Neutron superfluidity in extremely neutron-rich nuclei

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1. Strong-coupling pairing in dilute neutron matter

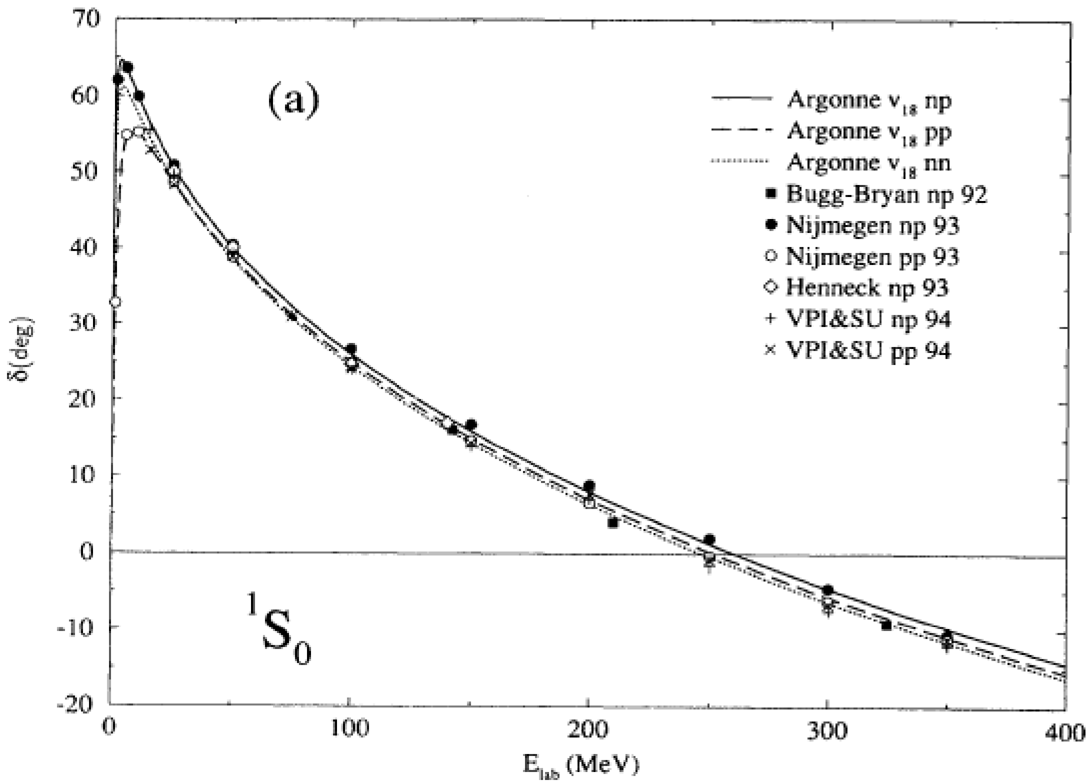
2. Spatially correlated Cooper pair at dilute surface of n-rich nuclei

3. Enhanced pair transfer at surface

4. Pairing dominance in the asymptotics

Properties of nuclear force

Phase shift in ^1S channel



Wiringa et al, PRC51, 38 (1995)

1. Strong E-,p-dependence

More attractive at lower E,p

2. Large scattering length

a = -18.5 fm

r_{eff} = 2.8 fm

Cf. Saturation density

$$\rho_n = 0.085\,\mathrm{fm}^{-3}$$

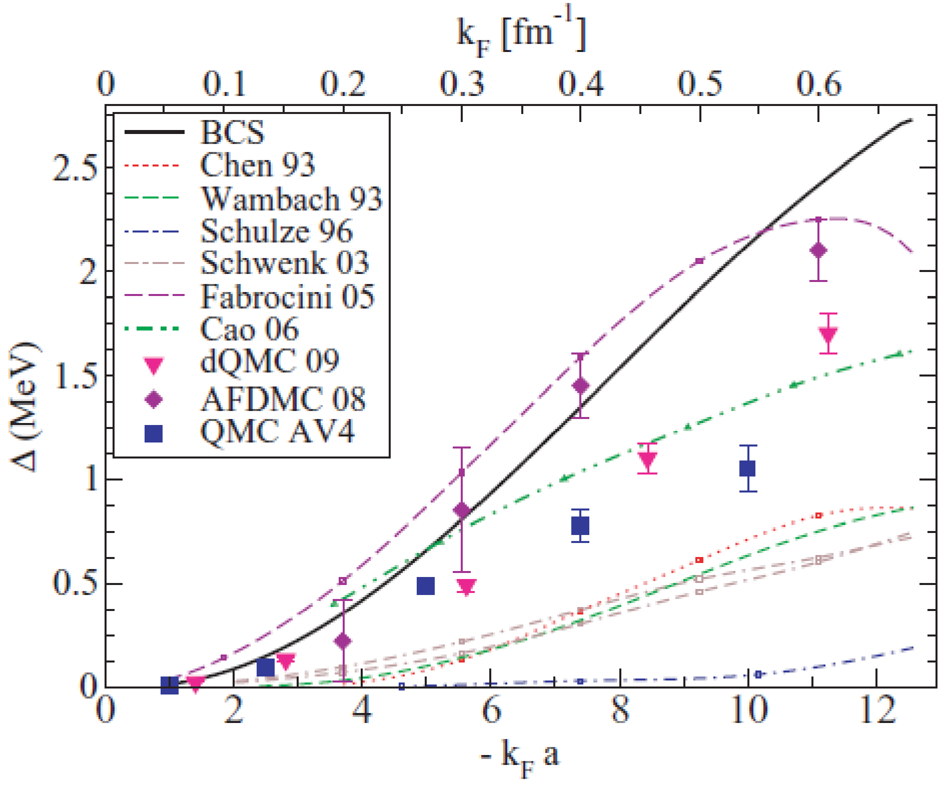
$$k_F = 1.36\,\mathrm{fm}^{-1}$$

$$d = \rho^{-1/3} = 2.3\,\mathrm{fm}$$

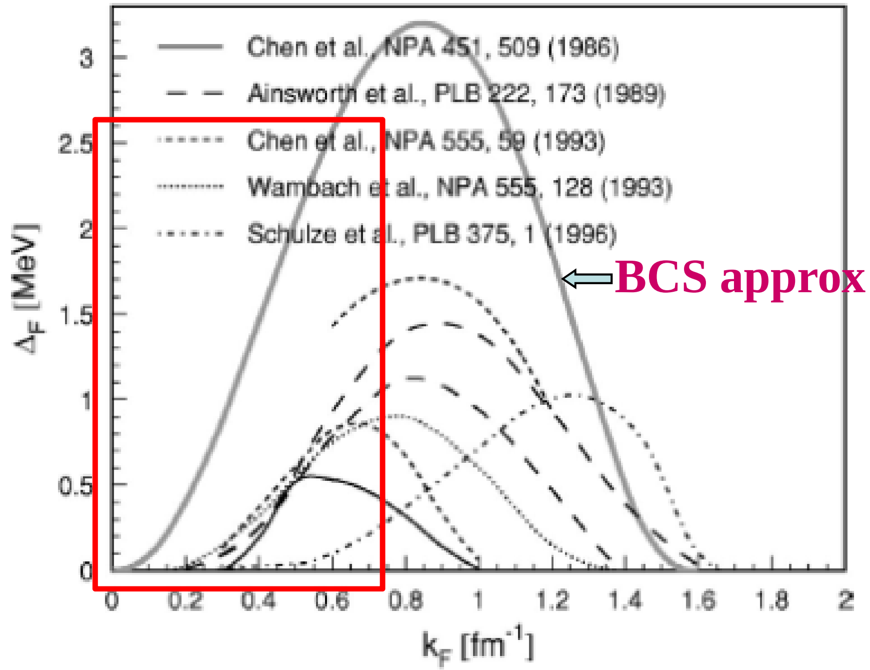
pairing gap in neutron matter

BCS calculation & ab initio calculations

- Strong density dependence
- $\Delta_{\text{ab initio}} = (1 \sim 0.5)\ \Delta_{\text{BCS}}$



Gezerlis & Carlson, PRC81 (2010)



Lombardo & Schulze 2001

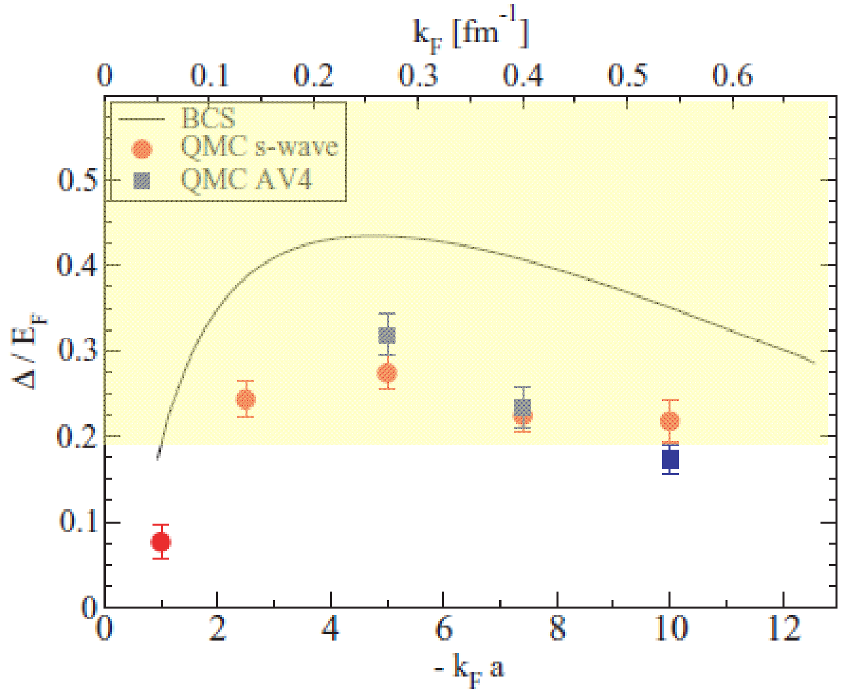
Strong pairing in dilute neutron matter

Large scattering length **a=-18fm** for nn-attraction

"Large" pair gap vs. Fermi energy $\Delta/e_F > 0.2$ at low-densities

Monte-Carlo calculation

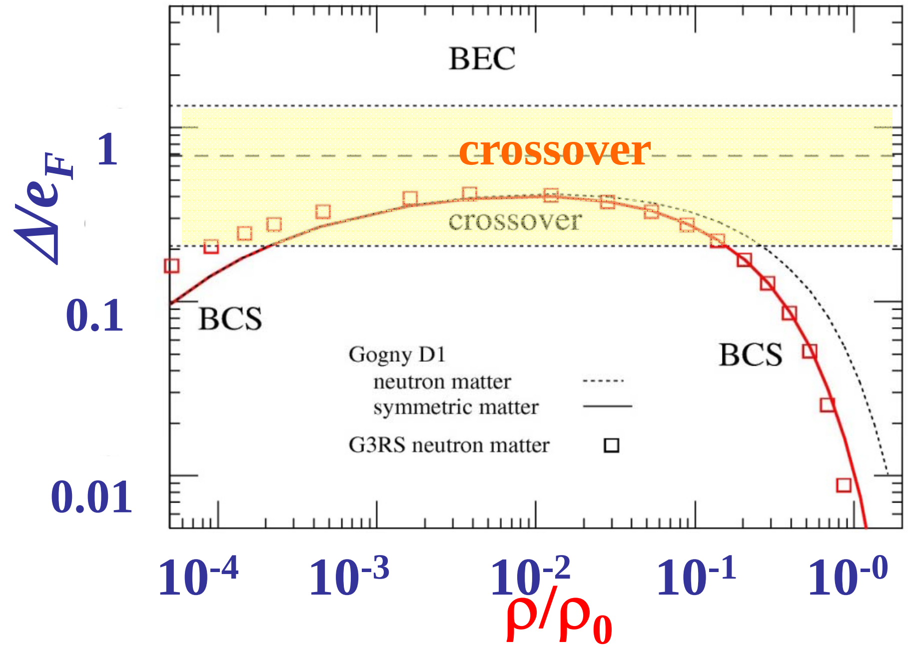
$\rho/\rho_0 = 10^{-3} \sim 0.5\times10^{-1}$



Gezerlis & Carlson, PRC81,025803 (2010)

Mean-field calculation (BCS approx.)

