Application of quantum number projection method to tetrahedral shape and high-spin states in nuclei

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  wobbling and chiral rotational band
Efficient method of projection calculation

S. Tagami and Y. R. Shimizu,
General quantum number projection and configuration mixing (GCM)

Final wave function

$$|\Psi_{M;\alpha}^{INZ(\pm)}\rangle = \sum_{K,n} g_{Kn,\alpha}^{INZ(\pm)} \hat{P}_{MK}^{I} \hat{P}^{N} \hat{P}^{Z} \hat{P}_{\pm} \Phi_{n}\rangle$$

projectors:

$$\hat{P}_{MK}^{I} = \frac{2I + 1}{8\pi^2} \int d^3 \omega D_{MK}^{I*}(\omega) \hat{R}(\omega), \quad \hat{P}^{N} = \frac{1}{2\pi} \int d\varphi e^{i\varphi(N-N)}$$

$$\omega = \text{Euler angle} (\alpha, \beta, \gamma)$$

Hill-Wheeler equation (generalized eigenvalue problem)

$$\sum_{K',n'} \mathcal{H}_{Kn;K'n'}^{INZ(\pm)} g_{K'n',\alpha}^{INZ(\pm)} = E_{\alpha}^{INZ(\pm)} \sum_{K',n'} \mathcal{N}_{Kn;K'n'}^{INZ(\pm)} g_{K'n',\alpha}^{INZ(\pm)}$$

kernels:

$$\begin{pmatrix} \mathcal{H}_{Kn;K'n'}^{INZ(\pm)} \\ \mathcal{N}_{Kn;K'n'}^{INZ(\pm)} \end{pmatrix} = \langle \Phi_{n} | \begin{pmatrix} \hat{H} \\ 1 \end{pmatrix} \hat{P}_{KK'}^{I} \hat{P}^{N} \hat{P}^{Z} \hat{P}_{\pm} \Phi_{n'} \rangle$$
Projection calculation

kernels: \( \left( \mathcal{H}_{K^N K'; n; n'}^{I N Z(\pm)} \right) = \langle \Phi_n | \begin{pmatrix} \hat{H} & \hat{P}^I_{K K'} \\ 1 & \hat{P}^N \hat{P}^Z \hat{P}_\pm \end{pmatrix} | \Phi_{n'} \rangle \)

\[ \hat{P}^I_{MK} = \frac{2I + 1}{8\pi^2} \int d^3 \omega D_{MK}^{I,*}(\omega) \hat{R}(\omega), \quad \hat{P}^N = \frac{1}{2\pi} \int d\varphi e^{i\varphi(\hat{N} - N)} \]

\( \hat{R}(\alpha, \beta, \gamma) \equiv e^{i\gamma J_z} e^{i\beta J_y} e^{i\alpha J_z} \)

General projection

\( |\alpha\rangle = \hat{P}_\alpha |\Phi\rangle, \quad \hat{P}_\alpha = \int g_\alpha(x) \hat{D}(x) dx \)

need to calculate:

\( \langle \Phi | \hat{P}_\alpha \hat{O} \hat{P}_\alpha' | \Phi' \rangle \) or \( \langle \Phi | \hat{D}(x) \hat{O} \hat{D}(x') | \Phi' \rangle \)

arbitrary observable

\( \langle \Phi | \hat{O} \hat{D}(x) | \Phi' \rangle \) between HFB-type states
Efficient method of projection and GCM

Calculation of $\langle \Phi | \hat{O} \hat{D}(x) | \Phi' \rangle$ HFB-type states with large basis size

- **Truncation in terms of canonical basis**
  
  c.f. P.Bonche et al., NPA510(1990),466. Appendix.

  \[ \beta_k |\Phi\rangle = 0 \quad (k = 1, 2, \ldots, M) \]
  
  \[ c_l |0\rangle = 0 \quad (l = 1, 2, \ldots, M) \]

  \[ \beta_k^\dagger = \sum_l (U_{lk} c_l^\dagger + V_{lk} c_l) \]

  Canonical basis $\rightarrow$ diagonalize

  \[ \rho_{\nu l} = \langle \Phi | c_l^\dagger c_{\nu} | \Phi \rangle \]

