Phase structure of many flavor lattice QCD at finite temperature

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in collaboration with

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Introduction

- The nature of the chiral phase transition of Many Flavor QCD [MFQCD] depends on the number of flavors and the masses.
  Pisarski and Wilczek, PRD 29, 338 (1984) and many evidences from lattice.

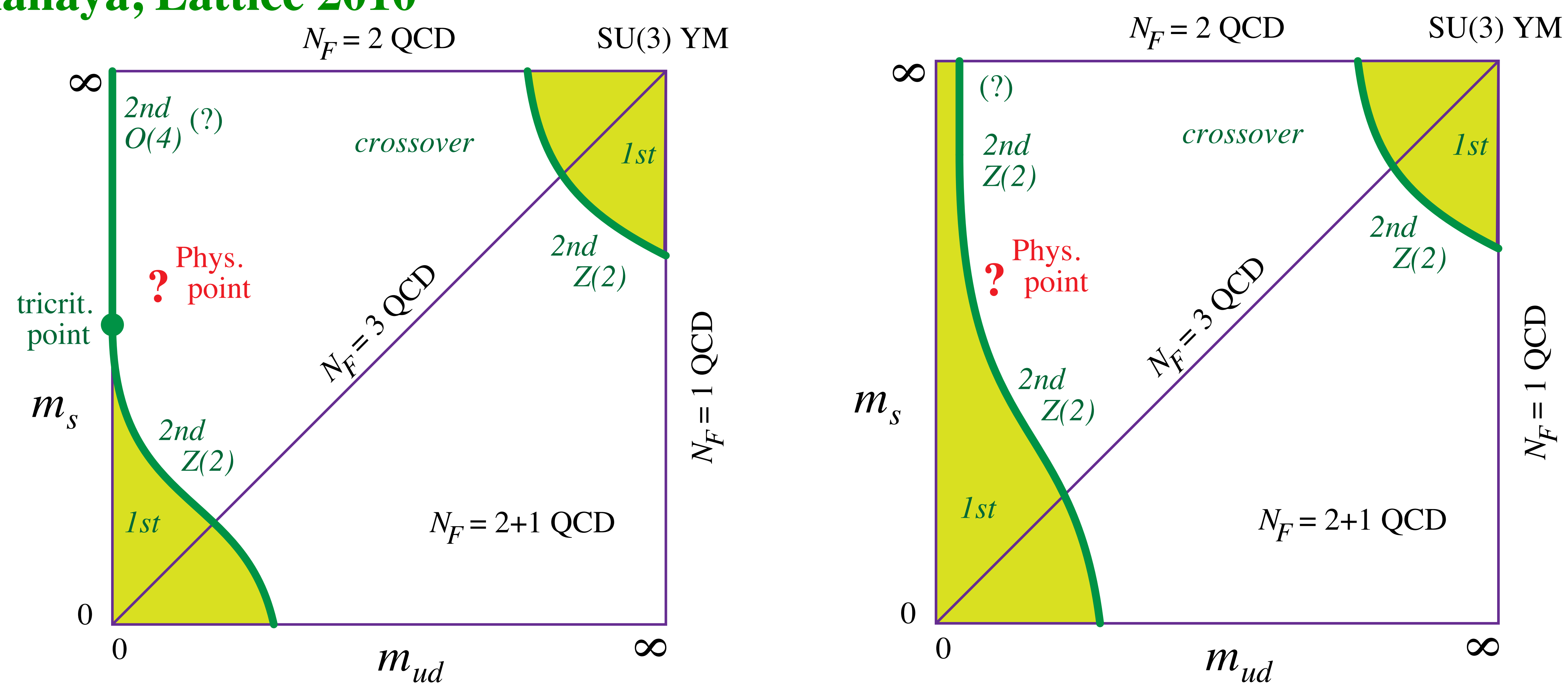
- EW baryogenesis in TC models. (TC is strongly interacting vector-like gauge theory. Its SχSB triggers EWSB.)
  Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995);
  Kikukawa, Kohda and Yasuda, PRD77 (2008) 015014

- As the first step toward the exploration of this possibility, we consider 2(light)+N_f(heavy)-flavor QCD, propose an easy method to explore the phase structure of this system and demonstrate the feasibility of the method.

