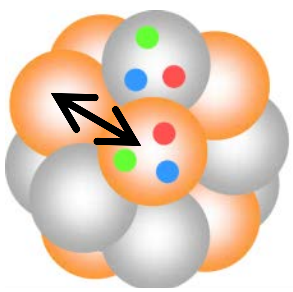
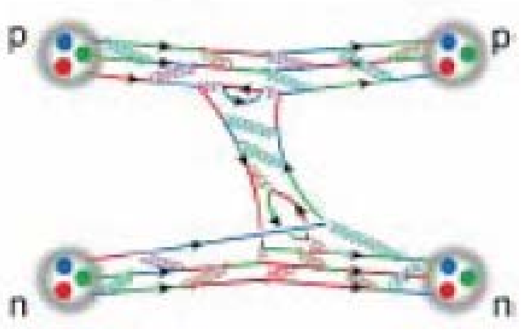
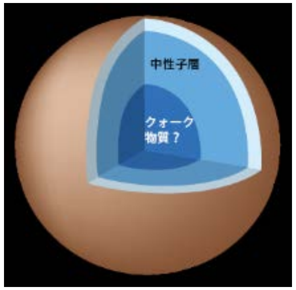
Hadron Interactions on the Lattice

Takumi Doi

(Nishina Center, RIKEN)



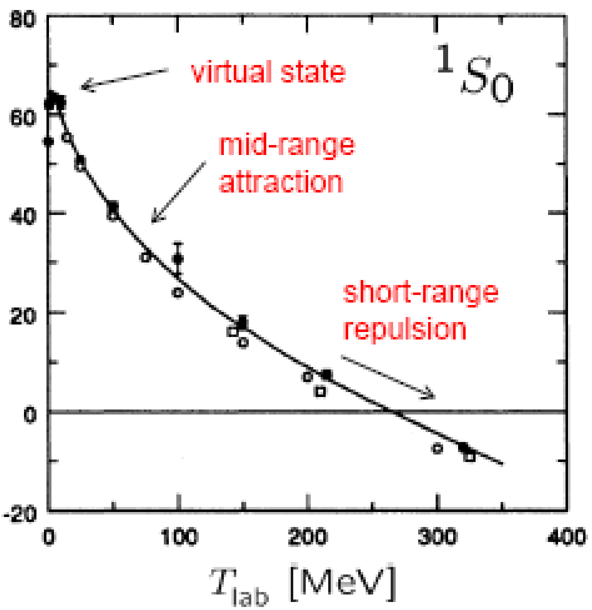




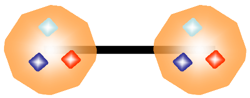
- Hadron Interactions
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics
  - **Frontier:** 1st principle calc by Lattice simulations

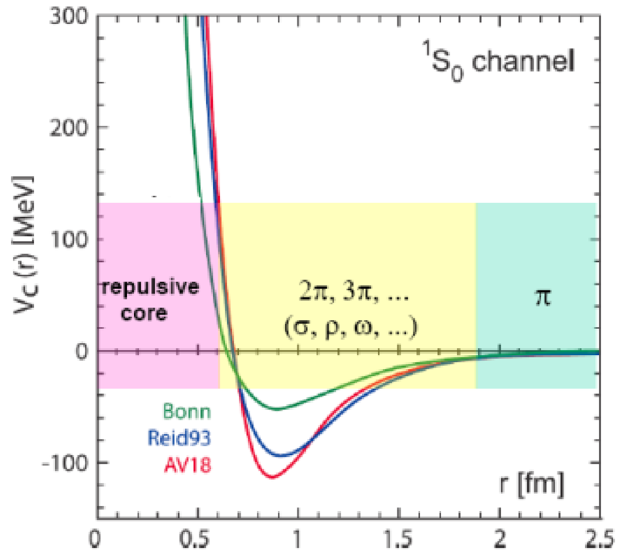
- <u>Outline</u>
  - Introduction
  - Theoretical framework for lattice hadron forces
  - Lattice results at heavier quark masses
  - Challenges toward physical quark mass point
    - ➔ **Nuclear Physics on the Lattice**
  - Summary / Prospects

(1) Build a foundation for nuclear physics

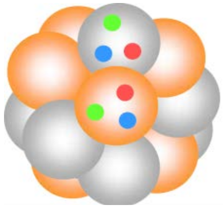


NN phase shifts from experiments

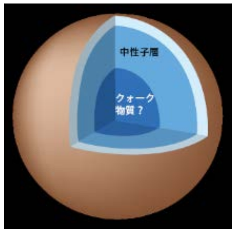




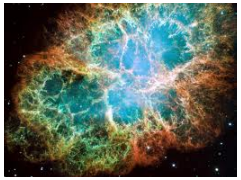
Phenomenological Nuclear Forces



Nuclei



Neutron Stars



Super Novae

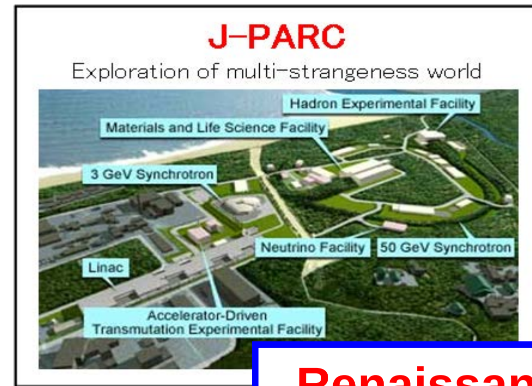
Various applications

- ***Nuclear Forces*** ***play*** ***crucial roles***
  - ***Yet, no clear connection to QCD so far***

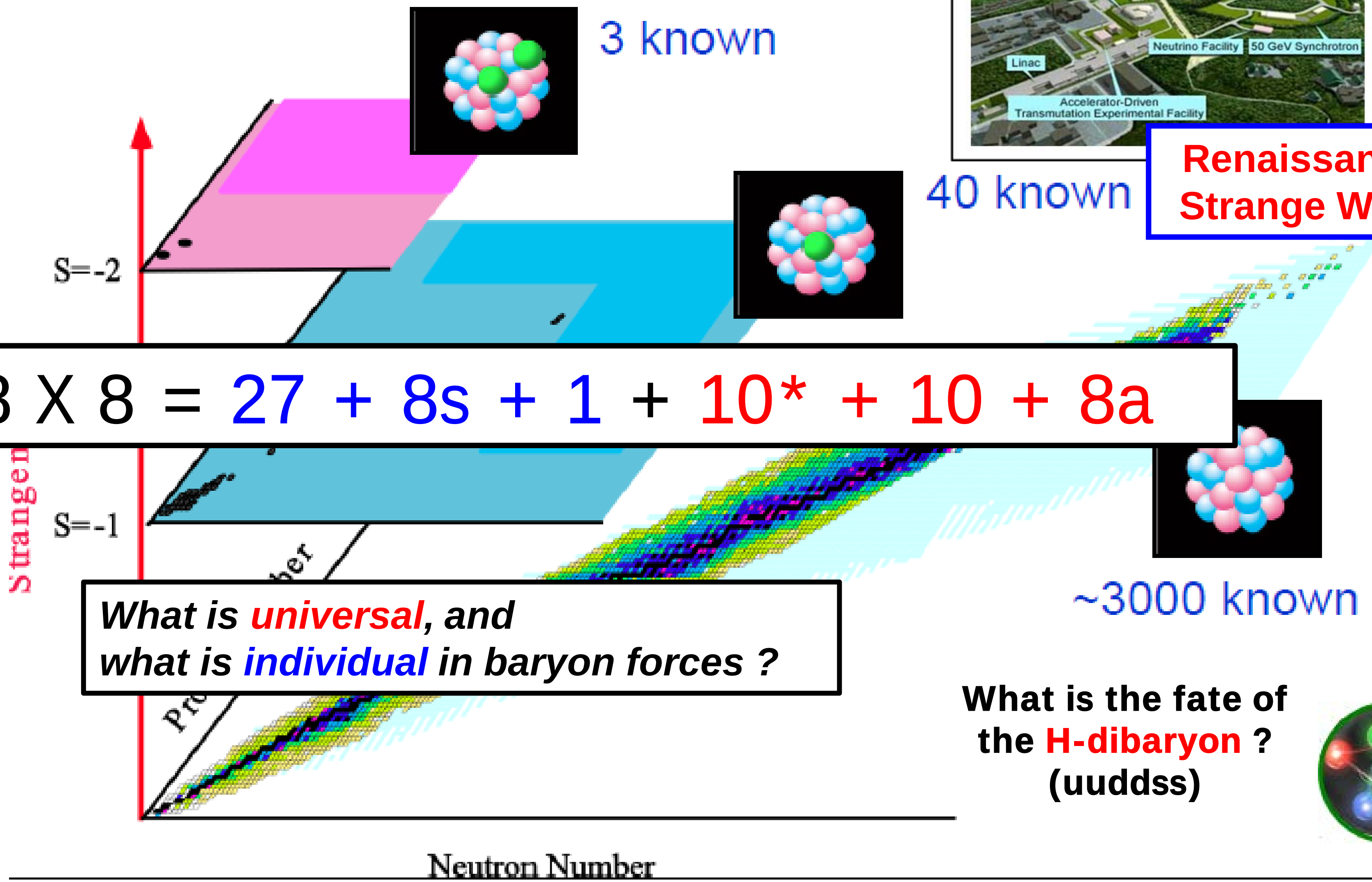
→ Lattice QCD

(2) Predict Unknown Interactions (YN, YY, NNN)

3D Nuclear Chart



Renaissance in Strange World !

