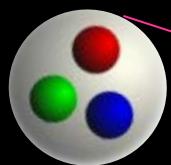
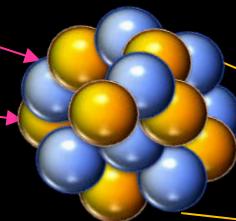


# Dense QCD and Compact Stars

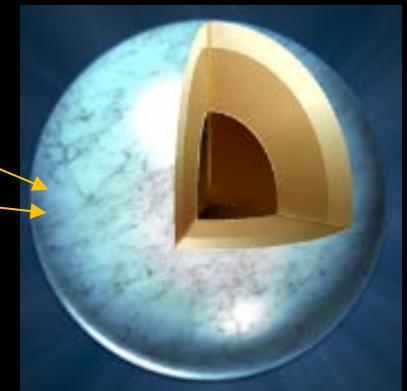
nucleon  $\sim 1$  [fm]



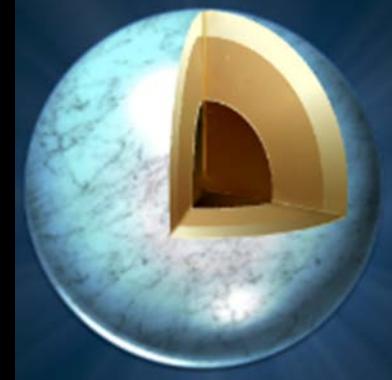
nucleus  $\sim 10$  [fm]



Neutron star  $\sim 10$  [km]



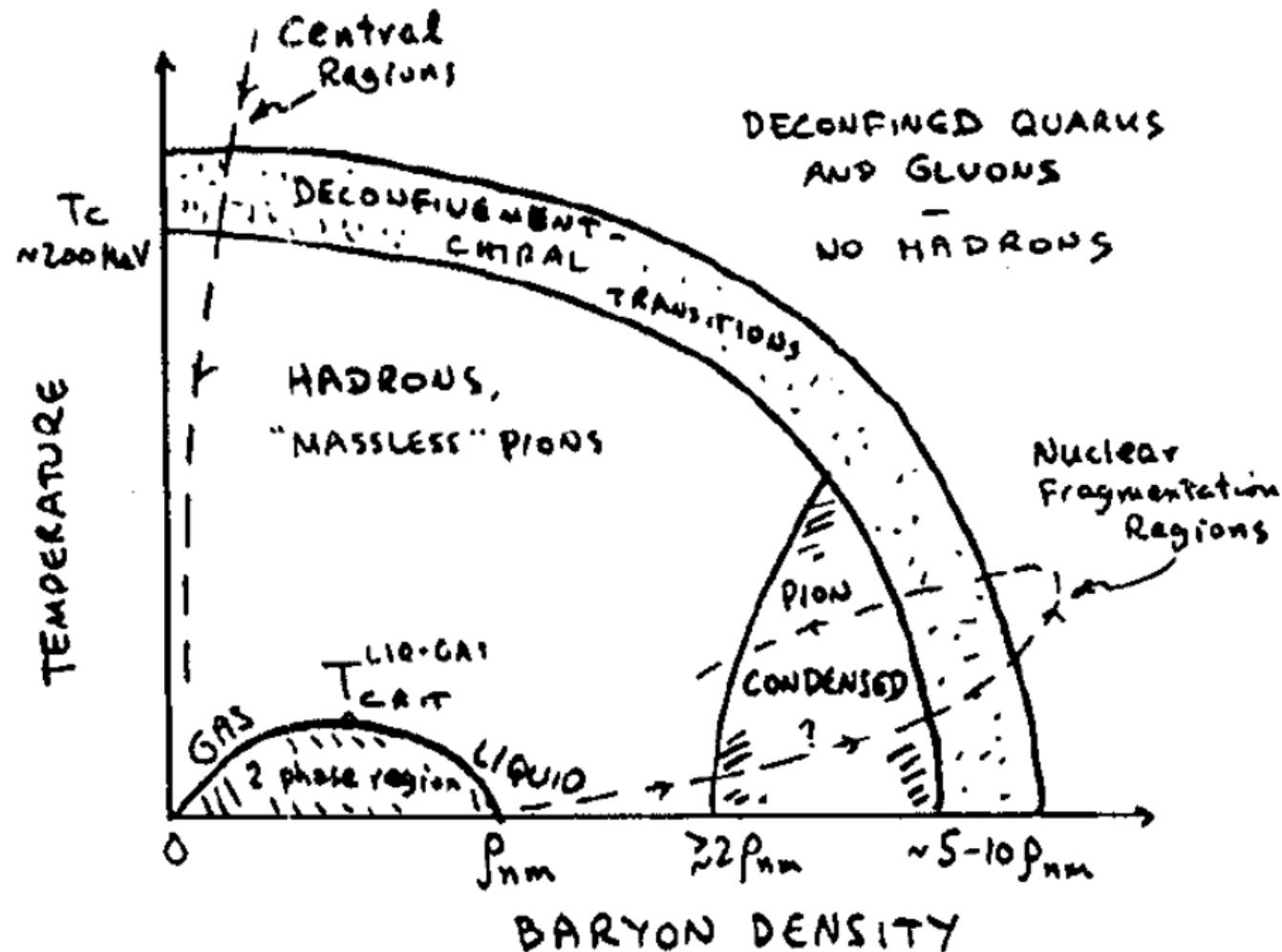
## Plan of this Talk



1. QCD Phase Structure
2. Neutron Star and Dense EOS
3. Dense EOS and Lattice QCD
4. Neutron Star and Hadron–Quark Crossover
5. Summary

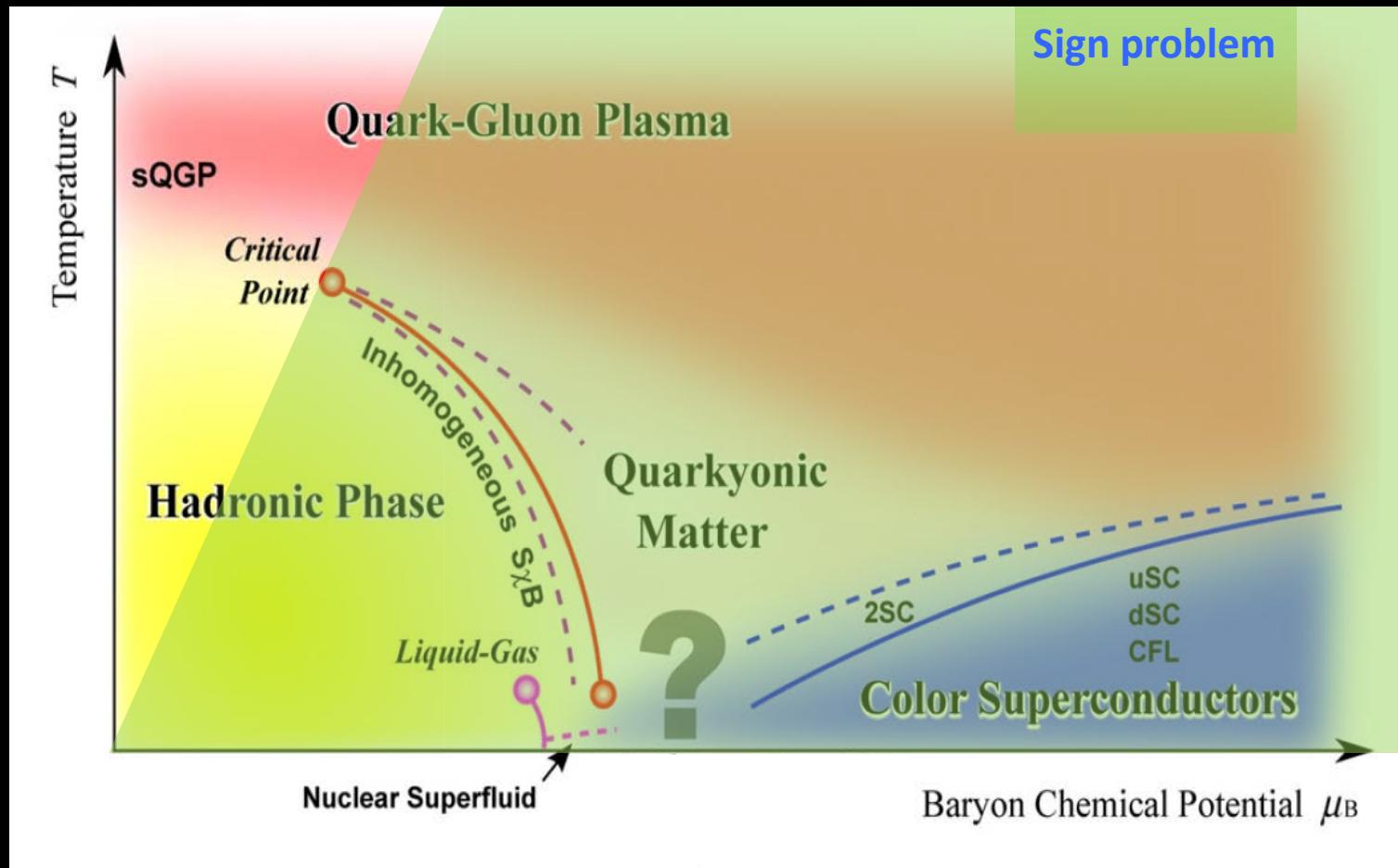
# QCD Phase Diagram (30 years ago)

## PHASE DIAGRAM OF NUCLEAR MATTER.



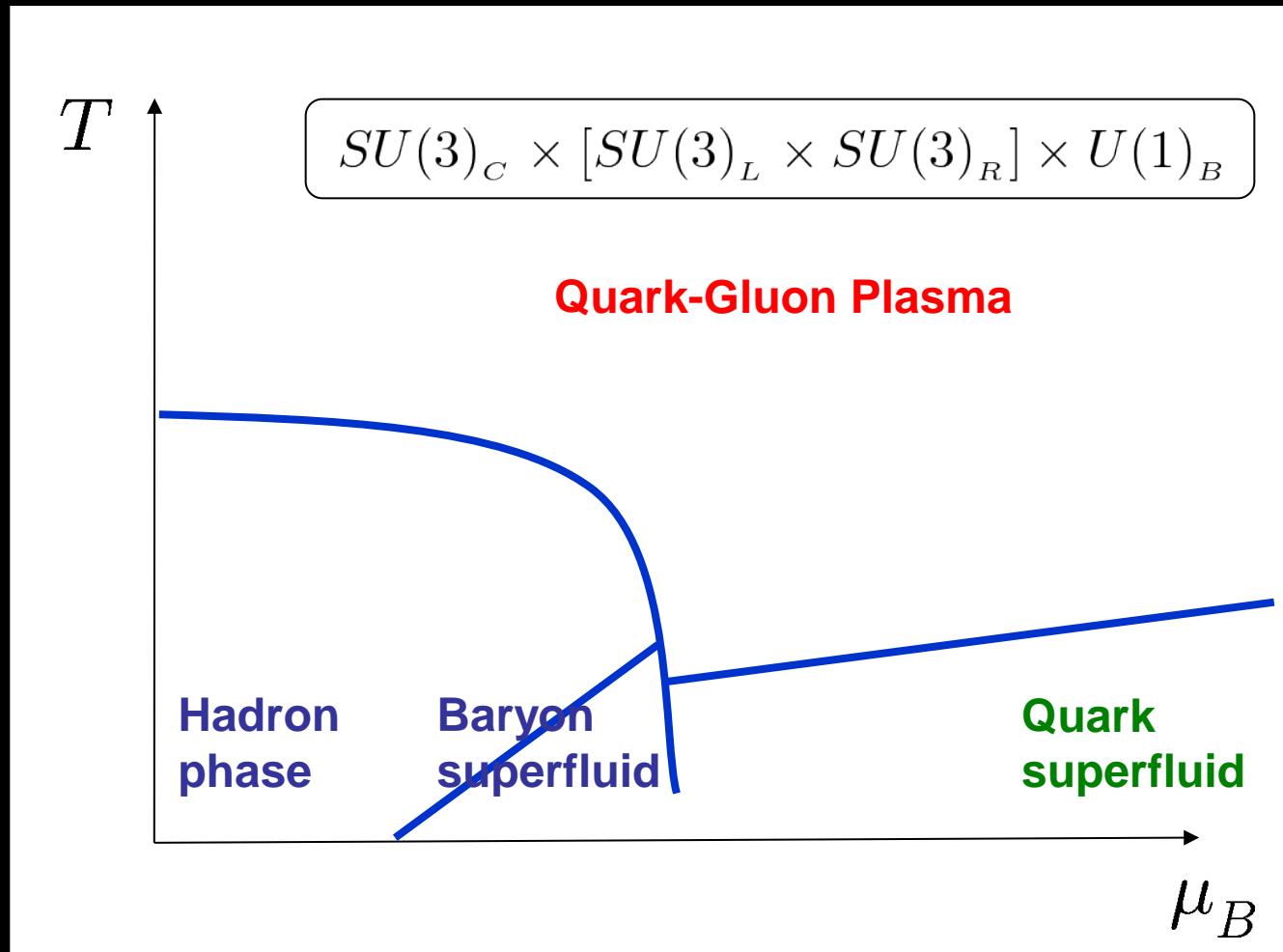
E. Fermi (1953) → G. Baym (1983)