Columbia plot for 2+1 QCD

Brown, Butler, Chen, Christ, Dong, Schaffer, Unger, and Vaccarino (90), N.H. Christ, Z. Dong (92) and N.H. Christ(92)

Kanaya, Lattice 2010

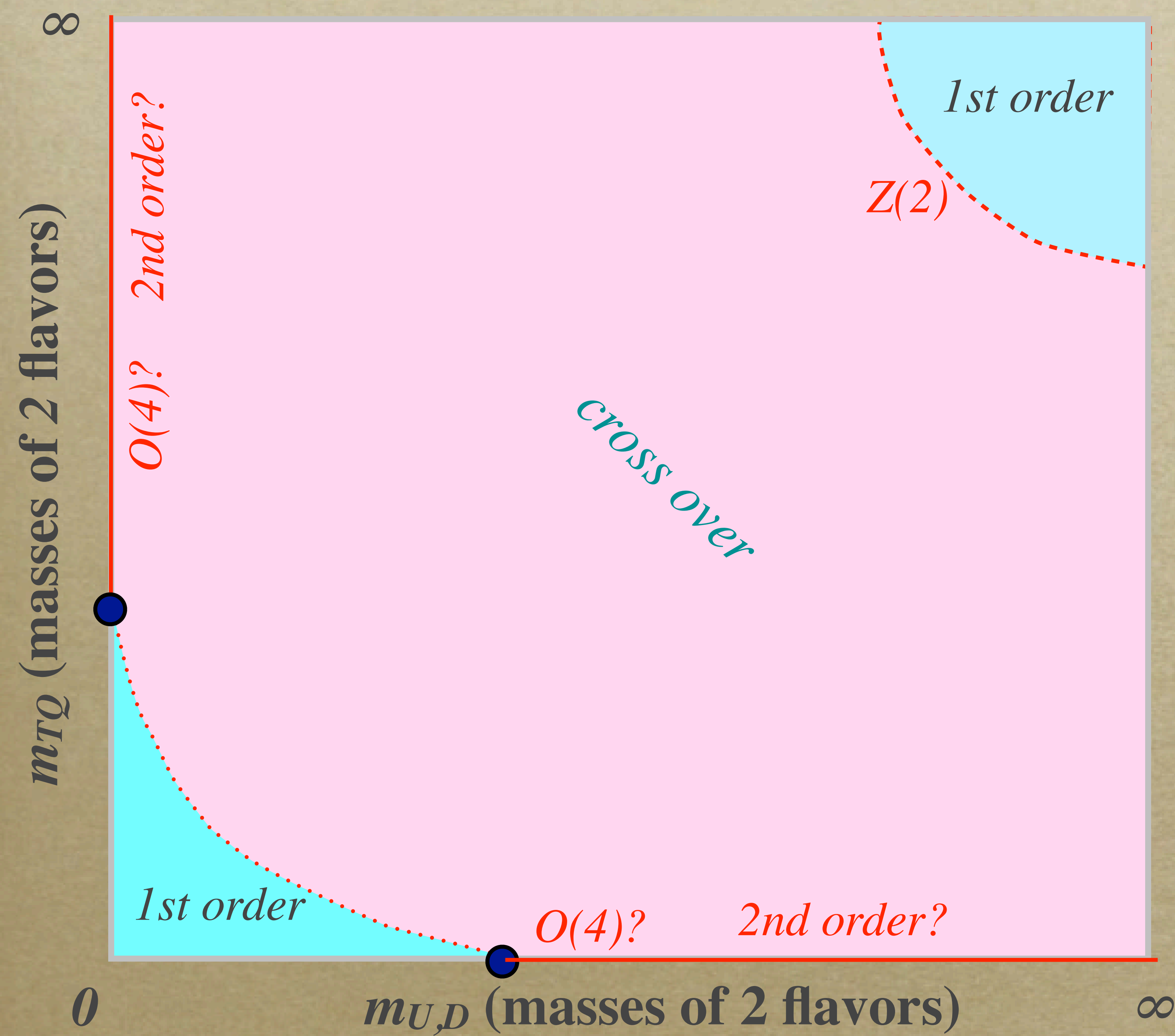


When $N_F \geqq 3$ (or 2?), Chiral Phase Transition around the massless limit is 1st order.

