熱的な量子純粋状態を用いた統計力学の定式化

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SS and A.Shimizu, PRL 108, 240401 (2012)
SS and A.Shimizu, PRL 111, 010401 (2013)

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Principle of Equal Weight

Boltzmann formula

Microscopic View

All the microstates that have energy E

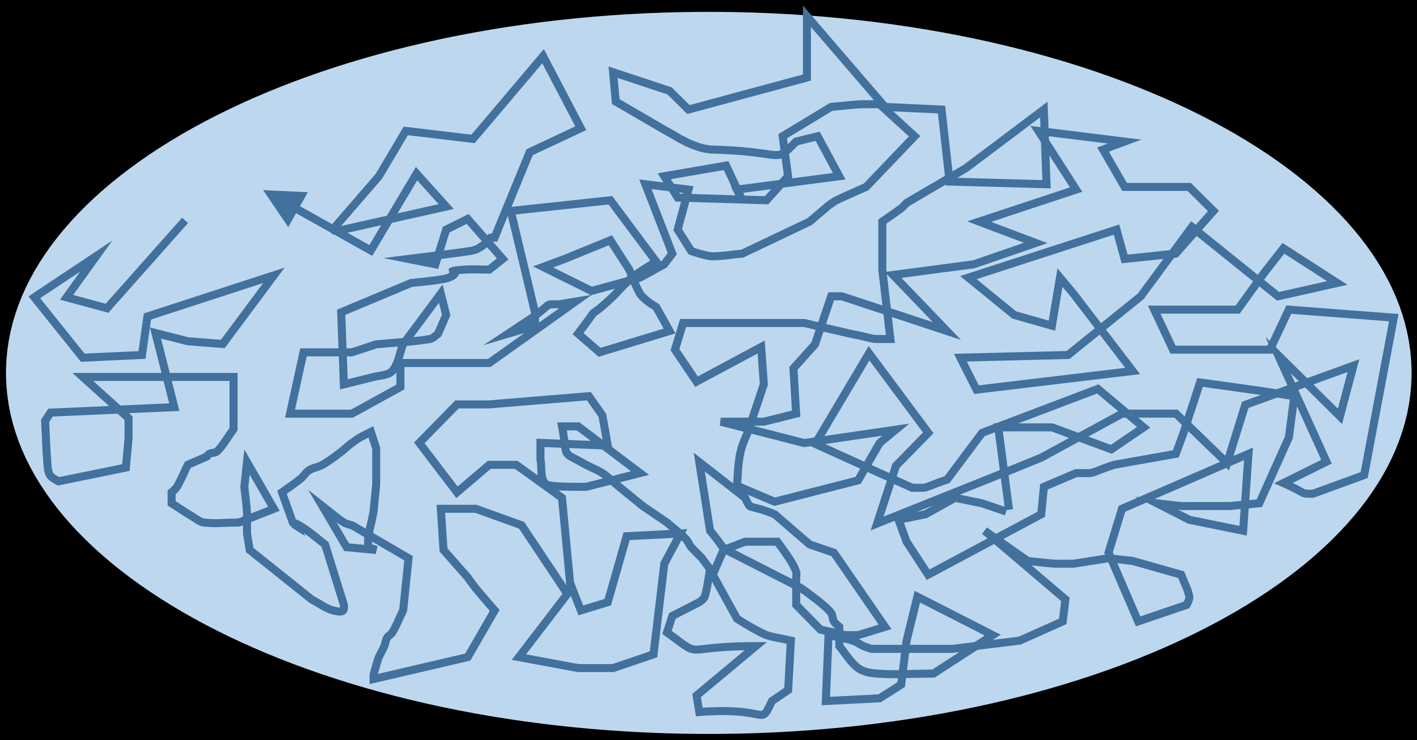
Principle of **Equal Weight**:

When all the microstates emerge in the same probability, the average value give the equilibrium value.

How can we justify this principle?

Explanation using the **Ergordic Hypothesis**

All the microstates that have energy *E*



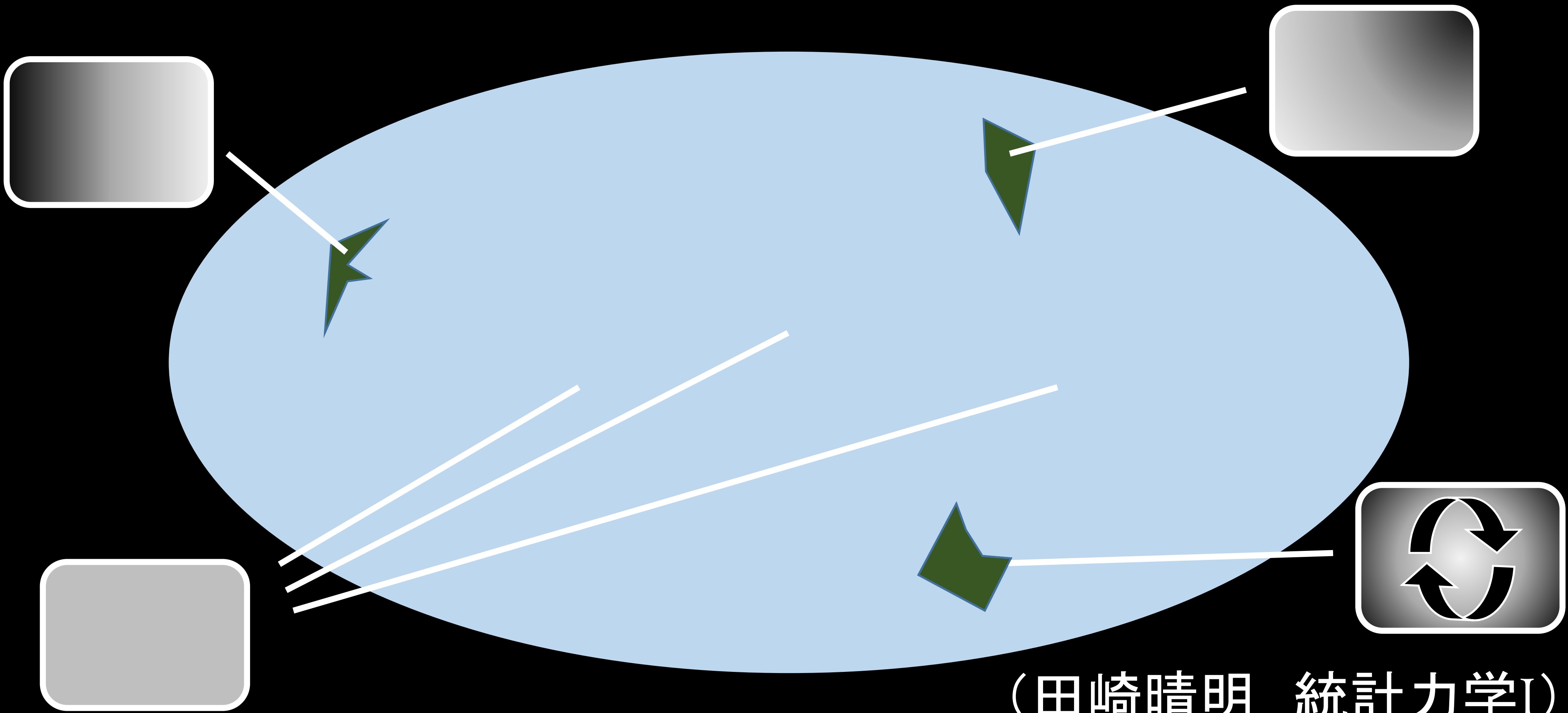
Ergordic theory gives the fruitful mathematics

But…

It takes too much time (physically nonsense)