$\rho/\rho_0 = 10^{-4} \sim 2\times10^{-1}$

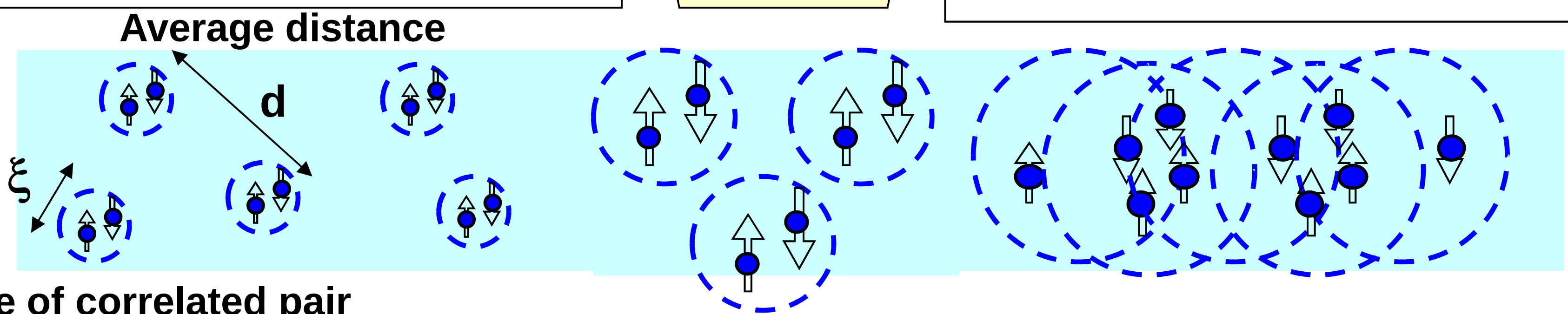


MM, PRC73,044309(2006)

Bose condensation and strong coupling pairing: BCS-BEC crossover

Leggett 1980, Nozieres & Schmitt-Rink 1985

BEC	Crossover	BCS
Interaction: **Strong**		Interaction: **Weak**
In free space: **Bound pair (boson) is already formed**		In free space: **No bound pair is formed**
In medium: **Condensate of bosons**		In medium: **Bound pairs (Cooper pairs) are formed, then condensate**



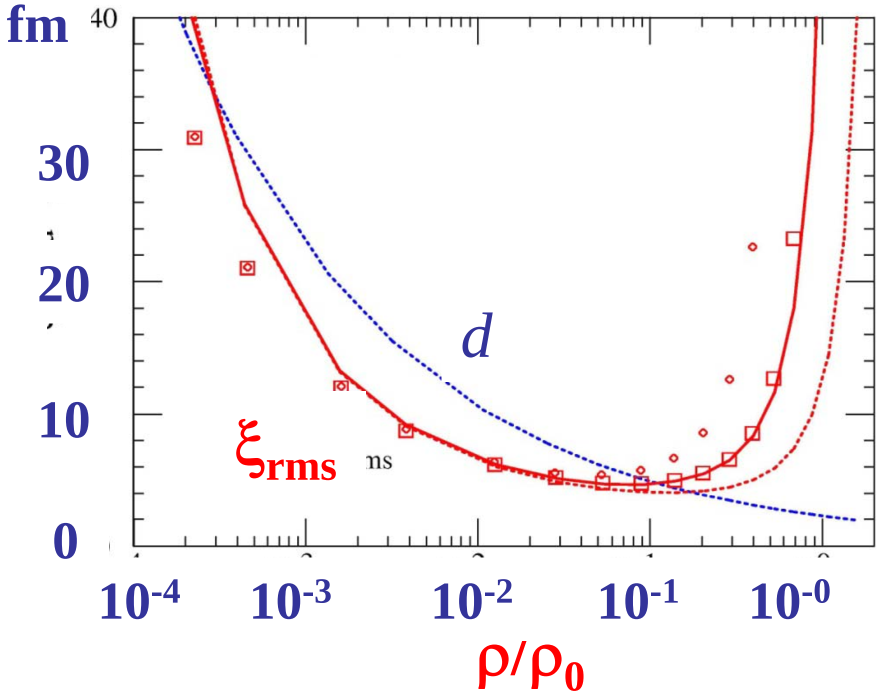
	BEC	Crossover	BCS
Size of correlated pair	$\xi/d \ll 1$	$\xi/d = 0.2 \sim 1.10$	$\xi/d > 1$
Pairing gap	$\Delta/e_F > 1$	$\Delta/e_F = 1.3 \sim 0.2$	$\Delta/e_F \ll 1$
Scattering length a & density	$1/k_F a > 1$	$1/k_F a = -1 \sim 1$	$1/k_F a < -1$

$$\rho = \frac{k_F^3}{3\pi^2}$$

Small Cooper pair in dilute neutron matter

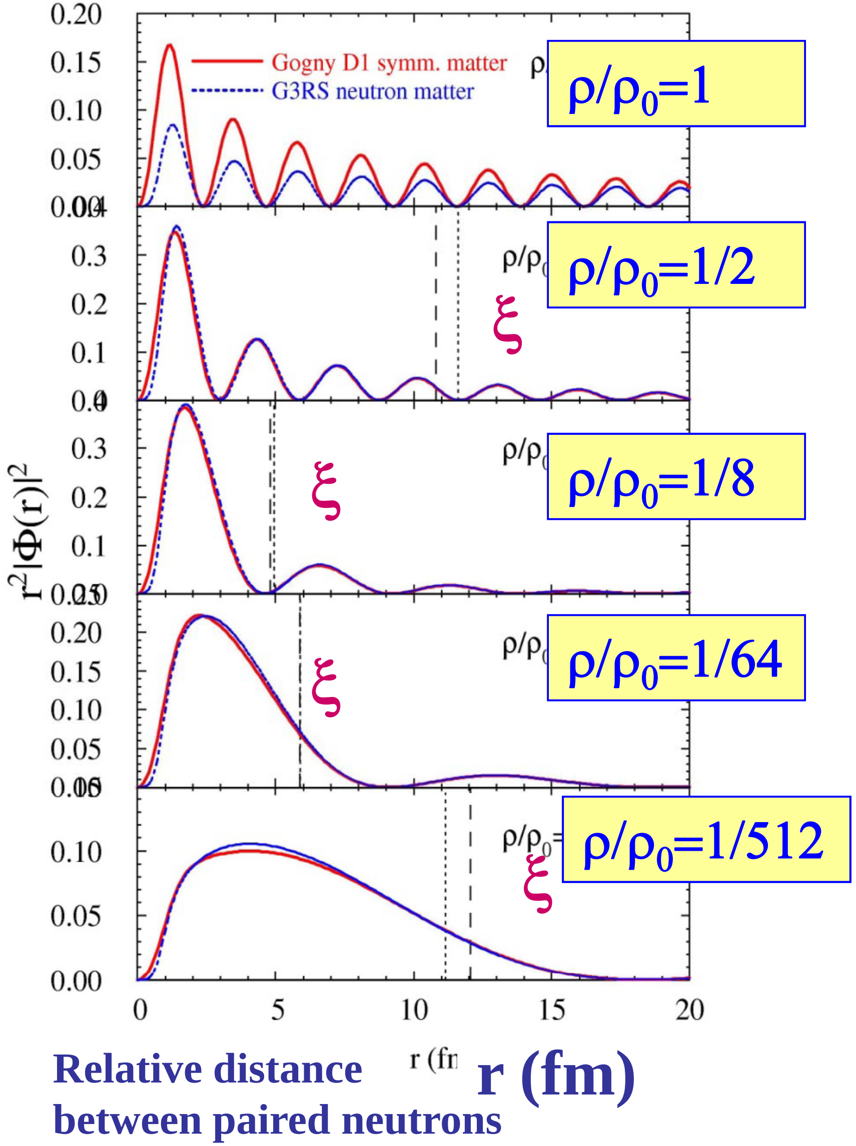
BCS calculation using A bare force (G3RS) / Gogny force (D1)

Neutron pairing gap



Pair wave function has large amplitude at short relative distances r∼2-3 fm

Cooper pair wave function

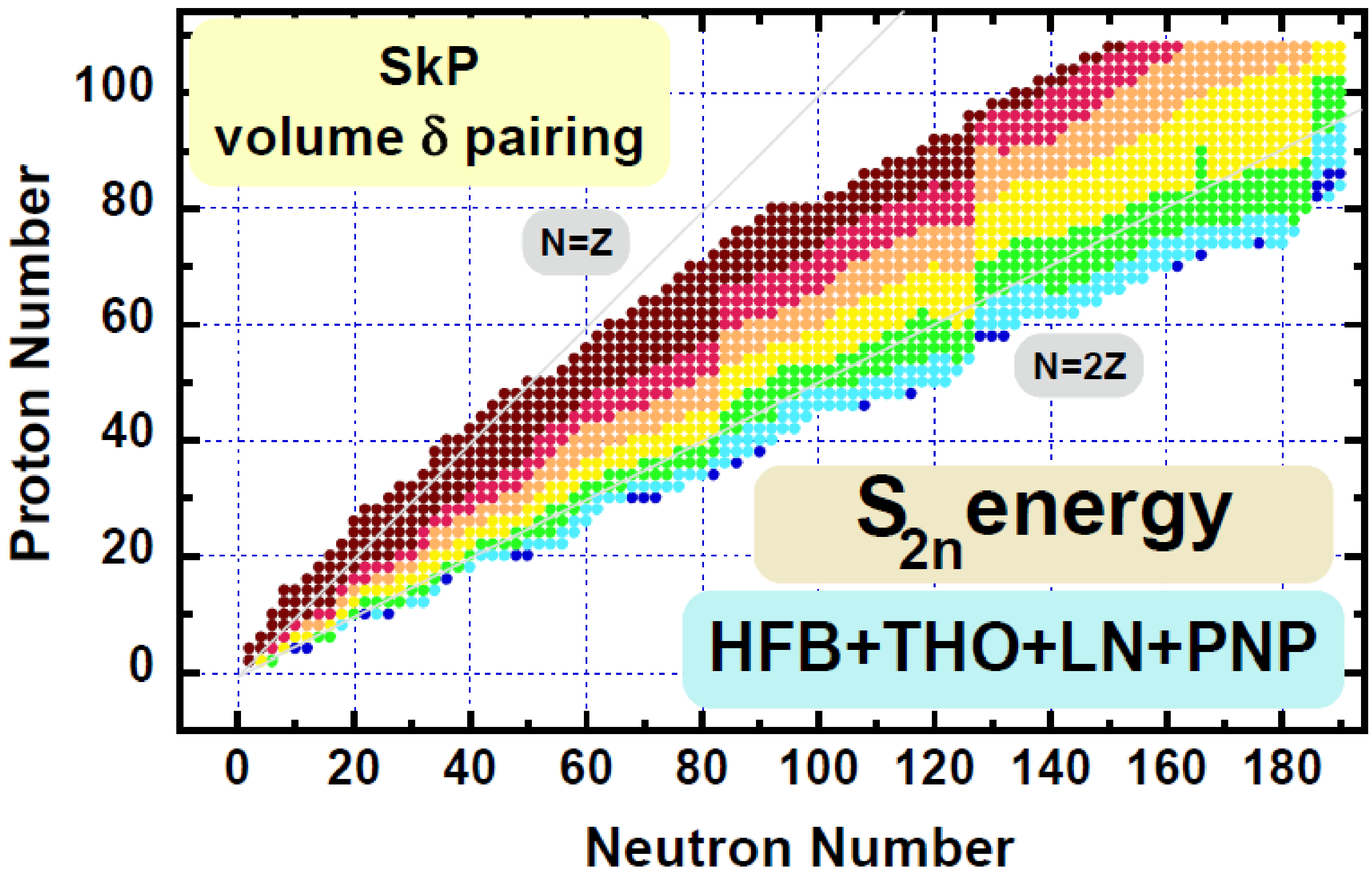


MM, PRC73,044309(2006)
Margueron et al, PRC77,054309(2008)

Our viewpoint

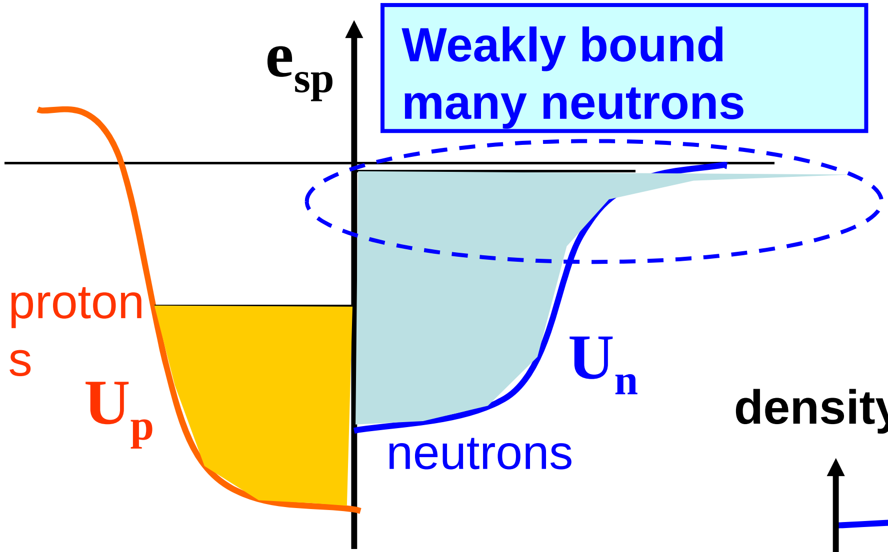
Neutron-rich nuclei may exhibit characteristic surface properties originating from the strong-coupling neutron pairing

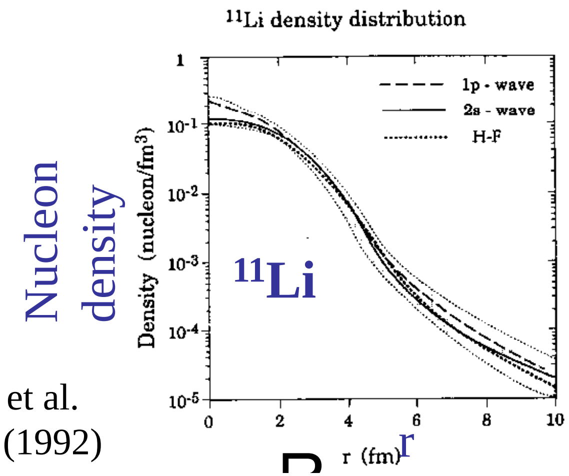
A DFT prediction of nuclear biding energy



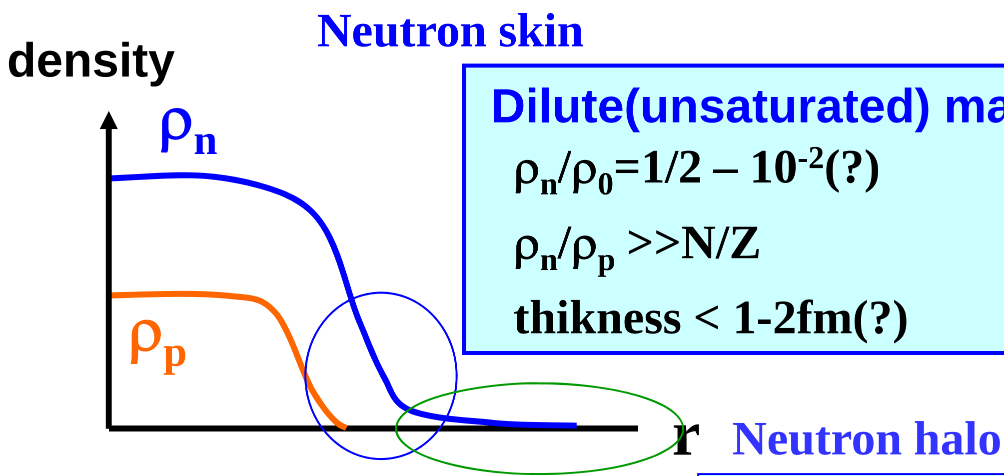
"Dilute matter" in n-rich nuclei

Weakly bound many neutrons





Tanihata et al.
PLB287 (1992)



Dilute(unsaturated) matter

ρ_n/ρ_0=1/2 – 10^{-2}(?)

