Shell Model Monte Carlo Methods
and their Applications to Nuclear Level Densities

H. Nakada (*Chiba U., Japan*)

@ RIKEN (Feb. 14, 2013)

Contents :

I. Introduction
II. Shell model Monte Carlo methods
III. Brief survey of nuclear level densities
IV. SMMC level densities (in medium-mass nuclei)
V. SMMC in rare-earth nuclei
VI. Summary & future prospect

Collaborators :

Y. Alhassid	(***Yale U.***,	***U.S.A.***)
C. Özen	(***Kadir Has U.***,	***Turkey***)
S. Liu	(***Yale U.***,	***U.S.A.***)*
L. Fang	(***Yale U.***,	***U.S.A.***)*
E. Akyüz	(***Istanbul U.***,	***Turkey***)
G.F. Bertsch	(***U. Washington***,	***U.S.A.***)

(* quit)

I. Introduction

Nuclear structure

- global nature (*e.g.* saturation, binding energies, shell structure)
  $\Rightarrow$ MF (or EDF) approaches
- 2-body correlations — often important for quantitative description
  $\Rightarrow$ GCM, Interacting shell model (or CI), *etc.*

Advantages & disadvantages of ISM

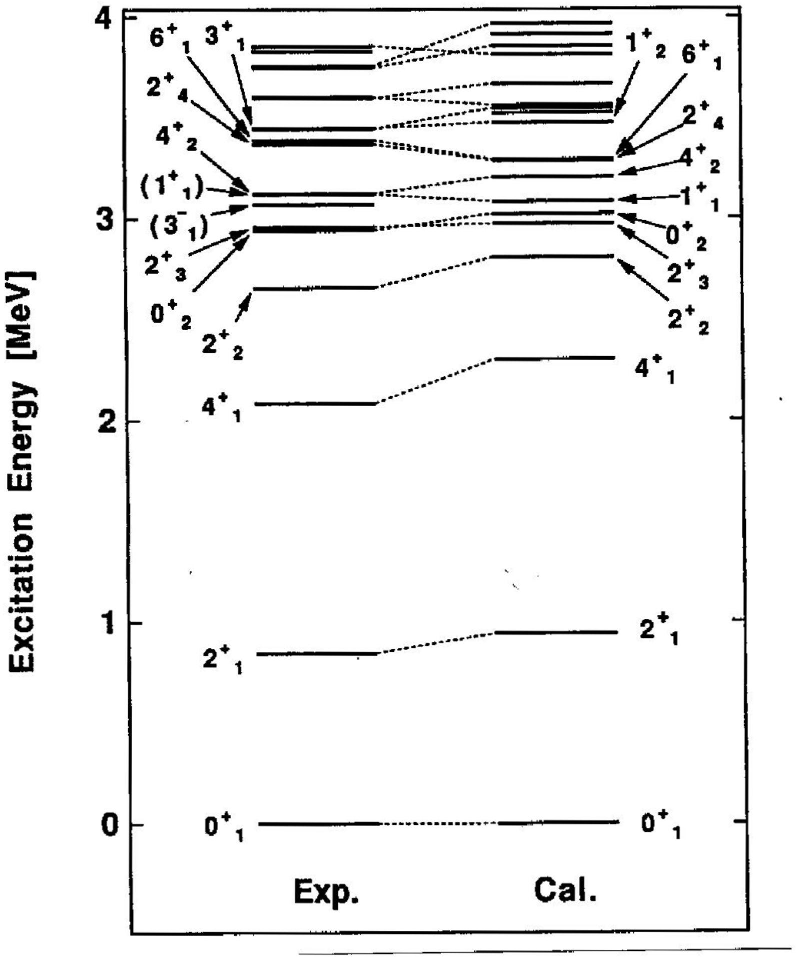
A full quantum theory ($\rightarrow$ collective + non-collective d.o.f)
good accuracy ! — up to (relatively) high excitation energies
cf. GCM

D model space $\rightarrow$ renormalization, energy limit
rapid increase of computational intensiveness

$\Rightarrow$ application & development of QMC technique (SMMC)

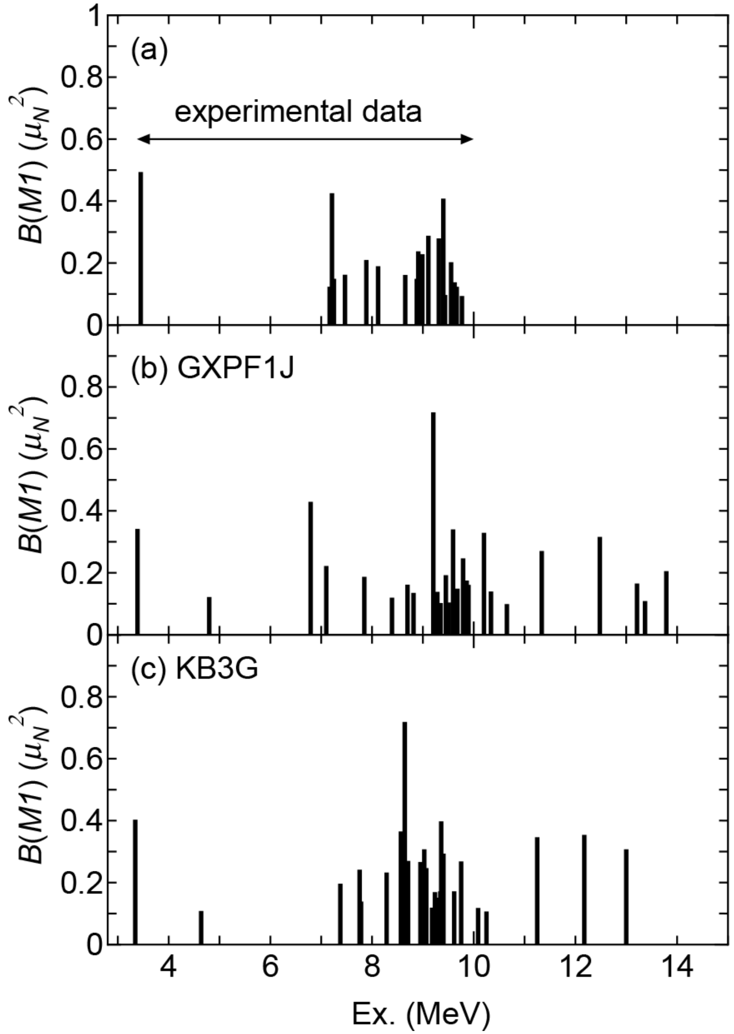
Ref. : G.H. Lang *et al.*, P.R.C 48, 1518 ('93)

Level scheme of ^{56}Fe :



Ref. : H.N. *et al.*,
N.P.A 571, 467 ('94)

$M1$ distribution in ^{56}Fe :



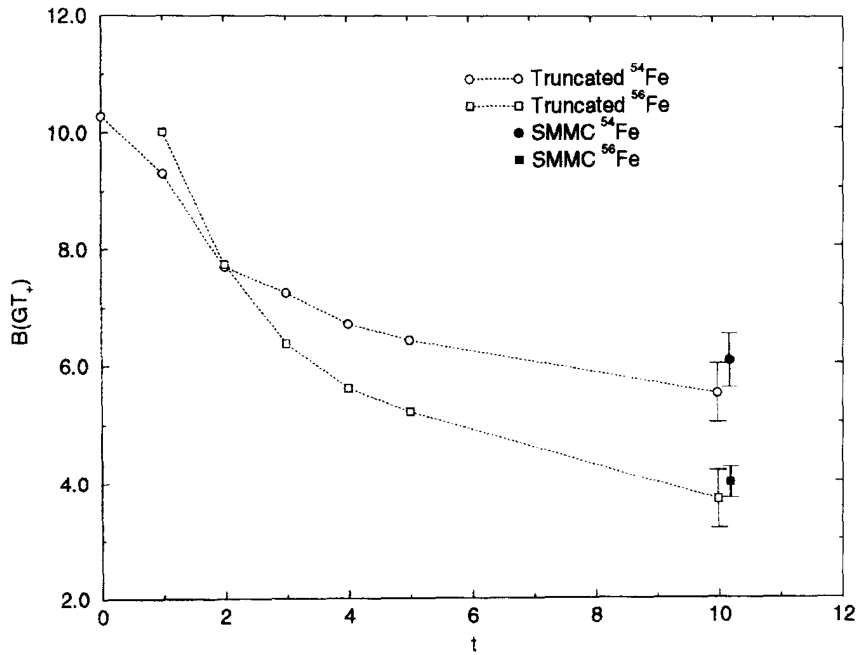
Ref. : T. Shizuma *et al.*,
P.R.C 87, 024301 ('13)

Advantages & disadvantages of SMMC

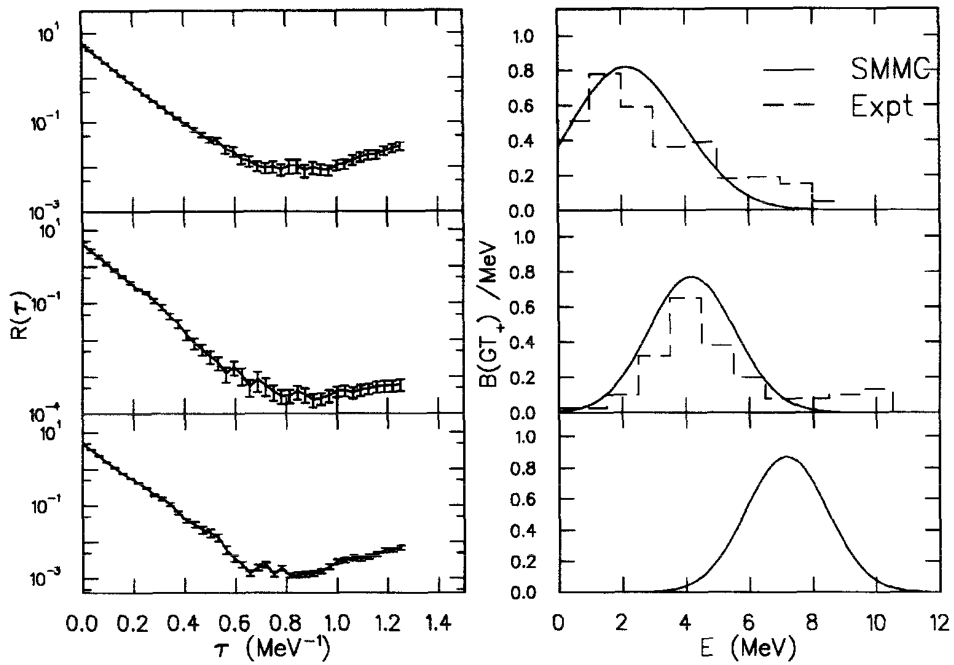
A implementable in large model space
'exact' quantum mechanical method
able to cover up to high excitation energies
including statistical properties (← finite T)

D no explicit wave functions
difficult to obtain detailed spectroscopic information
statistical errors
sign problem

Summed $B(\mathrm{GT}_+)$:
(comparison with diag.)



