Variational Study of a Nuclear Equation of State for Core-Collapse Supernovae

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1. Introduction

The aim of this study is

To construct *a new nuclear Equation of State (EOS)*

for supernova (SN) simulations

based on the realistic nuclear force.

The nuclear EOS plays an important role for astrophysical studies.

1. Lattimer-Swesty EOS : *The compressible liquid drop model* (NPA 535 (1991) 331)
2. Shen EOS : *The relativistic mean field theory* (NPA 637 (1998) 435)

- K. Nakazato EOS : (PRD 77(2008) 103006)
- G. Shen EOS : (PRC 83 (2010) 015806)
- C. Ishizuka EOS : (J. Phys. G 35(2008)085201)
- M. Hempel EOS : (NPA 837 (2010) 210)
- S. Furusawa EOS : (APJ 738 (2011) 178)

These EOSs are based on **phenomenological models** for uniform matter.

*There is **no** nuclear EOS based on **the microscopic many-body theory**.*

We aim at **a new EOS for SN** with the variational method.

Our Plan to Construct the EOS for SN Simulations

Uniform Nuclear Matter

EOS constructed with *the cluster variational method*

CLEAR

Non-uniform Nuclear Matter

EOS constructed with *the Thomas-Fermi (TF) calculation*

1. EOS for non-uniform matter at **zero temperature**

★We are here.★

2. EOS for non-uniform matter at **finite temperature**

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Completion of a Nuclear EOS table for SN simulations

Density ρ : $10^{5.1} \leq \rho_m \leq 10^{16.0}$g/cm^3	110 point
Temperature T : $0 \leq T \leq 400$ MeV	92 point
Proton fraction x : $0 \leq x \leq 0.65$	66 point

2. EOS for Uniform Nuclear Matter

The Nuclear Hamiltonian

$$H = H_2 + H_3$$

Two-body Hamiltonian

$$H_2 = -\sum_{i=1}^{N} \frac{\hbar^2}{2m}\nabla_i^2 + \sum_{i<j}^{N} V_{ij}$$

the AV18 two-body nuclear potential

Three-body Hamiltonian

$$H_3 = \sum_{i<j<k}^{N} V_{ijk}$$

the UIX three-body nuclear potential

We assume the Jastrow wave function.

$$\Psi = \mathrm{Sym}\left[\prod_{i<j} f_{ij}\right]\Phi_{\mathrm{F}}$$

Φ_{F}: The Fermi-gas wave function at zero temperature

P_{ts}^{μ}: Spin-isospin projection operators

f_{ij} : Correlation function

$$f_{ij} = \sum_{t=0}^{1}\sum_{\mu}\sum_{s=0}^{1}\left[\underbrace{f_{\mathrm{C}ts}^{\mu}(r_{ij})}_{\text{Central}} + s\underbrace{f_{\mathrm{T}t}^{\mu}(r_{ij})S_{\mathrm{T}ij}}_{\text{Tensor}} + s\underbrace{f_{\mathrm{SO}t}^{\mu}(r_{ij})(\boldsymbol{L}_{ij}\cdot\boldsymbol{s})}_{\text{Spin-orbit}}\right]P_{tsij}^{\mu}$$

Two-Body Energy

E_2/N is the expectation value of H_2 with the Jastrow wave function in *the two-body cluster approximation*.

$$\frac{E_2}{N}(\rho, x) = \frac{< H_2 >_2}{N}$$

ρ : Total nucleon number density

ρ_p : Proton number density $x = \rho_\mathrm{p}/\rho$: Proton fraction

E_2/N is minimized with respect to $f_{\mathrm{C}ts}{}^{\mu}(r)$, $f_{\mathrm{T}t}{}^{\mu}(r)$ and $f_{\mathrm{SO}t}{}^{\mu}(r)$ with the following two constraints.

1. Extended Mayer's condition

$$\rho \int \left[F_{ts}^{\mu}(r) - F_{\mathrm{F}ts}^{\mu}(r) \right] d\boldsymbol{r} = 0$$