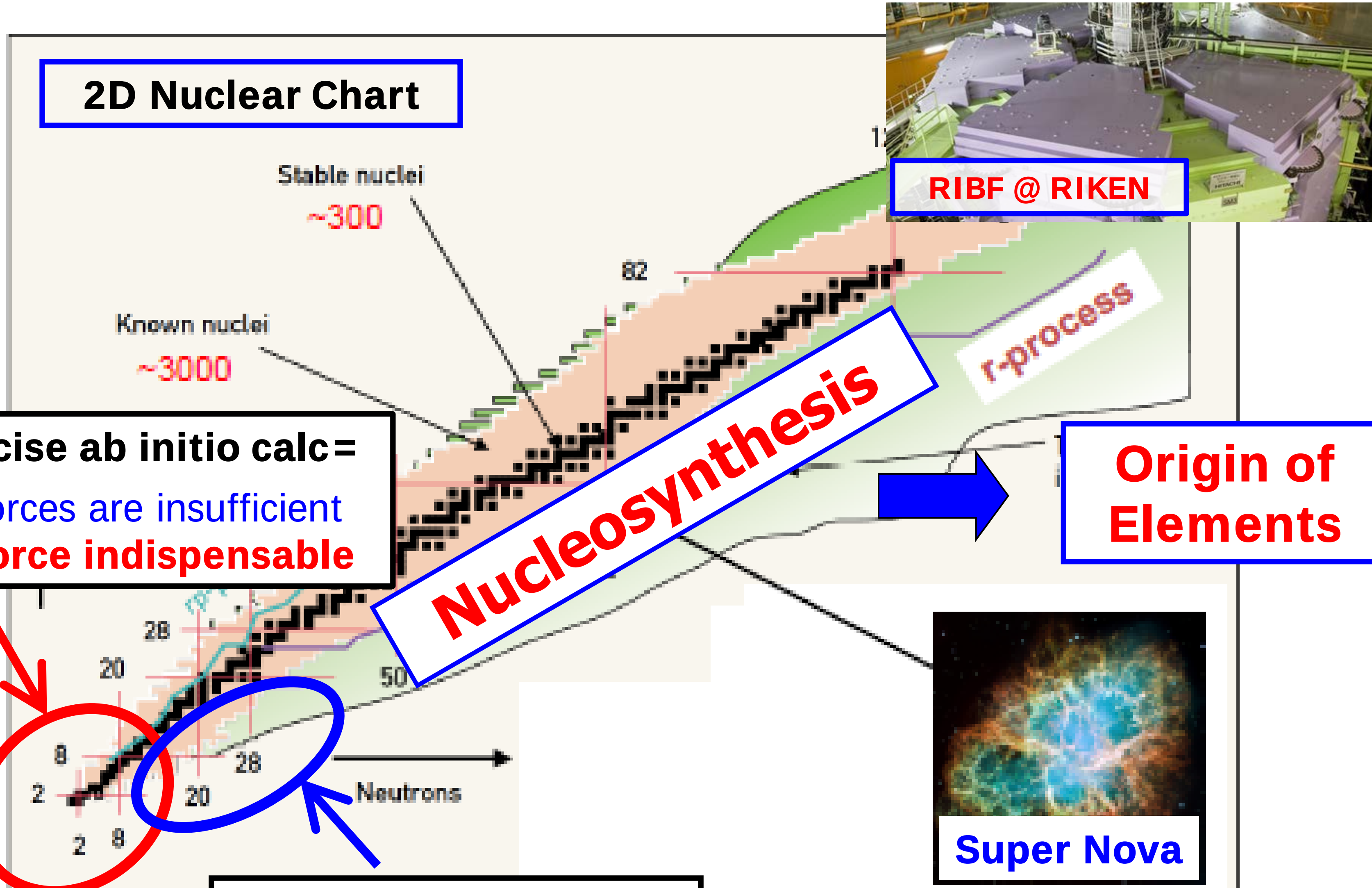


8 X 8 = 27 + 8s + 1 + 10* + 10 + 8a

What is universal, and
what is individual in baryon forces ?

What is the fate of the H-dibaryon ? (uuddss)

(2) Predict Unknown Interactions (YN, YY, NNN)



=Precise ab initio calc=

2N-forces are insufficient

3N-force indispensable

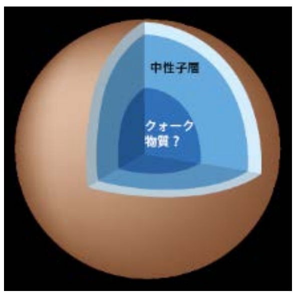
Origin of Elements

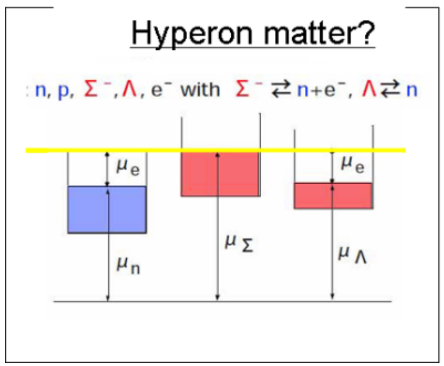
Paradigm Shift in
Unstable Nuclei
(New Magic Numbers !)

← **Important role of 3N-force**

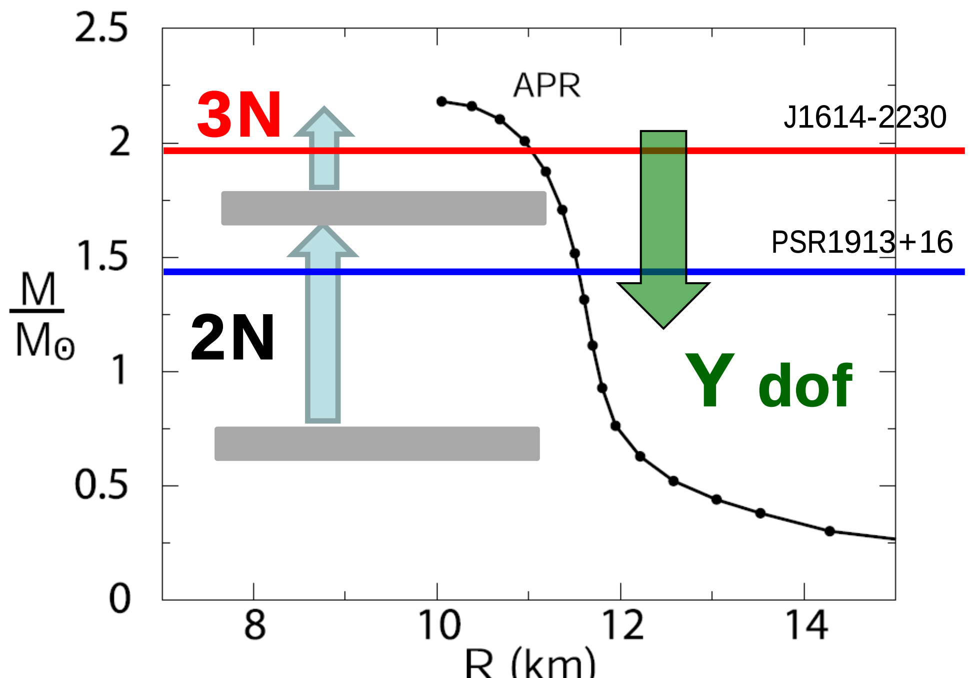
Dense Matter ← Interactions of YN, YY, NNN,… are crucial

- Neutron Stars, Super Novae ↔ EoS

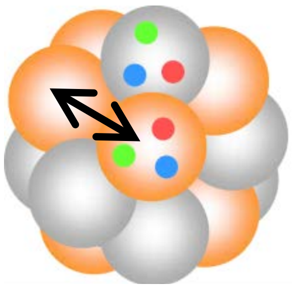


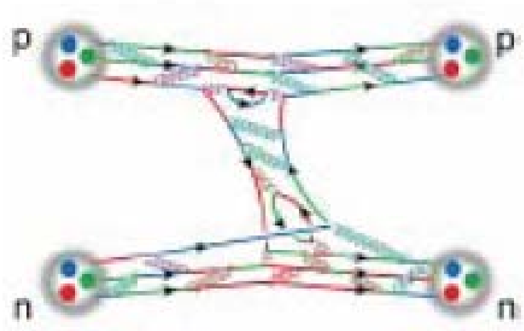


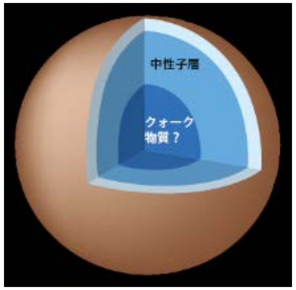
How to sustain a neutron star against gravitational collapse ?



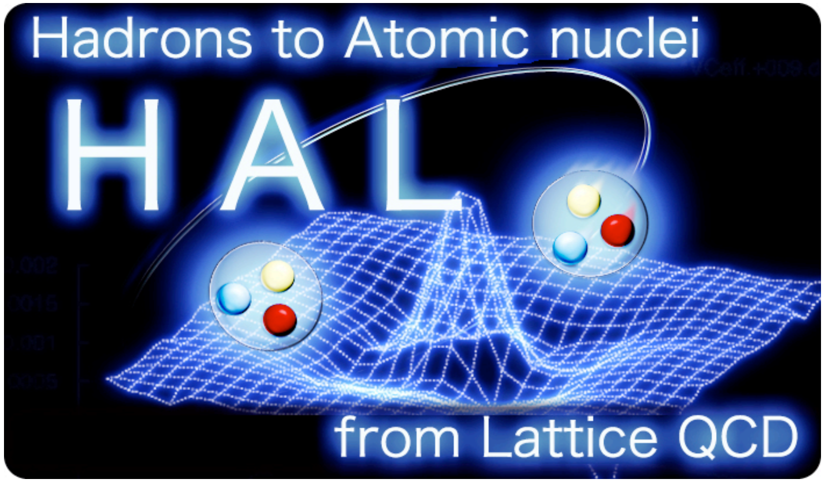
Akmal et al.('98), Nishizaki et al.('02), Takatsuka et al.('08)







- **<u>Outline</u>**
  - Introduction
  - Theoretical framework for lattice hadron forces
    - Phase shifts (a la Luscher) / Light nuclei on the lattice T.Yamazaki (Sat.)
    - Potentials from NBS wave functions on the lattice
  - Lattice results at heavier quark masses
  - Challenges toward physical quark mass point
  - Summary / Prospects



Hadrons to Atomic nuclei from Lattice QCD
(HAL QCD Collaboration)

S. Aoki, N. Ishii, H. Nemura, K. Sasaki, M. Yamada (Univ. of Tsukuba)

B. Charron (Univ. of Tokyo)

T. Doi, T. Hatsuda , Y. Ikeda, K. Murano (RIKEN)

T. Inoue (Nihon Univ.)

Nuclear Forces from Lattice QCD [HAL QCD method]

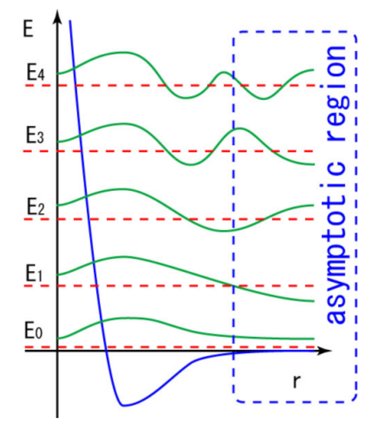
- Potential is constructed so as to reproduce the NN phase shifts (or, S-matrix)

- Nambu-Bethe-Salpeter (NBS) wave function

$$\psi(\vec{r}) = \langle 0 | N(\vec{x}+\vec{r}) N(\vec{x}) | 2N \rangle$$

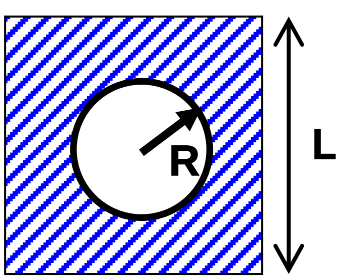
$$E = 2\sqrt{m^2 + k^2}$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$