$B(\mathrm{GT}_+)$ distribution in ^{54}Fe, ^{59,55}Co :



Ref. : S.E. Koonin *et al.*, Phys. Rep. 278, 1 ('97)

(One of) the most successful applications of SMMC so far

$\cdots$ nuclear level densities

H.N. & Y. Alhassid, P.R.L. 79, 2939 ('97)

W.E. Ormand, P.R.C 56, R1678 ('97)

H.N. & Y. Alhassid, P.L.B 436, 231 ('98)

K. Langanke, P.L.B 438, 235 ('98)

Y. Alhassid, S. Liu & H.N., P.R.L. 83, 4265 ('99)

Y. Alhassid, G.F. Bertsch, S. Liu & H.N., P.R.L. 84, 4313 ('00)

J.A. White, S.E. Koonin & D.J. Dean, P.R.C 61, 034303 ('00)

Y. Alhassid, S. Liu & H.N., P.R.L. 99, 162504 ('07)

C. Özen *et al.*, P.R.C 75, 064307 ('07)

H.N. & Y. Alhassid, P.R.C 78, 051304(R) ('08)

A. Mukherjee & Y. Alhassid, P.R.L. 109, 032503 ('12)

+ 'deformation' within ISM (← certain spectroscopic information)

Y. Alhassid, L. Fang & H.N., P.R.L. 101, 082501 ('08)

C. Özen, Y. Alhassid & H.N., P.R.L. 110, 042502 ('13)

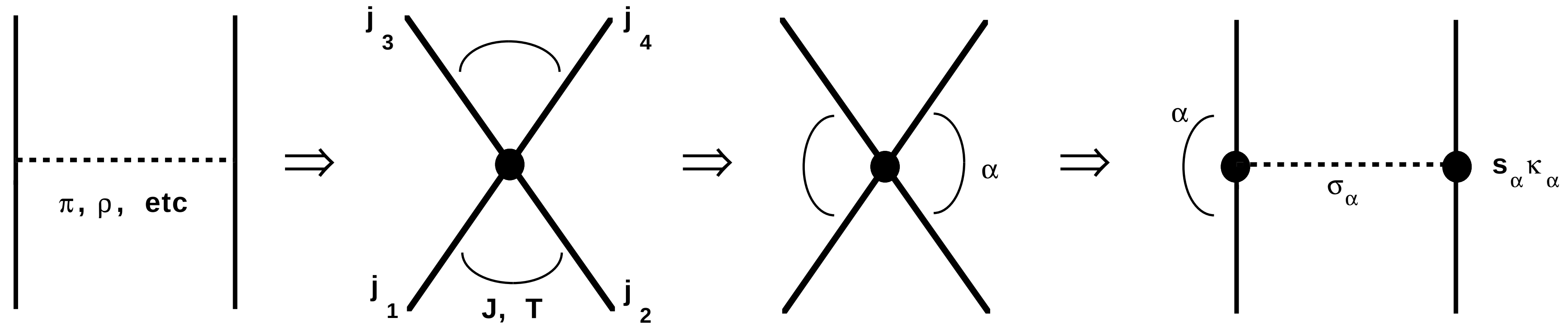
II. Shell model Monte Carlo methods

SMMC $\rightarrow$ **evaluate** $\langle \mathcal{O} \rangle_\beta = \mathrm{Tr}(\mathcal{O} e^{-\beta H})/Z(\beta)$ (***e.g.*** $E(\beta) = \langle H \rangle_\beta$)

H **: shell model Hamiltonian (1- + 2-body)**

$e^{-\beta H} \rightarrow$ **auxiliary-fields** $(\sigma_\alpha(\tau))$ **path integral rep.**

- **2-body int.** $\rightarrow$ **Pandya transform.**
- $e^{-\beta H}$ **: Suzuki-Trotter decomp.**
  $\rightarrow$ **Hubbard-Stratonovich transform.**



$$e^{-\beta H} = \left(e^{-\Delta\beta H}\right)^{n_t}; \quad e^{-\Delta\beta H} \approx \prod_j \left[\exp(-\Delta\beta\epsilon_j \hat{N}_j)\right] \prod_\alpha \left[\exp(-\Delta\beta\tfrac{\kappa_\alpha}{2}\hat{\rho}_\alpha^2)\right] + O\left((\Delta\beta)^2\right)$$

HS transform.: $\exp(-\Delta\beta\frac{\kappa_\alpha}{2}\hat{\rho}_\alpha^2) \propto \int d\sigma_\alpha \, \exp\left[-\Delta\beta(\frac{|\kappa_\alpha|}{2}\sigma_\alpha^2 + s_\alpha\kappa_\alpha\sigma_\alpha\hat{\rho}_\alpha)\right]$

$s_\alpha = \pm 1$ **(if** $\kappa_\alpha < 0$**) or** $\pm i$ **(if** $\kappa_\alpha > 0$**)**

$\Rightarrow$ $\mathrm{Tr}(\mathcal{O}e^{-\beta H}) \approx \int \mathcal{D}[\sigma] \, G(\sigma) \, \mathrm{Tr}(\mathcal{O}U_\sigma)$ **(AF path integral)**

$$G(\sigma) = \exp(-\Delta\beta\tfrac{|\kappa_\alpha|}{2}\sigma_\alpha^2), \quad U_\sigma = \prod \exp(-\Delta\beta h_\sigma);$$

$$h_\sigma = \sum_j \epsilon_j \hat{N}_j + \sum_\alpha s_\alpha\kappa_\alpha\sigma_\alpha\hat{\rho}_\alpha$$

$\mathrm{Tr}_{\mathrm{gc}}(U_\sigma) = \det(1 + \mathcal{U}_\sigma)$ $\mathcal{U}_\sigma$**: s.p. matrix for** U_σ

$\cdots$ **calculable only via the s.p. matrices**

σ_α**: auxiliary field** $\leftrightarrow$ **'mean field'**

cf. Mean-field approx. — $\sigma_\alpha = \langle\hat{\rho}_\alpha\rangle$ $\cdots$ **static mean-field**

(no fluctuation, no τ**-dependence)**

Static-path approx. — $\exp(-\beta\frac{\kappa_\alpha}{2}\hat{\rho}_\alpha^2) \propto \int d\sigma_\alpha \, \exp\left[-\beta(\frac{|\kappa_\alpha|}{2}\sigma_\alpha^2 + s_\alpha\kappa_\alpha\sigma_\alpha\hat{\rho}_\alpha)\right]$

σ_α $\cdots$ **static auxiliary-field with fluctuation**

MC sampling $\sigma_k = \{\sigma_\alpha(\tau)\}_k \to \langle\mathcal{O}\rangle_\beta \approx \frac{1}{N_k}\sum_k \langle\mathcal{O}\rangle_{\sigma_k}$ **(with stat. error)**

$\hookleftarrow$ **random walk of** $\{\sigma_\alpha(\tau)\}_k$

Metropolis algorithm (& amendment) $\to$ **detailed balance**

cf. $N_k \gtrsim 4000$ **in practical SMMC calculations**

- **advantage: easier to handle large model space**

  CPU time $\propto M_{\mathrm{s.p.}}^3 \times n_t$ **,** **suitable for parallel computing**

- **finite-T method** $\to$ **suitable for statistical properties**

  (but not for distinguishing discrete levels)

- **conservation laws** $\to$ **projections**
  - Z **&** N **proj.** $\to$ **canonical ens.** $(\mathrm{Tr}_{\mathrm{gc}} \to \mathrm{Tr})$
  - π**,** J (J_z)**, isospin proj.**

- $E_0 = \lim_{\beta\to\infty}\langle H\rangle_\beta$ $\cdots$ **evaluated also by SMMC**

  (stabilization method required for heavy nuclei)

- **disadvantage: sign problem**

  — propagator $\mathrm{Tr}(e^{-\beta h(\sigma_k)})$ **: not necessarily positive-definite**

  dominant part of nuclear int. — sign good !

III. Brief survey of nuclear level densities

Nuclear level densities (↔ partition fn.)