$F_{ts}{}^{\mu}(r)$: Radial distribution functions

$F_{\mathrm{F}ts}{}^{\mu}(r)$: $F_{ts}{}^{\mu}(r)$ for the degenerate Fermi gas

2. Healing distance condition

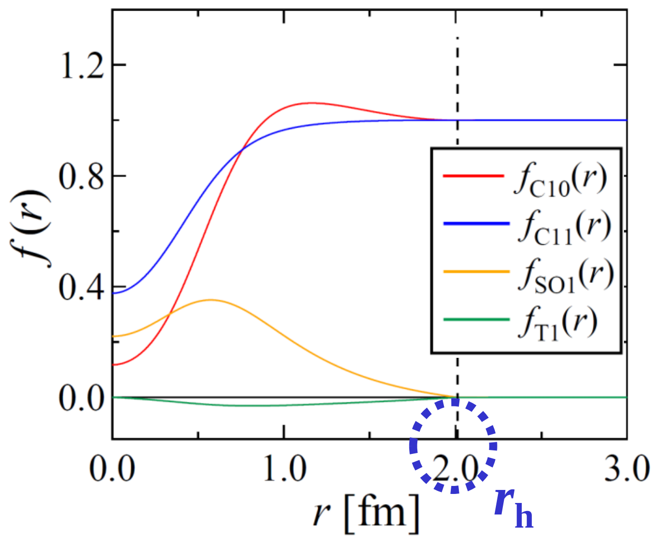
Healing distance

$$r_\mathrm{h} = a_\mathrm{h} r_0$$

a_h : adjustable parameter

Mean distance between nucleons

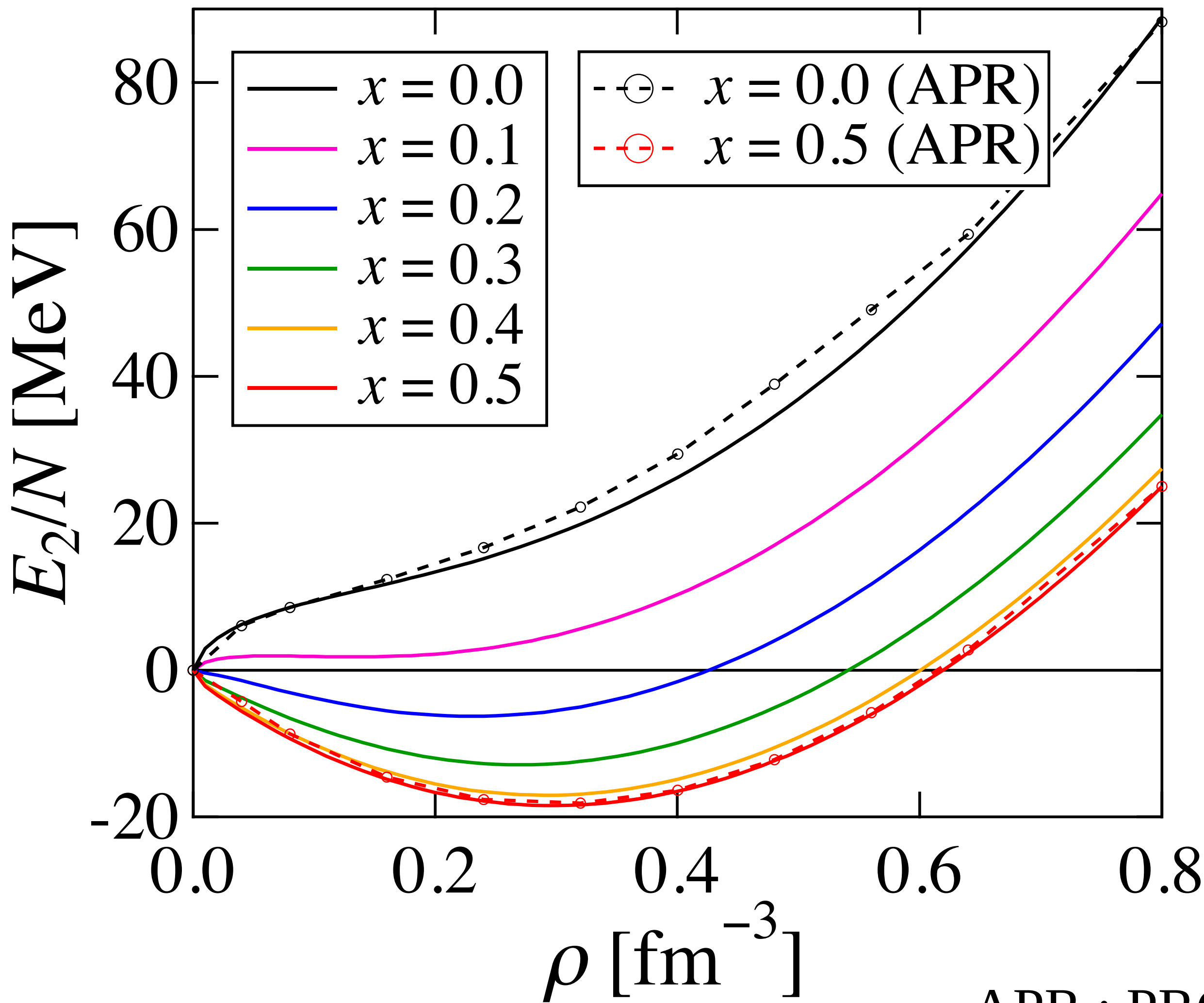
$$r_0 = \left(\frac{3}{4\pi\rho} \right)^{1/3}$$



a_h is determined so that E_2/N reproduces the results by APR(Akmal, Pandharipande and Ravenhall)

APR : PRC58(1998)1804

Two Body Energy



$a_h = 1.76$

APR : PRC58(1998)1804

Our results are in good agreement with the results by APR (FHNC method).

Three-Body Energy

UIX potential

$$V_{ijk} = V_{ijk}^{2\pi} + V_{ijk}^{R}$$

$V_{ijk}^{2\pi}$:2-pion exchange part

V_{ijk}^{R} :Repulsive part

Three Body Energy

$$\frac{E_3}{N}(\rho, x) = \frac{1}{N}\langle \alpha \sum_{i<j<k}^{N} V_{ijk}^{\mathrm{R}} + \beta \sum_{i<j<k}^{N} V_{ijk}^{2\pi} \rangle_{\mathrm{F}} + \gamma \rho^2 e^{-\delta\rho}\left[1 - (1-2x)^2\right]$$

Correction term

Expectation value with the Fermi-gas wave function

$\alpha, \beta, \gamma, \delta$: adjustable parameters

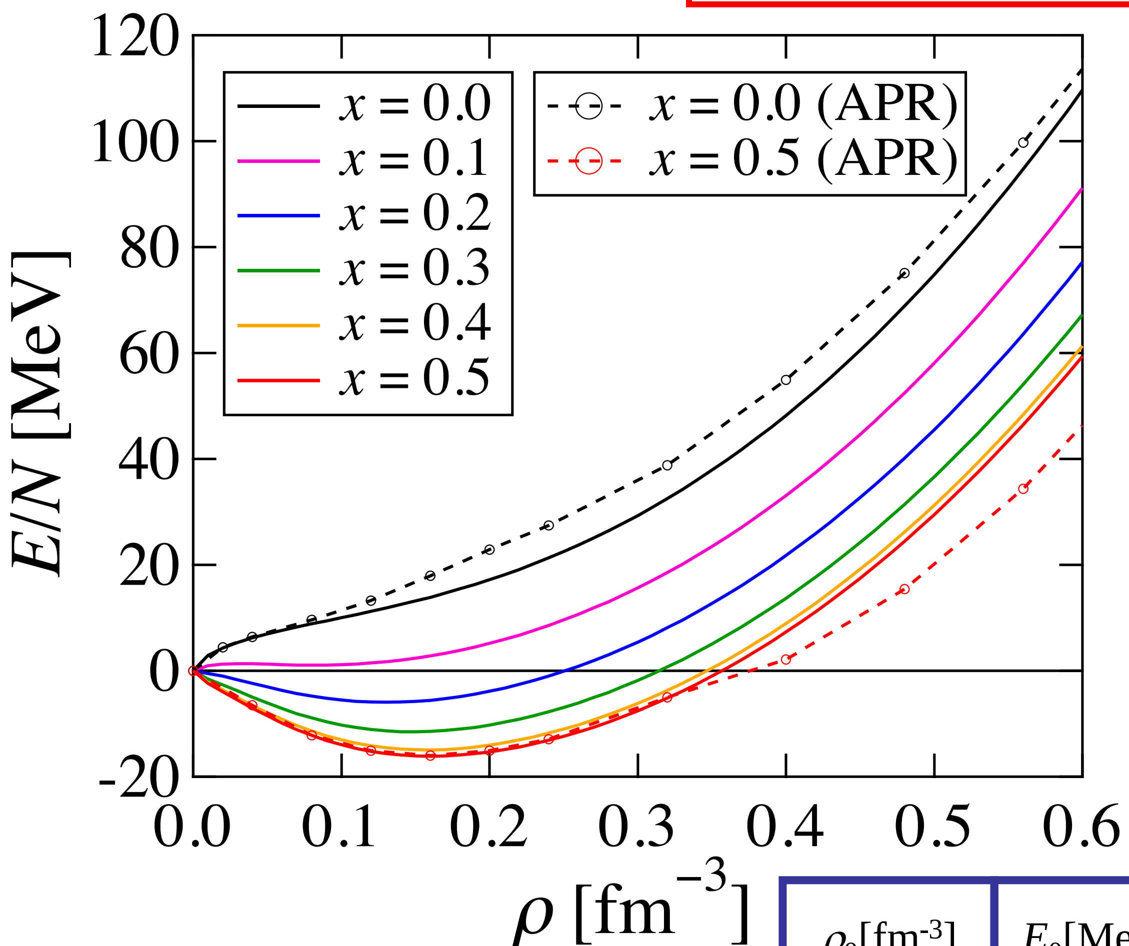
Parameters of E_3/N are determined so as to reproduce the empirical data.

TF calculation for atomic nuclei reproduces the gross feature of the experimental data.