It gets harder as the system size increases

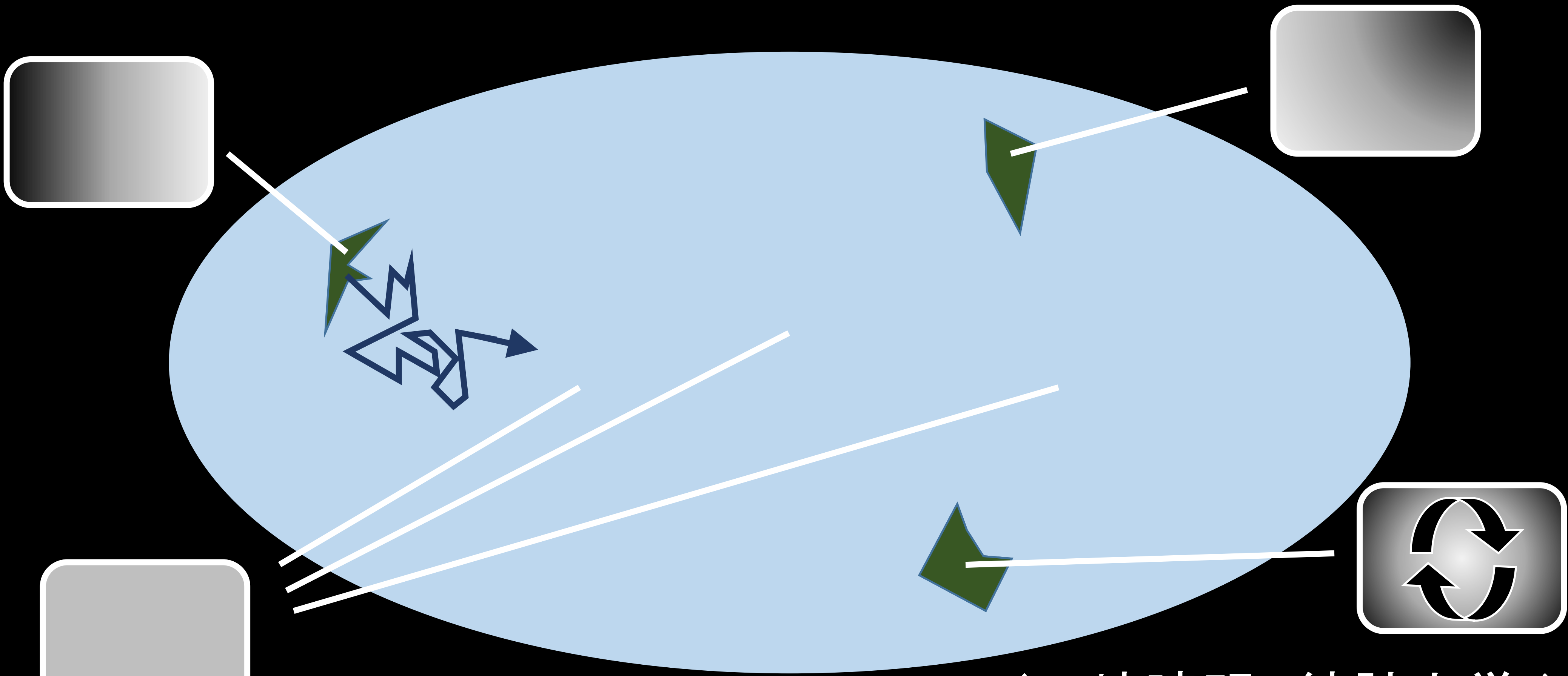
Explanation using the **Typicality**



（田崎晴明　統計力学I）

Almost all the microstate at energy E are macroscopically indistinguishable!

Explanation of the **approach** to equilibrium



（田崎晴明　統計力学I）

Previous Works

	Total System	Sub System
Ensemble Formulation	Mixed $\frac{1}{W(E)}\sum \lvert n\rangle\langle n\rvert$	Mixed $\exp(-\beta\hat{H})/Z$
Previous Works	Pure $\lvert\psi\rangle = \sum_n c_n \lvert n\rangle$ $\langle\psi\rvert\hat{M}_z\lvert\psi\rangle = \langle\hat{M}_z\rangle^{\mathrm{ens}}_{E,N}$ + (exponentially small error) A.Sugita (2007), P.Rieman (2008)	Mixed "Canonical Typicality" $\simeq \exp(-\beta\hat{H})/Z$ S.Popescu et al. (2006) S.Goldstein et al. (2006)

S.Popescu et al. (2006),
S.Goldstein et al. (2006)

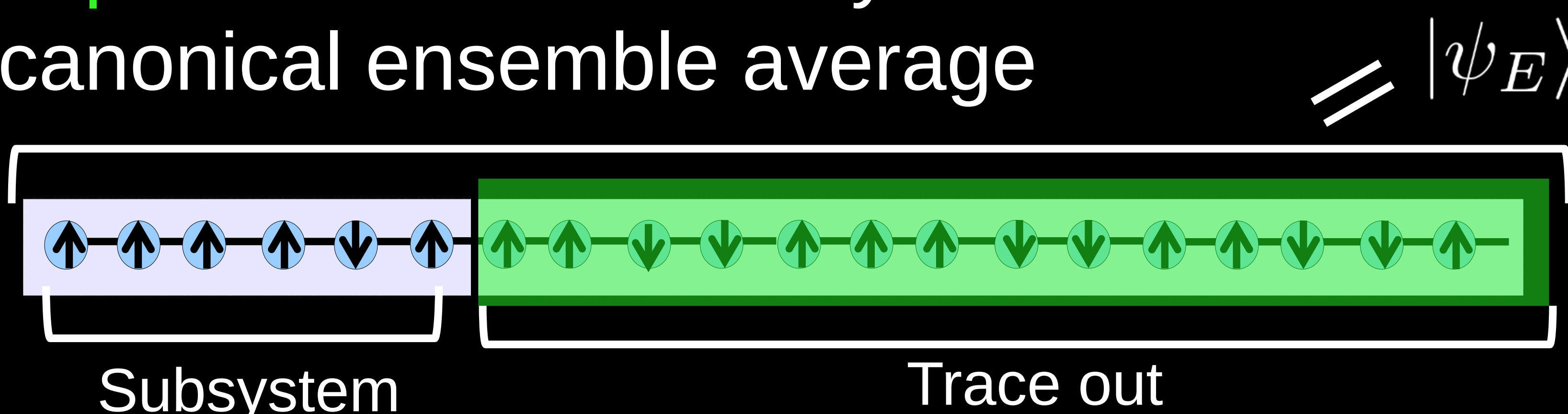
Take a random vector $|\psi_E\rangle \equiv \sum_i c_i |E_n\rangle$

i.e. the random vector in the specified energy shell

$\{|E_n\rangle\}_n$: an arbitrary orthonormal basis spanning the enegy shell $[E, E+\Delta E)$

$\{c_i\}_i$: a set of random complex numbers with $\sum_i |c_i|^2 = 1$.

When we see the subsystem of $|\psi_E\rangle$,
the expectation value is very close to
the canonical ensemble average



S.Popescu et al. (2006),
S.Goldstein et al. (2006)

When we see the subsystem of $|\psi_E\rangle$,
the expectation value is very close to
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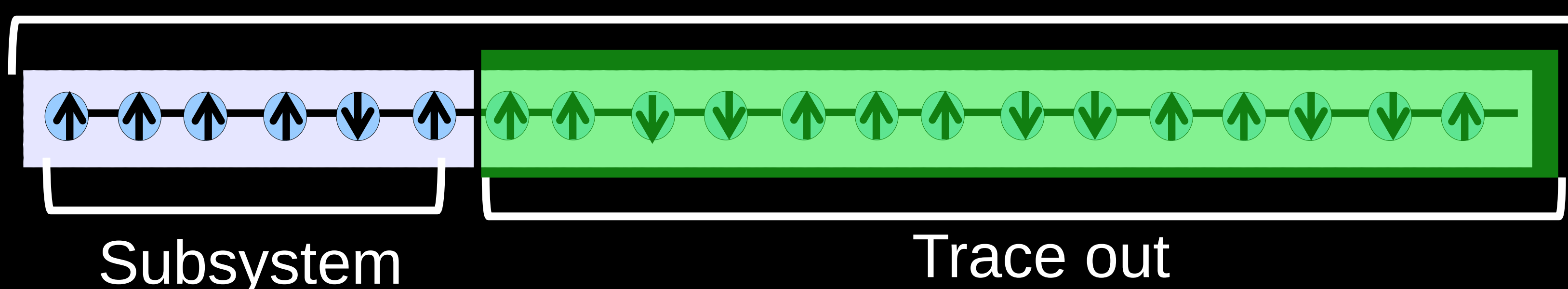


That is, the deviation ratio satisfies

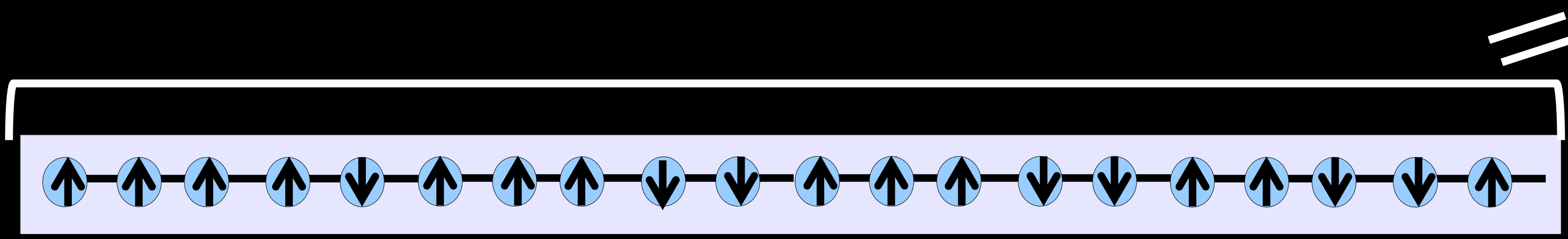
$$\mathrm{P}\left(\left|\langle\psi_E|\hat{A}|\psi_E\rangle - \langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N}\right| \geq \epsilon\right) \leq \frac{\|\hat{A}\|}{d}$$

$\left[\hat{A}\right.$: mechanical variable on the subsystem$\left.\right]$

$\longrightarrow$ Conditon for $\hat{A}$ can be weaken

A.Sugita (2007),
P.Rieman (2008)

When we see the observables which are low-degree polynomials of local operators, the expectation value is very close to the microcanonical ensemble average.



