Fluctuations of Conserved Charges
- Theory, Experiment, and Lattice -

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KEK, 2014/Jan./20
QCD @ nonzero $T$

Fluctuations of conserved charges

Theory (Motivation)

Lattice

Heavy Ion Collisions
QCD @ nonzero $T$

Theory
(Motivation)

QCD @ nonzero $T$

Lattice

Heavy Ion Collisions
Why QCD @ nonzero $T$ and $\mu$?

- Form of the matter under extreme conditions
  - QCD Phase diagram
  - New many body properties
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- State of the matter realized in
  - Early Universe
  - Compact stars
Why QCD @ nonzero $T$ and $\mu$?

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- State of the matter realized in
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- Relativistic heavy ion collisions
Relativistic Heavy Ion Collisions
Particle yields can be well described only by $T$, $\mu_B$!
Beam-Energy Scan Program

Grand Canonical Ensemble

Au+Au

STAR Preliminary

Color SC

Hadrons

beam energy

T

Tch (GeV)

μB (GeV)

μ

0

0.1

0.12

0.14

0.16

0.18

200 GeV

39 GeV

11.5 GeV

7.7 GeV

0

0.1

0.2

0.3

0.4

0.5
Hadron Resonance Gas (HRG) Model

The HRG model well describes thermodynamics calculated on the lattice.

“Trace Anomaly”

Baryon # fluctuation
Lattice and HIC : EoS

Equation of states

- Robust modelling of space-time evolution
- Small shear viscosity

Input

Lattice

Heavy Ion Collisions

Challenge!
Lattice and HIC: Heavy Quarkonia

Heavy quarkonia will disappear in QGP
Matsui, Satz, 1986

Charmonium SPC
Asakawa, Hatsuda, 2004

Input

Heavy Ion Collisions
Fluctuations of Conserved Charges
Fluctuations

Observables in equilibrium are fluctuating.
Fluctuations

Observables in equilibrium are fluctuating.

- Variance: $\langle \delta N^2 \rangle = V \chi_2 = \sigma^2$
- Skewness: $S = \frac{\langle \delta N^3 \rangle}{\sigma^3}$
- Kurtosis: $\kappa = \frac{\langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2}{\chi_2 \sigma^2}$

$\delta N = N - \langle N \rangle$

Non-Gaussianity
Conserved Charge Fluctuations

- Definite definition of the operator $\mathcal{O}$
  - as a Noether current
  - Expectation value: $\langle \mathcal{O} \rangle = \text{Tr}[\rho \mathcal{O}] = \int D\mathcal{U} \mathcal{O} e^{-S}$
  - Fluctuation: $\langle \delta \mathcal{O}^2 \rangle = \langle \mathcal{O}^2 \rangle - \langle \mathcal{O} \rangle^2$

Simple thermodynamic relation

$$\langle \delta \mathcal{O}^n \rangle_c = \frac{T^n}{V} \frac{\partial^n}{\partial \mu^n} \ln Z(\mu) \quad Z(\mu) = \text{Tr} e^{-\beta(H - \mu \mathcal{O})}$$
Taylor Expansion Method & Cumulants

\[ P(T, \mu) = \frac{T}{V} \ln Z(\mu) \]

\[ = P(T, 0) + \frac{\mu}{T} \frac{\partial P(T, 0)}{\partial (\mu/T)} + \frac{1}{2} \left( \frac{\mu}{T} \right)^2 \frac{\partial^2 P(T, 0)}{\partial (\mu/T)^2} + \ldots \]

\[ \langle N \rangle \]

\[ \langle \delta N^2 \rangle_c \]

Baryon number cumulants = Taylor expansion coeffs.
Recent Progress in Lattice Simulations

From LATTICE2013 presentations
QCD @ nonzero $T$

- Theory
  - (Motivation)

- Lattice

- Heavy Ion Collisions
Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.
Event-by-Event Analysis @ HIC

Fluctuations can be measured by e-by-e analysis in experiments.

STAR, PRL 105 (2010)

\[ \langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c \]
What are Fluctuations observed in HIC?