  – Wave function ←→ phase shifts

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

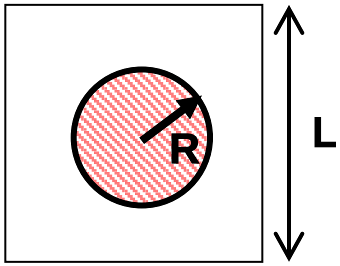


M.Luscher, NPB354(1991)531
CP-PACS Coll., PRD71(2005)094504
C.-J.Lin et al., NPB619(2001)467
Ishizuka, PoS LAT2009 (2009) 119

"Potential" as a representation of S-matrix [HAL QCD method]

- Consider the wave function at "interacting region"

$$(\nabla^2 + k^2)\psi(\boldsymbol{r}) = m \int d\boldsymbol{r}' U(\boldsymbol{r}, \boldsymbol{r}')\psi(\boldsymbol{r}'), \quad r < R$$



  – U(r,r'): faithful to the phase shift by construction
    - U(r,r') below inelastic threshold is

$$U(\boldsymbol{r}, \boldsymbol{r}') = \frac{1}{m} \sum_{n,n'}^{n_{\mathrm{th}}} (\nabla_{\boldsymbol{r}}^2 + k_n^2)\psi_n(\boldsymbol{r}) \mathcal{N}_{nn'}^{-1} \psi_{n'}^*(\boldsymbol{r}') \qquad \mathcal{N}_{nn'} = \int d\boldsymbol{r} \psi_n^*(\boldsymbol{r}) \psi_{n'}(\boldsymbol{r})$$

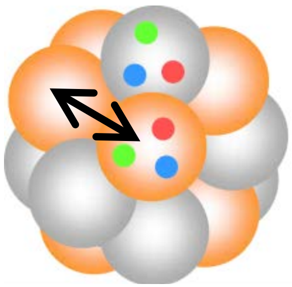
    - U(r,r'): E-independent, while non-local in general
    - Non-locality ➔ derivative expansion

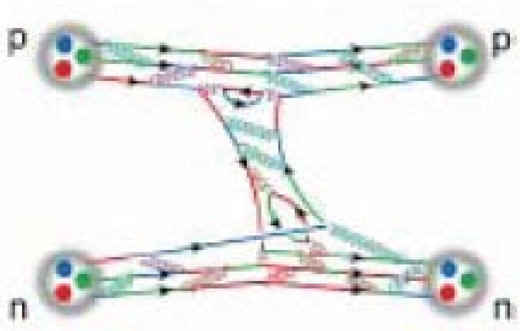
Okubo-Marshak(1958)

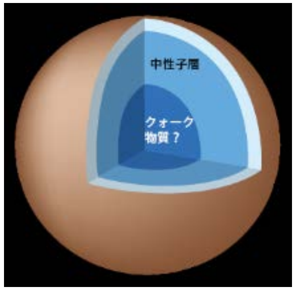
$$U(\vec{r}, \vec{r'}) = \underbrace{V_c(r)}_{\textbf{LO}} + \underbrace{S_{12}V_T(r)}_{\textbf{LO}} + \underbrace{\vec{L}\cdot\vec{S}V_{LS}(r)}_{\textbf{NLO}} + \underbrace{\mathcal{O}(\nabla^2)}_{\textbf{NNLO}}$$

Aoki-Hatsuda-Ishii PTP123(2010)89

Check on convergence: K.Murano et al., PTP125(2011)1225





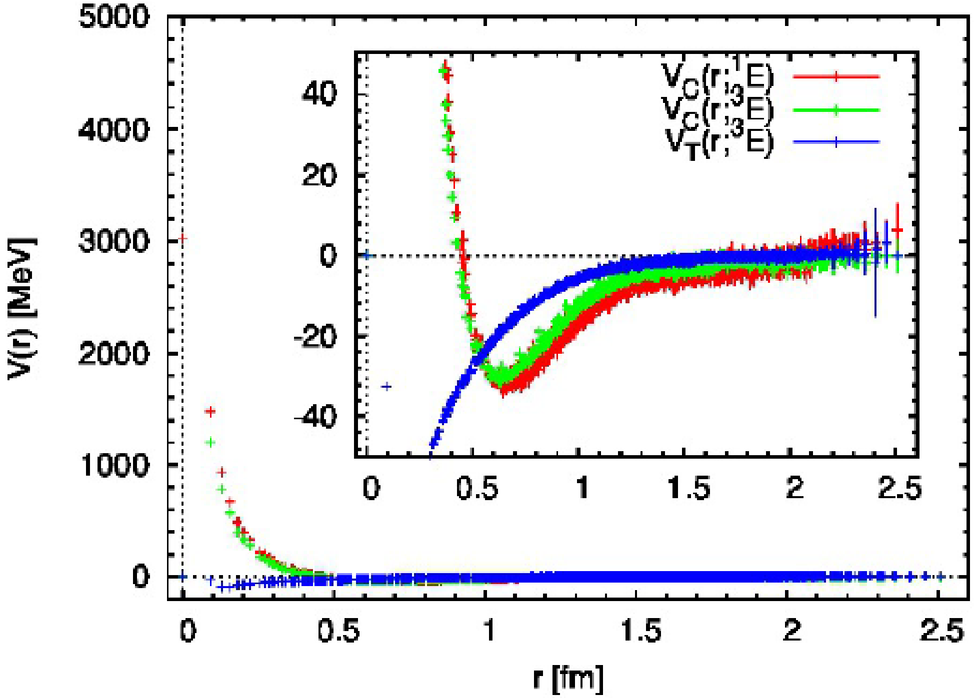


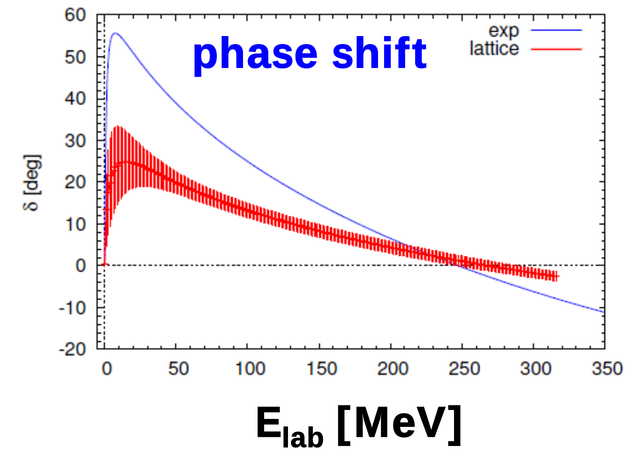
- **<u>Outline</u>**
  - Introduction
  - Theoretical framework for lattice hadron forces
  - Lattice results at heavier quark masses
    - (1) nuclear-, (2) hyperon-, (3) 3N- forces
  - Challenges toward physical quark mass point
  - Summary / Prospects

(1) NN potential on the lattice
(positive parity)

$^{2S+1}L_J$

- "di-neutron" channel $^1S_0$ ➔ central force
- "deuteron" channel $^3S_1 - ^3D_1$ ➔ central & tensor force





Not Bound $a(^1S_0) = 1.6(1.1)$ fm

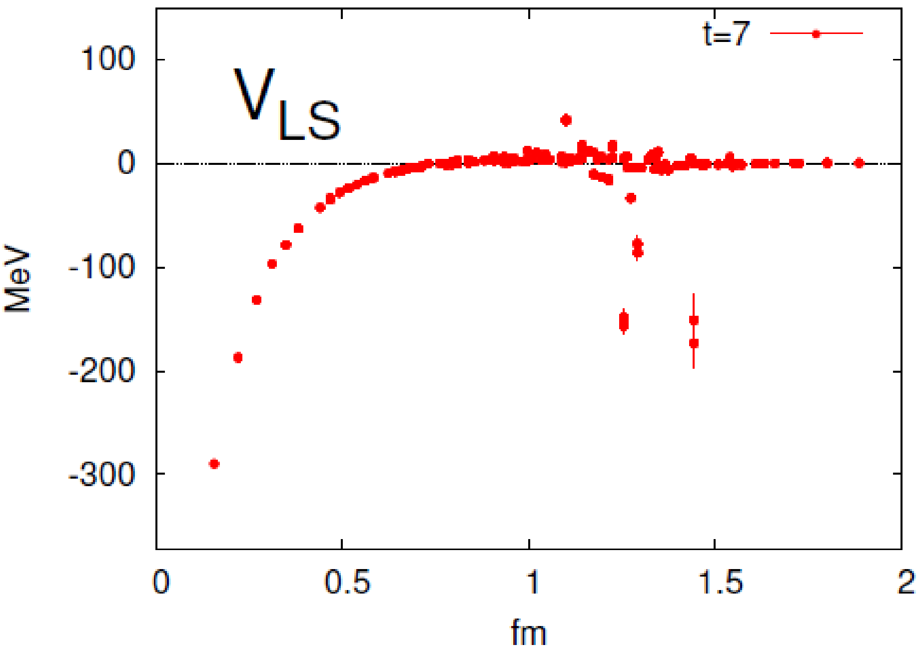
Nf=2+1 clover (PACS-CS), 1/a=2.2GeV,
L=2.9fm, m_π=0.7GeV, m_N=1.6GeV

[N.Ishii (Poster)]

NN potential on the lattice (negative parity)

$^{2S+1}L_J$

- S=1 channel: $^3P_0,\ ^3P_1,\ ^3P_2{-}^3F_2$
  - Central & tensor forces in LO
  - Spin-orbit force in NLO
    - Inject a momentum ➔ $J^P = A_1^-, T_1^-, T_2^-$



Nf=2 clover (CP-PACS), L=2.5fm, mπ=1.1GeV

$\vec{L}\cdot\vec{S} = +1$ for $^3P_2$

Superfluidity $^3P_2$ in neutron star ←→ neutrino cooling

←→ **Cas A NS: cooling is being measured !**

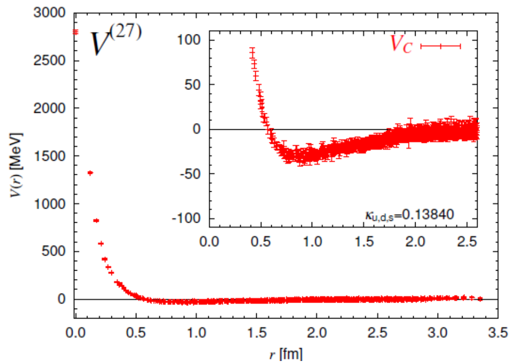
[K.Murano (Poster)]