# QCD Phase Diagram (now)

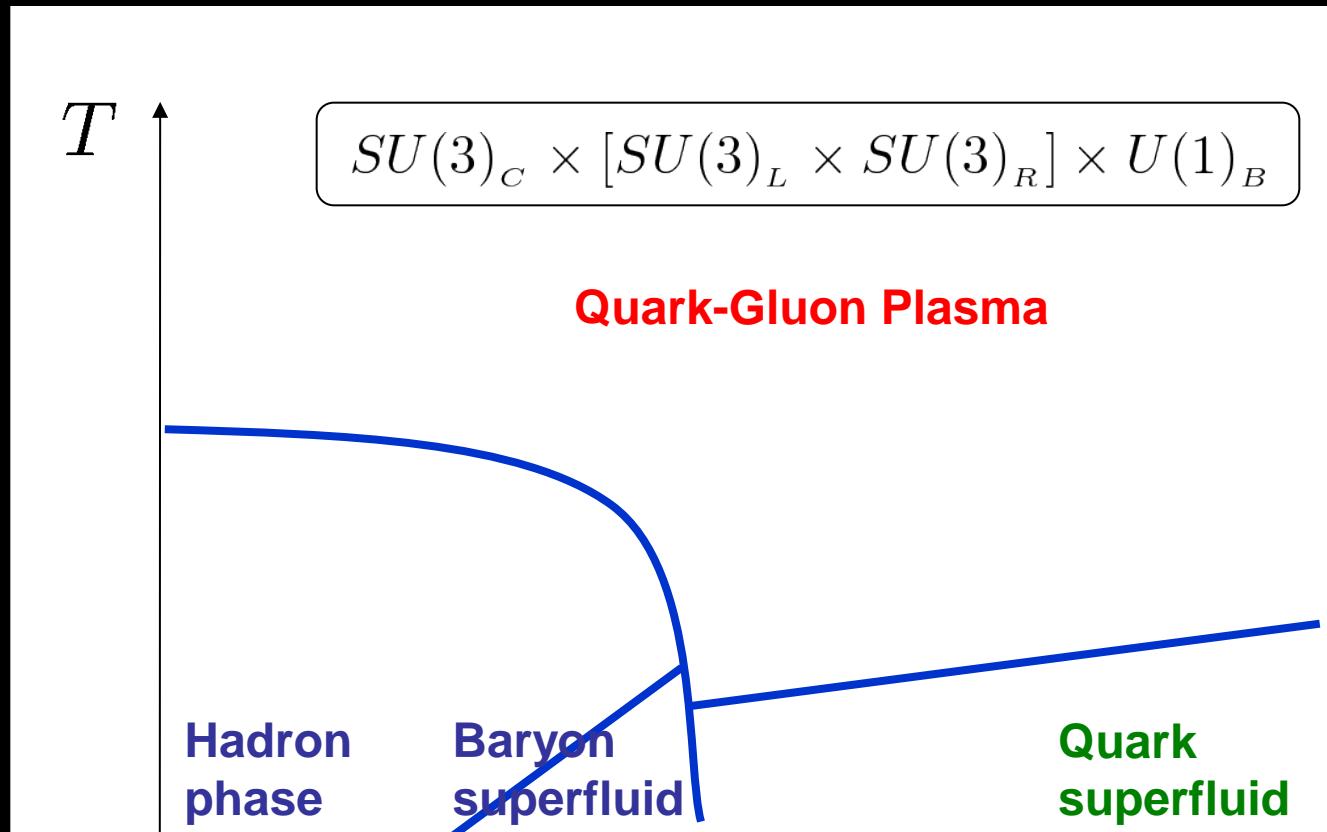


K. Fukushima and T. Hatsuda,  
“The Phase Diagram of Dense QCD”  
Rep. Prog. Phys. 74 (2011) 014001

# Symmetry Realization in dense QCD ( $N_c=3$ , $N_f=3$ )



# Symmetry Realization in dense QCD ( $N_c=3$ , $N_f=3$ )



$$\langle \bar{q}_L q_R \rangle \neq 0$$

$$SU(3)_C \times SU(3)_{L+R} \times U(1)_B$$

$$\langle BB \rangle \neq 0$$

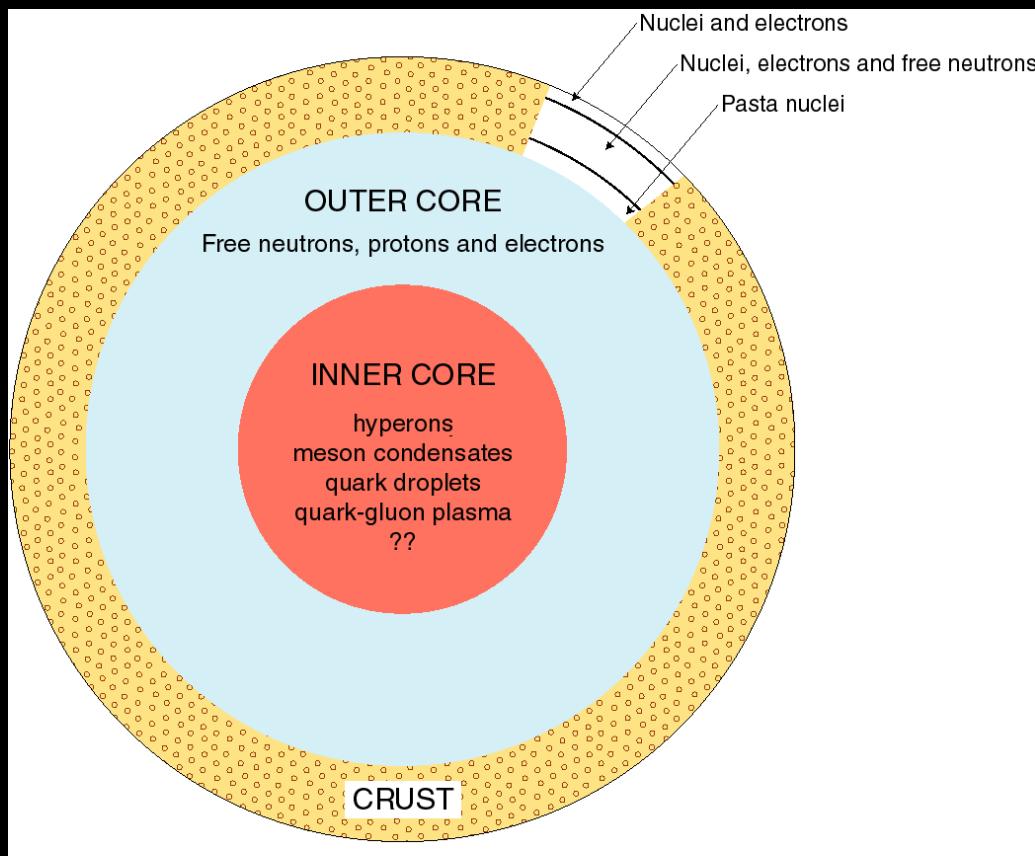
$$SU(3)_C \times SU(3)_{L+R}$$

$$\langle (q_L q_L)(\bar{q}_R \bar{q}_R) \rangle \neq 0$$

$$SU(3)_C \times SU(3)_{L+R}$$

Chiral symmetry is always broken at finite density

# Dense Matter and Neutron Star

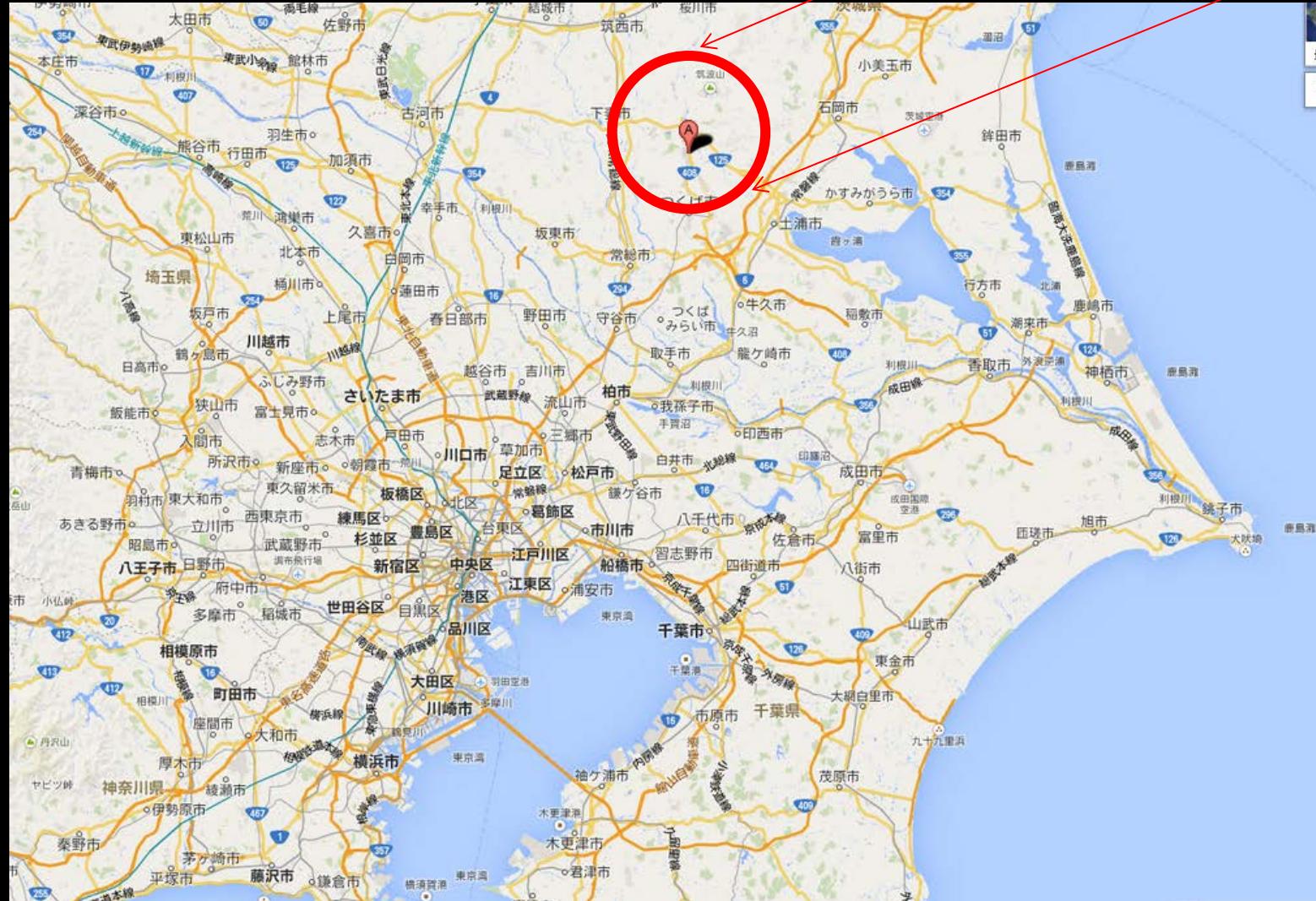
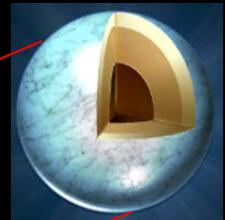


- $M \sim (1-2)M_{\odot}$
- $R \sim 10\text{ km}$
- $0 < \rho < 10 \rho_0$

## composition

- nuclei
- neutrons & protons
- mesons ( $\pi, K$ )
- hyperons ( $\Lambda, \Sigma^-, \Xi^-$ )
- quarks (u,d,s)
- + leptons (e,  $\mu$ )

# Neutron Star on top of KEK



# Basic equations for compact star

1. Tolman–Oppenheimer–Volkoff equation  $\leftarrow$  GR

(TOV)

$$\frac{d\mathcal{M}(r)}{dr} = 4\pi r^2 \varepsilon(r),$$

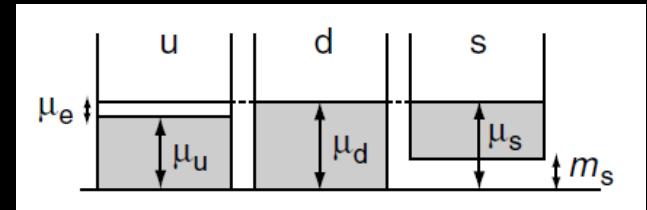
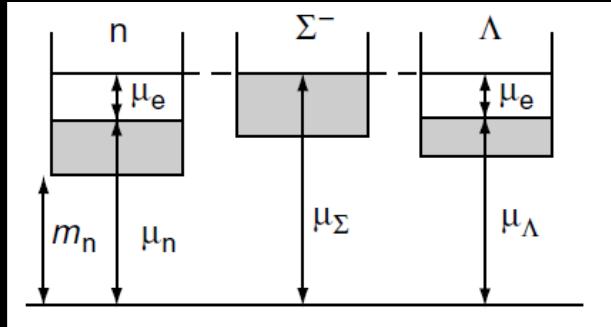
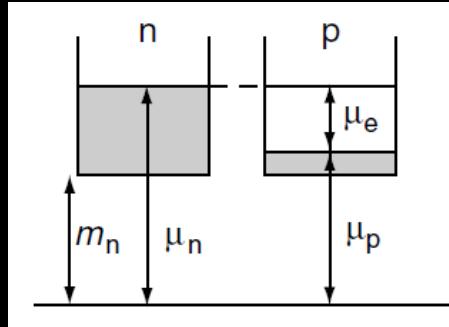
$$-\frac{dP(r)}{dr} = \frac{G\varepsilon\mathcal{M}}{r^2} \left(1 - \frac{2G\mathcal{M}}{r}\right)^{-1} \left(1 + \frac{P}{\varepsilon}\right) \left(1 + \frac{4\pi r^3 P}{\mathcal{M}}\right),$$

2. Equation of state  $P=P(\varepsilon)$   $\leftarrow$  Strong int.

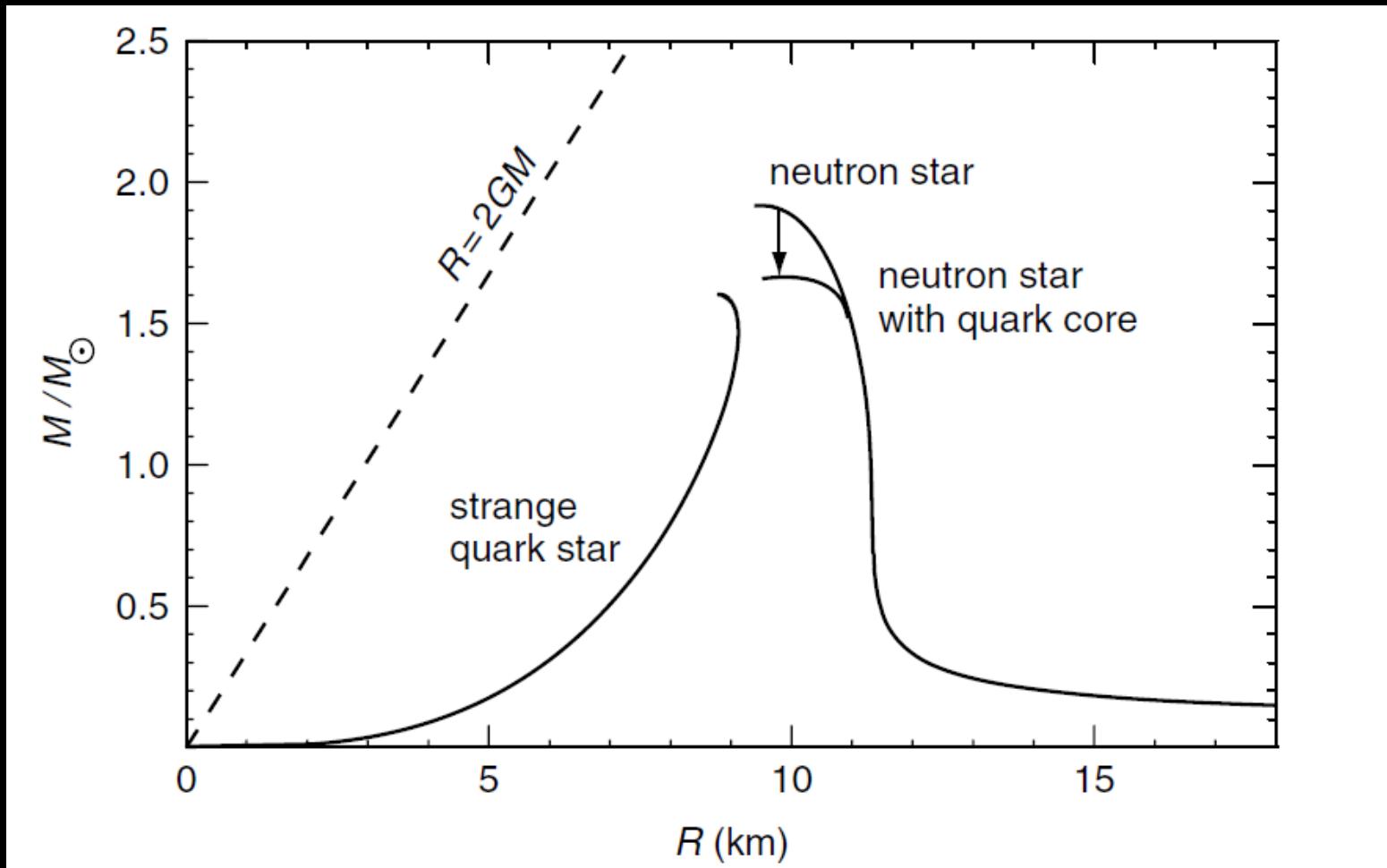
(EOS)

EM int. (charge neutrality)

Weak int. ( $\beta$  equilibrium)



## Schematic Mass-Radius relation



From Yagi, Miake and Hatsuda,  
“Quark-Gluon Plasma”, Cambridge Univ. Press (2008)

# $N_{\star}$ observations

Current:

$M = (1.97 \pm 0.04) M_{\odot}$  (Nature 2010)

$M = (2.01 \pm 0.04) M_{\odot}$  (Science 2013)

