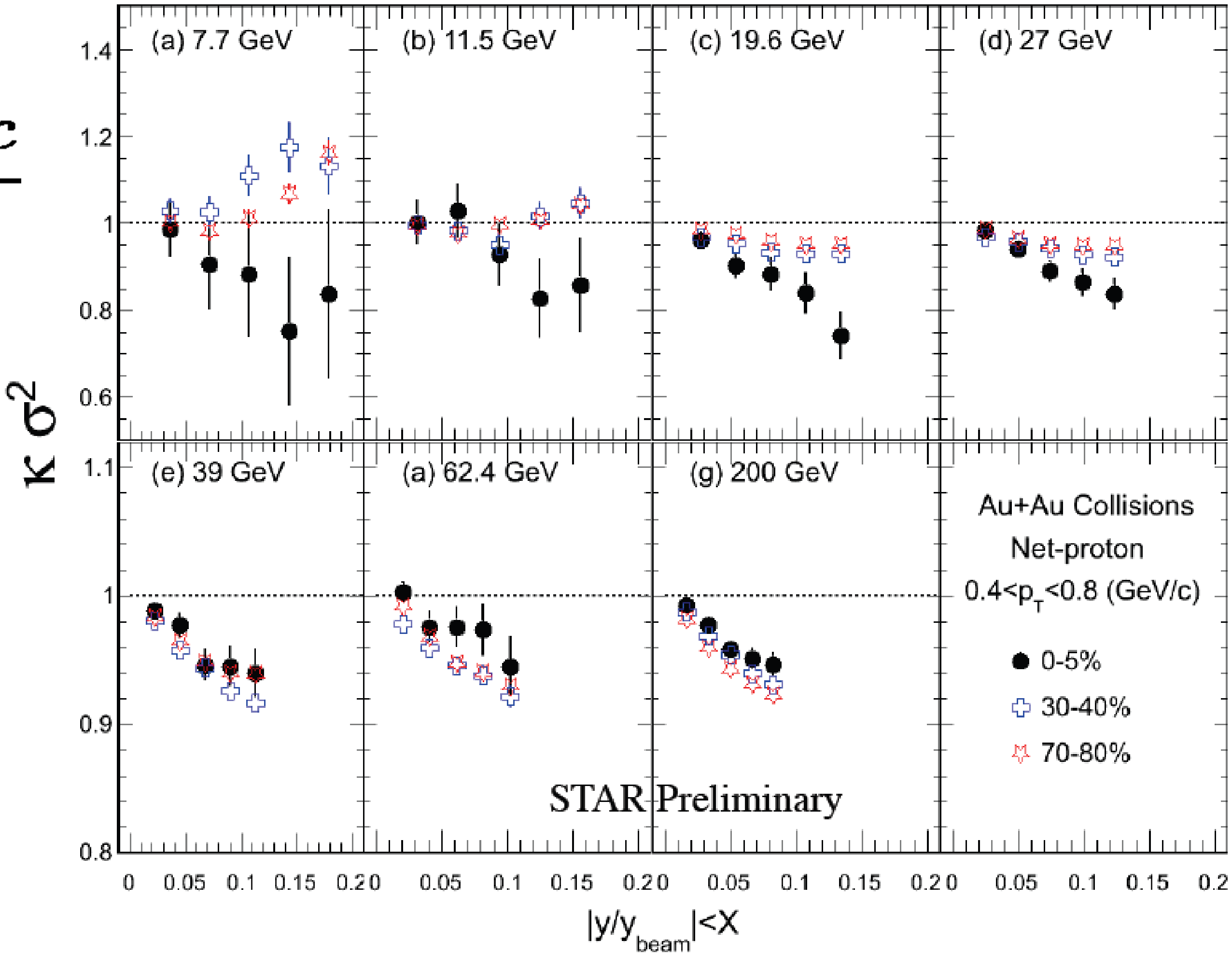
4th-order cumulant will be suppressed at LHC energy!

Δη dependences encode various information on the dynamics of HIC!

Δη Dependence at STAR

STAR, QM2012

$$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$$



$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as Δη becomes larger at RHIC energy.

Many Things to do …

Theory (Motivation)

- Better understanding on non-thermal nature
- Critical phenomena
- Other ideas?

Heavy Ion Collisions

- $\Delta\eta$ dependence of 4th order cumulant
- Baryon number cumulants
- Acceptance effect, etc.