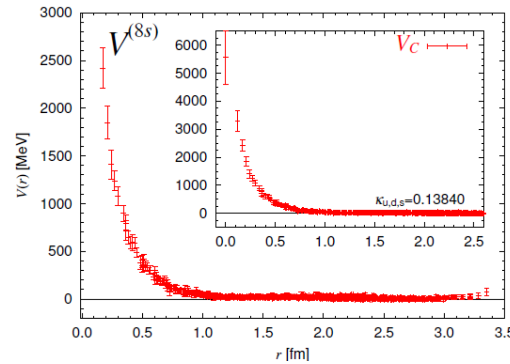
SU(3) study

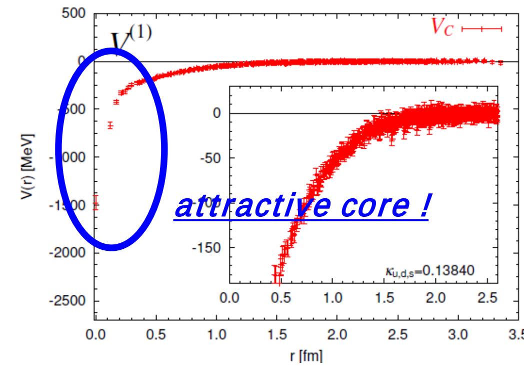
(2) Hyperon forces

a=0.12fm, L=3.9fm,
m(PS)= 0.47-1.2GeV

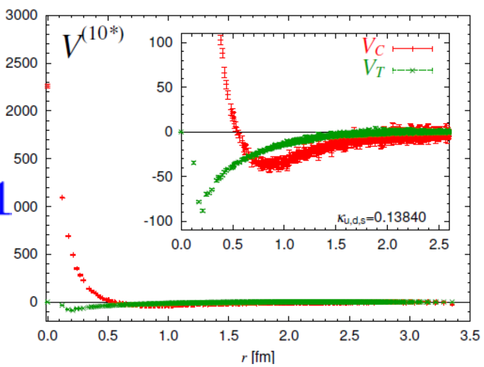
$^{1}S_{0}$

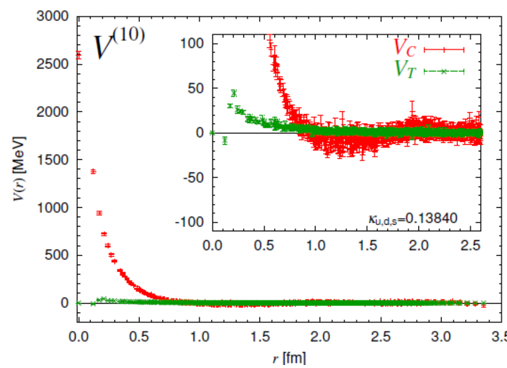


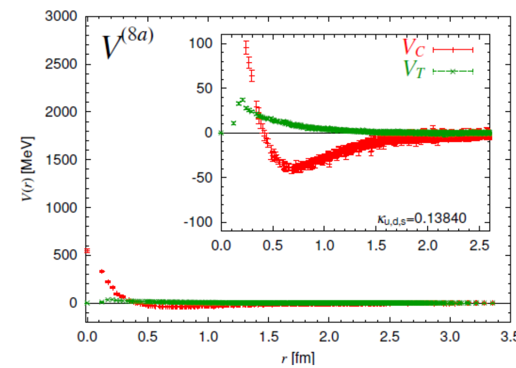




$^{3}S_{1}-^{3}D_{1}$







27,10*:
Same as NN

8s,10:
strong repulsive core

1s: deep attractive pocket
8a: weak repulsive core

Repulsive core
← Pauli principle !

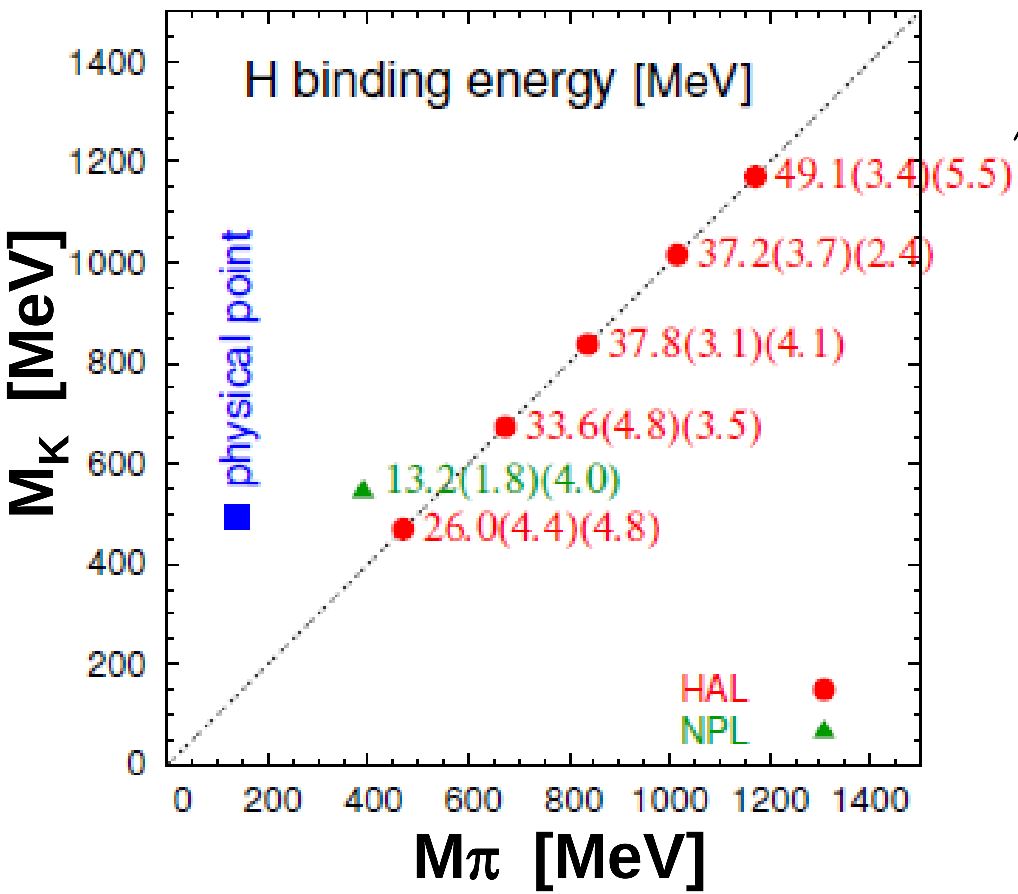
M.Oka et al., NPA464(1987)700

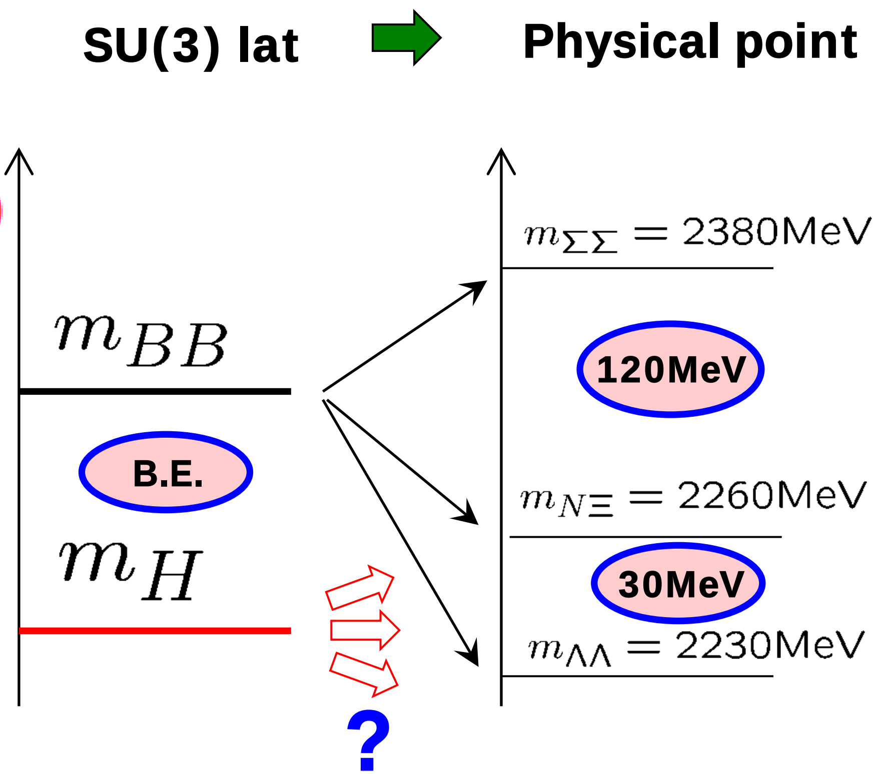
T.Inoue et al. (HAL QCD Coll.), NPA881(2012)28

Also seen in Takahashi et al. (2010), Kawanai et al. (2010)
Meson-baryon, **Y.Ikeda** et al., arXiv:1111.2663, **(Poster)**

➔ Study of baryonic matter & Neutron Star [T.Inoue]

H-dibaryon (uuddss, I=0,$^1S_0$)





Coupled channel study is essential

$$\Lambda\Lambda – N\Xi – \Sigma\Sigma$$

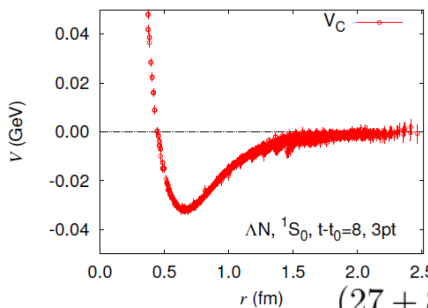
➔ Poster by K.Sasaki

Hyperon Interactions in S= -1

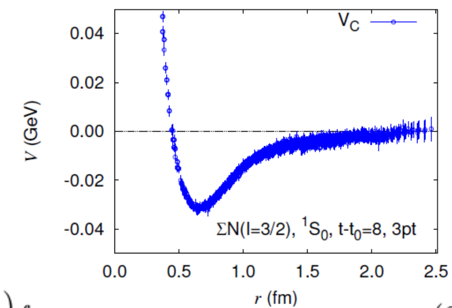
ΛN $\Sigma N(I=3/2)$

(single channel study)

$^1S_0$



$(27+8_s)_f$



(27_f)