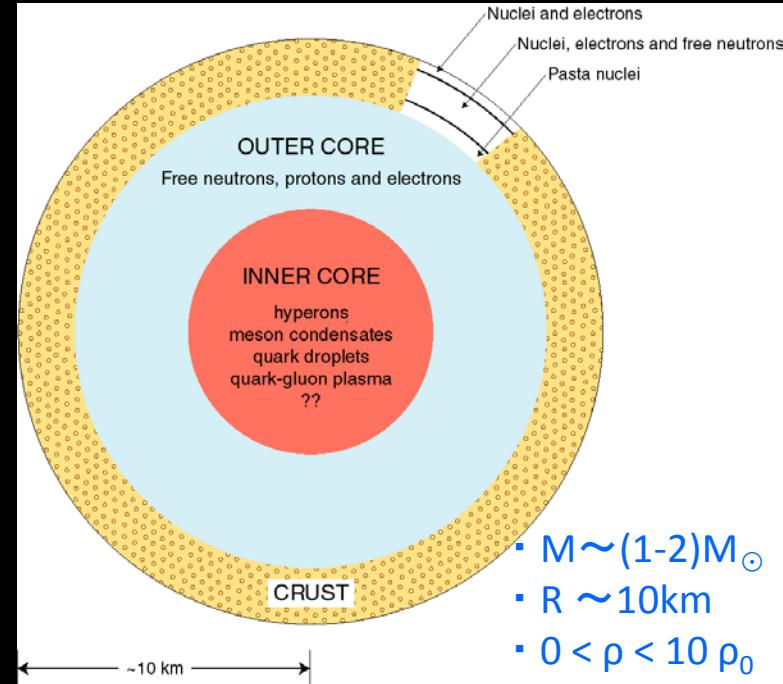
$\Leftrightarrow$  cold EOS

X-ray bursts  $\Leftrightarrow$  cold EOS

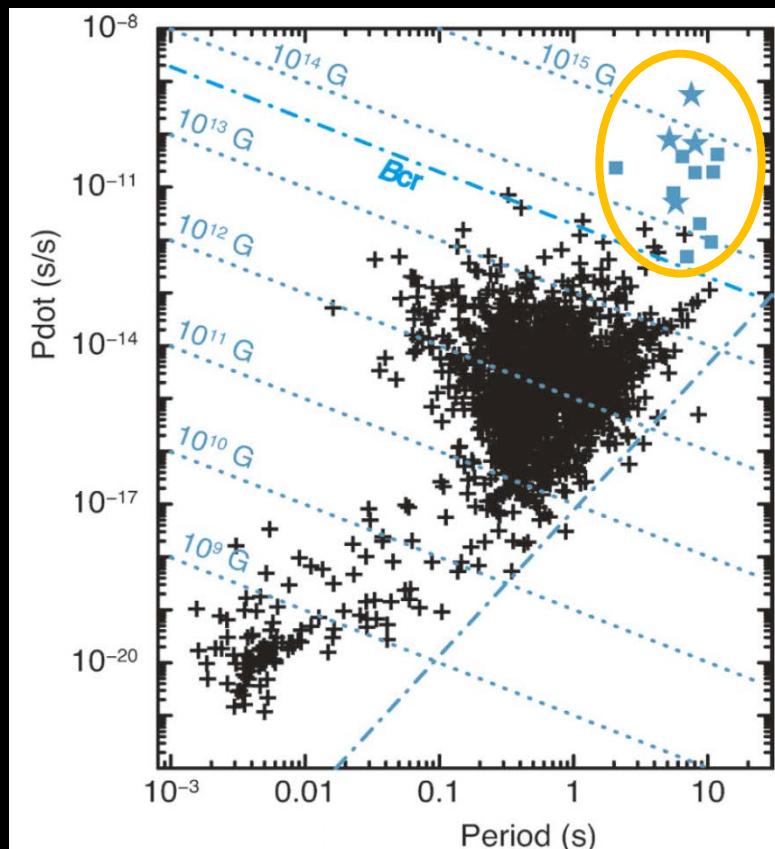
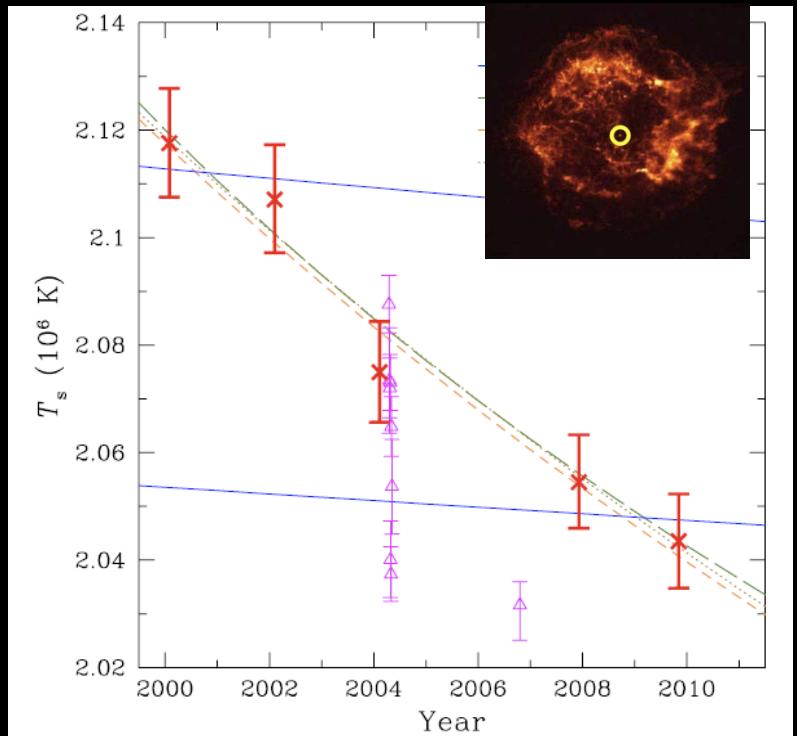
Cooling of CAS-A  $\Leftrightarrow$   ${}^3P_2$  superfluid?  
Magnetars  $\Leftrightarrow$  ferromagnetic core?

Near Future:

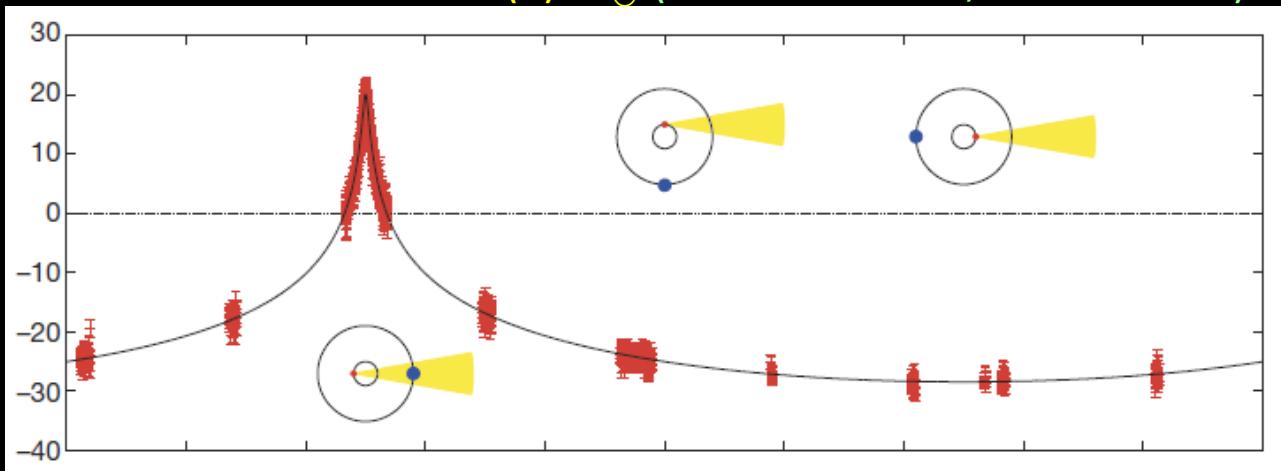
GW from  $N_{\star}$  merger  $\Leftrightarrow$  hot EOS



Cassiopeia A cooling: T decreases by 4% in 9 years  
 (Heinke & Ho, ApJ 2010)



PSR J1614-2230 :  $M=1.97(4) M_\odot$  (Demorest et al., Nature 2010)



Magnetars:  $B \sim 10^{14-15}$  G  
 (from Enoto, 2012)  
 $B_s = 3.2 \times 10^{19} \sqrt{P P_{\text{dot}}} [G]$

$T, B, M$

# Gravitational wave from $N_{\star}$ merger

## -- Detectors --

LIGO: 2015 ~

Design sensitivity: 2017 ~



KAGRA: 2018 ~

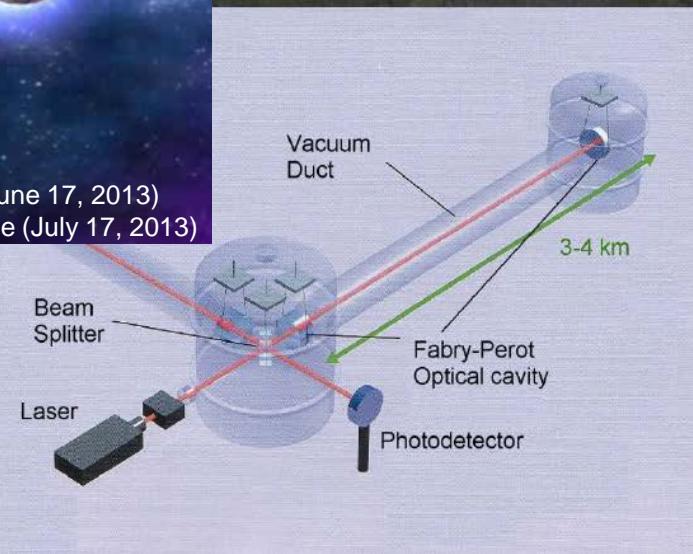
Design sensitivity ?



VIRGO: 2016 ~

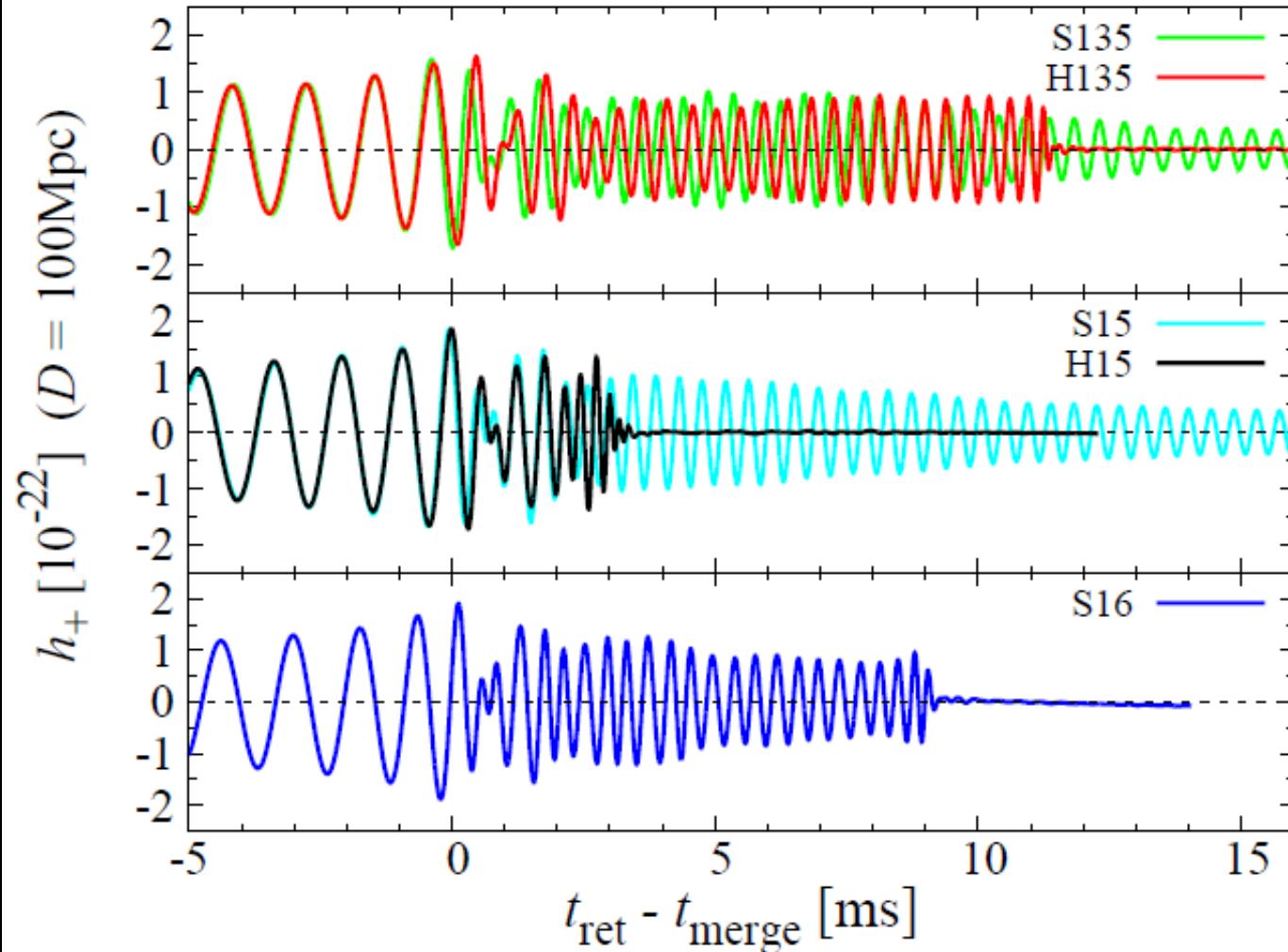
Design sensitivity: 2019 ~

Berger et al., arXiv:1306.3960[astro-ph.HE] (June 17, 2013)  
Animation in Harvard-Smithsonian press release (July 17, 2013)



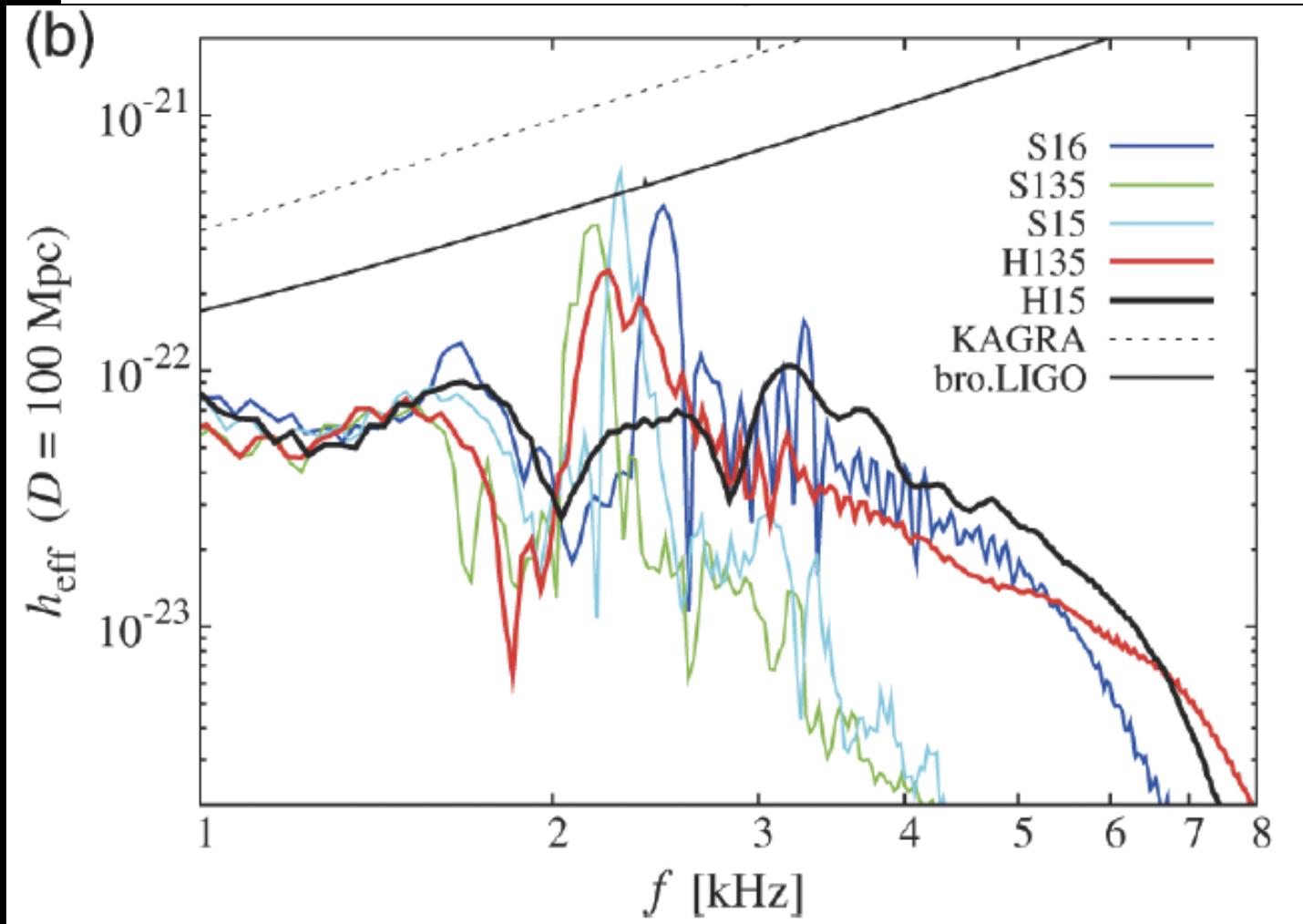
# Gravitational wave from $N_\star$ merger