ρ_n/ρ_p >>N/Z

thikness < 1-2fm(?)

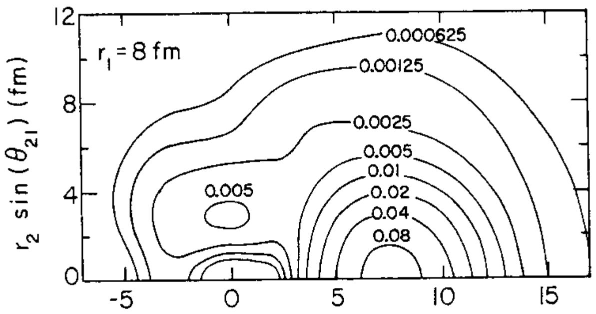
Long tail

$\rho_n/\rho_0 < 10^{-2}$(?)

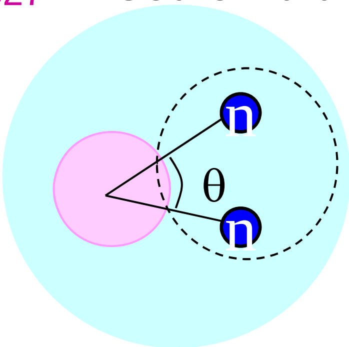
Enhancement of pairing at surface and tail in neutron-rich nuclei

1. Spatially correlated halo neutrons in ^{11}Li and ^{6}He, etc

G.F.Bertsch, H.Esbensen, Ann. Phys. 209(1991) 327



Coulomb break-up exp. on 11Li



Nakamura et al.
PRL96,252502 (2006)

$\theta_{nn}=48^{+14}_{-18}$ deg

$R_{c,2n}$=5.01+-0.32 fm

Charge radius Mueller et al.
PRL99,252501 (2007)

2. Medium & heavy mass n-rich nuclei with several weakly bound neutrons

Surface enhancement of pairing

Dobaczewski et al, PTPS146,70(2002), EPJA15,21(2002)

No evidence so far

Spatial correlation

In n-rich nuclei

Matsuo, Mizuyama, Serizawa PRC71,064326(2005)
Pillet, Sandulescu, Schuck, PRC76, 024310 (2007)
Pankratov, et al. PRC79, 024309 (2009)
etc

NB. In stable nuclei

Ibarra et al. NPA288, 397 (1977)
Janouch & Liotta PRC27,896 (1983)
etc

Pairing in the Bogoliubov's scheme & QRPA

Ground state pairing

Solving the Bogoliubov-de Genne /HFB equation in the coordinate-space

Nuclear mean-field.   pair pot.

$$\begin{pmatrix} -\frac{\hbar^2}{2m}\nabla^2+\underline{V(\boldsymbol{r})-\lambda} & \underline{\Delta(\boldsymbol{r})} \\ \Delta(\boldsymbol{r}) & \frac{\hbar^2}{2m}\nabla^2-V(\boldsymbol{r})+\lambda \end{pmatrix}\begin{pmatrix} \psi^{(1)}(\boldsymbol{r},E) \\ \psi^{(2)}(\boldsymbol{r},E) \end{pmatrix}=E\begin{pmatrix} \psi^{(1)}(\boldsymbol{r},E) \\ \psi^{(2)}(\boldsymbol{r},E) \end{pmatrix}$$

Nuclear density functional method for the mean-field

$$E = E_{Skyrme}[\rho, \vec{\nabla}\rho, \Delta\rho, \tau, \vec{j}, \vec{s}, \overleftrightarrow{J}]$$

Skyrme functional,   Parameter set: SLy4

+ Effective pairing force for the pair correlation

Density dependent delta interaction (DDDI)

Pairing collective excitations   *Continuum QRPA method*

Normal modes of time-dependent density functional theory
with Bogoliubov extension

Effective pairing force DDDI

nuclear force

1. contact force with cut-off p (or E)

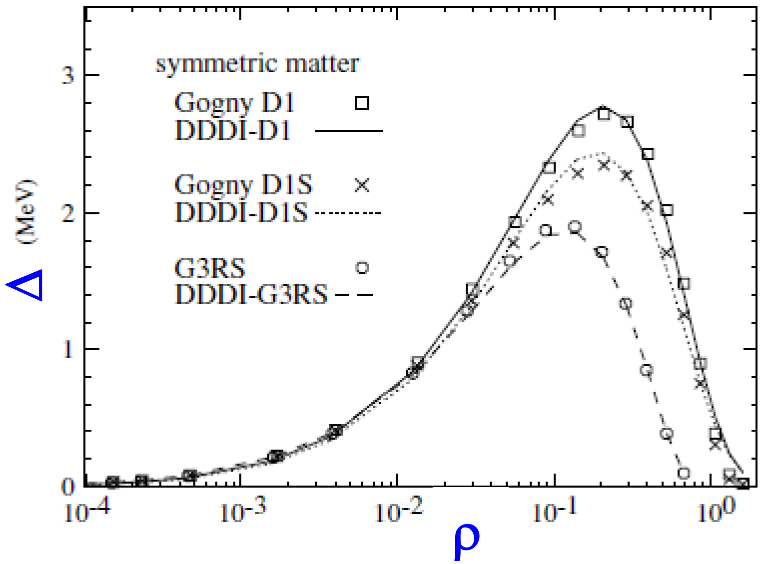
$$v(r-r') \Rightarrow v_0\delta(r-r') \qquad v_0 = -\frac{2\pi^2\hbar^2 m^{-1}}{k_c - \pi/2a},$$

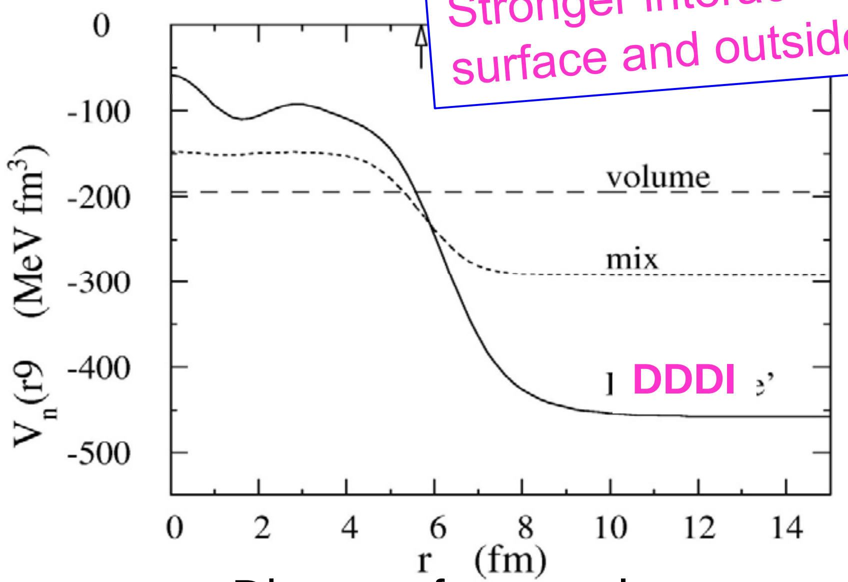
Bertsch & Esbensen,
Ann. Phys. 209, 327 (1991)

