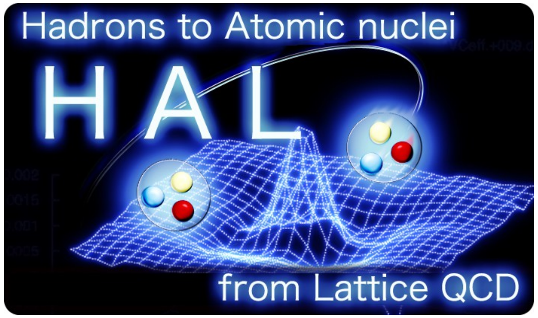
# Coupled channel approach to S=-2 baryon-baryon system in lattice QCD

Kenji Sasaki (*CCS, University of Tsukuba*)

for HAL QCD collaboration



***HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration***

**S. Aoki**
(*Univ. of Tsukuba*)

**B. Charron**
(*Univ. of Tokyo*)

**T. Doi**
(*RIKEN*)

**F. Etminan**
(*Univ. of Tsukuba*)

**T. Hatsuda**
(*RIKEN*)

**Y. Ikeda**
(*Tokyo Inst. Tech.*)

**T. Inoue**
(*Nihon Univ.*)

**N. Ishii**
(*Univ. of Tsukuba*)

**K. Murano**
(*RIKEN*)

**H. Nemura**
(*Univ. of Tsukuba*)

**M. Yamada**
(*Univ. of Tsukuba*)

# *Introduction*

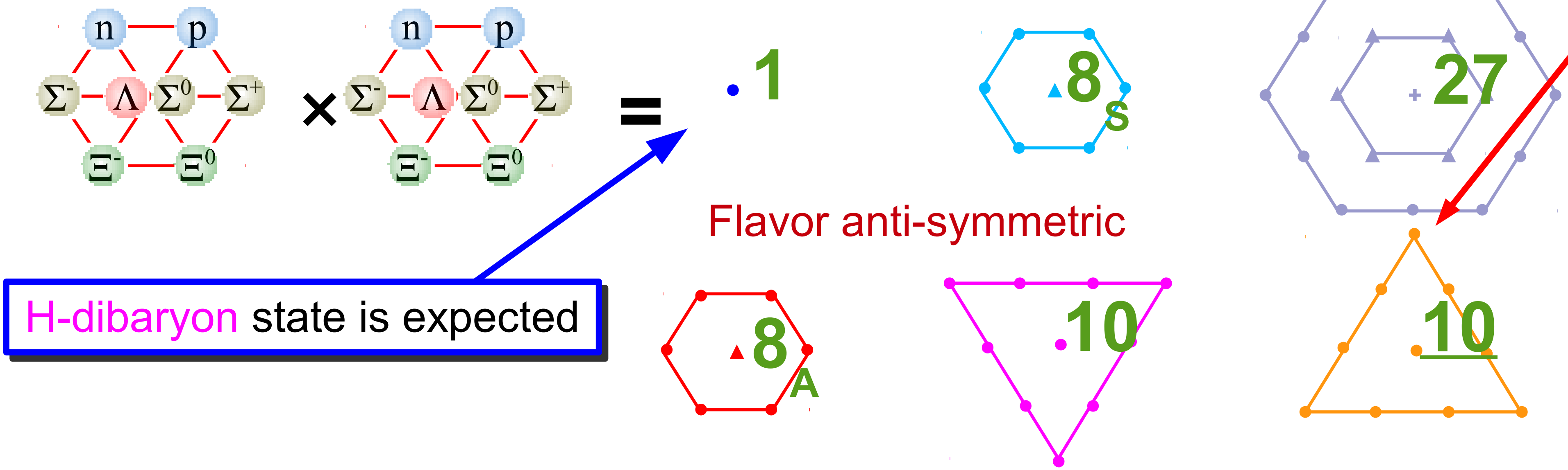
Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

One of the most important subject in the (hyper-) nuclear physics

- key to understand atomic nuclei,
- structure of neutron stars
- supernova explosions, etc

We need the way from quarks to hadrons

Three flavor (u,d,s) world



Flavor symmetric

Flavor anti-symmetric

NN sector

H-dibaryon state is expected

Wide variety of BB interaction

Coupled channel treatment is indispensable!

