Recent progress in 2d Causal Dynamical Triangulations

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September 29th @ KEK Workshop

J. Ambjørn, L. Glaser, Y. S. and Y. Watabiki, Physics Letters B 722, 2013

J. Ambjørn, L. Glaser, A. Görlich and Y.S., Phys. Lett. B 712, 2012

Quantum Gravity

Question: How did the Universe at the very beginning look like?

Its energy was about 10^{19} GeV (short scale).

Its dynamics was governed by strong gravity.

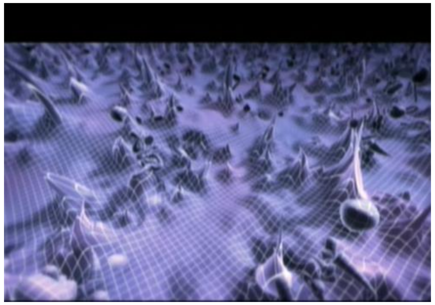
Physics at short scale → quantum mechanics

Gravitational physics at large scale → general relativity

To answer the question,
one needs to combine two theories in a consistent manner…
→ Quantum gravity

However, naïve construction doesn't work
because quantum fluctuations cannot be handled.

Non-renormalizabe



Causal Dynamical Triangulations (CDT)

→ A non-perturbative way to quantise Einstein gravity

lattice

Small scale structure regularized by CDT

UV --------------→ **IR**

Outcome:

1. 4d de Sitter Universe pops up

   (J. Ambjørn, Jurkiewicz and R. Loll, 2004)

2. 2nd-order phase transition

   (J. Ambjørn, A. Gorlich, S. Jordan, Jurkiewicz and R. Loll, 2010)

Causal Dynamical Triangulations (CDT)

→ A non-perturbative way to quantise Einstein gravity

lattice

Questions:
(1) What's the field theory at IR?
(2) Matter coupling?

Small scale structure regularized by CDT

UV ------------→ IR

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2d model is a nice playground because…

→ (1) CDT can be solved analytically only in 2 dimensions (so far).

→ (2) 2d CDT may be a ``good” toy model for 4d CDT.

Recent progress in 2d CDT:

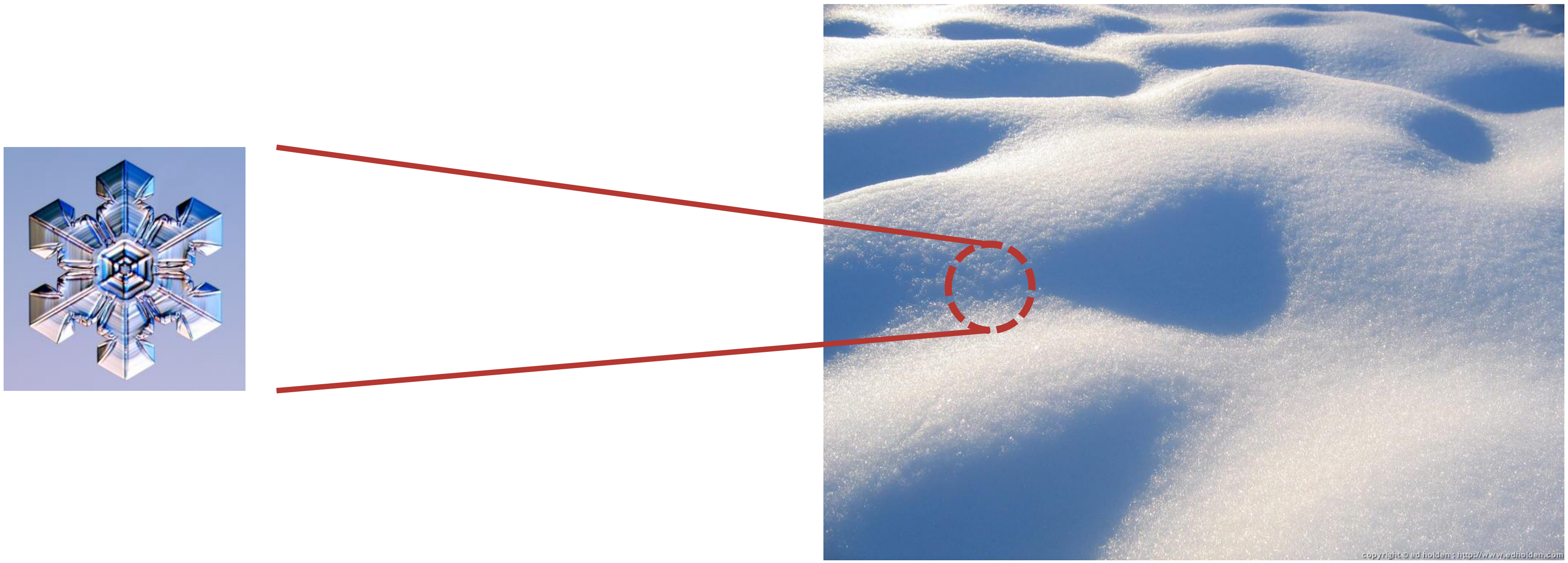
1. 2d CDT and 2d Horava-Lifshitz gravity are in the same universality class.

J. Ambjørn, L. Glaser, Y. S. and Y. Watabiki, 2013.

2. A generic method for coupling matters to 2d CDT has been proposed.

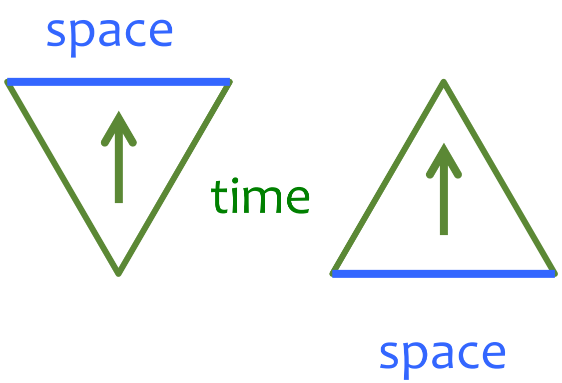
J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

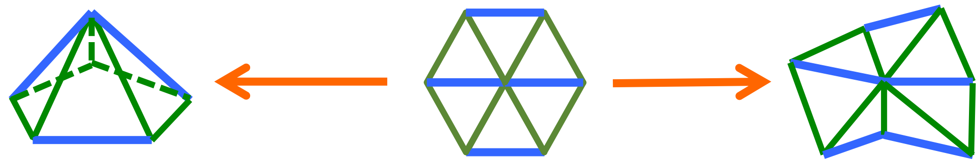
CDT **Horava-Lifshitz**



2d CDT J. Ambjørn, R. Loll, 1998

Lorentzian triangle lattice:





lattice spacing → fixed

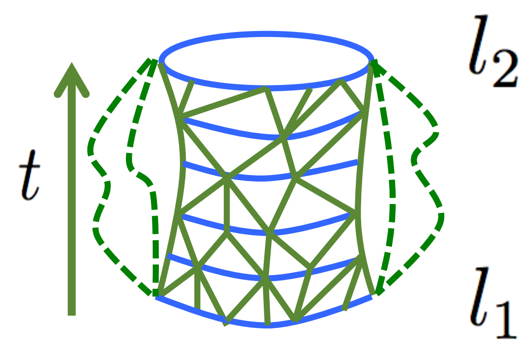
triangulations (T) → dynamical
(= how to divide geometry by triangles)

space $a_s^2 = \varepsilon^2$

time $a_t^2 = -\alpha\varepsilon^2$

Metric path-integral → sum over triangulations (T): $\int \mathcal{D}g \to \sum_T$

$$G(l_2, l_1; t) = \sum_{T(l_1,l_2)} e^{iS_L(\alpha)}$$



2d CDT

J. Ambjørn, R. Loll, 1998

Discrete action for CDT:

$$S_L(\Lambda; g) = -\Lambda \int d^2x \sqrt{-g}$$

$\downarrow$ discretise

$$S_L(\lambda; T) = -\lambda \frac{\sqrt{4\alpha+1}}{4} n(T)$$

$\downarrow$ rotation to Euclidian

$$S_E(\lambda; T) = \lambda \frac{\sqrt{4\alpha-1}}{4} n(T) \equiv \lambda n(T)$$

Area (△) = $\frac{\sqrt{4\alpha+1}}{4}\varepsilon^2$

(△) = n

Lorentzian → Euclidean

$$\boxed{\begin{gathered} \alpha \to -\alpha \\ iS_L(\alpha) \to -S_E(\alpha) \end{gathered}}$$

After the rotation, CDT amplitude becomes

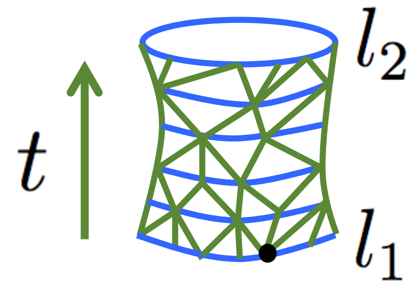
$$G(l_2, l_1; t) = \sum_{T(l_1, l_2)} e^{-\lambda n(T)}$$

λ : dimensionless cosmological constant

2d CDT

J. Ambjørn, R. Loll, 1998

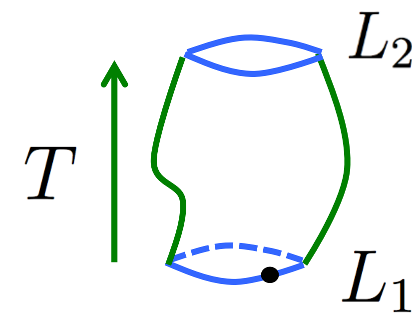
$$G(l_2, l_1; t) = \sum_l G(l_2, l; 1) G(l, l_1; t-1)$$



Continuum limit

$$\lambda \to \lambda_* \quad \& \quad \varepsilon \to 0$$

$$\frac{\partial}{\partial T} G(L_2, L_1; T) = -\boxed{\hat{H}(L_1)} G(L_2, L_1; T)$$



Quantum Hamiltonian

$$\boxed{\hat{H}(L_1) = -L_1 \frac{\partial^2}{\partial L_1^2} + \Lambda L_1}$$

$$\lambda - \lambda_* \sim \varepsilon^2 \Lambda$$

$$L_1 := \varepsilon l_1 \quad L_2 := \varepsilon l_2, \quad T := \varepsilon t \qquad G(L_2, L_1; T) = \lim_{\varepsilon \to 0} \varepsilon^{-1} G(l_2, l_1; t)$$

2d HL
J. Ambjørn, L. Glaser, YS, Y. Watabiki, 2013

2D **projectable** Horava-Lifshitz gravity (HL):

$$S_{\rm HL} = \int dt\, dx\, N\gamma\left[(1-\boxed{\lambda})K^2 - 2\Lambda\right]$$

isotropic limit

$$\lambda \to 1$$

projectable lapse

$$N = N(t)$$

where $\gamma := \sqrt{h}$ & $K = \frac{1}{N}\left(\frac{1}{\gamma}\partial_0\gamma - \frac{1}{\gamma^2}\partial_1 N_1 + \frac{N_1}{\gamma^3}\partial_1\gamma\right)$

$$\boxed{\{\gamma(x,t), \pi^\gamma(y,t)\} = \delta(x-y)}$$

$$H = \int dx\, [N\mathcal{H} + N_1\mathcal{H}^1]$$

"Hamiltonian constr." **momentum constr.**

$$\mathcal{H} = \gamma\frac{(\pi^\gamma)^2}{4(1-\lambda)} + 2\Lambda\gamma, \quad \mathcal{H}^1 = -\frac{\partial_1\pi^\gamma}{\gamma}$$

