A new look at instantons in THE large-N limit

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Azeyanagi-M.H.-Honda(KEK→HRI, India)-Matsuo(KEK)-Shiba(KEK), 1307.0809[hep-th]

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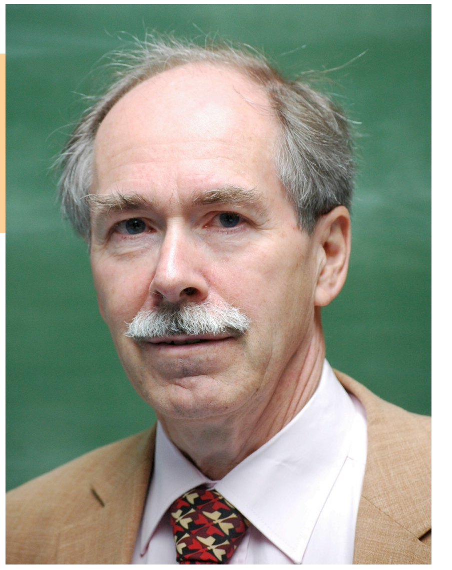
Introduction & summary

- The standard folklore on the large-N limit is wrong.
- Nonperturbative effects can be treated straightforwardly in the large-N limit.
- Now you can easily derive full instanton partition functions of a class of non-SUSY theories.
- A conceptual difficulty for the gauge/gravity duality with the M-theory gravity dual is resolved.
- Now we have an appropriate set up to play with instantons/monopoles with 11d gravity dual -- qualitatively new applications of the gauge/gravity duality to QCD!

what is “large-N” ?

usual answer $\lambda = g^2 N$ *fixed,* $N \to \infty$ *('t Hooft limit)*



why?

- 1/N expansion = genus expansion. (string theory!)

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

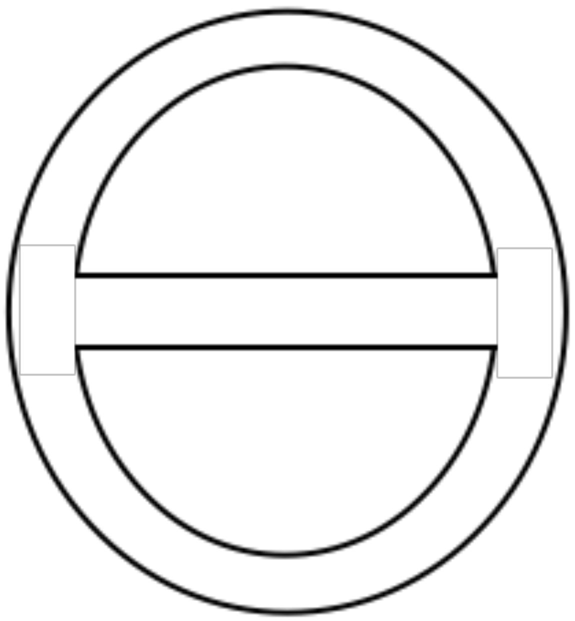
- perturbative series may have a finite radius of convergence at large-N → analytic continuation to strong coupling ?

- Various nice properties (factorization,

$$S = \frac{N}{4\lambda}\int d^4x\, Tr F_{\mu\nu}^2 \quad \Rightarrow$$

vertex $\sim$ N
index loop $\sim$ N
propagator $\sim$ 1/N

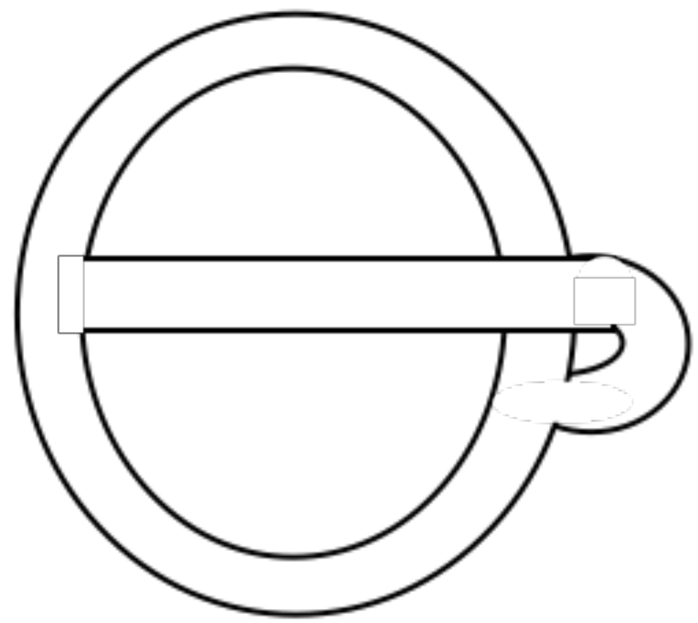
planar diagram



$N^2 \times N^{-3} \times N^3 =$

N^2

nonplanar diagram
(genus one)