Lattice

- More accurate data
- Various channels
- Nonzero μ

Summary

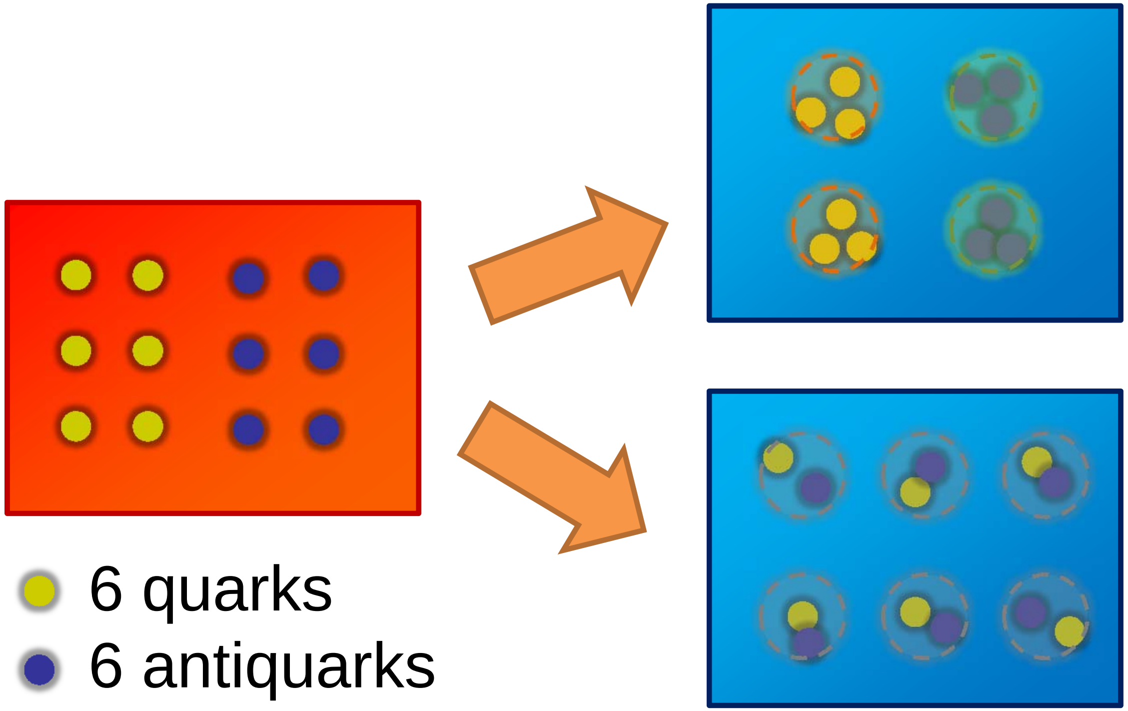
- Conserved charge fluctuations are observable both in lattice simulations and heavy ion collisions. The comparison of the results in these two “experiments” will provide us many information to understand the QCD at nonzero T/μ.

- A lot of efforts are required both sides:
  - Lattice: Higher statistics
  - HIC: reconstructing baryon #, acceptance, etc.

- Rapidity window dependences of cumulants in HIC are valuable tools to understand the non-thermal nature of fluctuations.