Total Energy per Nucleon at Zero Temperature

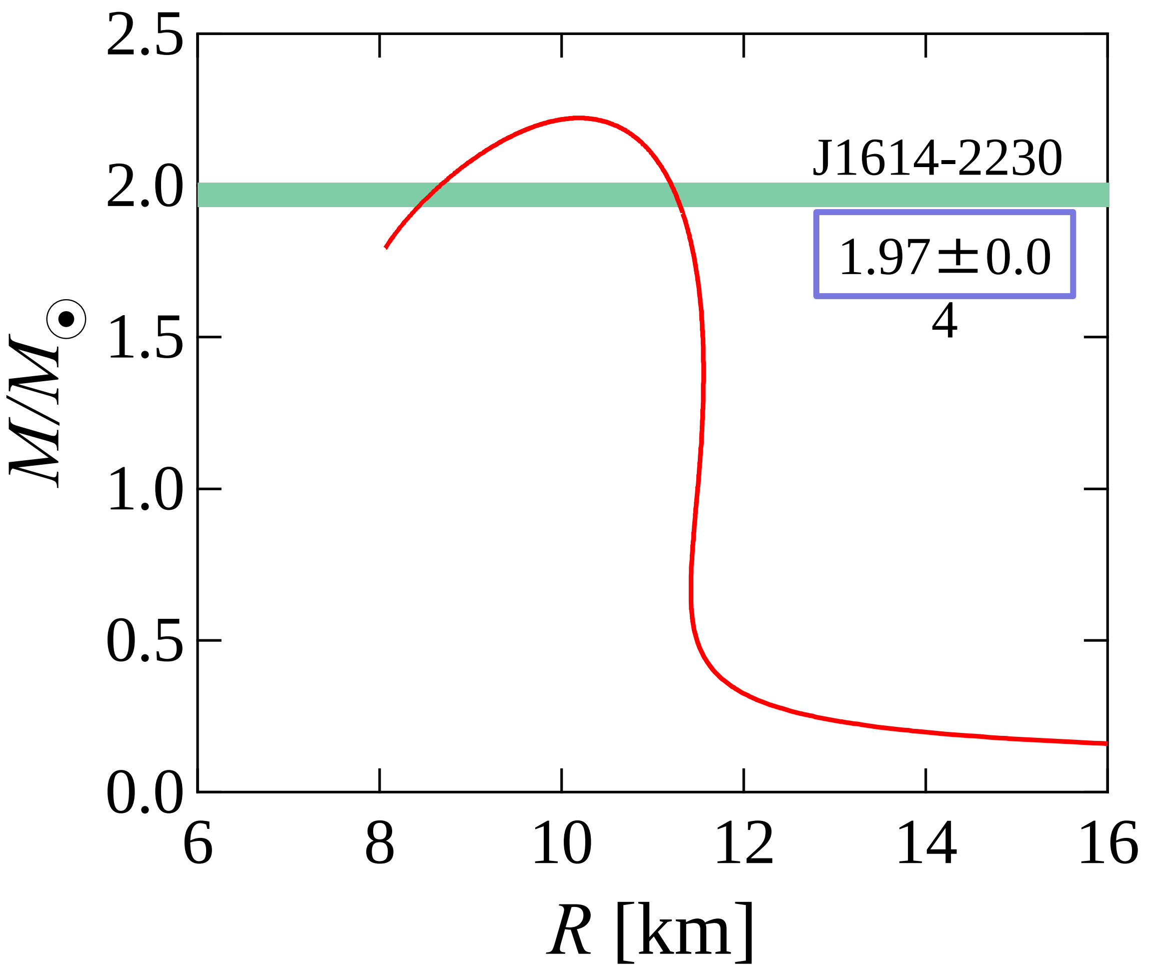
Total energy per nucleon

$$\frac{E}{N} = \frac{E_2}{N} + \frac{E_3}{N}$$



ρ_0[fm^{-3}]	E_0[MeV]	K[MeV]	E_{sym}[MeV]
0.16	–16.1	240	30.0

Application to Neutron Star



The NS mass-radius relation is consistent with observational data .

Free Energy at Finite Temperatures I

We follow the prescription proposed by *Schmidt and Pandharipande.*

(Phys. Lett. 87B(1979) 11) (A. Mukherjee et al., PRC 75(2007) 035802)

Free Energy

$$\frac{F}{N} = \frac{E_{0\mathrm{T}}}{N} - T\frac{S_0}{N}$$

$\frac{E_{0\mathrm{T}}}{N}$: Approximate Internal Energy

$\frac{S_0}{N}$: Approximate Entropy

S_0/N is expressed with the averaged occupation probabilities $n_i(k)$

Approximate Internal Energy

$$\frac{E_{0\mathrm{T}}}{N} = \frac{E_{2\mathrm{T}}}{N} + \frac{E_{3\mathrm{T}}}{N}$$

chosen to be the same as at 0 MeV

$E_{2\mathrm{T}}/N$:Two-body internal energy at finite temperatures

$$E_2/N[f_{ij}, n_{T=0}(k)] \longrightarrow E_{2\mathrm{T}}/N[f_{ij}, n(k)]$$

Correlation function f_{ij} is chosen to be the same at 0MeV.

Frozen-Correlation Approximation

Free Energy at Finite Temperatures II

The averaged occupation probability

$$n_i(k) = \left\{ 1 + \exp\left[\frac{\varepsilon_i(k) - \mu_i}{k_\mathrm{B} T} \right] \right\}^{-1} \qquad (i = \mathrm{p, n})$$

μ_i is determined with the normalization condition.