# “H-dibaryon”

Study of baryon-baryon interactions with strangeness S=-2

- Structures of double-$\Lambda$ hypernuelei and $\Xi$-hypernuclei.
- Fate of “H-dibaryon“ at physical point.

Recent Lattice QCD studies

- HAL QCD: SU(3) limit

  BE = 26MeV m$\pi$ = 470MeV

- NPLQCD: SU(3) breaking

  BE = 13MeV m$\pi$ = 390MeV

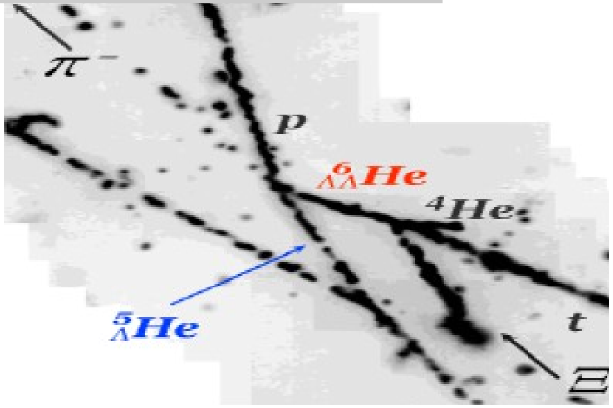
Conclusions of the “NAGARA Event”

K.Nakazawa and KEK-E176 & E373 collaborators

$\Lambda$–N attraction

$\Lambda$–$\Lambda$ weak attraction

$m_H \geq 2m_\Lambda$ - 6.9MeV



What happens on the physical point?

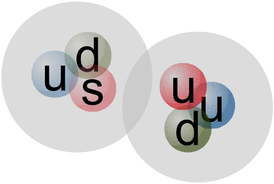
# *Quarks to hadrons*

$$L_{QCD}=\bar{q}\left(i\gamma_{\mu}D^{\mu}-m\right)q+\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}$$

Missing link

Due to the nonperturbative nature of low energy QCD

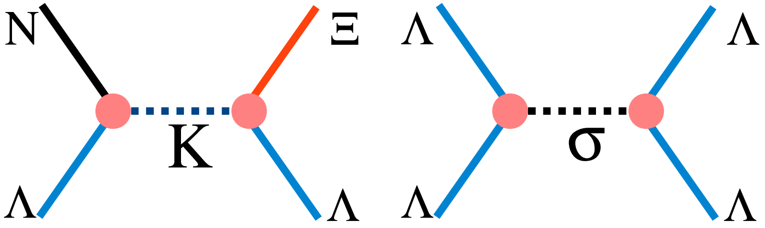
**Constituent quark model**



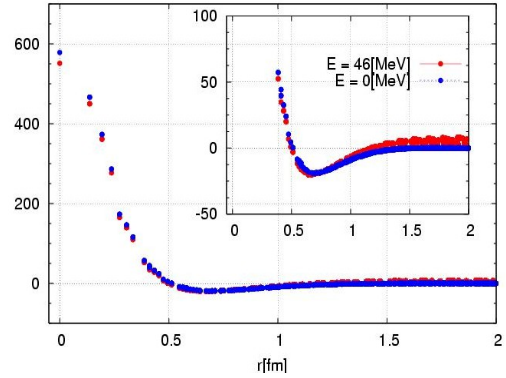
Quark Pauli effects are taking into account

**Meson exchange model**

Described by hadron dof with phenomenological repulsive core



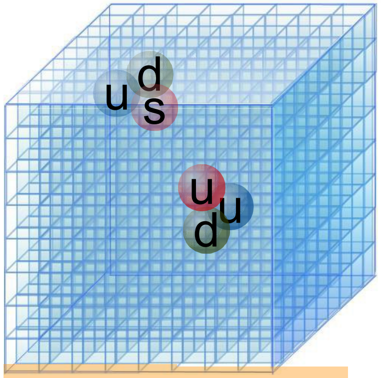
BB interaction (potential)