v_0 is related to the scattering length via the T-matrix

2. Density dependent strength
   which reproduces $\Delta(\rho)$ in uniform neutron matter

$$v_0 \Rightarrow V_0[\rho] = v_0\left(1-0.845(\rho_n/0.08)^{0.59}\right)$$



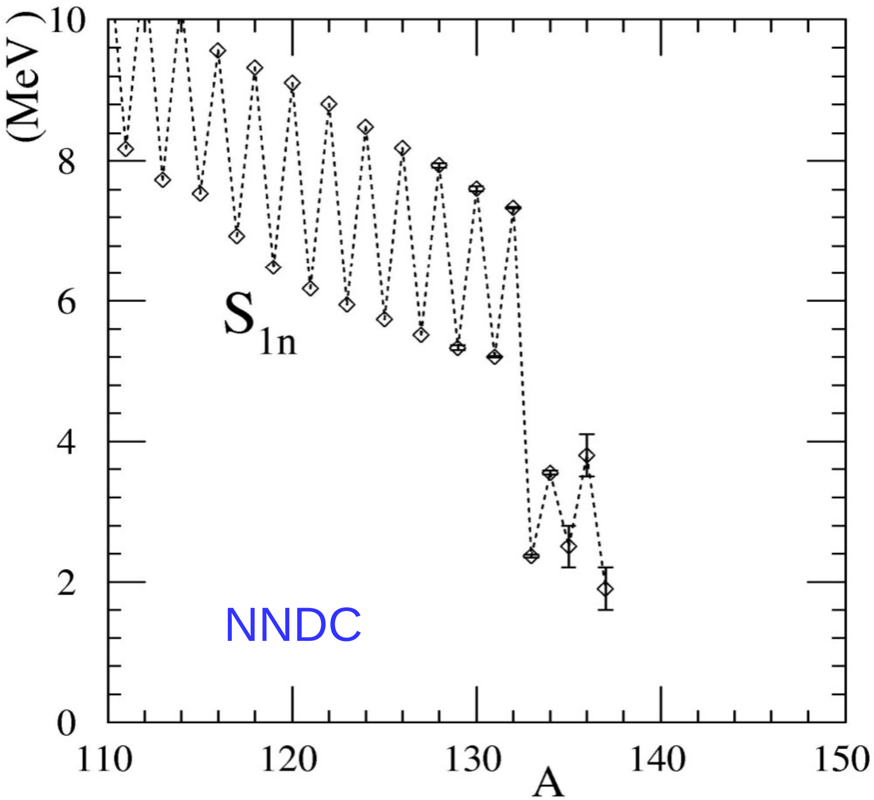


Stronger interaction at surface and outside

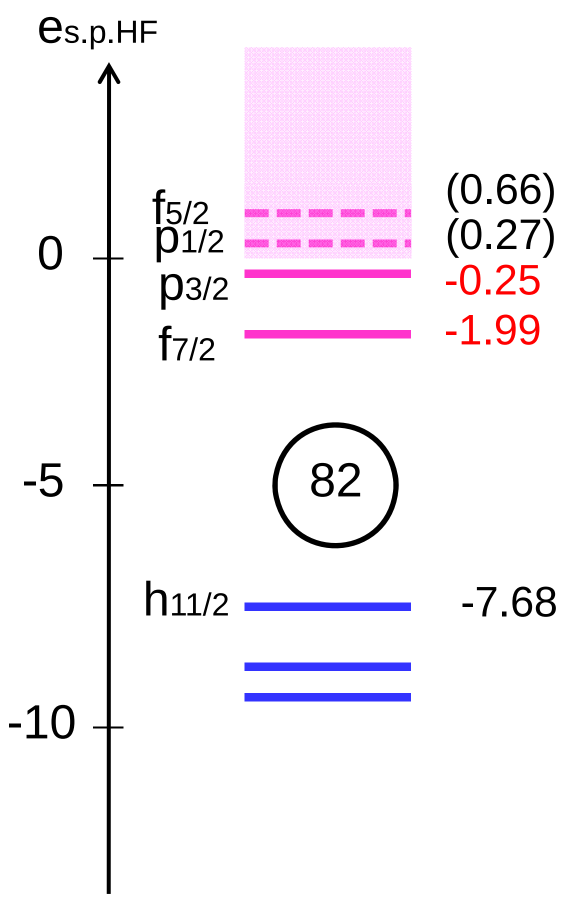
Distance from nuclear center

^{132}Sn and beyond

One-neutron separation energy



Neutron orbits

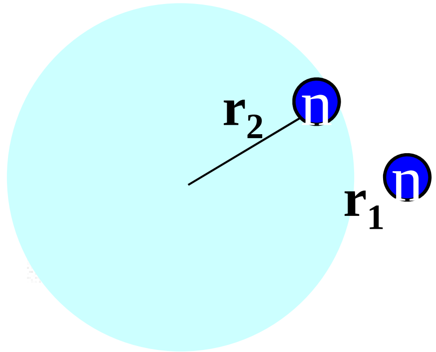


Hartree-Fock for 132Sn
Skyrme SLy4

1. Spatially correlated Cooper pair at dilute surface

Cooper pair wave function in ^{142}Sn

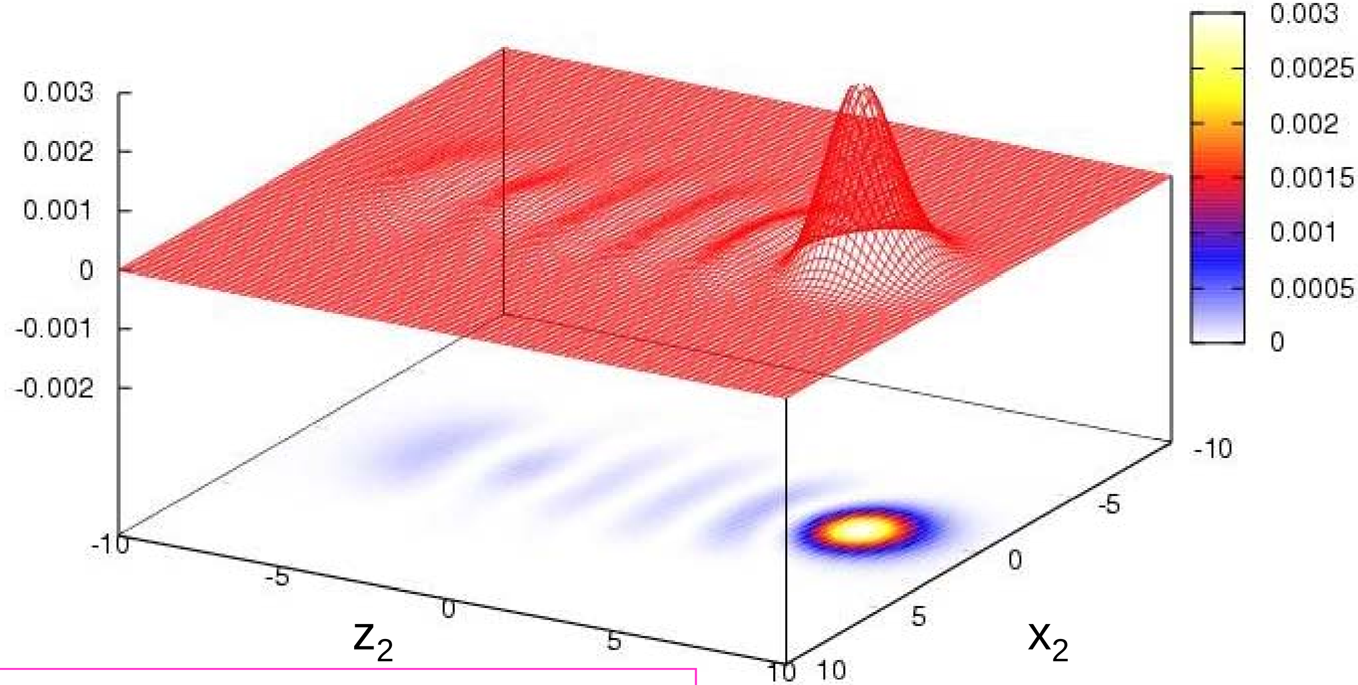
Probability distribution of the Cooper pair wave function



$$\left|\Psi_{pair}(r_1, r_2)\right|^2 = \left\langle \psi^+(r_1 \downarrow)\psi^+(r_2 \uparrow)\right\rangle^2$$

One neutron is fixed at $r_1 = 7\ fm$ (slightly outside)

<u>Plotted on the plane of $\mathbf{r_2}$</u>



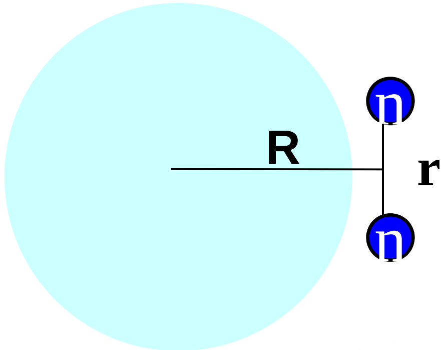
Probability for Cooper pair to be correlated at short distances <u>*r < a few fm*</u> is significantly enhanced *at* <u>*R > Rsurf*</u>

DDDI a-18

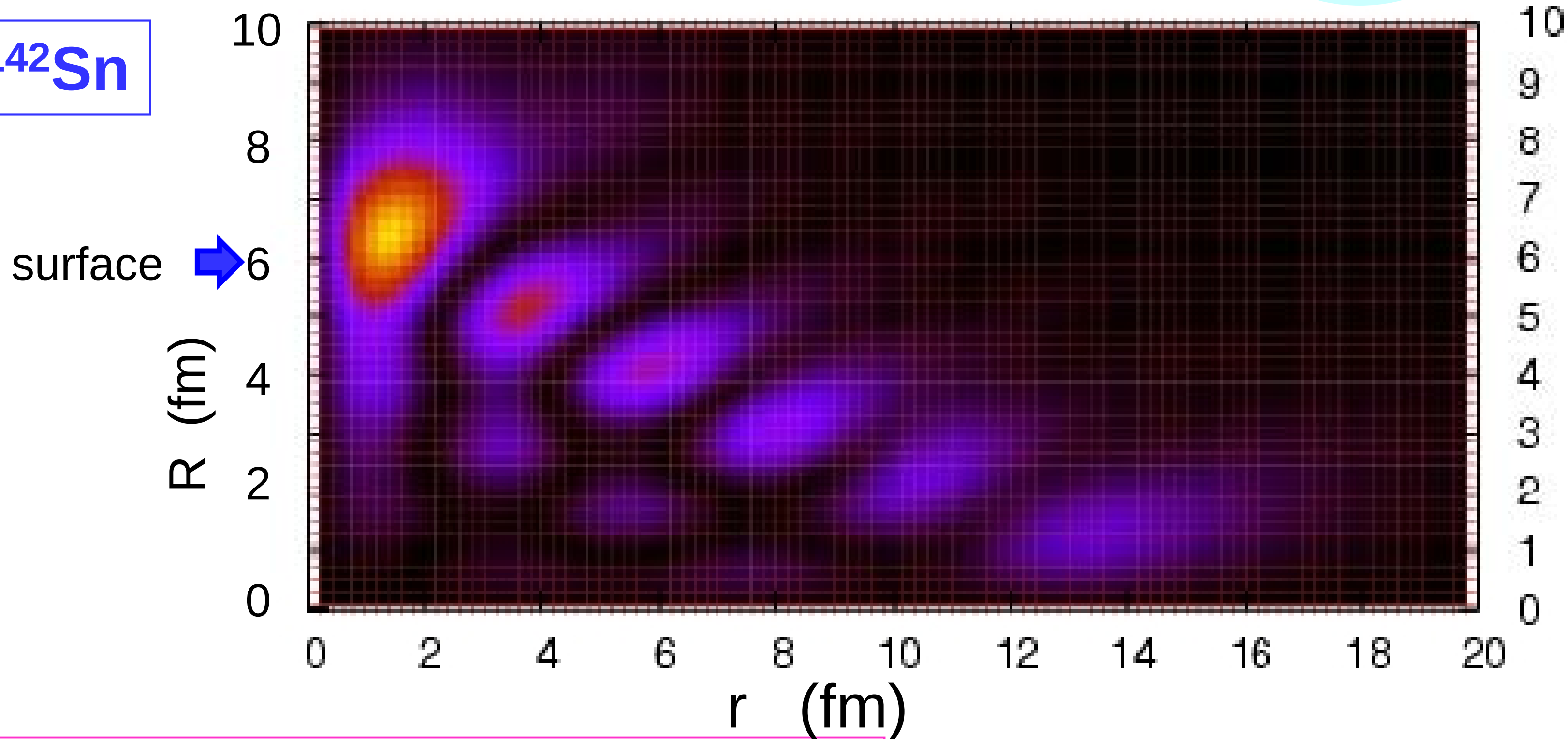
Large probability at short relative distances

Probability distribution in R-r

$$P_c(R,r) = R^2 r^2 \int d\Omega \, |\Psi_{pair}(\vec{R} - \tfrac{\vec{r}}{2}, \vec{R} + \tfrac{\vec{r}}{2})|^2$$



^{142}Sn

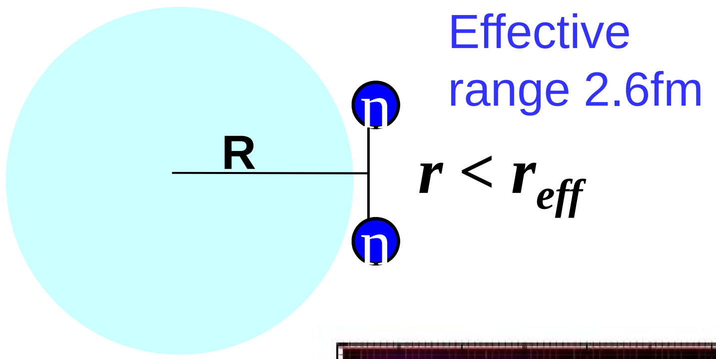


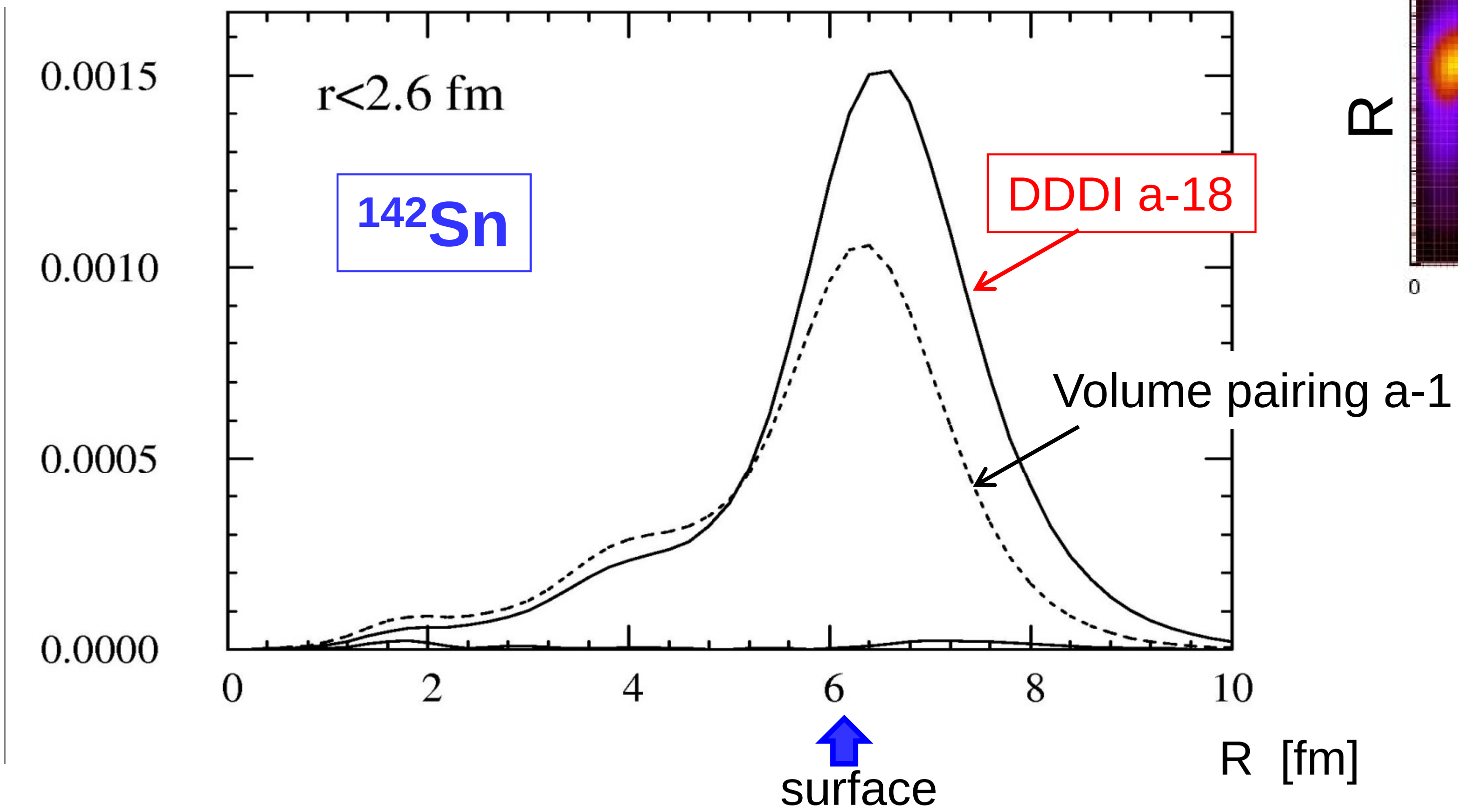
Probability for Cooper pair to be correlated at short distances *r < a few fm* is significantly enhanced *at* *R > Rsurf*