$\varepsilon_i(k)$: Single particle energy

$$\varepsilon_i(k) = \frac{\hbar^2 k^2}{2m_i^*}$$

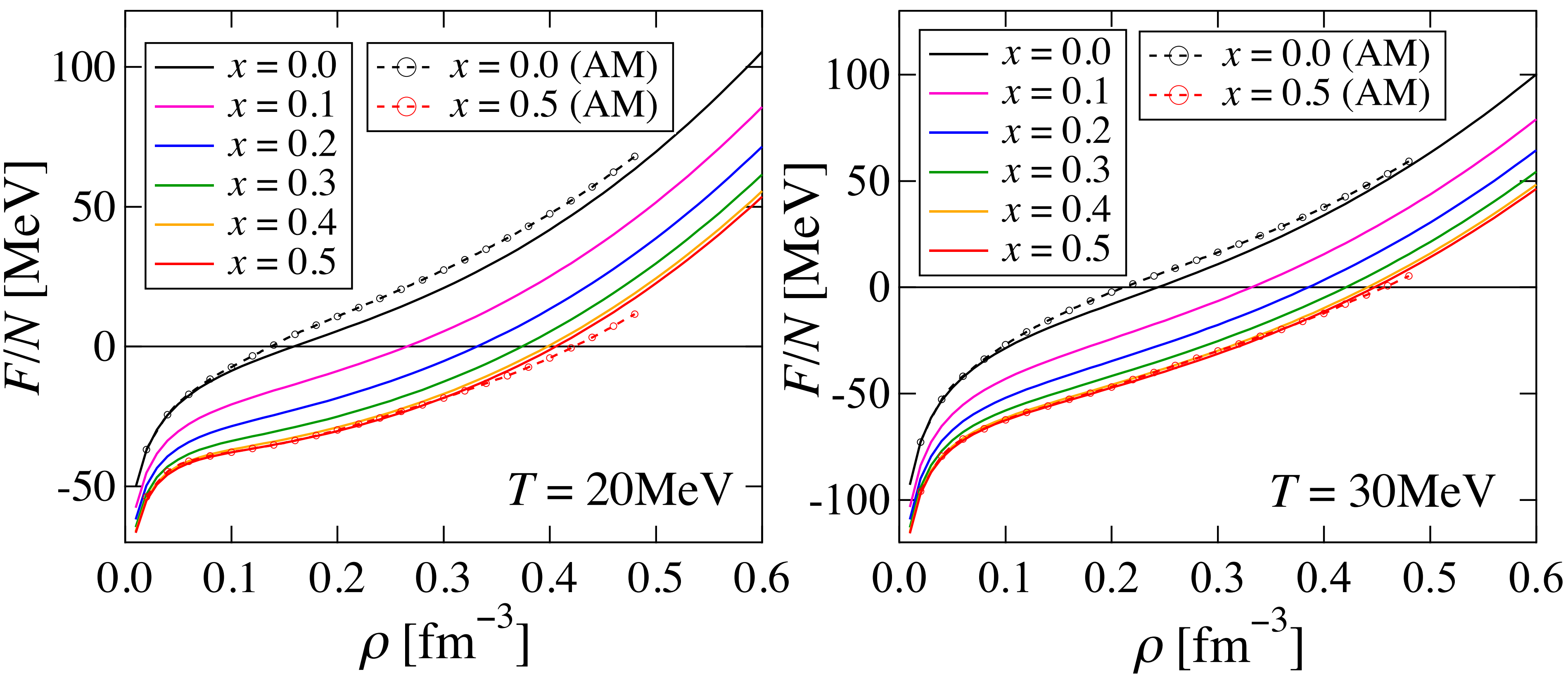
m_i*: Effective mass of nucleons

Approximate Entropy

$$\frac{S_0}{N} = -\frac{k_\mathrm{B}}{N} \sum_{i=\mathrm{p,n}} \sum_{\mathrm{spin}} \sum_{k} \left\{ [1 - n_i(k)] \ln [1 - n_i(k)] + n_i(k) \ln n_i(k) \right\}$$