-- Expected signal --



# Gravitational wave from $N_\star$ merger

## -- Expected signal --



# From QCD to Hot/Dense Matter

## Quantum Chromo Dynamics

Lattice  
gauge theory

sign  
problem

Phenomenological  
nuclear force

Equation of State for Hot Matter

Equation of State for Dense Matter

Relativistic  
hydrodynamics

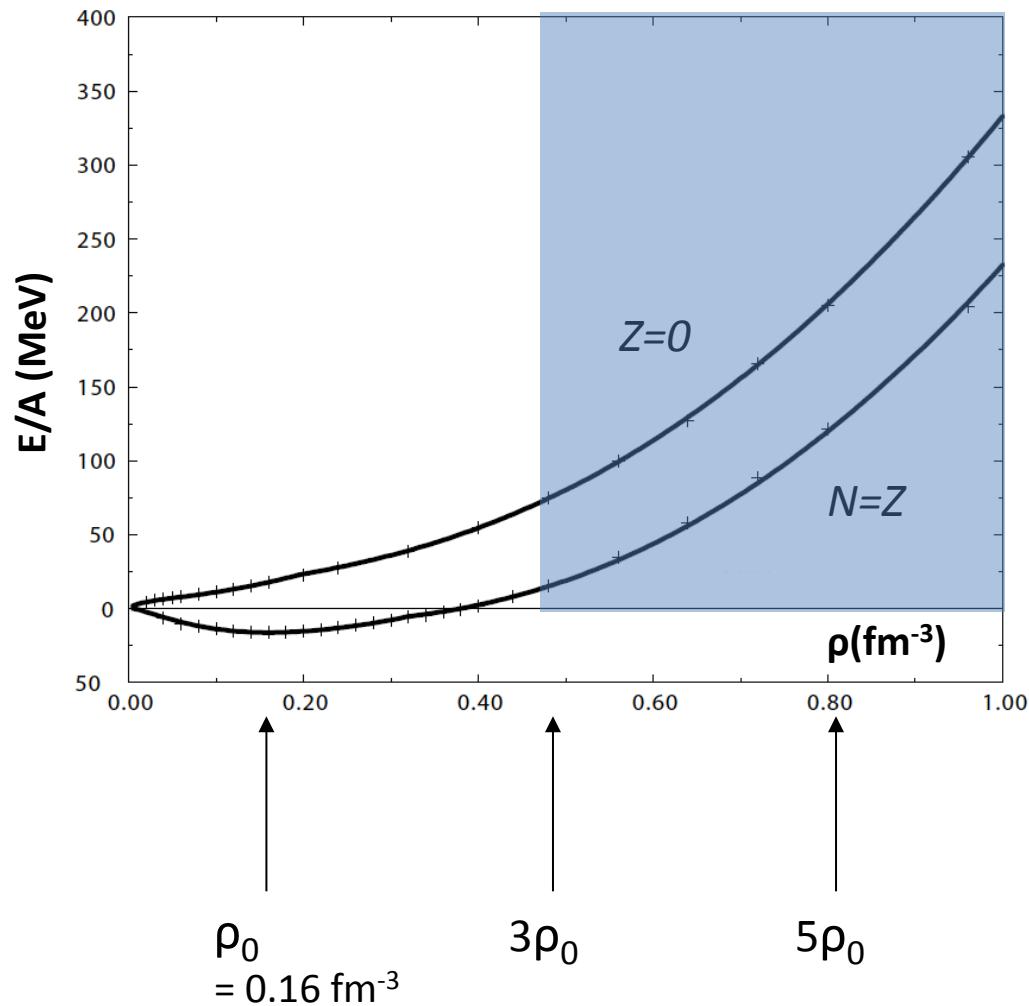
General relativity

Relativistic heavy-Ion collisions

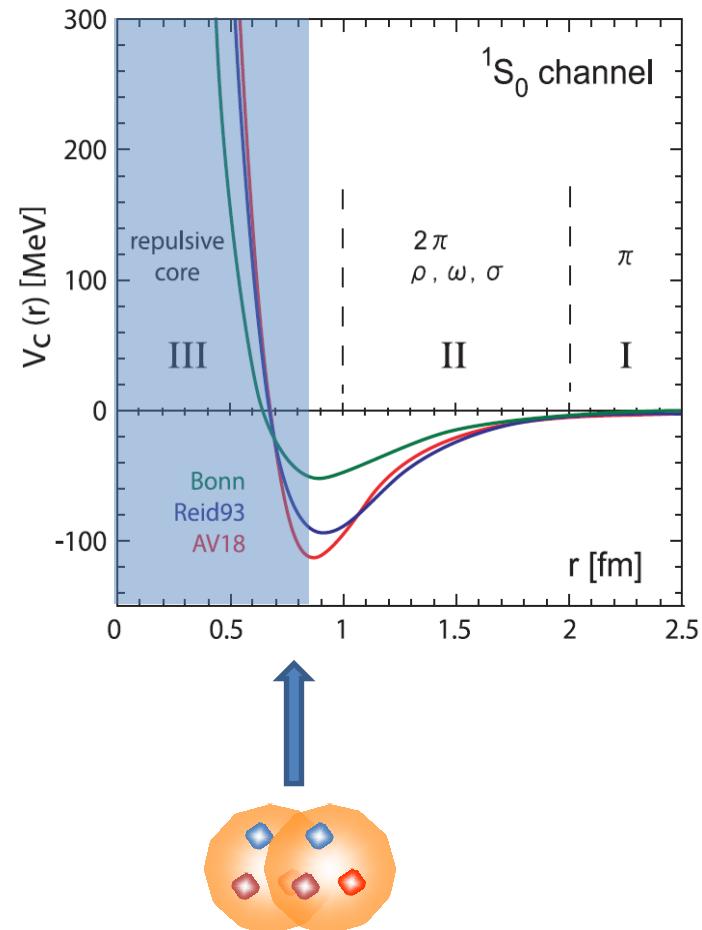
Neutron stars

# Nuclear Force and dense EOS (nucleons only)

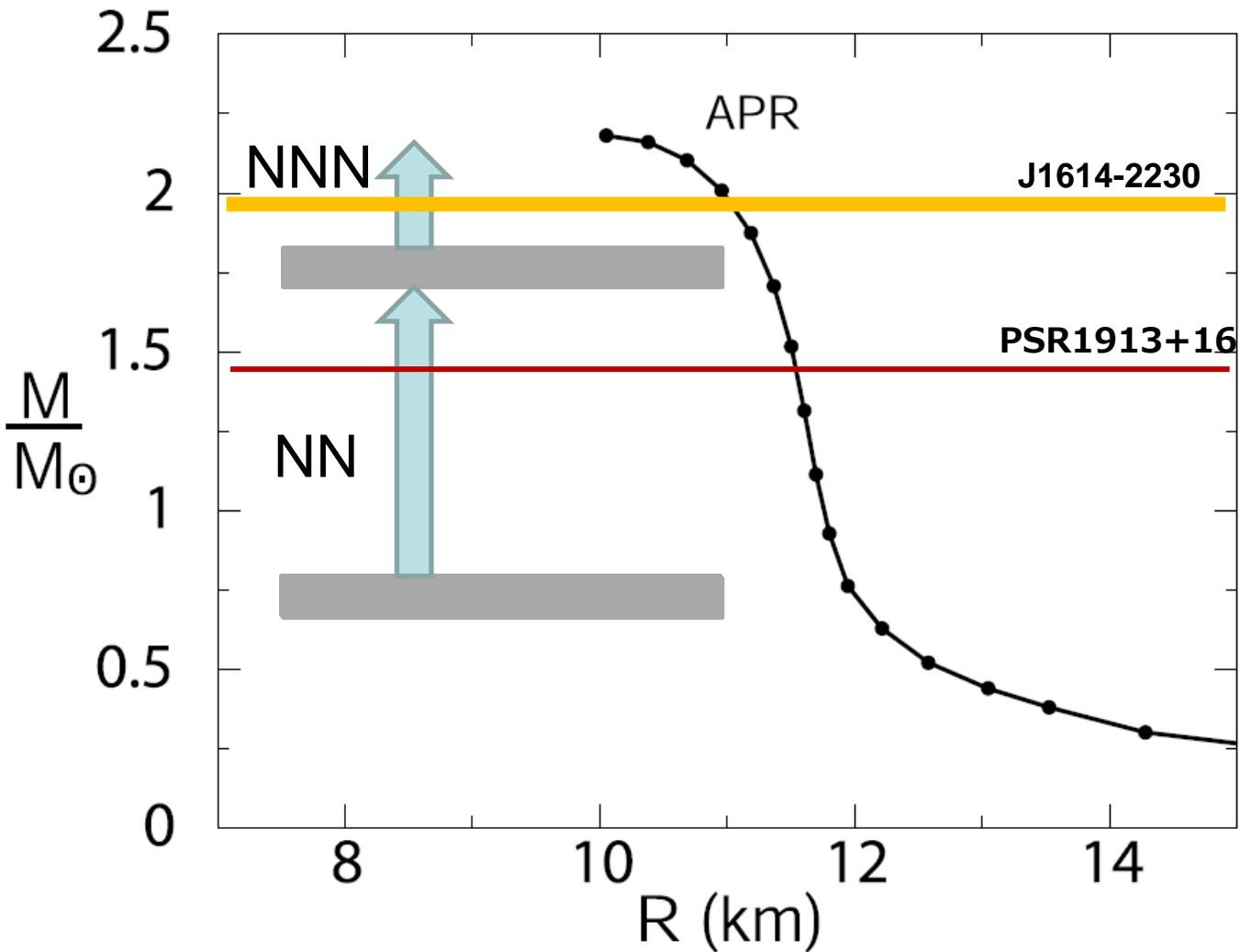
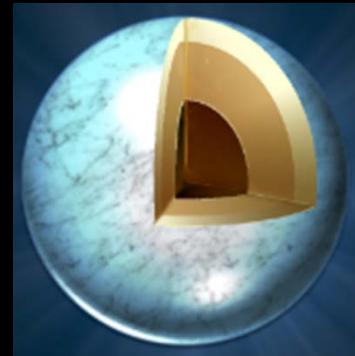
Akmal, Pandharipande & Ravenhall, PRC58 ('98)



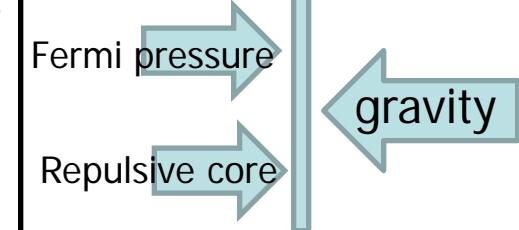
Phenomenological nuclear force



# Mass-Radius relation of $N_{\star}$ (nucleons only)



Pressure balance



# From QCD to Hot/Dense Matter

## Quantum Chromo Dynamics

Lattice  
gauge theory

sign  
problem

Equation of State for Hot Matter

Relativistic  
hydrodynamics

Relativistic heavy-Ion collisions

Equation of State for Dense Matter

General relativity

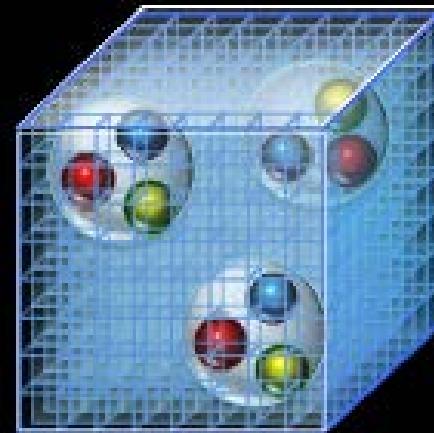
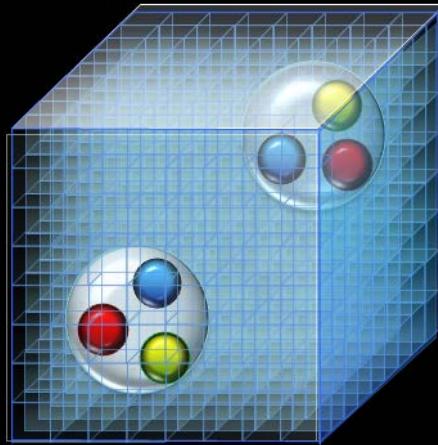
Neutron stars

Phenomenological  
nuclear force

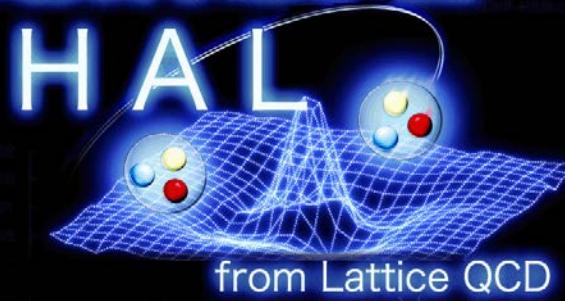
Baryon interactions

Many-body  
techniques

# *Lattice QCD and Multi-baryon*



Hadrons to Atomic nuclei



Univ. Tsukuba  
RIKEN  
Nihon Univ.  
Kyoto Univ.  
Univ. Tokyo

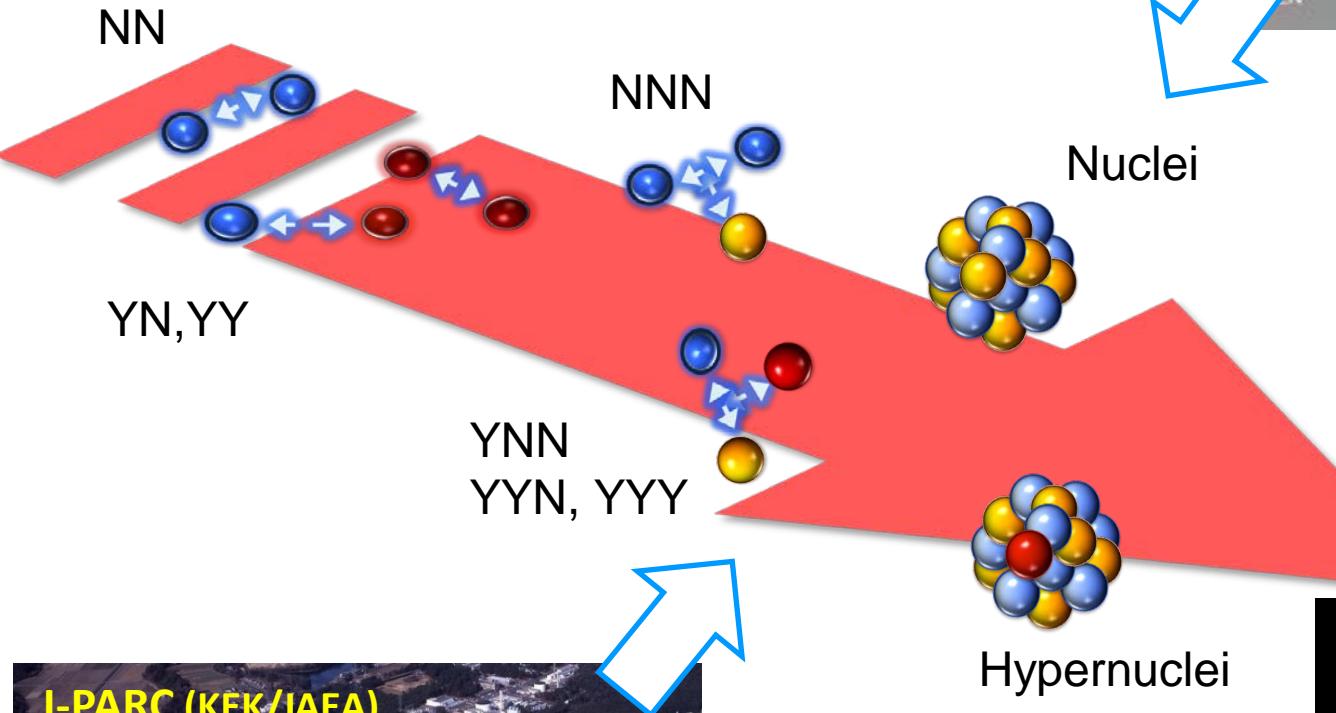
N. Ishii, H. Nemura, K. Sasaki  
T. Doi, T. Hatsuda, Y. Ikeda  
T. Inoue  
S. Aoki, K. Murano  
B. Charron

Review: “Lattice QCD Approach to Nuclear Physics”  
HAL QCD Collaboration, Prog. Theor. Exp. Phys. 2012 (2012) 01A105

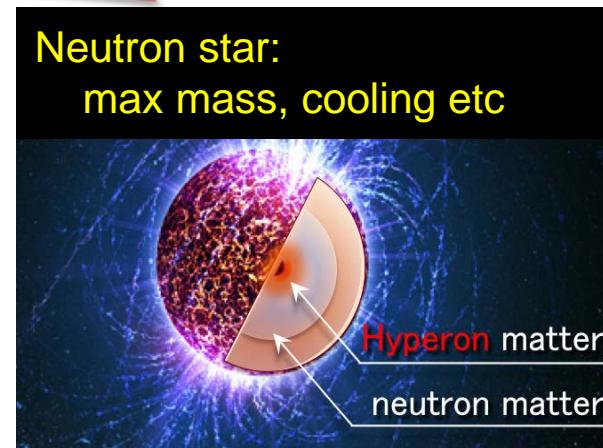
# HAL QCD Strategy : From QCD to Compact stars



BG/L  $\rightarrow \dots \rightarrow$  K computer  $\rightarrow$  post K  
(10TF  $\rightarrow \dots \rightarrow$  10PF  $\rightarrow$  1 EF)