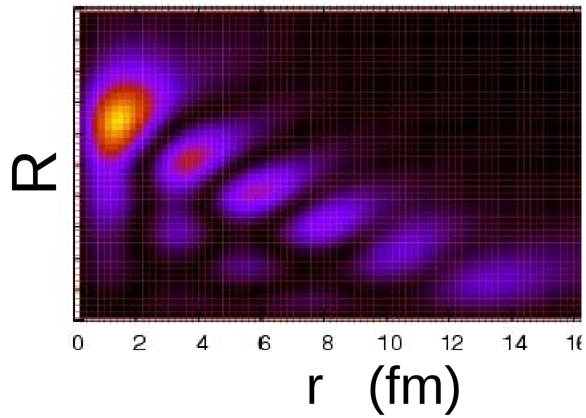
Pair contact probability $r < 2.6$ fm

Probability of pair at short relative distances within the interaction range

$$p(R) = \int_0^{r_{eff}} P_c(R,r)dr$$

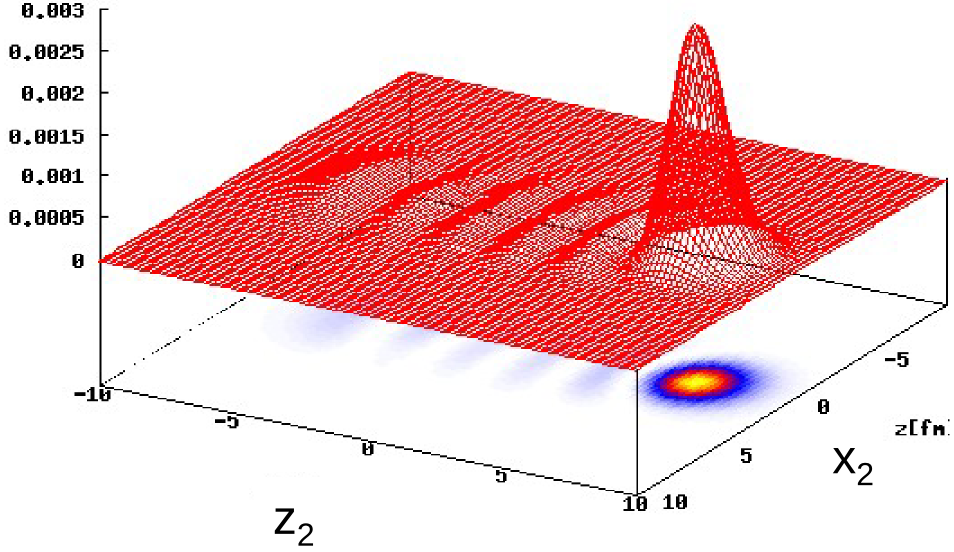






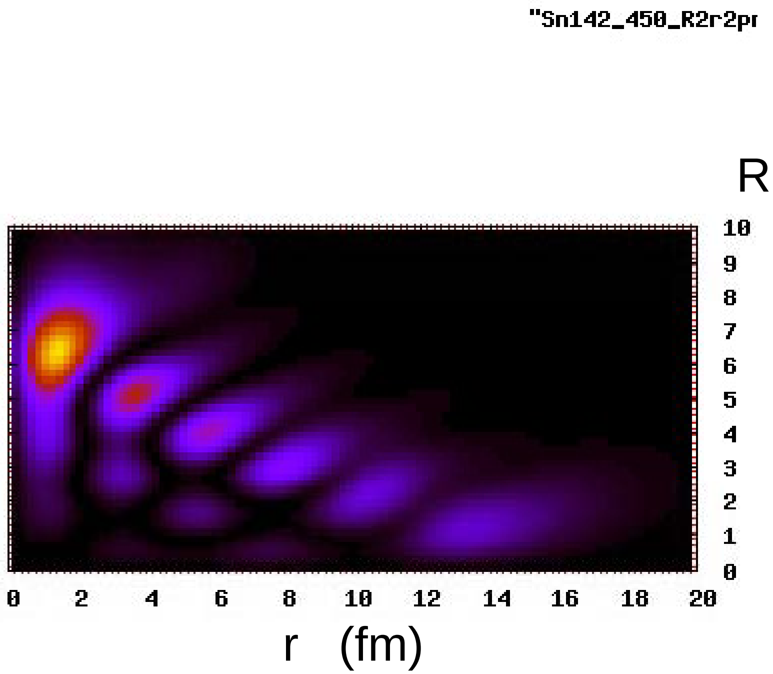
V=450, Δ_{uv}=0.856 MeV

^{142}Sn



$$\left|\Psi_{pair}(\vec{r}_1,\vec{r}_2)\right|^2$$

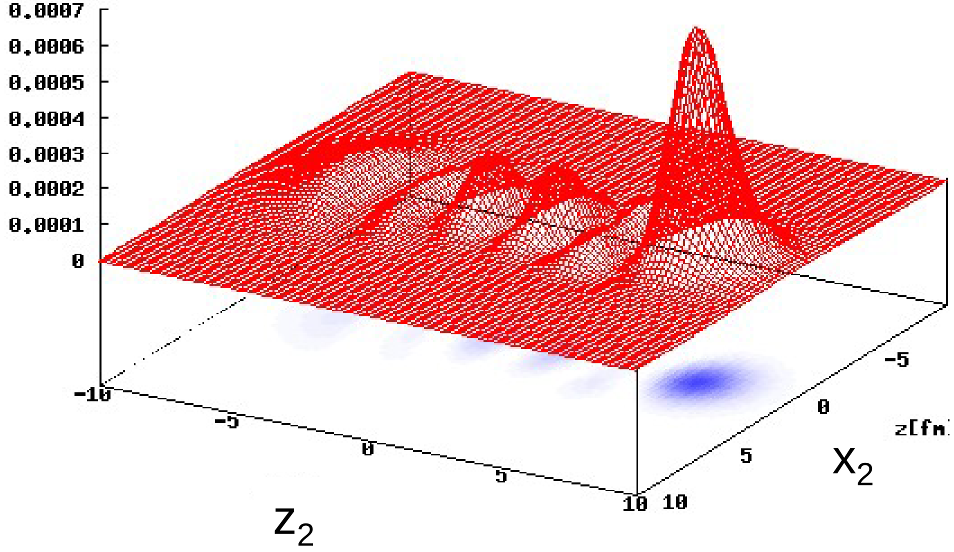
at r_1=7 fm



$$P_c(R,r)$$

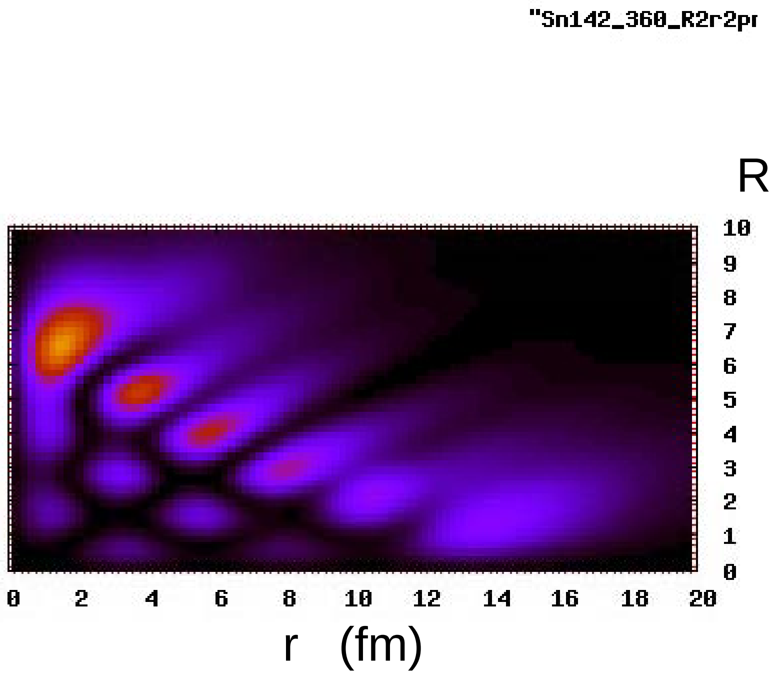
V=360, Δ_{uv}=0.32 MeV

^{142}Sn



$$\left|\Psi_{pair}(\vec{r}_1,\vec{r}_2)\right|^2$$

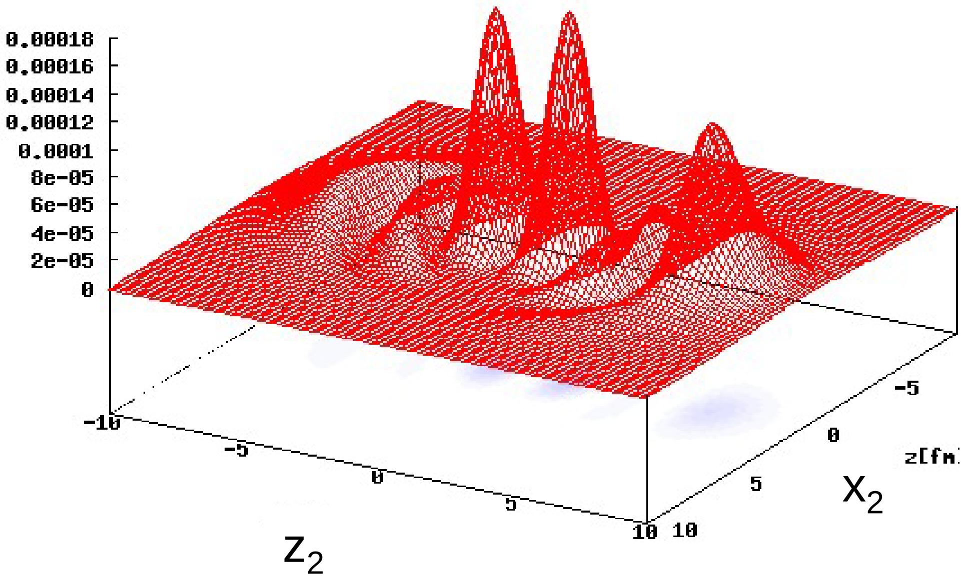
at r_1=7 fm



$$P_c(R,r)$$

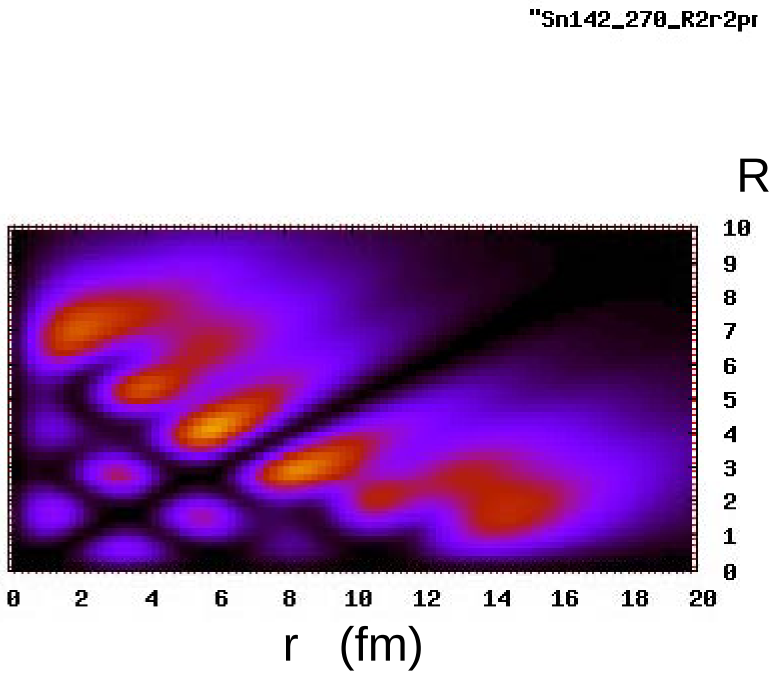
V=270, Δ_{uv}=0.10 MeV

^{142}Sn



$\left|\Psi_{pair}(\vec{r}_1, \vec{r}_2)\right|^2$

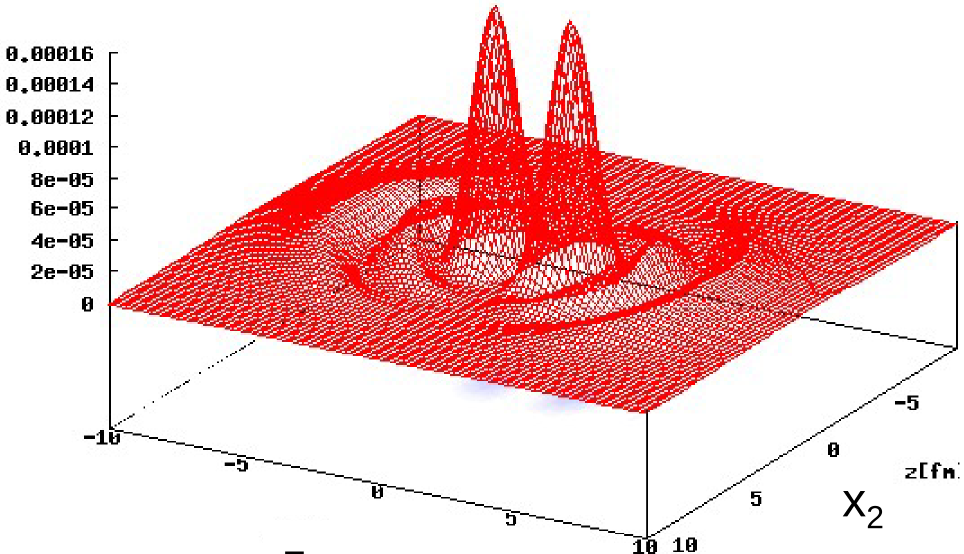
at r_1=7 fm



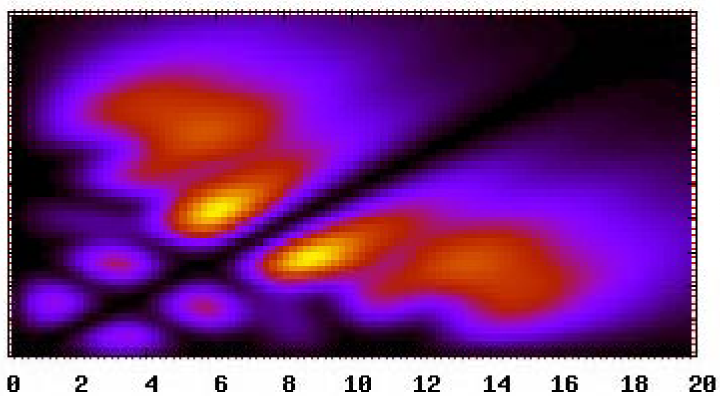
$P_c(R, r)$

Single-j J=0 pair $(3p_{3/2})^2$

^{142}Sn



z_2 x_2



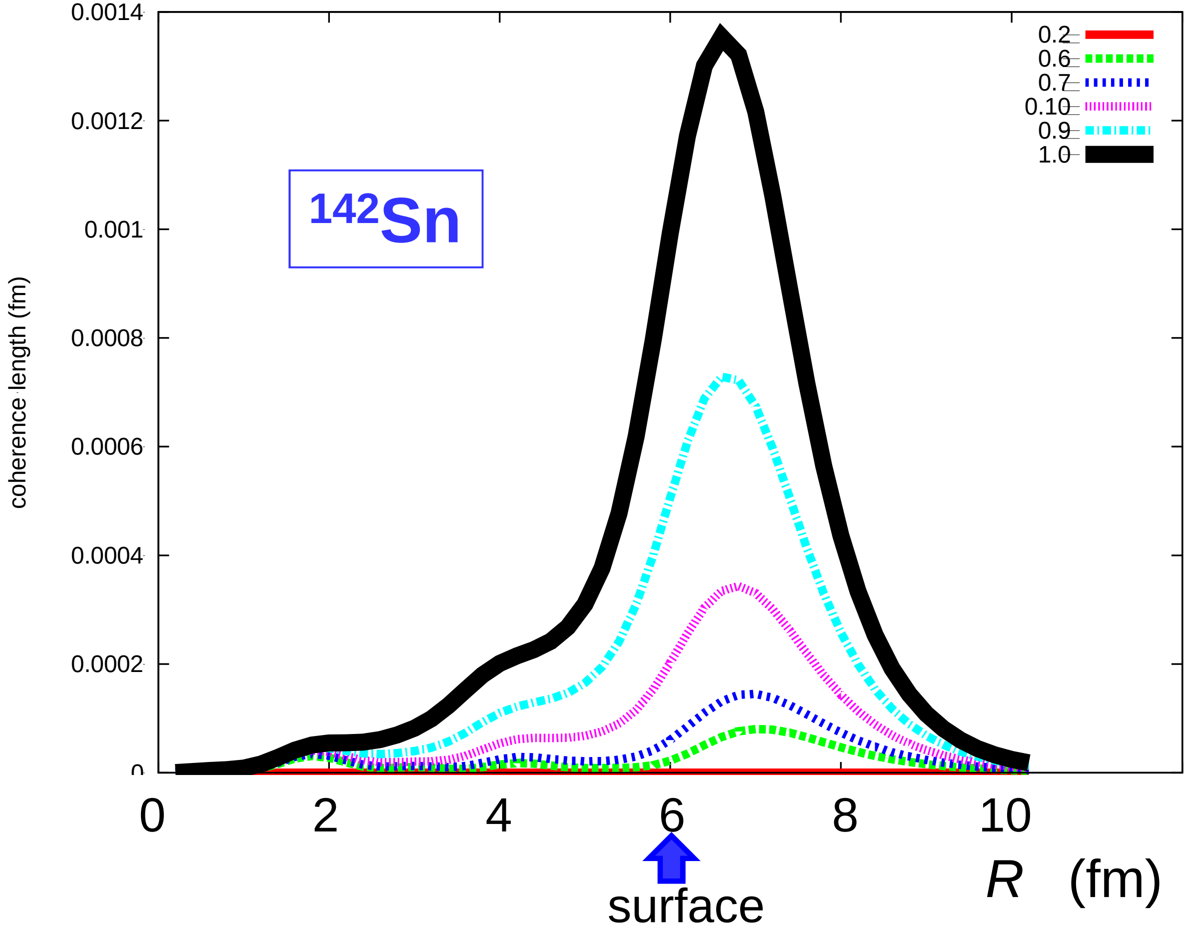
$$\left|\Psi_{pair}(\vec{r}_1,\vec{r}_2)\right|^2$$

at r_1=7 fm

$$P_c(R,r)$$

Pair contact probability *r* < 2.6 fm

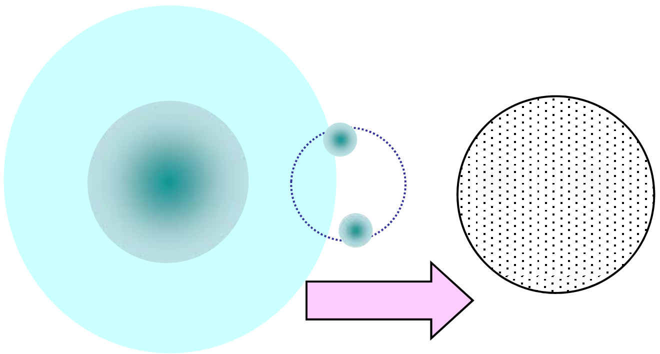
$$p(R) = \int_0^{r_{eff}} P_c(R, r)\,dr$$



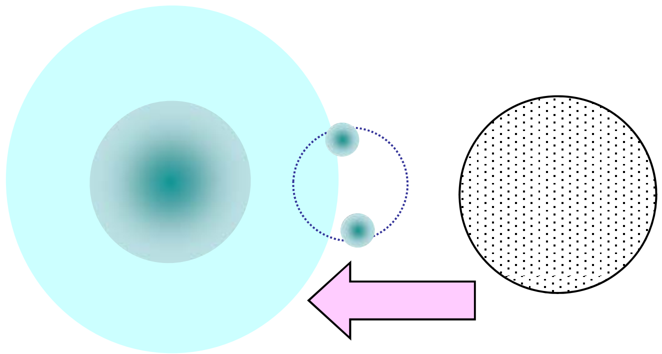
V=
450,
405,
360,
315,
270,

2. Surface enhanced 2n transfer

Pair transfers in neutron-rich nuclei



Pair removal

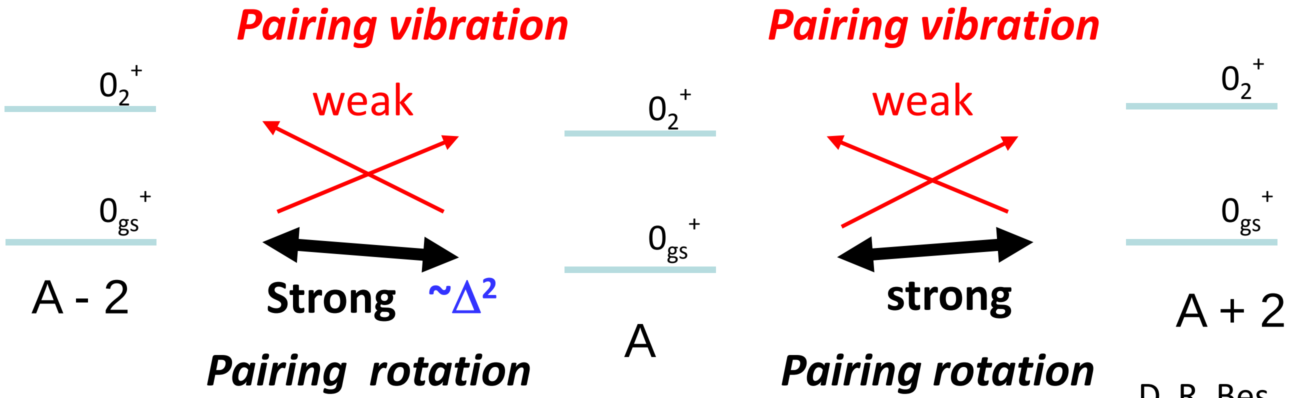


Pair addition

Pair transfer process, for Sn istopes, **132Sn~**