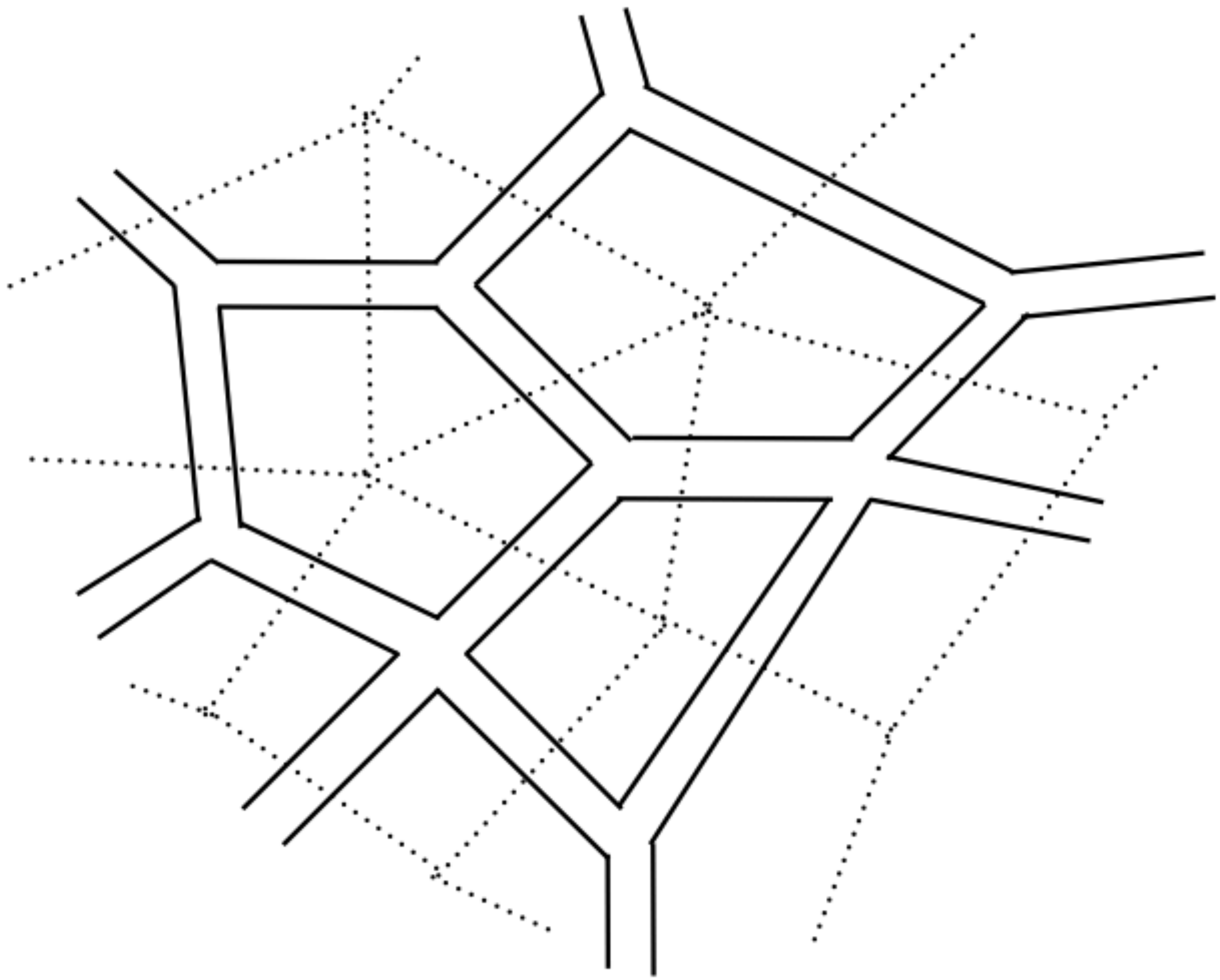
$N^2 \times N^{-3} \times N^1 =$

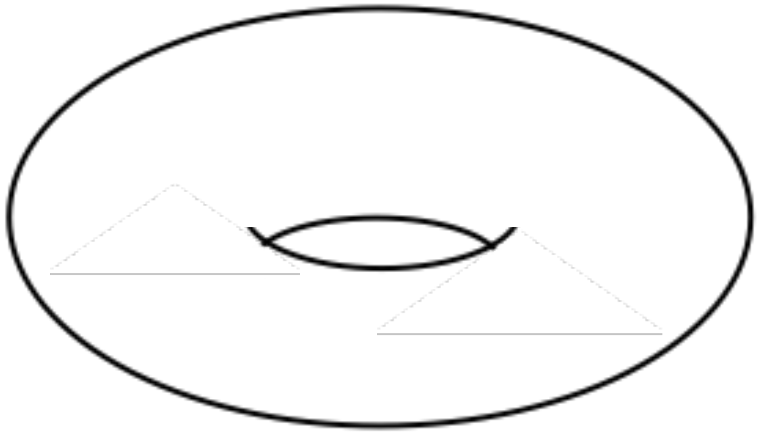
N^0

vertex ∼ N∼triangle/rectangle

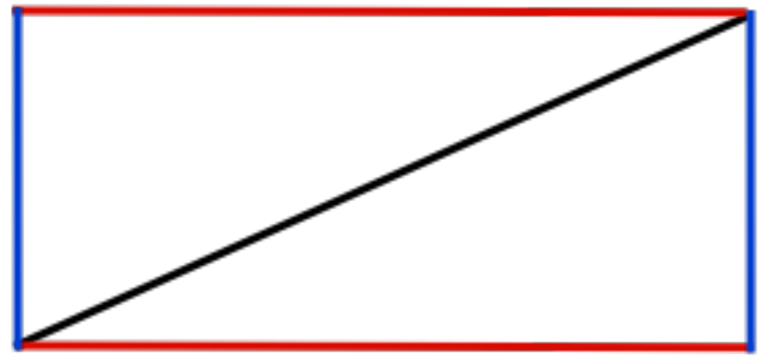
index loop ∼ N∼vertex

propagator ∼ 1/N∼side





torus



triangulation of torus

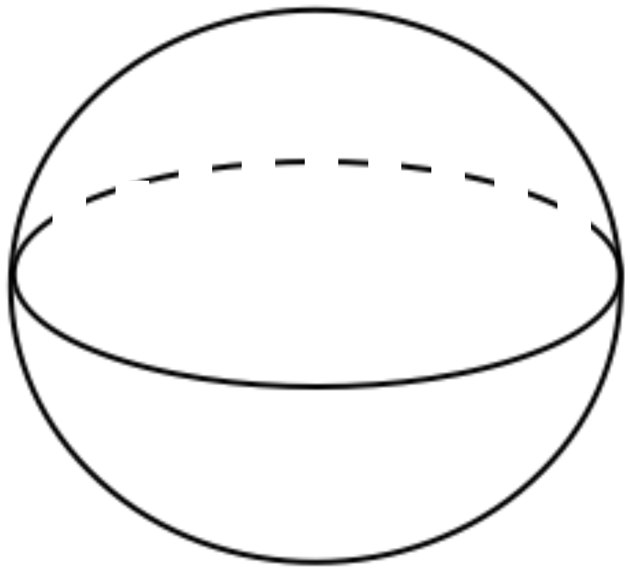
$\boxed{\text{Euler number}}$

$$(\#\text{triangles}) - (\#\text{sides}) + (\#\text{vertices}) = 2 - 3 + 1 = 0$$