Pisarski and Wilczek, PRD 29, 338 (1984). See also Cossu et al.; Aoki, Fukaya, Taniguchi (2012)

Extend Columbia plot to 2+N_f QCD.

$N_f = 2$ ($2+2 < N_f^{\text{crit}}$)



In TC, two flavors must be exact massless.

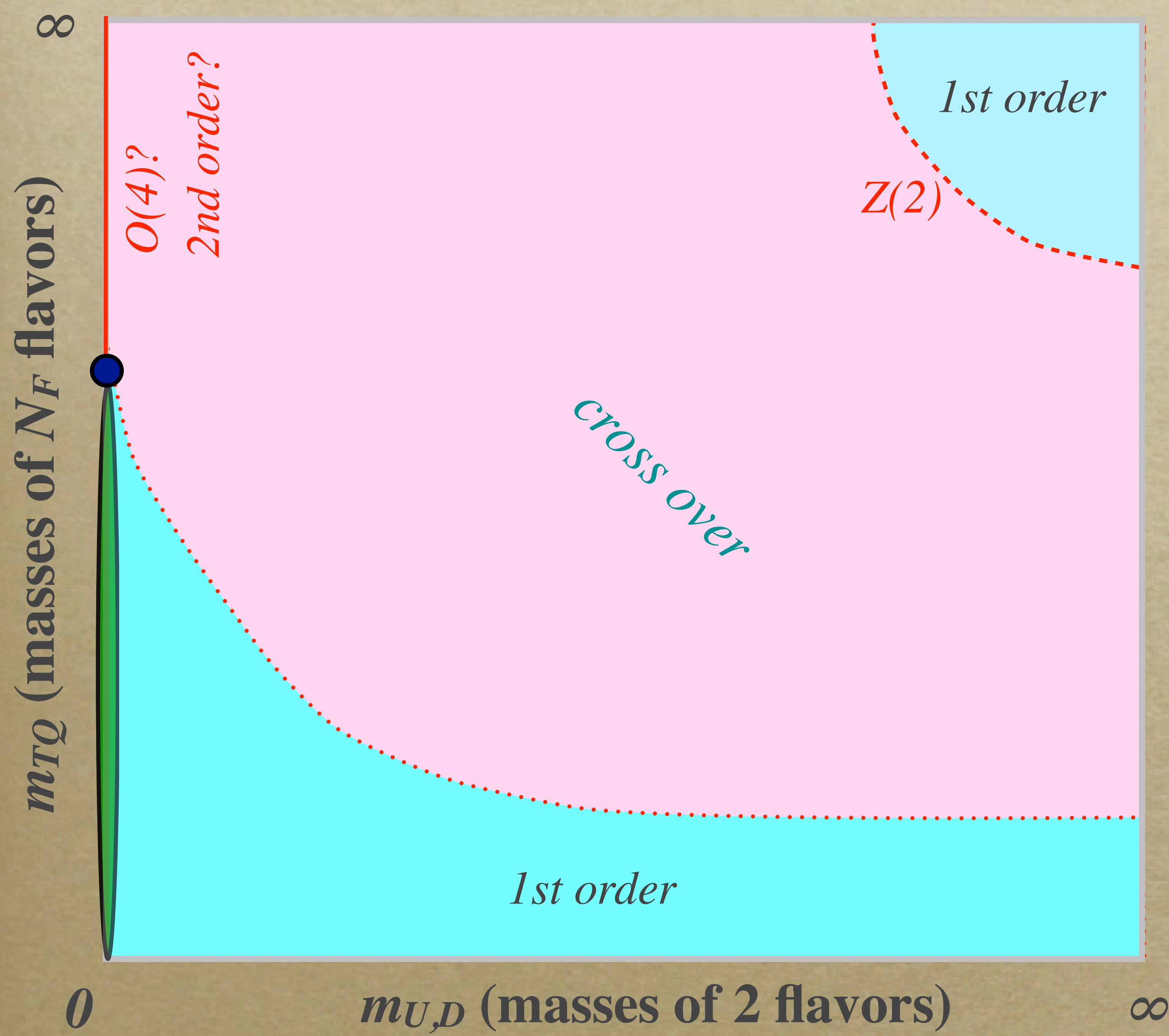
N_f^{crit} separating confining and conformal theories

Symmetric under reflection w.r.t. diagonal line.