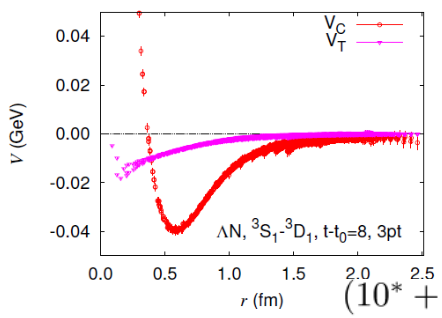
$\Lambda N(^1S_0, ^3S_1)$

$\Sigma N(I=3/2, ^1S_0)$

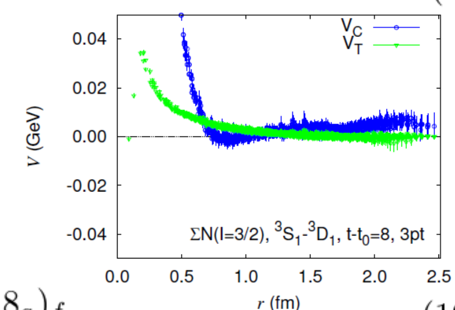
➔Attractive

Not bound

$^3S_1{-}^3D_1$



$(10^* + 8_a)_f$



(10_f)

$\Sigma N(I=3/2, ^3S_1)$

➔Repulsive

Nemura et al.
Nf=2+1, L=2.9fm,
mπ= 0.70GeV
arXiv:1203.3320

Crucial input for the core of neutron star and hyper-nuclei

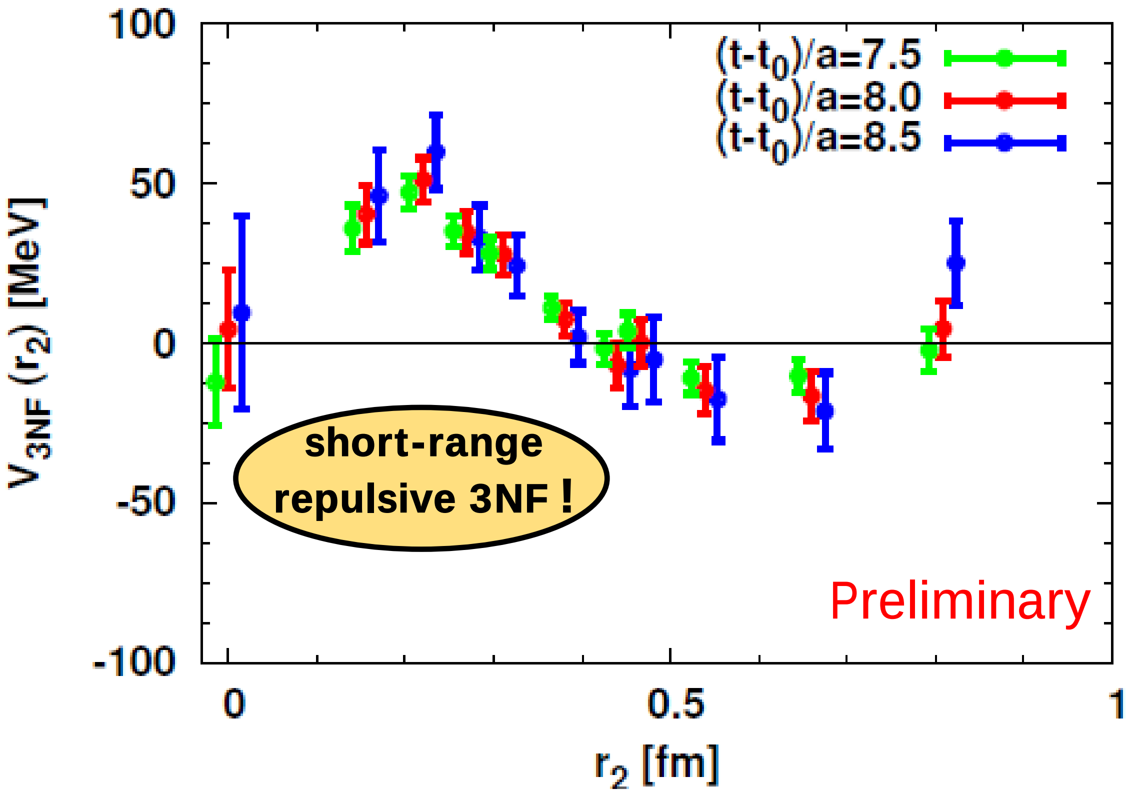
Poster by H. Nemura

ΛN-ΣN coupled channel study is also in progress

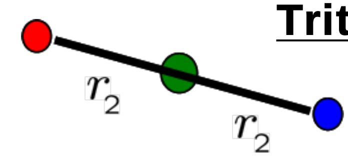
(3) 3N-forces (3NF) on the lattice

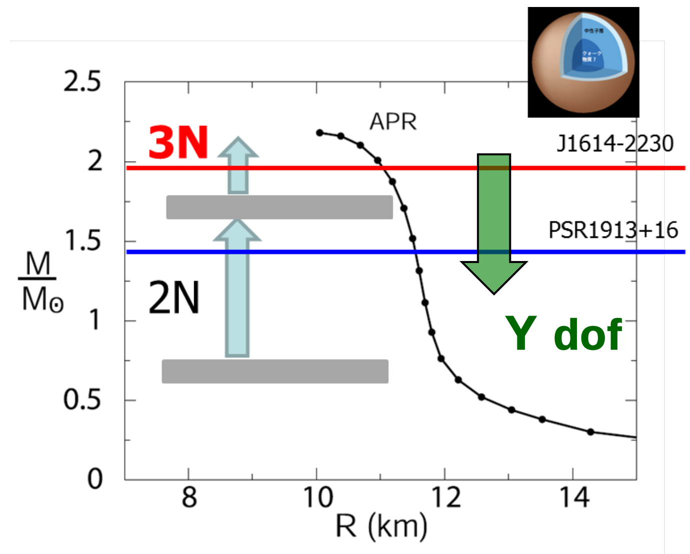
T.D. et al. (HAL QCD Coll.) PTP127(2012)723

+ t-dep method updates



Triton channel

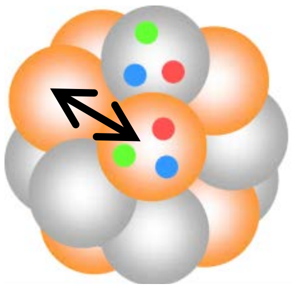


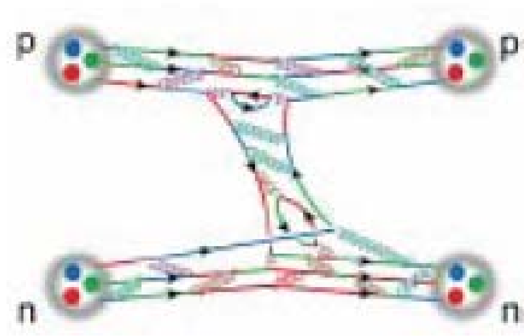


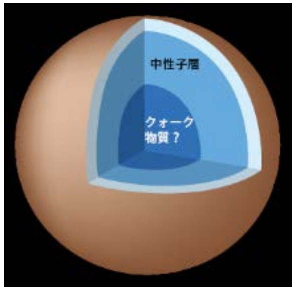
Nf=2 clover (CP-PACS), 1/a=1.27GeV,
L=2.5fm, m_π=1.1GeV, m_N=2.1GeV

How about other geometries ?

How about YNN, YYN, YYY ?





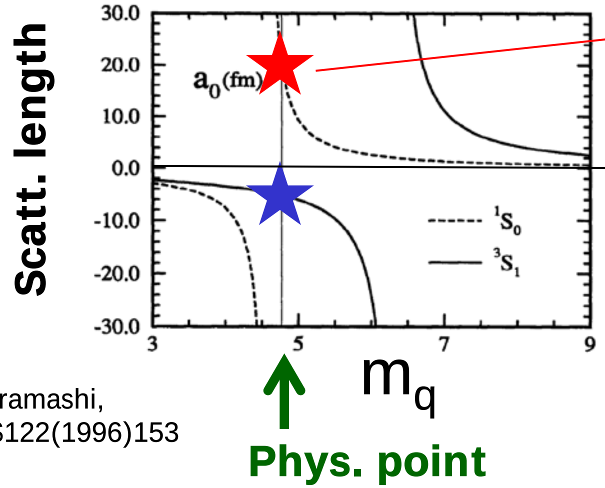


- **<u>Outline</u>**
  - – Introduction
  - – Theoretical framework for lattice hadron forces
  - – Lattice results at heavier quark masses
  - – Challenges toward realistic potentials
    - Major challenges: (1) S/N issue (2) comput. cost
  - – Summary / Prospects

K computer

Towards realistic potential by the K computer

- **Physical mass point**, **Infinite V limit**, **continuum limit**
  - Physical $m\pi$ crucial for OPEP, chiral extrapolation won't work

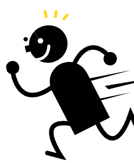


Y.Kuramashi, PTPS122(1996)153

$a_0(^1S_0) \sim 20\text{fm}$

"Unitary Region"

➔ m_q



We are here

  - Gauge confs generation at $m\pi$ = 140MeV, L=~10fm @ K

➔ **Challenge in the "measurement": S/N issue**

Challenges toward the physical point (1)

- S/N issue at light mass

Parisi, Lepage (1989)

  – To achieve ground state saturation, take $t \to \infty$

Single nucleon

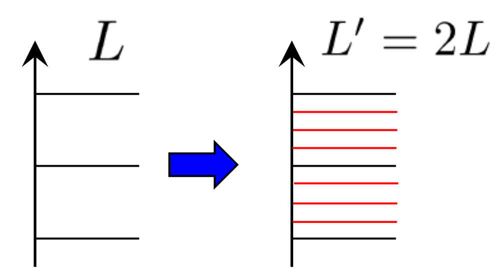
$$\frac{\text{Signal}}{\text{Noise}} \sim \frac{\langle N(t)\bar{N}(0)\rangle}{\sqrt{\langle N\bar{N}(t)N\bar{N}(0)\rangle}} \sim \frac{\exp(-m_N t)}{\sqrt{\exp(-3m_\pi t)}} \sim \mathbf{exp}[-(\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

Nucleons w/ mass number = A

$$\frac{\text{Signal}}{\text{Noise}} \sim \mathbf{exp}[-\mathbf{A} \times (\mathbf{m_N} - \mathbf{3/2m_\pi}) \times \mathbf{t}]$$

  – Situation gets worse for larger volume
    ← large spectral density by scatt. states

$$\Delta E \simeq \frac{\vec{p}^2}{m_N} \simeq \frac{1}{m_N}\left(\frac{2\pi}{L}\right)^2 \simeq 15\text{MeV} \quad \text{for } L = 10\text{fm}$$



→ Very large t >~ 100 would be required !

Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations, but signals***

➔ **Schrodinger Eq. : time-independent ➔ time-dependent**

$$\left(-\frac{\partial}{\partial t}+\frac{1}{4m}\frac{\partial^2}{\partial t^2}-H_0\right)R(\boldsymbol{r},t)=\int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2+k_n^2}=E_n=-\frac{\partial}{\partial_t}$$

Grand State (G.S.) saturation is NOT necessary !