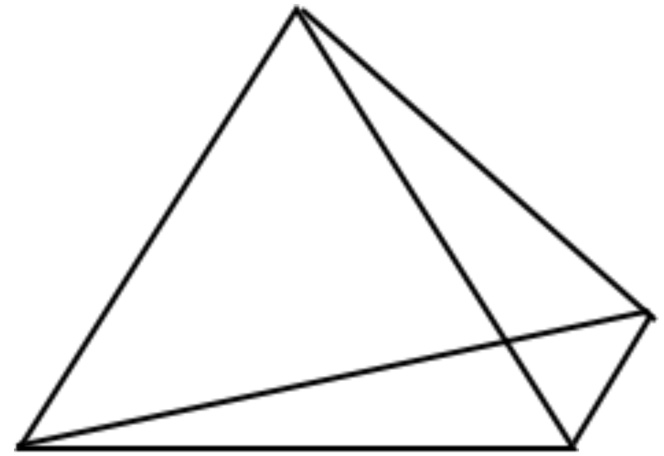
more generally,

$$(\#\text{triangles}) - (\#\text{sides}) + (\#\text{vertices}) = 2 - 2g$$

where $\quad g = (\#\text{genus})$

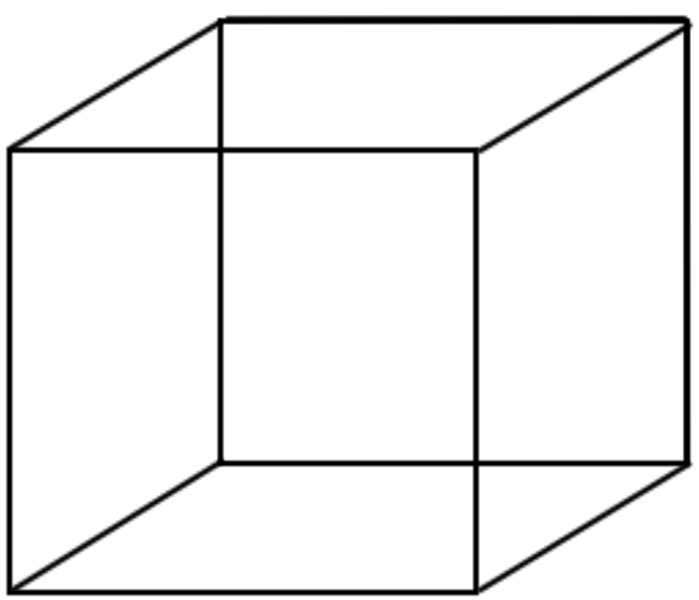


two-sphere ($g=0$)



4 triangles
6 sides
4 vertices

$$\boxed{4-6+4 = 2 = 2-2g}$$

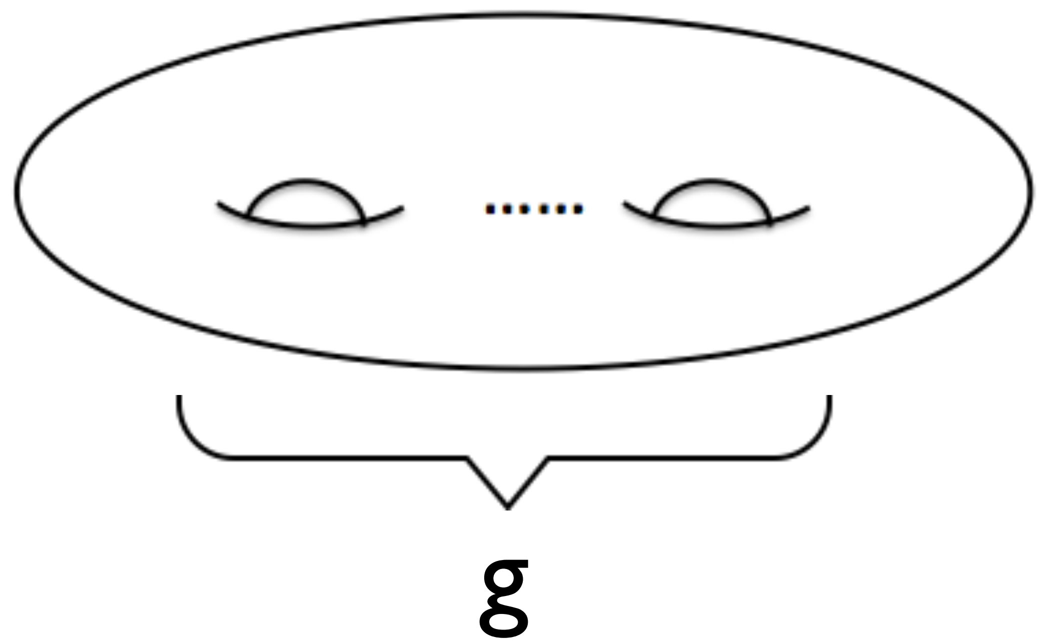


6 squares
12 sides
8 vetices

$$\boxed{6-12+8 = 2 = 2-2g}$$

genus-g diagram = diagram which can be drawn on genus-g surface

$\Rightarrow$ $\boxed{\text{g closed string loops}}$



$$(1/N)^{2g-2} = g_s^{2g-}$$

$$1/N^2 = g_s$$

classical gravity = planar limit

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

In AdS/CFT,

$$\boxed{\begin{array}{c} 1/N \text{ correction} = g_s \text{ correction} \\ 1/\lambda \text{ correction} = \alpha' \text{ correction} \end{array}}$$

But what about M-theory?

$$\boxed{(g_{YM})^2 \sim 1}$$

Let's consider Another large-N limit:

$g^2 \sim N^{-\alpha}$;α=1 is the 't Hooft limit

α<1: 'very strongly coupled'