— one of the key inputs

in calculations of low-energy nuclear reactions

for $A(a,b)B$ reaction $$\sigma_{(a,b)} \propto \sum_{J_f\pi_f} \int dE_f \, T^{(a)}_{J_i\pi_i}(E_i) \, T^{(b)}_{J_f\pi_f}(E_f) \, \rho_{J_f\pi_f}(E_f)$$

↔ Hauser-Feshbach formula

$T^{(a/b)}_{J\pi}(E)$ **: transmission coefficient from compound state**

$\rho_{J\pi}(E)$ **: level density**

⇒ application to nuclear-astrophysics

e.g. s**- &** r**-processes** $\cdots$ (n,γ) ***vs.*** β**-decay**

$$\sigma_{(n,\gamma)} \quad \leftarrow \quad (a=n,\ b=\gamma)$$

$$E_i = E_f + E_\gamma - S_n \quad (\to E_f \lesssim S_n)$$

rp**-process** $\cdots$ (p,γ) ***vs.*** β**-decay**

Experimental methods to measure nuclear level densities

1. **Direct counting of levels — lowest-lying states or light nuclei**
   ($E_x \lesssim 2-3\,\mathrm{MeV}$)
2. **Level spacing among neutron resonances** ($\rho = \bar{D}^{-1}$)
   — **small energy range around** $E_x = S_n \sim 8\,\mathrm{MeV}$**, restricted to** s**-wave**
3. **Ericson fluctuation** — $E_x \sim 20\,\mathrm{MeV}$
4. **Charged particle reactions** (← **reaction model**)
5. **'Oslo method'** $\cdots$ γ**-ray matrix** $P(E_x, E_\gamma) = C(E_x)\,T^{(\gamma)}(E_\gamma)\,\rho(E_x - E_\gamma)$
   (← **Brink-Axel hypothesis**)
   — $E_x \sim 3-7\,\mathrm{MeV}$

Conventional approach to nuclear level densities

★ **Backshifted Bethe's formula** (← **Fermi-gas model**)

$$\rho(E_x) = \frac{\sqrt{\pi}}{12} a^{-1/4}(E_x - \Delta)^{-5/4} \exp\left[2\sqrt{a(E_x - \Delta)}\right] \qquad \textbf{(for state density)}$$

$\cdots$ **fits well to experimental data (except at very low** E_x**),**

if the parameter a **(&** Δ**) is adjusted**

(Δ **: backshift** $\leftrightarrow$ **pairing & shell effects)**

$$\rho_{J,\pi}(E_x) = \rho(E_x)\, \frac{2J+1}{4\sqrt{2\pi}\sigma^3} \exp\left[-J(J+1)/2\sigma^2\right] \; ; \qquad \sigma = \mathcal{I}\sqrt{(E_x - \Delta)/a}$$

$$\left(\rho(E_x) = \sum_{J,\pi} (2J+1)\, \rho_{J,\pi}(E_x) \right)$$

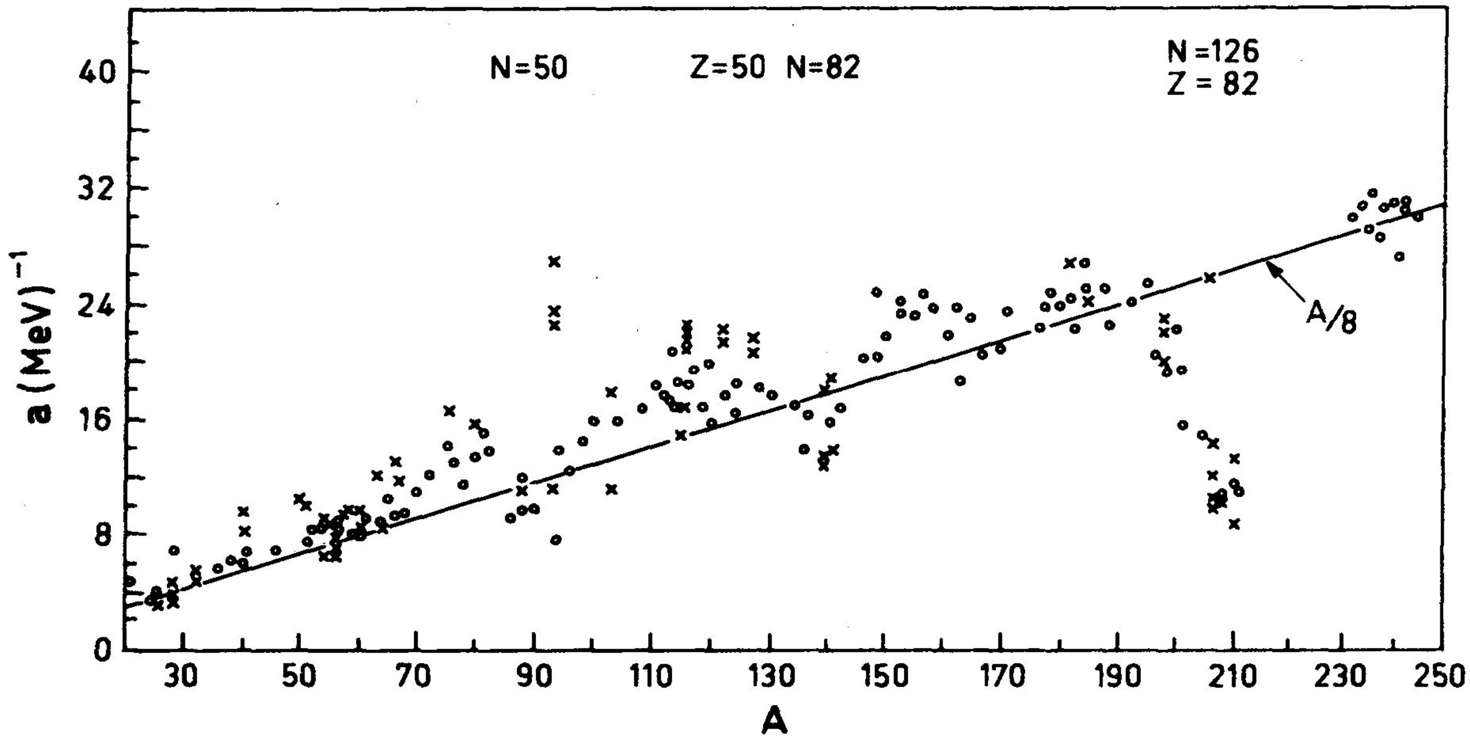
However, **1)** $a = A/6 \sim A/10\,\mathrm{MeV}^{-1}$**,**

in contrast to the Fermi-gas prediction $a \approx A/15$

2) a **: nucleus-dependent (not only** A**-dependent)**

— shell effects, ***etc.***

fitted values of a:



Ref.: Bohr & Mottelson, vol. 1

Note: 10% **change in** $a \rightarrow$ **change in** $\rho(E_x)$ **by greater than factor** 10**!**

(for $A \sim 150$**,** $E_x \sim 8\,\mathrm{MeV}$**)**

For better E_x-dep. — **correction for low E_x part**

★ **Constant-T formula** ($\leftrightarrow$ T-dep. of pairing)

$$\rho(E_x) \propto \exp[(E_x - E_1)/T_1] \quad \text{for } E_x < E_M$$

$\rightarrow$ **matching to BBF at** $E_x = E_M$

To get less A-dep. parameters — **nucl.-dep. corrections**

★ $a \rightarrow E_x$**-dep.**: $a(E_x) = \tilde{a}\left(1 + \delta W \frac{1-\exp[-\gamma(E_x-\Delta)]}{E_x-\Delta}\right)$

δW : **shell correction energy,** $\gamma = \gamma_1 A^{-1/3}$

★ **Collective enhancement factor** $K_{\rm vib}(E_x)$, $K_{\rm rot}(E_x)$

$$\textit{\textbf{e.g.}}\ K_{\rm rot}(E_x) = \max\left(\left[0.01389 A^{5/3}\left(1+\frac{\beta_2}{3}\right)\sqrt{\frac{E_x-\Delta}{a}} - 1\right]\frac{1}{1+\exp(\frac{E_x-E_c}{d_c})} + 1, 1\right)$$

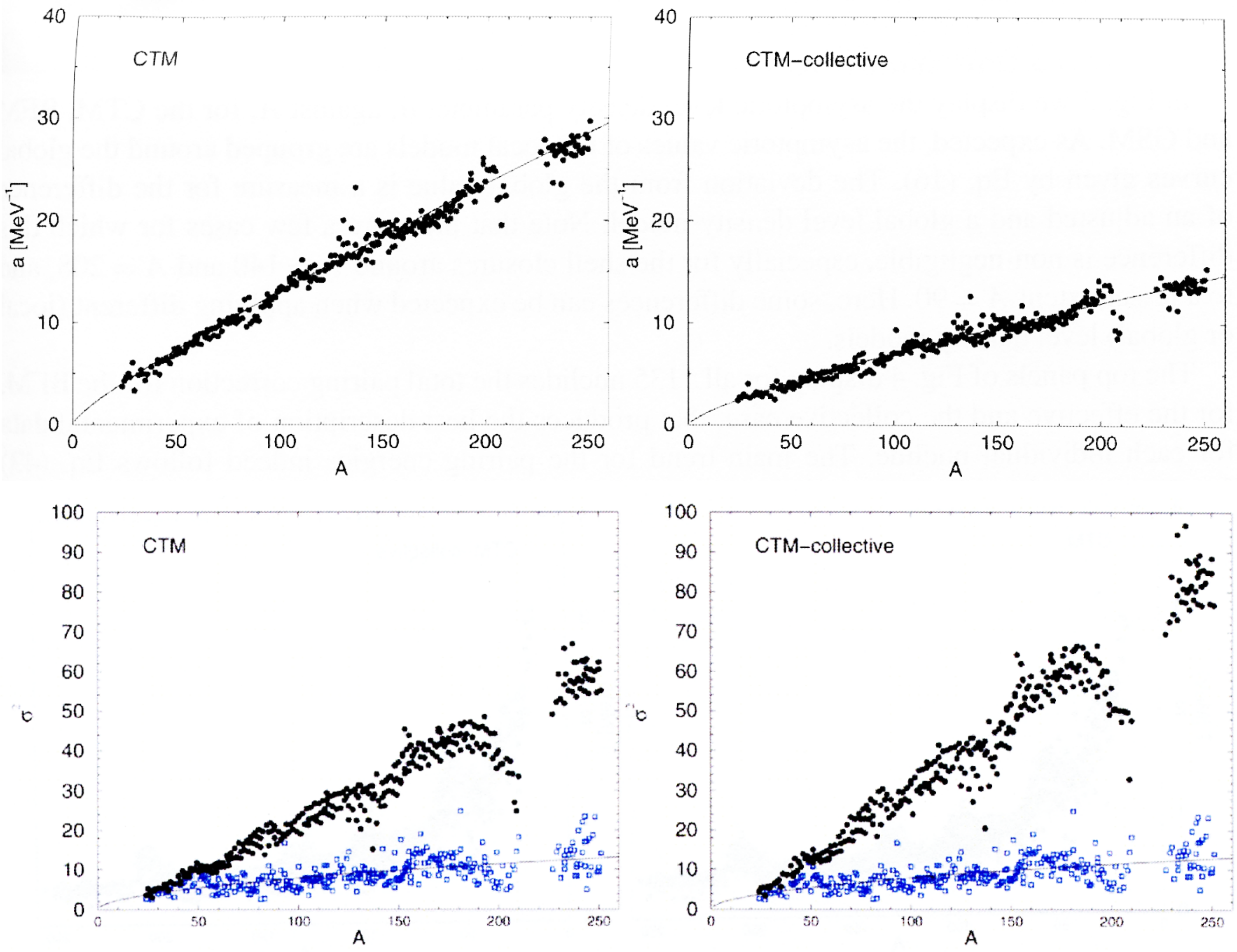
$\rightarrow$ **can be harmonious with** $a \approx A/15\,{\rm MeV}^{-1}$ **(Fermi-gas value)**