Running of g^2 is not slow enough.

Less interesting.

$3 \leqq N_f \; (2+N_f < N_f^{crit})$



Walking with γ_m~O(1) is expected at an appropriate N_f.

For EW baryogenesis, important to identify the location of the 1st order region and its strength.

Appelquist, Schwetz and Selipsky, PRD52, 4741 (1995).
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Phenomenologically interesting!

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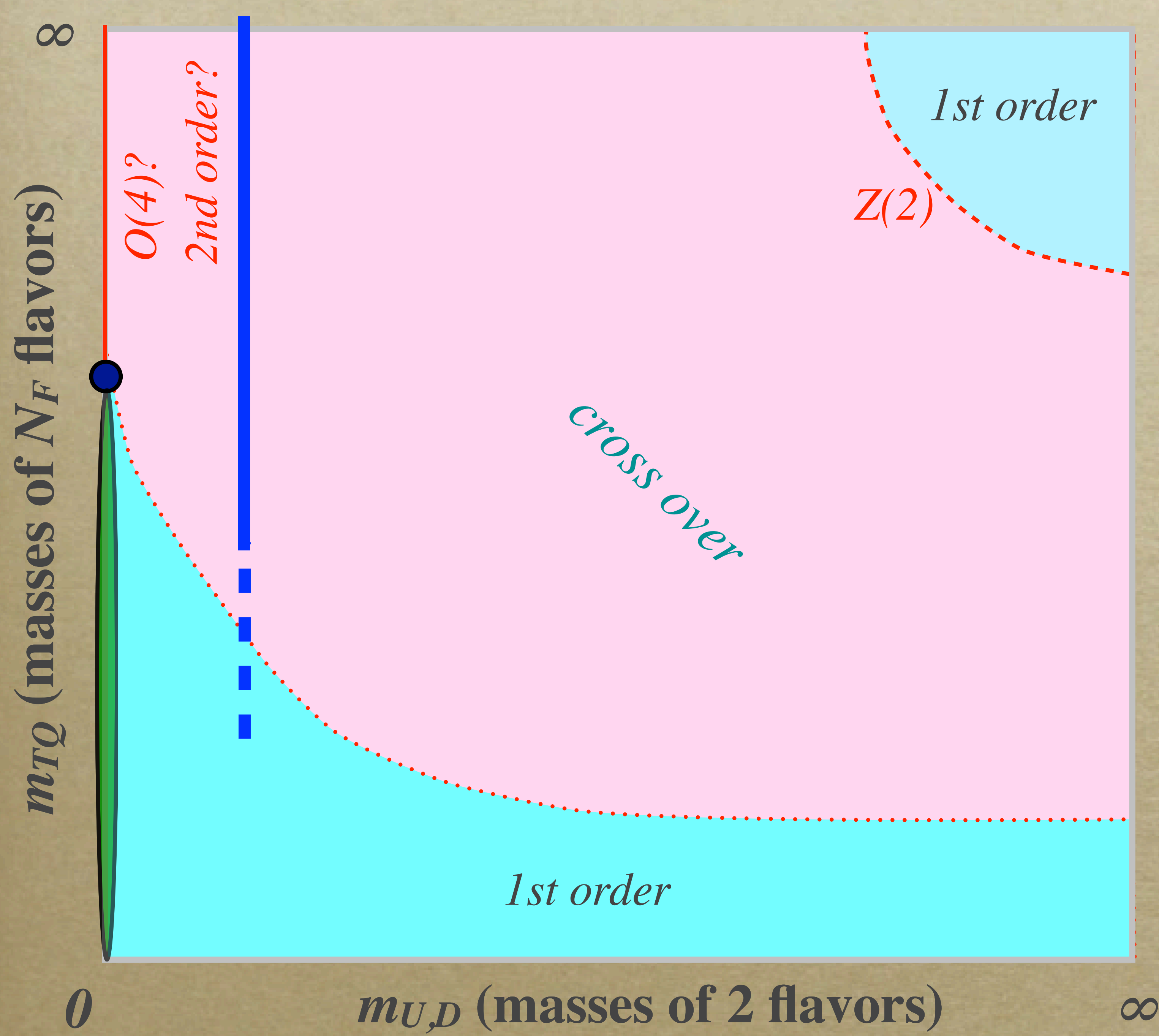