Significant advantage of potential method:

$$\Delta E \simeq E_{\rm th}-E\simeq m_\pi \simeq 140\mathrm{MeV}$$

➔ Moderate t >~ 10 would be fine

Explicit Lat calc for I=2 pipi phase shift

Beautiful agreement between

(1) Luscher's formula w/ g.s. saturation

(2) the HAL QCD method w/ & w/o g.s. saturation

T.Kurth et al. (HAL-BMW Coll.) @ Lat2012

Challenges toward the physical point (2)

- **Enormous computational cost for correlators**
  - # of Wick contraction (permutations) $\sim [\,(\frac{3}{2}A)!\,]^2$
  - # of color/spinor contractions $\sim 6^A \cdot 4^A \;\; \text{or} \;\; 6^A \cdot 2^A$
    (color) (spinor) (half-spin)
    - **Total cost:**
      - ^{2}H : 9 x 144 = 1 x 10^3
      - ^{3}H : 360 x 1728 = 6 x 10^5
      - ^{4}He : 32400 x 20736 = 7 x 10^8

Improvement:
T.Yamazaki et al.,
PRD81(2010)111504

- ➔ **[Unified contraction algorithm]**

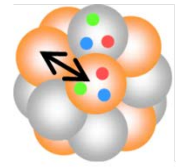
TD, M.Endres, arXiv:1205.0585
CPC184(2013)117

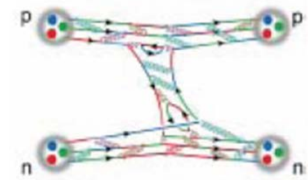
  - Treat Wick/color/spinor contractions in a unified index space
    - ➔ huge redundancies can be eliminated systematically
    - ➔ permutation finished BEFORE any lattice calc
    - **Significant improvement**

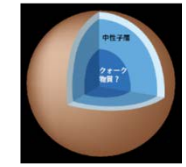
$$\boxed{\times 192 \text{ for } {}^3\text{H}/{}^3\text{He}, \; \times 20736 \text{ for } {}^4\text{He}, \; \times 10^{11} \text{ for } {}^8\text{Be}}$$

(x add'l. speedup)

Summary and Prospects







- Hadron Interactions by 1st principle Lat calc
  - Bridging different worlds:
    Particle Physics / Nuclear Physics / Astrophysics

- Lattice QCD results for NN, **YN/YY**, NNN
  - Intriguing physics even at heavy quark masses

- **On the K computer:** physical quark mass point !
  - Breakthroughs in S/N issue & Comput. cost issue

Thermodynamic limit & continuum limit

→ **Realistic hadron interactions**

→ **Nuclear Physics on the Lattice !**

Backup Slides

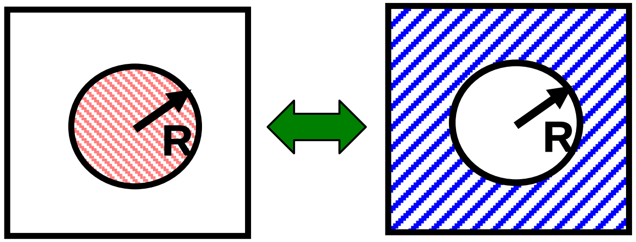
A few remarks on the Lattice Potential

- Potential is NOT an observable and is not unique: They are, however, phase-shift equivalent potentials.
  - Choosing the pot. (sink op.) ⇐⇒ choosing the "scheme"

- We study potential (+ phase shifts), since:
  - Convenient to understand physics
  - Essential to study many-body

$$Lat \rightarrow \boxed{\delta_E \rightarrow U(\boldsymbol{r})} \rightarrow \text{many-body}$$

$$\boxed{Lat \rightarrow \underset{\delta_E}{\rightarrow} \overset{\rightarrow}{\leftarrow} U(\boldsymbol{r})} \rightarrow \text{many-body}$$



  - Finite V artifact better under control
  - Excited states better under control

Effective Schrodinger equation with E-independent potential

$$K(\vec{x};E) \equiv \left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E)$$ **[START] local but E-dep pot. ($L^3 x L^3$ dof)**

(1) We assume ψ(x; E) for different E is linearly independent with each other.

(2) ψ(x; E) has a **"left inverse"** as an integration operator as

$$E \equiv 2\sqrt{m_N^2 + k^2}$$

$$\int d^3x \tilde{\psi}(\vec{x};E')\psi(\vec{x};E) = 2\pi\delta(E-E')$$

(3) K(x; E) can be factorized as

$$\equiv m_N U(\vec{x},\vec{y})$$

$$\begin{aligned} K(\vec{x};E) &= \int \frac{dE'}{2\pi} K(\vec{x};E') \times \int d^3y \tilde{\psi}(\vec{y};E')\psi(\vec{y};E) \\ &= \int d^3y \left\{ \sum_\alpha \int \frac{dE'}{2\pi} K(\vec{x};E')\tilde{\psi}(\vec{y};E') \right\} \psi(\vec{y};E) \end{aligned}$$

(4) We are left with **an effective Schrodinger equation with an $\boxed{\text{E−independent}}$ potential U.**

$$\left(\vec{\nabla}^2 + k^2\right)\psi(\vec{x};E) = m_N \int d^3y \boxed{U(\vec{x},\vec{y})} \psi(\vec{y};E)$$

Intuitive understanding

[GOAL] non-local but E-indep pot. ($L^3 x L^3$ dof)

(44)

Asymptotic form of BS wave function

[C.-J.D.Lin et al., NPB619,467(2001)]

For simplicity, we consider BS wave function of two pions

$$\psi_{\vec q}(\vec x) \equiv \left\langle 0 \middle| N(\vec x)\, N(\vec 0) \middle| N(\vec q) N(-\vec q), in \right\rangle$$

complete set

$$1 = \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec p)} \left| N(\vec p) \right\rangle \left\langle N(\vec p) \right| + \cdots$$

$$= \int \frac{d^3 p}{(2\pi)^3 2E_N(\vec p)} \left\langle 0 \middle| N(\vec x) \middle| N(\vec p) \right\rangle \left\langle N(\vec p) \middle| N(\vec 0) \middle| N(\vec q) N(-\vec q), in \right\rangle + I(\vec x)$$

$$Z^{1/2} e^{i\vec p\cdot\vec x}$$

$$disc. + Z^{1/2} \frac{T(\vec p; \vec q)}{m_N^2 - \left(2E_N(\vec q) - E_N(\vec p)\right)^2 + \vec p^2 - i\varepsilon}$$

$$= Z\left(e^{i\vec q\cdot\vec x} + \frac{1}{(2\pi)^3} \int \frac{d^3 p}{2E_N(\vec p)} \frac{T(\vec p; \vec q)}{4E_N(\vec q)\cdot\left(E_N(\vec p) - E_N(\vec q) - i\varepsilon\right)} e^{i\vec p\cdot\vec x} \right)$$

Integral is dominated by the on-shell contribution $E_N(\vec p) \approx E_N(\vec q)$

➔ T-matrix becomes the on-shell T-matrix

$$T^{(s-wave)}(s) = \frac{E(\vec q)}{2|\vec q|}(-i)\left(e^{2i\delta_0(s)} - 1\right)$$

$$= Z\left(e^{i\vec q\cdot\vec x} + \frac{1}{2i}\left(e^{2i\delta_0(s)} - 1\right)\frac{e^{iqr}}{qr} \right) + \cdots$$

The asymptotic form

$$\boxed{\psi_{\vec q}(\vec x) = Z e^{i\delta_0(s)} \frac{\sin(qr + \delta_0(s))}{qr} + \cdots \quad \text{(s-wave)}}$$

This is analogous
to a non-rela. wave function

Lattice QCD Setup

- **HAL QCD Coll.**
  - Nf=3 clover, 1/a=1.6GeV, L= (2,3),4 fm,
    m_{PS} = 0.47,.0.67, 0.84, 1.0, 1.2GeV, m_B=1.2, 1.5, 1.7, 2.0, 2.3GeV
  - Nf=2+1 clover, 1/a=2.2GeV, L=2.9fm, $m\pi$=0.7GeV, m_N=1.6GeV (PACS-CS)
  - Nf=2, clover, 1/a=1.3GeV, L=2.5fm, $m\pi$=1.1GeV, m_N=2.1GeV (CP-PACS)

- **PACS-CS Coll.**
  - quenched clover, 1/a=1.5GeV, L=3,6,12 fm, $m\pi$=0.8GeV, m_N=1.6GeV
  - Nf=2+1 clover, 1/a=2.2GeV, L= 3 – 6 fm, $m\pi$=0.5GeV, m_N=1.3GeV

- **NPLQCD Coll.** (a_s/a_t=3.5)
  - Nf=2+1 clover, $1/a_s$=1.6GeV, L=(2, 2.5), 3, 4fm, $m\pi$=0.39GeV, m_N=1.2GeV
  - Nf=3 clover, 1/a=1.4GeV, L=(3.4,4.5), 6.7fm, m_{PS}=0.81GeV,m_B=1.6GeV

NN spectra on the lattice

- **PACS-CS Coll.** T. Yamazaki et al. PRD84(2011)054506
  - quenched, $m\pi$= 0.8GeV, L= 3, 6, 12fm, (variational study)
    - di-neutron ($^1S_0$) : B.E. = 5.5(1.1)(1.0)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 9.1(1.1)(0.5)MeV

Bound

  - Nf=2+1, $m\pi$= 0.5GeV, L= 3 – 6 fm T. Yamazaki et al. arXiv: 1207.4277
    - Both channels still bound w/ similar B.E.
    - di-neutron ($^1S_0$) : B.E. = 7.4(1.3)(0.6)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 11.5(1.1)(0.6)MeV
- **NPLQCD Coll.** S.Beane et al. PRD85(2012)054511
  - Nf=2+1, $m\pi$= 0.39GeV, L= (2, 2.5), 3, 4 fm
    - di-neutron ($^1S_0$) : B.E. = 7.1(5.2)(7.3)MeV
    - deuteron ($^3S_1$ –$^3D_1$) : B.E. = 11 (05)(12)MeV