# *Quarks to hadrons*

$$L_{QCD}=\bar{q}\left(i\gamma_{\mu}D^{\mu}-m\right)q+\frac{1}{4}F^{a}_{\mu\nu}F^{a\mu\nu}$$

Lattice QCD simulation

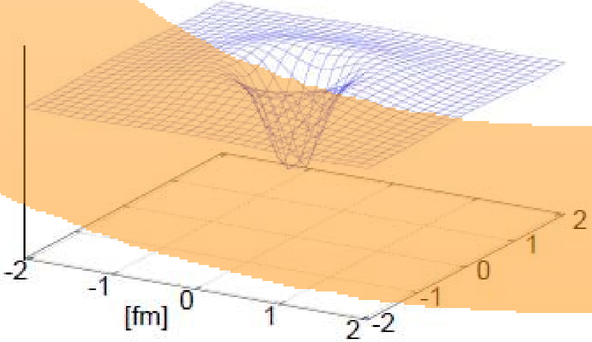


Lattice QCD simulation can connect the fundamental QCD with nuclear physics

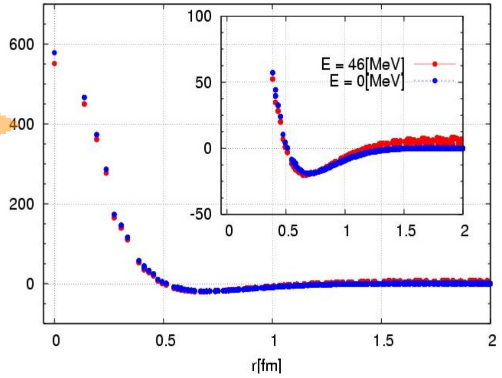
The potential through our method is faithful to the phase shift by QCD

**HAL QCD method**

NBS wave function



BB interaction (potential)



N. Ishii, S. Aoki and T. Hatsuda, Phys. Rev. Lett. **99** (2007) 022001

# Nambu-Bethe-Salpeter wave function

**Definition : equal time NBS w.f.**

$$\Psi_\nu(E,t-t_0,\vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t,\vec{x}+\vec{r}) B_j(t,\vec{x}) | E,\nu,t_0 \rangle$$

$$B = \epsilon^{abc}(q_a^T C \gamma_5 q_b) q_c$$

The ket stands for the eigenstate of the complete set of observables

E : Total energy of system
ν : other observables which needs to form the complete set

Local composite interpolating operators

$$p_\alpha = \epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) u(\xi_3)$$

$$\Sigma^0_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\sqrt{\frac{1}{2}}\left[d(\xi_1)s(\xi_2)u(\xi_3) + u(\xi_1)s(\xi_2)d(\xi_3)\right]$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3}(C\gamma_5)_{d_1 d_2}\delta_{d_3 \alpha}\sqrt{\frac{1}{6}}\left[d(\xi_1)s(\xi_2)u(\xi_3) + s(\xi_1)u(\xi_2)d(\xi_3) - 2u(\xi_1)d(\xi_2)s(\xi_3)\right]$$

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

$$\Psi(t-t_{0,}\vec{r}) \simeq A\frac{\sin(pr+\delta(E))}{pr}$$

N. Ishii *et al.* [HAL QCD Collab.], Phys. Lett. B **712** (2012) 437

# Schrödinger equation

- Define the energy-independent potential in Schrödinger equation (most general form)

$$\left(\frac{k^2}{2\mu}-H_0\right)\Psi(\vec{x})=\int U(\vec{x},\vec{y})\Psi(\vec{y})d^3y$$

  - Recent development : Time dependent method.