  *Density matrix*

  \[ b_k^\dagger = \sum_l W_{lk} c_l^\dagger, \quad \langle \Phi | b_k^\dagger b_{k'} | \Phi \rangle = \delta_{kk'} v_{k}^2 \]

  *Occupation probabilities*

  \[ Q = 1 - P \]

  **P-space:** \[ k = 1, 2, \ldots, L_p(\epsilon); \quad v_k^2 > \epsilon \]
by the way, in the canonical basis, a HFB-type state can be written:

\[ |\Phi\rangle = \prod_{i>0, \neq B} (u_i + v_i b_i^\dagger b_i^\dagger) \prod_{i_B} b_{i_B}^\dagger |0\rangle \]

canonical form

(Bloch-Messiah Theorem)

pair of orbits with small \(v_i\) does not contribute to HFB-type mean-field!
Efficient method of projection and GCM

Calculation of \( \langle \Phi | \hat{O} \hat{D}(x) | \Phi' \rangle \) HFB-type states with large basis size

- Truncation in terms of canonical basis
c.f. P.Bonche et al., NPA510(1990),466. Appendix.

\[
\begin{align*}
\beta_k |\Phi\rangle &= 0 \quad (k = 1, 2, \ldots, M) \\
\beta_k^\dagger &= \sum_l (U_{lk} c_l^\dagger + V_{lk} c_l) \\
\end{align*}
\]

Canonical basis \(\rightarrow\) diagonalize

\[
\rho_{ll'} = \langle \Phi | c_l^\dagger c_{l'} | \Phi \rangle
\]

Density matrix

\[
Q = 1 - P
\]

P-space: \(k = 1, 2, \ldots, L_p(\epsilon)\); \(v_k^2 > \epsilon\)

- Utilize Thouless amplitude with respect to a Slater-determinantal state

\[
|\phi_0\rangle = \prod_{k=1}^{N} b_k^\dagger |0\rangle \quad \text{N: particle number}
\]

\[
|\Phi\rangle = \exp \left( \sum_{ll'} Z_{ll'} a_l^\dagger a_{l'}^\dagger \right) |\phi_0\rangle
\]

\[
a_k^\dagger = \begin{cases} 
    b_k^\dagger & (1 \leq k \leq N) \\
    b_k^\dagger & (N + 1 \leq k \leq M)
\end{cases}
\]
Truncation in canonical basis (1)

\[ b_k^\dagger = \sum_l W_{lk} c_l^\dagger, \quad \langle \Phi | b_k^\dagger b_{k'} | \Phi \rangle = \delta_{kk'} v_k^2 \]

occupation probabilities of canonical basis

\[ \beta_k | \Phi \rangle = 0 \quad (k = 1, 2, \ldots, M) \]

total num. \( M = 3,542 \) !!

for \( N_{\text{osc}}^{\text{max}} = 20 \)

P-space: \( k = 1, 2, \ldots, L_P(\epsilon); \quad v_k^2 > \epsilon \)

\[ \beta_k^\dagger = \sum_l (U_{lk} c_l^\dagger + V_{lk} c_l) \]

\[ U = W \bar{U} C, \quad V = W^* \bar{V} C \]

(Bloch-Messiah Theorem)

Thouless form

\[ | \Phi \rangle = \exp \left( \sum_{ll'} Z_{ll'} b_l^\dagger b_{l'}^\dagger \right) |0\rangle \]

\[ Z \equiv (\bar{V} \bar{U}^{-1})^* = \begin{pmatrix} Z_{pp} & 0 \\ 0 & 0 \end{pmatrix} \]

\( L_P \) is much smaller !!
Truncation in canonical basis (2)

Calculation of norm overlap

\[ \langle \Phi | \hat{D} | \Phi' \rangle = \langle 0 | \hat{D} | 0 \rangle \left( \det \hat{D} \det \hat{U}_D^* \right)^{1/2} \]

\[ \hat{D} = W^\dagger \hat{D} W' = \begin{pmatrix} \tilde{D}_{pp} & \tilde{D}_{pq} \\ \tilde{D}_{qp} & \tilde{D}_{qq} \end{pmatrix} \]

\[ \hat{U}_D = \tilde{U}^\dagger \hat{D} \tilde{U}' + \tilde{V}^\dagger \hat{D}^* \tilde{V}' \]