2d HL J. Ambjørn, L. Glaser, YS, Y. Watabiki, 2013

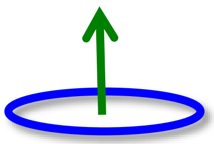
Solve momentum constraint → System reduces to be of 1 dimension

$$H = \int dx \, [N\mathcal{H} + N_1 \mathcal{H}^1]$$

Gauge fixing spatial Diff

$$\boxed{\mathcal{H}^1 = 0 \quad \text{i.e.} \quad \pi^\gamma(x,t) = \pi^\gamma(t)}$$

$$H = N(t)\left(L(t)\,\frac{(\pi^\gamma(t))^2}{4(1-\lambda)} + 2\Lambda L(t)\right), \qquad L(t) := \int dx\,\gamma(x,t)$$



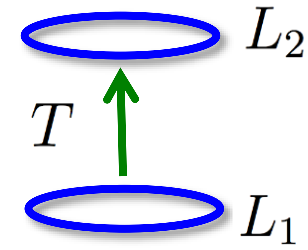
Quantise the 1d system based on the following action:

$$S = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} - \tilde{\Lambda} N(t) L(t)\right), \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

(from Hamiltonian constr., $\Lambda > 0 \quad \lambda < 1$)

2d HL J. Ambjørn, L. Glaser, YS, Y. Watabiki, 2013

Quantum amplitude (after a rotation to Euclidean signature):



$$G(L_2, L_1; T) = \int \frac{\mathcal{D}N(t)}{\mathrm{Diff}[0,1]} \int \mathcal{D}L(t)\; \mathrm{e}^{-S_E[N(t),L(t)]}$$

where $$S_E = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} + \tilde{\Lambda} N(t) L(t) \right)$$ $$\int_0^1 dt\, N(t) = T;$$

$$G(L_2, L_1; T) = \langle L_2 | e^{-T\hat{H}} | L_1 \rangle$$
$$= \int [dL] \boxed{\langle L_2 | e^{-\varepsilon \hat{H}} | L \rangle} G(L, L_1; T - \varepsilon)$$

integrate

$$\boxed{\exp\left(-\frac{(L_2 - L)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)}$$

completeness:

$$\int [dL_1] | L_1 \rangle \langle L_1 | = 1$$

2d HL

J. Ambjørn, L. Glaser, YS, Y. Watabiki, 2013

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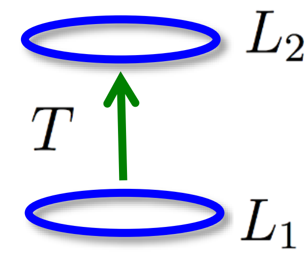

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where $$S_E = \int dt \left(\frac{\dot{L}^2}{4N(t)L(t)} + \tilde{\Lambda} N(t) L(t) \right) \qquad \int_0^1 dt\, N(t) = T$$

$$G(L_2, L_1; T) = \langle L_2 | e^{-T\hat{H}} | L_1 \rangle$$

compare

$$= \int [dL] \boxed{\langle L_2 | e^{-\varepsilon \hat{H}} | L \rangle} G(L, L_1; T - \varepsilon)$$ integrate

$$= G(L_2, L_1; T - \varepsilon) - \varepsilon \boxed{\hat{H}(L_2)} G(L_2, L_1; T) + \cdots$$ expand

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$$\boxed{\exp\left(-\frac{(L_2 - L)^2}{4\varepsilon L_2} - \varepsilon \tilde{\Lambda} L_2 \right)}$$

completeness:

$$\int [dL_1] |L_1\rangle \langle L_1| = 1$$

2d HL   J. Ambjørn, L. Glaser, YS, Y. Watabiki, 2013

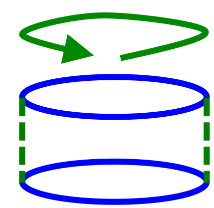
Quantum Hamiltonian for HL

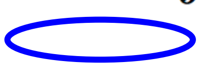
$$\hat{H} = -L\frac{\partial^2}{\partial L^2} - 2a\frac{\partial}{\partial L} - \frac{a(a-1)}{L} + \tilde{\Lambda}L \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

What is a ?

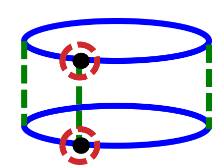
Completeness: $\displaystyle\int [dL]|L\rangle\langle L| = \int \boxed{L^a dL}|L\rangle\langle L| = 1$

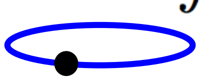
a = 1 $\displaystyle\int LdL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \frac{1}{L_1}\delta(L_1 - L_2)$





a = 0 $\displaystyle\int dL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \delta(L_1 - L_2)$





2d HL

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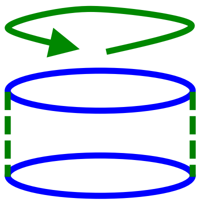
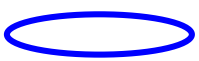
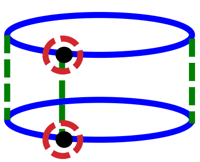


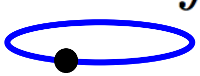
a = 1 $$\int L dL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \frac{1}{L_1}\delta(L_1 - L_2)$$



→ CDT Hamiltonian for an **<u>unmarked</u>** loop

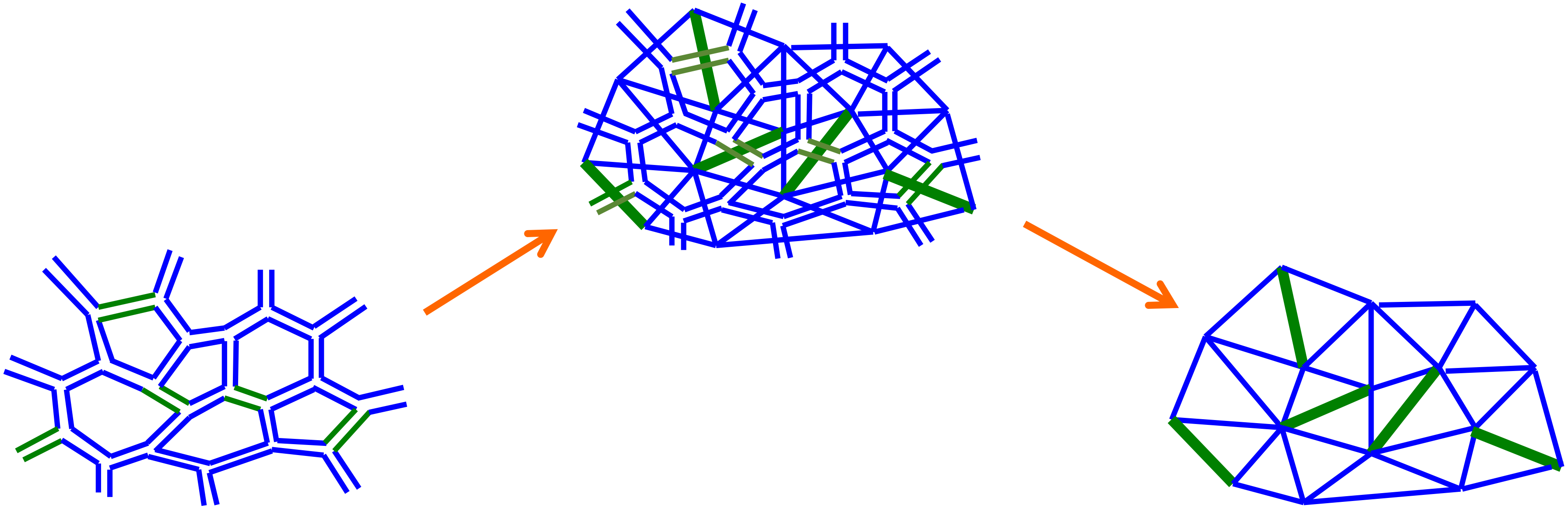


a = 0 $$\int dL|L\rangle\langle L| = 1 \quad \leftrightarrow \quad \langle L_2|L_1\rangle = \delta(L_1 - L_2)$$



→ CDT Hamiltonian for **<u>a marked</u>** loop

Matter-coupled CDT

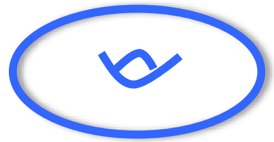
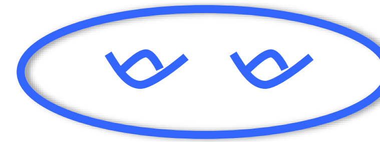


2d quantum gravity

Trivial classical dynamics:

$$\boxed{\int d^2x\sqrt{g}R = 4\pi\chi}$$

χ : Euler number

$\chi = 2$  $\chi = 0$  $\chi = -2$ 

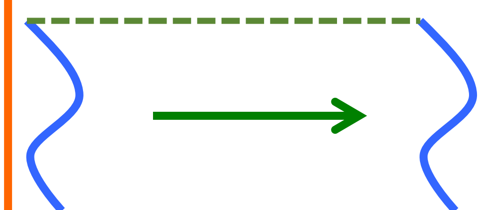
Gauss-Bonnet's theorem, 1848

Non-trivial quantum dynamics:

$$Z = \int [\mathcal{D}g] \exp\left[-\int d^2x\sqrt{g}\left(\frac{\kappa}{4\pi}R + \Lambda\right)\right]$$

$$= \sum_\chi \underbrace{\left(e^{-\kappa}\right)}_{N}{}^{\chi} \int [\mathcal{D}g(\chi)] \underbrace{\left(e^{-\Lambda}\right)}_{\lambda}{}^{\int d^2x\sqrt{g(\chi)}}$$