    We replace ψ to *R* defined below

$$\partial_t R_\alpha(\vec{x},E) \equiv \partial_t\left(\frac{A\Psi_\alpha(\vec{x},E)e^{-Et}}{e^{-m_A t}e^{-m_B t}}\right) \propto -\frac{p_\alpha^2}{2\mu_\alpha}R_\alpha(\vec{x},E)$$

  - Performing the derivative expansion for the interaction kernel

$$\left(\frac{-\partial}{\partial t}-H_0\right)R(\vec{x})=\int U(\vec{x},\vec{y})R(\vec{y})d^3y$$

- Taking the leading order of derivative expansion of non-local potential

$$U(\vec{x},\vec{y}) \simeq V_0(\vec{x})\delta(\vec{x}-\vec{y}) + V_1(\vec{x},\nabla)\delta(\vec{x}-\vec{y}) \cdots$$

- Finally local potential was obtained as

$$\boxed{V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{v})}+\frac{1}{2\mu}\frac{\nabla^2 R(\vec{x})}{R(\vec{x})}}$$

S.Aoki [HAL QCD collab.] Proc. Jpn. Acad., Ser. B, **87** 509

# *Coupled channel Schrödinger equation*

**Preparation for the NBS wave function**

$$\Psi^{\alpha}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\alpha}(t,\vec{r}) | E \rangle$$

$$\Psi^{\beta}(E,t,\vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^{\beta}(t,\vec{x}) | E \rangle$$

Two-channel coupling case

**The same “in” state**

**Inside the interaction range**

In the *leading order of velocity expansion* of non-local potential,

**Coupled channel Schrödinger equation.**

$$\left( \frac{p_{\alpha}^2}{2\mu_{\alpha}} + \frac{\nabla^2}{2\mu_{\alpha}} \right) \psi^{\alpha}(\vec{x},E) = V^{\alpha}{}_{\alpha}(\vec{x}) \psi^{\alpha}(\vec{x},E) + V^{\alpha}{}_{\beta}(\vec{x}) \psi^{\beta}(\vec{x},E)$$

$\mu_{\alpha}$ : *reduced mass*

$p_{\alpha}$ : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of *R*.

$$R_I^{B_1 B_2}(t,\vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t,\vec{x}+\vec{r}) B_2(t,\vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t}$$

$$\begin{pmatrix} V^{\alpha}{}_{\alpha}(\vec{r}) & V^{\alpha}{}_{\beta}(\vec{r}) x \\ V^{\beta}{}_{\alpha}(\vec{r}) x^{-1} & V^{\beta}{}_{\beta}(\vec{r}) \end{pmatrix} = \begin{pmatrix} (\frac{\nabla^2}{2\mu_{\alpha}} - \frac{\partial}{\partial t}) R^{\alpha}_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_{\beta}} - \frac{\partial}{\partial t}) R^{\beta}_{I2}(\vec{r},E) \\ (\frac{\nabla^2}{2\mu_{\alpha}} - \frac{\partial}{\partial t}) R^{\alpha}_{I1}(\vec{r},E) & (\frac{\nabla^2}{2\mu_{\beta}} - \frac{\partial}{\partial t}) R^{\beta}_{I2}(\vec{r},E) \end{pmatrix} \begin{pmatrix} R^{\alpha}_{I1}(\vec{r},E) & R^{\beta}_{I1}(\vec{r},E) \\ R^{\alpha}_{I2}(\vec{r},E) & R^{\beta}_{I2}(\vec{r},E) \end{pmatrix}^{-1}$$