That is,

$$\mathrm{P}\left(\left| \langle \psi_E | \hat{A} | \psi_E \rangle - \langle \hat{A} \rangle^{\mathrm{ens}}_{E,N} \right| \geq \epsilon \right) \leq \frac{\|\hat{A}\|}{d}$$

($\hat{A}$: mechanical variable on the subsystem)

(Conditon for $\hat{A}$ can be weaken even more)

Previous Works

	Total System	Sub System
Ensemble Formulation	Mixed $\frac{1}{W(E)}\sum \vert n\rangle\langle n\vert$	Mixed $\exp(-\beta\hat{H})/Z$
Previous Works	Pure $\vert\psi\rangle = \sum_n c_n \vert n\rangle$ $\langle\psi\vert\hat{M}_z\vert\psi\rangle = \langle\hat{M}_z\rangle^{\mathrm{ens}}_{E,N}$ + (exponentially small error) A.Sugita (2007), P.Rieman (2008) $S=?\ \ T=?$	Mixed "Canonical Typicality" $\simeq \exp(-\beta\hat{H})/Z$ S.Popescu et al. (2006) S.Goldstein et al. (2006) $\vert\psi\rangle = ?$

Purpose of Our Work

Establish the formulation of statistical mechanics using a **single** pure quantum state.

Macroscopic Variables

Mechanical Variables

・Low-degree polynomials of local operators

(i.e. their degree $\leq m = o(N)$)

Ex) Magnetization, Spin-spin correlation function

Genuine Thermodynamic Variables

Ex) Temperature T, Entropy S

・Cannot be represented as mechanical variables

・All genuine thermodynamic variables can be derived from entropy S.

Thermal Pure Quantum (TPQ) State

When $|\Psi\rangle$ is generated from some probability measure, we call $|\Psi\rangle$ a TPQ state if

$$\langle \hat{A} \rangle_N^{\Psi} \equiv \frac{\langle \Psi | \hat{A} | \Psi \rangle}{\langle \Psi | \Psi \rangle} \xrightarrow{P} \langle \hat{A} \rangle_N^{\mathrm{ens}}$$

uniformly for every mechanical variable $\hat{A}$ as $N \to \infty$

($\langle \hat{A} \rangle_N^{\mathrm{ens}}$: ensemble average of $\hat{A}$
$\xrightarrow{P}$: convergence in probability)

Independent variables

$u, N : \langle \hat{A} \rangle_{u,N}^{\Psi} \xrightarrow{P} \langle \hat{A} \rangle_{u,N}^{\mathrm{ens}}$ Microcanonical TPQ state

energy density E/N

$\beta, N : \langle \hat{A} \rangle_{\beta,N}^{\Psi} \xrightarrow{P} \langle \hat{A} \rangle_{\beta,N}^{\mathrm{ens}}$ Canonical TPQ state

New Formulation

	Total System	Sub System
Ensemble	Mixed	Mixed
Previous Works	Pure $|\psi\rangle = \sum_n c_n |n\rangle$ $S = ? \quad T = ?$	Mixed "Canonical Typicality" $|\psi\rangle = ?$

New Formulation

	Total System	Sub System
Ensemble	Mixed	Mixed
Previous Works	Pure $\vert\psi\rangle = \sum_n c_n \vert n\rangle$ $S = ?\quad T = ?$	Mixed "Canonical Typicality" $\boxed{\vert\psi\rangle = ?}$

New Formulation

	Total System	Sub System
Ensemble	Mixed	Mixed
Previous Works	Pure $\lvert\psi\rangle = \sum_n c_n \lvert n\rangle$	Mixed "Canonical Typicality"
Microcanonical TPQ state	Pure $\lvert k\rangle = (l - \hat{h})^k \lvert\psi_0\rangle$ $S = ?\quad T = ?$	Mixed
Canonical TPQ state		$\lvert\beta, N\rangle \equiv e^{-N\beta\hat{h}/2}\lvert\psi_0\rangle$

New Formulation

	Total System	Sub System
Ensemble	Mixed	Mixed
Previous Works	Pure $\|\psi\rangle = \sum_n c_n \|n\rangle$	Mixed "Canonical Typicality"
Microcanonical TPQ state	Pure $\|k\rangle = (l - \hat{h})^k \|\psi_0\rangle$ $S = ?\quad T = ?$	Mixed
Canonical TPQ state		$\|\beta, N\rangle \equiv e^{-N\beta\hat{h}/2}\|\psi_0\rangle$ $F = ?$

New Formulation

	Total System	Sub System
Ensemble	Mixed	Mixed
Previous Works	Pure $\lvert\psi\rangle = \sum_n c_n \lvert n\rangle$	Mixed "Canonical Typicality"
Microcanonical TPQ state	Pure $\lvert k\rangle = (l - \hat{h})^k \lvert\psi_0\rangle$ $\langle k\lvert k\rangle \leftrightarrow S$	Mixed
Canonical TPQ state		$\lvert\beta, N\rangle \equiv e^{-N\beta\hat{h}/2}\lvert\psi_0\rangle$ $\langle\beta, N\lvert\beta, N\rangle \leftrightarrow F$

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Setup

System:

- Discrete quantum system composed of N sites,
- The dimension of the Hilbert space is D.
- The ensemble formulation gives correct results, which are consistent with thermodynamics in $N \to \infty$

- Assume every mechanical variable $\hat{A}$ is normalized as $\|\hat{A}\| \leq K N^m$

  (To exclude foolish operators (ex. $N^N \hat{H}$)
  K : Constant independent of $\hat{A}$ and N.)

- We use quantities per site, $u \equiv E/N$ and $\hat{h} \equiv \hat{H}/N$
- The spectrum of $\hat{h}$ is $e_{\min} \leq u \leq e_{\max}$

Canonical TPQ State

PRL 111, 010401 (2013)

Firstly, take a random vector $|\psi_0\rangle \equiv \sum_i c_i |i\rangle$ from the **whole** Hilbert space.

$\{|i\rangle\}_i$: an arbitrary orthonormal basis of the **whole** Hilbert space

$\{c_i\}_i$: a set of random complex numbers with $\sum_i |c_i|^2 = D$.

Notice:

This random vector $|\psi_0\rangle$ is independent of the choice of the basis set $\{|i\rangle\}_i$.

$\longrightarrow$ Preparation of $|\psi_0\rangle$ is easy.

Canonical TPQ State

PRL 111, 010401 (2013)

Firstly, take a random vector $|\psi_0\rangle \equiv \sum_i c_i |i\rangle$

from the **whole** Hilbert space.

($\{|i\rangle\}_i$: an arbitrary orthonormal basis of the **whole** Hilbert space

$\{c_i\}_i$: a set of random complex numbers with $\sum_i |c_i|^2 = D$.)