***Anomalous 0+ state*, a new kind of pair vibration**

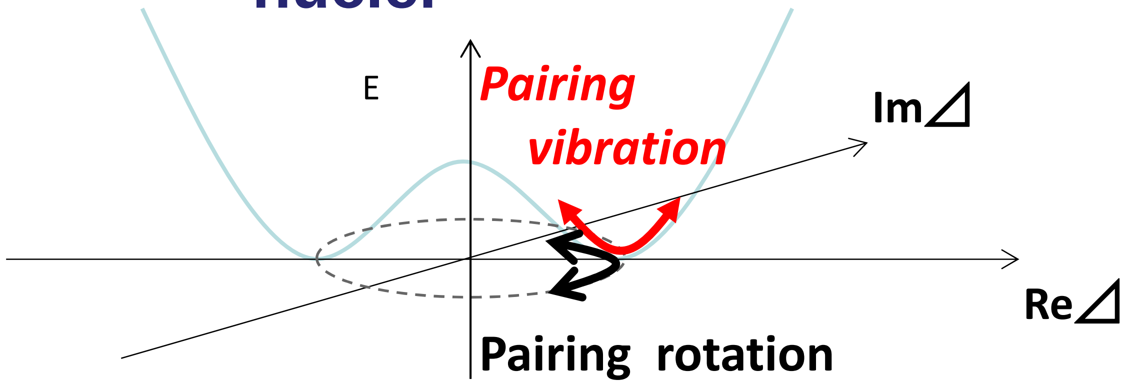
Pairing collectivity and two-neutron transfers in superfulid nuclei



D. R. Bes , et. al, NP(1966)
R. A. Broglia , at. Al, NP(1973)

superfluid phase

open-shell nuclei



Pair transfer and Cooper pair w.f.

Two-neutron transfer reaction model

1. Single-step DWBA calculation
2. Zero-range approximation

code TWOFNR

$$\frac{d\sigma}{d\Omega} \propto \left| \int d\vec{R} \chi_f(\vec{R}') F(\vec{R}) \chi_i(\vec{R}) \right|^2$$

Nuclear structure inputs: transfer form factor

1. Ground-to-ground transfer: pair-addition/removal

$$F(\vec{R}) = \int d\vec{r} \left\langle 0_{gs,N\pm 2} \middle| \psi^+(\vec{R}+\tfrac{\vec{r}}{2}\uparrow)\psi^+(\vec{R}-\tfrac{\vec{r}}{2}\downarrow) \middle| 0_{gs,N} \right\rangle \phi_{2n}(\vec{r})$$

$$\approx C \left\langle 0_{gs,N\pm 2} \middle| \psi^+(\vec{R}\uparrow)\psi^+(\vec{R}\downarrow) \middle| 0_{gs,N} \right\rangle$$

Cooper pair condensate at short rel. distances

2. Ground-to-excited states

pair-addition transition density

$$F(\vec{R}) \propto \left\langle L^+, N+2 \middle| Y_{LM} \psi^+(\vec{R}\uparrow)\psi^+(\vec{R}\downarrow) \middle| 0_{gs}, N \right\rangle$$

pair-removal transition density

$$F(\vec{R}) \propto \left\langle L^+, N-2 \middle| Y_{LM} \psi(\vec{R}\uparrow)\psi(\vec{R}\downarrow) \middle| 0_{gs}, N \right\rangle$$

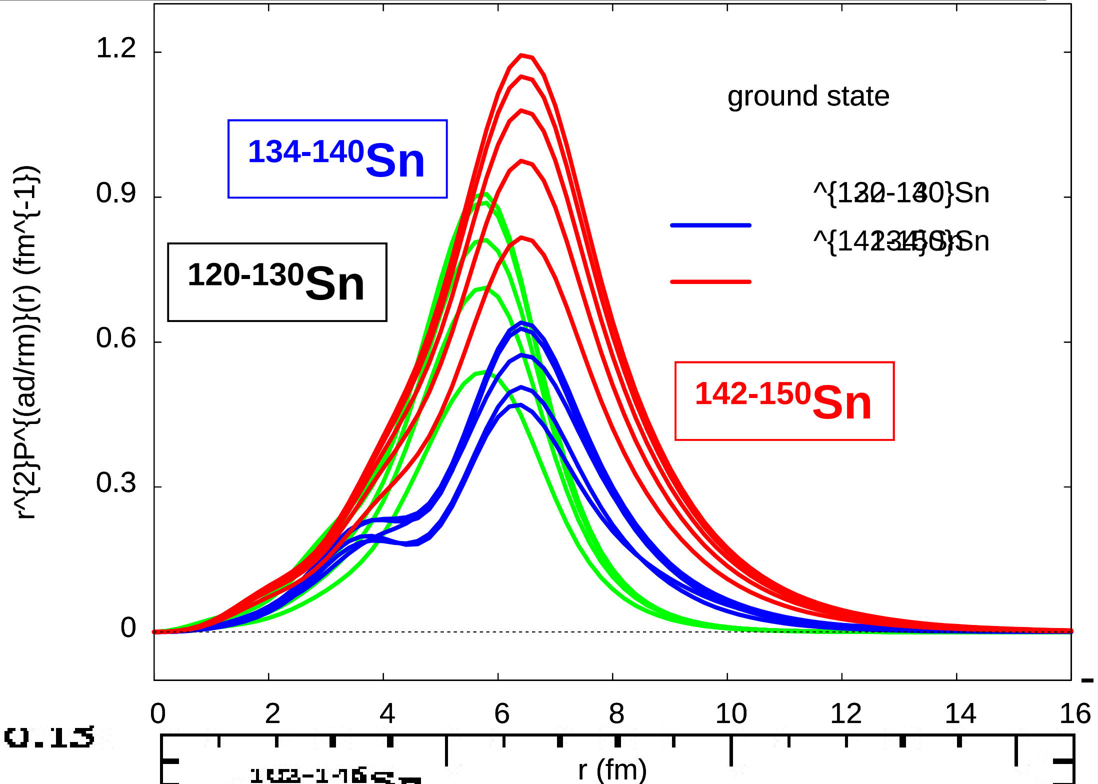
Continuum QRPA based on the same Skyrme-HFB model