Supernova explosion  
Neutron star merger

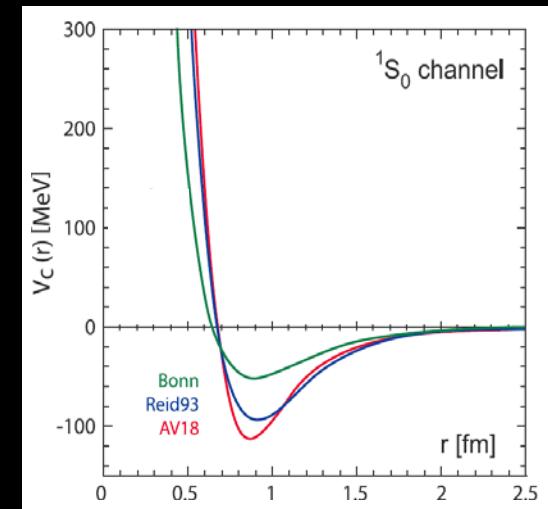


# Baryon force: From phenomenology to 1<sup>st</sup> principle

- NN int.: about 4500 np and pp scatt. data

“high precision” NN interactions	# of parameters
CD Bonn (p space)	38
AV18 (r space)	40
EFT in N <sup>3</sup> LO (n $\pi$ +contact)	24

R. Machleidt, arXiv:0704.0807 [nucl-th]

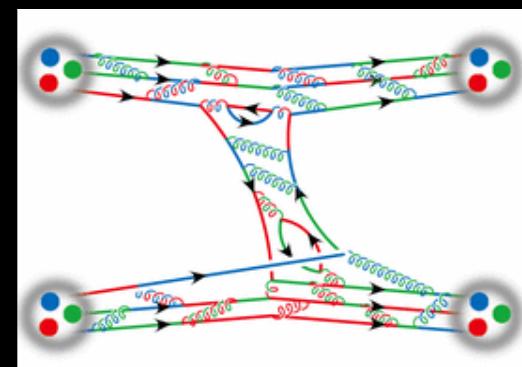


- NNN, YN, YY : data very limited
- YNN, YYN, YYY : none

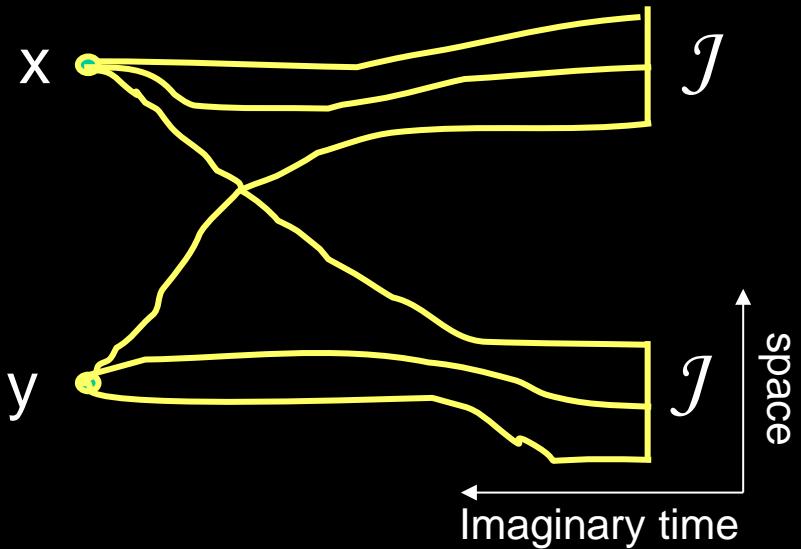


QCD has only four parameters :

$$m_u, m_d, m_s, \Lambda_{\text{QCD}}$$



# Hadronic correlations in LQCD



$$\begin{aligned} \langle N_1(\mathbf{x}, t) N_2(\mathbf{y}, t) \mathcal{J}_1^\dagger(0) \mathcal{J}_2^\dagger(0) \rangle \\ = \sum_n \langle 0 | N_1(\mathbf{x}) N_2(\mathbf{y}) | n \rangle a_n e^{-E_n t} \\ \xrightarrow{t > t^*} \phi(\mathbf{r}, t) = \sum_{n < n^*} b_n \phi_n(\mathbf{r}) e^{-E_n t} \end{aligned}$$

Finite Volume Method :

$E_0(L) \rightarrow$  phase shift

Luescher, Nucl. Phys. B354 (1991) 531

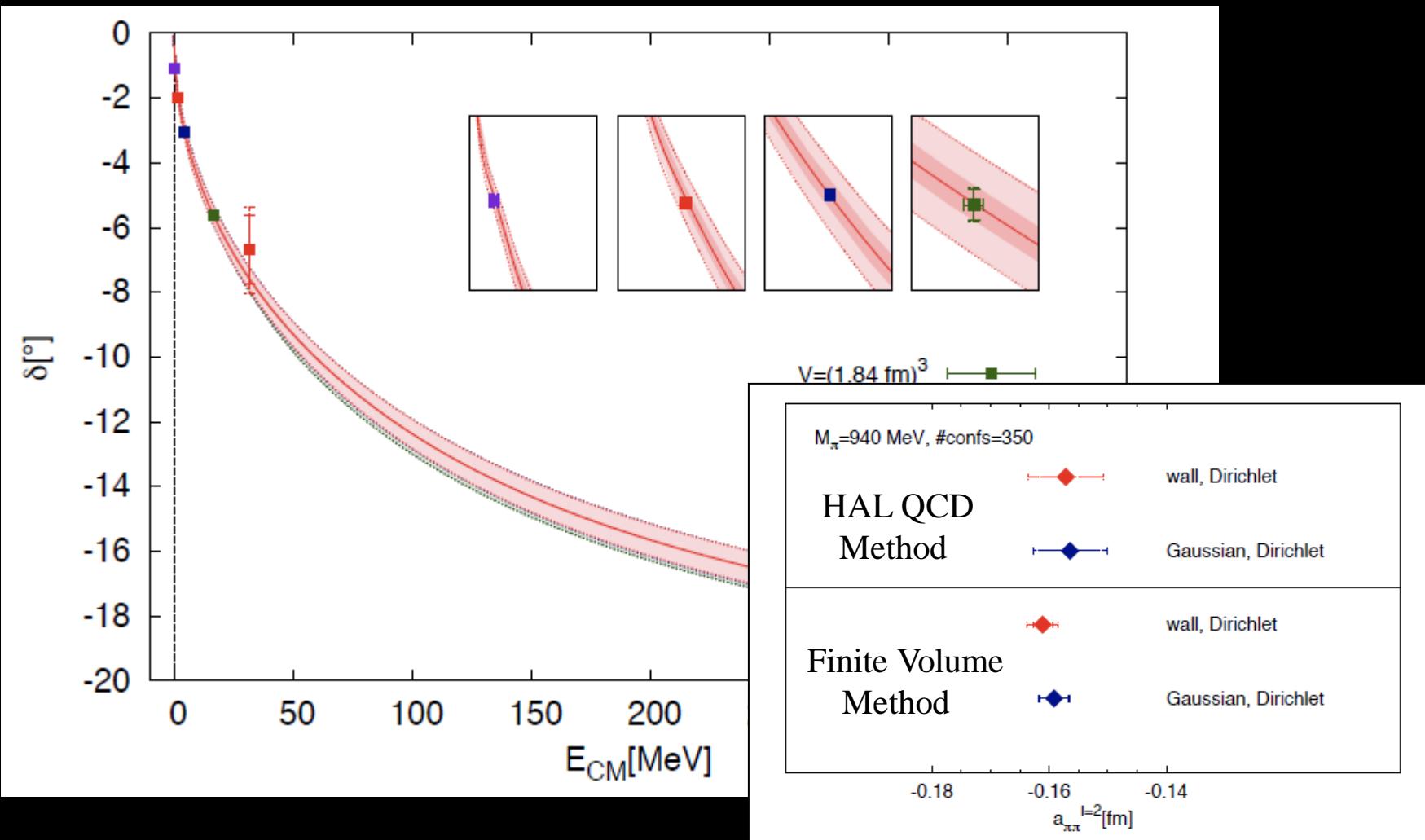
HAL QCD Method

$\phi(r,t) \rightarrow$  kernel  $\rightarrow$  phase shift

Ishii, Aoki & Hatsuda, PRL 99 (2007) 022001  
Comp. Sci. Dis. 1 (2009) 015009  
HAL QCD Coll., PLB 712 (2012) 437

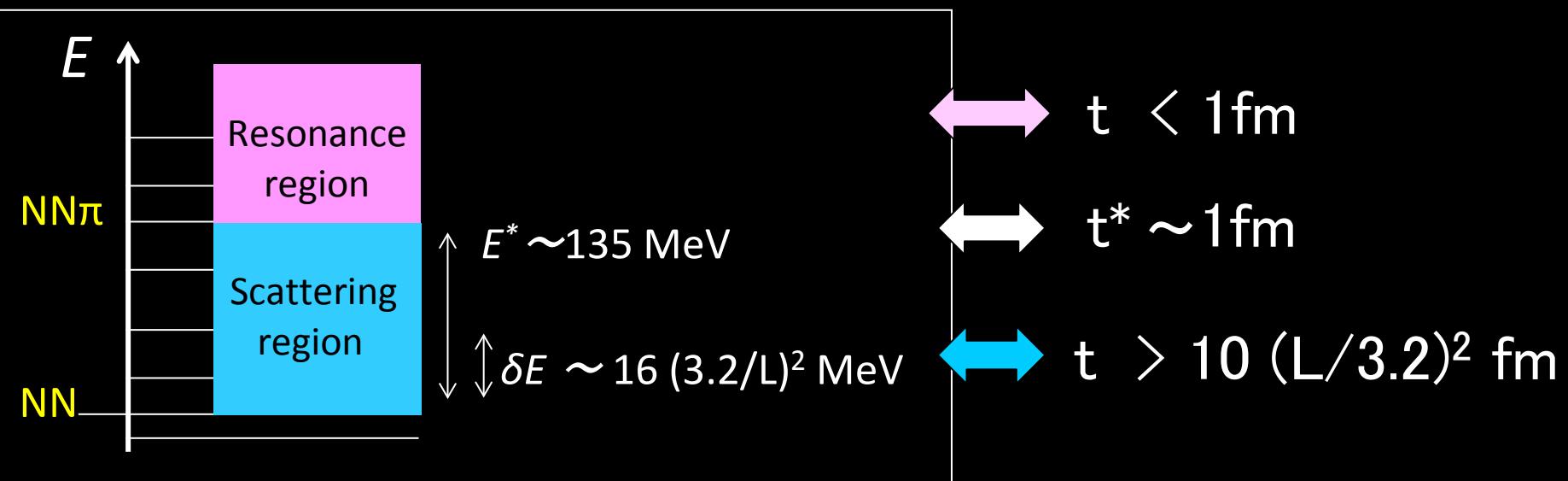
# FV Method vs. HAL QCD Method : do they agree ?

## $\pi\pi$ scattering in I=2 channel



$N_s = 16, 24, 32, 48, N_t = 128, a = 0.115 \text{ fm}$ , (quenched QCD,  $m_\pi = 940 \text{ MeV}$ )  
Kurth, Ishii, Doi, Aoki & Hatsuda, arXiv: JHEP 1312 (2013) 015

# What about NN ?

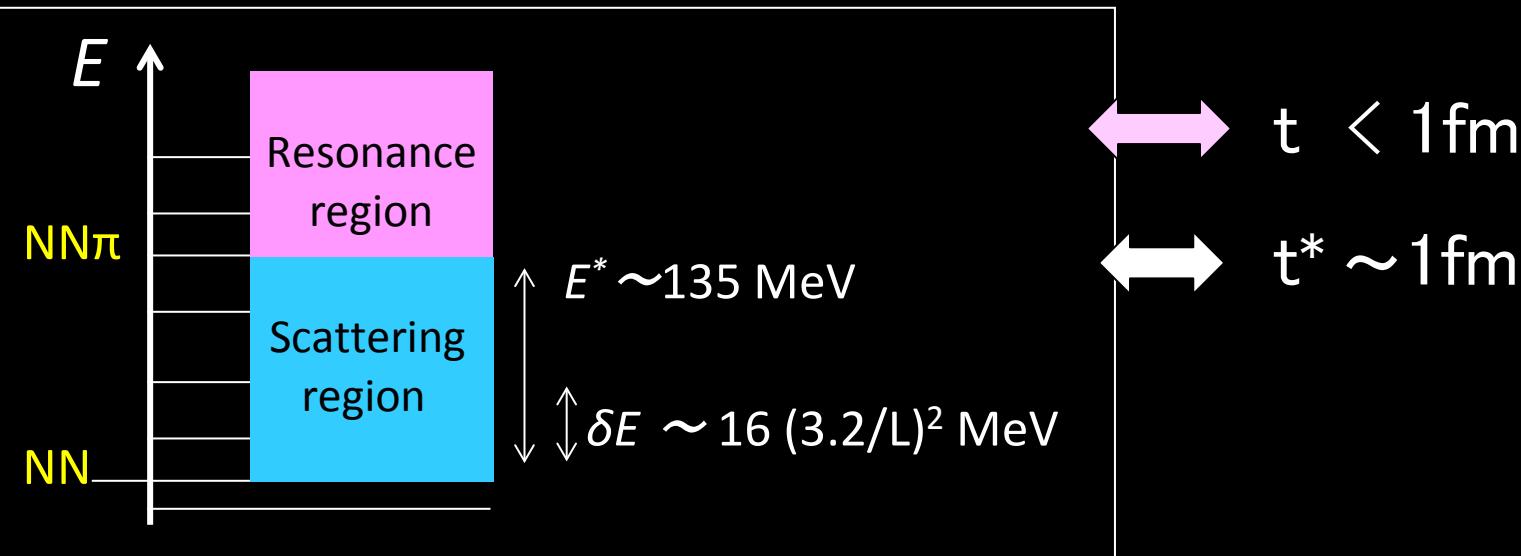


$$\left(\frac{\mathcal{S}}{\mathcal{N}}\right)_{NN} \sim \sqrt{N_{\text{gc}}} e^{-2(m_N - 3m_\pi/2)t}$$

**Finite Volume Method**  
guaranteed to fail  
for NN system  
with small  $m_\pi$  and large L

$$\left(\frac{\mathcal{S}}{\mathcal{N}}\right)_{\pi\pi} \sim \sqrt{N_{\text{gc}}}$$

# What about NN (cont'd) ?



$$\phi(\mathbf{r}, t > t^*) = \sum_{n < n^*} b_n \phi_n(\mathbf{r}) e^{-E_n t} \equiv \varphi(\mathbf{r}, t) e^{-2m_N t}$$

$$\left[ \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{m_N} \right] \varphi(\mathbf{r}, t) = \int U(\mathbf{r}, \mathbf{r}') \varphi(\mathbf{r}') d^3 r'$$

$t > t^*$  is only necessary  
**Non-local kernel  $U(\mathbf{r}, \mathbf{r}')$**

- energy independent
- $L$ -insensitive
- consistent 3-body force
- not an observable

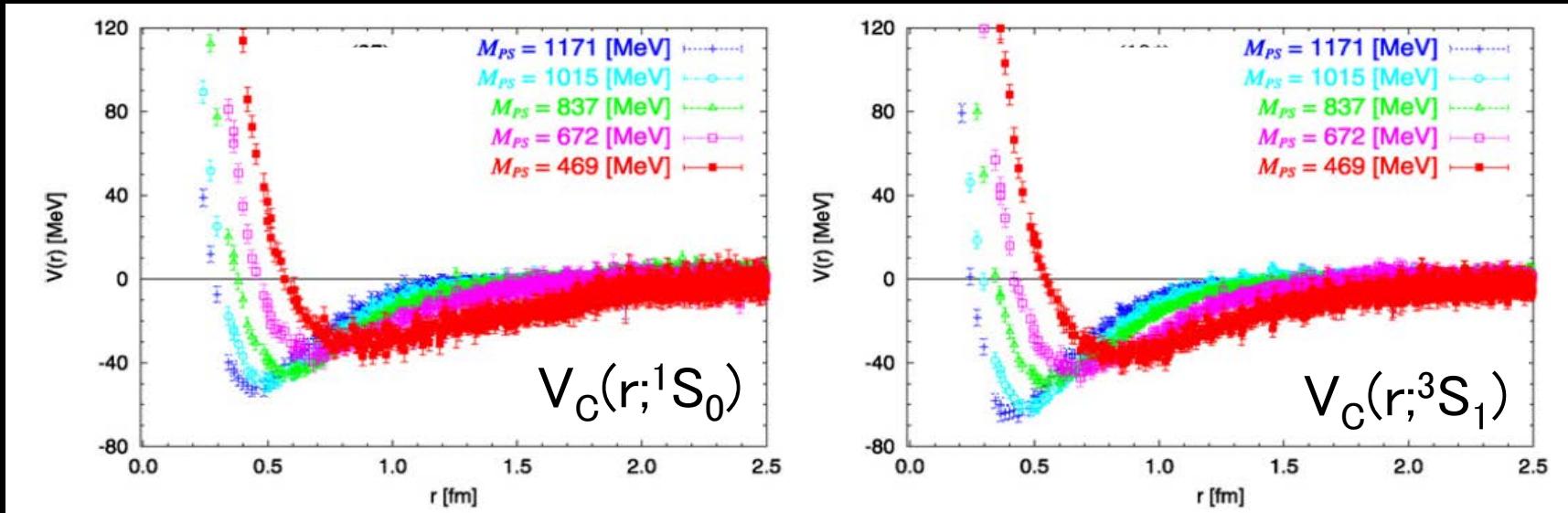
$$V(\vec{r}, \nabla) = V_C(r) + S_{12}V_T(r) + \vec{L} \cdot \vec{S} V_{LS}(r) + \{V_D(r), \nabla^2\} + \dots$$