**QUESTION:**
When the experimentally-observed fluctuations are formed?

- at chemical freezeout?
- at kinetic freezeout?
- or, much earlier?
QCD @ nonzero $T$

- Theory (Motivation)
- Lattice
- Heavy Ion Collisions
Fluctuations

- Fluctuations reflect properties of matter.
  - Enhancement near the critical point
    Stephanov, Rajagopal, Shuryak (’98); Hatta, Stephanov (’02); Stephanov (’09); …
  - Ratios between cumulants of conserved charges
    Asakawa, Heintz, Muller (’00); Jeon, Koch (’00); Ejiri, Karsch, Redlich (’06)
  - Signs of higher order cumulants
    Asakawa, Ejiri, MK (’09); Friman, et al. (’11); Stephanov (’11)

![Diagram showing fluctuations and phase transitions]
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$\langle \delta N^n \rangle_c = \langle N \rangle$

$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$

$\Rightarrow \langle \delta N_{B}^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$

$3N_B = N_q$

$\langle \delta N_{B}^n \rangle_c = \langle N_B \rangle$
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

$$\langle \delta N^q \rangle_c = \langle N_q \rangle$$

$$\rightarrow \langle \delta N^q_B \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

RBC-Bielefeld '09
Fluctuations

Free Boltzmann $\rightarrow$ Poisson

$$\langle \delta N^n \rangle_c = \langle N \rangle$$

$$\langle \delta N_q^n \rangle_c = \langle N_q \rangle$$

$$\langle \delta N_B^n \rangle_c = \frac{1}{3^{n-1}} \langle N_B \rangle$$

$$3N_B = N_q$$

WB [1305.5161]
Skellam Distribution

- Poisson + Poisson = Poisson
  \[\langle N_1 \rangle + \langle N_2 \rangle = \langle N_1 + N_2 \rangle\]

- Poisson - Poisson = Skellam distribution
  \[\langle N_1 \rangle - \langle N_2 \rangle = \begin{cases} \langle N_1 + N_2 \rangle & (n:\text{even}) \\ \langle N_1 - N_2 \rangle & (n:\text{odd}) \end{cases}\]

In the HRG model, (Net-)baryon and electric charge fluctuations are of Skellam distribution.
Search of QCD Critical Point

- Fluctuations diverge at the QCD critical point.

Example: \( \langle \delta N_B^2 \rangle \)

- Higher order cumulants are more sensitive to correlation length

\[
\begin{aligned}
\langle \delta N^2 \rangle &\sim \xi^2 \\
\langle \delta N^3 \rangle &\sim \xi^{4.5} \\
\langle \delta N^4 \rangle_c &\sim \xi^7
\end{aligned}
\]

- Stephanov, PRL, 2010
- Athanasiou, Rajagopal, Stephanov, 2010
Sign of Higher Order Cumulants

- $\chi_B$ has an edge along the phase boundary

$$\frac{\partial \chi_B}{\partial \mu_B} \text{ changes the sign at the QCD phase boundary!}$$

\[
\begin{align*}
\chi_B &= -\frac{1}{V} \frac{\partial^2 \Omega}{\partial \mu_B^2} = \frac{\langle (\delta N_B)^2 \rangle}{VT} \\
\frac{\partial \chi_B}{\partial \mu_B} &= -\frac{1}{V} \frac{\partial^3 \Omega}{\partial \mu_B^3} = \frac{\langle (\delta N_B)^3 \rangle}{VT^2}
\end{align*}
\]

Asakawa, Ejiri, MK, PRL, 2009
Impact of Negative Third Moments

Once negative $m_3(BBB)$ is established, it is evidences that

1. $\chi_B$ has a peak structure in the QCD phase diagram.
2. Hot matter beyond the peak is created in the collisions.

- No dependence on any specific models.
- Just the sign! No normalization (such as by $N_{ch}$).
Various third moments, $\langle \delta N_B^3 \rangle$, $\langle \delta N_Q^3 \rangle$, $\langle \delta E^3 \rangle$ become negative near the phase boundary.

The behaviors can be checked by lattice and HIC!