\( \hat{U} = \begin{pmatrix} U_{pp} & 0 \\ 0 & 1 \end{pmatrix} \)

\( \hat{V} = \begin{pmatrix} V_{pp} & 0 \\ 0 & 0 \end{pmatrix} \)

But, using Thouless amplitude

\[ \langle \Phi | \hat{D} | \Phi' \rangle = \langle 0 | \hat{D} | 0 \rangle \left| \det \hat{U} \det \hat{U}' \right|^{1/2} \left( \det \left[ 1 + Z^\dagger Z_D' \right] \right)^{1/2} \]

\[ Z = \begin{pmatrix} Z_{pp} & 0 \\ 0 & 0 \end{pmatrix} \]

\[ Z'_D = \tilde{D} Z' \tilde{D}^T = \begin{pmatrix} Z'_{Dpp} & Z'_{Dpq} \\ Z'_{Dqp} & Z'_{Dqq} \end{pmatrix}, \quad 1 + Z^\dagger Z_D = \begin{pmatrix} 1 + Z_{pp}^\dagger Z_{Dpp} & Z_{pp}^\dagger Z_{Dpq} \\ Z_{pp} Z_{Dqp} & 1 \end{pmatrix} \]

\[ \langle \Phi | \hat{D} | \Phi' \rangle = \langle 0 | \hat{D} | 0 \rangle \left| \det \hat{U}_{pp} \det \hat{U}'_{pp} \right|^{1/2} \left( \det \left[ 1 + Z_{pp}^\dagger Z_{Dpp}' \right] \right)^{1/2} \]

\( L_p \times L_p \) determinant!

The matrix dimension reduced from \( M \times M \) to \( L_p \times L_p \)

Here \( M \approx O(1,000) \) \( L_p \approx O(100) \) !!

Including one-body operator, \( \langle \Phi | \hat{O} \hat{D}(x) | \Phi' \rangle \), it can be shown that the calculational effort: \( O(M^3) \rightarrow O(M L_p^2) \)

LARGE REDUCTION OF CALCULATION \( \sim 100 \) times
by the way, the actual calculation is performed with using the pfaffian rather than determinant,

\[ \hat{D} |\Phi\rangle \rightarrow |\Phi_D\rangle \] another HFB-type state
generally for overlap of two HFB-type states

\[
\langle \Phi | \Phi' \rangle = | \det \bar{U} \det \bar{U}' |^{1/2} \left( \det \left[ 1 + Z^\dagger Z' \right] \right)^{1/2}
\]

\[
\left( \det \left[ 1 + Z^\dagger Z' \right] \right)^{1/2} = \left[ \det \begin{pmatrix} Z' & -1 \\ 1 & Z^\dagger \end{pmatrix} \right]^{1/2}
\]

\[
\Rightarrow (-)^{M(M+1)/2} \text{pf} \left( \begin{pmatrix} Z' & -1 \\ 1 & Z^\dagger \end{pmatrix} \right)
\]

For 2Nx2N anti-symmetric matrix R,

\[
\text{pf}(R) \equiv \frac{1}{2^n n!} \sum_{\sigma \in S_{2N}} \text{sgn}(\sigma) \prod_{i=1}^{N} R_{\sigma(2i-1)\sigma(2i)}
\]

L.M. Robledo, PRC79(2009),021302(R).

In order to calculate pfaffian, Thouless form is necessary!
Utilize Thouless form with Slater-det.(1)

Slater-determinant in canonical basis:

\[ |\Phi\rangle = \exp \left( \sum_{ll'} (Z_a)_{ll'} a_l^{\dagger} a_{l'}^{\dagger} \right) |\phi_0\rangle \]

\[ |\Phi\rangle = \exp \left( \sum_{ll'} (Z_b)_{ll'} b_l^{\dagger} b_{l'}^{\dagger} \right) |0\rangle \]

\[ Z_a = (V_a U_a^{-1})^{*} \] \[ \text{pairing correlations} \]

\[ Z_a \rightarrow 0 \text{ as } \Delta \rightarrow 0 \text{ while } Z_b \rightarrow \infty \text{ as } \Delta \rightarrow 0 \]

One can take the no-pairing limit !!