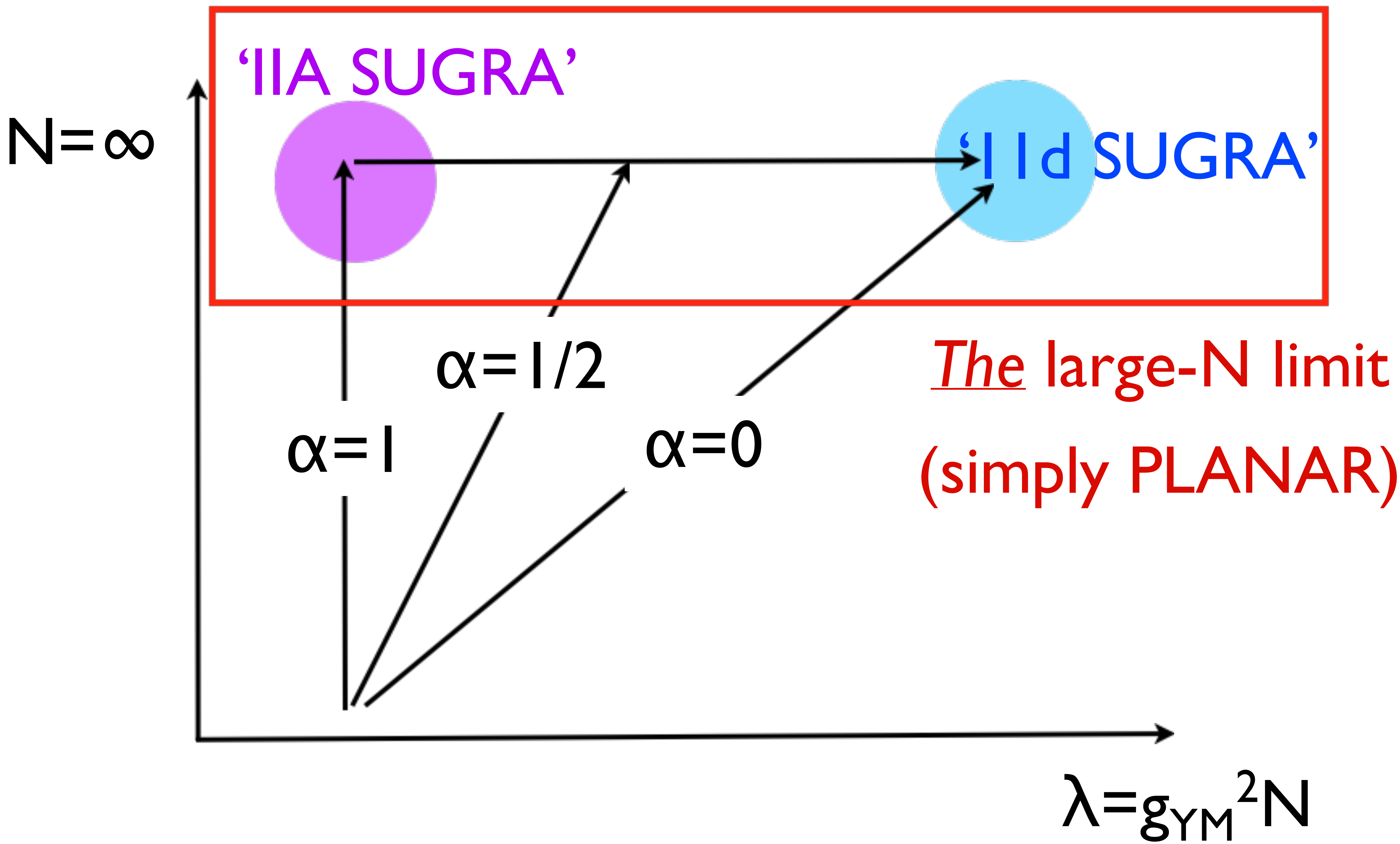
<u>why?</u>

- It is possible.
- application to M-theory $\boxed{(g_{YM})^2 \sim 1}$
- Instanton effect remains finite. $\boxed{\exp(-8\pi^2/g_{YM}^2) = O(1)}$

λ is N-dependent.
1/N expansion and genus expansion are different.

Our conjecture

- The very strongly coupled large-N limit is well-defined and essentially the same as the ‘t Hooft limit.
- More precisely: large-N limit and strong coupling limit commute.
- When there is no ‘phase transition’ (or as long as one considers the same point in the moduli space), the analytic continuation from the planar limit gives the right answer.



(Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013))

A typical (wrong) objection

- Planar and nonplanar diagrams are mixed in such a limit!

$$\begin{aligned}
F(\lambda, N) &= N^2 F_0(\lambda) + F_1(\lambda) + F_2(\lambda)/N^2 + \cdots \\
&= N^2 \left(f_{0,0} + f_{0,1}\lambda + f_{0,2}\lambda^2 + \cdots\right) \\
&\quad + \left(f_{1,0} + f_{1,1}\lambda + f_{1,2}\lambda^2 + \cdots\right) \\
&\quad + N^{-2} \left(f_{2,0} + f_{2,1}\lambda + f_{2,2}\lambda^2 + \cdots\right) + \cdots \\
&= N^2 \left(f_{0,0} + f_{0,1}N + f_{0,2}N^2 + \cdots\right) \\
&\quad + \left(f_{1,0} + f_{1,1}N + f_{1,2}N^2 + \cdots\right) \\
&\quad + N^{-2} \left(f_{2,0} + f_{2,1}N + f_{2,2}N^2 + \cdots\right) + \cdots
\end{aligned}$$

Nonplanar diagrams contribute as well, so the limits do not commute!

Why are you using a perturbative expression at strong coupling???????