Free energies are minimized with respect to m_p* and m_n*

Free Energy per Nucleon at Finite Temperatures

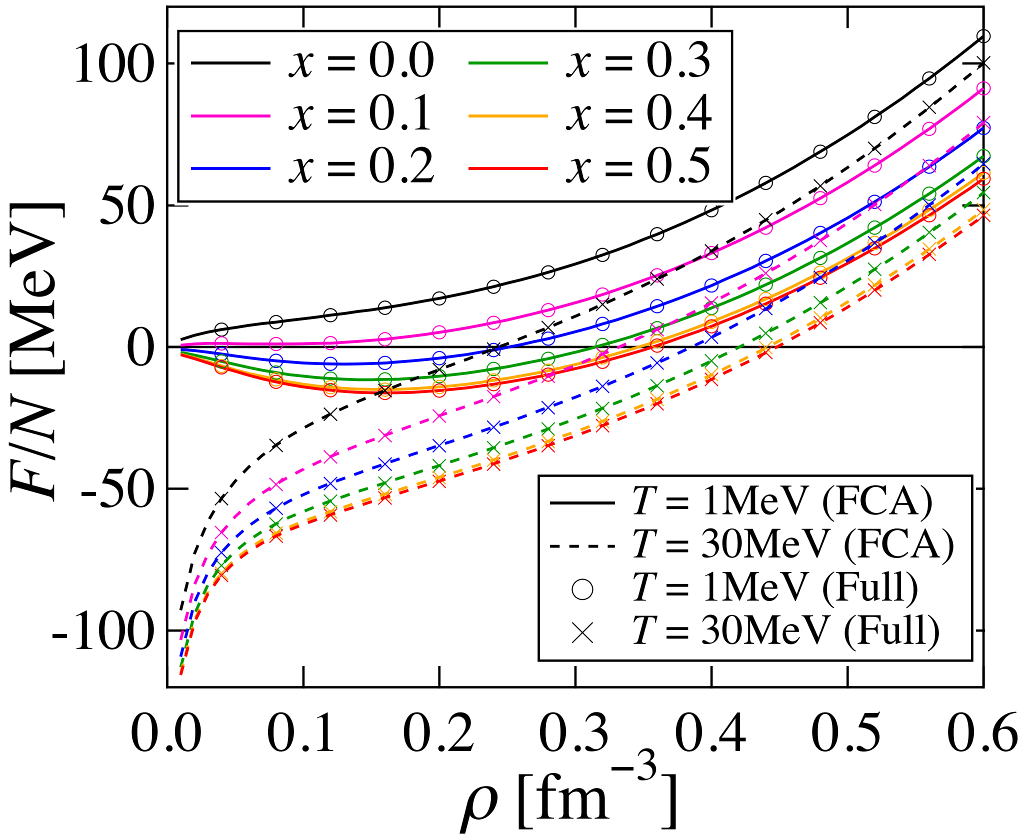


Free energy per nucleon at T=20MeV

Free energy per nucleon at T=30MeV

AM : A. Mukherjee, PRC 79(2009) 045811

Validity of the Frozen-Correlation Approximation

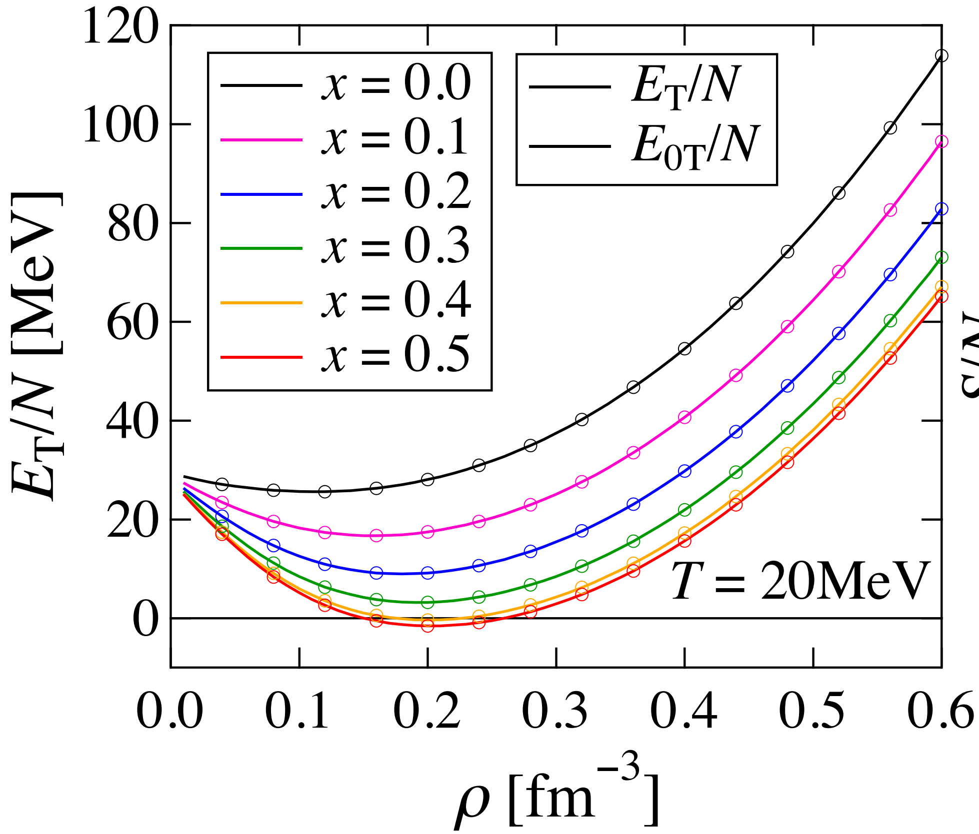


Full minimization

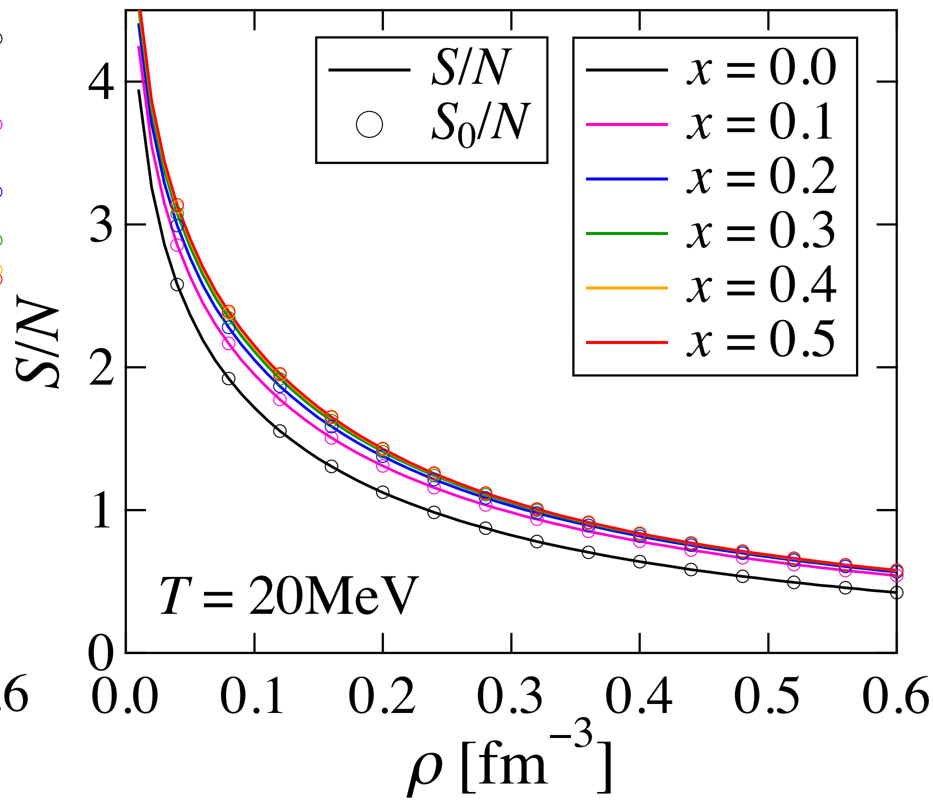
F/N is minimized with respect to $f_{Cts}{}^{\mu}(r)$, $f_{Tt}{}^{\mu}(r)$, $f_{SOt}{}^{\mu}(r)$ and m_i*
with
1.Extended Mayer's condition
and
2. Healing distance condition.

The free energies with the frozen-correlation approximation are in good agreement with those with the full minimization.