Furthermore,

\[ a_k^{\dagger} = \begin{cases} b_k & (1 \leq k \leq N) \\ b_k^{\dagger} & (N + 1 \leq k \leq M) \end{cases} \]

\[ u_k \leftrightarrow v_k \text{ for hole states} \]

\[ (k = 1, 2, \ldots, N) \]

\[ u_k^2 = 1 - v_k^2 \]

Reduction by core truncation is restricted (need to calculate \[ \langle \phi_0 | \hat{\mathcal{D}}(x) | \phi_0' \rangle \]), but effective for heavy nuclei !!

projection from HF states
Utilize Thouless form with Slater-det.(2)

occupation and empty probabilities of canonical basis

\[ \epsilon \approx 10^{-4} \]

P-space and core-space dimensions

Reduction of effective number of space:
\[ M \approx 3,000 \Rightarrow L_p - L_o \approx 100 \]  about 2-3 orders of magnitude!
Test of convergence

Rotational excitation spectra: $^{226}\text{Th}$ octupole deformed

$\epsilon \approx 10^{-4} - 10^{-5}$ is enough

$N_{\text{osc}}^{\text{max}} \approx 16 - 18$ is enough
Choice of hamiltonian

Schematic multi-separable type, consistent with Woods-Saxon mean-field

\[ \hat{H} = \hat{h} + \hat{H}_F + \hat{H}_G, \quad \hat{h} = \sum_{\tau=n,p} \left( \hat{t}_\tau + \hat{V}_{\text{WS}}^\tau \right), \]

\[ \hat{H}_F = -\frac{1}{2} \chi \sum_{\lambda \geq 2} : \hat{F}_\lambda \cdot \hat{F}_\lambda : \text{isoscalar}, \]

\[ \hat{F}_{\lambda \mu} = \sum_{\tau=n,p} \sum_{ij} \langle i | \hat{F}_{\lambda \mu}^\tau | j \rangle \hat{c}_i^\dagger \hat{c}_j \]

Bohr-Mottelson textbook Vol.II

\[ \chi = \chi_{\text{self}} = (\kappa_n + \kappa_p)^{-1}, \quad \kappa_\tau \equiv (R_0^\tau)^2 \int \rho_0^\tau (r) \frac{d}{dr} \left( r^2 \frac{dV_c^\tau (r)}{dr} \right) dr \]

pairing channel \( \lambda = 0, 2 \)

\[ \hat{H}_G = - \sum_{\tau, \lambda \geq 0} g_\lambda^\tau \hat{G}_{\lambda \mu}^\dagger \cdot \hat{G}_{\lambda \mu} \]

\[ \hat{G}_{\lambda \mu}^\dagger = \frac{1}{2} \sum_{ij} \langle i | \hat{G}_{\lambda \mu}^\tau | j \rangle \hat{c}_i^\dagger \hat{c}_j \]

\[ g_\lambda^\tau : \text{even-odd mass diff.} \quad \frac{g_2^\tau}{g_0^\tau} = 13.6 : \text{moment of inertia} \]

Mean-field state \( |\Phi\rangle \) is generated with cranking

\[ \hat{h}' = \hat{h}_{\text{def}} - \sum_{\tau=n,p} \Delta_\tau \left( \hat{P}_\tau^\dagger + \hat{P}_\tau \right) - \sum_{\tau=n,p} \lambda_\tau \hat{N}_\tau - \omega_{\text{rot}} \hat{J}_x, \]

Monopole pair field

\[ F_{\lambda \mu}^\tau (r) \equiv R_0^\tau \frac{dV_c^\tau}{dr} Y_{\lambda \mu}(\theta, \phi), \quad V_c^\tau, R_0^\tau : \text{WS central pot., radius} \]

Woods-Saxon -Strutinsky cal.