LO

LO

NLO

NNLO



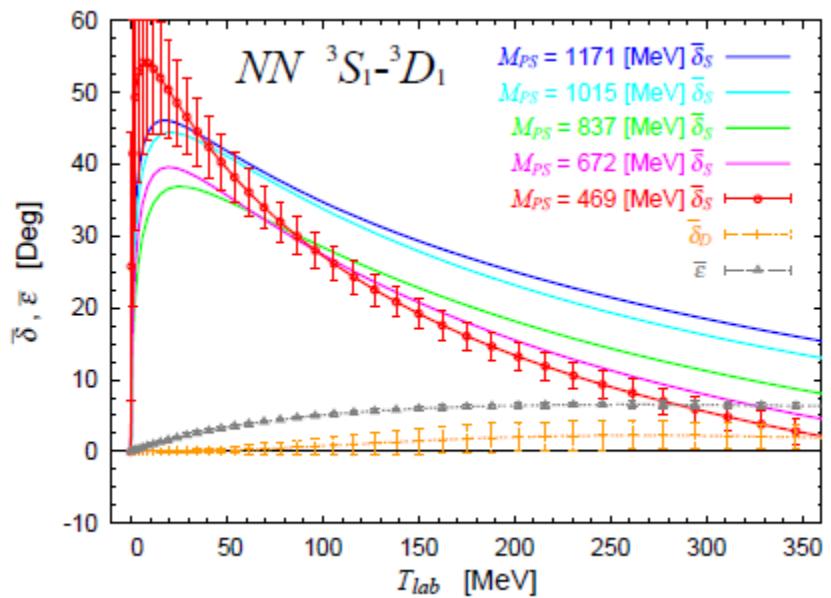
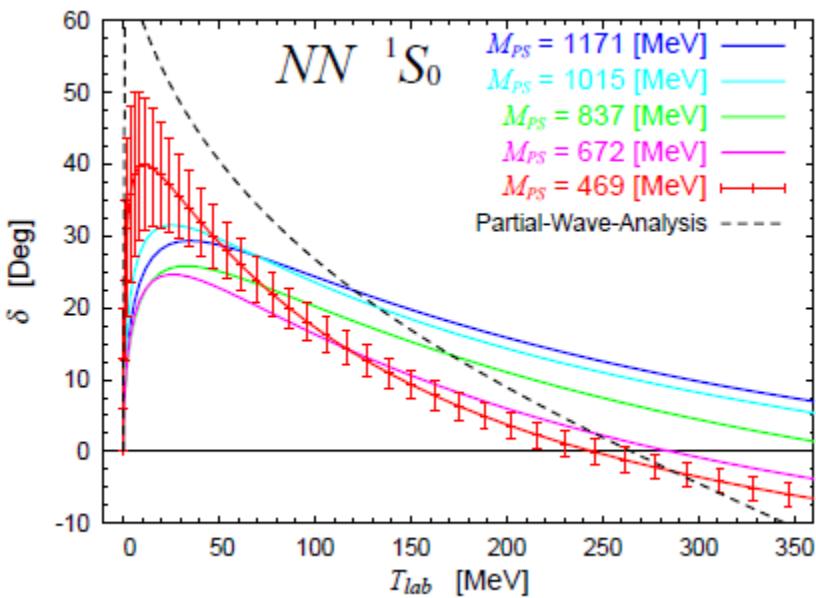
## NN Central & Tensor Forces in 3-flavor QCD

HAL QCD Coll.

Phys. Rev. Lett. 106 (2011) 162002

Nucl. Phys. A881 (2012) 28

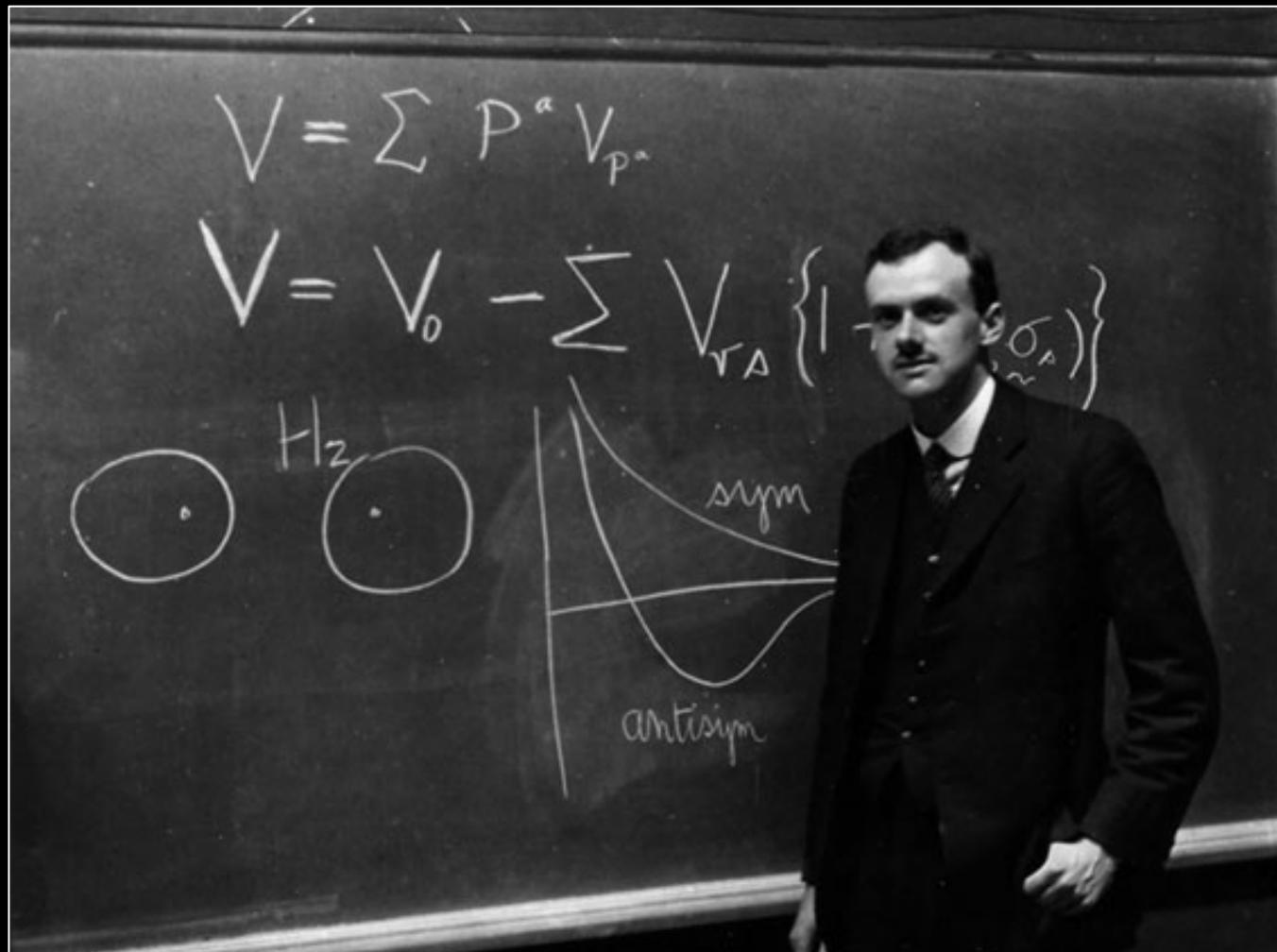
# NN phase shifts in 3-flavor QCD



Stronger attraction in the deuteron channel

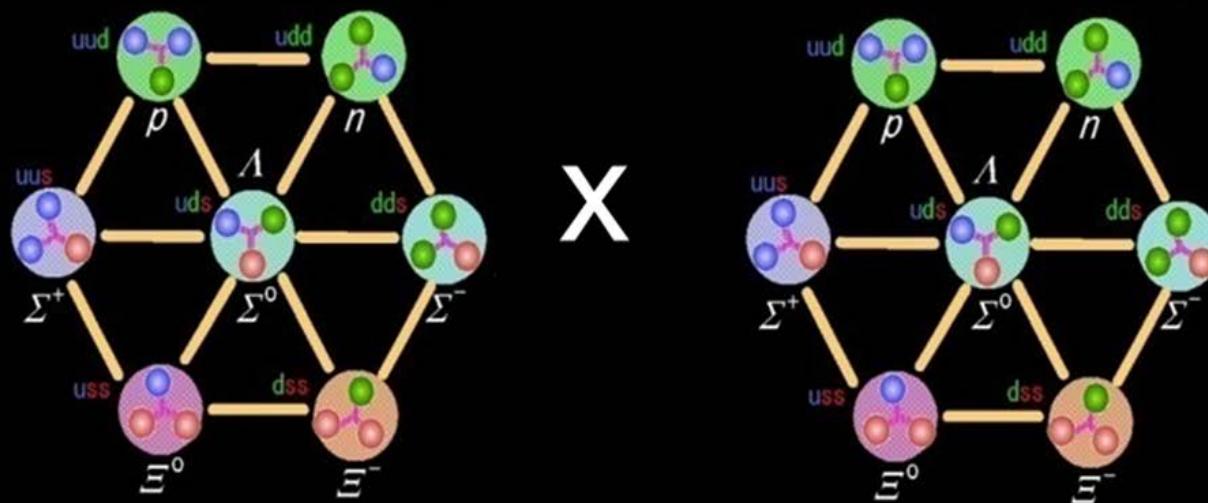
HAL QCD Coll.,  
Phys. Rev. Lett. 106 (2011) 162002  
Nucl. Phys. A881 (2012) 28

# Symmetry Matters



Heisenberg (1926), Dirac (1926)  
Heitler-London (1927)

## ⇒ Baryon-baryon force in flavor SU(3)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric                    Anti-symmetric

Six independent potentials in the flavor-basis

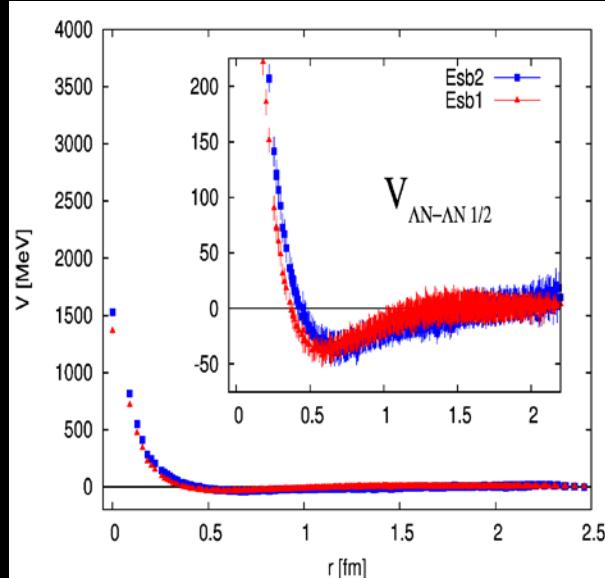
# SU(3) breaking: coupled channel LQCD

Sasaki et al.  
[HAL QCD Coll.] (2012)

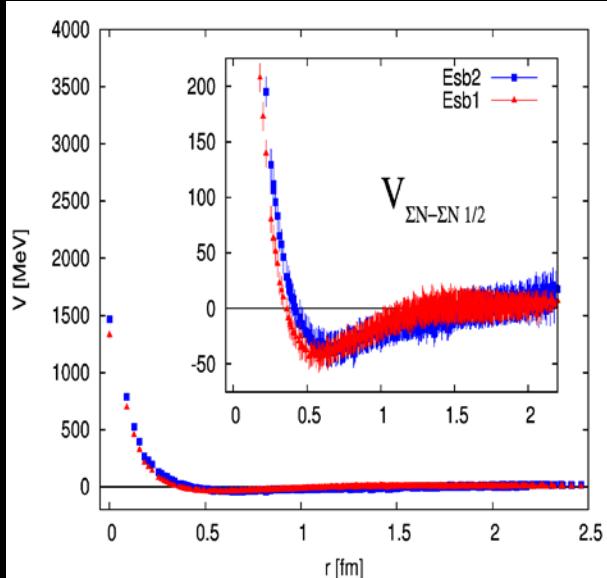
$$(k_n^2 + \nabla^2) \phi_n^\alpha(\vec{r}, t) = \int U(\vec{r}, \vec{r}')^{\alpha\beta} \phi_n^\beta(\vec{r}', t) d^3 r'$$

Example:  $S=-1, {}^3S_1, l=1/2$  ( $m_\pi/m_K=0.89, 0.8$ )