Then, calculate

$$|\beta, N\rangle \equiv \exp[-N\beta\hat{h}/2]|\psi_0\rangle \qquad \left(\hat{h} \equiv \hat{H}/N\right)$$

$|\beta, N\rangle$ is the canonical TPQ state at temperature $1/\beta$

Canonical TPQ State

PRL 111, 010401 (2013)

Mechanical Variables

$$\langle \hat{A} \rangle^{\mathrm{TPQ}}_{\beta,N} \equiv \frac{\langle \beta,N|\hat{A}|\beta,N\rangle}{\langle \beta,N|\beta,N\rangle} \xrightarrow{P} \langle \hat{A} \rangle^{\mathrm{ens}}_{\beta,N}$$

$$\left(|\beta,N\rangle \equiv \exp[-N\beta\hat{h}/2]|\psi_0\rangle \right)$$

Genuine Thermodynamic Variables

Free energy

$$-\frac{1}{N}\ln\langle \beta,N|\beta,N\rangle \xrightarrow{P} \beta f(1/\beta;N).$$

$\left(\lambda^N \right.$: Dimension of total Hilbert space $\left. \right)$

Error Probabilisty of Canonical TPQ State

$$\overline{\langle \beta, N | \beta, N \rangle} = Z(\beta, N)$$

$$\left(\begin{array}{l} \overline{\cdots} : \text{Random average over } \{c_i\}_i \\ Z(\beta, N) \equiv \mathrm{Tr}[\exp(-N\beta \hat{h})] \\ \lambda^N : \text{Dimension of total Hilbert space} \end{array} \right)$$

$$\mathrm{P}\left(\left| \langle \beta, N | \beta, N \rangle / \overline{\langle \beta, N | \beta, N \rangle} - 1 \right| \geq \epsilon \right)$$
$$\leq \frac{1}{\epsilon^2} \frac{1}{\exp[2N\beta\{f(1/2\beta; N) - f(1/\beta; N)\}]}$$
$$\leq \frac{1}{\epsilon^2} \frac{1}{\exp[\Theta(N)]}$$

$$\left(f(\beta; N) : \text{free energy density} \right)$$

Error Probability of Canonical TPQ State

$$\overline{\langle \hat{A} \rangle_{\beta,N}^{\mathrm{TPQ}}} \equiv \overline{\langle \beta, N | \hat{A} | \beta, N \rangle} / \overline{\langle \beta, N | \beta, N \rangle}$$
$$= \langle \hat{A} \rangle_{\beta,N}^{\mathrm{ens}}$$

$\left(\overline{\cdots} \right.$: Random average over $\left. \{c_i\}_i \right)$

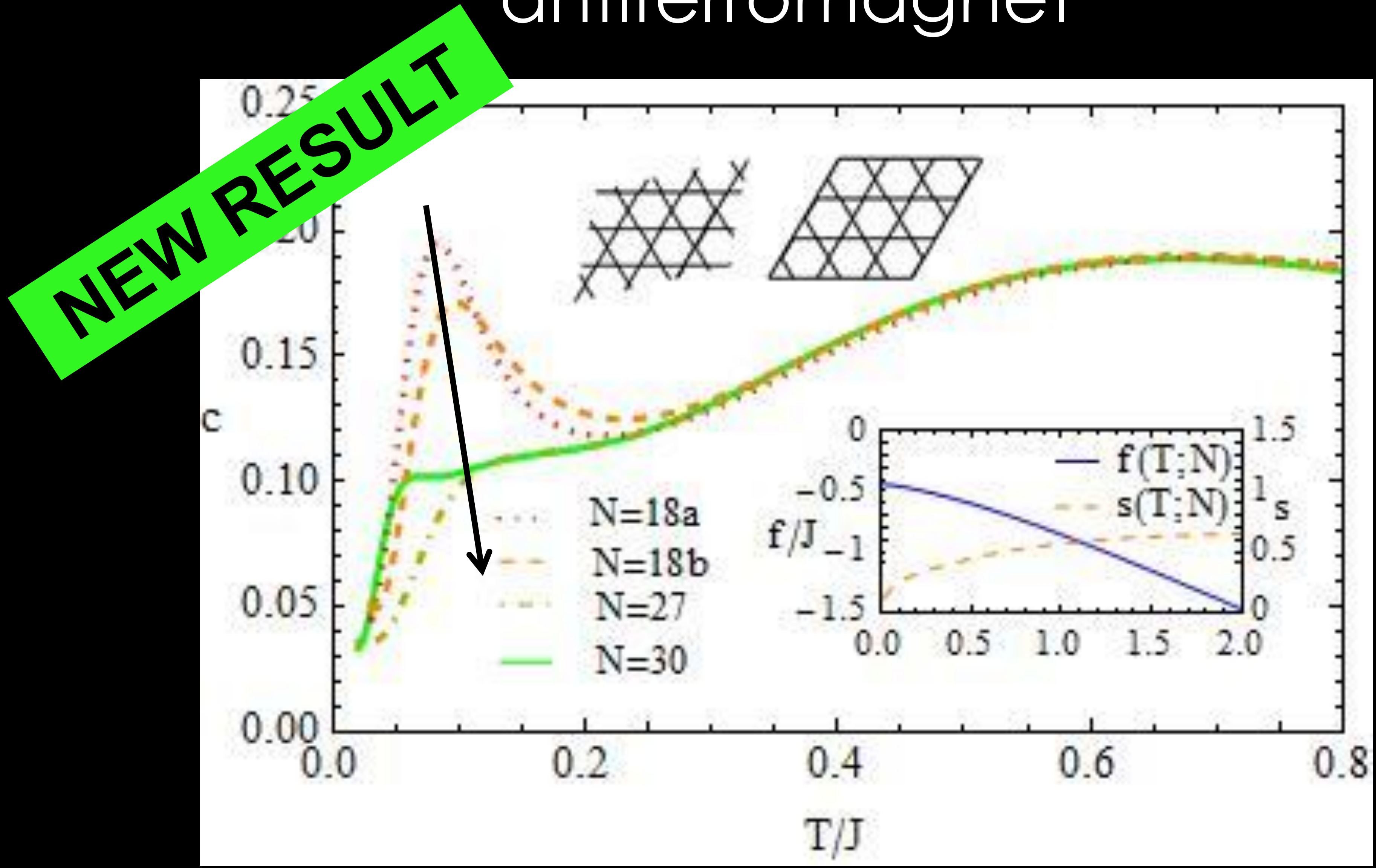
Error Probability of Canonical TPQ State

$$\overline{\langle \hat{A} \rangle^{\mathrm{TPQ}}_{\beta,N}} \equiv \overline{\langle \beta, N | \hat{A} | \beta, N \rangle} / \overline{\langle \beta, N | \beta, N \rangle}$$
$$= \langle \hat{A} \rangle^{\mathrm{ens}}_{\beta,N}$$

$$\mathrm{P}\left(\left| \langle \hat{A} \rangle^{\mathrm{TPQ}}_{\beta,N} - \langle \hat{A} \rangle^{\mathrm{ens}}_{\beta,N} \right| \geq \epsilon \right)$$
$$\leq \frac{1}{\epsilon^2} \frac{\langle (\Delta \hat{A})^2 \rangle^{\mathrm{ens}}_{2\beta,N} + (\langle A \rangle^{\mathrm{ens}}_{2\beta,N} - \langle A \rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N) - f(1/\beta;N)\}]}$$
$$\leq \frac{1}{\epsilon^2} \frac{N^{2m}}{\exp[\Theta(N)]} \qquad \left[\langle (\Delta \hat{A})^2 \rangle^{\mathrm{ens}}_{\beta,N} \text{ :Variance of } \hat{A} \right]$$

A single realization of a TPQ state gives the equilibrium values of all mechanical variables.

S=1/2 kagome-lattice Heisenberg antiferromagnet



Second peak vanishesas as $N \to \infty$?