Total Charge Number

In recombination model,

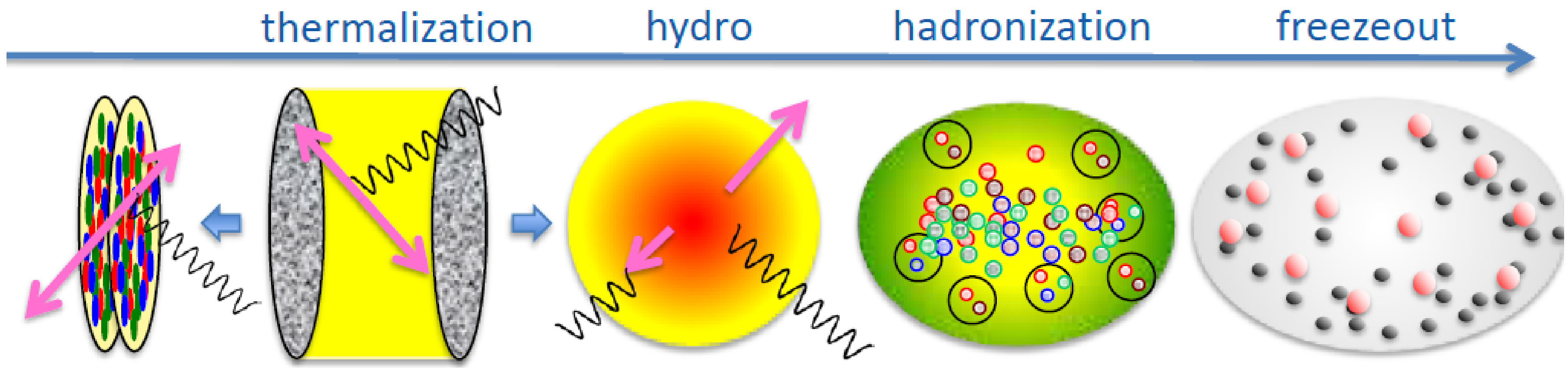


$$N_B^{(\mathrm{net})} = 0 \qquad N_B^{(\mathrm{tot})} = 4$$

$$N_B^{(\mathrm{net})} = 0 \qquad N_B^{(\mathrm{tot})} = 0$$

- $N_B^{(\mathrm{tot})}$ can fluctuate, while $N_B^{(\mathrm{net})}$ does not.

