Numerical Relativity Simulations of NS-NS binary merger

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Why NS-NS mergers are interesting?

- Promising source of gravitational wave (GW)
  - Direct detection of GW within 5-10 years by adv. LIGO (USA), adv. VIRGO (ITA/FRA), KAGRA (JPN)

- Laboratory for fundamental physics
  - Verification of GR in strong field regime
  - Physics of dense nuclear matter
    - NS-NS merger as a cosmological collider

- Theoretical candidate of gamma-ray bursts (GRB)
  - Central engine: BH + accretion disk
    - Energy source: neutrino pair annihilation?

General relativistic gravity is important
Highly nonlinear and dynamical

**Numerical Relativity**
Current & up-coming GW detectors

- LIGO
- VIRGO
- GEO
- KAGRA
- LIGO India?
BNS 1.35-1.35Msolar optimal @ 100Mpc

Strain amplitude [Hz^{-1/2}]

- Inspiral charp signal Post Newton
- Initial LIGO
- Merger HMNS formation NR
- KAGRA
- Broadband Adv. LIGO
- Einstein Telescope

f [Hz]
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**Numerical Relativity**
NS structure ⇔ Theoretical model

- For given equation of state, structure of NS is uniquely determined
- Information of NS structure ⇒ constraining EOS model

F. Weber (2005)
For given equation of state, structure of NS is uniquely determined

Information of NS structure $\Rightarrow$ constraining EOS model

F. Weber (2005)

Lattimer & Prakash (2007)
NS structure $\Leftrightarrow$ Theoretical model

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- Information of NS structure $\Rightarrow$ constraining EOS model
Open Question

- Given the theoretical uncertainty, which one is the right one?
- Traditional method to constrain the models
  - **Mass-Radius relation**: Estimation of mass and radius observation of X-ray binary, Large systematic error
  - **Maximum mass**: Just find a massive NS
  - **PSR J1614-2230 (NS-WD)**: NS of 1.97M\(_{\odot}\), Mass measurement by Shapiro time delay, Too soft EOSs are excluded
  - Still we have a number of theoretical models

Lattimer & Prakash (2007)
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**Numerical Relativity**
What is Numerical Relativity?

- Solving Einstein eq. and source field eqs. to clarify dynamical phenomena in the universe where strong gravity plays a role

\[ G_{ab} = \frac{8\pi G}{c^4} T_{ab} \]

\[ \nabla_a T^{ab} = 0 \quad (T^{ab} = (T_{\text{Fluid}} + T_{\text{EM}} + T_{\nu} + \ldots)^{ab}) \]

\[ \nabla_a J^a = 0 \quad (J^a \sim (n_{\text{baryon}}, n_{\text{lepton}}(n_e, n_\nu, \ldots), \ldots)u^a) \]

- All four known interactions play important roles
  - **Gravity**: GR, BH formation, ISCO, etc
  - **Strong**: EOS (equation of state) of dense nuclear/hadronic matter
  - **EM**: MHD phenomena, EOS of dense matter
  - **Weak**: Electron capture, Neutrino production, neutrino pair annihilation
    - 99% gravitational binding energy released is carried away by neutrinos in SNe
NR simulations with a physical modeling is now possible!

- **Einstein’s equations**: Shibata-Nakamura (BSSN) formalism
  - 4\(^{th}\) order finite difference in space, 4\(^{th}\) order Runge-Kutta time evolution
  - Gauge conditions: 1+log slicing, dynamical shift

- **GR Hydrodynamics with neutrinos** (Sekiguchi 2010)
  - Nuclear-theory-based finite temperature EOS table
  - EOM of Neutrinos (leakage scheme, moment scheme)
  - Lepton Conservations
  - Weak Interactions
    - e\(^{\pm}\) captures, pair annihilation, plasmon decay, Bremsstrahlung
  - A detailed neutrino opacities
  - High-resolution-shock-capturing scheme