Skyrme,Gogny : density-dep. → some problems
Application to tetrahedral shape

Tetrahedral nuclear states

Usual quadrupole def. $\leftrightarrow D_{2h}$-symmetry

Higher point-group symmetry in nuclei?

symmetry $\leftrightarrow$ degeneracies of single-particle orbits $\leftrightarrow$ stability (shell-effect)

\[
R(\theta, \varphi) = R_0 c_v(\{\alpha\}) \left( 1 + \sum_{\lambda,\mu} \alpha^*_{\lambda\mu} Y_{\lambda\mu}(\theta, \varphi) \right)
\]

Tetrahedral $T_d$ : only $\alpha_{32} \equiv t_3$ (no $\alpha_{2\mu}, \alpha_{4\mu}, \alpha_{5\mu}, \alpha_{6\mu}$)

$\rightarrow$ 4-fold degenerate orbits appears (usually 2-fold!)

X.Li and J.Dudek, PRC49(1994),R1250.

Various predictions so far:

- Onishi-Shline, NPA165(1971),180. $^{16}\text{O}$ 4-\(\alpha\) states
- Yamagami-Matsuyanagi-Matsuo, NPA693(2001),579. $^{80}\text{Zr}$

systematic calculations with Woods-Saxon Strutinsky and Skyrme HF/HFB approaches
Tetrahedral shape and magic numbers

$T_3 = \alpha_{32}$

$\alpha_{\lambda\mu}$

$\lambda$: odd, $\mu \neq 0$

parity broken and non-axial deformation

Tetrahedral Magic numbers:
16, 20, 32, 40,
56(58), 64, 70,
90, 112, 136

$^{160}_{70}$Yb

4-fold orbits
Example of potential surface calculations

Woods-Saxon universal-compact potential, Strutinsky cal. with finite-range droplet model

Projection of Multi-dimensional ($\lambda < 9$) surface to ($\alpha_{32}, \alpha_{20}$) (Strasbourg-Lublin-Krakow collaboration)

$^{154}_{64}\text{Gd}_{90}$ E(fyu)+Shell[e]+Correlation[PNP] $^{160}_{70}\text{Yb}_{90}$ E(fyu)+Shell[e]+Correlation[PNP]

Quadrupole ground state! $^{154}_{64}\text{Gd}_{90}$ E$_{\text{min}}$=2.41, $^{160}_{70}\text{Yb}_{90}$ E$_{\text{min}}$=2.13, E$_{\text{o}}$= 1.20

Tetrahedral ground state! (Z=70,N=90)
What kind of spectra is expected for tetrahedral rotor?

Known in molecules

defformation \rightarrow quantum rotor, but “spherical rotor” in a sense,

\[ J_1 = J_2 = J_3 = J, \quad E(I) = \frac{I(I + 1)}{2J} \]

But, no quadrupole moment \( Q_2 \), no E2 transitions at all!

Group theory consideration
(irreducible representations of point-group \( T_d \))
even-even nucleus with pairing

\[ A_1 : \quad 0^+, \ 3^-, \ 4^+, \ 6^+, \ 6^-, \ 7^-, \ 8^+, \ 9^+, \ 9^-, \ 10^+, \ 10^-, \ 11^-, \ 2 \times 12^+, \ 12^-, \ \cdots . \]

as the lowest band
Result of angular momentum and parity projection for tetrahedral shape (1)

\[ A_1 : \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2 \times 12^+, 12^-, \ldots \]
Result of angular momentum and parity projection for tetrahedral shape (2)

\[ E_I - E_{0+} \propto I(I+1) \]

\[ \alpha_{32} = 0.10 \]
\[ \pi=+ \]
\[ \pi=- \]
\[ I(I+1) \]

\[ 160\text{Yb} \]

vibrational

\[ \alpha_{32} = 0.15 \]

rotational
\[ \propto I(I+1) \]

tetrahedral rotor spectra realized!!

\[ \alpha_{32} = 0.20 \]

\[ \alpha_{32} = 0.25 \]

\[ \alpha_{32} = 0.30 \]

\[ \alpha_{32} = 0.35 \]
What kind of spectra is expected for tetrahedral rotor? (2)