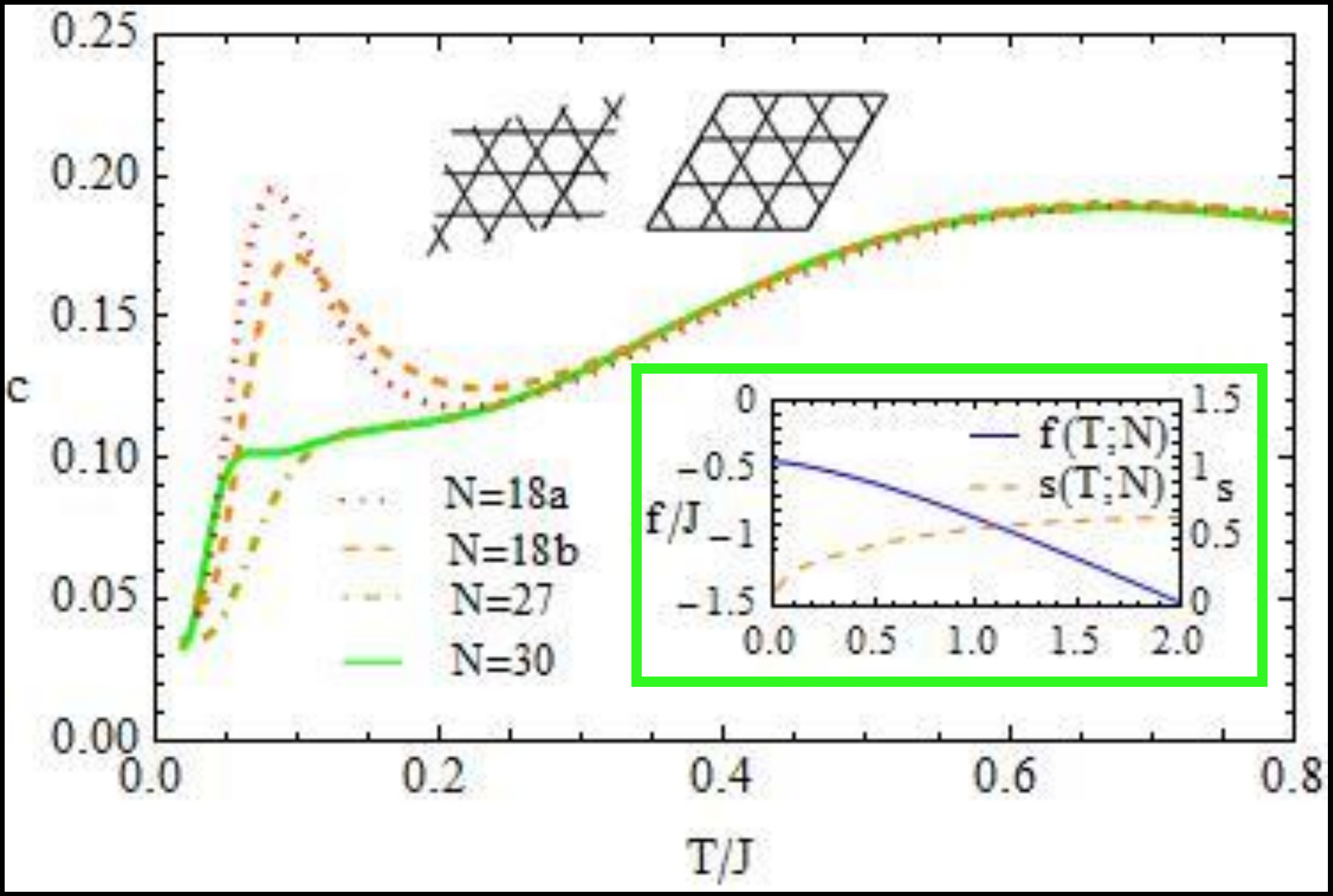
Error Probability of Canonical TPQ State

$$\mathrm{P}\left(\left|\langle \hat{A}\rangle^{\mathrm{TPQ}}_{\beta,N} - \langle \hat{A}\rangle^{\mathrm{ens}}_{\beta,N}\right| \geq \epsilon\right)$$

$$\leq \frac{1}{\epsilon^2}\,\frac{\langle(\Delta \hat{A})^2\rangle^{\mathrm{ens}}_{2\beta,N} + (\langle A\rangle^{\mathrm{ens}}_{2\beta,N} - \langle A\rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N) - f(1/\beta;N)\}]}$$

$$\boxed{\text{Almost self-validating!}}$$

S=1/2 kagome-lattice Heisenberg antiferromagnet



Error Probability of Canonical TPQ State

$$\mathrm{P}\left(\left|\langle\hat{A}\rangle_{\beta,N}^{\mathrm{TPQ}} - \langle\hat{A}\rangle_{\beta,N}^{\mathrm{ens}}\right| \geq \epsilon\right)$$

$$\leq \frac{1}{\epsilon^2}\,\frac{\langle(\Delta\hat{A})^2\rangle_{2\beta,N}^{\mathrm{ens}} + (\langle A\rangle_{2\beta,N}^{\mathrm{ens}} - \langle A\rangle_{\beta,N}^{\mathrm{ens}})^2}{\exp[2N\beta\{f(1/2\beta;N) - f(1/\beta;N)\}]}$$

$$\left(\langle(\Delta\hat{A})^2\rangle_{\beta,N}^{\mathrm{ens}} \equiv \langle\ (\hat{A} - \langle\hat{A}\rangle_{\beta,N}^{\mathrm{ens}})^2\ \rangle_{\beta,N}^{\mathrm{ens}}\right)$$

Error Probability of Canonical TPQ State

$$\mathrm{P}\left(\left|\langle\hat{A}\rangle^{\mathrm{TPQ}}_{\beta,N}-\langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N}\right|\geq\epsilon\right)$$

$$\left(\leq\frac{1}{\epsilon^2}\,\frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{2\beta,N}+(\langle A\rangle^{\mathrm{ens}}_{2\beta,N}-\langle A\rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N)-f(1/\beta;N)\}]}\right)$$

$$\leq\frac{1}{\epsilon^2}\,\frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{\beta,N}}{\exp[N\beta\{f(0;N)-f(1/\beta;N)\}]}$$

$$\left(\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{\beta,N}\equiv\left\langle\ (\hat{A}-\langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N})^2\ \right\rangle^{\mathrm{ens}}_{\beta,N}\right)$$

Error Probability of Canonical TPQ State

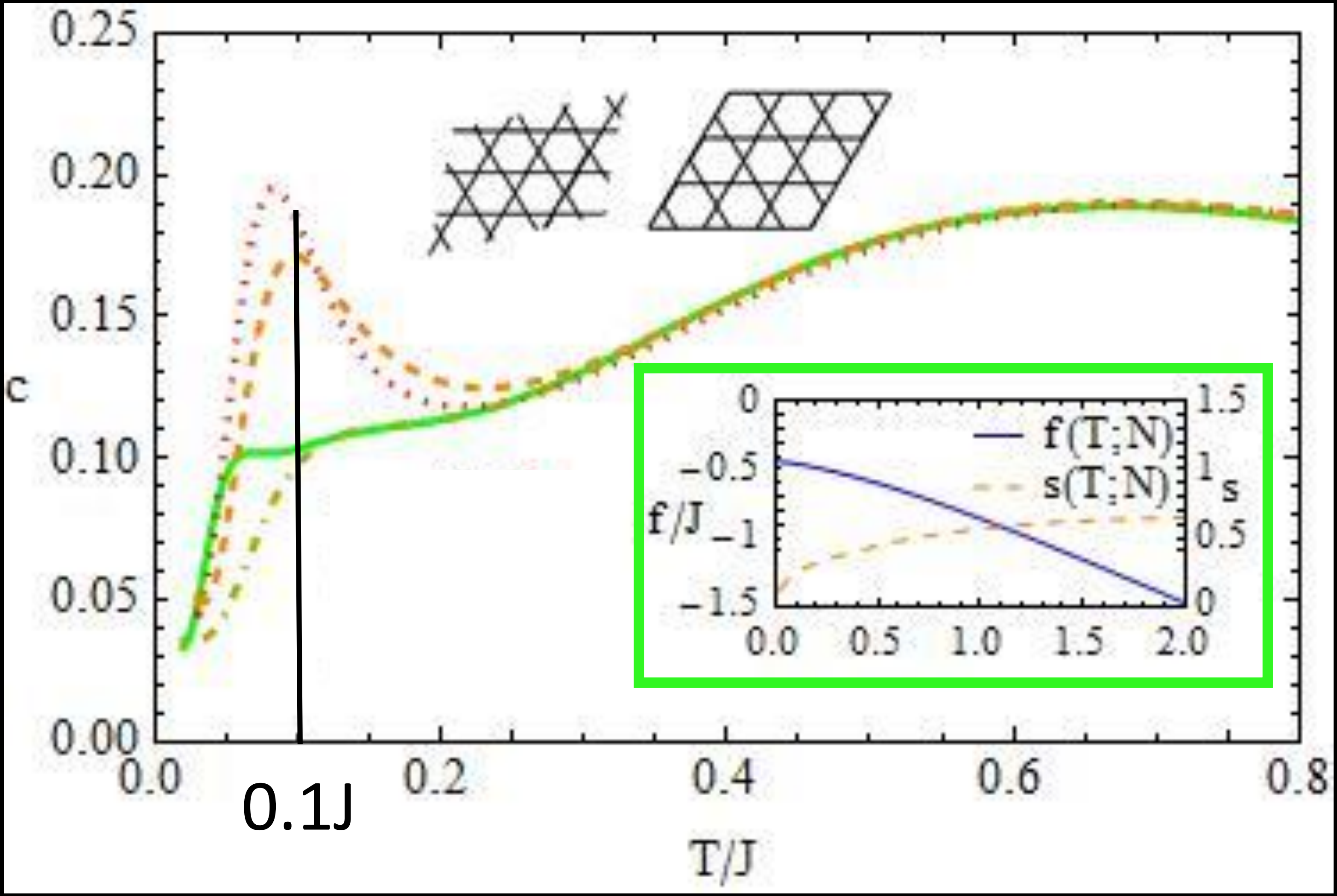
$$\mathrm{P}\left(\left|\langle\hat{A}\rangle^{\mathrm{TPQ}}_{\beta,N} - \langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N}\right| \geq \epsilon\right)$$

$$\left(\leq \frac{1}{\epsilon^2} \frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{2\beta,N} + (\langle A\rangle^{\mathrm{ens}}_{2\beta,N} - \langle A\rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N) - f(1/\beta;N)\}]} \right)$$

