Calculation of light nuclei from lattice QCD

Takeshi Yamazaki

Kobayashi-Maskawa Institute for the Origin of Particles and the Universe

Nagoya University

K.-I. Ishikawa, Y. Kuramashi, A. Ukawa

Refs: PRD81:111504(R)(2010); PRD84:054506(2011); PRD84:054506(2012)

QUCS 2012 @ the Reception Hall in Nara Prefectural New Public Hall,
December 13–16 2012

Supports from Scientific Research on Innovative Areas
「素核宇宙融合による計算科学に基づいた重層的物質構造の解明」

1. Researcher of A01 group at Univ. of Tsukuba
2. Proposed project grants
   2009-2010
     Study of light nuceli using lattice quantum chromodynamics
   2011-2012
     Study of calculation method of light nuclei
     using lattice quantum chromodynamics
3. $\cdots$

Appreciate all supports

Outline

1. Introduction
2. Problems of multi-nucleon bound state
3. Simulation parameters
4. $N_f = 2+1$ results at $m_\pi = 0.5$ GeV
   ^{4}He, ^{3}He, NN channels
5. Summary and future work

1. Introduction

Strong interaction

$$\text{Bind} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

Spectrum of nuclei

success of Shell model: Jensen and Mayer(1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

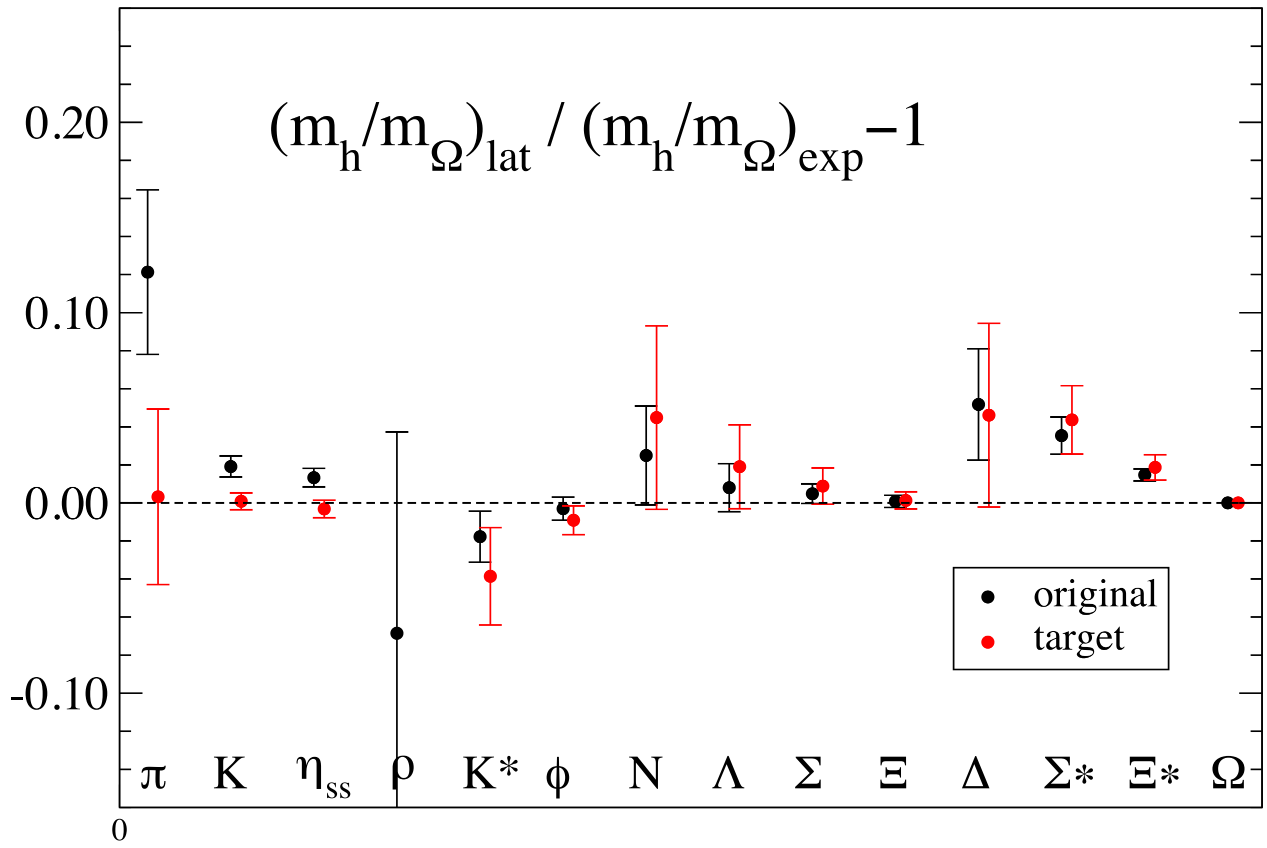
such as lattice QCD

degrees of freedom of quarks and gluons

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons}}^{}}_{\text{(lattice) QCD}}\! \quad \overbrace{\phantom{\text{protons and neutrons}} \rightarrow \text{nuclei}}^{\text{Shell model}}$$

Lattice QCD result at physical pion mass '10 PACS-CS

target = physical pion mass



Consistent nucleon mass with experiment

1. Introduction

Strong interaction

$$\text{Bind}\begin{cases}\text{protons and neutrons} & \to & \text{nuclei}\\ \text{quarks and gluons} & \to & \text{protons and neutrons}\end{cases}$$

Spectrum of nuclei

success of Shell model: Jensen and Mayer(1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

$$\underbrace{\text{quarks and gluons} \to \overbrace{\text{protons and neutrons}}}_{\text{(lattice) QCD}}\overbrace{\to \text{nuclei}}^{\text{Shell model}}$$

1. Introduction

Strong interaction

$$\text{Bind} \left\{ \begin{array}{rcl} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{array} \right.$$

Spectrum of nuclei

success of Shell model: Jensen and Mayer(1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

Motivation: Understand property of nuclei from (lattice) QCD directly

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}}_{\text{(lattice) QCD}}$$

1. Introduction

Motivation :

Understand property of nuclei from (lattice) QCD directly

through different approach from nuclear force, HALQCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

such as neutron rich nuclei

So far only few works for multi-baryon bound states
Before studying such difficult problems, we should check

$\rightarrow$ Can we calculate known binding energy in a-few-nucleon systems?