Internal Energy and Entropy



Internal energy at T=20MeV

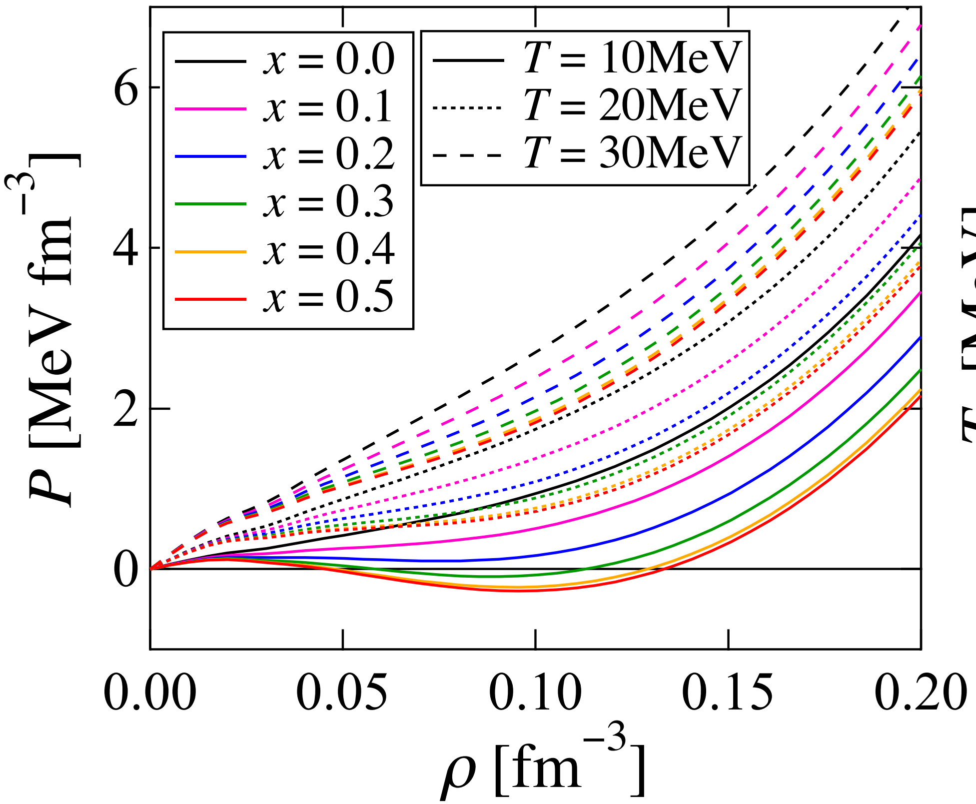


Entropy at T=20MeV

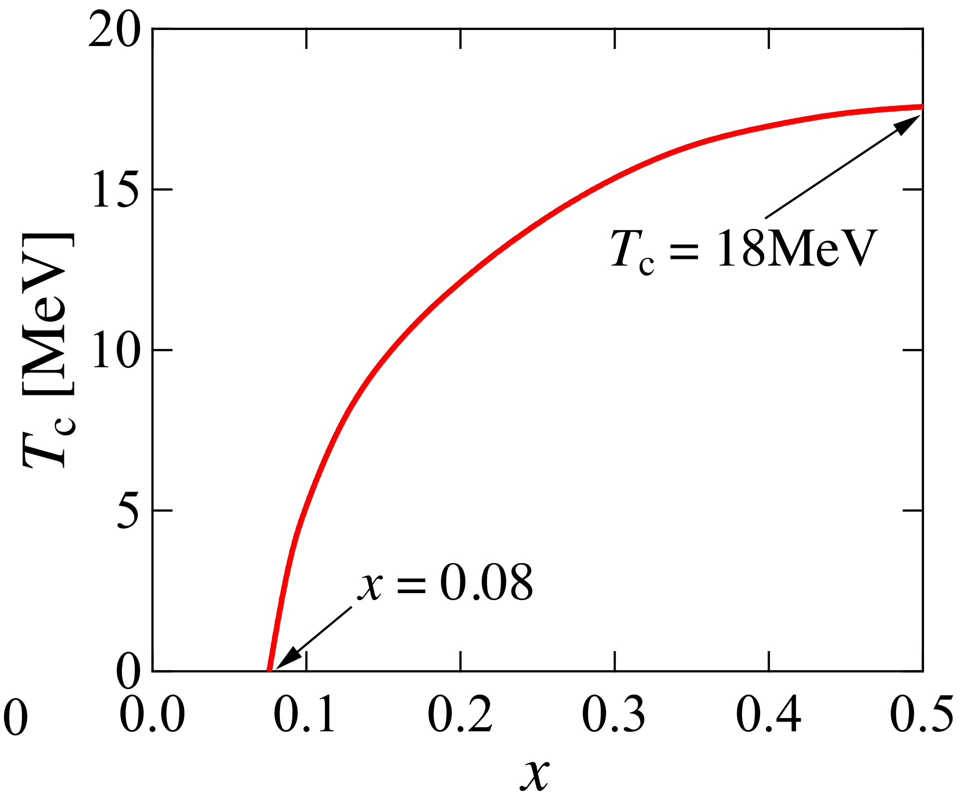
Entropies are in good agreement with the approximate entropies.

This variational calculation is Self Consistent.

Pressure and Critical Temperature



Pressure



Critical temperature

Critical Temperature T_C is defined by

$$\left.\frac{\partial P}{\partial \rho}\right|_{x,T=T_C} = \left.\frac{\partial^2 P}{\partial \rho^2}\right|_{x,T=T_C} = 0$$

3. EOS for Non-uniform Nuclear Matter

We follow the TF method by Shen et. al. (NPA637(1998)435)

Free energy in the Wigner-Seitz (WS) cell

Bulk energy **Gradient energy**

$$F = \int d\boldsymbol{r} f(n_\mathrm{p}(r), n_\mathrm{n}(r)) + F_0 \int d\boldsymbol{r} |\nabla n(r)|^2$$
$$+ \frac{e^2}{2} \int d\boldsymbol{r} \int d\boldsymbol{r}' \frac{[n_\mathrm{p}(r) - n_\mathrm{e}][n_\mathrm{p}(r') - n_\mathrm{e}]}{|\boldsymbol{r} - \boldsymbol{r}'|} + c_\mathrm{bcc} \frac{(Ze)^2}{a}$$

Coulomb energy

$F_0 = 68.00$ MeV fm^5

Nucleon density distribution

$$n_i(r) = \begin{cases} n_i^\mathrm{in}[1 - (r/R_i)^{t_i}]^3 + n_i^\mathrm{out} & (0 \le r \le R_i) \\ n_i^\mathrm{out} & (R_i \le r \le R_\mathrm{cell}) \end{cases} \quad (i = \mathrm{p, n})$$

a : Lattice constant

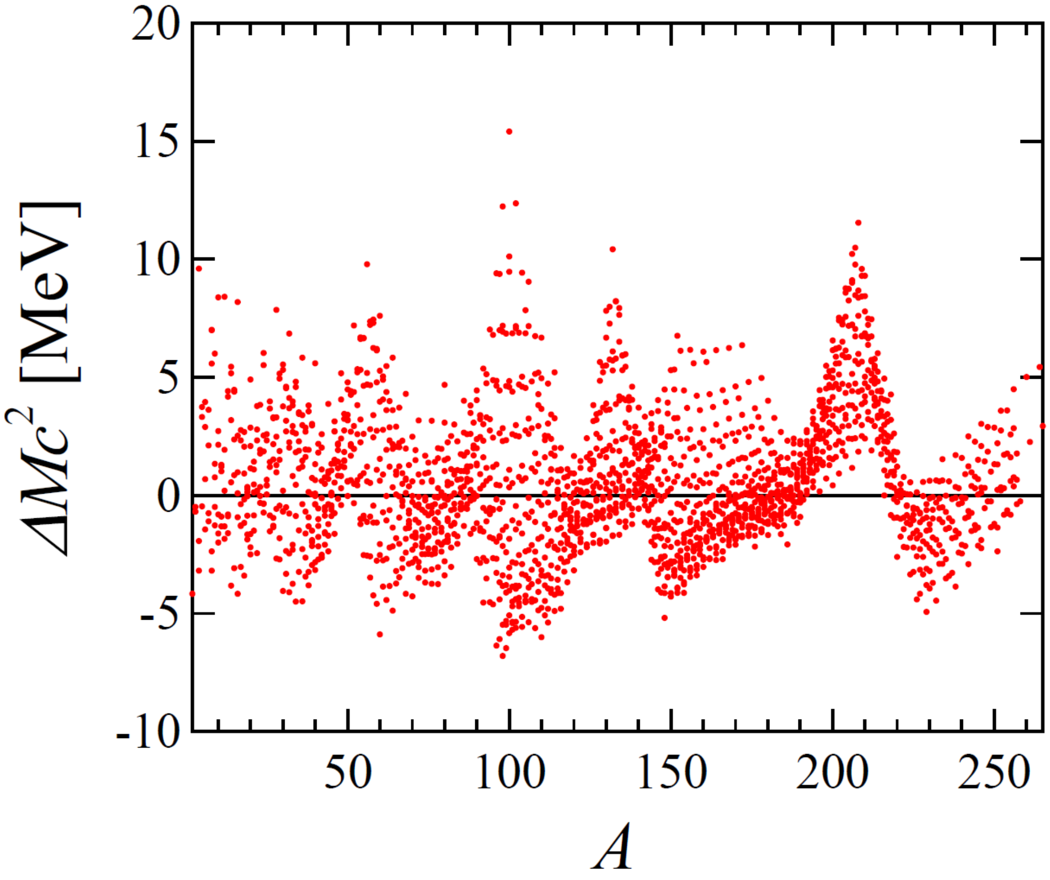
$$V_\mathrm{cell} = \frac{4\pi R_\mathrm{cell}^3}{3} = a^3$$

f **: Free energy density of uniform nuclear matter**

Parameter	Minimum	Maximum	Number
$\log_{10}(T)$ [MeV]	–1.24	1.40	23 + 1
x	0.0	0.5	213
ρ [fm^{-3}]	0.000001	0.18	1980