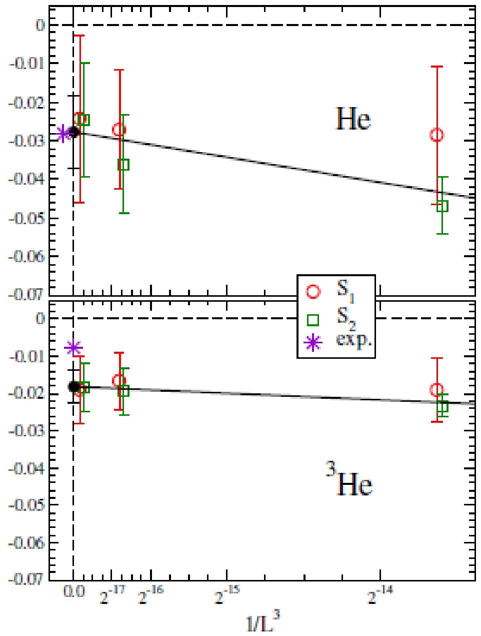
Suggestion of bound states

Spectroscopy on the lattice

- **PACS-CS Coll.**

  Yamazaki et al. PRD81(2010)111504
  Yamazaki et al. arXiv: 1207.4277

  - quenched, mπ= 0.8GeV, L= 3, 6, 12fm
    - ^{3}He=(^{3}H) : B.E. = 18.2 (3.5)(2.9) MeV
    - ^{4}He : B.E. = 27.7 (7.8)(5.5) MeV
  - Nf=2+1, mπ= 0.5GeV, L= 3 – 6 fm
    - Both nuclei are still bound w/ similar B.E.
    - ^{3}He=(^{3}H) : B.E. = 20.3 (4.0)(2.0) MeV
    - ^{4}He : B.E. = 43 (12) (8) MeV



- **NPLQCD Coll.**

  S.Beane et al. PRD80(2009)074501
  Prog.Part.Nucl.Phys 66(2010)1

  - Nf=2+1, mπ= 0.39GeV, L= 2.5 fm only
    - Study $\Xi^0\Xi^0 n$ and pnn $E_{pnn} - 3m_N = +40(21)(38)\text{MeV}$
  - Nf=3, mπ= 0.81GeV, L= (3.4, 4.5), 6.7fm

    S.Beane et al., arXiv:1206.5219

    - many (hyper) nuclei bound, e.g.,

      (version 2 ??)
    - ^{3}He=(^{3}H) : B.E. = 71 (6) (5) MeV ➔ 53.9(7.1)(8.0)(0.6) MeV
    - ^{4}He : B.E. = 110 (20)(15) MeV ➔ 107(12)(21)(1) MeV

^{3}H ($=^3$He)

NPLQCD: SU(3) study

Beane et al., arXiv:1206.5219

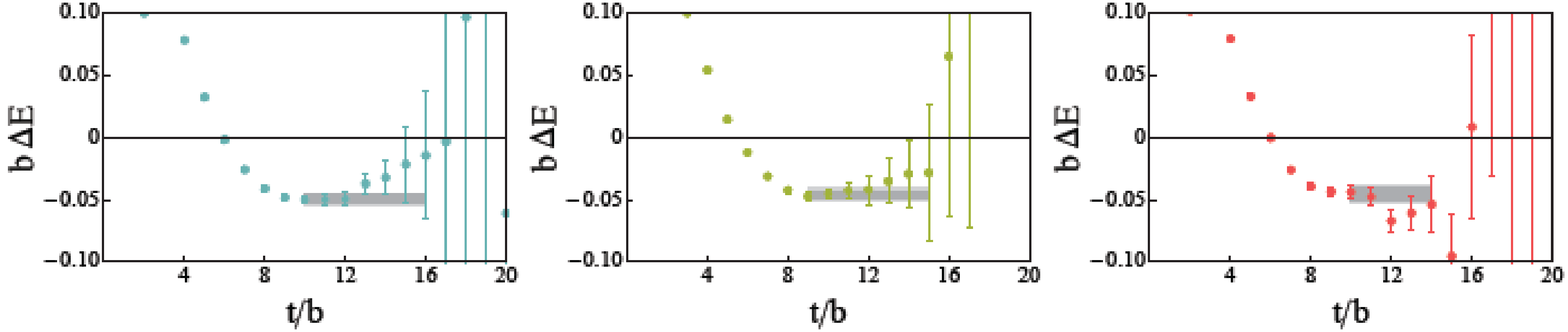


FIG. 6: EMPs associated with $J^\pi = \frac{1}{2}^+$ ^{3}He (^{3}H) $|\mathbf{P}| = 0$ correlation functions computed with the $24^3 \times 48$ (left), $32^3 \times 48$ (center) and $48^3 \times 64$ (right) en

G.S. saturated or not, that is the question

^{4}He

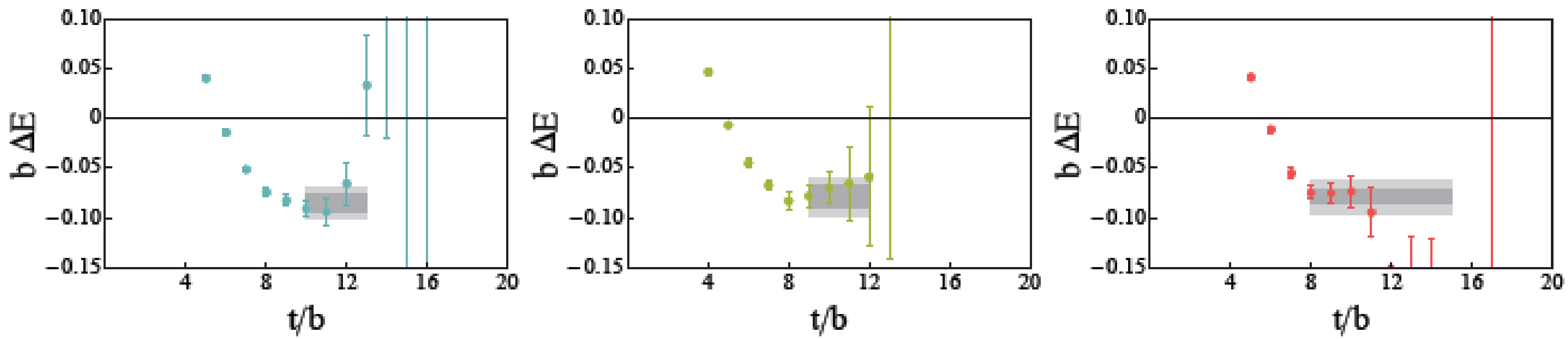
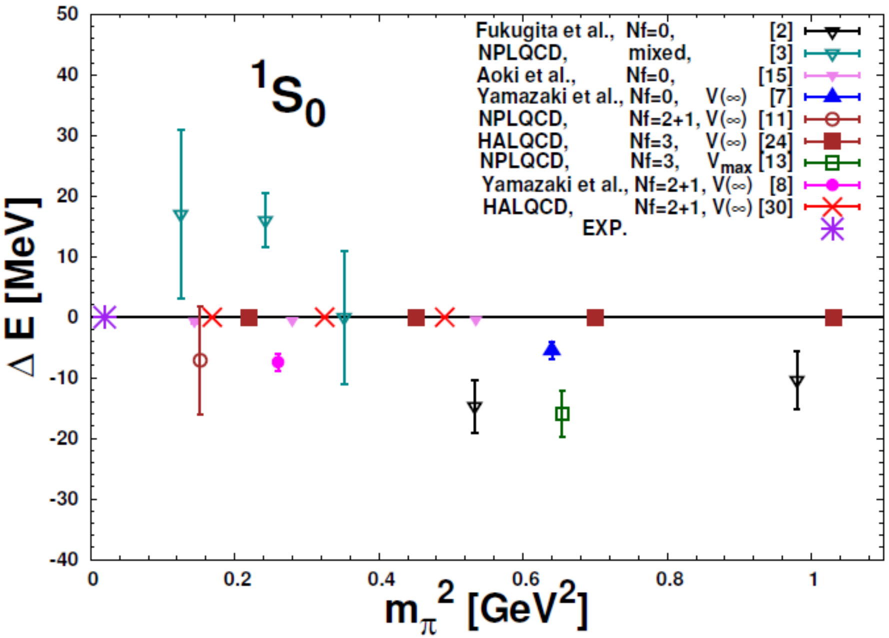


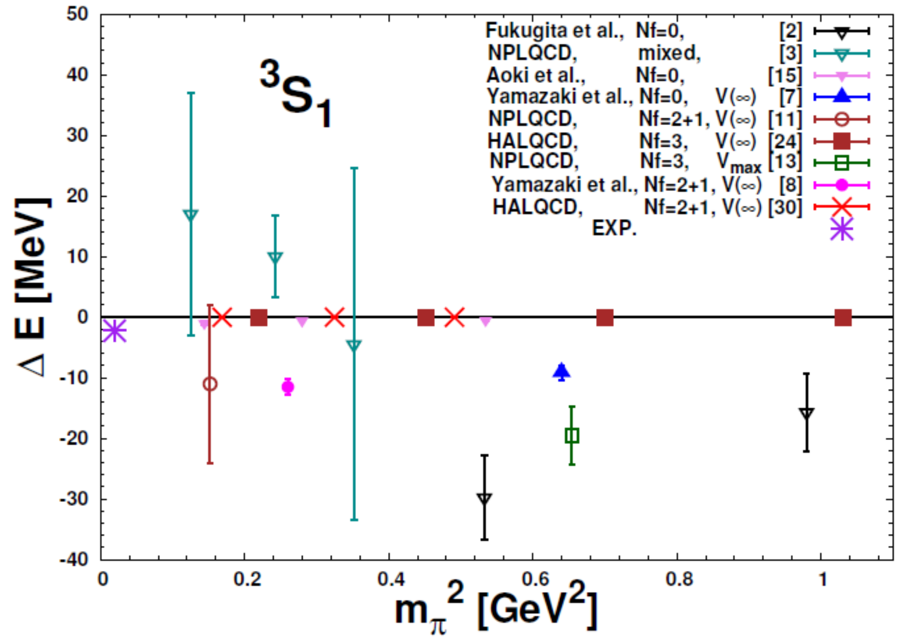
FIG. 14: EMPs associated with a $|\mathbf{P}| = 0$ $J^\pi = 0^+$ ^{4}He correlation function computed with the $24^3 \times 48$ (left), $32^3 \times 48$ (center) and $48^3 \times 64$ (right) ensembles. The inner (darker) shaded region

Spectroscopy on the lattice (v2)

“di-neutron”