See also, Friman, et al. ('11); Stephanov ('11)
Exploring Medium Properties

Hadronic

B=0,1

strangeness with baryon number

Quark-Gluon

B=1/3
Exploring Medium Properties

Hadronic

\[ B = 0, 1 \]

strangeness with baryon number

Quark-Gluon

\[ B = \frac{1}{3} \]

Combinations of cumulants which vanish in the HRG model

BNL-Bielefeld, PRL 2013
QCD @ nonzero $T$

- Theory (Motivation)
- Lattice
- Heavy Ion Collisions
Proton # Fluctuations @ STAR-BES

STAR, PRL2010

(a) $m_1$

(b) $m_2$

$S\sigma = \frac{\langle (\delta N_p^{(net)})^3 \rangle}{\langle (\delta N_p^{(net)})^2 \rangle}$,

$\kappa \sigma^2 = \frac{\langle (\delta N_p^{(net)})^4 \rangle_c}{\langle (\delta N_p^{(net)})^2 \rangle}$
Proton # Fluctuations @ STAR-BES

STAR, PRL2010

(a) $m_1$

$S \sigma = \frac{\langle (\delta N_p^{(net)})^3 \rangle}{\langle (\delta N_p^{(net)})^2 \rangle}$, \hspace{1cm} \kappa \sigma^2 = \frac{\langle (\delta N_p^{(net)})^4 \rangle_c}{\langle (\delta N_p^{(net)})^2 \rangle}$

(b) $m_2$

$\sqrt{s_{NN}}$ (GeV)

STAR, 2011

Au+Au Collisions

- 0-5%
- 30-50%
- 70-80%

$S \sigma$

HRG

STAR Preliminary

$\sqrt{s_{NN}}$ (GeV)

$k \sigma^2$

high $\mu$ \hspace{2cm} low $\mu$
Proton # Fluctuations @ STAR-BES

STAR, 2012 (Quark Matter)

Net-proton

0.4<p_T<0.8 (GeV/c),|y|<0.5

S σ

K σ^2

UrQMD 0-5% p+p

Au+Au Poisson

0-5%  
5-10%  
30-40%  
70-80%

STAR, 2011

Au+Au Collisions

0-5%  30-50%  70-80%

HRG

STAR Preliminary

K σ^2

√s_{NN} (GeV)

3 4 5 10 20 30 100 200

high μ  low μ
Proton # Cumulants @ STAR-BES

\[
\frac{C_4}{C_2} = \frac{C_3}{C_1} = \frac{C_3}{C_2 \text{ Poissonian}}
\]

CAUTION!

Proton number ≠ baryon number

MK, Asakawa, 2011; 2012
Proton # Cumulants @ STAR-BES

\[
\frac{C_4}{C_2} = \frac{C_3}{C_2}
\]

\[\kappa \sigma^2\]

Something interesting??

Athanasiou, Rajagopal, Stephanov, 2010
Electric Charge Fluctuation @ LHC

ALICE, PRL110, 152301 (2013)

D-measure

\[ D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle} \]

\begin{itemize}
  \item \( D \sim 3-4 \) Hadronic
  \item \( D \sim 1-1.5 \) Quark
\end{itemize}

significant suppression from hadronic value at LHC energy!

\[ \langle \delta N_Q^2 \rangle \] is not equilibrated at freeze-out at LHC energy!
$\Delta \eta$ Dependence @ ALICE

\[
D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}
\]
Dissipation of a Conserved Charge

\[ \frac{\langle \Delta N^2 \rangle}{\Delta x} \]

$t = 0$

$t \rightarrow \infty$

$\Delta x$

$\Delta x$

$N$
Dissipation of a Conserved Charge

$t = 0$

$t \rightarrow \infty$

$\frac{\langle \Delta N^2 \rangle}{\Delta x}$

$\frac{\langle \Delta N^2 \rangle}{\Delta x}$
Variation of a conserved charge is achieved only through diffusion.