Group theory consideration
odd nuclei (half-integer spins)

three irrep.'s

\[ E_{1/2} : \begin{array}{c}
\frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{11}{2} \\
\frac{13}{2}, 2 \times \frac{13}{2}, \frac{15}{2}, \frac{15}{2}, \ldots
\end{array} \]

parity conjugate

\[ E_{5/2} : \begin{array}{c}
\frac{1}{2}, \frac{5}{2}, \frac{7}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{11}{2} \\
\frac{13}{2}, 2 \times \frac{13}{2}, \frac{15}{2}, \frac{15}{2}, \ldots
\end{array} \]

two-fold states

\[ G_{3/2} : \begin{array}{c}
\frac{3}{2}, \frac{3}{2}, \frac{5}{2}, \frac{5}{2}, \frac{7}{2}, \frac{7}{2}, 2 \times \frac{9}{2}, 2 \times \frac{9}{2} \\
2 \times \frac{11}{2}, 2 \times \frac{11}{2}, 2 \times \frac{13}{2}, 2 \times \frac{13}{2}, \ldots
\end{array} \]

four-fold states

which becomes lowest depends on the last odd-nucleon orbit!

completely different from quadrupole rotor!
Example of calculations (1)

odd nuclear states around $^{80}$Zr (N=Z=40)

$\alpha_{32}=0.4$ and without pairing ($\Delta=0$)

Group theory consideration OK, but projection necessary for precise spectra!!
Example of calculations (2)

odd and two-particle states around $^{80}\text{Zr}$ (N=Z=40) 
$\alpha_{32}=0.4$ and without pairing ($\Delta=0$)

Group theory consideration OK, but projection necessary for precise spectra!!

$G_{3/2}^\pi$:

$3^+, \frac{3}{2}^-, \frac{5}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+, \frac{7}{2}^-, 2 \times \frac{9}{2}^+, 2 \times \frac{9}{2}^-, 2 \times \frac{11}{2}^+, 2 \times \frac{11}{2}^-, 2 \times \frac{13}{2}^+, 2 \times \frac{13}{2}^-$, ….

$A\{G_{3/2} \times G_{3/2}\} = A_1 + E + F_2$

$F_2$ seems split into three bands
Irreducible representations of $T_d$-rotor for even-even nuclei (integer spins)

$A_1$:
- $0^+$, $3^-$, $4^+$, $6^+$, $6^-$, $7^-$, $8^+$, $9^+$, $9^-$,
- $10^+$, $10^-$, $11^-$, $2 \times 12^+$, $12^-$, $\cdots$.

$A_2$:
- $0^-$, $3^+$, $4^-$, $6^-$, $6^+$, $7^+$, $8^-$, $9^-$, $9^+$,
- $10^-$, $10^+$, $11^-$, $2 \times 12^-$, $12^+$, $\cdots$.

$E$:
- $2^+$, $2^-$, $4^+$, $4^-$, $5^+$, $5^-$, $6^+$, $6^-$, $7^+$, $7^-$,
- $2 \times 8^+$, $2 \times 8^-$, $9^+$, $9^-$, $2 \times 10^-$, $2 \times 10^+$, $\cdots$.

$F_1$:
- $1^+$, $2^-$, $3^+$, $3^-$, $4^+$, $4^-$, $2 \times 5^+$, $5^-$, $6^+$, $2 \times 6^-$,
- $2 \times 7^+$, $2 \times 7^-$, $2 \times 8^+$, $2 \times 8^-$, $3 \times 9^+$, $2 \times 9^-$, $\cdots$.

$F_2$:
- $1^-$, $2^+$, $3^-$, $3^+$, $4^-$, $4^+$, $2 \times 5^-$, $5^+$, $6^-$, $2 \times 6^+$,
- $2 \times 7^-$, $2 \times 7^+$, $2 \times 8^-$, $2 \times 8^+$, $3 \times 9^-$, $2 \times 9^+$, $\cdots$. 
How demanding for computer

Full angular momentum projection is demanding! because of three dimensional integrals

Typical tetrahedral calculation with cranking: all the symmetries broken

Medium heavy nuclei with mesh of Euler angles,
\[ N_\alpha = N_\beta = N_\gamma \approx 100 \]
(for sizable tetrahedral deformation)