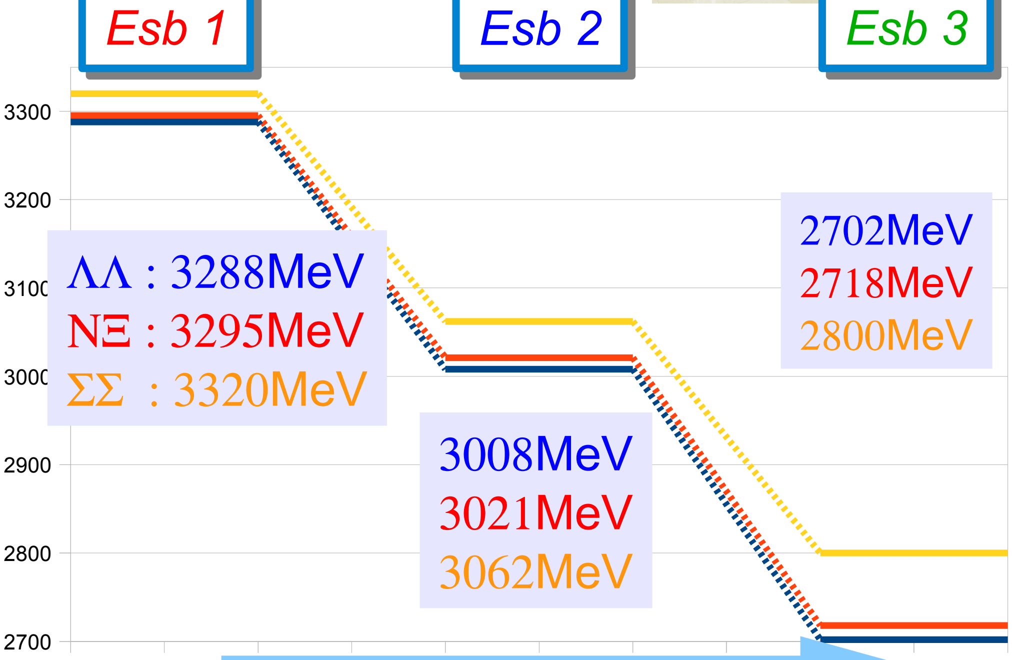
$$x = \frac{\exp(-(m_{\alpha_1}+m_{\alpha_2})t)}{\exp(-(m_{\beta_1}+m_{\beta_2})t)}$$

# *Numerical setup*

- 2+1 flavor gauge configurations by PACS-CS collaboration.
  - RG improved gauge action & O(a) improved clover quark action
  - $\beta = 1.90$, $a^{-1} = 2.176$ [*GeV*], $32^3$x64 lattice, $L = 2.902$ [*fm*].
  - $\kappa_s = 0.13640$ is fixed, $\kappa_{ud}$ = 0.13700, 0.13727 and 0.13754 are chosen.
- Flat wall source is considered to produce S-wave B-B state.
- The KEK computer system A resources are used.

| In unit of MeV | *Esb 1* | *Esb 2* | *Esb 3* |
|---|---|---|---|
| $\pi$ | 701±1 | 570±2 | 411±2 |
| K | 789±1 | 713±2 | 635±2 |
| $m_\pi/m_K$ | 0.89 | 0.80 | 0.65 |
| N | 1585±5 | 1411±12 | 1215±12 |
| $\Lambda$ | 1644±5 | 1504±10 | 1351± 8 |
| $\Sigma$ | 1660±4 | 1531±11 | 1400±10 |
| $\Xi$ | 1710±5 | 1610± 9 | 1503± 7 |