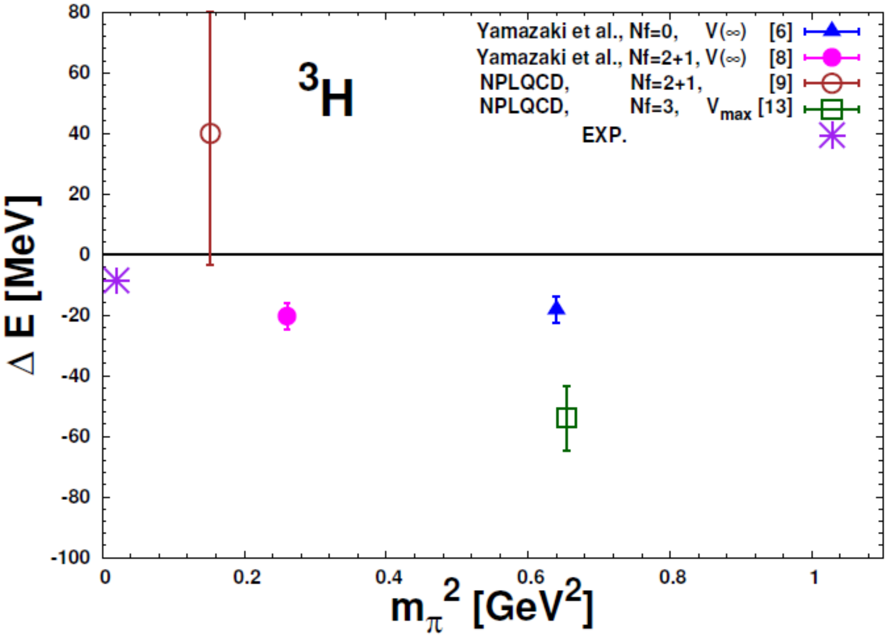


“deuteron”

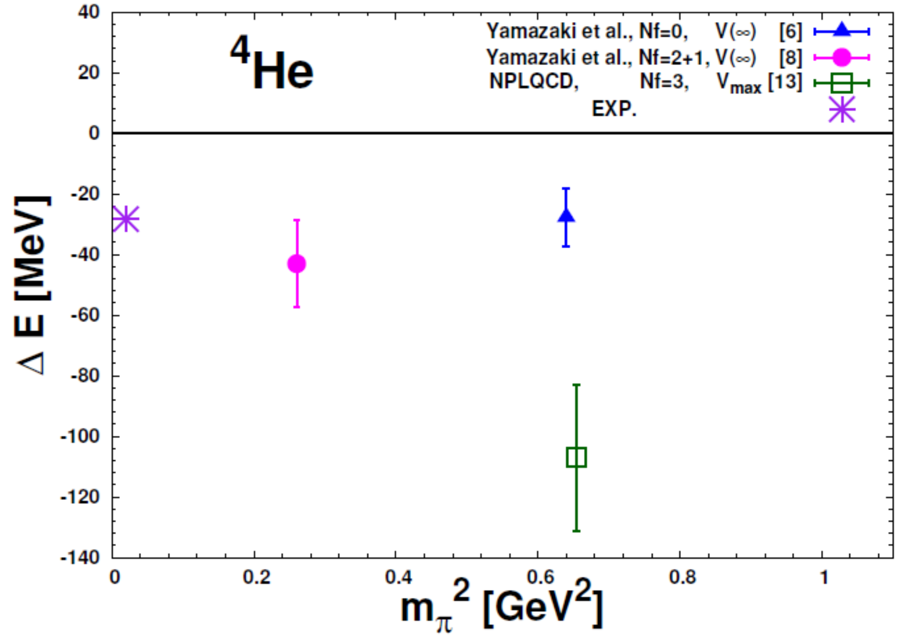


Spectroscopy on the lattice (v2)

$^3\mathrm{H}\ (=^3\mathrm{He})$



$^4\mathrm{He}$



Solution: Extract the signal from excited states

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437

E-indep of potential U(r,r') ➔ *(excited) scatt states share the same U(r,r')*

They are ***not contaminations, but signals***

$$\left(k_n^2/m - H_0\right)\psi_n(\boldsymbol{r}) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}')\psi_n(\boldsymbol{r}')$$

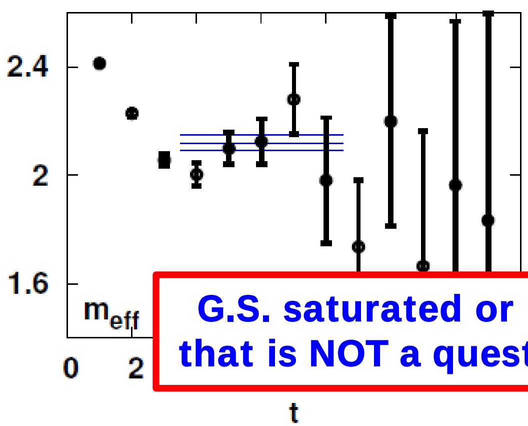
$$\Downarrow$$

$$\left(-\frac{\partial}{\partial t} + \frac{1}{4m}\frac{\partial^2}{\partial t^2} - H_0\right) R(\boldsymbol{r},t) = \int d\boldsymbol{r}' U(\boldsymbol{r},\boldsymbol{r}') R(\boldsymbol{r}',t)$$

$$2\sqrt{m^2+k_n^2} = E_n = -\frac{\partial}{\partial_t}$$

$$R(\boldsymbol{r},t) \equiv C_{NN}(\boldsymbol{r},t)/C_N(t)^2$$

Grand State (G.S.) saturation is NOT necessary !



G.S. saturated or not, that is NOT a question !

Explicit Lat calc for I=2 pipi phase shift

Beautiful agreement between

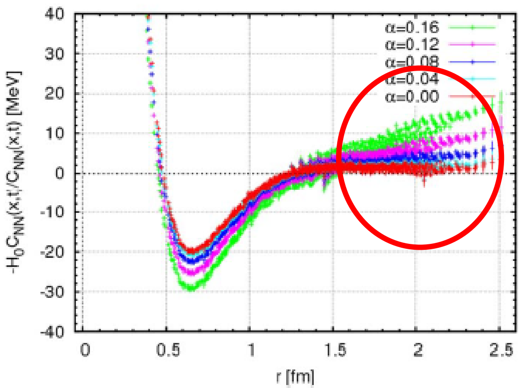
(1) Luscher's formula w/ g.s. saturation

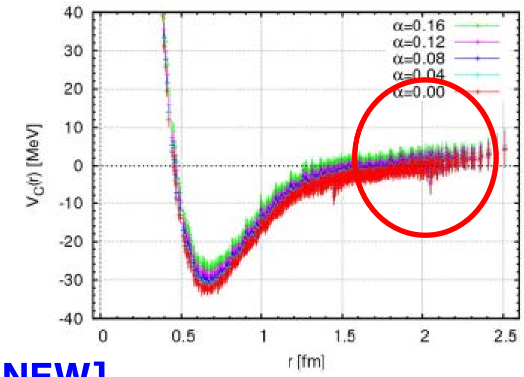
(2) the HAL QCD method w/ & w/o g.s. saturation

Explicit check on the new t-dep HAL method

NN system

N.Ishii et al. (HAL QCD Coll.) PLB712(2012)437





[OLD]

Different sources (creation op.) ➔ different results

“contaminations” from excited states

[NEW]

Results converged

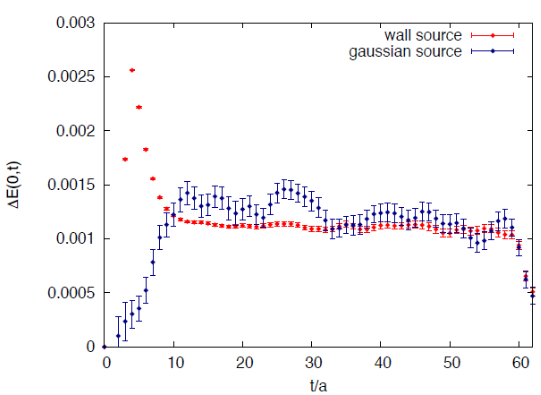
“signals” from excited states

pipi (I=2) system

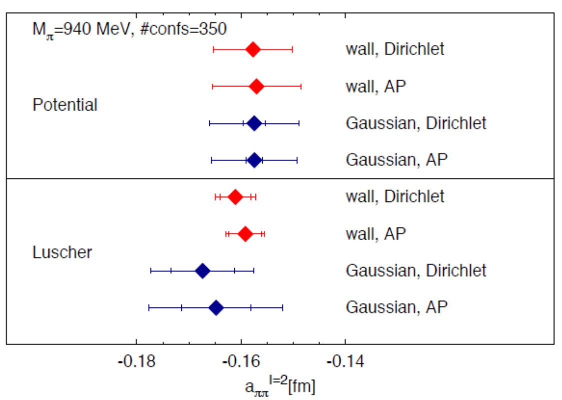
T.Kurth et al. (HAL-BMW Coll.) @ Lat2012

(Preliminary)

Effective mass



Scatt. length



Beautiful agreement between

(1) Luscher’s formula

w/ g.s. saturation

(2) the HAL QCD method

w/ & w/o g.s. saturation

Challenges toward the physical point (2)

- Enormous computational cost for correlators
  - # of Wick contraction (permutation)

$$N_{\rm perm} = N_u! \times N_d! \sim [\,(\tfrac{3}{2}A)!\,]^2$$ for mass number A

(🡐 **a factor of 2^A speedup** by inner-baryon exchange)

  - # of color / spinor contractions

$$N_{\rm loop} = \underset{\text{(color)}}{6^A} \cdot \underset{\text{(spinor)}}{4^A}$$

(🡐 **a factor of 2^A speedup** by "half-spin" method)

$$N = \epsilon_{abc}(q^T C\gamma_5 q)q$$

  - Total cost: $N_{\rm perm} \times N_{\rm loop}$
    - ^{2}H : 9 x 144 = 1 x 10^3
    - ^{3}H : 360 x 1728 = 6 x 10^5
    - ^{4}He : 32400 x 20736 = 7 x 10^8

c.f. T.Yamazaki et al., PRD81(2010)111504
$N_{\rm perm} = 1107$ for ^{4}He in the isospin limit

See also TD (HAL QCD Coll.) PoS LAT2010, 136

Solution: Unified contraction algorithm

TD, M.Endres, arXiv:1205.0585, CPC184(2013)117

- Traditional algorithm

color/spinor contractions (ξ')

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

color ϵ_{abc}, spinor $(C\gamma_5)$, etc.

Permutations

- New algorithm

[impose the same spacial label at source]

– Permutation applies to color/spinor indices at “Coeff”

Permuted Sum

$$\Pi^{2N} \simeq \langle qqqqqq(t)\bar{q}(\xi'_1)\bar{q}(\xi'_2)\bar{q}(\xi'_3)\bar{q}(\xi'_4)\bar{q}(\xi'_5)\bar{q}(\xi'_6)(t_0)\rangle \times \mathrm{Coeff}^{2N}(\xi'_1,\cdots,\xi'_6)$$

Sum over color/spinor unified list

– **Permutation DONE beforehand**
  - (Wick contraction and color/spinor contractions are unified)

– Significant improvement

$\boxed{\times 192 \text{ for } {}^3\mathrm{H}/{}^3\mathrm{He},\ \times 20736 \text{ for } {}^4\mathrm{He},\ \times 10^{11} \text{ for } {}^8\mathrm{Be}}$

(x add'l. speedup)