Example (1)
4d N=4 SYM

AdS_5/CFT_4 conjectre

(Maldacena, 1997)

Bunch of D3-branes

4d N=4 SYM $\longleftrightarrow$ IIB string on $AdS_5 \times S^5$

equivalent

$g_{YM}^2 \sim g_{string}$, $\alpha'/R_{AdS}^2 \sim \lambda^{-1/2}$

perturbative string picture is valid when

$g_{YM}^2 << 1$, $\lambda >> 1$

The right picture

[Azeyanagi-Fujita-M.H., Phys.Rev.Lett. 110 (2013)]

When there is a gravity dual:

$$\begin{aligned} F(\lambda, N) &= F(g_s, \alpha') \\ &= g_s^{-2} F_0(\alpha') + F_1(\alpha') + g_s^2 F_2(\alpha') + \cdots \\ &= g_s^{-2}\left(f_{0,0} + f_{0,1}\alpha' + f_{0,2}\alpha'^2 + \cdots\right) \\ &\quad + \left(f_{1,0} + f_{1,1}\alpha' + f_{1,2}\alpha'^2 + \cdots\right) \\ &\quad + g_s^2\left(f_{2,0} + f_{2,1}\alpha' + f_{2,2}\alpha'^2 + \cdots\right) + \cdots \end{aligned}$$

($g_s \sim g_{YM}^2 \sim \lambda/N$, $\alpha' \sim \lambda^{-1/2}$ for 4d N=4 SYM)

$f_{0,0}$ dominates as long as $g_{YM}^2 \ll 1$ and $\lambda \gg 1$

The same expression at $1 \ll \lambda \ll N$, simply supergravity!

(By using the S-dual, we can show it even at $N < \lambda$.)

perturbative string picture is valid when $g_{YM}^2 << 1$, $\lambda >> 1$

- Usually one takes the 't Hooft limit first and then consider strong 't Hooft coupling. (tree-level string)

- Or one consider large-but-finite-N with $\lambda = O(1)$, so that 1/N expansion and string loop expansion coincide.

- But according to Maldacena's conjecture, such limit is ***not*** required for the validity of the weakly coupled gravity description.

- So, the very strongly coupled limit exists, and at $\lambda << N$ it is simply the same as the planar limit: *supergravity*!

- Analytic continuation to $\lambda >> N$ can be confirmed by using S-duality.

Example (2)
ABJM theory

(Analytic continuation to M-theory)

ABJM theory

(Aharony-Bergman-Jafferis-Maldacena, 2008)