$$= \sum_\chi N^\chi \int \boxed{[\mathcal{D}g(\chi)]} \lambda^{A(g)}$$

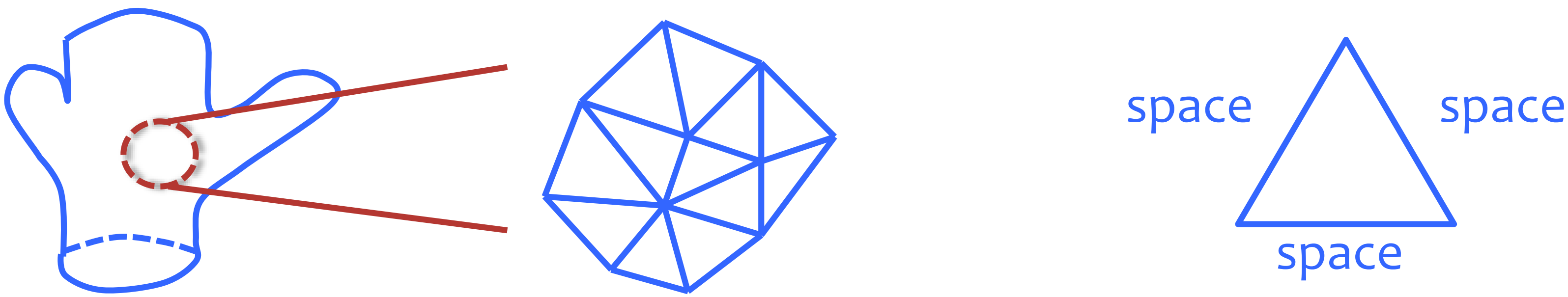


World sheet of string
= 2d geometry
→ Liouville gravity

2d quantum gravity

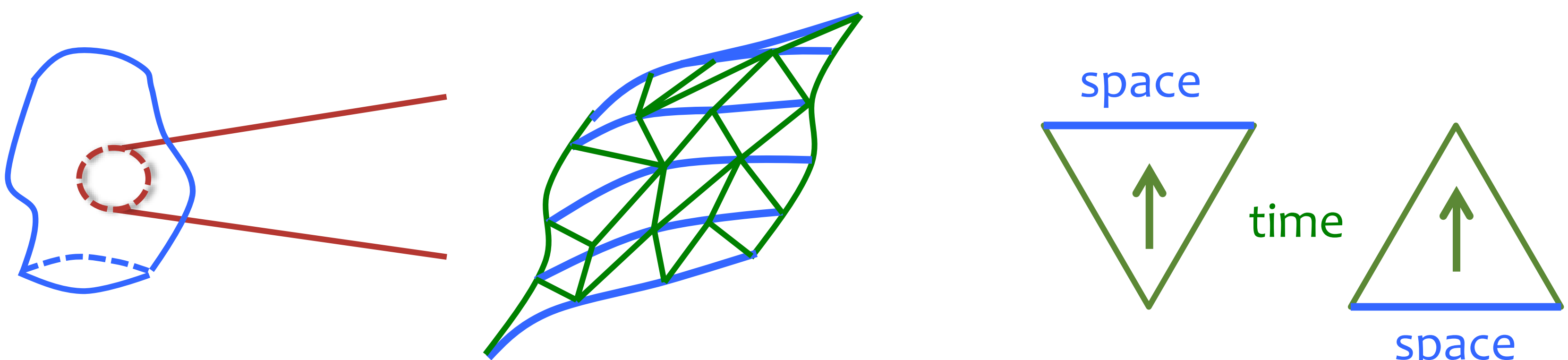
Dynamical Triangulation (DT) → Coupling to matters (Matrix Model)

Path-integral in DT = Sum over all triangulated geometries



Causal Dynamical Triangulation (CDT) → Coupling to matters (?)

Path-integral in CDT = Sum over all triangulated geometries w/ causality



2d quantum gravity

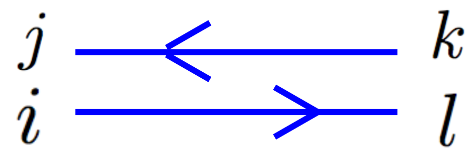
Define the matrix integral (Φ: N X N Hermitian matrix):

$$\int d\phi e^{-N\mathrm{tr}V(\phi)} \quad \text{where} \quad V(\phi) = \frac{1}{2}\phi^2 - \frac{\lambda}{3}\phi^3$$

<u>*Feynman's rule*</u>

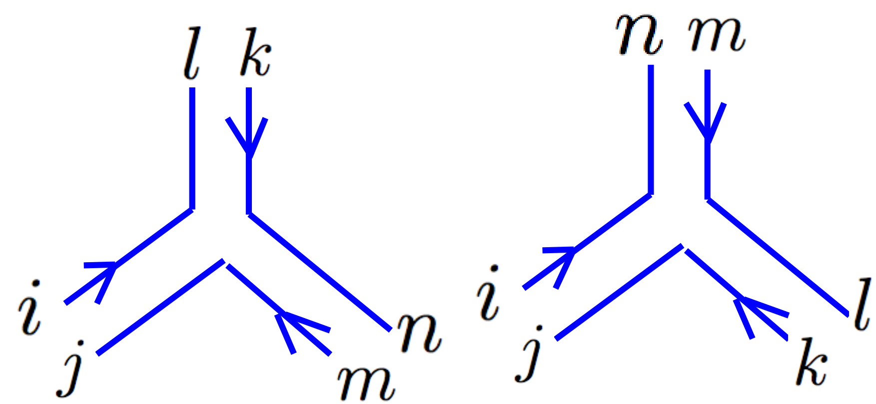
Propagator:

$$\int d\phi\ \phi_i{}^j \phi_k{}^l e^{-\frac{N}{2}\mathrm{tr}(\phi^2)} = \frac{1}{N}\delta_i^l \delta_k^j$$



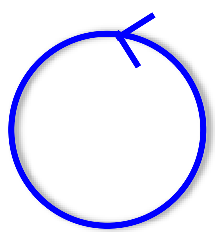
3-point vertices:

$$N\lambda\left(\delta_i^l \delta_k^n \delta_m^j + \delta_i^n \delta_k^j \delta_m^l\right)$$



Loop:

$$\delta_i^i = N$$

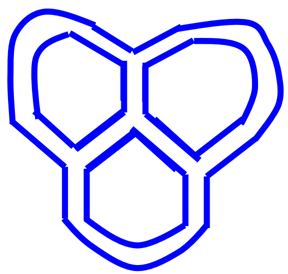


2d quantum gravity

$$Z = \sum_{\chi} \int [\mathcal{D}g(\chi)] \lambda^A N^\chi$$

Liouville

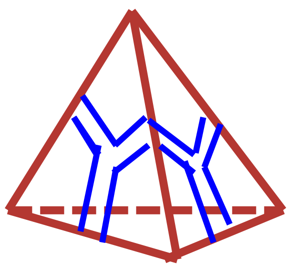
Vacuum diagram:



$$(N\lambda)^{\#(\text{vertex})} \left(\frac{1}{N}\right)^{\#(\text{prop})} N^{\#(\text{loop})}$$

dual

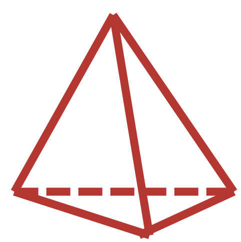
Triangulated surface:

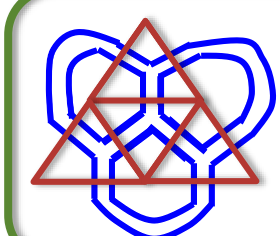


$$(N\lambda)^F \left(\frac{1}{N}\right)^E N^V = \lambda^F N^\chi$$

$$\chi = F - E + V$$

#(face) #(edge) #(vertex)





$\#(\text{vertex}) = F$

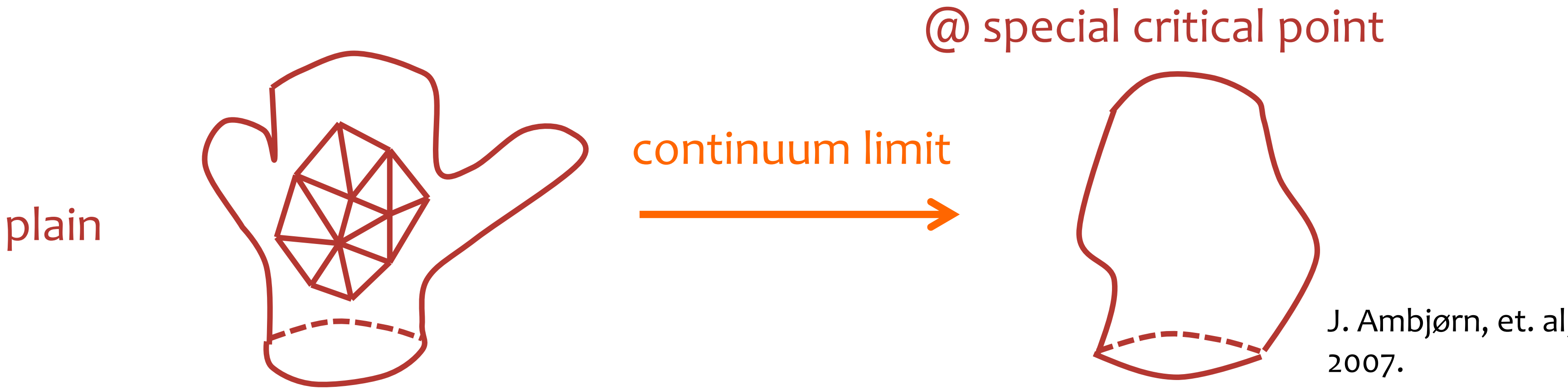
$\#(\text{prop}) = E$

$\#(\text{loop}) = V$

Matter-coupled CDT

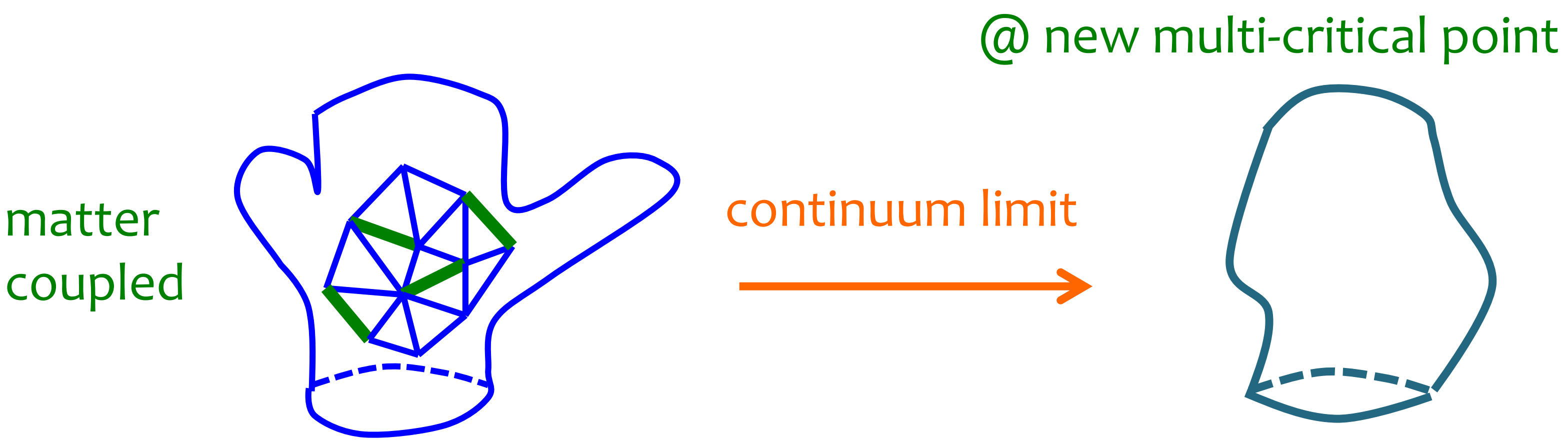
J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

Matter-coupled CDT can be obtained at least in a continuum limit:



Discretized geometry by DT

CDT in continuum limit



Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

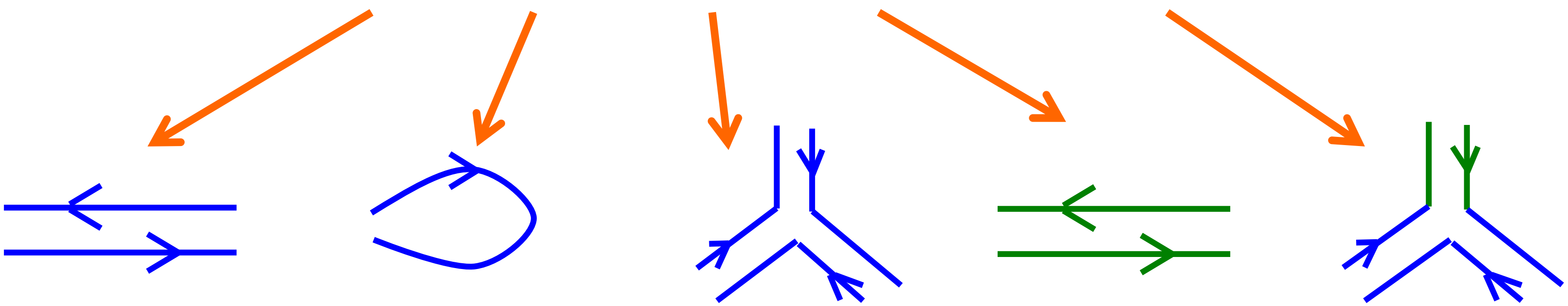
Define the matrix integral (Φ, M: N X N Hermitian matrices):

$$\int d\phi dM e^{-\frac{N}{g_s}\mathrm{tr}U(\phi,M)}$$

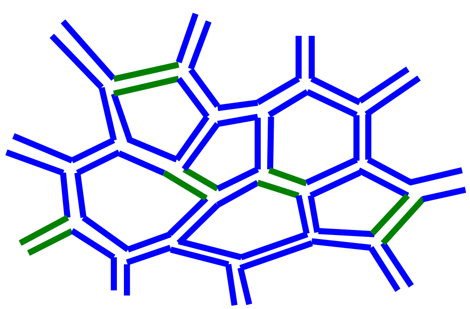
g_s : string coupling

where

$$U(\phi, M) = \frac{1}{2}\phi^2 - \lambda\phi - \frac{\lambda}{3}\phi^3 + \frac{1}{2}M^2 - \lambda\sqrt{\zeta}M\phi^2$$



Sample diagram



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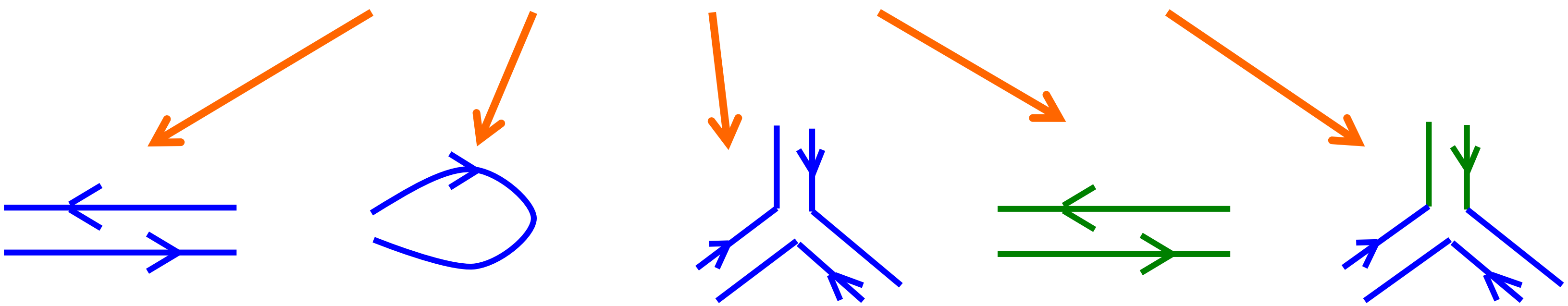
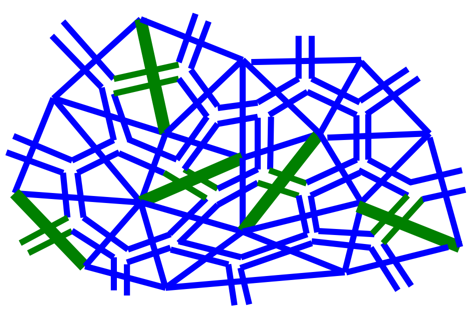


Dual picture



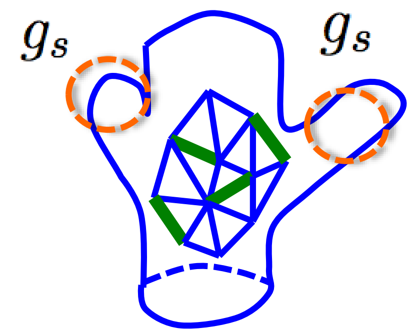
Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

Define the matrix integral (Φ, M: N X N Hermitian matrices):

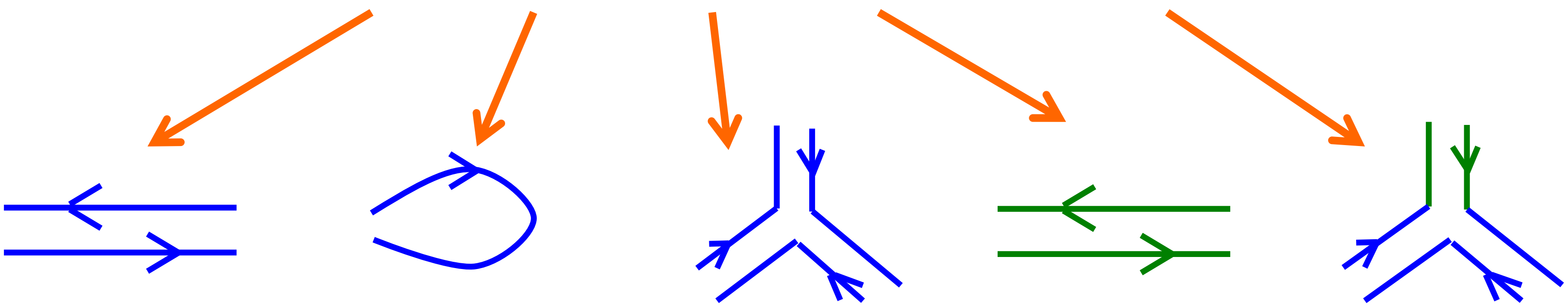
$$\int d\phi dM e^{-\frac{N}{g_s}\mathrm{tr}U(\phi,M)}$$

g_s : string coupling

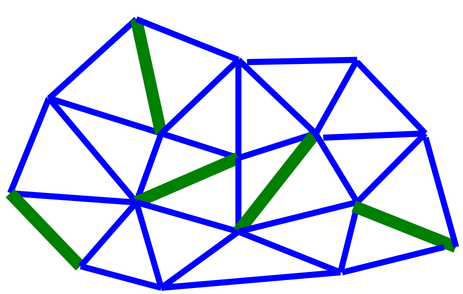


where

$$U(\phi, M) = \frac{1}{2}\phi^2 - \lambda\phi - \frac{\lambda}{3}\phi^3 + \frac{1}{2}M^2 - \lambda\sqrt{\zeta}M\phi^2$$



Sample geometry



→ **Hard dimers** on a dynamical lattice

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

After the Gaussian integration over M, we find

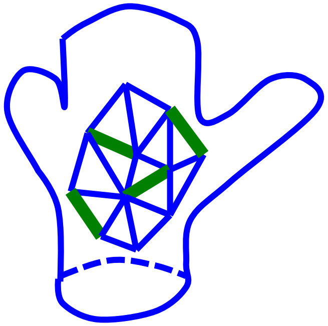
$$\int d\phi e^{-\frac{N}{g_s}\mathrm{tr}V(\phi)} \quad \text{where} \quad V(\phi) = \frac{1}{2}\phi^2 - \lambda\phi - \frac{\lambda}{3}\phi^3 - \frac{1}{2}\lambda^3\zeta\phi^4$$

In the large-N limit, the so-called loop equation is obtained:

$$g_s w(z)^2 - V'(z)w(z) + P_2(z) = 0$$

where

$$w(z) \equiv \frac{1}{N}\mathrm{tr}\left\langle \frac{1}{z-\phi} \right\rangle = \frac{1}{N}\sum_{i=1}^{N}\left\langle \frac{1}{z-\tau_i} \right\rangle$$



One-cut solution:

$$w(z) = \frac{1}{2g_s}\left(V'(z) - \sum_{k=1}^{3} M_k (z-a)^{k-1}\sqrt{(z-a)(z-b)} \right)$$

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

One-cut solution:

$$w(z) = \frac{1}{2g_s}\left(V'(z) - \sum_{k=1}^{3} M_k (z-a)^{k-1}\sqrt{(z-a)(z-b)}\right)$$

Since w(z) = 1/z + O(1) in |z| >> a-b,

$$w(z) = \sum_{k=0}^{\infty} C_{3-k} z^{3-k}$$

$$w(z) \equiv \frac{1}{N}\mathrm{tr}\left\langle \frac{1}{z-\phi}\right\rangle = \frac{1}{N}\sum_{i=1}^{N}\left\langle \frac{1}{z-\tau_i}\right\rangle$$

where

$$C_3 = C_2 = C_1 = C_0 = 0, \quad C_{-1} = 1.$$

5 equations

5 unknowns, $\{a, b, M_1, M_2, M_3\}$

can be determined by 3 couplings, $\{g_s, \lambda, \zeta\}$

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

Approach to the multi-critical point!

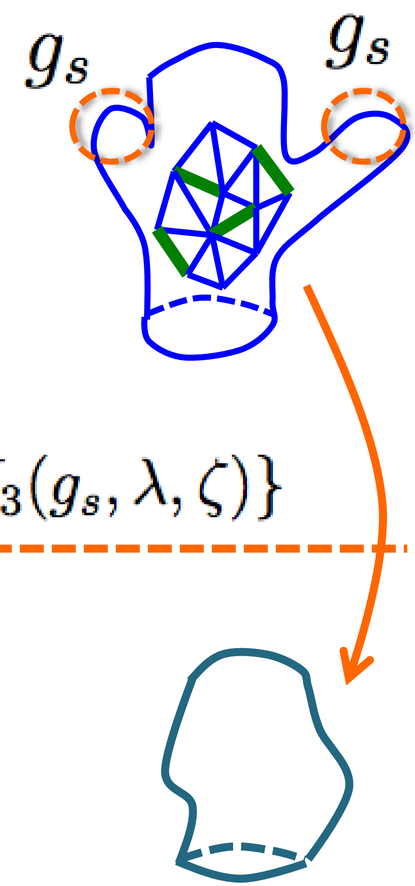
$\{a, b, M_1, M_2, M_3\}$ 5 unknowns

$\{g_s, \lambda, \zeta\}$ 3 coupling constants

+

$$\boxed{C_3 = C_2 = C_1 = C_0 = 0, \quad C_{-1} = 1.}$$

5 equations



$$\{a(g_s,\lambda,\zeta), b(g_s,\lambda,\zeta), M_1(g_s,\lambda,\zeta), M_2(g_s,\lambda,\zeta), M_3(g_s,\lambda,\zeta)\}$$