Two to three days: 50 — 70 hours by a machine with Xeon E5645(6cores)x2=12 CPU cores

c.f. If the system is nearly axially symmetric, then the calculation is much faster!
Application to high-spin states

- "Problem" of projection from cranked mean-field
- Wobbling rotational band
- Chiral doublet band
State-of-the-art high-spin yrast spectra
E.S. Paul et al., PRL 98, 012501 (2007)

bulk rotational energy (rigid inertia) subtracted
Application to high-spin states

Mean-field approximation ↔ Cranking method

Mean-field state $|\Phi\rangle$ is generated with cranking

$$\hat{h}' = \hat{h}_{\text{def}} - \sum_{\tau=n,p} \Delta_{\tau} \left( \hat{P}_{\tau}^+ + \hat{P}_{\tau} \right) - \sum_{\tau=n,p} \lambda_{\tau} \hat{N}_{\tau} - \omega_{\text{rot}} \hat{J}_x,$$

Rotational frequency

$\omega_{\text{rot}} \to \text{large} \iff \langle \hat{J}_x \rangle = I(\omega_{\text{rot}}) \to \text{large}$

$$|\Psi_{IM=I}\rangle \approx |\Phi(\omega_{\text{rot}})\rangle$$

Semiclassical quantization

Angular momentum projection → rotational band:

how about high-spin states?
Some results of projection calculation
(no configuration-mixing!)

Importance of cranking for moment of inertia

difference between $\omega_{\text{rot}} = 0$ and $\omega_{\text{rot}} \neq 0$

$\omega_{\text{rot}} = 0 \rightarrow E_I = \mathcal{H}^I_{00}/\mathcal{N}^I_{00}$

only K=0 contributes (axially symm.), but

$\omega_{\text{rot}} \neq 0 \rightarrow \det (\mathcal{H}^I_{KK'} - E_I \mathcal{N}^I_{KK'}) = 0$

K-mixing contributes, even if $\omega_{\text{rot}} \to 0$ → larger mom. of inertia

small $\omega_{\text{rot}}$ is enough
“Problem” of projection from cranked mean-field
Which frequency $\omega_{\text{rot}}$ should be used?

$$|\Psi_{IM}(\omega_{\text{rot}})\rangle = \sum_K g_K^I \hat{P}_{MK}^I |\Phi(\omega_{\text{rot}})\rangle$$

extra-ambiguity!

$\omega_{\text{rot}} = 0.01 \text{ MeV}$

needs improvement of Hamiltonian

exp.
Possible solutions

• Variation after projection:
  search best $\omega_{\text{rot}}$ for each spin-values
  $\rightarrow$ different projection calculation for each spin
  (very inefficient!)

• Do not use cranking!
  use projection to generate basis for shell model
  (not only 0-qp. but 2,4,...-qp. states coupled)
  $\rightarrow$ Projected Shell Model (PSM)
    by K.Hara and Y.Sun and collaborators
  Most successfull application of projection

• No other possibility?
  e.g., use two mean-field states for g- and s-bands
Wobbling rotational bands

Quantum mechanical motions of asymmetric-top → How triaxial nucleus rotates collectively?

strong triaxial def. → coll. rot. around three PA-axes → phonon-like multiple bands from one intrinsic configuration

superposition of rotations around 3PA-axes → rotation-axis tilts and precesses

No-breaking of chiral symm.
TSD: triaxial superdeformed

\[ ^{163}\text{Lu}_{92} \]

Wobbling Spectra  TSD bands

(Triaxial Superdeformed Band)


First identified by Ødegård et al. (2001)

\[ \begin{align*}
E & \uparrow \\
I & \rightarrow
\end{align*} \]

Wobbling

\[ \Delta I = +1 \]

\[ \Delta I = +2 \]

Very large \( B(E2)_{\text{in}} \) and \( B(E2)_{\text{out}} \) are observed!!