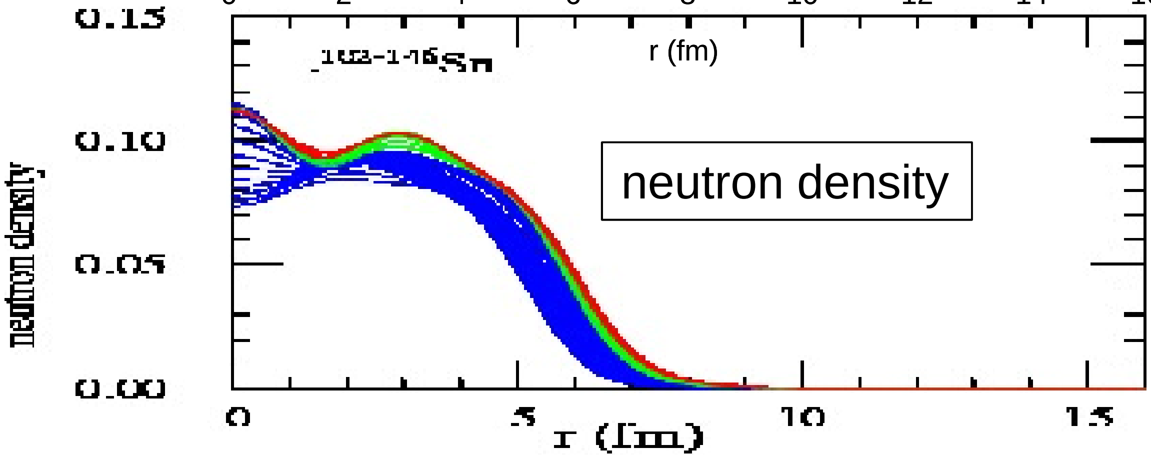
MM, Y.Serizawa PRC82, 024318 (2010)

H. Shimoyama, MM, PRC84, 044317 (2011)

Cooper pair condensate

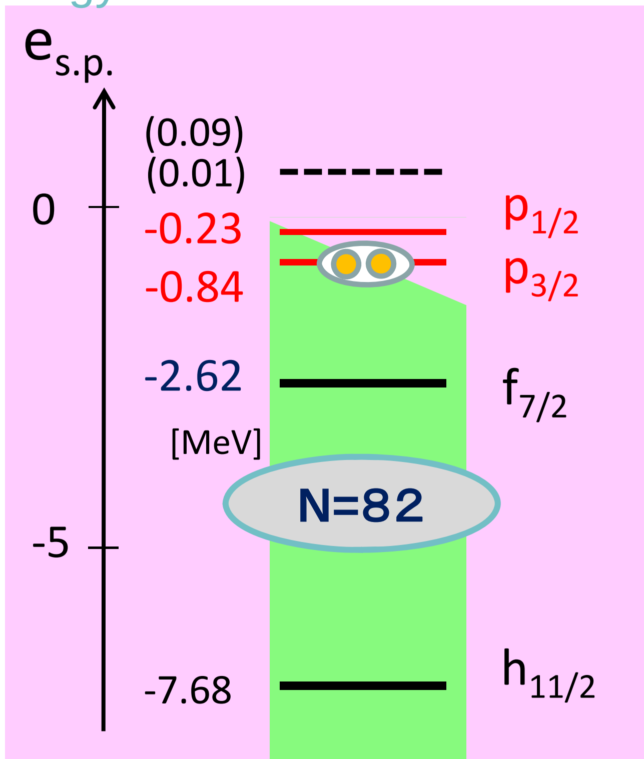
$$\langle 0_{gs}, N \pm 2 | \psi^+(\vec{r})\psi^+(\vec{r}) | 0_{gs}, N \rangle \approx \langle 0_{HFB} | \psi^+(\vec{r}\uparrow)\psi^+(\vec{r}\downarrow) | 0_{HFB} \rangle = \tilde{\rho}(\vec{r})$$





Extended far outside the nucleus with A>140

Hartree-Fock single-particle energy in ^{142}Sn

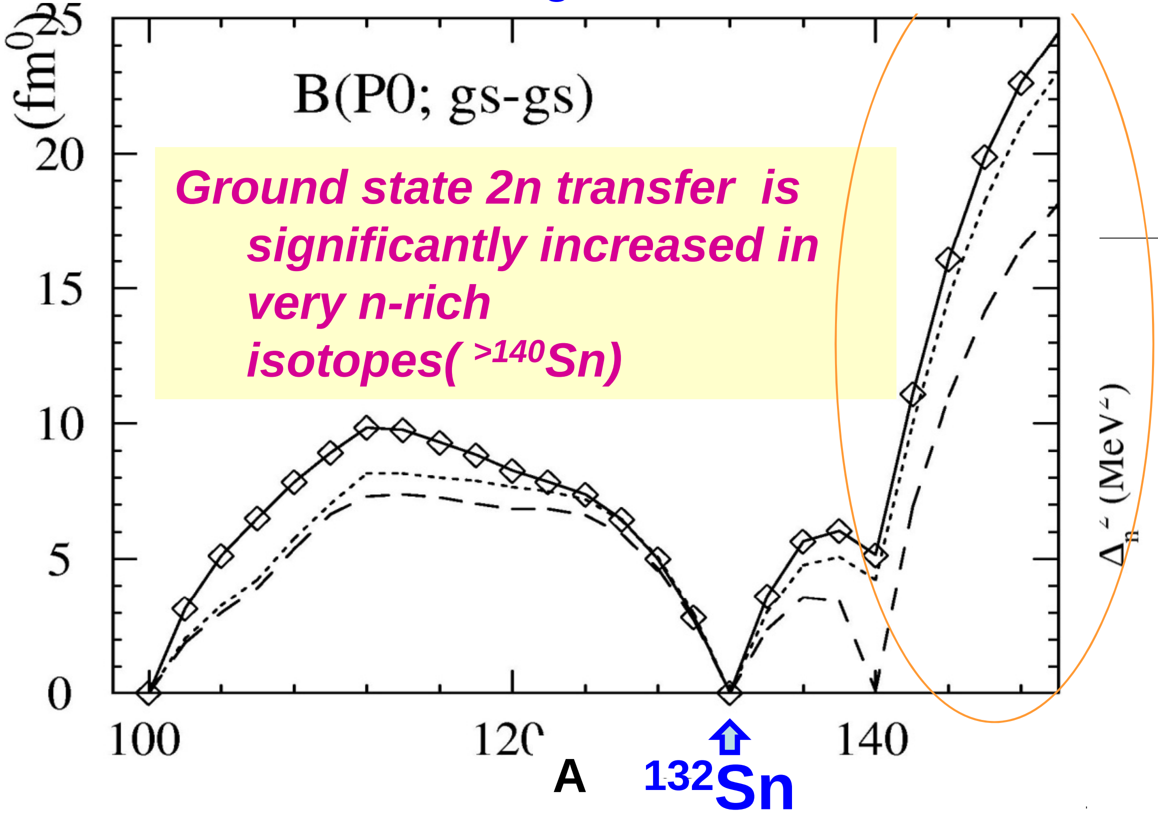


Pair-transfer strength : a simple measure

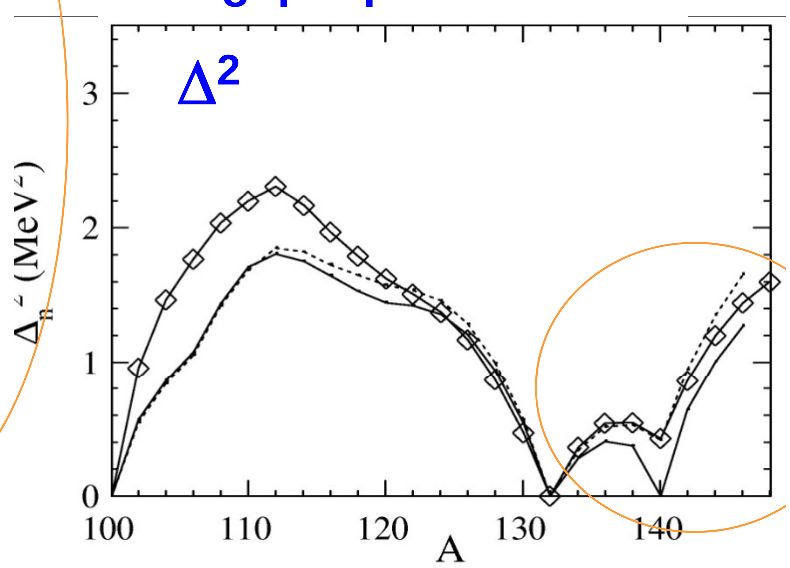
Ground-ground pair transfer strength

$$B(\mathrm{P0}) = \left| \int Y_{00} \langle 0_{gs} | \psi^+(\vec{r}) \psi^+(\vec{r}) | 0_{gs} \rangle d\vec{r} \right|^2$$