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. ^{4}He and ^{3}He channels
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV
   - '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
3. NN channels
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

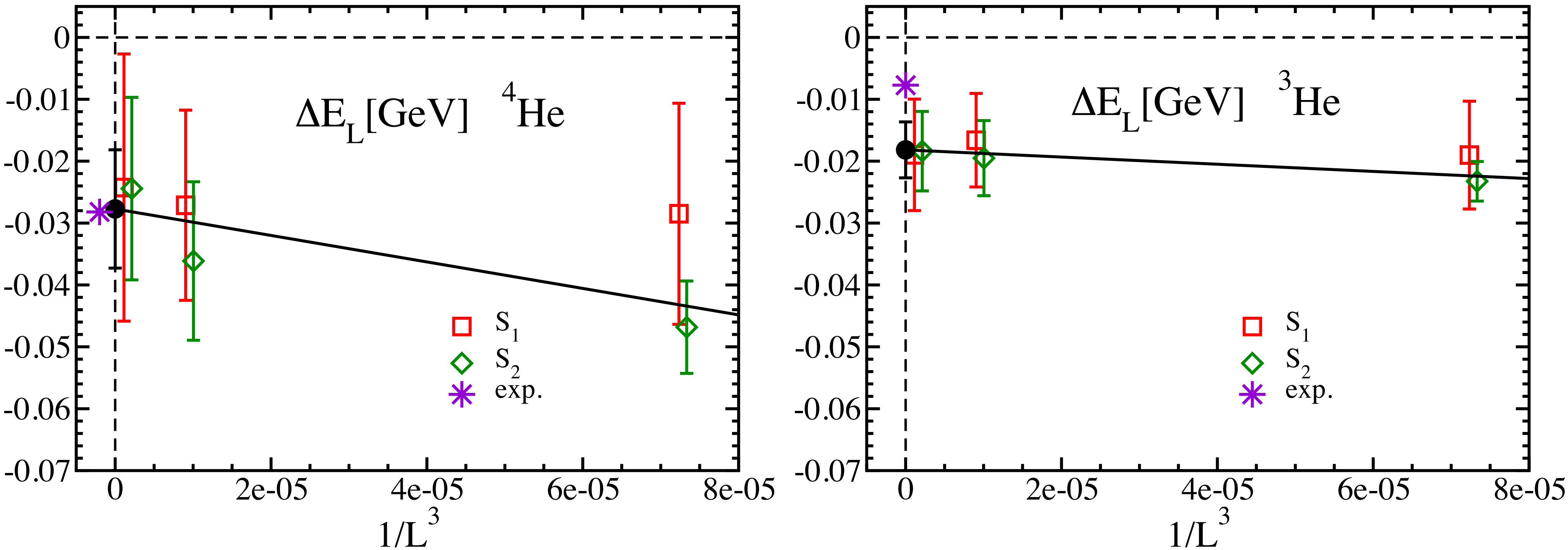
Other channels: '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV; '12 $N_f = 3$ $m_\pi = 0.81$ GeV

Related studies: BB forces HALQCD [Doi and Inoue]; ΩΩ scattering Buchoff *et al.*

Expoloratory study of 3-, 4-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE



$\Delta E_{^4\mathrm{He}} = 27.7(7.8)(5.5)$ MeV $\qquad$ $\Delta E_{^3\mathrm{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

Containing several systematic errors, e.g., $m_\pi = 0.8$ GeV

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. ^{4}He and ^{3}He channels
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV
   - '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
3. NN channels
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

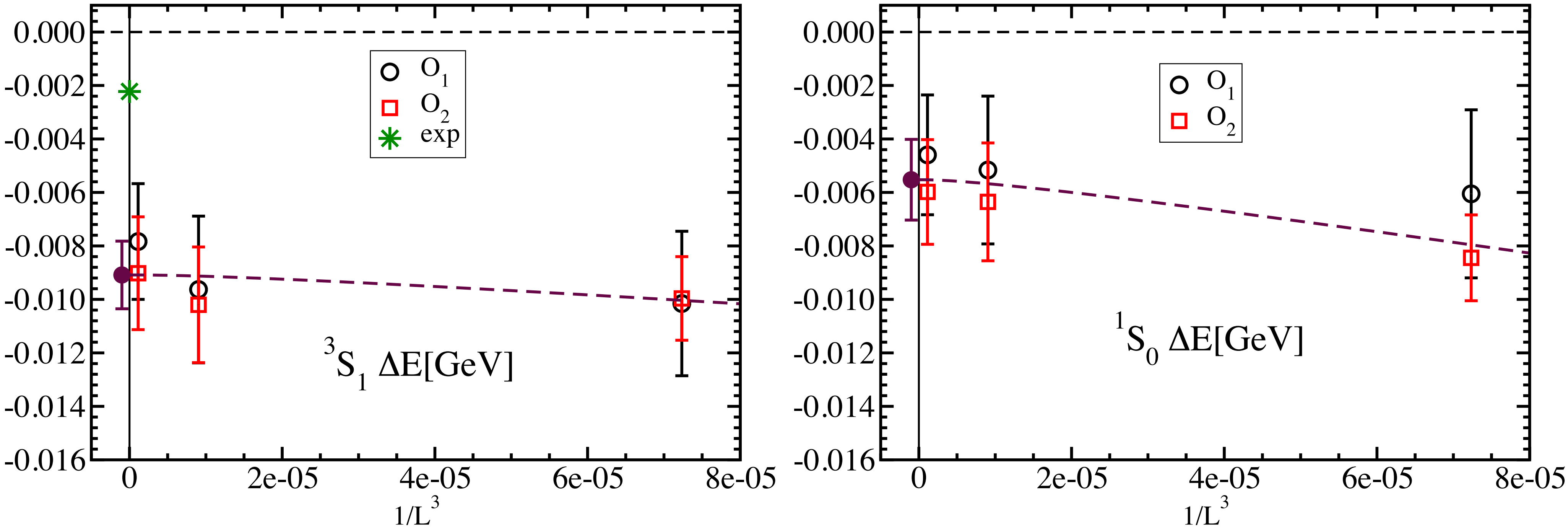
Other channels: '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV; '12 $N_f = 3$ $m_\pi = 0.81$ GeV