u,d quark masses lighter

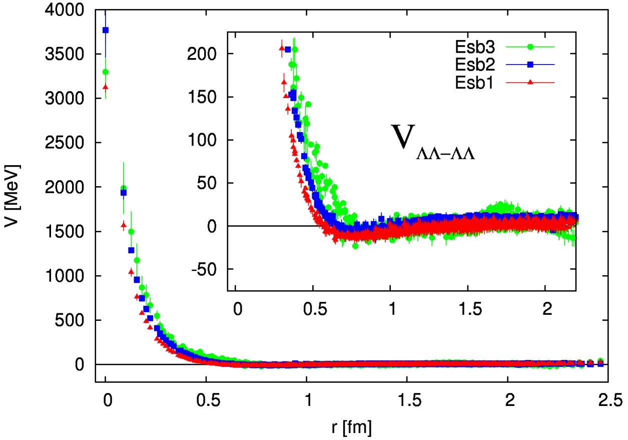


SU(3) breaking effects becomes larger

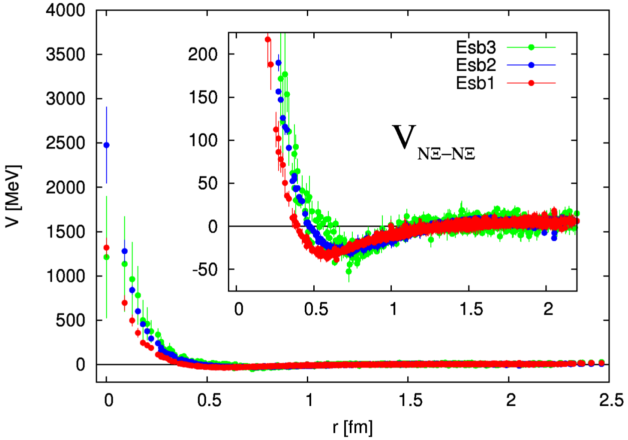
# *ΛΛ, NΞ, ΣΣ (I=0)* $^1S_0$ *channel*

***Esb1 : mπ= 701 MeV***
***Esb2 : mπ= 570 MeV***
***Esb3 : mπ= 411 MeV***

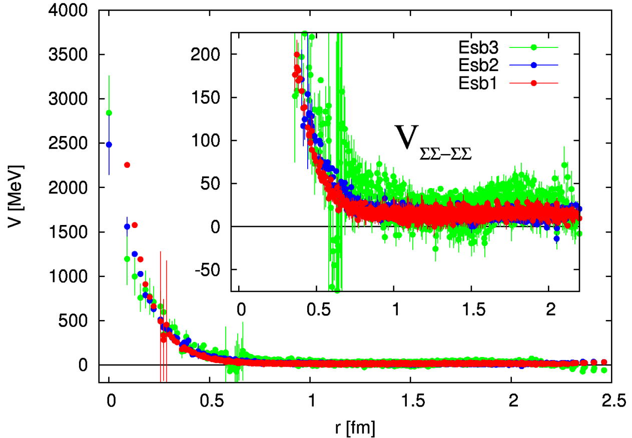
## **Diagonal elements**



shallow attractive pocket



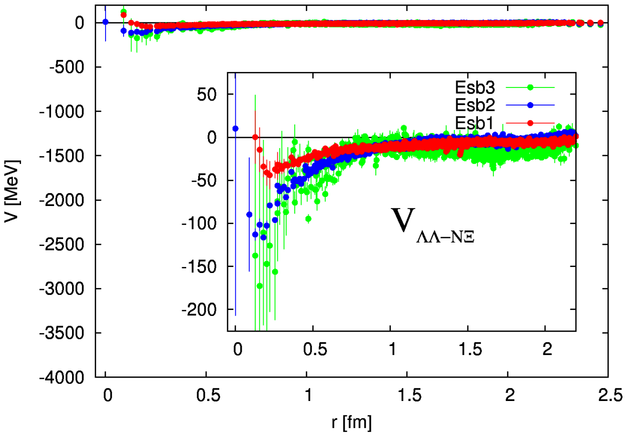
Deeper attractive pocket