+

$M_1 = 0$ Gravity limit

$M_2 = 0$ Matter limit

$g_{s*} = 0$ CDT limit

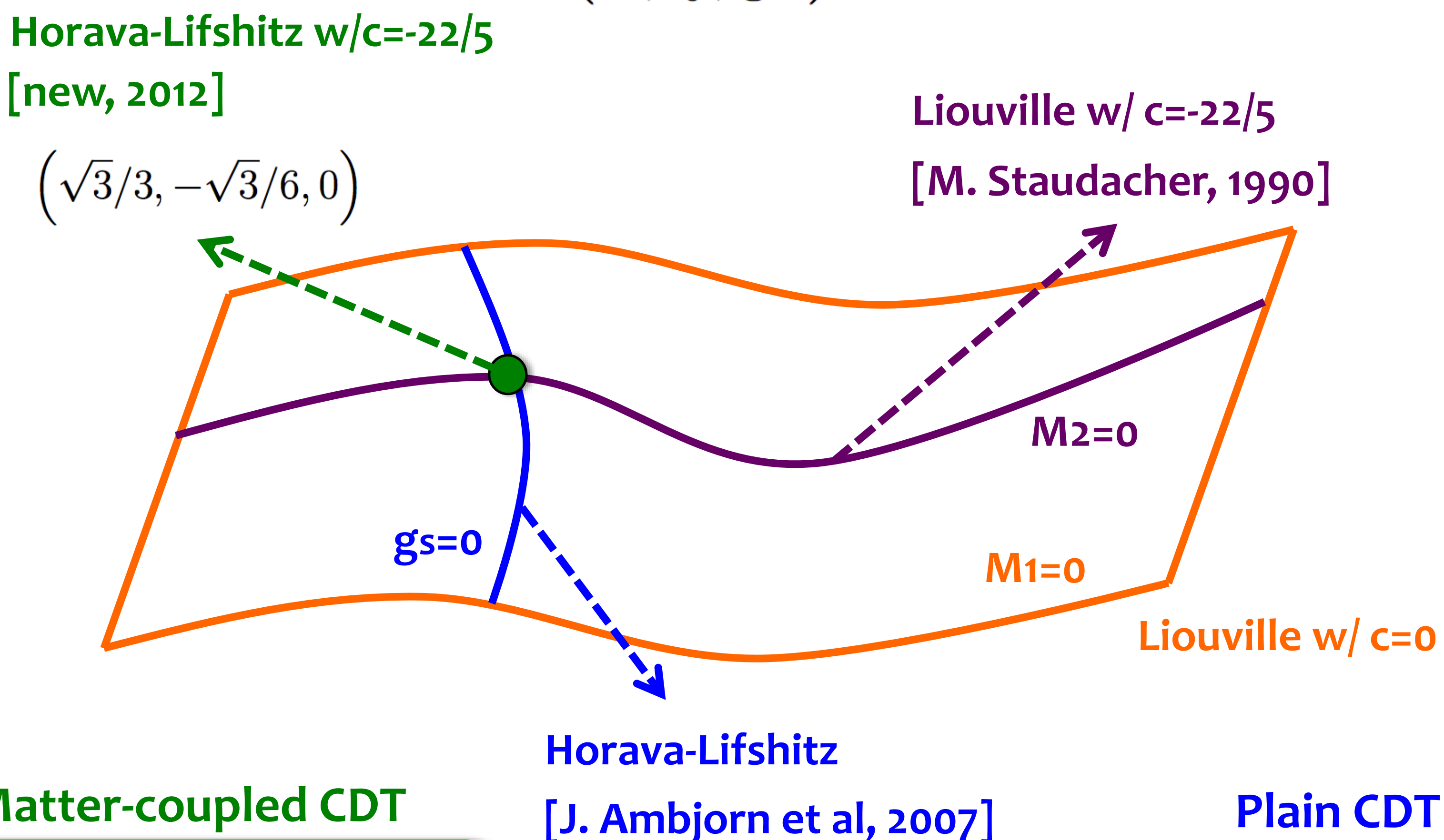
$$\{g_{s*}, \lambda_*, \zeta_*\} = \{0, \sqrt{3}/3, -\sqrt{3}/6\}$$

a new 3rd-order multi-critical point!

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

$$(\lambda, \zeta, g_s)$$



Matter-coupled CDT

$$W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \Lambda_{\rm cdt}^{1/3}}$$

Plain CDT

$$W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \sqrt{2\Lambda_{\rm cdt}}}$$

Where to go

2d CDT has been understood quite well:

[1] A corresponding field theory is the projectable Horava-Lifshitz gravity.

→ 2d black hole in CDT K. Hirochi, T. Katsuragawa, H. Suenobu and Y.S.

→ non-projectable HL = Generalised CDT?

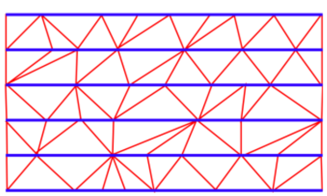
[2] Matters can be coupled using matrix model techniques.

→ CDT coupled to Ising model

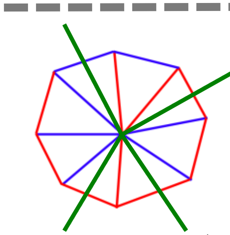
What is an implication from 2d toy model to 4d world?

→ 4d CDT = 4d (projectable) Horava-Lifshitz?

It seems not… (CDT w/o foliation, S. Jordan and R. Loll, 2013.)



CDT (global causality)



CDT w/o foliation (local causality)

Where to go

At least in 3 dimensions,
CDT and CDT w/o foliation are in the same universality class.

S. Jordan and R. Loll, arXiv: 1307.5469.

Status of 2d CDT:

GOOD: Exactly solvable (w/ or w/o matters)

BAD: May not be a good toy model for 4d CDT

Future directions:

[1] Construct an analytic method for 2d CDT w/o foliation.

T. Tanaka and Y.S. → **Generalisation of Matrix Model**

[2] Construct an analytic method for higher-dimensional CDT.

N. Sasakura and Y.S. → **Tensor Models!!**

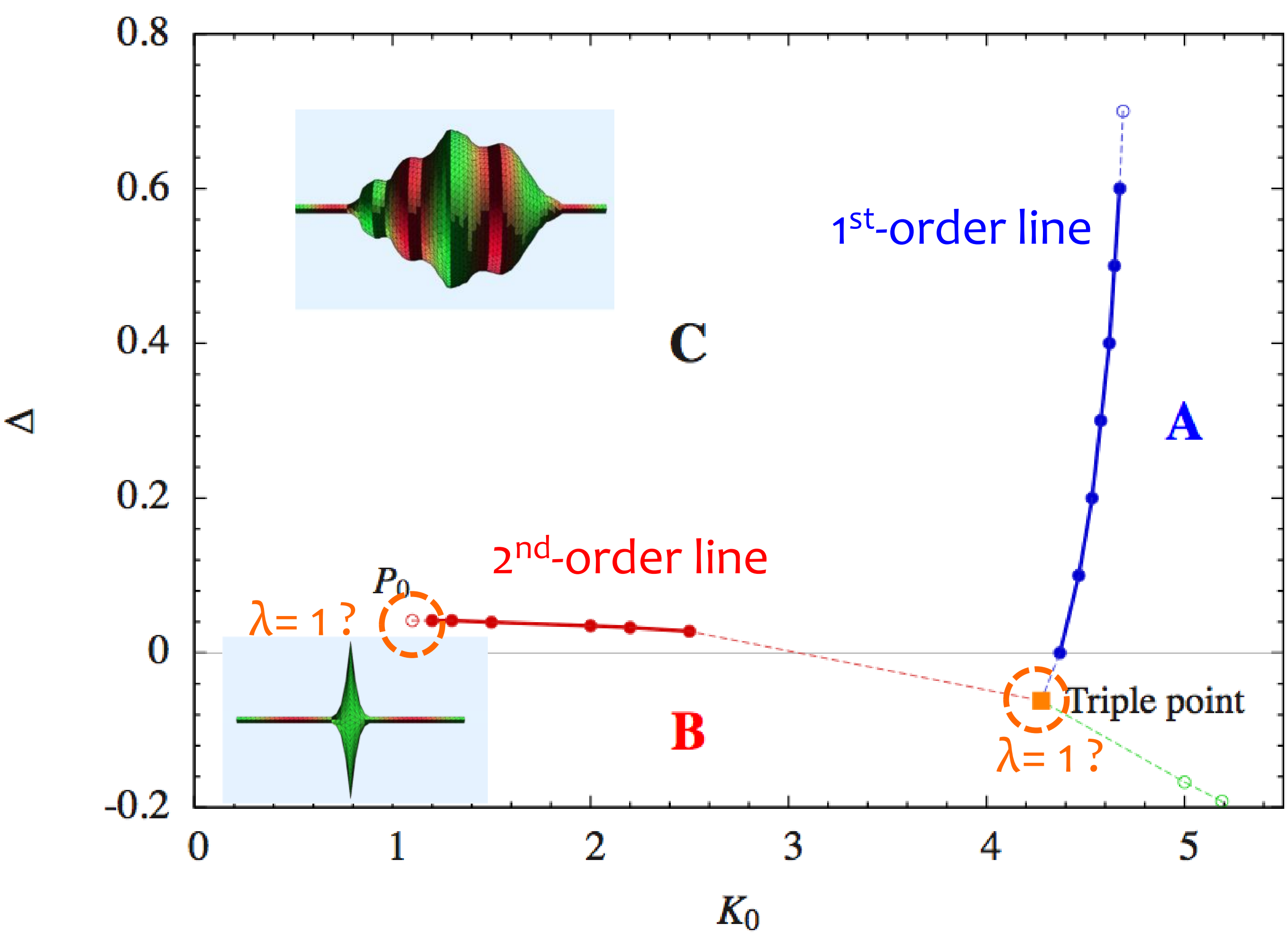
Summary (2)

	$W(Z_{\rm cdt})$	σ	d_H
Horava-Lifshitz (plain CDT)	$W(Z_{\rm cdt}) = \dfrac{1}{Z_{\rm cdt} + \sqrt{2\Lambda_{\rm cdt}}}$	-	2
Horava-Lifshitz coupled to non-unitary CFT (CDT coupled to dimers)	$W(Z_{\rm cdt}) = \dfrac{1}{Z_{\rm cdt} + \Lambda_{\rm cdt}^{1/3}}$	1/2	3/2

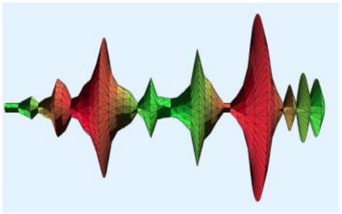
Magnetization: $$\frac{d\log\lambda_*}{d\zeta} \sim (\zeta - \zeta_*)^{\sigma}$$

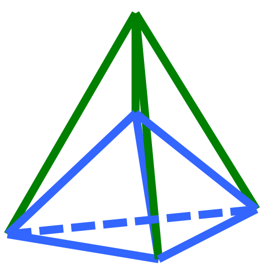
Hausdorff dimension: $$d_H = \frac{m}{m-1}$$ m=2 (plain), m=3 (dimer)