Related studies: BB forces HALQCD [Doi and Inoue]; ΩΩ scattering Buchoff *et al.*

Expoloratory study of NN nuclei

PACS-CS Collaboration, PRD84:054506(2011)

Identification of bound state from volume dependence of ΔE_0 and ΔE_1



Observe bound state in both channels

$\Delta E_{^3\mathrm{S}_1} = 9.1(1.1)(0.5)$ MeV
Same order to experiment

$\Delta E_{^1\mathrm{S}_0} = 5.5(1.1)(1.0)$ MeV
Not observed in experiment

might be caused by heavy quark mass in calculation

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. ^{4}He and ^{3}He channels
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.17 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
3. NN channels
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

Other channels: '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV; '12 $N_f = 3$ $m_\pi = 0.81$ GeV

Related studies: BB forces HALQCD [Doi and Inoue]; ΩΩ scattering Buchoff *et al.*

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. ^{4}He and ^{3}He channels
   - '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$ GeV, $m_\pi > 1$ GeV ^{4}He
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   - '11 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV
   - '12 HALQCD $N_f = 3$ $m_\pi = 0.47$–1.17 GeV
   - '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV
3. NN channels
   - '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   - '12 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV (Possibility)
   - '12 NPLQCD $N_f = 3$ $m_\pi = 0.81$ GeV

Purpose: Extend our works to $N_f = 2 + 1$ QCD with smaller m_π and a

2. Problems of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

2. Calculation cost

Wick contraction for $^4\mathrm{He} = p^2 n^2 = (udu)^2(dud)^2$: 518400

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state $\Delta E_0 = E_0 - N_N m_N < 0$

2. Problems of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\to$ heavy quark mass $m_\pi = 0.5$ GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for $^4\mathrm{He} = p^2n^2 = (udu)^2(dud)^2$: 518400 $\to$ 1107

$\to$ reduction using $p(n) \leftrightarrow p(n)$, $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

Multi-meson: '10 Detmold and Savage

Multi-baryon: '12 Doi and Endres; Detmold and Orginos

3. Identification of bound state on finite volume

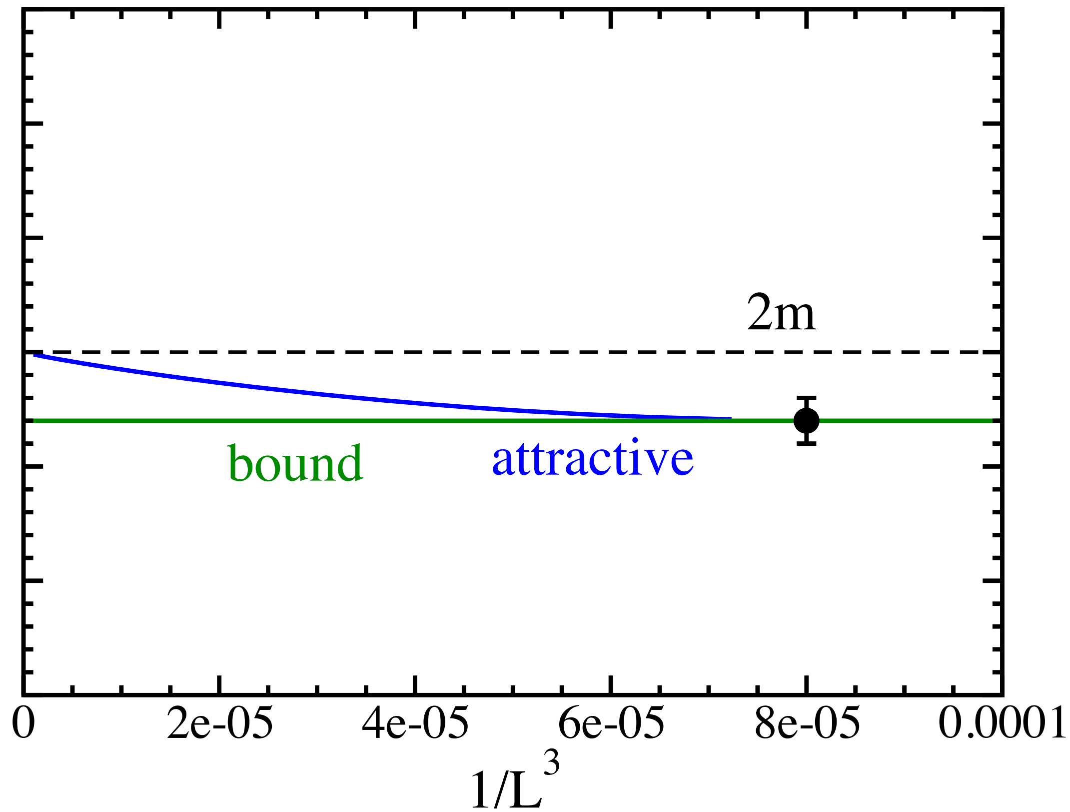
Finite volume effect of attractive scattering state $\Delta E_0 = E_0 - N_N m_N < 0$