- **many corrections & parameters introduced**

  — **origin ? estimate ? (physics ?)**

- **significant nucleus-dependence still remains**

  e.g. **for** $\tilde{a}$(: **"asymptotic value" of** a) & σ



Ref. : A.J. Koning *et al.*, N.P.A 810, 13 ('08)

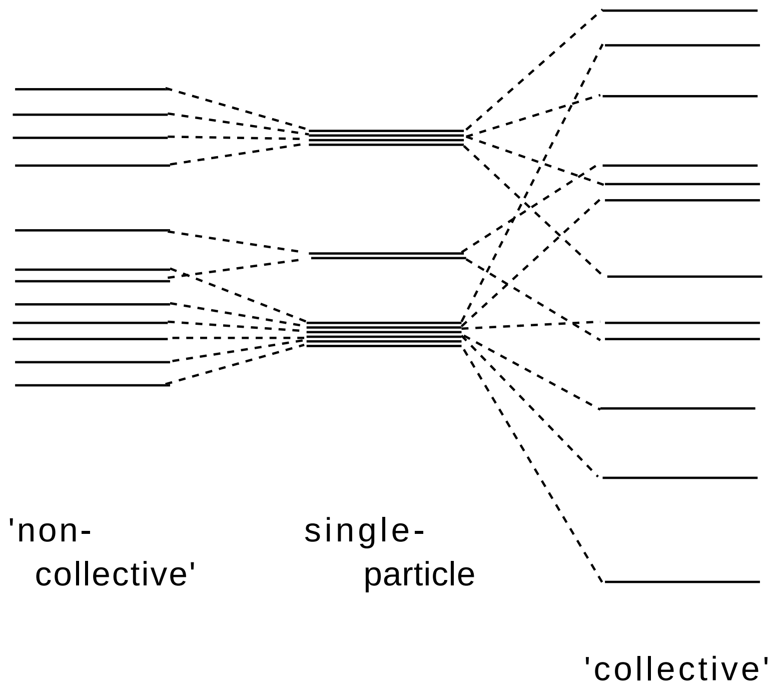
$\Longrightarrow$ **It has been difficult to predict nuclear level densities to good accuracy**

What is needed ?

(1) shell effects (2) 'collective' 2-body correlations

e.g. $V = -\frac{\kappa}{2}\,\hat{\rho}^2$ ($\hat{\rho}$: 1-body op.)

typically, κ : large $\leftrightarrow$ collective



(Both coll. & non-coll. d.o.f. relevant !)

full shell model $\rightarrow$ Both (1) & (2) are fully taken into account within the model space

$\cdots$ large model space required $\longrightarrow$ SMMC (H.N. & Y. Alhassid)

IV. SMMC level densities (in medium-mass nuclei)

'**Nuclear temperature**' $\leftrightarrow$ (average) excitation energy

$\rightarrow$ finite-T formalism
- Fermi gas $\rightarrow$ BBF
- MF

} $\cdots$ grand-can. ens.

- SMMC $\cdots$ can. ens.

$\rightarrow$ statistical properties — thermodynamics in finite systems

state density ⟷ partition fn.

$$\rho(E) = \mathrm{Tr}\,\delta(E-H) \quad \longleftrightarrow \quad Z(\beta) = \mathrm{Tr}(e^{-\beta H}) = \int dE\,\rho(E)e^{-\beta E}$$

Laplace transform.

(Tr : can. trace)

saddle-point approx. ($\leftrightarrow$ smoothening for E, nuclear temp.)

$$\rightarrow \rho(E) \approx \frac{e^S}{\sqrt{2\pi\beta^{-2}C}}; \quad S = \beta E + \ln Z(\beta),\ \ \beta^{-2}C = -\frac{dE}{d\beta}$$

(S : entropy, C : heat capacity)

cf. grand-can. $\cdots$ $\rho_{\mathrm{gc}}(E) \approx \dfrac{e^{S_{\mathrm{gc}}}}{\sqrt{(2\pi)^3(\Delta N_p)^2(\Delta N_n)^2\beta^{-2}C_{\mathrm{gc}}}}$ $\rightarrow$ nucl.-dep. ?

★ Nuclei around Fe-Ni region

- **setup**

  model space: full $pf + 0g_{9/2}$

  Hamiltonian: s.p. energy ← W-S pot.

  int. $\left\{\begin{array}{l}\text{monopole pairing (SE channel)} \\ \quad \text{strength} \leftarrow \text{even-odd mass difference} \\ \text{surface-peaked multipole (isoscalar, } \lambda = 2, 3, 4) \\ \quad \text{strength} \leftarrow \text{self-consistency + renorm.} \\ \quad \text{renorm. factor} \leftarrow \text{realistic int.}\end{array}\right.$

  no adjustable parameters!

  $\cdots$ sign good (for even-even nuclei) → small stat. error

- **applications**

  $\rho(E_x)$ → $\left\{\begin{array}{l}\text{comparison to BBF} \\ \text{extension to higher } E_x \text{ (via connection with HF)}\end{array}\right.$

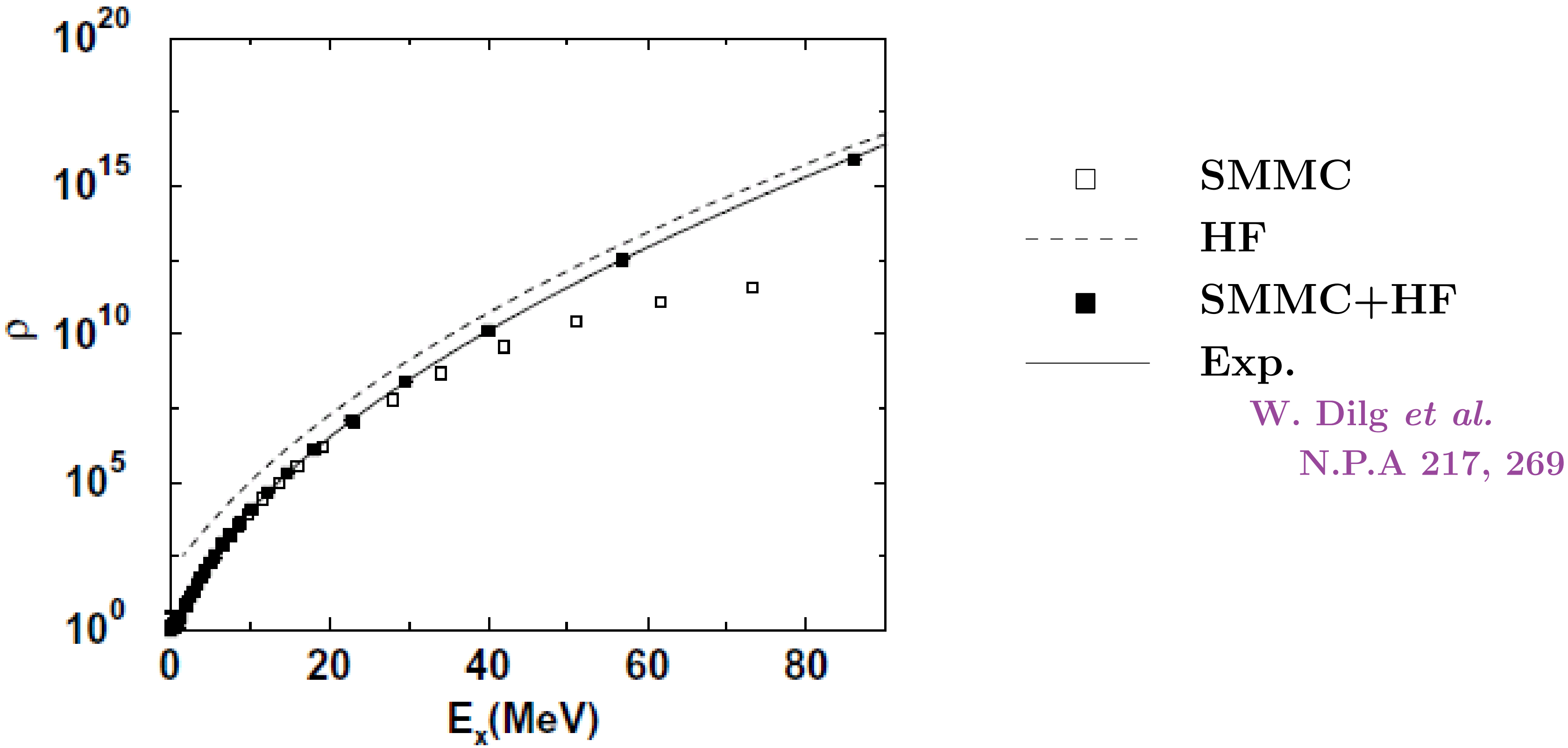
  $\rho_\pi(E_x)$ (← π proj.)