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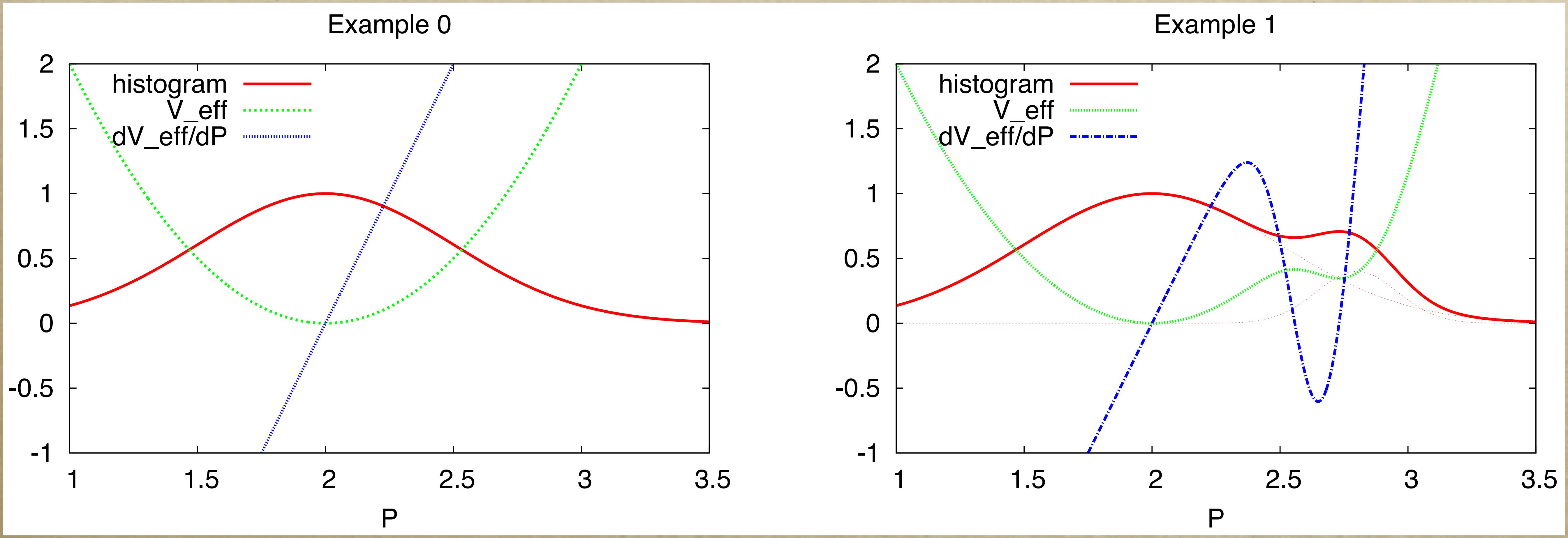
Effective potential through histogram

S. Ejiri, PRD77, 014508 (2008);Saito et al, [WHOT-QCD], PRD84, 054502 (2011)



$$
\begin{aligned}
w(P,\beta) &= \langle \delta(P-\hat{P}) \rangle_\beta \\
V_{\text{eff}}(P,\beta) &= -\ln w(P,\beta) \\
&= -\ln w(P,\beta_0) + 6(\beta-\beta_0) N_{\text{site}} P
\end{aligned}
$$

Double well potential

=1st order PT

⇒look for $\partial^2 V_{\text{eff}}/\partial P^2 \leq 0$

Re-weighting

- Start with the partition function with 2+N_f-flavor QCD.

$$Z(\beta, m_f, \mu_f) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_q - S_g} = \int \mathcal{D}U\; e^{6\beta N_{\rm site}\hat{P}} \prod_{f=1}^{N_{\rm f}+2} (\det M(m_f, \mu_f)).$$

$$\prod_{f=1}^{N_{\rm f}+2} (\det M(m_f, \mu_f)) \to (\det M(m_{\rm l}, 0))^2 \left(\frac{\det M(m_{\rm l}, \mu)}{\det M(m_{\rm l}, 0)}\right)^2 \prod_{h=1}^{N_{\rm f}} \left(\frac{\det M(m_{\rm h}, 0)}{\det M(\infty, 0)}\right)$$