Outlook for 4D CDT – a wild guess



J. Ambjørn, et. al, Phys. Lett. B 690.





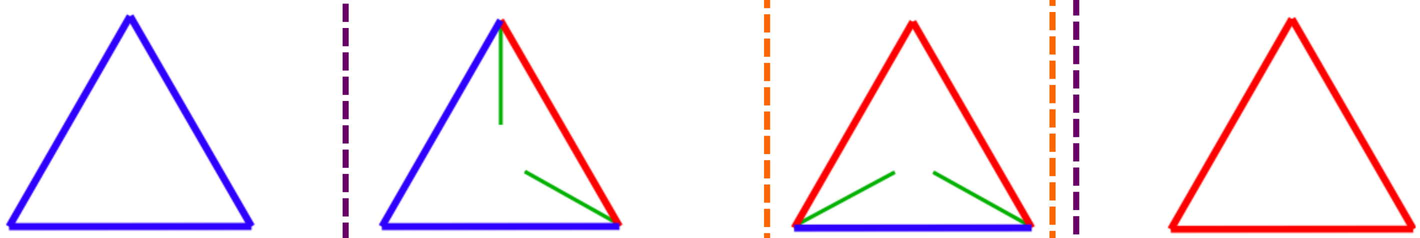
4d CDT: $$S^{(e)}_{\mathrm{CDT}} = -(\kappa_0 + 6\Delta) N_0 + \kappa_4 (N_4^{(4,1)} + N_4^{(3,2)}) + \Delta(2N_4^{(4,1)} + N_4^{(3,2)})$$

4d HL: $$S_{\mathrm{HL}} = \frac{1}{\kappa}\int dt d^3x \, \sqrt{h} N (K_{ij}K^{ij} - \boxed{\lambda} K^2 - \mathcal{V}[h_{ij}])$$

CDT without foliation

S. Jordan and R. Loll, 2013.

Four kinds of possible triangles with space-like and/or time-like edges:



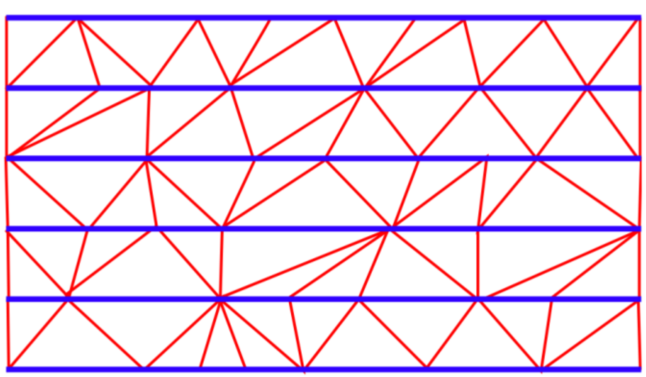
DT (Euclidean)

CDT

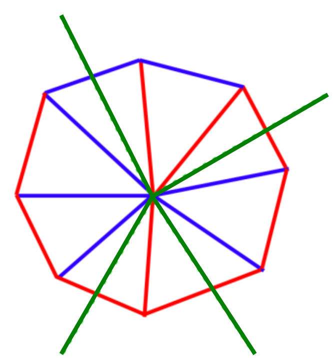
CDT w/o foliation (Lorentzian)

Global causality vs. Local causality:

CDT (global causality)



CDT w/o foliation (local causality)



Space-like edge:

$$\ell_s^2$$

Time-like edge:

$$\ell_t^2 = -\alpha \ell_s^2$$

Light ray:

CDT without foliation

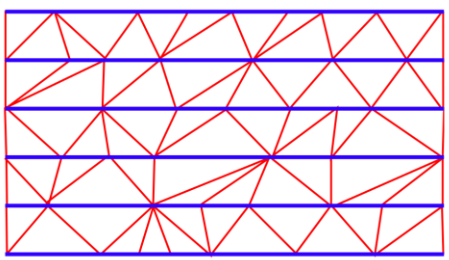
S. Jordan and R. Loll, 2013.

At least in 3 dimensions,
CDT and CDT w/o foliation are in the same universality class.

S. Jordan and R. Loll, arXiv: 1307.5469.

2d model may not be a good toy model for 4d CDT.

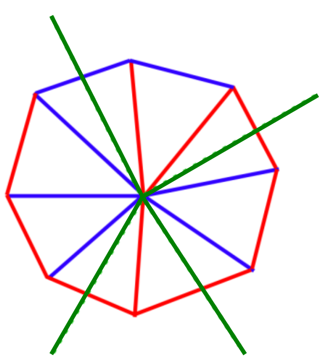
2d CDT



Same universality

2d Horava-Lifshitz

2d CDT
w/o foliation



NOT Horava-Lifshitz
NOT Liouville

Short Summary

2d CDT turns out to be the 2d projectable Horava-Lifshitz quantum gravity:

$$\boxed{S_{\rm HL} = \int dt\, dx\, N\gamma\left[(1-\lambda)K^2 - 2\Lambda\right]}$$

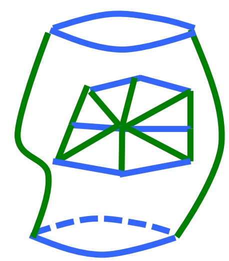
where

$$N = N(t)$$

$$\Lambda > 0 \quad \lambda < 1$$

DT

CDT



continuum

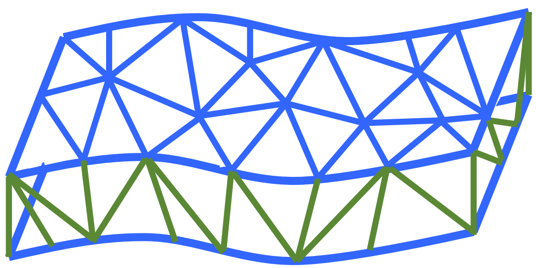
Liouville gravity

Known

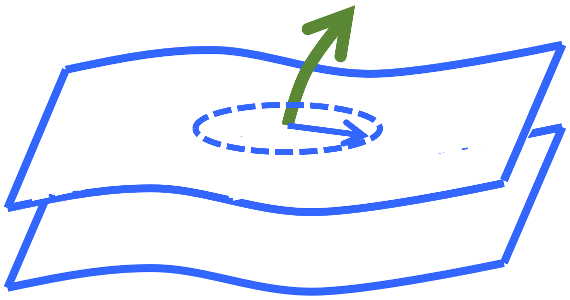
Projectable
Horava-Lifshitz gravity

Our work

Causal Dynamical Triangulations vs. Horava-Lifshitz gravity



vs.



Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

Solve

$$C_3 = C_2 = C_1 = C_0 = 0, \quad C_{-1} = 1 \quad \& \quad M_1 = M_2 = 0$$

around small g_s

$$g_s = G_s \epsilon^4$$

Then we find

$$\lambda = \lambda_* + \tilde{\Lambda}\epsilon^2 - \Lambda\epsilon^3, \qquad \zeta = \zeta_* - \frac{1}{2}\tilde{\Lambda}\epsilon^2,$$

$$a = a_* - A\epsilon, \qquad b = a_* - B\epsilon, \qquad z = a_* + \epsilon Z,$$

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S., 2012.

Continuum wave func. of the Universe:

$$w(z) = \frac{1}{2g_s}\left(V'(z) - \sum_{k=1}^{3} M_k (z-a)^{k-1}\sqrt{(z-a)(z-b)}\right)$$

Continuum limit

$$w(z) = \frac{1}{\epsilon} W(Z) + \cdots$$

$\tilde{\Lambda}_{\rm cdt} = 0$, CDT background

$G_s \to 0$, Classical limit

Matter-coupled CDT

$$\boxed{W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \Lambda_{\rm cdt}^{1/3}}}$$

Plain CDT

$$\boxed{W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \sqrt{2\Lambda_{\rm cdt}}}}$$

$$Z = Z_{\rm cdt} - \alpha G_s^{1/4}, \quad \Lambda = \Lambda_{\rm cdt} - \gamma G_s^{3/4}, \quad \tilde{\Lambda} = \tilde{\Lambda}_{\rm cdt} - \alpha G_s^{1/4} \quad \alpha = \frac{2}{5^{1/4}\sqrt{3}}, \quad \beta = \frac{\sqrt{5}}{9\sqrt{3}}, \quad \gamma = \frac{16\sqrt{5}}{27\sqrt{3}}$$

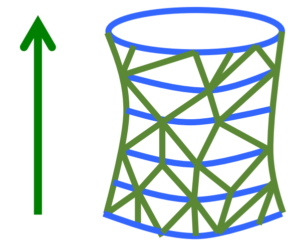
FAQ

Q1. Is the diffeomorphism broken in a lattice approach?

No. Because a lattice gravity is quantum gravity without coordinates.

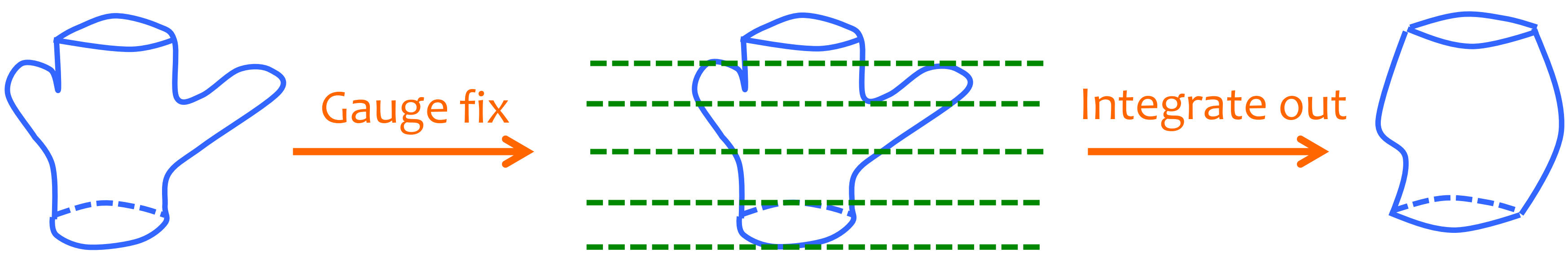
Q2. Is CDT a background-independent formulation?

It seems No. Because there is a global time direction.



Q3. So, then what's the status of CDT?

Probably, an effective theory arising from integrating out baby universes.



0. INTRODUCTION

What is the IR (field-theoretical) description of CDT?