  $\rho_J(E_x)$, $\rho_{J\pi}(E_x)$ (← J proj.) → comparison to spin cut-off model

  $\rho_T(E_x)$ (← isospin proj.) → correction subject to $Z \approx N$ nuclei

State density $\rho(E_x)$ of ^{56}Fe : Ref. : Y. Alhassid *et al.*, P.R.C 68, 044322 ('03)

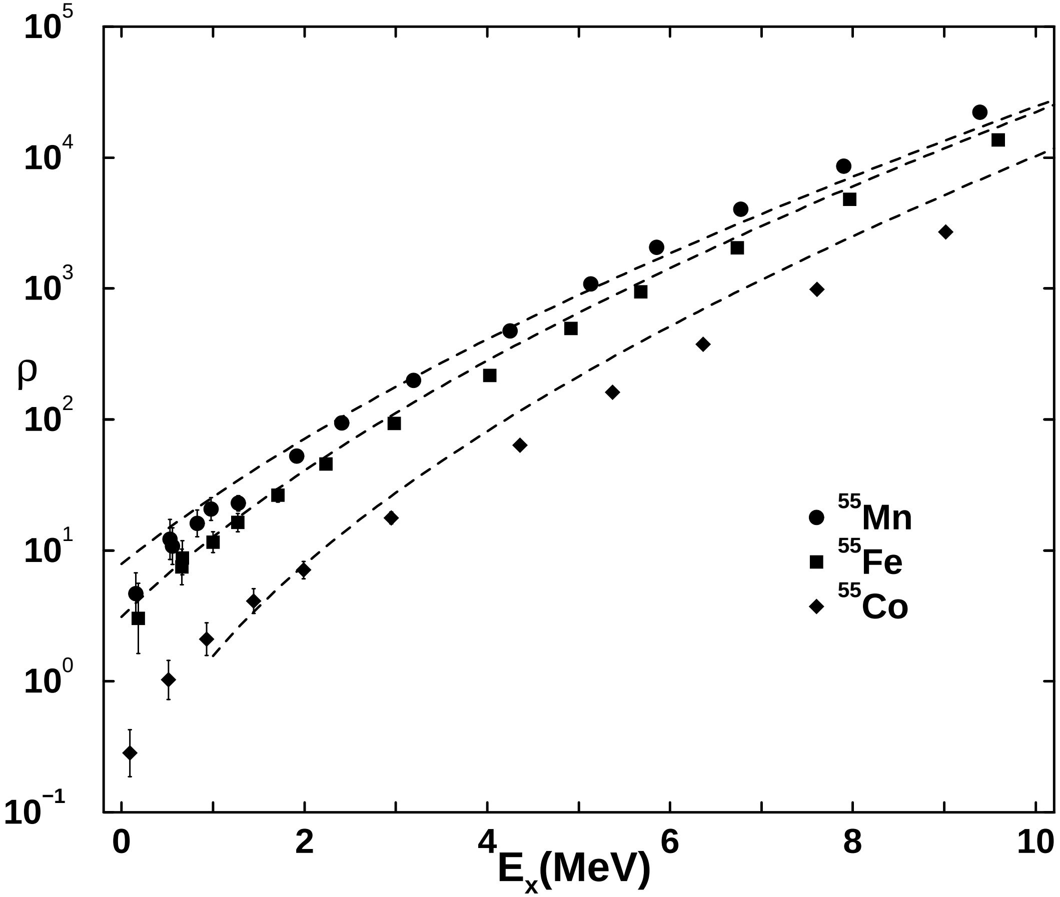
$\left.\begin{array}{l}\textbf{SMMC for } pf+g_{9/2} \\ \textbf{finite-}T\textbf{ HF}\end{array}\right\} \rightarrow$ **connection of** $F(\beta) \rightarrow \rho(E)$



$$\left\{\begin{array}{l} E_x \lesssim 10\,\mathrm{MeV} \text{ — strong correlations inside } pf+g_{9/2} \text{ shell} \\ E_x \gtrsim 25\,\mathrm{MeV} \text{ — weak correlation, s.p. d.o.f. is essential} \end{array}\right.$$

State density $\rho(E_x)$ of $A = 55$ isobars:

Ref.: Y. Alhassid, S. Liu, H.N., P.R.L. 83, 4265 ('99)



Exp.: W. Dilg *et al.*, N.P.A 217, 269 ('73)

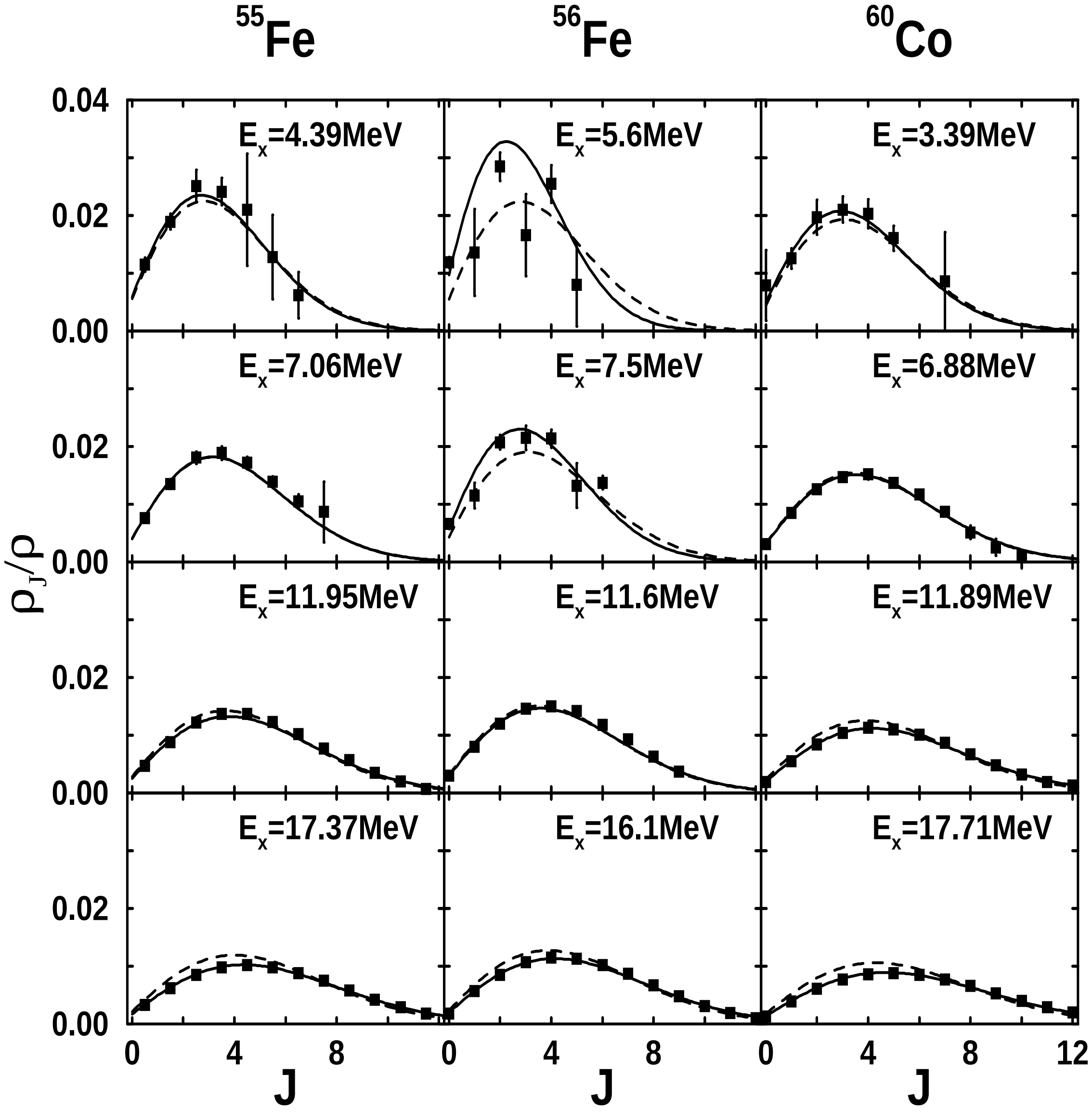
Many empirical formulas predict equal $\rho(E_x)$ among odd-A isobars

— not true!

(← exp. & micro. cal.)

J-dep. level density $\rho_{J\pi}(E_x)$ of ^{56}Fe :

Ref. : Y. Alhassid, S. Liu, H.N., P.R.L. 99, 162504 ('07)



■ SMMC results

spin-cutoff model with σ

- ——— fitted to SMMC
- - - - from rigid-body $\mathcal{I}$

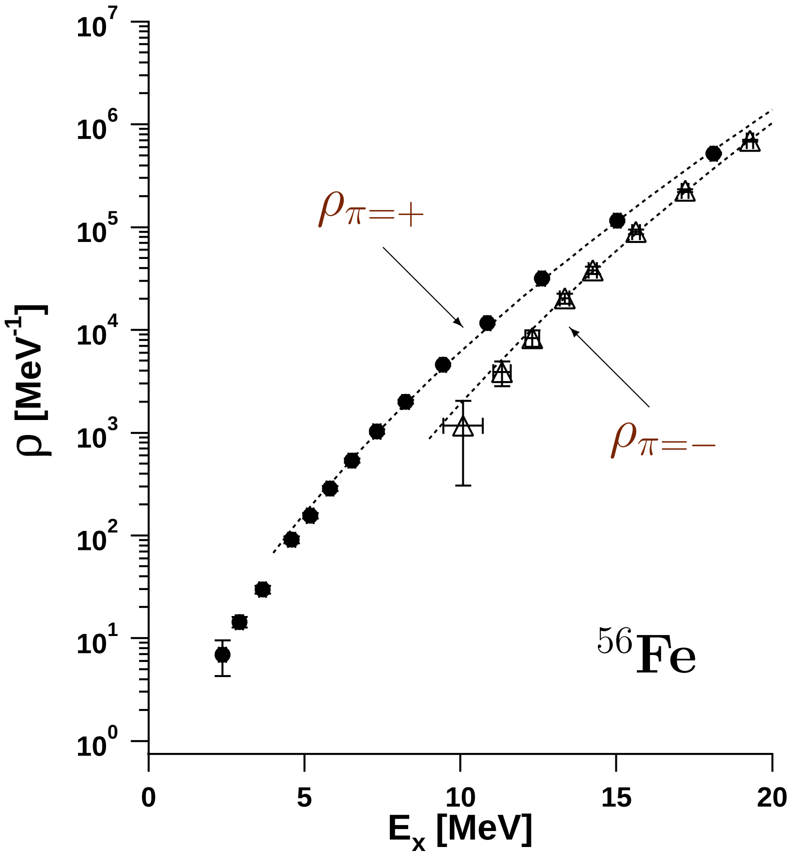
significant deviation from spin-cutoff model

- at low E_x &
- in even-even nuclei

$\leftrightarrow$ influence of pairing

π-dep. state density $\rho_\pi(E_x)$:

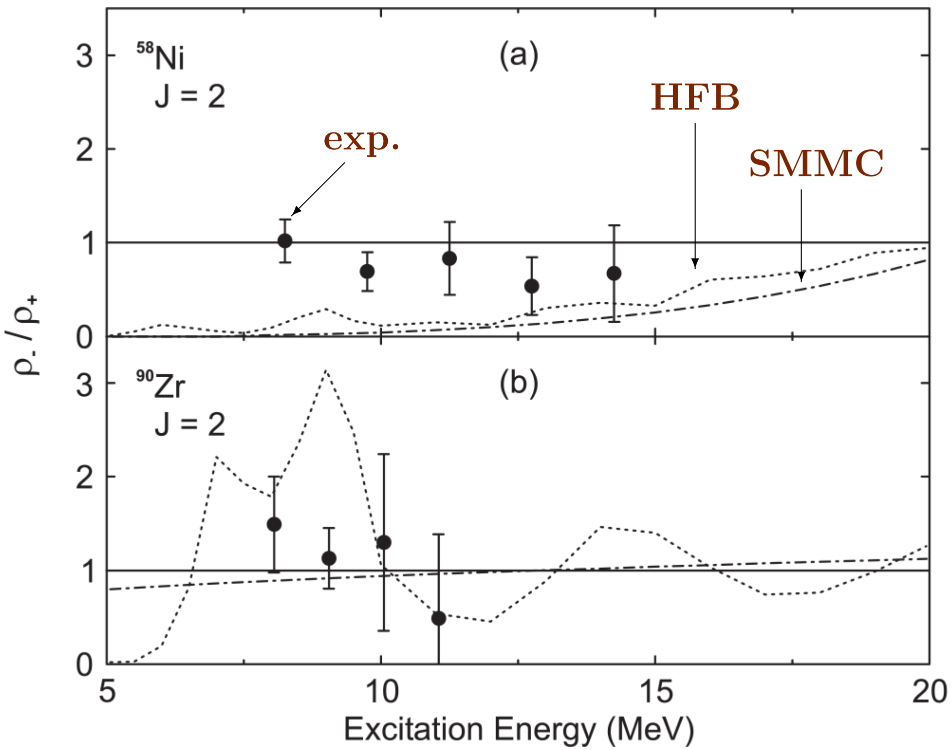
SMMC



Ref. : H.N. & Y. Alhassid, P.R.L. 79, 2939 ('97)

⇒ strong π-dep. ?

Exp.



Ref. : Y. Kalmykov *et al.* P.R.L. 99, 202502 ('07)

— exc. out of sd-shell plays a role

→ C. Özen, E. Akyüz & H.N., work in progress

V. SMMC in rare-earth nuclei

★ **Questions**

- **rotational (*i.e.* well-deformed) nuclei**

  ··· described within spherical shell model ?

  with how large model space ?

- **crossover transition from vibrational to rotational nuclei ?**
- **how to identify vibrational / rotational characters in SMMC ?**
- **nuclear shape & level densities ?**
- **micro. understanding of $K_{\rm vib}(E_x)$, $K_{\rm rot}(E_x)$ (for level density)**

★ **Nuclides :** ^{162}**Dy, even-**N **Nd-Sm isotopes**

Ref : Y. Alhassid, L. Fang & H.N., P.R.L. 101, 082501 ('08)
C. Özen, Y. Alhassid & H.N., P.R.L. 110, 042502 ('13)

★ **Setup**

model space : $\begin{cases} p : (Z = 50 - 82 \textbf{ shell}) + 1f_{7/2} \\ n : 0h_{11/2} + (N = 82 - 126 \textbf{ shell}) + 1g_{9/2} \end{cases}$

$\leftarrow$ **expand def. WS solutions by sph. WS orbitals**

$(0.1 < \langle \hat{n}_{\alpha j} \rangle / (2j + 1) < 0.9)$

$\cdots$ **biggest SMMC calculations to date !**

Hamiltonian : s.p. energy $\leftarrow$ **W-S pot. + HF-type correction**

int.
- pp & nn **monopole pairing** $(g_0 = \gamma\, g_0^{\mathrm{BCS}})$
  - **strength** $\leftarrow$ **even-odd mass difference**
  - $+$ **fit to** $\mathcal{I}_g$
- $(p + n)$ **surface-peaked multipole** $(\lambda = 2, 3, 4)$
  - **strength** $\leftarrow$ **self-consistency + renorm.**
  - **renorm. factor for** $\lambda = 2$ (k_2) $\leftarrow$ **fit**

adjust. parameters — smooth function of N

★ Technical developments for SMMC calculations

1) **Proton-neutron formalism** $\leftarrow p$ & n **occupying different shell**

2) **Stabilization of canonical propagator**

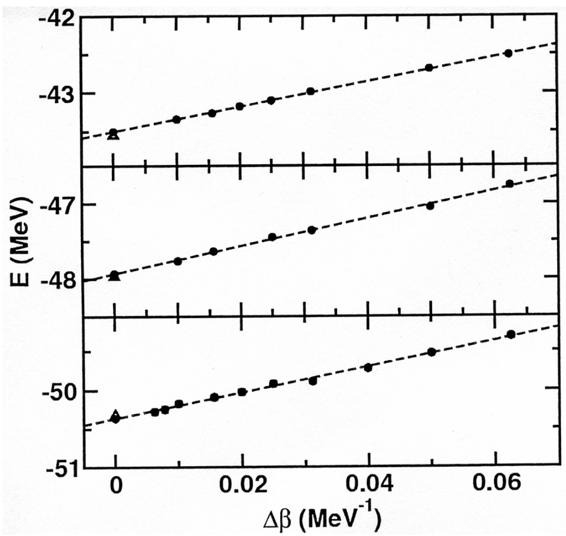
E_g, $\mathcal{I}_g$ $\cdots$ **needs calculations at** $\beta \gtrsim 10\,\mathrm{MeV}^{-1}$ **for heavy well-deformed nuclei**

propagator $U_\sigma\,(\leftrightarrow e^{-\beta H})$ $\cdots$ **ill-behaved at large** β

$\Rightarrow$ **stabilization for canonical propagator** ($\leftarrow$ n-**proj.**)

3) **Assessment of discretization effects** — **extrapolation to** $\Delta\beta = 0$

test case : ^{22}**Ne**



($\beta = 0.5,\ 0.75,\ 1\,\mathrm{MeV}^{-1}$)

for ^{162}**Dy**

$\cdots$ $\Delta\beta = 1/32$ & $1/64\,\mathrm{MeV}^{-1}$

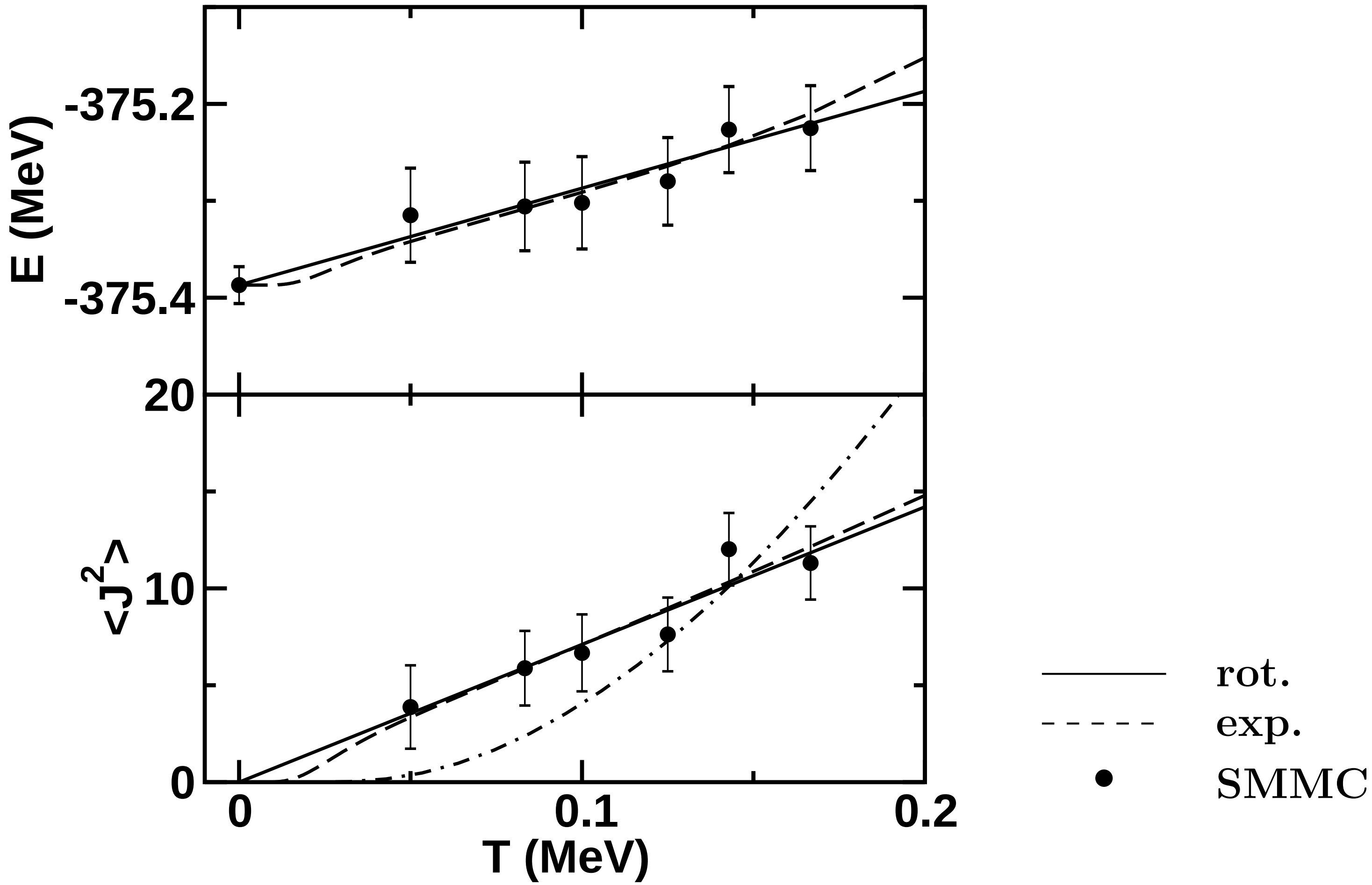
$\rightarrow$ { **linear extrapolation for** $\beta \leq 3.25\,\mathrm{MeV}^{-1}$; **average for** $\beta > 3.25\,\mathrm{MeV}^{-1}$ }