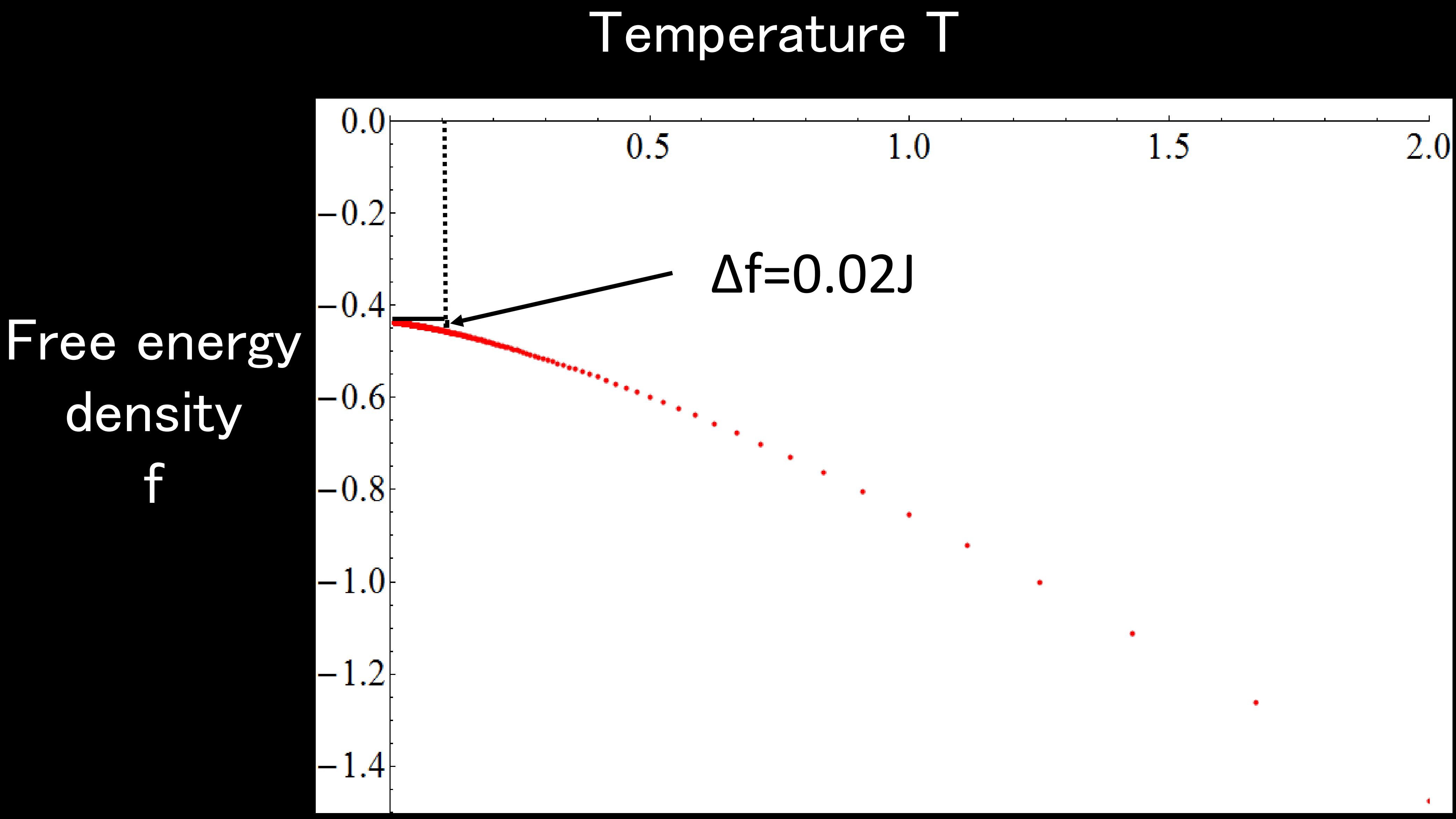
$$\boxed{\leq \frac{1}{\epsilon^2} \frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{\beta,N}}{\exp[N\beta\{f(0;N) - f(1/\beta;N)\}]}}$$

$$\left(\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{\beta,N} \equiv \langle\ (\hat{A} - \langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N})^2\ \rangle^{\mathrm{ens}}_{\beta,N} \right)$$

S=1/2 kagome-lattice Heisenberg antiferromagnet



Free energy density



Error Probability of Canonical TPQ State

$$\mathrm{P}\left(\left|\langle\hat{A}\rangle^{\mathrm{TPQ}}_{\beta,N}-\langle\hat{A}\rangle^{\mathrm{ens}}_{\beta,N}\right|\geq\epsilon\right)$$

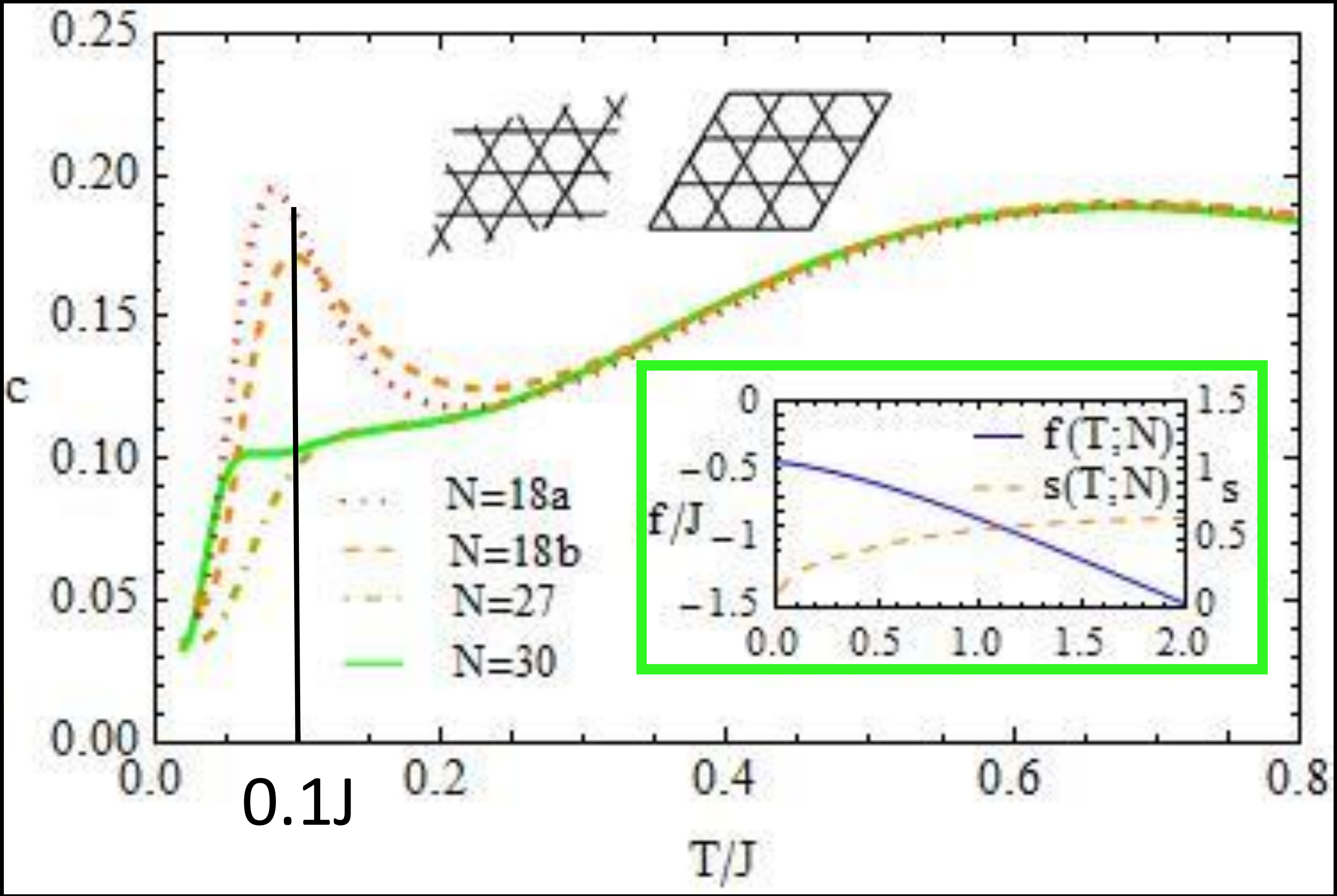
$$\left(\leq\frac{1}{\epsilon^2}\,\frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{2\beta,N}+(\langle A\rangle^{\mathrm{ens}}_{2\beta,N}-\langle A\rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N)-f(1/\beta;N)\}]}\right)$$

$$\leq\frac{1}{\epsilon^2}\,\frac{\langle(\Delta\hat{A})^2\rangle^{\mathrm{ens}}_{\beta,N}}{\boxed{\exp[N\beta\{f(0;N)-f(1/\beta;N)\}]}}$$