CDT looks like Horava-Lifshitz gravity (HL)…

CDT: Lattice quantum gravity w/ causality (**foliation)**

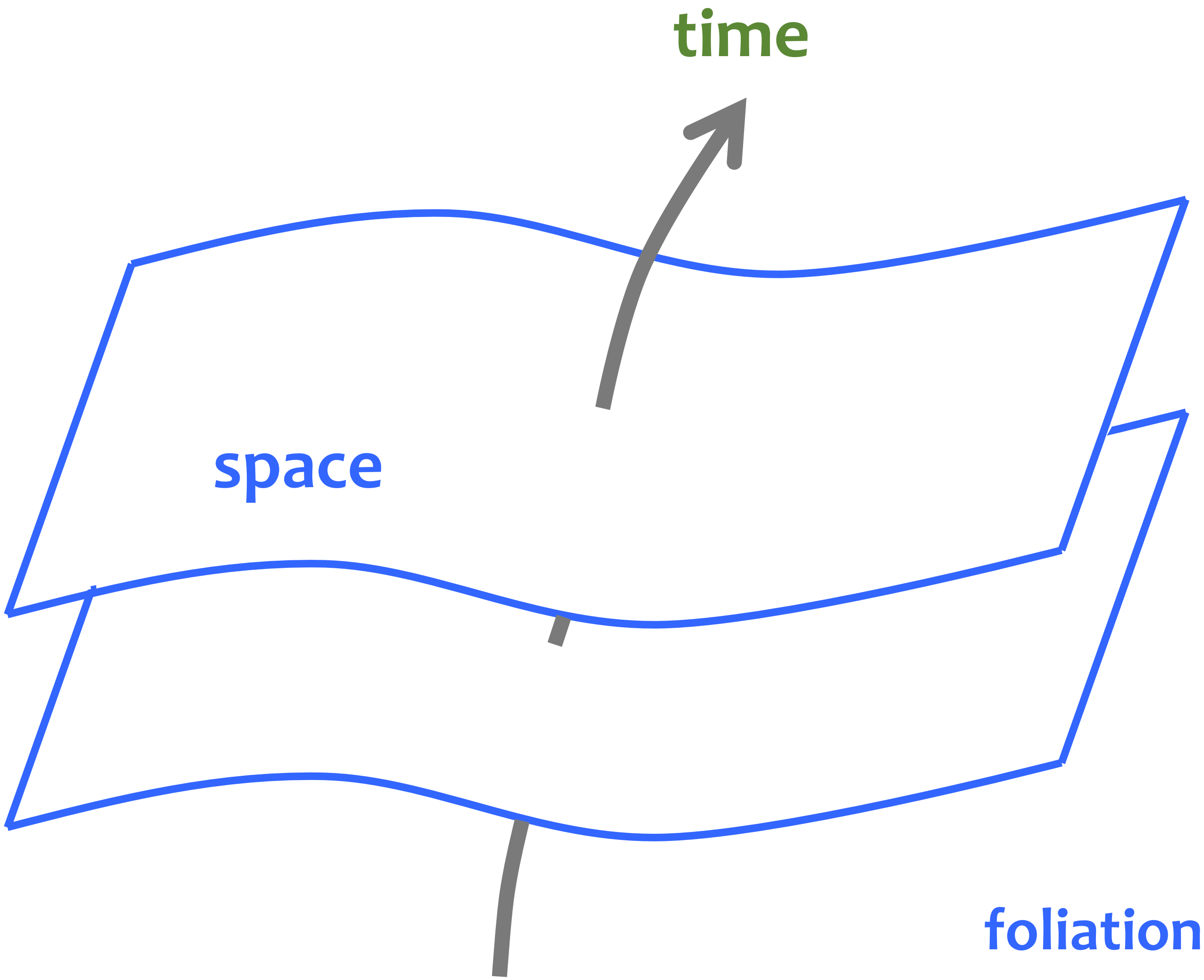
common

HL: Quantum gravity w/ anisotropic scaling (**foliation**)

In our work, we have determined that

2D CDT is 2D projectable HL quantum gravity!!

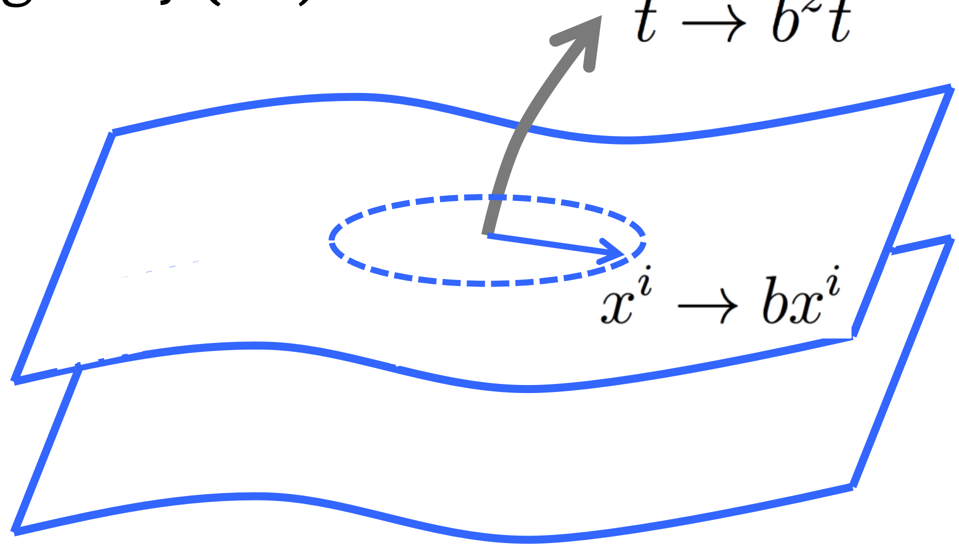
space-time → space [time]



foliation (葉層)

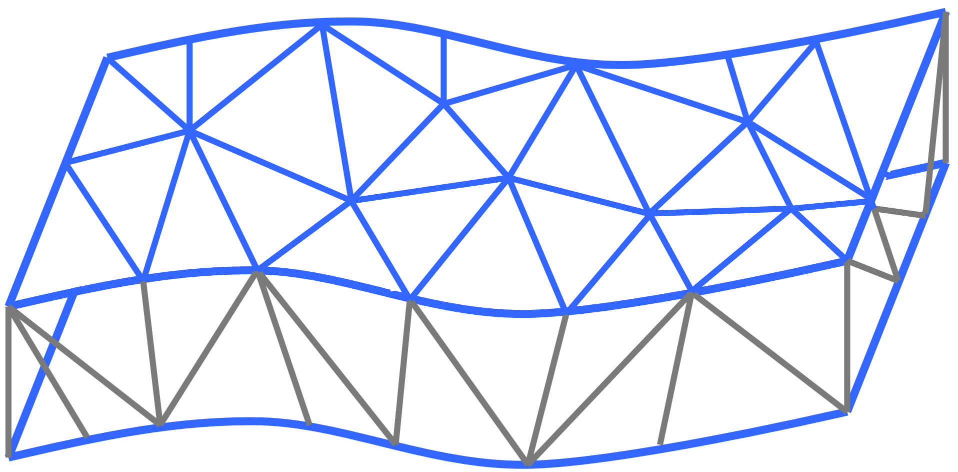
Horava-Lifshitz gravity (HL):

anisotropy



Causal Dynamical Triangulations (CDT):

causality



0. INTRODUCTION

What is the IR description of CDT?

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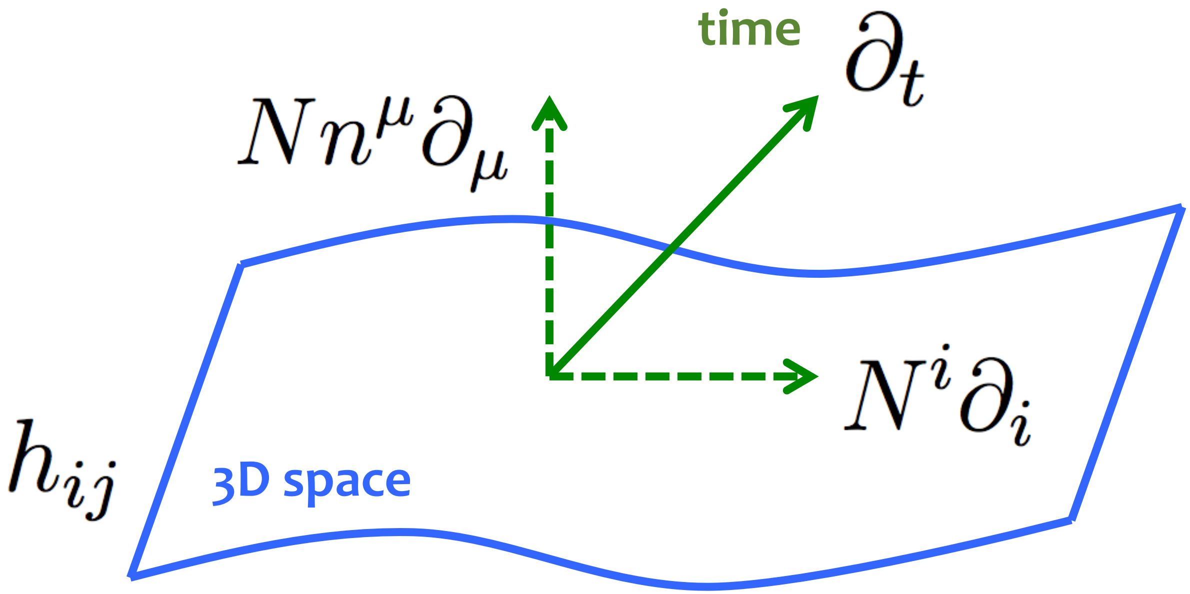
2D CDT is 2D projectable HL quantum gravity!!

2. 2D HL

ADM decomposition:



$$R^{(4)} = K_{ij}K^{ij} - K^2 + R$$

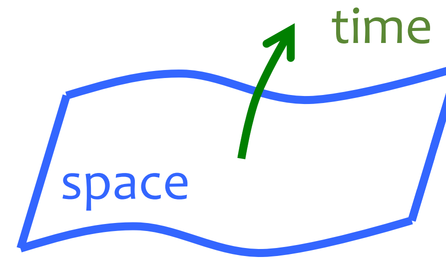
4D Einstein (covariant) gravity:

$$\boxed{S_{\rm ADM} = \frac{1}{\kappa}\int dt d^3x\ \sqrt{h}N(K_{ij}K^{ij} - K^2 + R - 2\Lambda)}$$

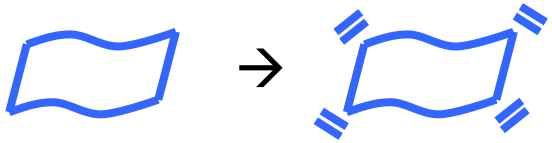
2. 2D HL

4D Einstein (covariant) gravity:

$$S_{\mathrm{ADM}} = \frac{1}{\kappa}\int dt d^3x \, \sqrt{h} N (K_{ij}K^{ij} - K^2 + R - 2\Lambda)$$



ADM

[symmetry]  changeable

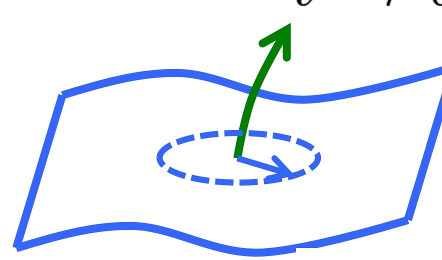
[Newton's constant] $[\kappa] = -2$

4D HL (anisotropic) gravity:

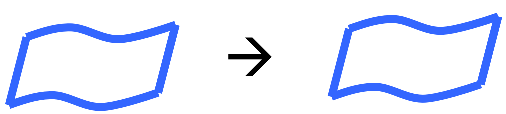
higher spatial curvature

$$S_{\mathrm{HL}} = \frac{1}{\kappa}\int dt d^3x \, \sqrt{h} N (K_{ij}K^{ij} - \lambda K^2 - \underline{\mathcal{V}[h_{ij}]})$$

$t \to b^z t$



$x^i \to b x^i$

[symmetry]  fixed

[Newton's constant] $[\kappa] = z - 3$ renormalizable

2. 2D HL

4D HL:

$$\boxed{S_{\rm HL} = \frac{1}{\kappa}\int dt d^3x\, \sqrt{h} N(K_{ij}K^{ij} - \lambda K^2 - \mathcal{V}[h_{ij}])}$$

$$\downarrow$$

2D HL:

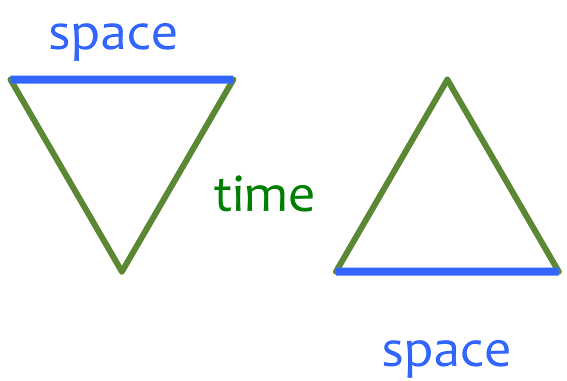
$$\boxed{S_{\rm HL} = \int dt dx \sqrt{h} N[(1-\lambda)K^2 - 2\Lambda]}$$

$$K_{11}K^{11} = K^2$$

No higher derivative terms

1. 2D CDT

Triangle lattice (UV cutoff):



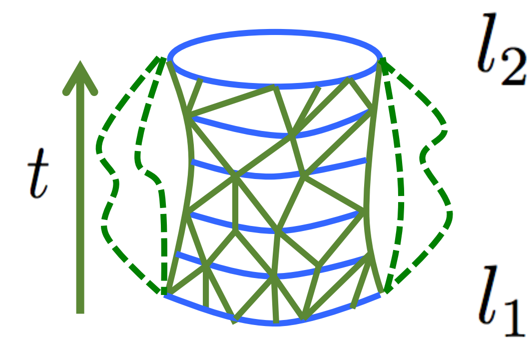
lattice spacing (ε) → fixed

triangulations (T) → dynamical

(= how to divide geometry by triangles)

Metric path-integral → sum over triangulations (T): $\int \mathcal{D}g \to \sum_T$

$$G(l_2, l_1; t) = \sum_{T(l_1,l_2)} e^{-\lambda n(T)}$$
$$= \sum_n e^{-\lambda n} w_{n,l_1,l_2}$$



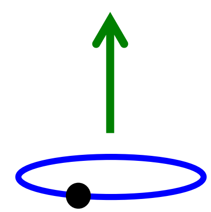
λ : cosmological constant   # (△) = n # (T) = $w_{l_1,l_2,n}$

2. 2D HL

Quantum Hamiltonian for HL

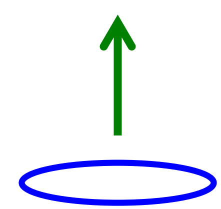
$$\boxed{\hat{H} = -L\frac{\partial^2}{\partial L^2} - 2a\frac{\partial}{\partial L} - \frac{a(a-1)}{L} + \tilde{\Lambda}L} \qquad \tilde{\Lambda} = \frac{\Lambda}{2(1-\lambda)}$$

$$a = 0 \;\leftrightarrow\; \underline{dL} \;\leftrightarrow\; \hat{H} = -L\frac{\partial^2}{\partial L^2} + \tilde{\Lambda}L$$



→ CDT Hamiltonian for **a marked** loop

$$a = 1 \;\leftrightarrow\; \underline{LdL} \;\leftrightarrow\; \hat{H} = -L\frac{\partial^2}{\partial L^2} - 2\frac{\partial}{\partial L} + \tilde{\Lambda}L$$



→ CDT Hamiltonian for an **unmarked** loop

3. SUMMARY

2D CDT turns out to be the 2D projectable Horava-Lifshitz quantum gravity:

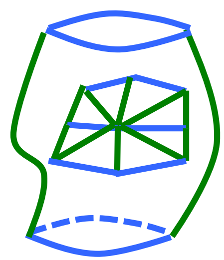
$$\boxed{S_{\mathrm{HL}} = \int dt\, dx\, N\gamma \left[(1-\lambda)K^2 - 2\Lambda\right]}$$

where $N = N(t)$

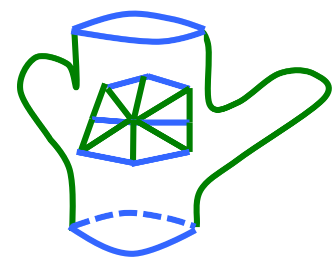
$\Lambda > 0 \quad \lambda < 1$

DT

CDT



Generalised CDT



continuum

continuum

?

Liouville gravity

Projectable Horava-Lifshitz gravity

Non-projectable Horava-Lifshitz gravity?

Known

Our work

Wild guess

3. SUMMARY & CONJECTURE

2D CDT turns out to be the 2D projectable Horava-Lifshitz quantum gravity:

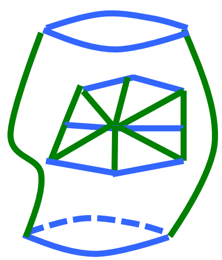
$$\boxed{S_{\mathrm{HL}} = \int dt\, dx\, N\gamma\left[(1-\lambda)K^2 - 2\Lambda\right]}$$

where

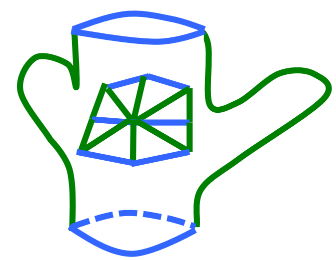
$$N = N(t)$$

$$\Lambda > 0 \quad \lambda < 1$$

CDT



Generalised CDT



continuum

?

Projectable
Horava-Lifshitz gravity

Our work

Non-projectable
Horava-Lifshitz gravity?

Wild guess

3. SUMMARY

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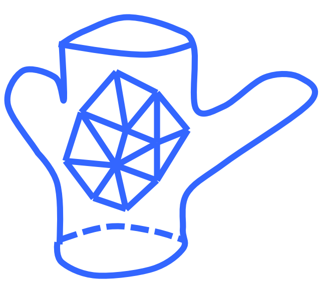
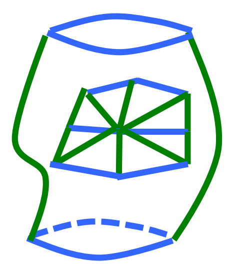
DT



CDT



continuum

Liouville gravity

Infinite outgrowth of baby universes

Projectable Horava-Lifshitz gravity

No baby universe

Future works:

(1) Non-projectable HL = Generalised CDT?

(2) BH in 2D CDT?

(3) What about 4D?

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S.

Solve

$$C_3 = C_2 = C_1 = C_0 = 0, \quad C_{-1} = 1 \quad \textbf{\&} \quad M_1 = M_2 = 0$$

around small gs

$$g_s = G_s \epsilon^4$$

Then we find

$$\lambda = \lambda_* + \tilde{\Lambda}\epsilon^2 - \Lambda\epsilon^3, \qquad \zeta = \zeta_* - \frac{1}{2}\tilde{\Lambda}\epsilon^2,$$

$$a = a_* - A\epsilon, \qquad b = a_* - B\epsilon, \qquad z = a_* + \epsilon Z,$$

Matter-coupled CDT

J. Ambjørn, L. Glaser, A. Görlich and Y.S.

Continuum wave func. of the Universe:

$$w(z) = \frac{1}{2g_s}\left(V'(z) - \sum_{k=1}^{3} M_k (z-a)^{k-1}\sqrt{(z-a)(z-b)}\right) \longrightarrow w(z) = \frac{1}{\epsilon} W(Z) + \cdots$$

where

$$W(Z_{\rm cdt}) = \frac{\left(\Lambda_{\rm cdt} + 2\sqrt{3}\tilde{\Lambda}_{\rm cdt} + \frac{1}{9} Z_{\rm cdt}^3\right) - P_2(Z_{\rm cdt})\sqrt{(Z_{\rm cdt}+A_{\rm cdt})(Z_{\rm cdt}+B_{\rm cdt})}}{2G_s}$$

$\tilde{\Lambda}_{\rm cdt} = 0$ CDT background

$G_s \to 0$ Classical limit

Pure case

$$W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \Lambda_{\rm cdt}^{1/3}}$$

$$W(Z_{\rm cdt}) = \frac{1}{Z_{\rm cdt} + \sqrt{2\Lambda_{\rm cdt}}}$$

$$Z = Z_{\rm cdt} - \alpha G_s^{1/4}, \quad \Lambda = \Lambda_{\rm cdt} - \gamma G_s^{3/4}, \quad \tilde{\Lambda} = \tilde{\Lambda}_{\rm cdt} - \alpha G_s^{1/4}$$

$$A = A_{\rm cdt} + \alpha G_s^{1/4}, \quad B = B_{\rm cdt} + \alpha G_s^{1/4}$$

$$\alpha = \frac{2}{5^{1/4}\sqrt{3}}, \quad \beta = \frac{\sqrt{5}}{9\sqrt{3}}, \quad \gamma = \frac{16\sqrt{5}}{27\sqrt{3}}$$

Wilsonian renormalization

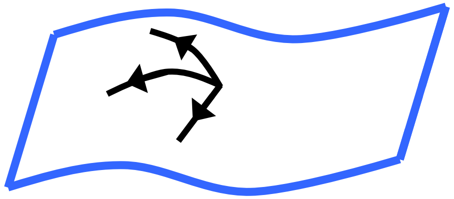
If and only if a cutoff theory is fundamental, it should possess

(1) Infinite UV cutoff limit characterized via a fixed point in RG flow:

(A) Gaussian fixed point (GF) → canonical scaling dim.

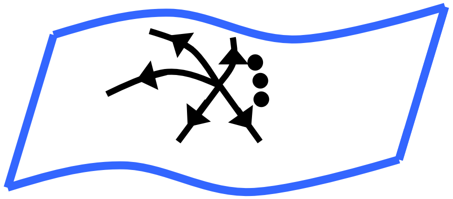
(B) Non-Gaussian fixed point (NGF) → non-canonical scaling dim.

(2) Predictability



Finite number of couplings

→ Predictable!!



Infinite number of couplings

→ Non predictable

Summary (1)

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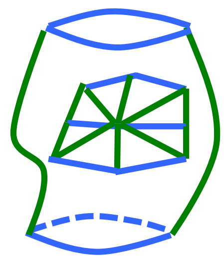
DT

Integrate out baby universes*

CDT



continuum

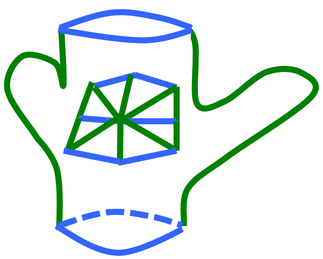
Liouville gravity

Known

Projectable Horava-Lifshitz gravity

Our work

Generalised CDT



?

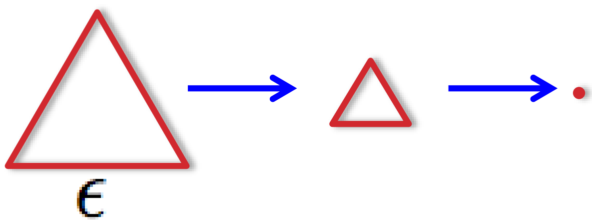
Non-projectable Horava-Lifshitz gravity?

Wild guess

(*) J. Ambjørn, J. Correia, C. Kristjansen and R. Loll, 1999

2d quantum gravity

Continuum limit:



&

(trangle) = k (= F) = infinity

under the fixed volume V.

This can be realized
by fine-tuning the coupling constants to critical values.

Physical volume:

$$V = \epsilon^2(\lambda)k(\lambda) = \epsilon^2(\lambda_*)k(\lambda_*)$$

critical value