★ **Rotational character ?** $\cdots$ ^{162}**Dy**

$E_x \lesssim 1\,\mathrm{MeV}$ ($\leftrightarrow$ $T \lesssim 0.2\,\mathrm{MeV}$) $\cdots$ **g.s. band only**

$\rightarrow$ $E(T) \approx E_0 + T$, $\langle \boldsymbol{J}^2 \rangle_T \approx 2\mathcal{I}_g T$ **cf.** $\langle \boldsymbol{J}^2 \rangle_T \propto e^{-E_x(2^+)/T}$ **for vib.**

SMMC :



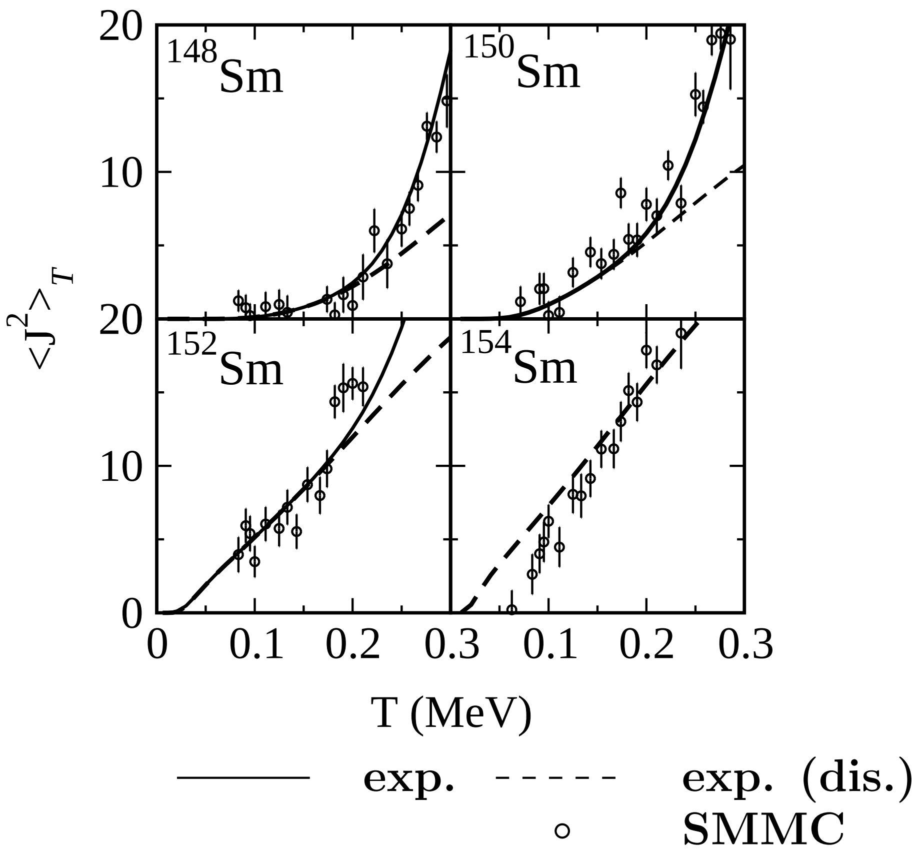
$\rightarrow$ $\mathcal{I}_g = 35.8 \pm 1.5\,\mathrm{MeV}^{-1}$ ***vs.*** $\mathcal{I}_g = 37.2$ **(exp.)**

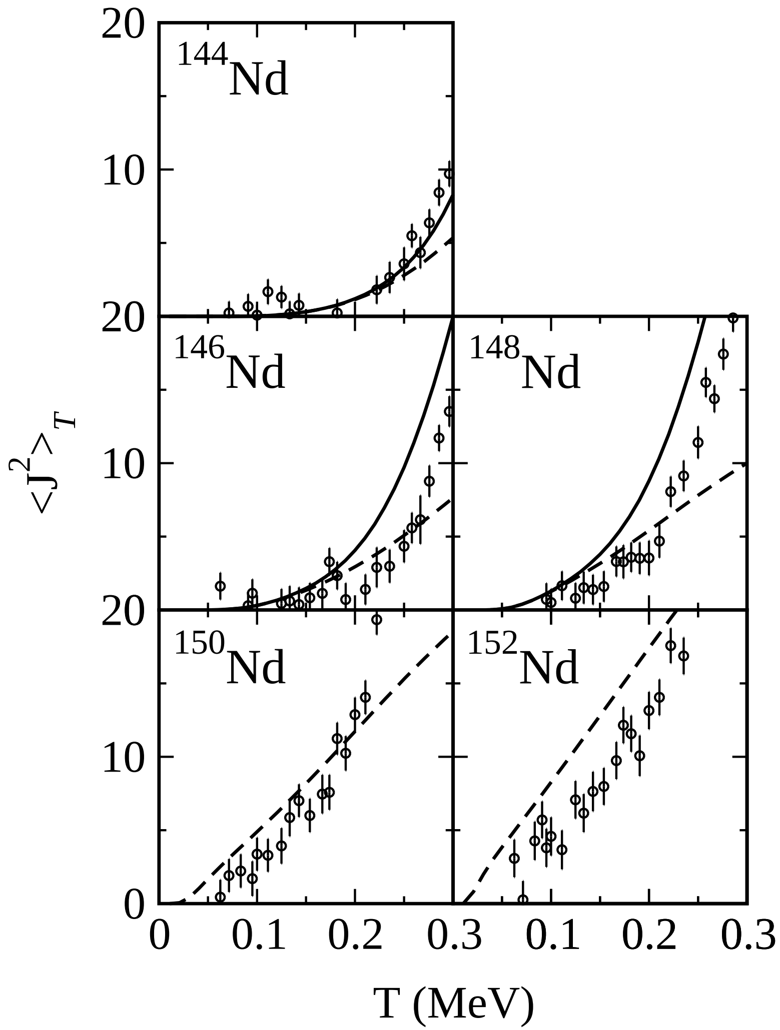
★ Crossover from vibrational to rotational nuclei ?

$\cdots$ **even-N Nd-Sm isotopes**

$$\langle \boldsymbol{J}^2 \rangle_T \approx \begin{cases} 30\, e^{-E_x(2^+)/T} \big/ \left(1 - e^{-E_x(2^+)/T}\right)^2 & \textbf{(vib.)} \\ 6\,T \big/ E_x(2^+) & \textbf{(rot.)} \end{cases}$$

(at low T) $\Rightarrow$ **vib. / rot.**





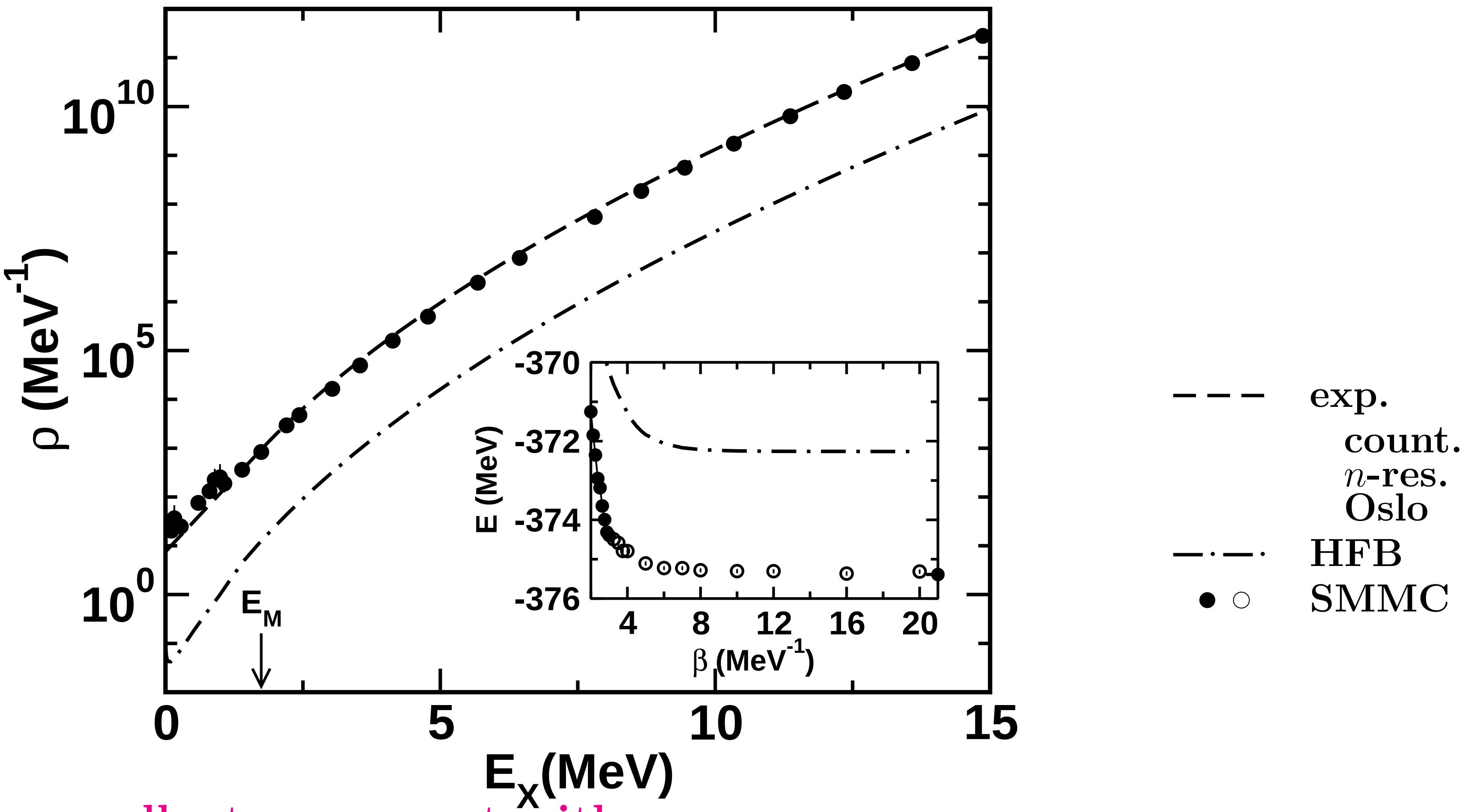
exp. $\leftarrow$ discrete levels + BBF

crossover from vib. to rot. well reproduced ! **(within shell model)**

$$\textbf{fit to } \langle \boldsymbol{J}^2 \rangle_T \rightarrow \begin{cases} ^{148}\textbf{Sm}: & E_x(2^+) = 0.538 \pm 0.031\,\text{MeV} \quad \textbf{\textit{vs.}} \quad 0.550\,\text{MeV} \quad \textbf{(exp.)} \\ ^{154}\textbf{Sm}: & 0.087 \pm 0.006\,\text{MeV} \quad 0.082\,\text{MeV} \\ ^{144}\textbf{Nd}: & 0.702 \pm 0.062\,\text{MeV} \quad 0.697\,\text{MeV} \\ ^{150}\textbf{Nd}: & 0.132 \pm 0.012\,\text{MeV} \quad 0.130\,\text{MeV} \\ ^{152}\textbf{Nd}: & 0.107 \pm 0.006\,\text{MeV} \quad 0.073\,\text{MeV} \end{cases}$$