≒1800

$$\left(\begin{array}{l}\Delta f \fallingdotseq 0.02\\ \beta=10J\\ N=30\end{array}\right)$$

S=1/2 kagome-lattice Heisenberg antiferromagnet



Error is less than 1% down to T=0.1J!

Error Probability of Canonical TPQ State

$$
\mathrm{P}\left(\left| \langle \hat{A} \rangle^{\mathrm{TPQ}}_{\beta,N} - \langle \hat{A} \rangle^{\mathrm{ens}}_{\beta,N} \right| \geq \epsilon \right)
$$

$$
\leq \frac{1}{\epsilon^2} \, \frac{\langle (\Delta \hat{A})^2 \rangle^{\mathrm{ens}}_{2\beta,N} + (\langle A \rangle^{\mathrm{ens}}_{2\beta,N} - \langle A \rangle^{\mathrm{ens}}_{\beta,N})^2}{\exp[2N\beta\{f(1/2\beta;N) - f(1/\beta;N)\}]}
$$

$$
\leq \frac{1}{\epsilon^2} \, \frac{N^{2m}}{\exp[\Theta(N)]}
$$

Even when we replace $\hat{A}$ by the dynamical quantities e.g. $\hat{A} e^{-i\hat{H}t} \hat{B}$, the error is still exponentially small, because $\|\hat{A} e^{-i\hat{H}t} \hat{B}\| = \|\hat{A}\| \, \|e^{-i\hat{H}t}\| \, \hat{B}\| \leq O(N^4 m)$

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Microcanonical TPQ State

PRL 108.240401 (2012)

Start from the same state $|\psi_0\rangle \equiv \sum_i c_i|i\rangle$, which is the random vector in the **whole** Hilbert space

Then, calculate

$$|k\rangle \equiv (l-\hat{h})^k|\psi_0\rangle$$
$$u_k \equiv \langle k|\hat{h}|k\rangle/\langle k|k\rangle$$

for k=1,2,...

$$\left(\begin{array}{l} \hat{h} \equiv \hat{H}/N \\ l \text{ : arbitrary constant of O(1) s.t. } l > e_{\max} \end{array} \right)$$

$|k\rangle$ is the microcanonical TPQ state at energy u_k

Microcanonical TPQ State

PRL 108.240401 (2012)

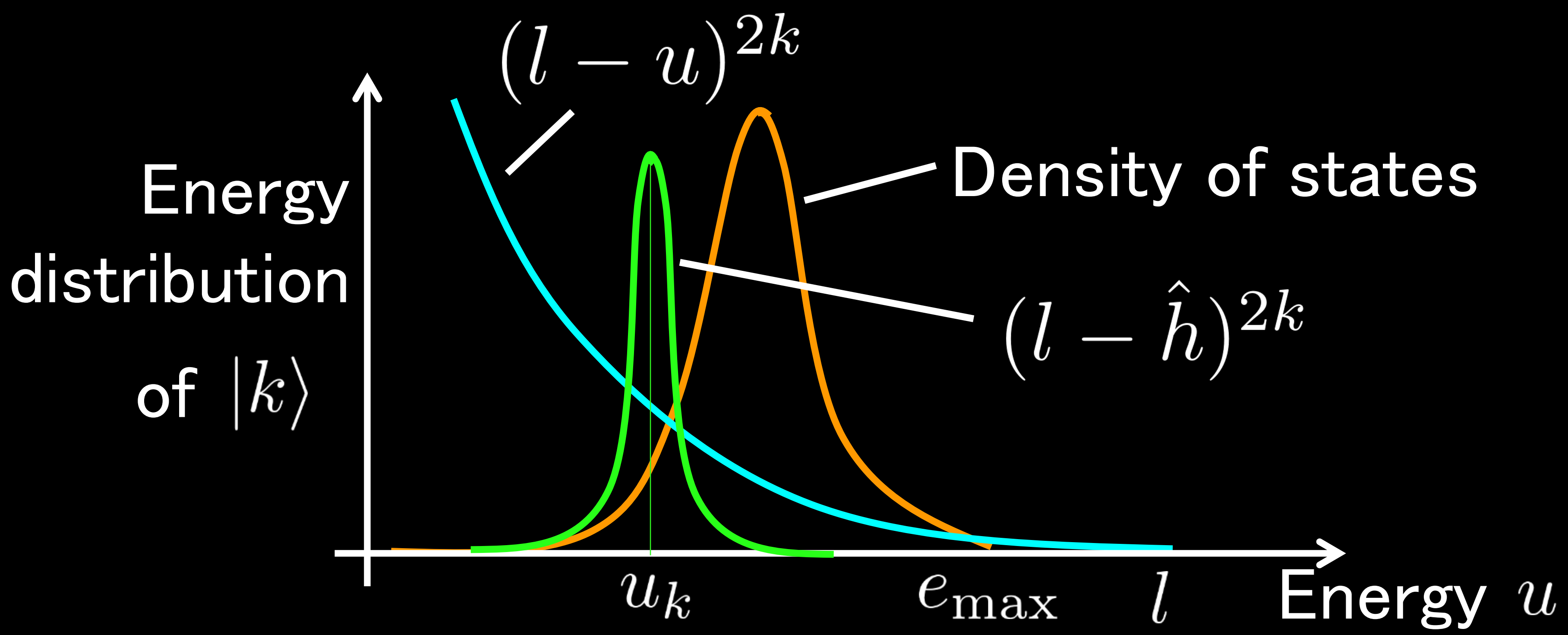
$$|k\rangle \equiv (l - \hat{h})^k |\psi_0\rangle$$

$$u_k \equiv \langle k|\hat{h}|k\rangle / \langle k|k\rangle$$

for k=1,2,...

$\hat{h} \equiv \hat{H}/N$

l : arbitrary constant of O(1) s.t. $l > e_{\max}$



Microcanonical TPQ State

PRL 108.240401 (2012)

Mechanical Variables

$$\frac{\langle k|\hat{A}|k\rangle}{\langle k|k\rangle} \xrightarrow{P} \frac{\mathrm{Tr}[\hat{A}\rho_{\mathrm{mc}}]}{\mathrm{Tr}[\rho_{\mathrm{mc}}]}$$