A

B

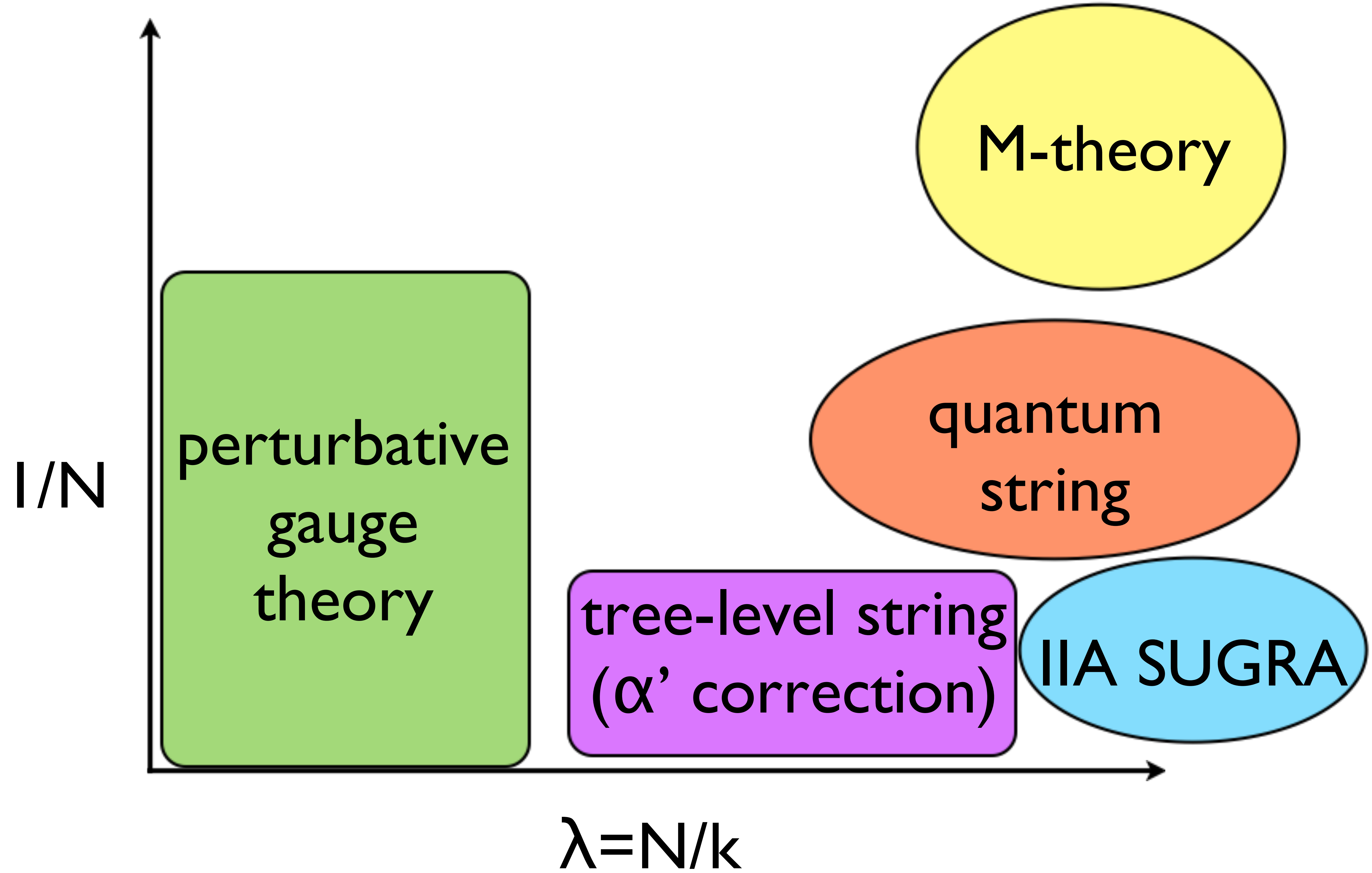
J

M

$$k\,Tr\Bigg\{\frac{\epsilon^{\mu\nu\rho}}{2}\left(-A_\mu\partial_\nu A_\rho-\frac{2}{3}A_\mu A_\nu A_\rho+\tilde{A}_\mu\partial_\nu\tilde{A}_\rho+\frac{2}{3}\tilde{A}_\mu\tilde{A}_\nu\tilde{A}_\rho\right)$$
$$+\left(-D_\mu\bar{\Phi}^\alpha D^\mu\Phi_\alpha+i\bar{\Psi}^\alpha \not{D}\Psi_\alpha\right)-i\epsilon^{\alpha\beta\gamma\delta}\Phi_\alpha\bar{\Psi}_\beta\Phi_\gamma\bar{\Psi}_\delta+i\epsilon_{\alpha\beta\gamma\delta}\bar{\Phi}^\alpha\Psi^\beta\bar{\Phi}^\gamma\Psi_\delta$$
$$+i\left(-\bar{\Psi}_\beta\Phi_\alpha\bar{\Phi}^\alpha\Psi^\beta+\Psi_\beta\bar{\Phi}_\alpha\Phi^\alpha\bar{\Psi}^\beta+2\bar{\Psi}_\alpha\Phi_\beta\bar{\Phi}^\alpha\Psi^\beta-2\Psi^\beta\bar{\Phi}^\alpha\Phi_\beta\bar{\Psi}_\alpha\right)$$
$$+\frac{1}{3}\left(\Phi_\alpha\bar{\Phi}^\beta\Phi_\beta\bar{\Phi}^\gamma\Phi_\gamma\bar{\Phi}^\alpha+\Phi_\alpha\bar{\Phi}^\alpha\Phi_\beta\bar{\Phi}^\beta\Phi_\gamma\bar{\Phi}^\gamma+4\Phi_\beta\bar{\Phi}^\alpha\Phi_\gamma\bar{\Phi}^\beta\Phi_\alpha\bar{\Phi}^\gamma-6\Phi_\gamma\bar{\Phi}^\gamma\Phi_\beta\bar{\Phi}^\alpha\Phi_\alpha\bar{\Phi}^\beta\right)$$

3d $U(N)_k \times U(N)_{-k}$ Superconformal Chern-Simons-Matter theory

(developed out of earlier works by Schwarz, Bagger-Lambert, etc etc...)


M-theory
quantum string
1/N
perturbative gauge theory
tree-level string (α' correction)
IIA SUGRA
λ=N/k

Prediction from gravity side

- Free energy in IIA string region ($k \ll N \ll k^5$)

$$F = \frac{\pi\sqrt{2}}{3}\frac{N^2}{\sqrt{\lambda}}$$

- Free energy in M theory region ($N \gg k^5$)

$$F = \frac{\pi\sqrt{2}}{3}\sqrt{k}N^{3/2}$$

the same expression
→ analytic continuation
from λ=O(1) to λ = O(N)

$$F = \sum_{g=0}^{\infty} F_g(\lambda)/N^{2g-2}$$

N-dependent

AdS/CFT tells us that, at strong coupling,
α'-expansion (1/λ-expansion) is good,
at least in IIA string region.

→ Only the leading term in each F_g is important.

$$\frac{\sqrt{2}\pi}{3}\frac{N^2}{\sqrt{\lambda}} \quad (g=0), \quad c_g(N^2/\lambda^2)^{1-g} \quad (g>0)$$

g=0 (planar) dominates even outside the planar limit.

More on the planar dominance in the M-theory limit

Large-N orbifold equivalence (Kachru-Silverstein 1998)

$$
\begin{tikzcd}[column sep=huge, row sep=huge]
\text{4d } \mathcal{N}=4 \text{ } U(kN) \text{ SYM} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] \arrow[d, leftrightarrow, "\text{orbifold equivalence}"] & \text{IIB string on } AdS_5\times S^5 \arrow[d, leftrightarrow, "\text{orbifold equivalence}"] \\
{[U(N)]^k} \text{ SYM} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] & \text{IIB string on } AdS_5\times S^5/Z_k
\end{tikzcd}
$$

- In the gravity side, Z_k-invariant modes do not distinguish these two theories.
- In the gauge theories, correlation functions of Z_k-invariant operators coincide with the counterparts in the orbifolded theory.

Large-N orbifold equivalence (Kachru-Silverstein 1998)

$$
\begin{array}{ccc}
\text{4d } \mathcal{N}\text{=4 U(kN) SYM} & \xleftrightarrow{\text{AdS/CFT}} & \text{IIB string on } AdS_5 \times S^5 \\
\updownarrow \text{ orbifold equivalence} & & \updownarrow \text{ orbifold equivalence} \\
[U(N)]^k \text{ SYM} & \xleftrightarrow{\text{AdS/CFT}} & \text{IIB string on } AdS_5 \times S^5/Z_k
\end{array}
$$

In the planar limit, it can be proven in the field theory language.

(Bershadsky-Johansen '98, Kovtun-Unsal-Yaffe '06,...)

From the gauge theory point of view,
the equivalence is gone as soon as the
nonplanar diagrams are taken into account.