$\to$ Volume dependence of ΔE '86,'91 Lüscher, '07 Beane *et al.*

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

Identification of bound state in finite volume

observe small $\Delta E_L = E - 2m < 0$ at one L is not enough



Bound state : $\Delta E_L = \Delta E_\infty + O\left(e^{-CL}\right) < 0$

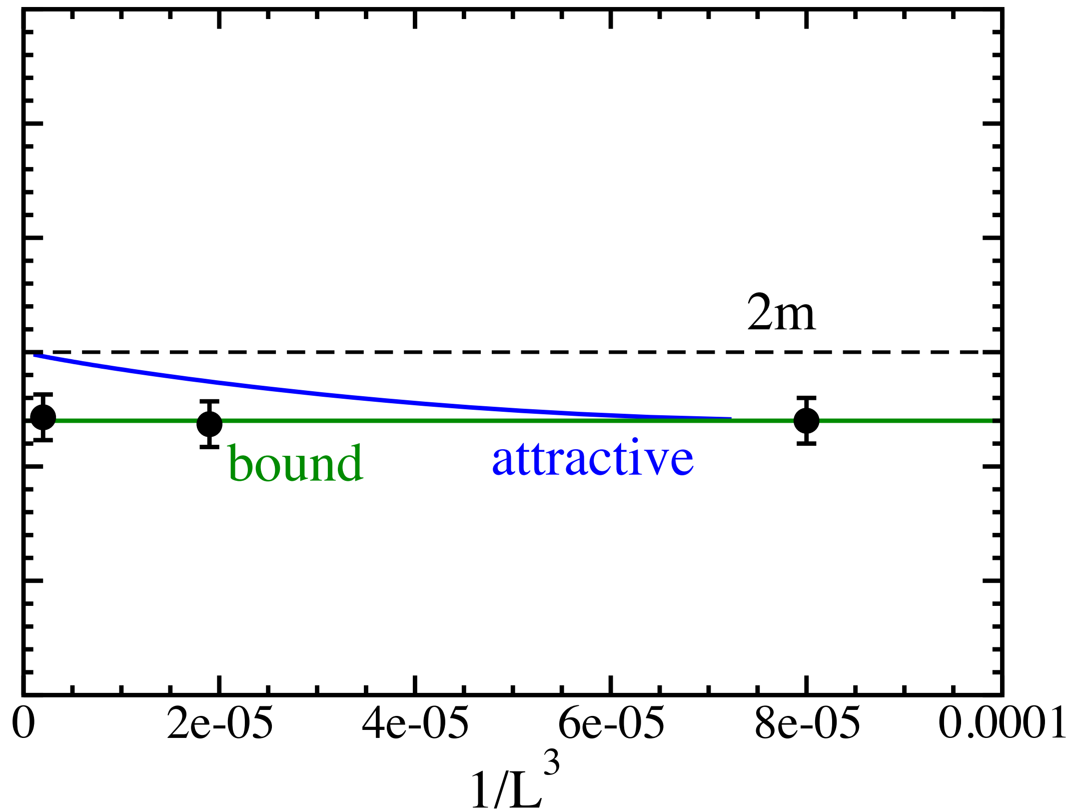
'04 Beane *et al.*, '06 Sasaki & TY

Attractive scattering state : $\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

'86, '91 Lüscher

Identification of bound state in finite volume

observe small $\Delta E_L = E - 2m < 0$ at several L



Bound state : $\Delta E_L = \Delta E_\infty + O\left(e^{-CL}\right) < 0$

'04 Beane *et al.*, '06 Sasaki & TY

Attractive scattering state : $\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

'86, '91 Lüscher

3. Simulation parameters

$N_f = 2+1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV

$m_s \sim$ physical strange quark mass

Finite volume dependence of ΔE_0 with four volumes

(^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ channels)

L	L [fm]	N_{Traj}	N_{conf}	N_{meas}
32	2.9	4000	200	192
40	3.6	2000	200	192
48	4.3	2000	200	192
64	5.8	1900	190	256

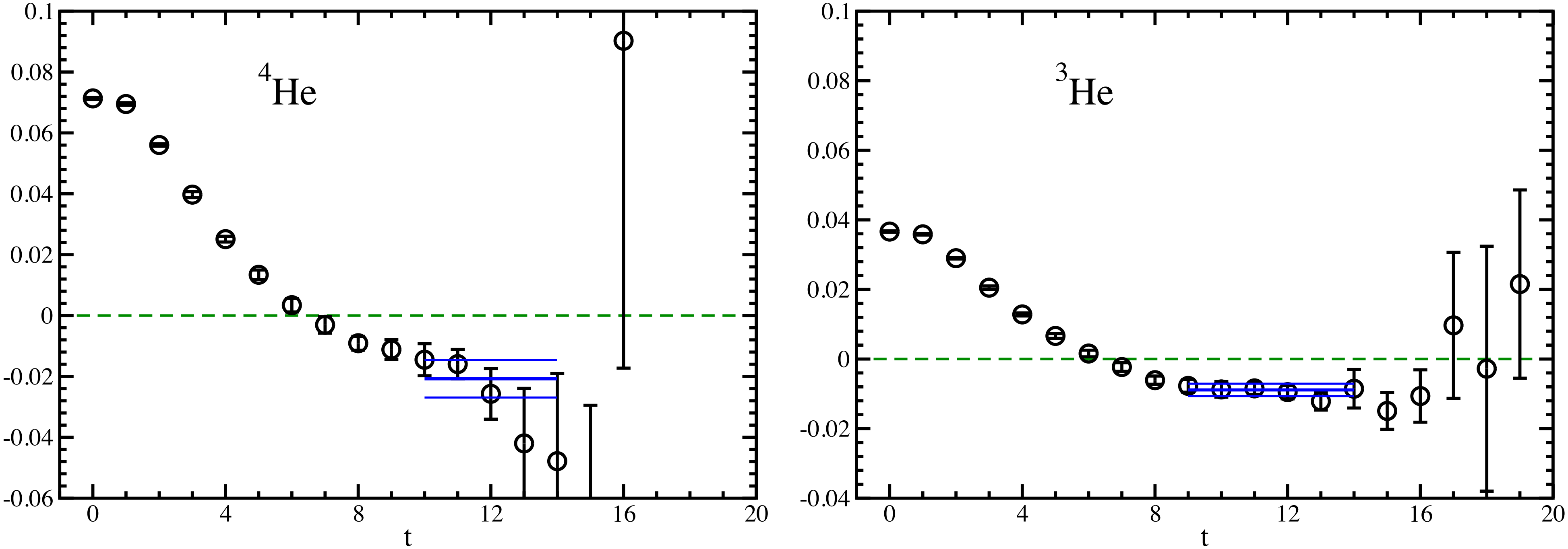
Simulations:

PACS-CS, T2K-Tsukuba, HA-PACS at Univ. of Tsukuba, HA8000 at Univ. of Tokyo and K at AICS

4. Results

ΔE_L in ^{4}He and ^{3}He channels at $L = 5.8$ fm

$$\Delta E_L = \log\left(\frac{R_{^4\text{He}}(t)}{R_{^4\text{He}}(t+1)}\right) \text{ with } R_{^4\text{He}}(t) = \frac{C_{^4\text{He}}(t)}{(C_N(t))^4}$$

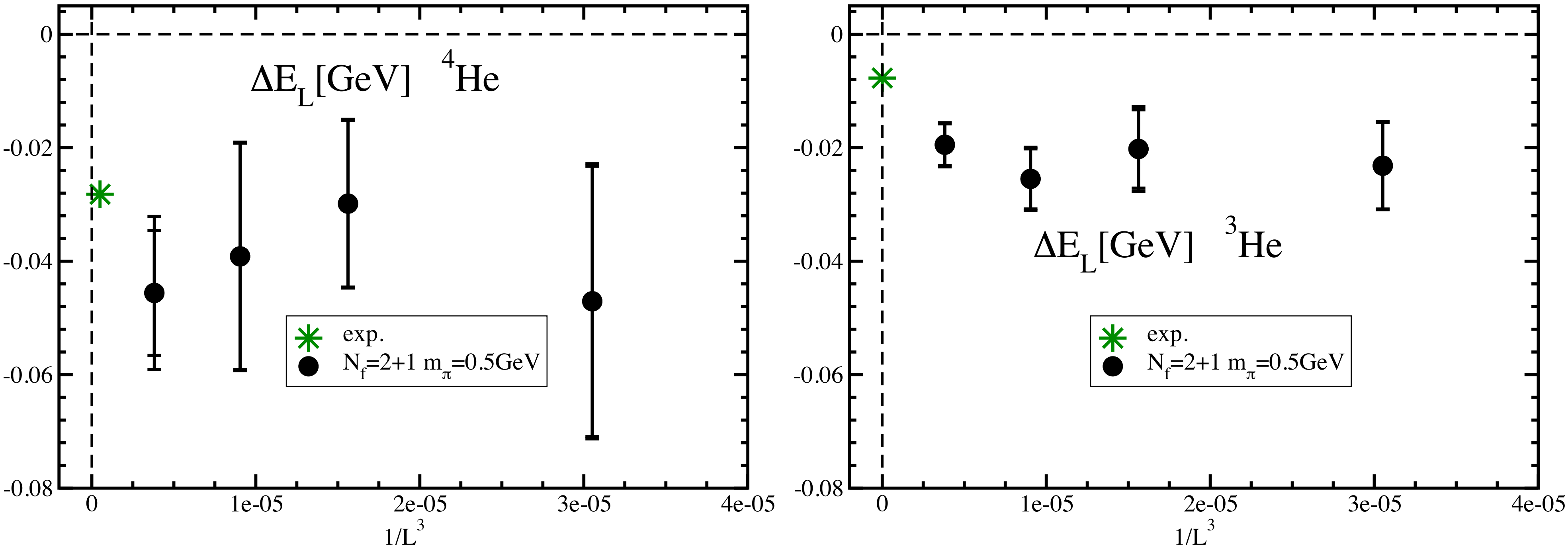


- Statistical error under control in $t < 12$
- Relatively smaller error in ^{3}He channel
- Negative ΔE_L in both channels