- **BH excision technique**
- **(Fixed) Mesh refinement technique**

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\[ \nabla_a T_b^a = -Q_b^{(\text{leak)}} \]
\[ \nabla_a T_b^a (\nu, \text{stream}) = Q_b^{(\text{leak)}} \]

\[ \nabla_a (\rho Y_e u^a) = -\gamma_{e \text{- cap}} + \gamma_{e \text{+ cap}} \]
\[ \nabla_a (\rho Y_{\nu e} u^a) = \gamma_{e \text{- cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} + \gamma_{\text{Brems}} - \gamma_{\nu_e \text{leak}} \]
\[ \nabla_a (\rho Y_{\bar{\nu} e} u^a) = \gamma_{e \text{+ cap}} + \gamma_{\text{pair}} + \gamma_{\text{plasmon}} + \gamma_{\text{Brems}} - \gamma_{\bar{\nu}_e \text{leak}} \]
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Sekiguchi (2010) Progress of Theoretical Physics **124**, 331
Neutrino transfer: a preliminary result

- Solving Boltzmann equation (6+1 dims. !) is not feasible at current status
- Approximate solution by Thorne’s Moment scheme with a closure relation
  - Neutrino heating (absorption on proton/neutron) can be treated
  - But some (approximate) treatment is required for $\nu$ annihilation
Evolution of NS-NS Binary

$M_{\text{total}} < M_{\text{crit}} \approx 2.9 - 3.2 M_{\odot}$

More Likely

Hyper Massive NS (HMNS)

$M_{\text{total}} < M_{\text{NS, max}}$

stable

$M_{\text{total}} > M_{\text{NS, max}}$

accretion

$M_{\text{total}} > M_{\text{crit}}$

ringdown

$M_{\text{total}} > M_{\text{crit}}$

GWs, neutrinos

Imre Bartos, GECo, Columbia University (Bartos et al. 2013, in prep.) with permission
Evolution of NS-NS Binary:

- Tidal deformation
- Inspiral
- Stable hyper massive NS (HMNS)
- Accretion

More Likely:

- $M_{\text{total}} < M_{\text{NS, max}}$
- $M_{\text{total}} > M_{\text{NS, max}}$

Hyper Massive NS (HMNS):

- Canonical mass = 1.35-1.4M_{\odot}

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Lattimer & Prakash (2007)

- GWs, neutrinos

Shibata et al. 2005, 2006

White dwarf - neutron star binaries

Main sequence - neutron star binary

Neutron star mass ($M_\odot$)
GW from NS-NS (long lived HMNS)

NS(1.2M\text{solar})-NS(1.5M\text{solar}) binary (APR EOS)

Density Contour

Gravitational Waveform

Inspiral
Charp signal
Tidal
defformation
Merger
HMNS

Animation by Hotokezaka

Sekiguchi et al. PRL (2011a, 2011b)
Kiuchi et al. PRL (2010); Hotokezaka et al. (2011); (2012)
Exploring Dense matter physics by GW

- **Inspiral phase**
  - Point particle approximation
  - Information of orbits, neutron star mass etc.

- **Tidal deformation**
  - Finite size effect
  - Deviation from charp \( \Rightarrow \) NS radius

- **Merger and oscillation of HMNS**
  - BH or NS \( \Rightarrow \) maximum mass
  - GW from rotating HMNS \( \Rightarrow \) NS radius (and EOS)

![Diagram showing density contour and time-amplitude relations](image)

Log density \([\text{g/cm}^3]\)

Amplitude

Time [ms]
Prompt BH formation
GW spectra: deviation from point particle approximation

GW spectra for prompt BH formation

Deviation due to finite size effects

Point particle approximation

$\text{APR2.9-1}$ $\text{APR2.9-0.8}$ $\text{SLy2.8-1}$ $\text{FPS2.6-1}$ $\text{FPS2.6-0.8}$

$h_{\text{eff}} (D = 100 \text{ Mpc})$

$10^{-22}$ $10^{-23}$

frequency [kHz]