However, though the planar limit is always assumed, it is not really necessary; classical gravity description is the key.

orientifold

~~Orbifold~~ equivalence in ABJM(1)

(Fujita-M.H.-Hoyos 2012)

$$\begin{tikzcd}
U(2N)_{2k}\times U(2N)_{-2k} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] \arrow[d] & AdS_4\times S^7/Z_{2k} \arrow[d, "\text{orientifold projection}"] \\
O(2N)_{2k}\times USp(2N)_{-k} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] & AdS_4\times (S^7/Z_{2k})/Z_2
\end{tikzcd}$$

It provides us with a natural generalization of the planar equivalence to the M-theory region.

orientifold ~~Orbifold~~ equivalence in ABJM(2)

(Fujita-M.H.-Hoyos 2012)

$$
\begin{tikzcd}
U(2N)_{2k}\times U(2N)_{-2k} \arrow[rr, leftrightarrow, "\text{AdS/CFT}"] \arrow[d, leftrightarrow] & & \text{IIA on } AdS_4\times CP_3 \arrow[d, leftrightarrow, "\begin{array}{c}\text{orientifold}\\ \text{equivalence}\end{array}"] \\
O(2N)_{2k}\times USp(2N)_{-k} \arrow[rr, leftrightarrow, "\text{AdS/CFT}"] & & \text{IIA on } AdS_4\times(CP_3/Z_2)
\end{tikzcd}
$$

$k<<N<<k^5$

~~Orbifold~~ orientifold equivalence in ABJM(3)

(Fujita-M.H.-Hoyos 2012)

$$\begin{tikzcd}
U(2N)_{2k}\times U(2N)_{-2k} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] \arrow[d, leftrightarrow] & \text{M on } AdS_4\times S^7/Z_{2k} \arrow[d, leftrightarrow, "\text{orientifold equivalence}"] \\
O(2N)_{2k}\times USp(2N)_{-k} \arrow[r, leftrightarrow, "\text{AdS/CFT}"] & \text{M on } AdS_4\times(S^7/Z_{2k})/Z_2
\end{tikzcd}$$

$N>>k^5$

orientifold

~~Orbifold~~ equivalence in ABJM(4)

(Fujita-M.H.-Hoyos 2012)

• Planar equivalence, which does not hold if the nonplanar contribution is taken into account, is naturally generalized to the M-theory region.

• No discontinuity at $N \sim k^5$ ($\leftarrow$loclization)

Why can it hold?

The planar dominance outside the planar limit!

More evidence

- Any field theory with a gravity dual
- 2d pure Yang-Mills (solvable thanks to Migdal)
- strong coupling expansion of the lattice gauge theory
- large-N reduction (Eguchi-Kawai 1982) in the M-theory limit (Honda-Yoshida 2012, Ishiki-Ohta-Shimasaki-Tsuchiya, private communication)

Instantons

(Azeyanagi-M.H.-Honda-Matsuo-Shiba, `1307.0809 [hep-th]`)

instantons

(Azeyanagi-M.H.-Honda-Matsuo-Shiba, 1307.0809 [hep-th])

- The same argument is valid at each instanton sector.

- Therefore 'planar dominance' holds there.

- Can be confirmed in various theories with N=2 SUSY by using the Nekrasov formula for the partition functions.

Nice properties in the planar limit holds where the instanton weight is finite!

$\exp(-8\pi^2/g_{YM}^2) = O(1)$

orbifold equivalence between instanton patition functions

(※ It is just one of various examples, which is almost trivial *from our new viewpoint.*)

'parent' 4d N=2 SYM $\xrightarrow{Z_k \text{ orbifolding}}$ 'daughter' YM with less SUSY

you can take it to be non-SUSY!

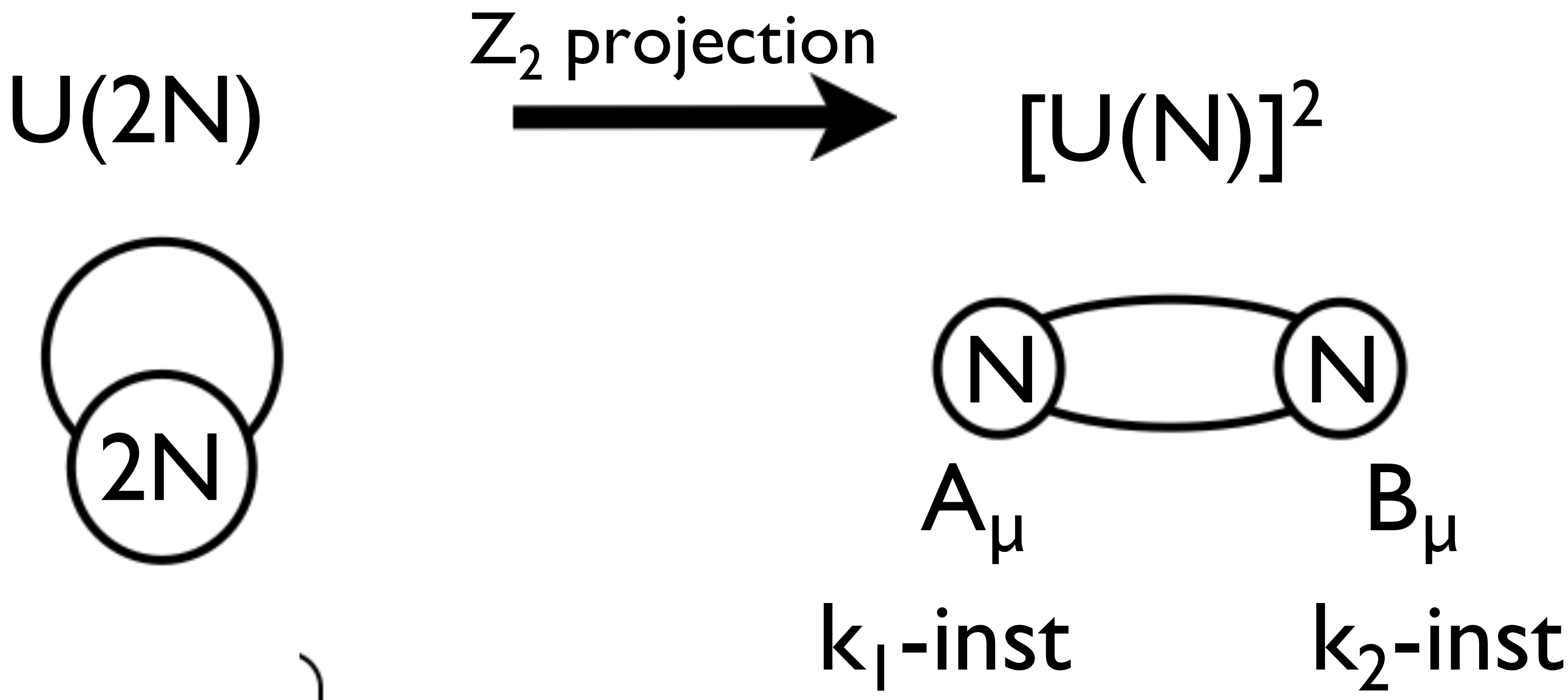
Nekrasov formula

$$\boxed{Z_{\text{p-inst, parent}} = (Z_{\text{p-inst, daughter}})^k}$$

in THE large-N limit

When the daughter keeps N=2 SUSY,
you can easily confirm it by using the Nekrasov formula.