4. Results

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$

Identification of bound state from volume dependence of ΔE

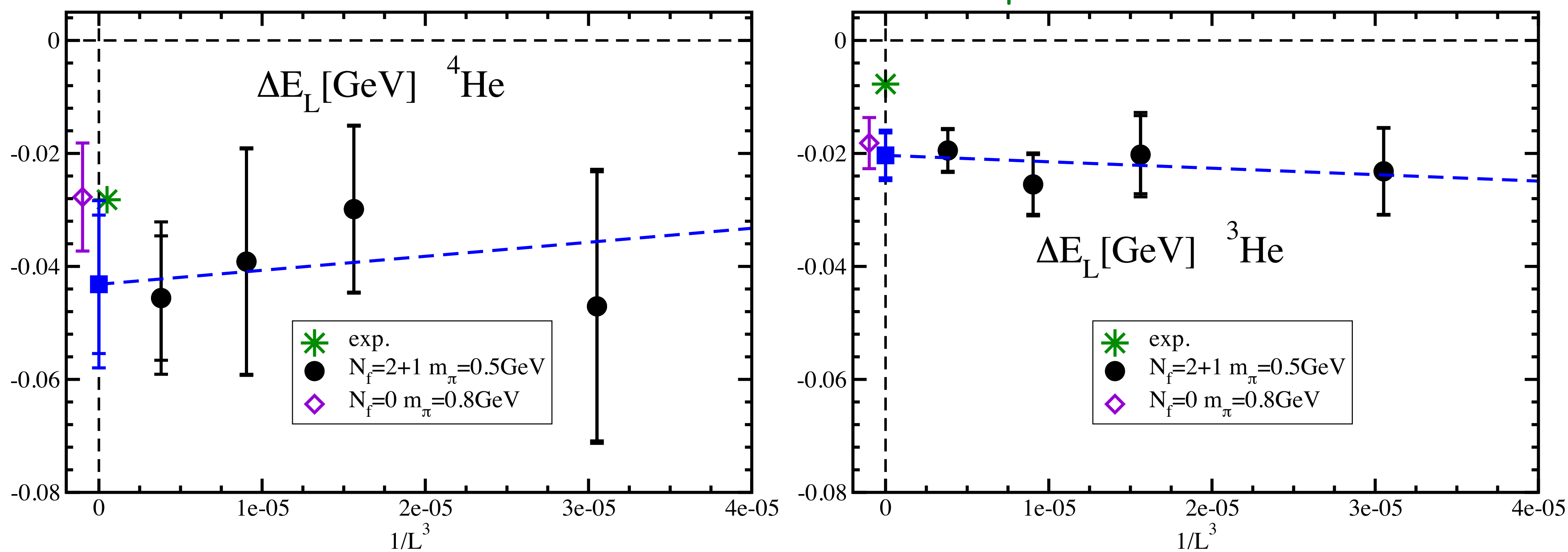


- $\Delta E_L < 0$ and small volume dependence
- Infinite volume extrapolation with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

4. Results

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$

Identification of bound state from volume dependence of ΔE



$\Delta E_{^4\text{He}} = 43(12)(8)$ MeV $\qquad$ $\Delta E_{^3\text{He}} = 20.3(4.0)(2.0)$ MeV

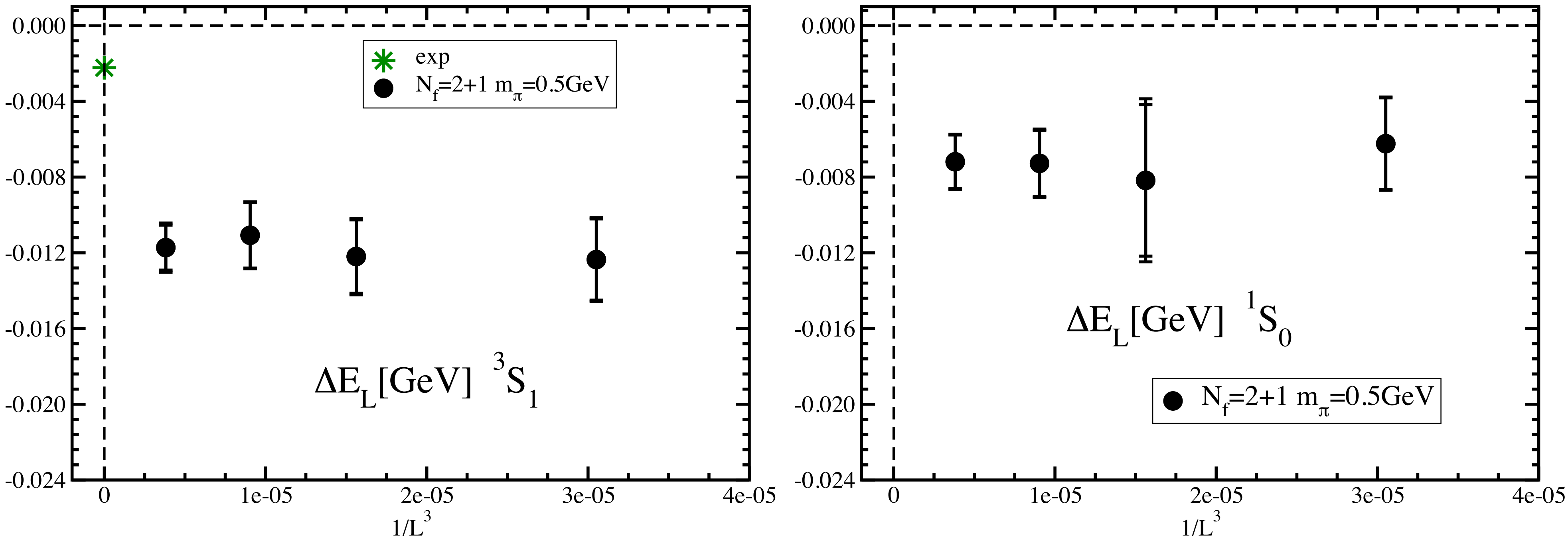
1. Observe bound state in both channels
2. Similar to quenched result

open symbols: quenched at $m_\pi = 0.8$ GeV, PRD81:111504(R)(2010)

4. Results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

Identification of bound state from volume dependence of ΔE



- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L '04 Beane *et al.*, '06 Sasaki & TY