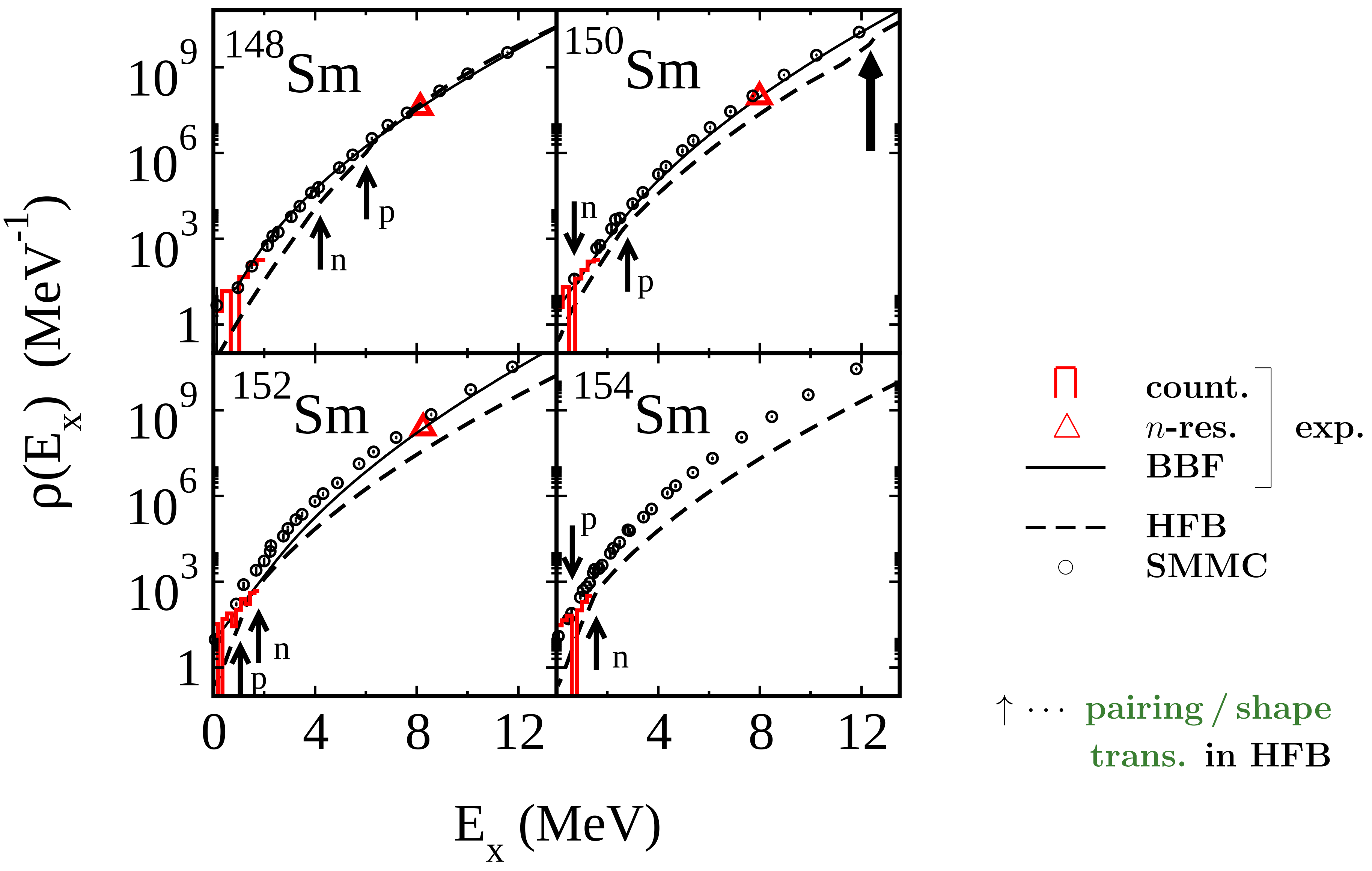
★ SMMC state densities (*vs.* exp. & finite-T HFB)

^{162}Dy :



- excellent agreement with exp.
- almost equal "slope" at high E_x, but factor 10^2 enhancement from finite-T HFB $\leftrightarrow$ collective rotation

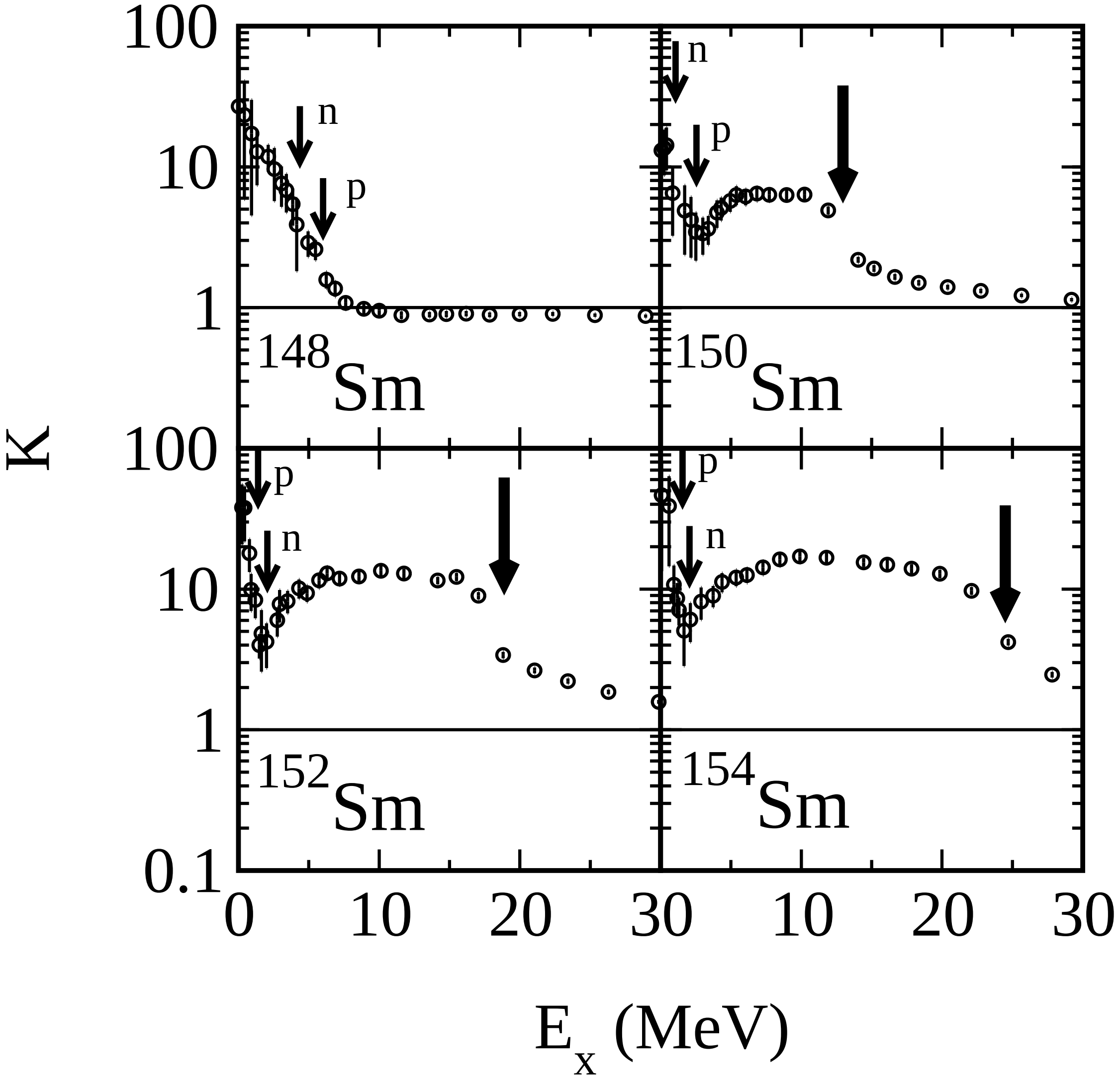
$^{148-154}$**Sm :**



↑ ⋯ pairing / shape trans. in HFB

★ **Collective enhancement** $K(E_x) := \rho_{\mathrm{SMMC}}(E_x) / \rho_{\mathrm{HFB}}(E_x)$

$\underset{(?)}{\longleftrightarrow} K_{\mathrm{vib}}(E_x),\ K_{\mathrm{rot}}(E_x)$



"decay" of K

↔ losing collectivity

pairing

rotation

VI. Summary & future prospect

Summary

- SMMC methods reviewed briefly
  - projections ← isolated finite system

- Applications to level densities — successful
  - rotational properties & crossover — O.K.
  - collective enhancement — obtained microscopically

Future prospect

- Systematic calculations !

  ← { connection of different model spaces !
powerful & massive CPUs (+ man-power) ?
simplification based on physics understanding ? }

  ⋯ "RIPL-4 hopefully contain SMMC level densities"

  (by Capote-Noy @ SNP2008)

- Quantitative derivation of collective enhancement factor ?