24 × 213 × 1980
= 10121760 points

TF Calculation for Atomic Nuclei



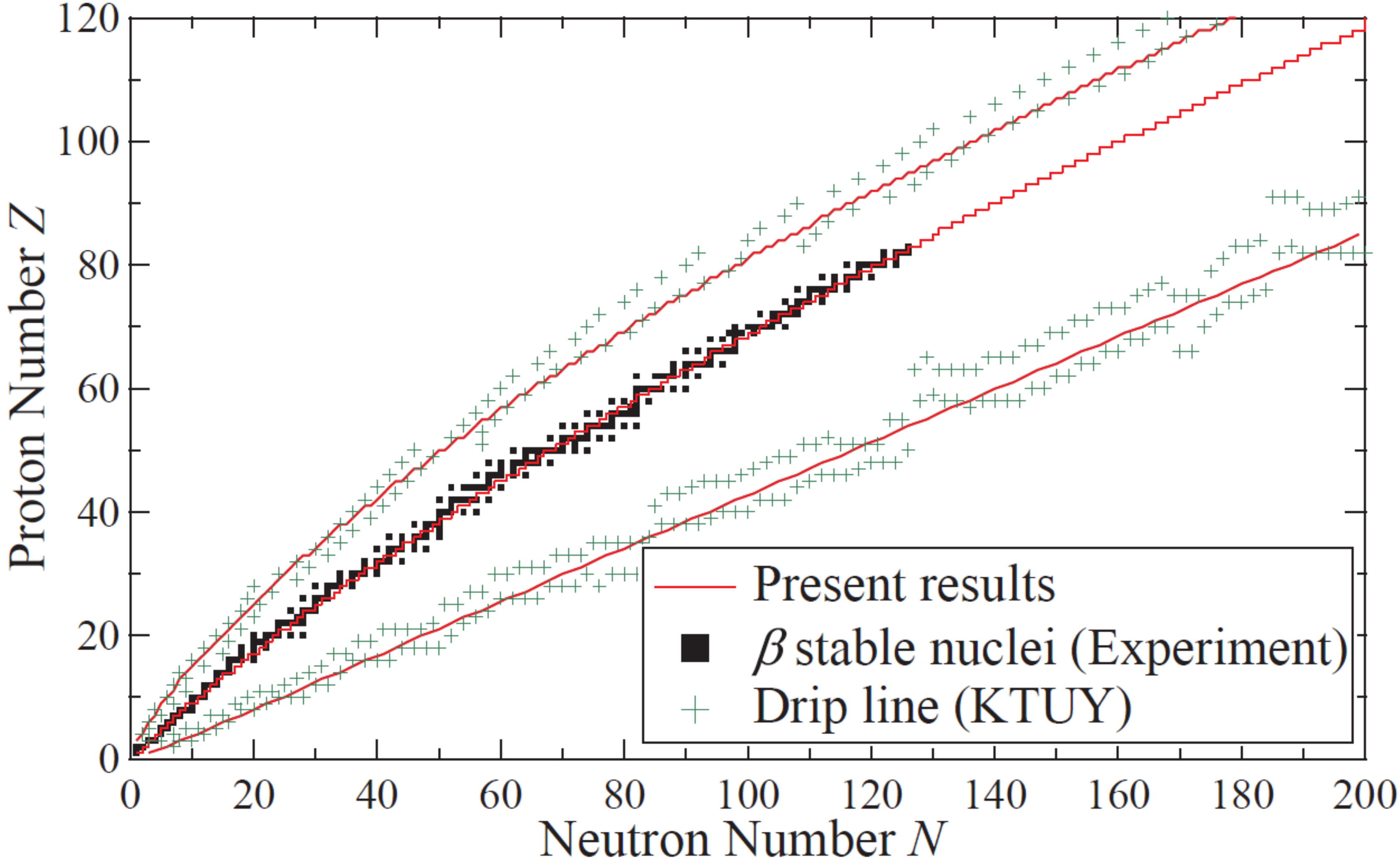
$\triangle M = M_{TF} - M_{exp}$

M_{TF} : Mass by the Thomas-Fermi calculation

M_{exp} : Experimental data

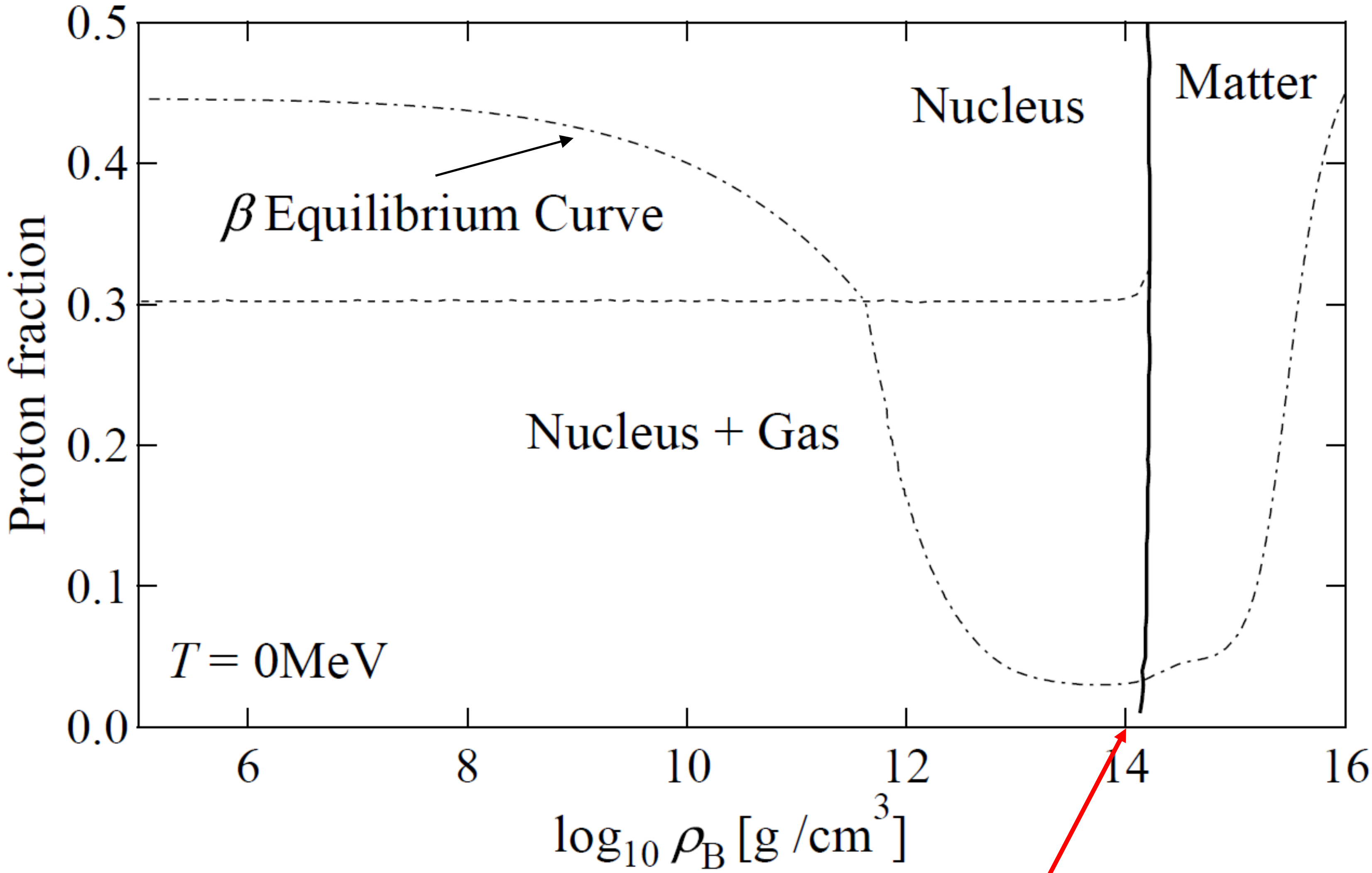
RMS deviation (for 2226 nuclei) 2.99 MeV

TF Calculation for Atomic Nuclei



Our results are in good agreement with
the experimental data and the sophisticated atomic mass formula.