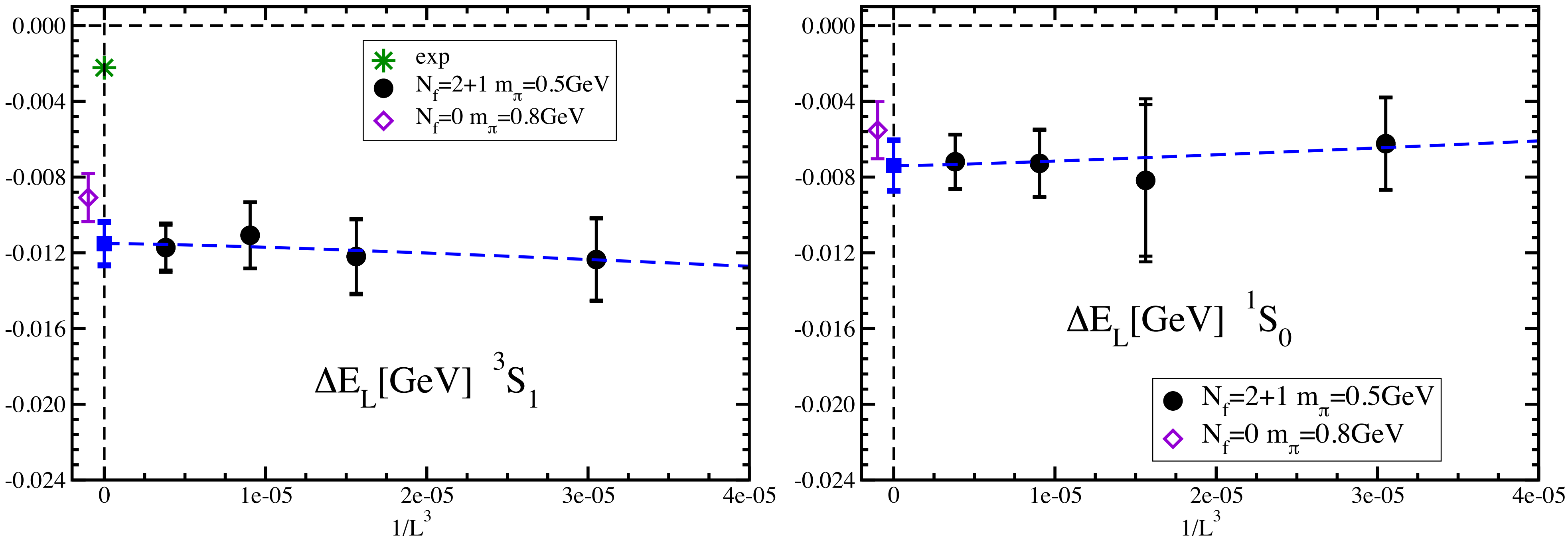
$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\}, \quad \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

based on Lüscher's finite volume formula

4. Results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

Identification of bound state from volume dependence of ΔE



Bound state in both channels

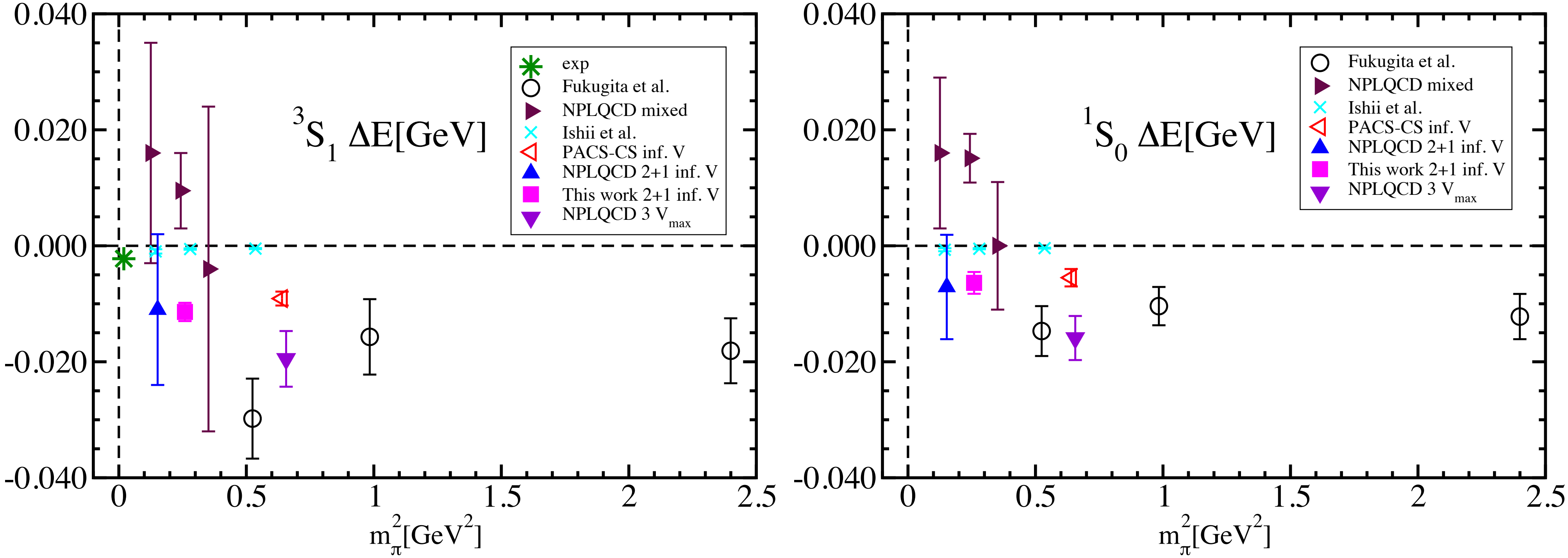
$$\Delta E_{^3\mathrm{S}_1} = 11.5(1.1)(0.6)\ \mathrm{MeV} \qquad \Delta E_{^1\mathrm{S}_0} = 7.4(1.3)(0.6)\ \mathrm{MeV}$$

Not observed in experiment

Similar to quenched result at $m_\pi = 0.8$ GeV PRD84:054506(2011)