The larger $\Delta \eta$, the slower diffusion.
$\Delta \eta$ dependences of conserved charge fluctuations encode history of dynamical evolution
Comparison b/w Lattice & HIC

Gupta, Xu, et al., Science, 2009

- Taylor expansion method
- Chemical freezeout $T, \mu$
- Pade approx.
Cumulants: HIC@RHIC vs Lattice

Parameter window constrained by lattice

BNL-Bielefeld, LATTICE2013

Fluctuations “exp + lattice”

\( \mu / T \) discrepancy

Particle abundance (chem. freezeout \( T \))
Many Things to Do

- Proton vs baryon number cumunants
- Are fluctuations generated with fixed $T$?
- Experimental environments
  - Acceptance, efficiency
  - Particle missid
  - Global charge conservation
Baryon vs Proton Number Fluctuations

\[ \frac{\langle \delta N_B^n \rangle_c}{\langle \delta N_B^m \rangle_c} \neq \frac{\langle \delta N_p^n \rangle_c}{\langle \delta N_p^m \rangle_c} \]

\[ \langle \delta N_B^n \rangle_c \] are experimentally observable

MK, Asakawa, PRC85, 021901C(2012); PRC86, 024904(2012)
Nucleon Isospin as Two Sides of a Coin

Nucleons have two isospin states.

MK, Asakawa, 2012
Nucleon Isospin as Two Sides of a Coin

Nucleons have two isospin states.

Coins have two sides.

MK, Asakawa, 2012
Slot Machine Analogy

\[ P(N) \]

\[ = \]

\[ N \]

\[ P(N) \]
Extreme Examples

Fixed # of coins

Constant probabilities
Reconstructing Total Coin Number

\[ P(N) = \sum P(N) B_{1/2}(N; N) \]

\[ B_p(k; N) = p^k (1 - p)^{N-k} C_N \] : binomial distr. func.
Isospin of baryons can vary after chemical freezeout via charge exchange reactions mediated by $\Delta(1232)$:

$$p, n \rightarrow \Delta(1232) \rightarrow p, n \quad I = \frac{3}{2}$$

$$\pi \rightarrow \Delta(1232) \rightarrow \pi \quad \Gamma \simeq 1.8 \text{ [fm] }$$

"200mb = 20fm$^2$"
$\Delta(1232)$

Cross sections of $p$

1. $n + \pi^+ \rightarrow \Delta^+ \rightarrow p + \pi^+$ (2:1)
2. $p + \pi^- \rightarrow \Delta^0 \rightarrow n + \pi^0$ (1:2)
3. $p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+$

Decay rates of $\Delta$

$n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^-$
\( \Delta(1232) \)

Cross sections of \( p \):

- \( p + \pi^+ \rightarrow \Delta^{++} \rightarrow p + \pi^+ \)
- \( p + \pi^0 \rightarrow \Delta^+ \rightarrow p + \pi^0 \)
- \( n + \pi^+ \rightarrow \Delta^+ \rightarrow n + \pi^+ \)
- \( p + \pi^- \rightarrow \Delta^0 \rightarrow p + \pi^- \)
- \( n + \pi^0 \rightarrow \Delta^0 \rightarrow n + \pi^0 \)
- \( n + \pi^- \rightarrow \Delta^- \rightarrow n + \pi^- \)

Decay rates of \( \Delta \):

- \( p + \pi \rightarrow \Delta^{+,0} \rightarrow p : n \)
- \( = 5 : 4 \)
Nucleons in Hadronic Phase

- Time
  - 10-20 fm
  - Rare NN collisions
  - No quantum corr.
  - Many pions

- Chemical freeze-out
- Kinetic freeze-out

\[ m_\pi \simeq T \ll m_N - \mu_N \]

- \( n_N \ll 1 \)
- Rare NN collisions
- No quantum corr.
- \( n_N \ll n_\pi \)
- Many pions

- Protons: \( p, \bar{p} \)
- Neutrons: \( n, \bar{n} \)
- Mesons
- Baryons: \( \Delta(1232) \)
Probability Distribution $\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}})$

$\begin{align*}
\mathcal{P}(N_p, N_n, N_{\bar{p}}, N_{\bar{n}}) &= F(N_N, N_{\bar{N}})B(N_p; N_N)B(N_{\bar{p}}; N_{\bar{N}}) \\
\text{for any phase space in the final state.}
\end{align*}$
Difference btw Baryon and Proton Numbers

(1) \( N_{B}^{\text{(net)}} = N_B - N_{\bar{B}} \) deviates from the equilibrium value.