- N_f flavors are heavy and the chemical potential for *u* and *d* quarks are small.

$$\ln\left[\frac{\det M(m_{\rm l}, \mu)}{\det M(m_{\rm l}, 0)}\right] = \sum_{n=1}^{N_\mu} \frac{1}{n!}\left[\frac{\partial^n (\ln\det M)}{\partial(\mu/T)^n}\right]\left(\frac{\mu}{T}\right)^n$$

$$\ln\left[\frac{\det M(\kappa_{\rm h}, 0)}{\det M(0, 0)}\right] = 288 N_{\rm site}\kappa_{\rm h}^4 P + 12 N_s^3 (2\kappa_{\rm h})^{N_t}\Omega + \cdots$$

M:quark matrix
P :plaquette
Ω:Re[Polyakov loop]

Re-weighting

Expectation value in 2+N_f-flavor QCD and its m_h-dependence can be calculated by ensemble average over two-flavor QCD configurations upto truncation errors.

$$V_\mathrm{eff}(P,\beta,m_\mathrm{l},m_\mathrm{h},\mu) \;=\; -\ln w^{(2+N_\mathrm{f})}(P,\beta,m_\mathrm{l},m_\mathrm{h},\mu)$$

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$$V_\mathrm{eff}(P,\beta,m_\mathrm{h},\mu) = V_0(P,\beta_0) - \ln R(P;\beta,m_\mathrm{l},m_\mathrm{h},\mu;\beta_0)$$

Two-flavor part
Always single well

Effect of heavy flavors

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Effect of heavy flavors

$$\ln R(P;\beta,m_{\rm h},\mu;\beta_0) = 6(\beta-\beta_0)N_{\rm site}P + \ln \frac{\left\langle \delta(P-\hat{P}) \left(\frac{\det M(m_{\rm l},\mu)}{\det M(m_{\rm l},0)}\right)^2 \prod_{h=1}^{N_{\rm f}} \left(\frac{\det M(m_{\rm h},0)}{\det M(\infty,0)}\right) \right\rangle_{\rm two-flavors,\beta}}{\left\langle \delta(P-\hat{P}) \right\rangle_{\rm two-flavors,\beta}}$$

Heavy quark mass and N_f dependence

$$\ln R(P;\beta,\kappa_{\rm h},0;\beta_0) = \ln \bar{R}(P;\kappa_{\rm h},0) + (\text{plaquette term}) + O(\kappa_{\rm h}^{N_t+2})$$

$$\bar{R}(P;\kappa_{\rm h},0) = \frac{\left\langle \delta(P-\hat{P}) \exp[6hN_s^3\Omega] \right\rangle_\beta}{\langle \delta(P'-P) \rangle_\beta}$$