Comparison of ΔE with previous works

NN ($^3S_1$ and $^1S_0$) channels with (possible) bound states



filled(open) symbols: dynamical(quenched) results

Roughly consistent

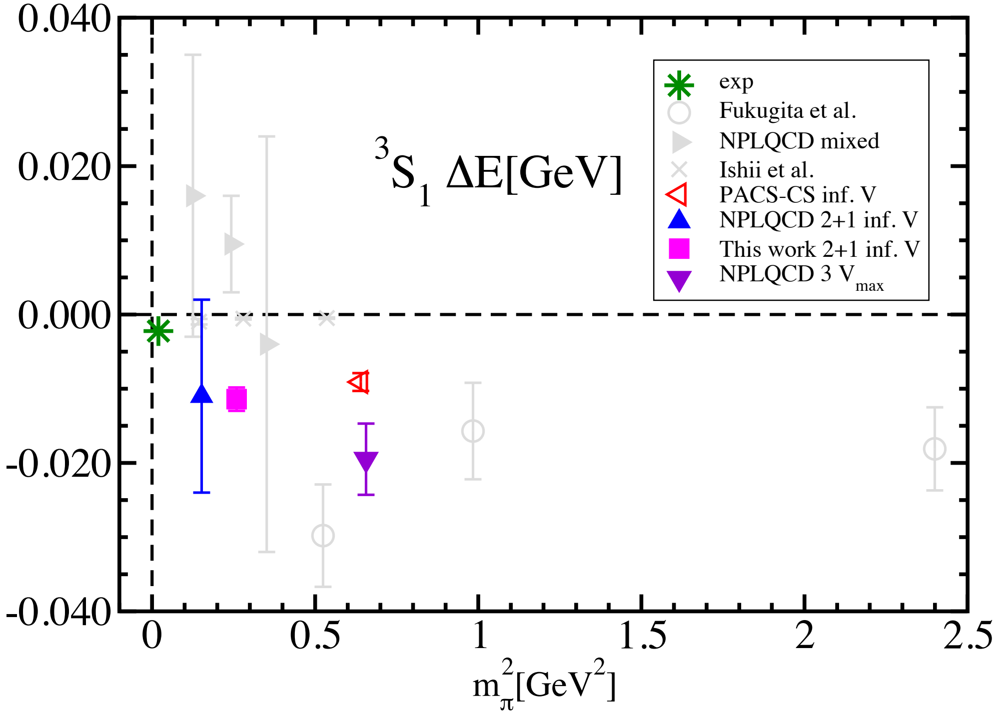
Our quenched and $N_f = 3 \to$ Large sea quark effect?

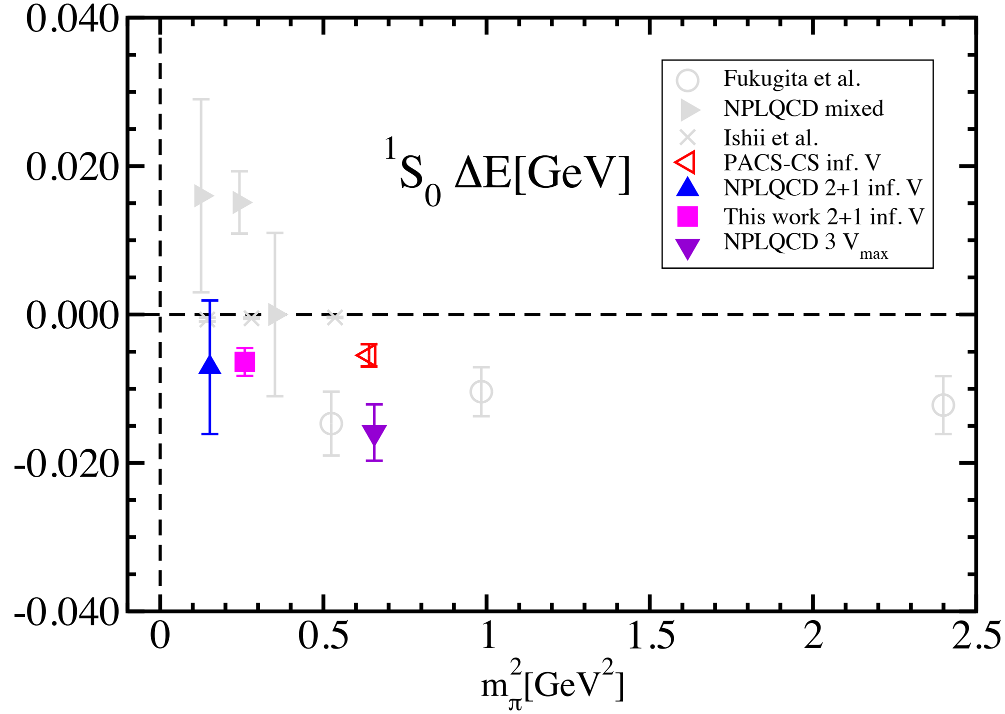
This work and $N_f = 3 \to$ Large quark mass effect?

Need further study of systematic errors
such as quark mass dependence

Comparison of ΔE with previous works

NN ($^3S_1$ and $^1S_0$) channels with (possible) bound states





filled(open) symbols: dynamical(quenched) results

Roughly consistent

Our quenched and $N_f = 3 \rightarrow$ Large sea quark effect?

This work and $N_f = 3 \rightarrow$ Large quark mass effect?

Need further study of systematic errors
such as quark mass dependence

5. Summary and future work

Extend our exploratory, quenched studies to $N_f = 2 + 1$ lattice QCD

- Heavy quark mass of $m_\pi = 0.5$ GeV
- Volume dependence of ΔE_0

  $\Delta E \neq 0$ of 0th state in infinite volume limit

$\rightarrow$ bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ at $m_\pi = 0.5$ GeV

- Similar result to quenched case $\rightarrow$ ΔE larger than experiment
- Bound state in ^{1}S$_0$ not observed in experiment

  Possibility in $N_f = 2 + 1$ at $m_\pi = 0.39$ GeV ('12 NPLQCD)

  Deeper bound state in $N_f = 3$ at $m_\pi = 0.8$ GeV ('12 NPLQCD)

Further investigations in $N_f = 2 + 1$, *e.g.* m_π dependence

(0.7 GeV $\leftarrow$) $m_\pi = 0.5$ GeV $\rightarrow$ 0.3 GeV $\rightarrow$ physical m_π(HPCI)