$\mathbf{f}_{\text{cut}}$
$f_{\text{cut}}$ may be used to estimate $R_{\text{NS}}$
Exploring Dense matter physics by GW

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  - Deviation from charp ⇒ NS radius

- **Merger and oscillation of HMNS**
  - BH or NS ⇒ maximum mass
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- Density contour
- Log density [g/cm³]
- Tidal deformation
- Merger and oscillation
- Amplitude
- Time [ms]
GWs from HMNS (1.3-1.4 Msolar Merger)

Hotokezaka et al. (2011); (2012)
GW spectra (1.35-1.35 Msolar)

GW spectra show characteristic peak frequency $f_{\text{peak}}$. $f_{\text{peak}}$'s are different for different EOS ⇒ constraining EOS

Hotokezaka et al. (2011); (2012)
Relation between $f_{\text{peak}}$ and NS structure

- The peak GW frequency depends strongly on EOS
- The frequency has correlation with NS radius and stiffness of EOS
  - Bauswein & Janka. 2011
  - Bauswein et al. 2012
  - Hotokezaka et al. 2012
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![Graph showing the relation between $f_{\text{peak}}$ and NS structure](image-url)
Relation between $f_{\text{peak}}$ and NS structure

Bauswein & Janka. 2011
Bauswein et al. 2012
Hotokezaka et al. 2012

Relation between $f_{\text{peak}}$ and NS structure
Stiffer EOS
Radius of 1.6M$_{\odot}$ solar NS

preliminary
$f_{\text{peak}}$ vs. NS radius

- $f_{\text{peak}}$ [kHz] vs. $R_{\text{max}}$ [km]
- $f_{\text{peak}}$ [kHz] vs. $R_{1.8}$ [km]
- $f_{\text{peak}}$ [kHz] vs. $R_{1.35}$ [km]
Emergence of Hyperon is putted in GW?

- Dynamics of HMNS formed after the merger
  - **Nucleonic**: HMNS shrinks by angular momentum loss in a long GW timescale
  - **Hyperonic**: GW emission $\Rightarrow$ HMNS shrinks $\Rightarrow$ More Hyperons appear $\Rightarrow$ EOS becomes softer $\Rightarrow$ HMNS shrinks more $\Rightarrow$ ....

- As a result, the characteristic frequency of GW increases with time
  - **Might** providing potential way to tell existence of hyperons (exotic particles)
Neutrino signal (w. and w.o. hyperons)

- There is no difference except for the duration until the BH formation
  - Difficult to tell the existence of hyperons using the neutrino signals alone
- Copious neutrinos are emitted from disk around BH
  - NS-NS merger as a progenitor of short GRBs?
Possible EM counterpart

- Expected electromagnetic (EM) wave emission from the merger
  - Detection of EM counterpart enhances reliability and detectability of GW

- **Ejecta** Sweeping inter stellar matter $\Rightarrow$ shock $\Rightarrow$ **Synchrotron rad.**
  - Nakar & Piran (2011) Nature
  - $\sim 90\mu$Jy $(E_0/10^{50}$erg$)(n_0/1$cm$^{-3})^{0.9}(v/0.3c)^{-2.8}(D/200$Mpc)$^{-2}(\nu_{\text{obs}}/1.4$ GHz)$^{-0.75}$

- Neutron rich **ejecta** $\Rightarrow$ R-process $\Rightarrow$ **radioactive decay** (talk by Wanajo san)
  - Li & Paczynski (1998)
  - $L_{\text{peak}} \sim 2.6 \times 10^{42}$ erg/s $(f/3 \times 10^{-6})(v/0.3c)^{1/2}(M_{\text{eje}}/10^{-2}$ M$_{\odot})^{1/2}$

- These transient event could be detected with upcoming radio or optical detectors
Mass ejection depends strongly on EOS