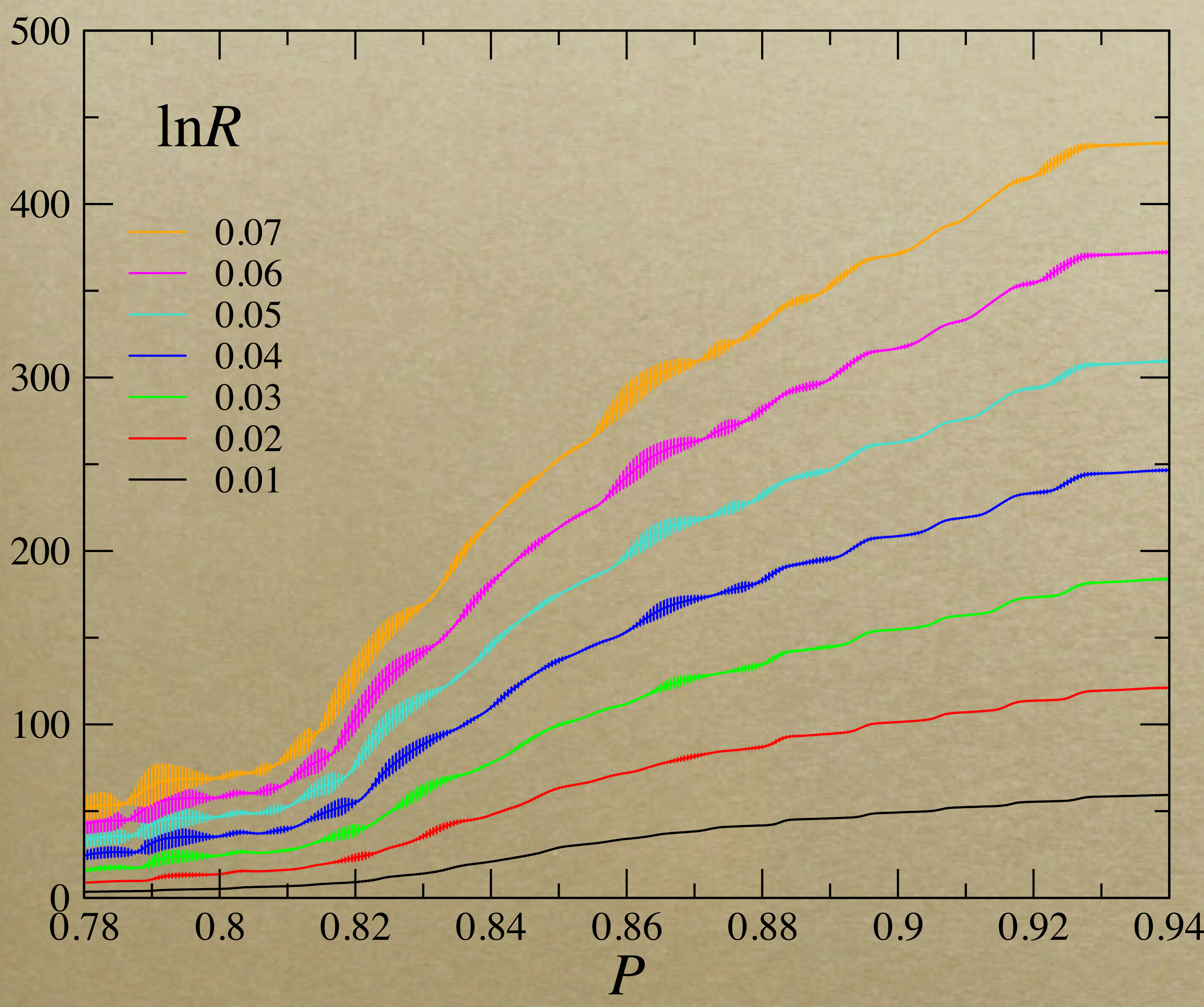
where

$h = 2N_{\rm f}(2\kappa_{\rm h})^{N_t}$ for the Wilson quark action

$h = N_{\rm f}/(4 \times (2m_{\rm h})^{N_t})$ for the staggered quark action

Heavy quark dependence and N_f-dependence are parameterized in a single parameter h.

Result: $\ln R$



Simulation Parameters:
N_f=2,
p4-improved staggered quark
the standard plaquette gauge
$a\, m_l = 0.1$,
10,000-40,000 trajs.
$V=16^3\times 4$,
β = [3.52, 4.00] (16 values),
T/T_c = [0.76,1.98],
$M_{PS}/M_V \sim 0.7$
[C.R. Allton, et al., PRD71,054508 (2005)]