Pair transfer strength

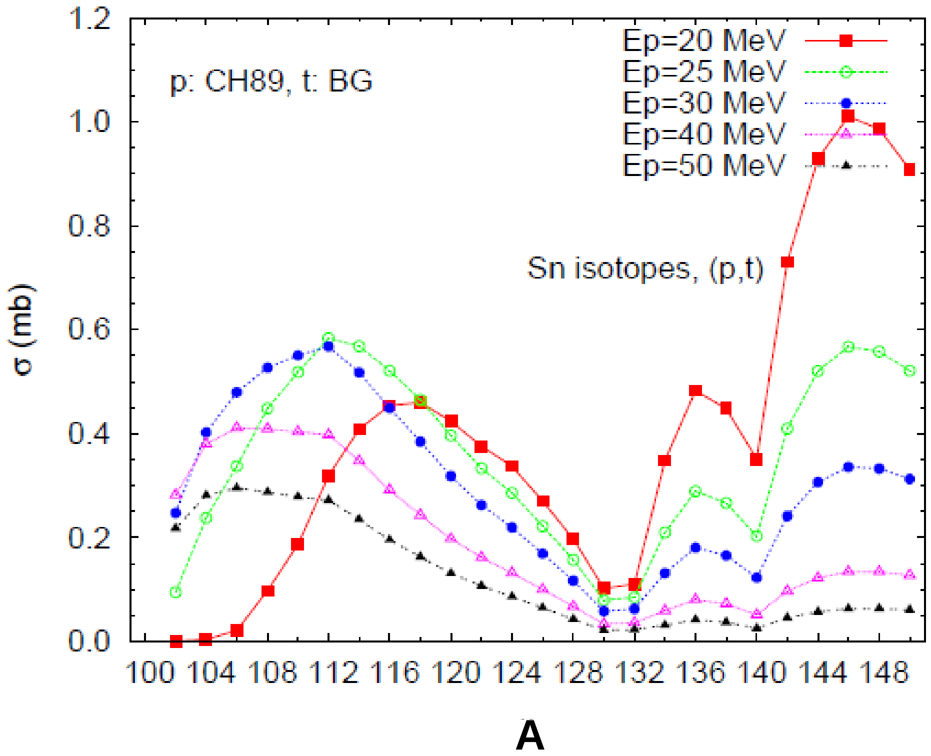


Pair gap squared

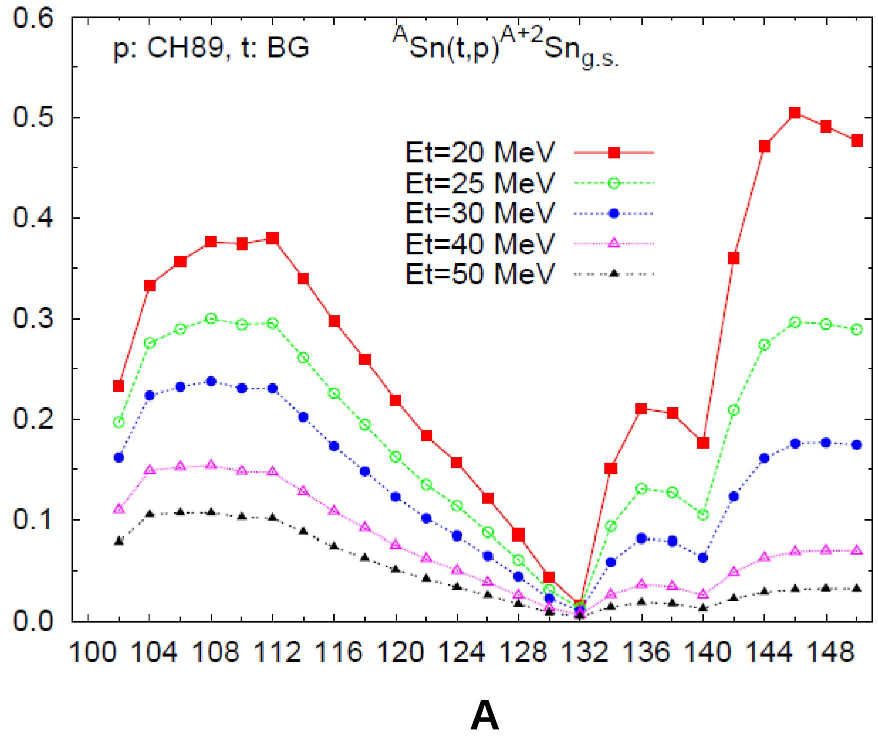


Pair transfer cross section:
Ground-to-ground

A_{gs} (p , t) $A\text{-}2_{gs}$

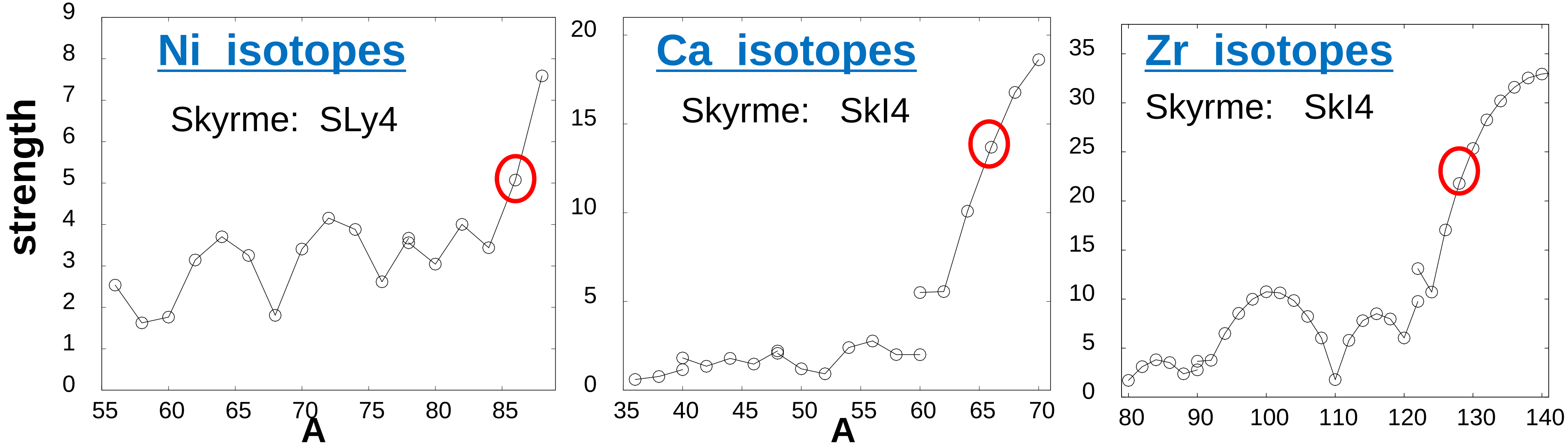


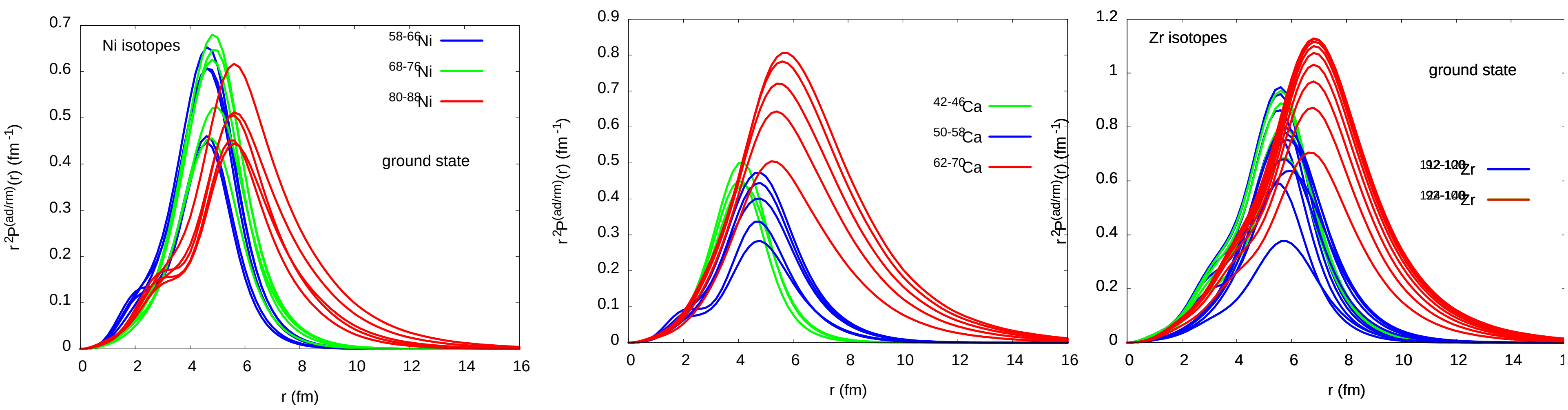
A_{gs} (t , p) $A+2_{gs}$



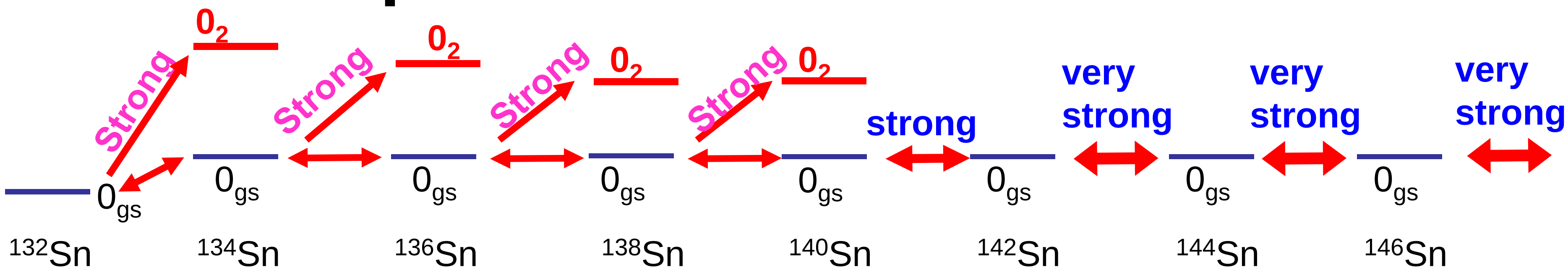
Surface enhancement is generic to nuclides with small separation energy

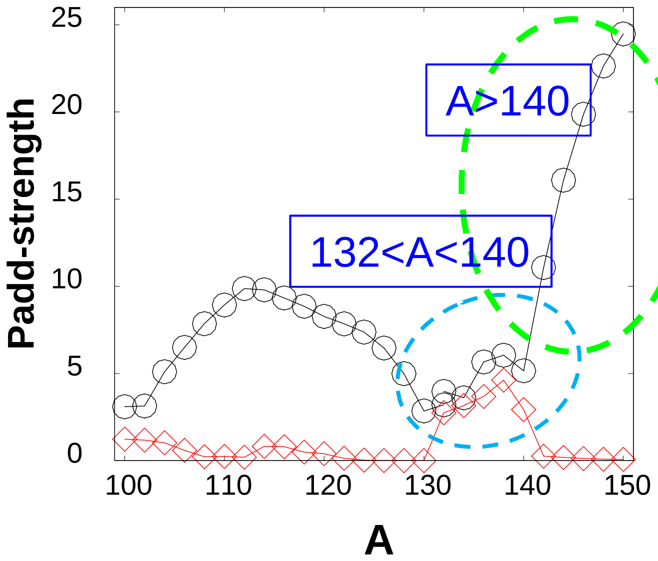
The strength of the ground state transfer

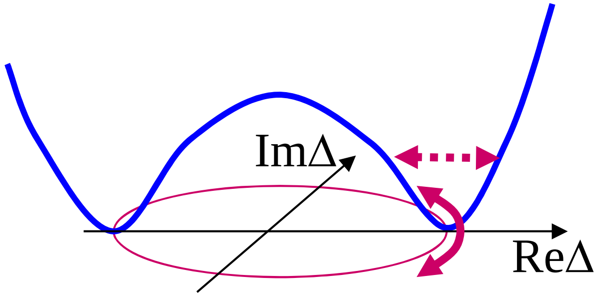




A precursor phenomenon: anomalous pair vibration

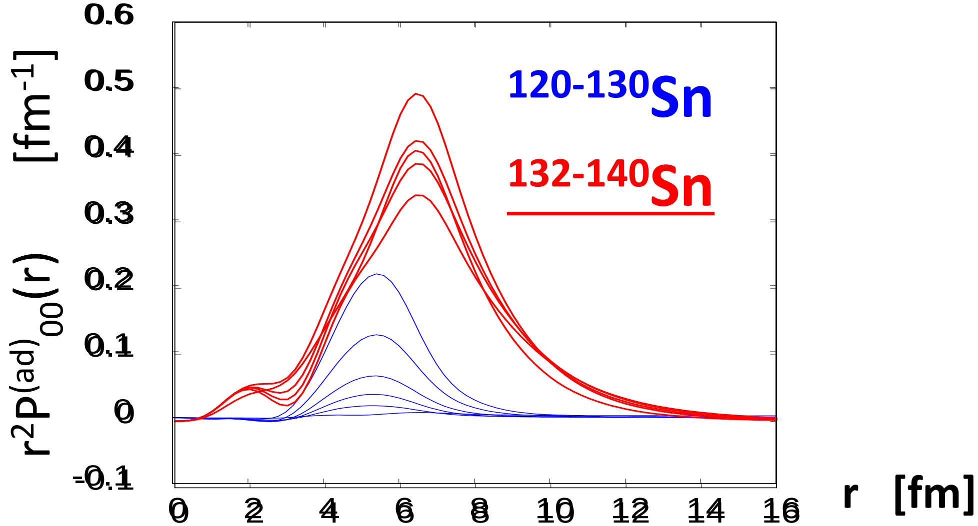




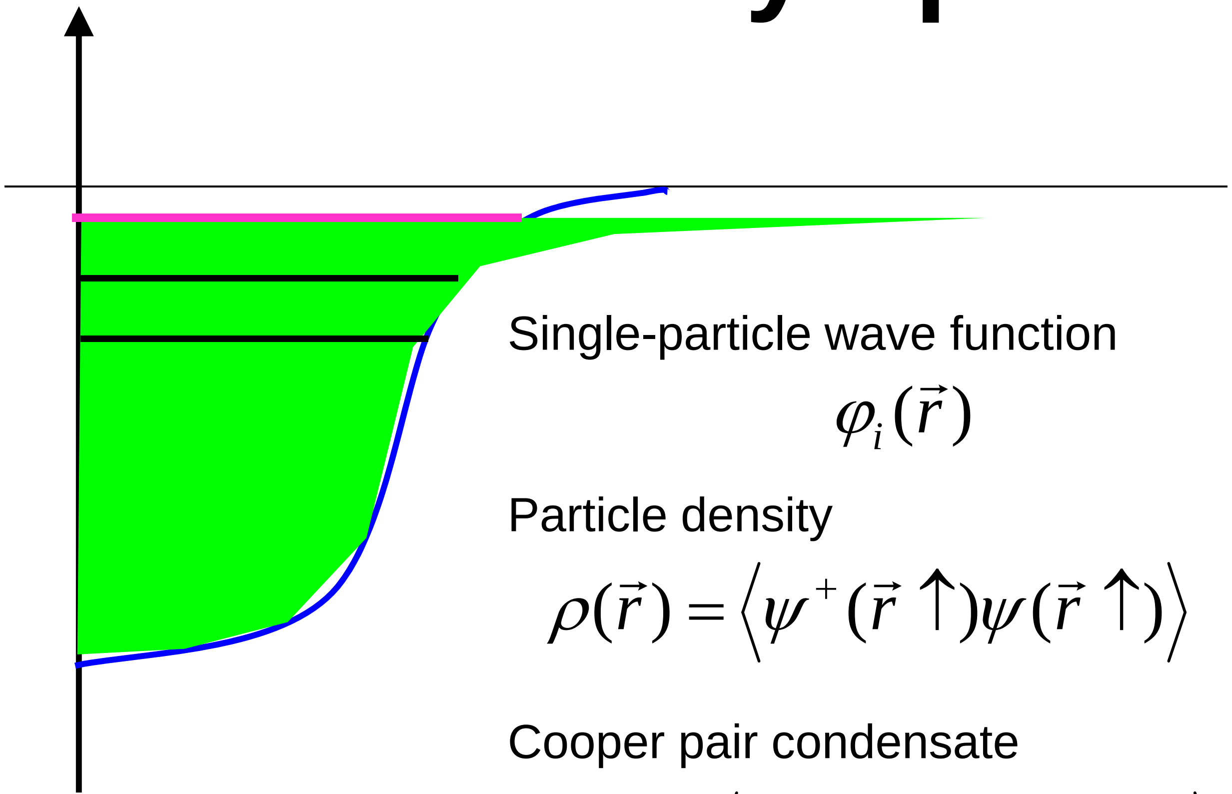


Pair rotation & pair vibration

$$P_{00}^{(ad)}(\boldsymbol{r}) = \langle \Psi_{PV} \mid \psi^\dagger(\boldsymbol{r}\uparrow)\psi^\dagger(\boldsymbol{r}\downarrow) \mid \Psi_{gs} \rangle$$



3. Pairing dominance in the asymptotics



$$r \to \infty$$

Single-particle wave function

$$\varphi_i(\vec{r}) \to e^{-\kappa_i r}$$

Particle density

$$\rho(\vec{r}) = \langle \psi^+(\vec{r}\uparrow)\psi(\vec{r}\uparrow) \rangle \to e^{-Cr}$$

~Least bound quasi particle

Cooper pair condensate

$$\tilde{\rho}(\vec{r}) = \langle \psi^+(\vec{r}\uparrow)\psi^+(\vec{r}\downarrow) \rangle \to e^{-Dr}$$

~Fermi energy

$D < C$

$$\tilde{\rho}(\vec{r}) > \rho(\vec{r})$$

Summary

1. Strong-coupling pairing in dilute neutron matter

MM, PRC73,044309(2006) and references theirin

2. Spatially correlated Cooper pair at dilute surface of n-rich nuclei

MM, K. Mizuyama, Y. Serizawa PRC71,064326(2005)
MM, H. Shimoyama, Y. Otaki, Phys. Scr. T150, 014024 (2012)

3. Enhanced pair transfer at surface

H. Shimoyama, MM, PRC84,044317(2011)

4. Pairing dominance in the asymptotics

Y. Zhang, MM, J. Meng, PRC 86, 054318 (2012)

A review: MM arXiv 1206.5870, in “Fifty years of nuclear BCS”