TF Calculation for Non-uniform Nuclear Matter

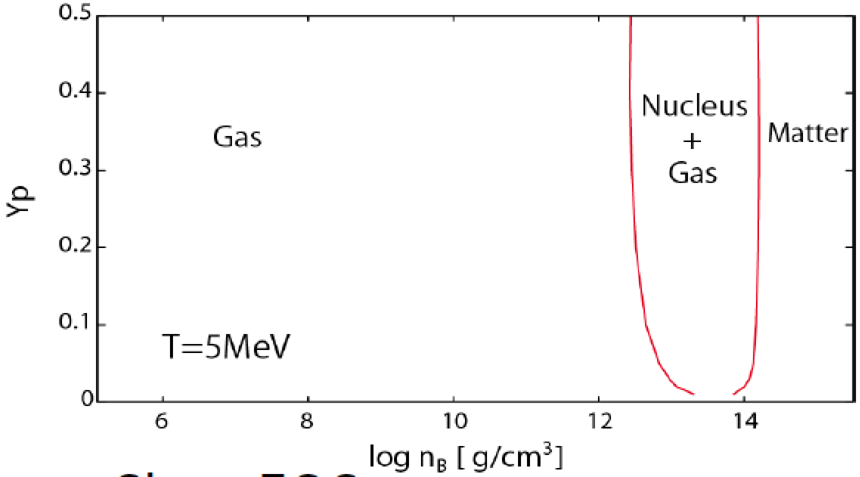


Phase diagram of nuclear matter at T = 0MeV

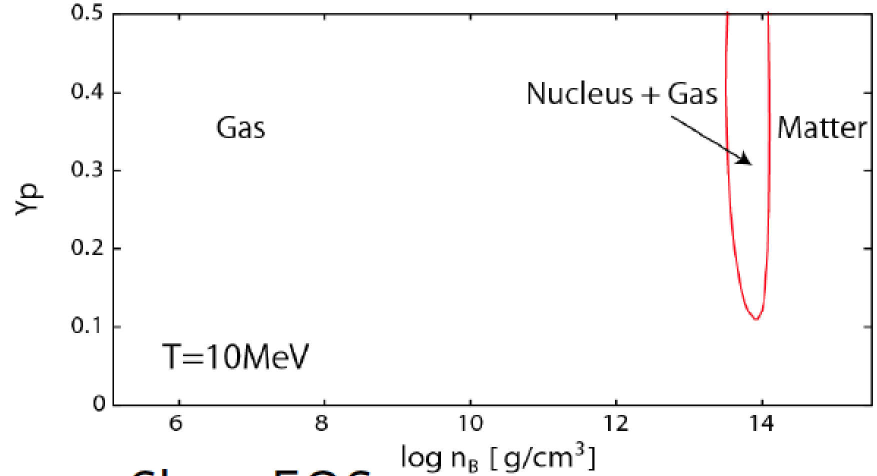
$\rho_B = 10^{14.23}$ g/cm^3

TF Calculation for Non-uniform Nuclear Matter

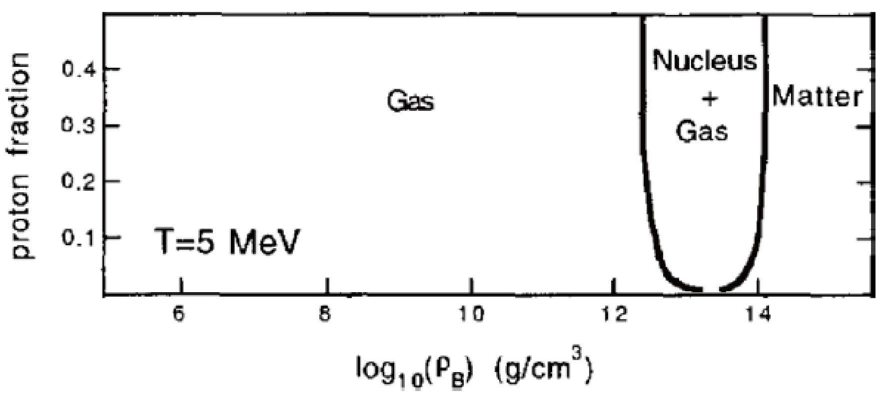
This Work



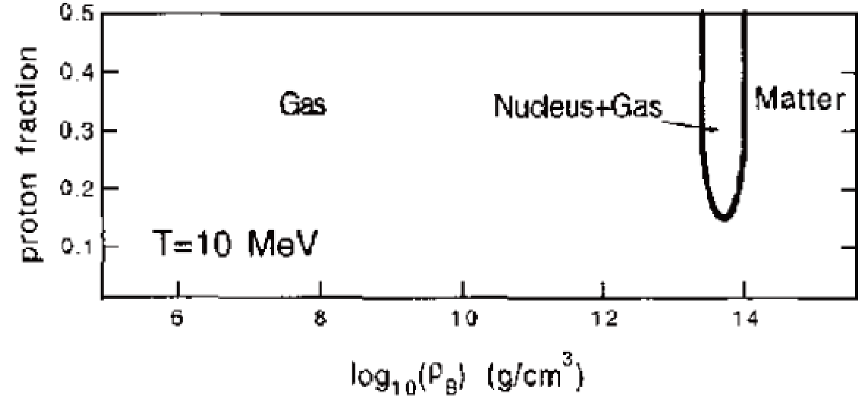
This Work



Shen EOS



Shen EOS



H.Shen et.al NuclPhysA 637 (1998) 435-450

Phase diagrams of nuclear matter at *finite temperatures*

F/V_{cell} is minimized with respect to n_i^{out}, n_i^{in} R_i, t_i, a at given density and proton fraction.

5. Summary

- The EOS for uniform nuclear matter is constructed with the cluster variational method. (zero and finite temperatures)
- The EOS for non-uniform nuclear matter is constructed in the Thomas-Fermi approximation. (zero and finite temperatures)

Uniform nuclear matter

The obtained thermodynamic quantities are reasonable.
This variational calculation is self consistent.
The validity of the frozen-correlation approximation is confirmed.

Non-uniform nuclear matter

RMS deviation for atomic nuclei is 2.99 MeV.
Phase diagrams are reasonable at zero and finite temperatures.

Ongoing Calculations

- Construction of the EOS table for non-uniform matter
- Contribution of the α-particle mixing

Construction of the EOS for supernova simulations