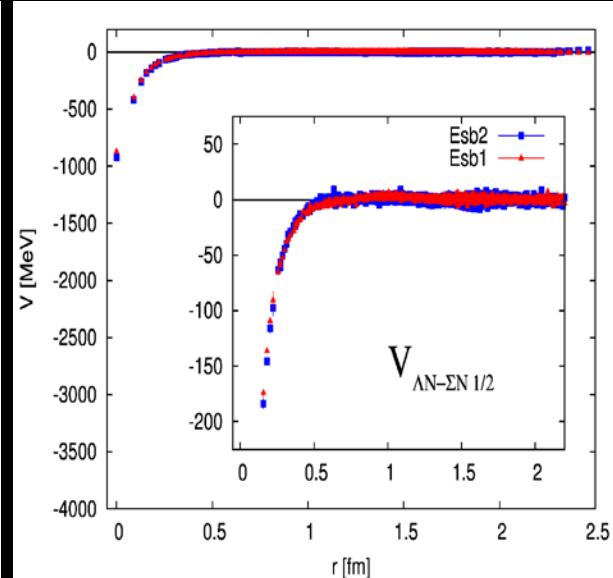
$\Lambda N - \Lambda N$



$\Sigma N - \Sigma N$



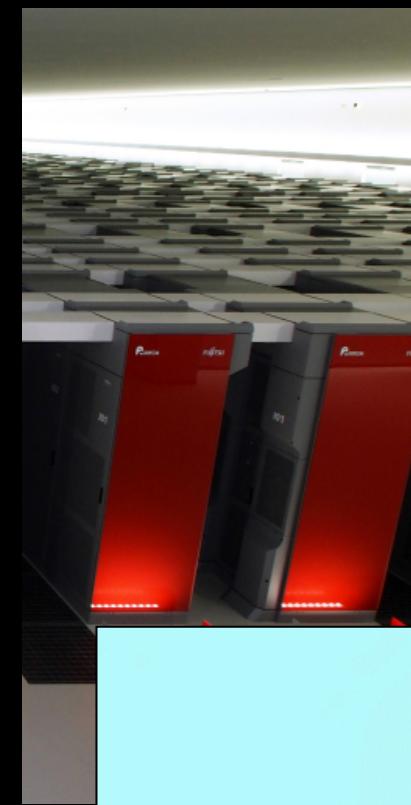
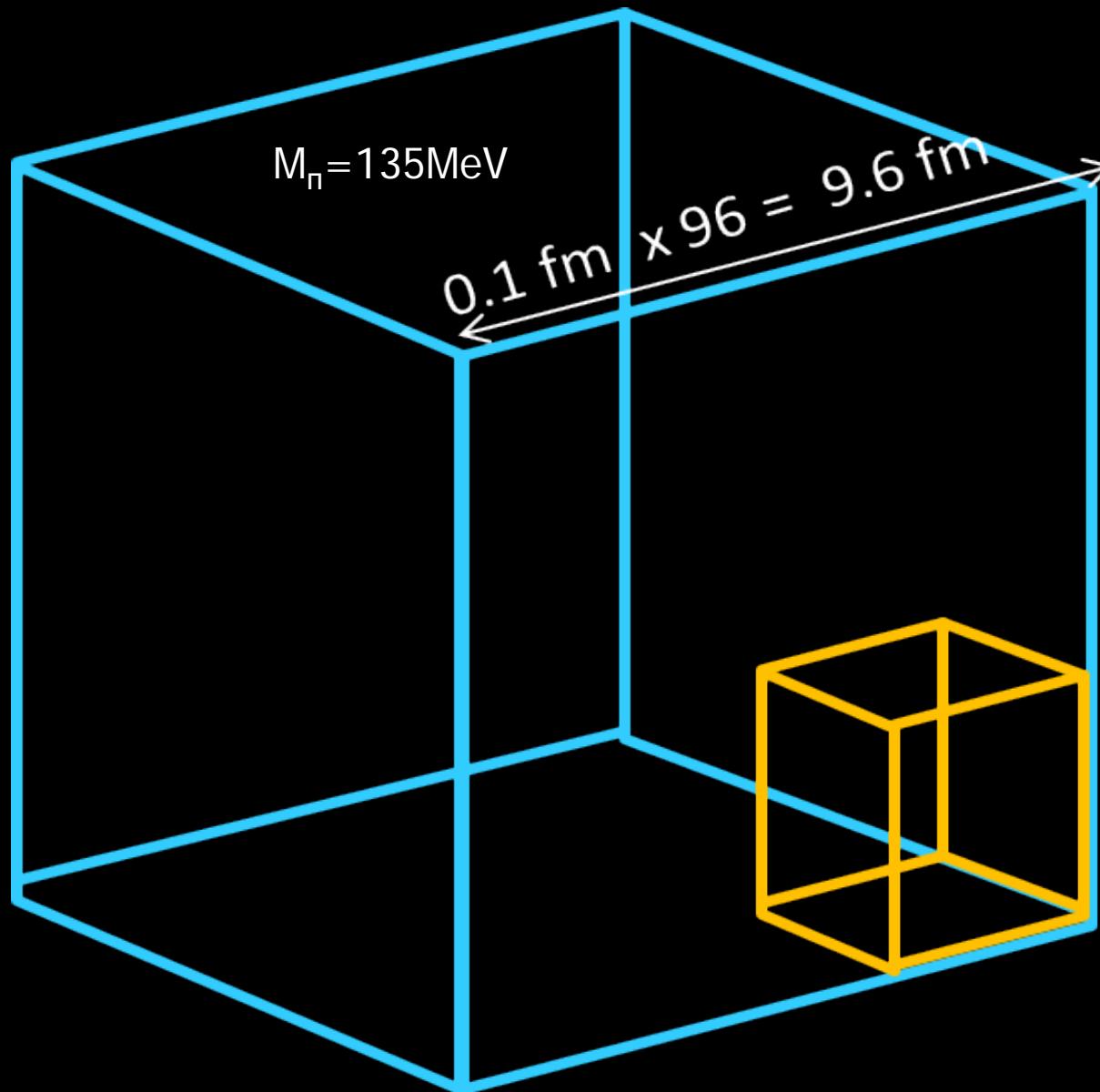
$\Lambda N - \Sigma N$



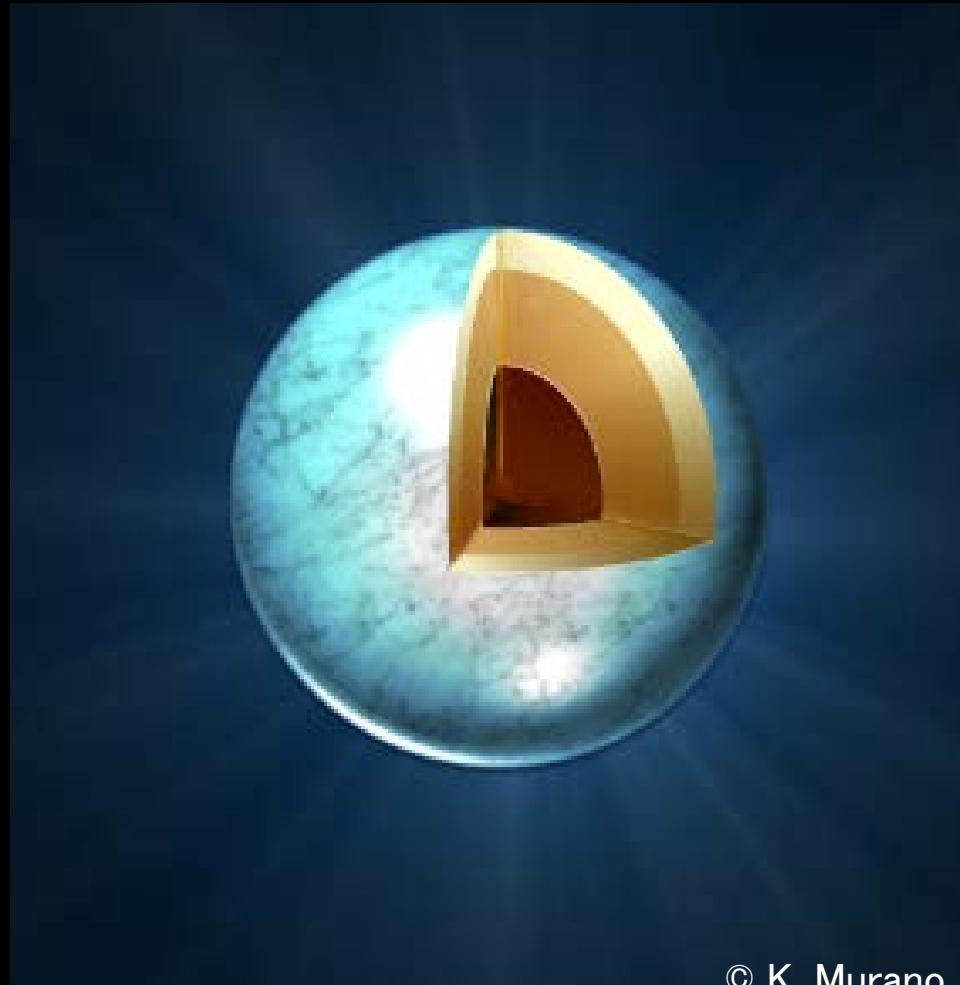
PACS-CS (2+1)-flavor config.  
 $L=2.9$  fm

# K computer @ RIKEN

(11.28 PFlops, 80,000 CPUs x 8 = 640,000 cores)



# *From Quarks to Cosmos*



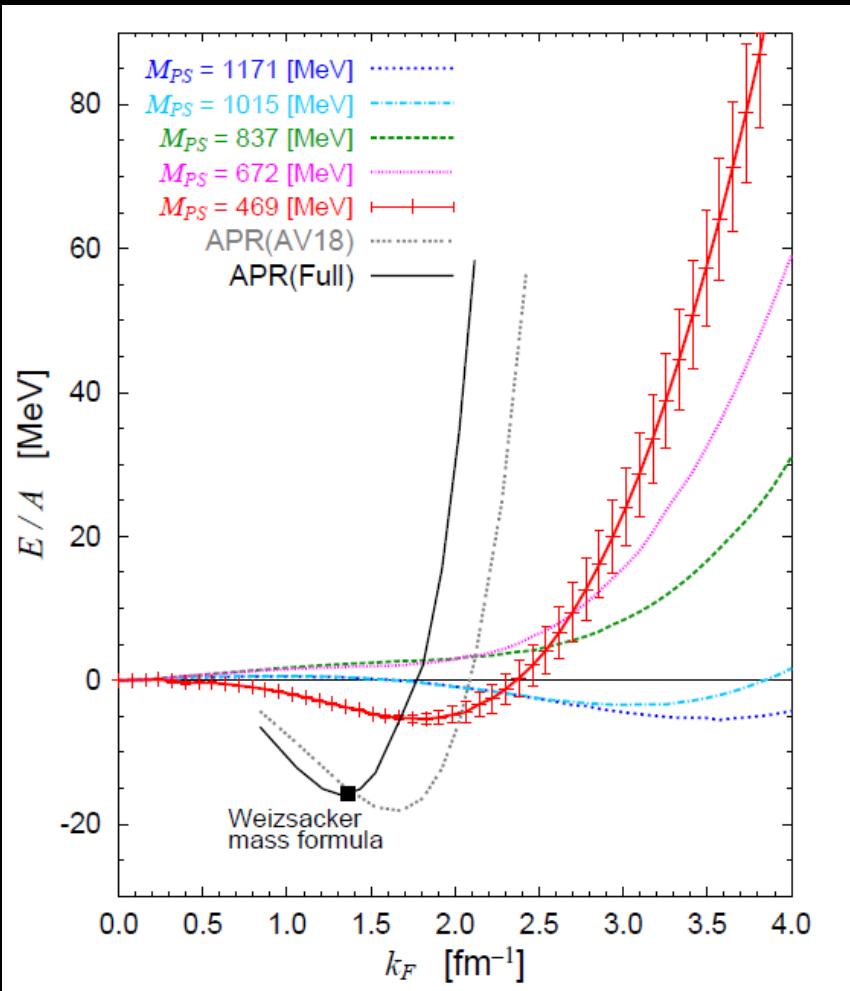
© K. Murano

# Nuclear EOS from Lattice NN force + BHF calculation

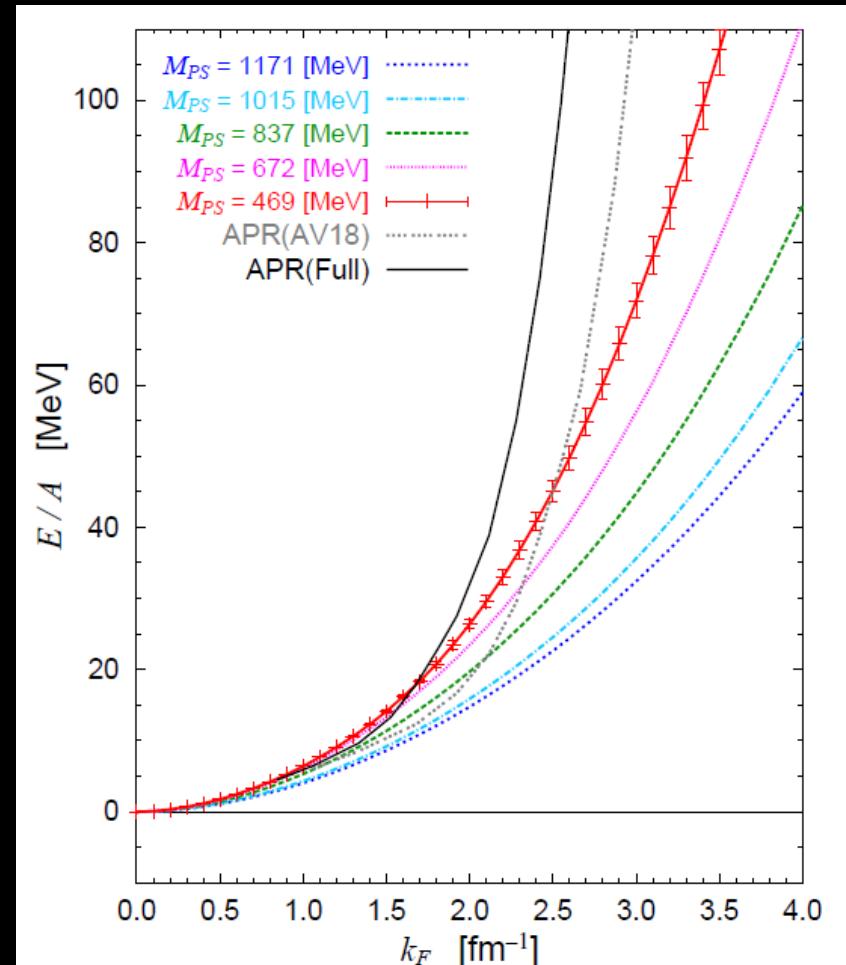
(NN force:  $^1S_0$ ,  $^3S_1$ ,  $^3D_1$  channels only)