\( B(E2)_{\text{in}} \approx 500 \text{W.u.}, \ B(E2)_{\text{out}} \approx 100 \text{W.u.} \)

observed in \( ^{161,163,165,167}\text{Lu} \)
Result of angular momentum projection for wobbling rotational bands (1)

projection from one intrinsic state

\[ \omega_{\text{rot}} = 0.2 \text{MeV} \]

- \(^{163}\text{Lu}_{92}\)
- \( \beta_2 = 0.42, \beta_4 = 0.02, \gamma = 0^\circ, \Delta = 0.5, \omega = 0.2 \)

\[ \beta_2 = 0.42, \beta_4 = 0.02, \gamma = 18^\circ, \Delta = 0.5, \omega = 0.2 \]

- no wobbling bands
- wobbling bands appear!!
  (although moment of inertia is too small)

predicted by Strutinsky cal.
Result of angular momentum projection for wobbling rotational bands (2)

projection from one intrinsic state
\[ \omega_{\text{rot}} = 0.2 \text{MeV} \]

relative spectra

\[ {}^{163}_{71} \text{Lu}_{92} \]

\[ \text{B(E2)}_{\text{out}} / \text{B(E2)}_{\text{in}} \]

rotor-model-like results obtained by the microscopic projection
Result of angular momentum projection for wobbling rotational bands (3)

projection from one intrinsic state

$^{163}_{71}$Lu$_{92}$

relative spectra

difficult to simultaneously reproduce both one- and two-phonon energies
Chiral doublet band

Breaking right-/left-handed symmetry ⇒ two degenerate bands

Importance of triaxial deformation

Tonev et al., PRL96(2006),052501.

3D rotation

$\vec{J}$: total angular momentum

$\vec{J}_\pi$: high-$j$ particle → short-axis

$\vec{J}_\nu$: high-$j$ hole → long-axis

$R$: irrotational inertia → inter-axis

c.f. $R \approx 0$ ⇒ magnetic rotation in the Pb region

$N=75$ isotones odd-odd $A \sim 130$ region

$^{130}$Cs

$^{132}$La

$^{134}$Pr

$^{136}$Pm

Starosta et al., PRL86(2001),971.

Tonev et al., PRL96(2006),052501.
Result of two-particle rotor coupling model

Macroscopic model analysis: the original paper

Rotor + particle-hole

\[ \gamma = -30 \text{ deg.} \]
Result of angular momentum projection for chiral doublet band

\[ ^{104}_{45} \text{Rh}_{59} \]

\[ \text{BAND DIAGRAM} \quad \omega = 0.0, \beta_2 = 0.30, \beta_4 = 0.0, \gamma = -30^\circ \]

Odd-odd

\[ ^{104}_{45} \text{Rh}_{59} \quad (\pi g_{9/2})^{-1} \]

\[ (\nu h_{11/2})^{1} \]

Odd-odd

Even spin

Odd spin

Two lowest bands nearly degenerate

\[ \gamma = -30 \text{ deg.} \]

Sig.-degenerate

Only spectra, BE2 \cdot BM1 not yet calculated!!
No doublet bands, e.g. if $\omega_{\text{rot}}$ increased!

\[ ^{104}_{45}\text{Rh}_{59} \quad \text{BAND DIAGRAM} \quad \omega = 0.3 \text{, } \beta_2 = 0.25 \text{, } \beta_4 = 0.0 \text{, } \gamma = -30^\circ \]

\[ 104\text{Rh}_{59} \quad \frac{(\pi g_{9/2})^{-1}}{(\nu\hbar_{11/2})^{1/2}} \]

$\omega_{\text{rot}} = 0.0 \rightarrow 0.3 \text{ MeV}$

$\beta_2 = 0.30 \rightarrow 0.25$

- red: even spin
- dashed: odd spin

sig.-splitting

$\gamma = -30 \text{ deg.}$
Summary

• Effective method for projection
  1) canonical-basis truncation
  2) full use of Thouless amplitude

• Application to nuclei with tetrahedral shape
  Td-symmetry $\rightarrow$ specific spin-parity combinations
  Expected spectra not only for closed-shell but also for one-particle and two-particle systems
  However, there are considerable splittings

• Application to high-spin states
  Wobbling bands: confirmed full-microscopically, but not easy for quantitative description
  Chiral doublet bands: seems to be existing in full-microscopic calculations, need more study, e.g., electromagnetic transitions etc.