(2) Boltzmann (Poisson) distribution for \( N_B, N_{\bar{B}} \).

\[
2\langle (\delta N_p^{\text{(net)}})^2 \rangle = \frac{1}{2}\langle (\delta N_B^{\text{(net)}})^2 \rangle + \frac{1}{2}\langle (\delta N_B^{\text{(net)}})^2 \rangle_{\text{free}} + \frac{3}{4}\langle (\delta N_B^{\text{(net)}})^3 \rangle_{\text{free}} + \cdots
\]

For free gas
\[
2\langle (\delta N_p^{\text{(net)}})^n \rangle_{c} = \langle (\delta N_N^{\text{(net)}})^n \rangle_{c}
\]
Time Evolution of Higher Order Cumulants

MK, Asakawa, Ono, PLB728, 386, 2014
$\Delta \eta$ Dependence @ ALICE

![Graph showing $\Delta \eta$ dependence with different rapidity windows and ALICE PRL 2013 text]
Dissipation of a Conserved Charge

$t = 0$

$t \to \infty$
How does $\langle \delta N_Q^4 \rangle_c$ behave as a function of $\Delta \eta$?

- suppression
- enhancement

Plot showing the behavior of $\langle \delta N_n(\eta) \rangle / \langle \delta N_n(0) \rangle$ as a function of $\Delta \eta$. The plot indicates a suppression or enhancement depending on the data points and the fit lines.
Hydrodynamic Fluctuations

Stochastic diffusion equation

\[ \partial_\tau n = D \partial_\eta^2 n + \partial_\eta \xi(\eta, \tau) \]

Markov (white noise) + continuity

Gaussian noise

Fluctuation of \( n \) is Gaussian in equilibrium

Landau, Lifshitz, Statistical Mechanics II
Kapusta, Muller, Stephanov, 2012
Stephanov, Shuryak, 2001

cf) Gardiner, “Stochastic Methods”
How to Introduce Non-Gaussianity?

**Stochastic** diffusion equation

\[ \partial_\tau n = D \partial^2_\eta n + \partial_\eta \xi(\eta, \tau) \]

- Choices to introduce non-Gaussianity in equil.:
  - \( n \) dependence of diffusion constant \( D(n) \)
  - colored noise
  - discretization of \( n \)
How to Introduce Non-Gaussianity?

Stochastic diffusion equation

\[ \partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau) \]

Choices to introduce non-Gaussianity in equil.:

- \( n \) dependence of diffusion constant \( D(n) \)
- colored noise
- discretization of \( n \) our choice

REMARK: Fluctuations measured in HIC are almost Poissonian.
Diffusion Master Equation

Divide spatial coordinate into discrete cells

\[ \cdots \quad n_{x-1} \quad n_x \quad n_{x+1} \quad n_{x+2} \quad \cdots \]

\[ \gamma \quad \gamma \quad \alpha \]

probability \[ P(n) \]

Hadronization

Freezeout

\[ \Delta \eta \]
Diffusion Master Equation

Divide spatial coordinate into discrete cells

Master Equation for $P(n)$

$$\frac{\partial}{\partial t} P(n) = \gamma \sum_x [(n_x + 1) \{ P(n + e_x - e_{x+1}) + P(n + e_x - e_{x-1}) \} - 2n_x P(n)]$$

Solve the DME exactly, and take $a \to 0$ limit

No approx., ex. van Kampen’s system size expansion
Baryons in Hadronic Phase

hadronize
chem. f.o.