HAL QCD Coll., Phys. Rev. Lett. 111 (2013) 112503

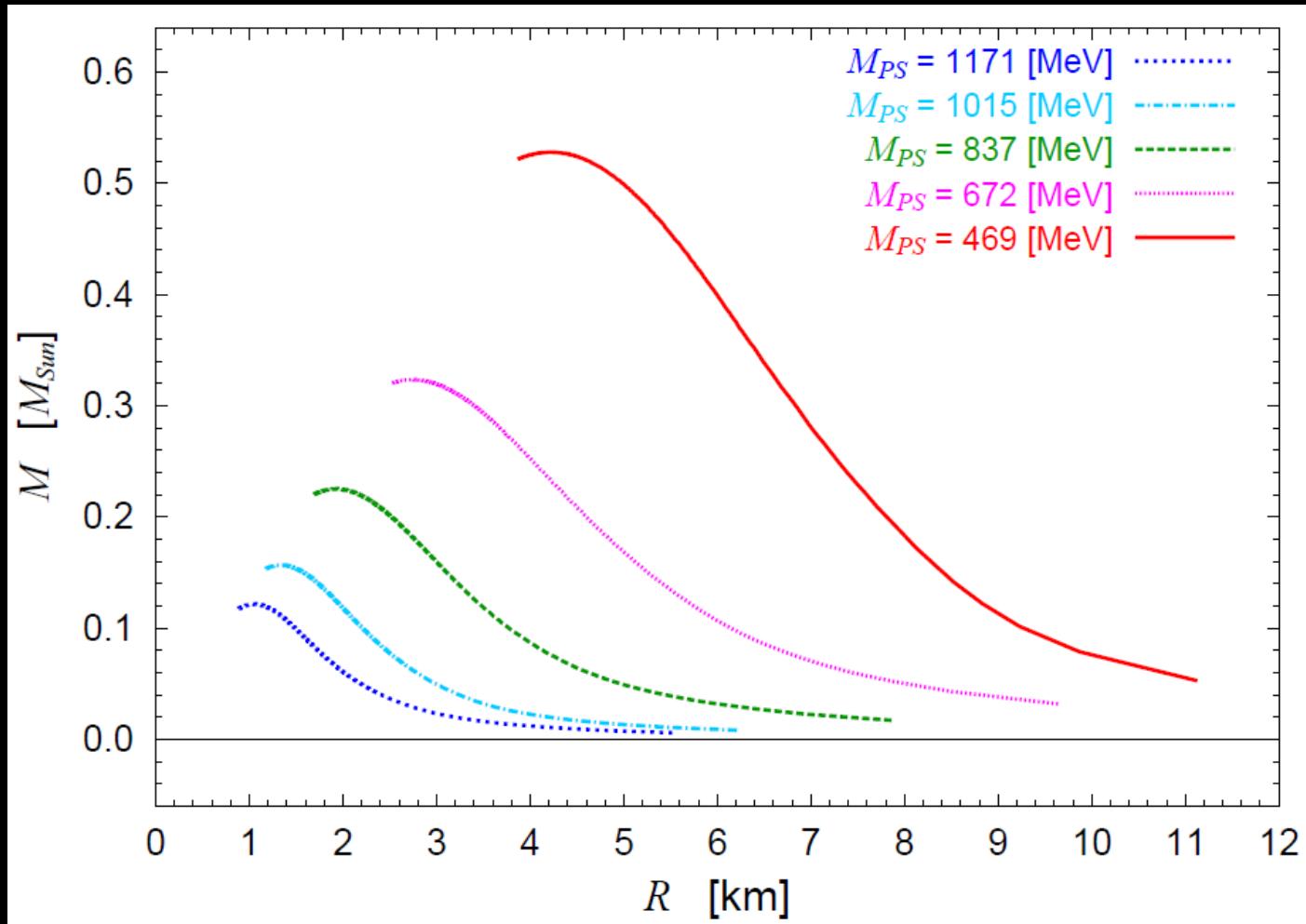
## Nuclear Matter



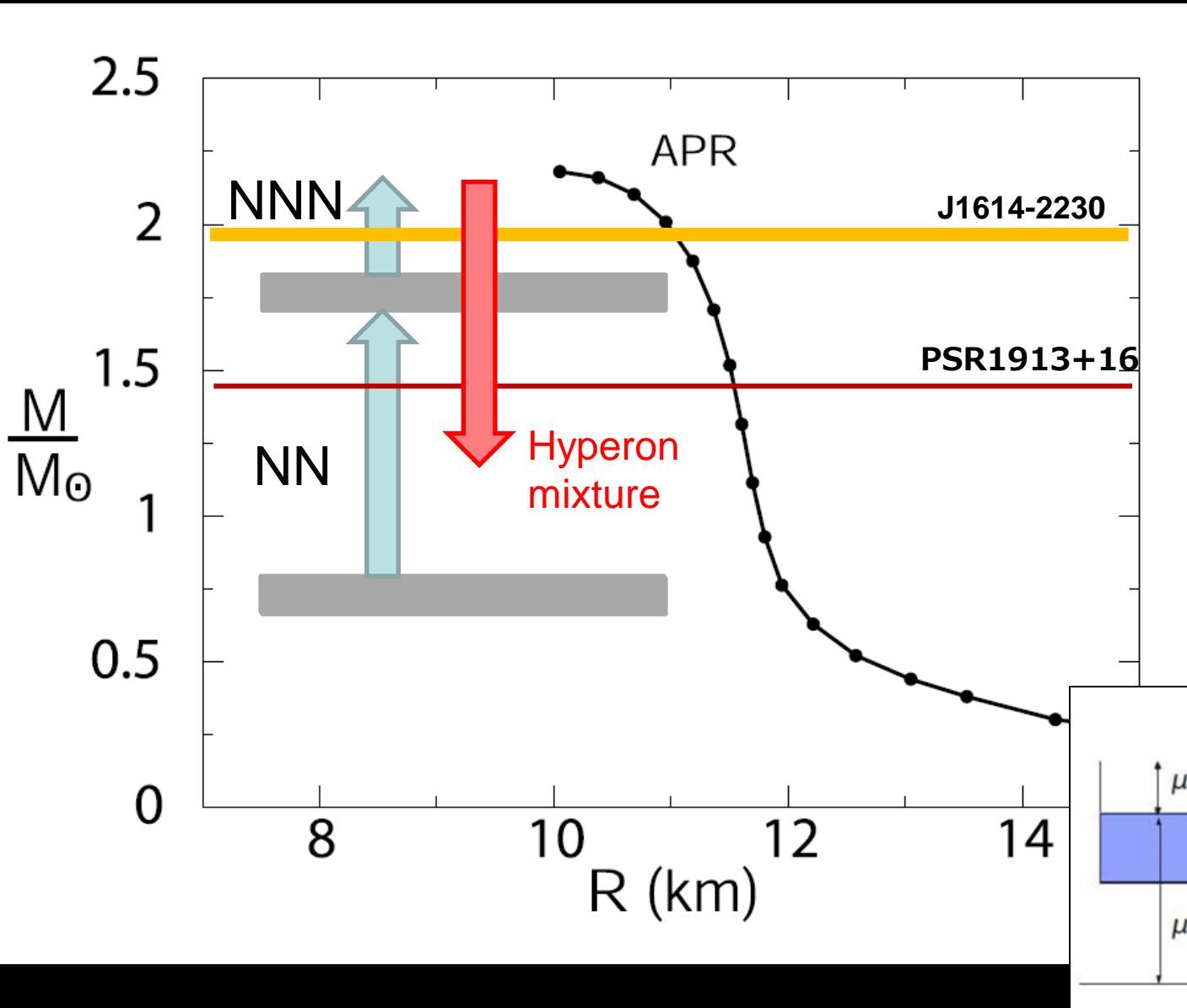
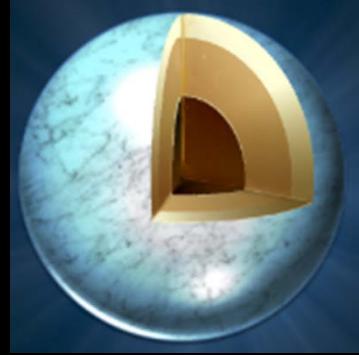
## Neutron Matter



# Neutron Star from “Lattice EOS”



# Hyperon Crisis



# Possible Resolution(s) of Hyperon Crisis

## 1. 2-body YN forces completely different from NN?

unlikely from lattice QCD studies

HAL QCD Coll., Nucl.Phys.A881 (2012) 28

## 2. Repulsive 3-body forces in YN too?

not enough even with  $V_{YNN} = V_{YYN} = V_{YYY} = V_{NNN}$

## 3. $n(>3)$ -body forces ?

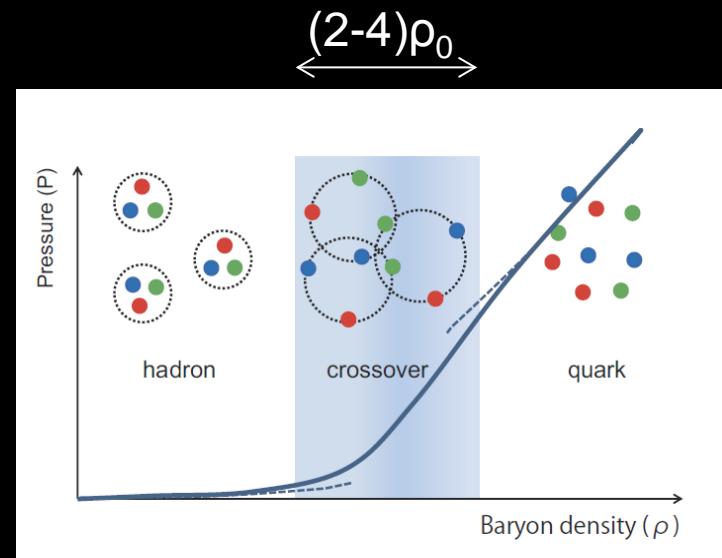
no information so far. convergence ?

## 4. Crossover to quark matter ?

Hatsuda, Tachibana, Yamamoto & Baym, PRL 97 (2006) 122001

$2M_\odot$  neutrons stars require,

- STIFF quark-matter EOS
- Smooth crossover (no 1<sup>st</sup> order transition)
- Crossover at  $\rho = (2-4) \rho_0$



Masuda, Hatsuda & Takatsuka, Astrophysical Journal Letters 764 (2013) 12

# From QCD to Hot/Dense Matter

## Quantum Chromo Dynamics

Lattice  
gauge theory

sign  
problem

Phenome. model

Equation of State for Hot Matter

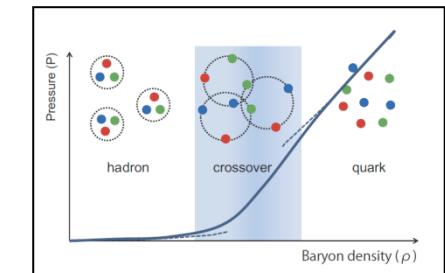
Equation of State for Dense Matter

Relativistic  
hydrodynamics

General relativity

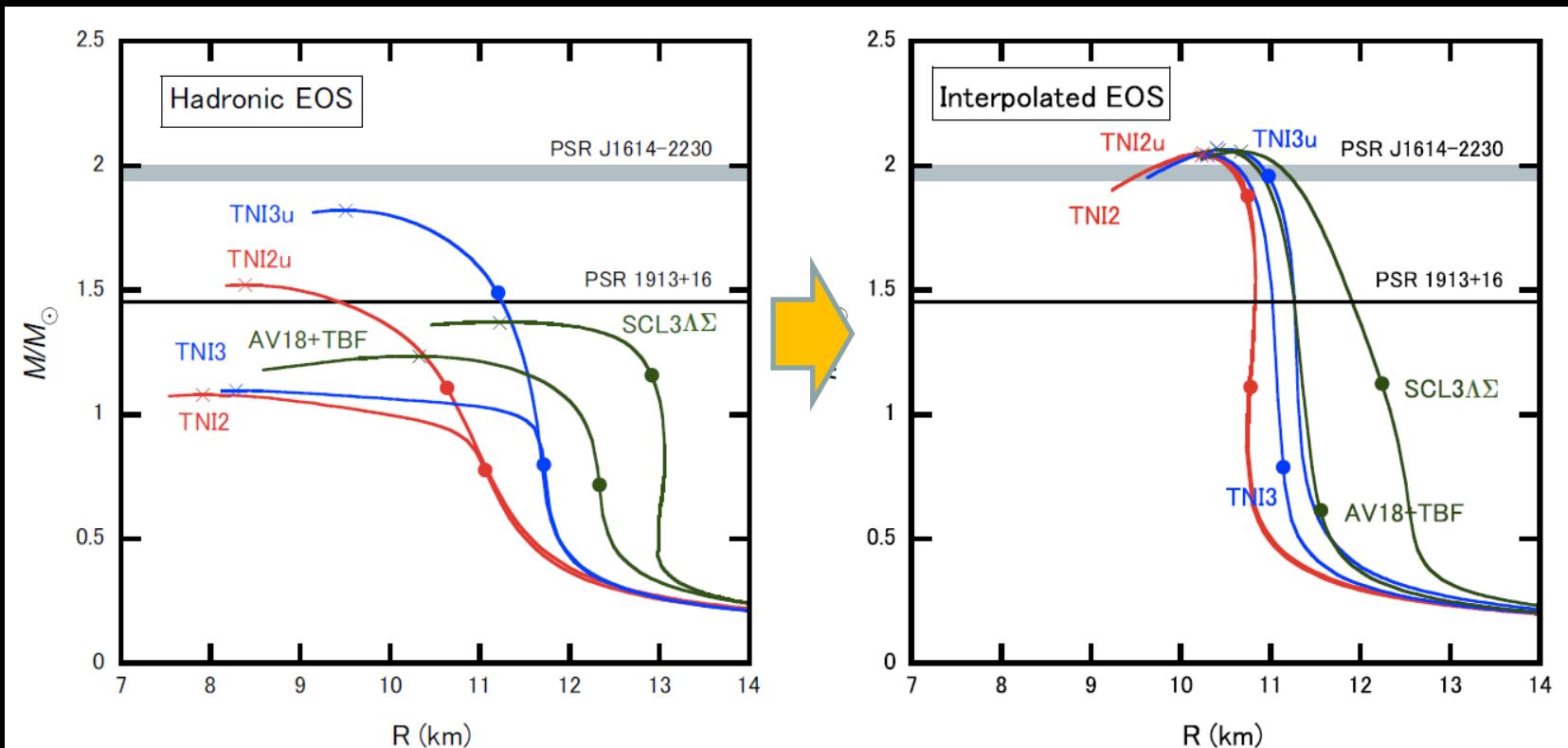
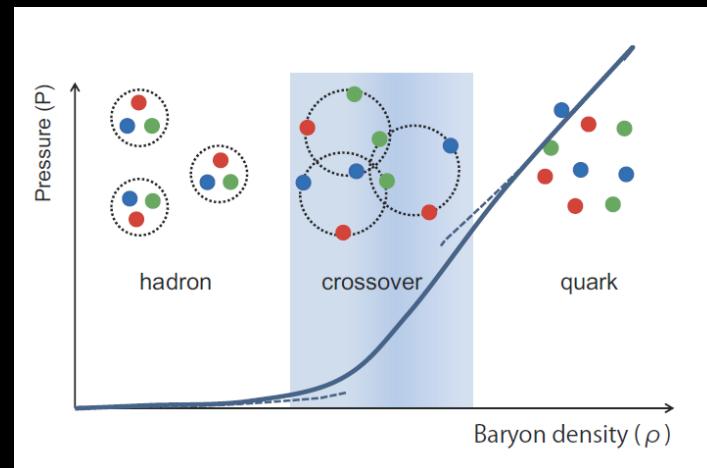
Relativistic heavy-Ion collisions

Neutron stars

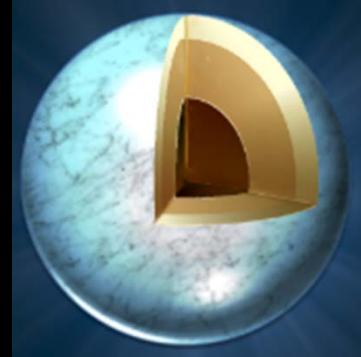


# Crossover from Soft Hyperon Core to Stiff Quark Core

Hatsuda, Tachibana, Yamamoto & Baym,  
Phys. Rev. Lett. 97, 122001 (2006)  
Bratovic, Hatsuda & Weise, PLB 719, 131 (2013)



# Summary



## 1. Dense QCD is a real challenge

theory:

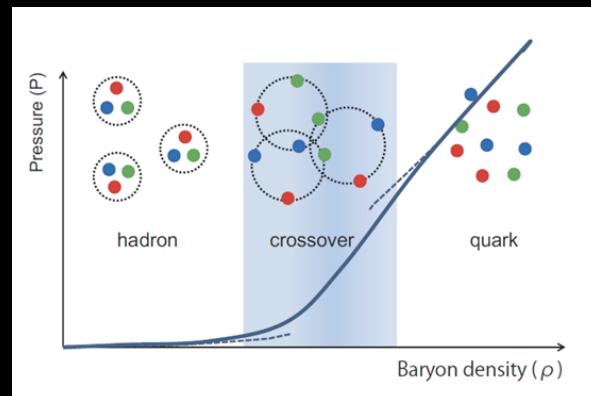
- BB and BBB forces from **lattice QCD** (HAL QCD Coll.)
  - physical point results with  $L \sim 10$  fm will come soon.
- **sign problem** unsolved

obs.:

- progresses in **M, R, T, B measurements**
  - $2M_{\odot}$  NStars, Magnetars, CAS-A cooling, X-ray bursts
- **Gravitational wave** detections will be ready soon

## 2. Hyperon Crisis

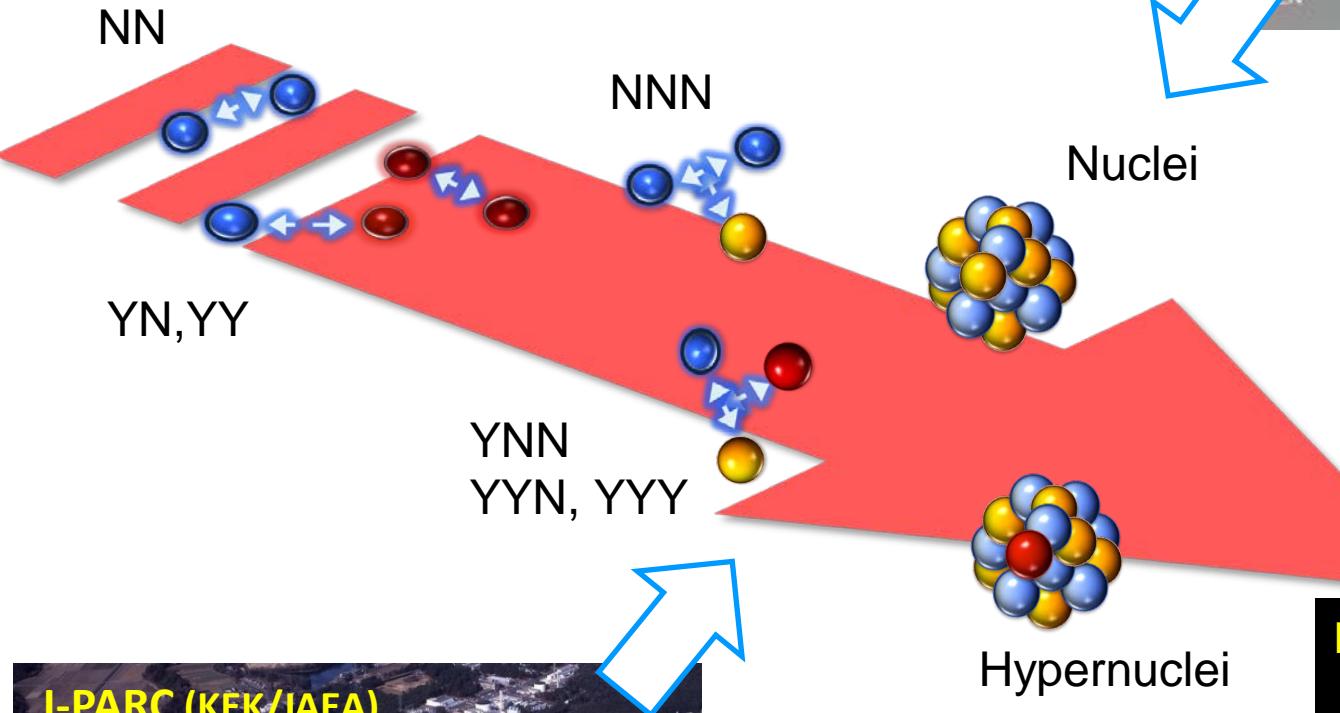
- no convincing resolution yet
  - higher body hyperon force ?
  - **crossover** to stiff quark matter  
→ may be studied by HIC ?



# HAL QCD Strategy : From QCD to Compact stars

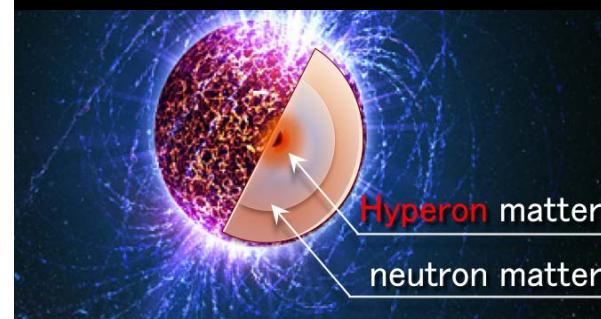


BG/L  $\rightarrow \dots \rightarrow$  K computer  $\rightarrow$  post K  
(10TF  $\rightarrow \dots \rightarrow$  10PF  $\rightarrow$  1 EF)



Hypernuclei

Neutron star:  
max mass, cooling etc



END