- More compact NS results in more massive ejecta
  - For mass ratio close to unity ($q \sim 1$) and more compact NS, mass ejection is driven by shocks. Tidal effects are relatively less important
  - For larger mass ratio and less compact NS, tidal effect is also important
- Coupled observations of GW and EM will be important!
Homologous mass ejection for $q \sim 1$
Tidal effects play role for $q<1$ and stiff EOS
More compact NS results in more massive ejecta

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Hotokezaka et al. (2012)
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  - For larger mass ratio and less compact NS, tidal effect is also important.
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Summary

- Numerical Relativity is the unique tool to study dynamical phenomena such as NS-NS merger where strong gravity plays a role
  - Recent developments enable us to perform simulations in physical modeling
- NS-NS merger is interesting both in physics and astrophysics
  - Promising sources of ground-based GW detectors
  - As laboratory for exploring physics of dense matter
    - It may be possible to constrain EOS by GW from the merger
  - Central engine of SGRB
    - A large number of neutrinos are emitted from the hot disk
  - Exploring EM counterpart will be also important
BH+Disk formation in stellar core collapse (2D)

- 100M$solar$ model by Umeda & Nomoto (2008) + rotation
- Torus-structured shock: accumulation of matter to the proto-NS
- Time varying, large ($\sim 10^{52}$ erg/s) neutrino luminosity after BH formation

After the core bounce
Standing shock wave is formed
BH+Disk formation in stellar core collapse (2D)

- 100Msolar model by Umeda & Nomoto (2008) + rotation (< ms NS)
- Torus-structured shock: accumulation of matter to the proto-NS
- **Time varying, large (~10^{52} erg/s) neutrino luminosity after BH formation**
Importance of Rotation: **Oblique Shock**

- Torus-structured shock
- Infalling materials are accumulated into the PNS due to the **oblique shock**
- Thermal energy is efficiently stored in the pole of PNS
  - Ram pressure ↓
  - ⇒ **Outflow**
- Flows hit central PNS
  - NS oscillation
  - ⇒ **PdV work**, Lν ↑
Importance of High Entropy/Rotation: Energy balance

- **Compact core / Oblique shock** ⇒ **high mass accretion rate**
- Energy balance may not be satisfied ......
  - Rotation decreases $|Q_{adv}|$ & $|Q_{v}|$ (dense disk)
  - Additional ‘cooling’ sources required

\[
\dot{Q}_{acc}^+ = \dot{Q}_{adv}^- + \dot{Q}_{v}^-
\]
\[
\Rightarrow \dot{Q}_{acc}^+ = \dot{Q}_{adv}^- + \dot{Q}_{v}^- + \dot{Q}_{\text{outflow/expansion}}^- + \dot{Q}_{\text{convection}}^-
\]

- Strong dependence of $Q_v$ ($v$-cooling) on $T$ (and $\rho$)
  ⇒ slight change of configuration leads to dynamically large change
  - Torus is partially supported by the (thermal) pressure gradient

- **Smaller amount of heavy nuclei** ⇒ **more energetic SNe**?
- Dissociation of 0.1 M$_{\odot}$ Fe costs $\sim 10^{51}$ erg
- Higher temperature: Less Pauli blocking in neutrino pair annihilation
Importance of Rotation: **BH spin**

- Energy conversion efficiency can change two orders of magnitude
- Disk properties to neutrinos strongly depend on BH spin
  - Slow rot. BH ⇒ ISCO (disk edge) located far ⇒ low density / opacity ⇒ Efficient cooling ⇒ the local valance satisfied ⇒ weak/no time variability

![Diagram showing the relationship between BH spin and disk properties](image)
Similarities to ordinary SN

- Same components: ‘stalled’ shock + neutrino sphere/torus
  - SASI-like activities are likely to occur?
  - The gain (neutrino-heated) regions do exist (Sumiyoshi+ 2012)

- Only topology is different
  - How will this system evolve in the presence of ν-heating
  - The next study using GR-vRad-Hydro Code (recently developed)