(Azeyanagi-M.H.-Honda-Matsuo-Shiba, 1307.0809 [hep-th])



$$\begin{pmatrix} A_\mu & 0 \\ 0 & B_\mu \end{pmatrix}$$

(k_1+k_2)-inst

$$\boxed{\color{red}{Z_{(k1+k2)\text{-inst, parent}}} = \color{blue}{(Z_{(k1+k2)\text{-inst, daughter}})^2}}$$

The equivalence holds at each instanton sector.

Speculations

M-theoretic holography

(in progress)

- 11-d SUGRA should know the instantons, monopoles, etc at large-N with g^2 fixed. This is nothing but 'planar' in the gauge theory side.

- It should be possible to study the dynamics of solitons by using 11d SUGRA!

- As a first nontrivial test:

On-shell action of 11d SUGRA (Gaiotto-Maldacena geometry) = Free energy of 4d N=2 gauge theory (Gaiotto theory)

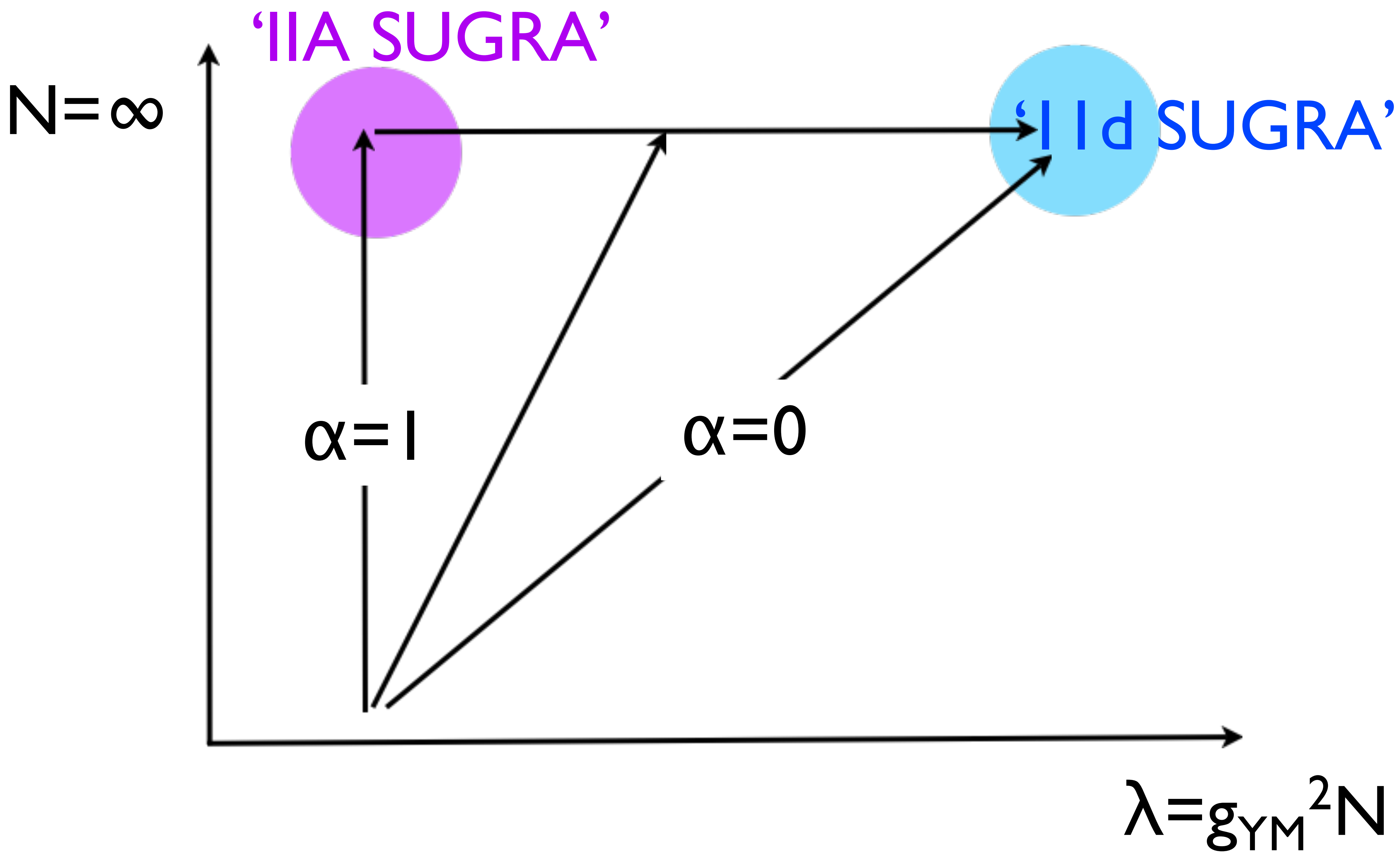
(conjecture; now checking it.)

Instantons in QCD

(not even in progress)

- The coupling constant runs with the scale.
- The 't Hooft coupling diverges when instanton size is of order $1/\Lambda_{QCD}$. $\rightarrow$The very strong coupling limit can be realized.
- Small instantons ($g^{-2} \sim N^{\#} \rightarrow \infty$) are naturally suppressed.
- So instantons with the radius $1/\Lambda_{QCD}$ can be dominant. Looks consistent with lattice data!

Conclusion



‘IIA SUGRA’
N=∞
‘11d SUGRA’
α=1
α=0
λ=g_YM^2 N