10~20fm

kinetic f.o.

time

Baryons behave like Brownian pollens in water
Net Charge Number

Prepare 2 species of (non-interacting) particles

\[ \bar{Q}(\tau) = \int_{0}^{\Delta \eta} d\eta \left( n_1(\eta, \tau) - n_2(\eta, \tau) \right) \]

Let us investigate

\[ \langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \text{at freezeout time } t \]
Solution of DME in a $a \rightarrow 0$ Limit

**1st order (deterministic) $\langle n \rangle$**
- consistent with diffusion equation with $D = \gamma a^2$
- Continuum limit with fixed $D = \gamma a^2$

**2nd order $\langle \delta n^2 \rangle$**
- consistent with stochastic diffusion eq.
  (for sufficiently smooth initial conditions)

Nontrivial results for non-Gaussian fluctuations

Shuryak, Stephanov, 2001
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle \bar{Q}^2 \rangle_c \quad \langle \bar{Q}^4 \rangle_c \quad \langle \bar{Q}^2 Q_{(tot)} \rangle_c \quad \langle Q_{(tot)}^2 \rangle_c
\]

suppression owing to local charge conservation

strongly dependent on hadronization mechanism
Time Evolution in Hadronic Phase

Hadronization (initial condition)

- Boost invariance / infinitely long system
- Local equilibration / local correlation

\[
\langle Q^2 \rangle_c \quad \langle Q^4 \rangle_c \quad \langle Q^2 Q_{(tot)} \rangle_c \quad \langle Q^2_{(tot)} \rangle_c
\]

suppression owing to local charge conservation

strongly dependent on hadronization mechanism

Freezeout
$\Delta \eta$ Dependence at Freezeout

Initial fluctuations:

$$\langle \bar{Q}^2 \rangle_c = \langle \bar{Q}^4 \rangle_c = \langle \bar{Q}^2 Q_{(tot)} \rangle_c = 0$$

Parameter sensitive to hadronization.
\(< \delta N_Q^4 > \) @ LHC

Assumptions

- boost invariant system
- small fluctuations of CC at hadronization
- short correlation in hadronic stage

4th-order cumulant will be suppressed at LHC energy!

\( \Delta \eta \) dependences encode various information on the dynamics of HIC!
$\frac{\langle \delta N_p^4 \rangle_c}{\langle \delta N_p^2 \rangle}$ decreases as $\Delta \eta$ becomes larger at RHIC energy.
Many Things to do …

Theory (Motivation)

- Better understanding on non-thermal nature
- Critical phenomena
- Other ideas?

Lattice

- More accurate data
- Various channels
- Nonzero $\mu$

Heavy Ion Collisions

- $\Delta\eta$ dependence of 4th order cumulant
- Baryon number cumulants
- Acceptance effect, etc.
Summary

- Conserved charge fluctuations are observable both in lattice simulations and heavy ion collisions. The comparison of the results in these two “experiments” will provide us many information to understand the QCD at nonzero $T/\mu$.

- A lot of efforts are required both sides:
  - Lattice: Higher statistics
  - HIC: reconstructing baryon #, acceptance, etc.

- Rapidity window dependences of cumulants in HIC are valuable tools to understand the non-thermal nature of fluctuations.
Total Charge Number

In recombination model,

\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 4 \]

\[ N_B^{(\text{net})} = 0 \]
\[ N_B^{(\text{tot})} = 0 \]

- \( N_B^{(\text{tot})} \) can fluctuate, while \( N_B^{(\text{net})} \) does not.
Evolution of Fluctuations

- Fluctuation in initial state
- Time evolution in the QGP
- Approach to HRG by diffusion
- Volume fluctuation
- Experimental effects (particle mis-ID, etc.)
Time Evolution in HIC

Quark-Gluon Plasma

Hadronization

Freezeout

\[ \langle \Delta N^2 \rangle \]
\[ \Delta \eta \]

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)

\[ \Delta \eta \]

\( \chi_{\text{HAD}} \)
\( \chi_{\text{QGP}} \)

\[ \Delta \eta \]
Time Evolution in HIC

- Pre-Equilibrium
- Quark-Gluon Plasma
- Hadronization
- Freezeout

\[ \langle \Delta N^2 \rangle / \Delta \eta \]

- \( \chi_{\text{HAD}} \)
- \( \chi_{\text{QGP}} \)

\[ \Delta \eta \]
Time Evolution in HIC

Pre-Equilibrium
Quark-Gluon Plasma
Hadronization
Freezeout

\[ \langle \Delta N^2 \rangle / \Delta \eta \]

\[ \chi_{\text{HAD}} \]
\[ \chi_{\text{QGP}} \]

\[ \Delta \eta \]

\[ \Delta \eta \]

\[ \Delta \eta \]