$$\left(\rho_{\mathrm{mc}} \equiv (l-\hat{h})^{2k}\right)$$

Genuine Thermodynamic Variables

Entropy

$$s(u_k;N) = \frac{1}{N}\ln\langle k|k\rangle - \frac{2k}{N}\ln(l-u_k) + O(N)$$

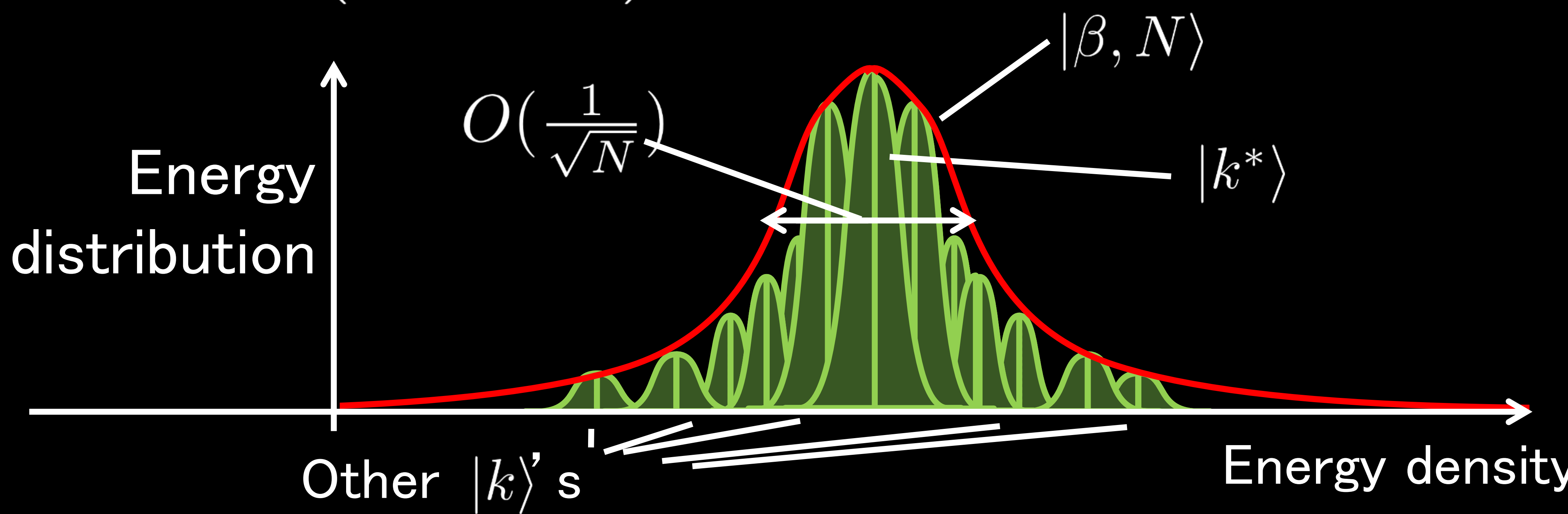
$\Big($ $|k\rangle \equiv (l-\hat{h})^k|\psi_0\rangle$: Unnormalized microcanonical TPQ state at energy u_k $\Big)$

Analytic Relations

Canonical and microcanonical TPQ states are related by simple analytic transformations.

$$\exp[-N\beta\hat{h}/2]|\psi_0\rangle = e^{-N\beta l/2}\sum_{k=0}^{\infty}\frac{(N\beta/2)^k}{k!}|k\rangle$$
$$= e^{-N\beta l/2}\sum_{k=0}^{\infty}R_k|\psi_k\rangle.$$

$$\left[\ |\psi_k\rangle \equiv \left(1/\sqrt{\langle k|k\rangle}\right)|k\rangle, \quad R_k \equiv (N\beta/2)^k\sqrt{\langle k|k\rangle}/k!\ \right]$$



Advantages for Numerical Method

$$\exp(-\beta\hat{H})/Z \longrightarrow |\beta, N\rangle \equiv \exp[-N\beta\hat{h}/2]|\psi_0\rangle$$

Many Advantages :

- Free from dimension and structure of Hamiltonian.

  Free From Negative Sign Problem

  Applicable to Higher Dimensional Systems

- Finite temperature.
- Less amount of calculation than a diagonalization of Hamiltonian.
- Only 2 vectors (i.e. Computer Memory) are needed

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Different Representations of the Same Equilibrium State

Conventional Formulation

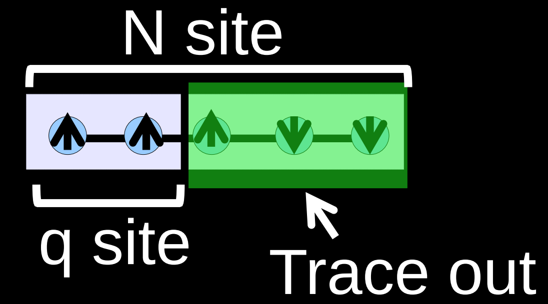
$$\sum_{n\in \mathrm{energy\ shell}} |n\rangle\langle n|, \ \exp(-\beta \hat{H})$$

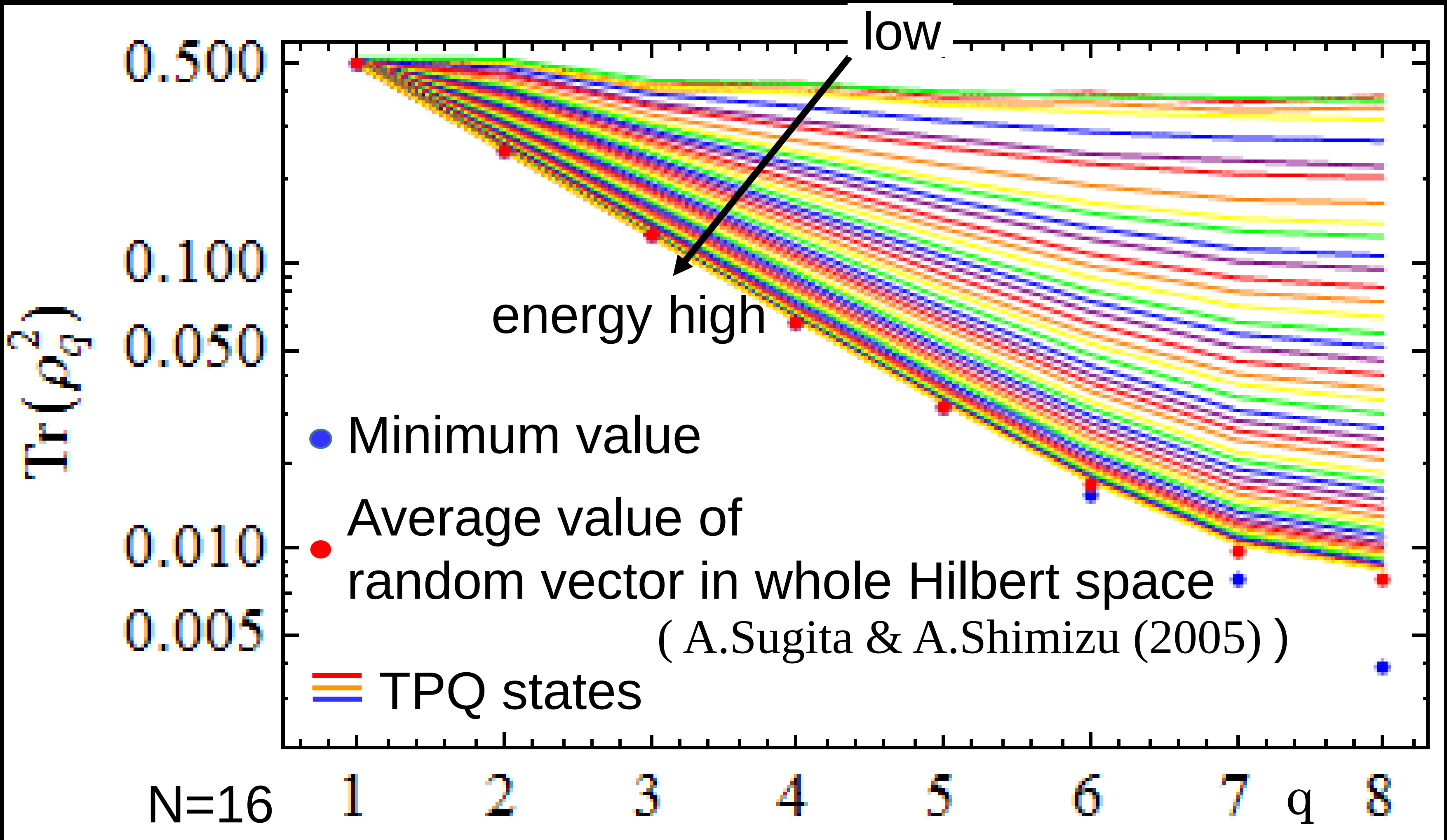
TPQ States Formulation

$$|k\rangle, \quad |\beta, N\rangle$$

As far as we see macroscopic quantities, we cannot distinguish them.

Entanglement -Purity





TPQ states are almost maximally entangled

Different Representations of the Same Equilibrium State

Conventional Formulation

$$\sum_{n\in\text{energy shell}} |n\rangle\langle n|, \; \exp(-\beta\hat{H})$$

$\longrightarrow$ At high temperature, they have little entanglement.

TPQ States Formulation

$$|k\rangle, \quad |\beta, N\rangle$$

$\longrightarrow$ TPQ states have almost maximum entanglement.

$$\boxed{\text{Microscopically completely different states represent the same equilibrium state.}}$$

TPQ State

$\langle k|k\rangle,\ \langle \beta, N|\beta, N\rangle$

Thank You!