Evolution of Fluctuations



Fluctuation in initial state

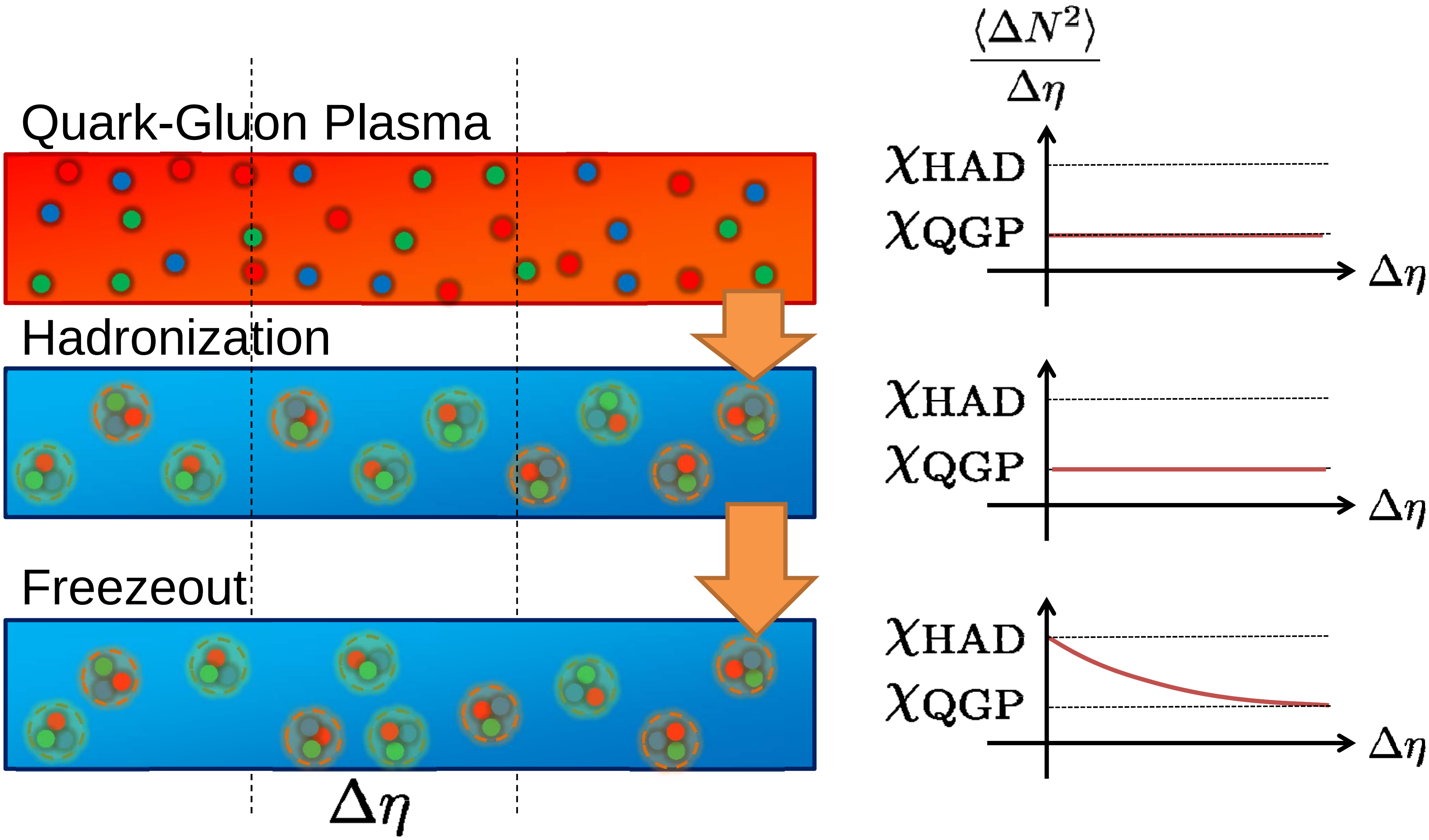
Time evolution in the QGP

approach to HRG by diffusion

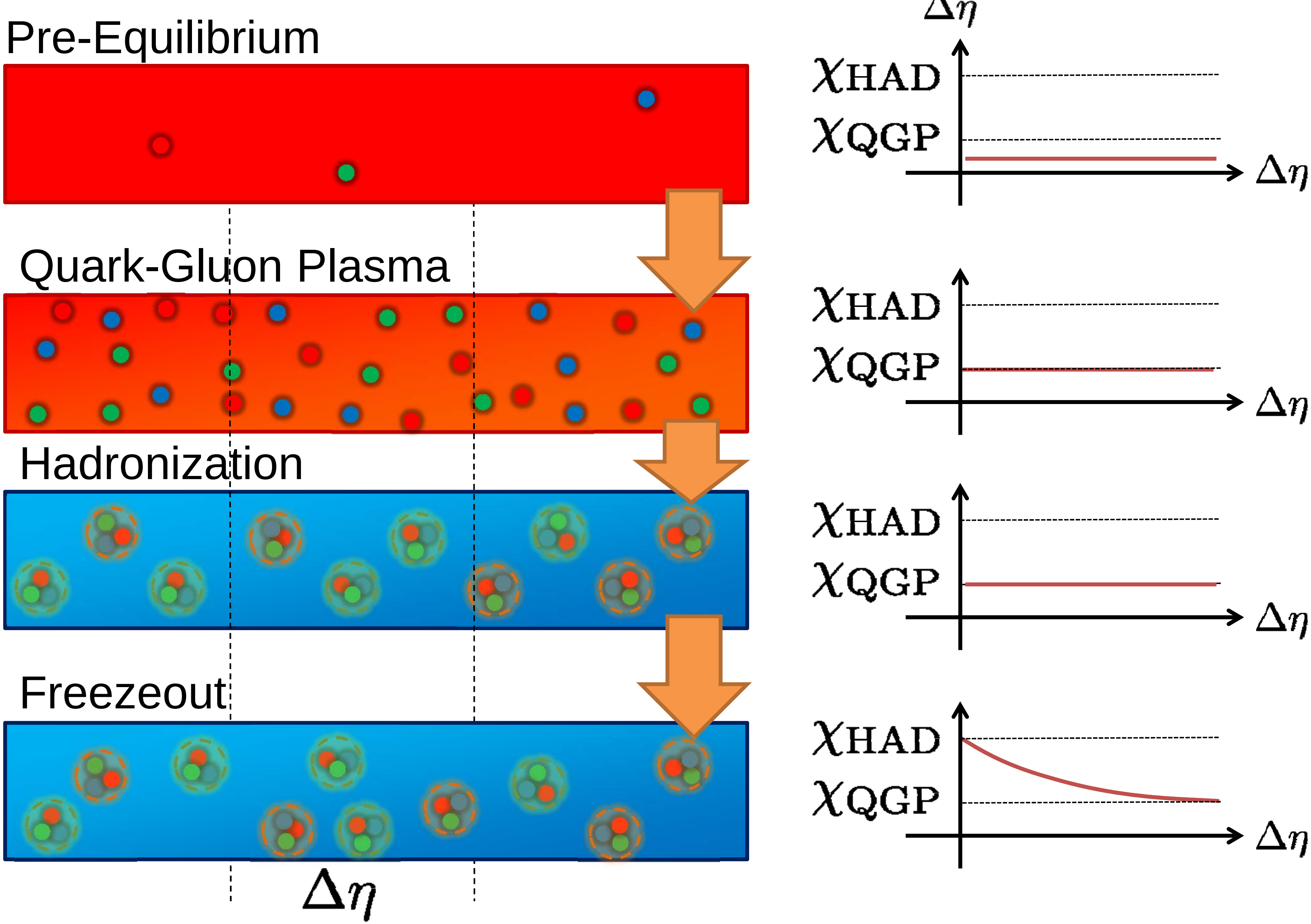
experimental effects
particle missID, etc.

volume fluctuation

Time Evolution in HIC



Time Evolution in HIC



Time Evolution in HIC