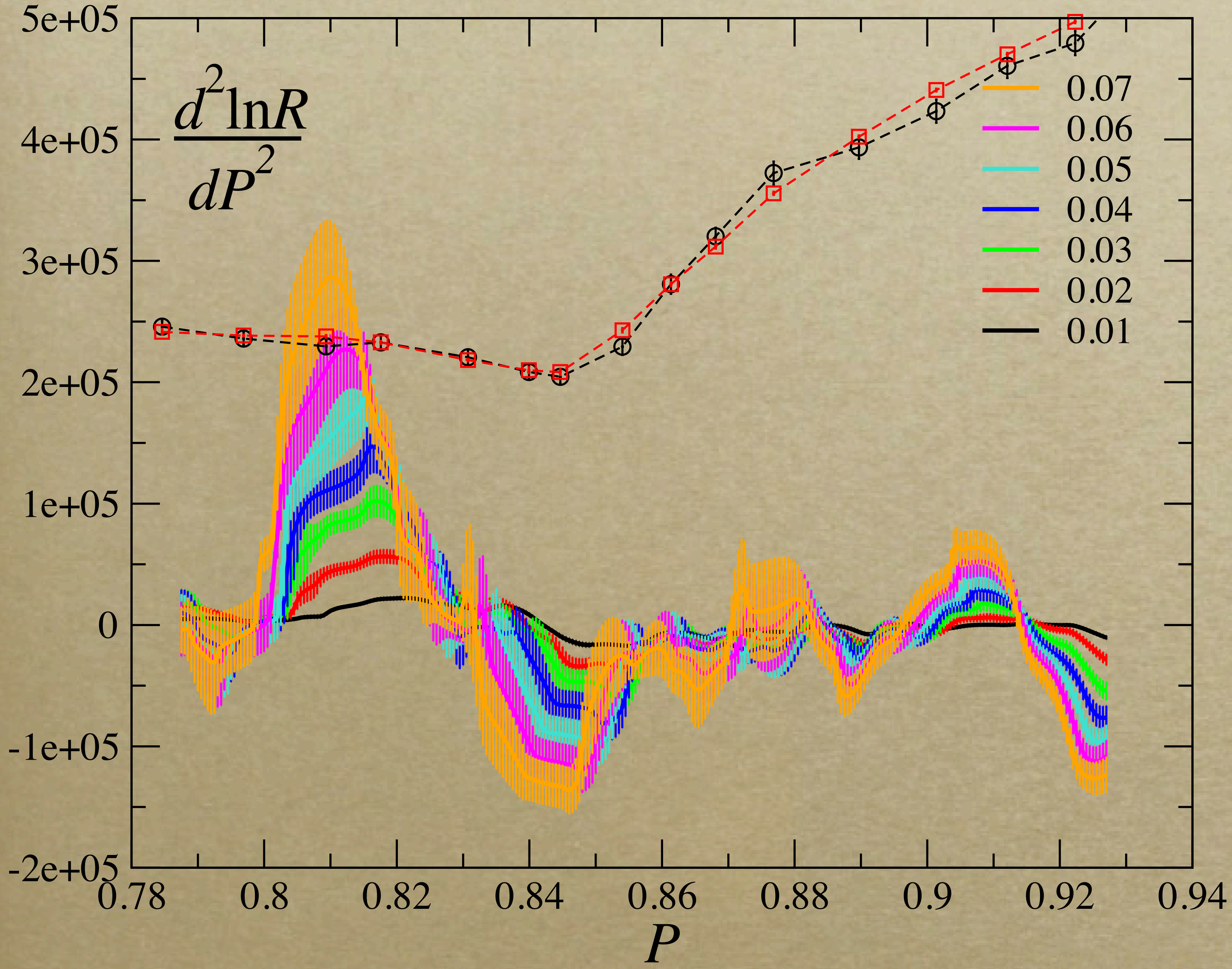
Calculated with h = [0.01, 0.07]

- ln$R$ increases with h.
- Rapid increase@P~0.81

⇒large curvature

Result:

$\partial^2 V_{\text{eff}}/\partial P^2 = \partial^2 V_0/\partial P^2 - \partial^2(\ln R)/\partial P^2$



1st term is calculate in two different ways (black and red).
Maximum of the 2nd term exceeds 1st term at P~0.81 for $h \gtrsim 0.06$.
There $\partial^2 V_{\text{eff}}/\partial P^2$ is negative.
⇒ **1st order phase transition**

$h_c = 0.0614(69)$

Comments

- $h_c = 0.0614(69) = 2\, N_\mathrm{f}\, (2K_\mathrm{hc})^{Nt}$
- Critical kappa K_hc becomes small as N_f increases.
- For example, for $N_\mathrm{f} = 10$, $K_\mathrm{hc} \sim 0.118$.
- Convergence of the HPE is studied in quenched theory in [Ejiri et al. in preparation], and found to be (LO)~(NLO) @$K_\mathrm{h} \sim 0.18$.
- Study of phase structure at finite density can be done in the same footing. [S. Ejiri, PRD77, 014508 (2008)]
- We found that a finite μ also makes K_hc small.

Summary and outlooks

- ✓ In general, QCD with many flavors is computationally demanding.
- ✓ We proposed an easy method to explore the phase structure of MFQ, and determined the critical kappa (⇒upper limit for heavy flavors mass).
- ✓ Future works:
  - Quantify the strength of 1st order transition
  - Check the universal power behavior at (or existence of) the tri-critical point