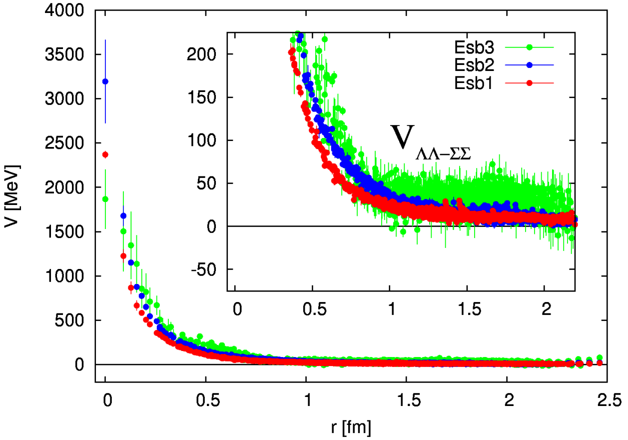


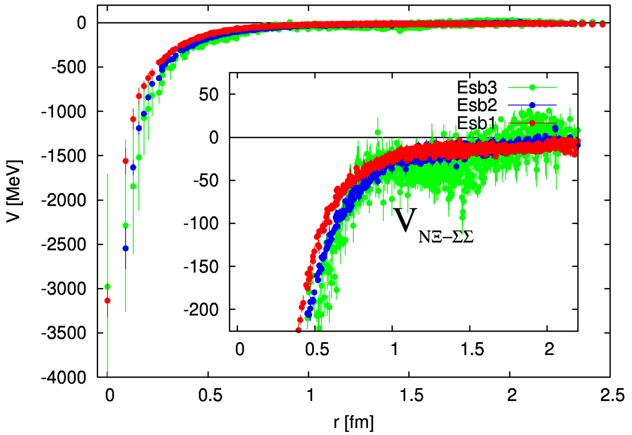
Strongly repulsive

All channels have repulsive core

## **Off-diagonal elements**





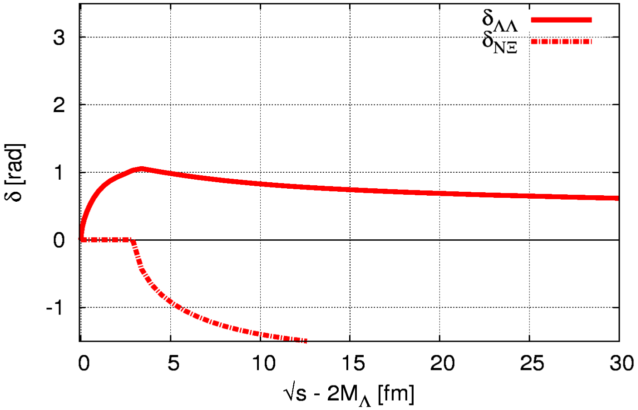


Relatively weaker than the others

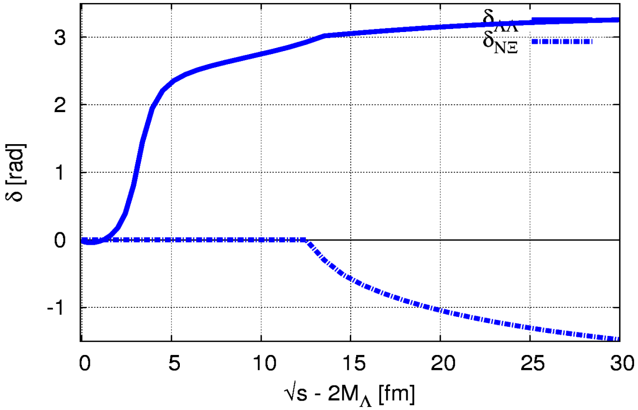
In this channel, our group found the "H-dibaryon" in the SU(3) limit.

# *$\Lambda\Lambda$ and $N\Xi$ phase shifts*

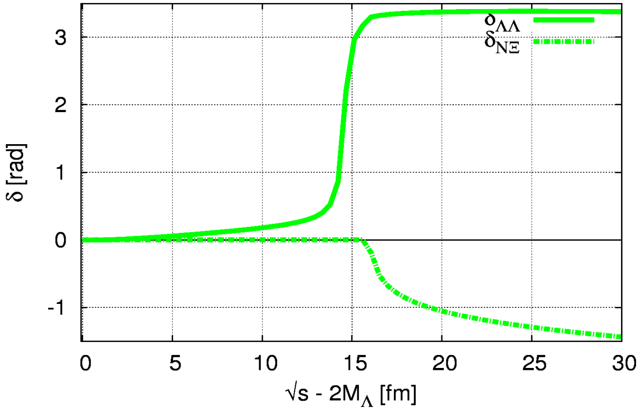
***Esb1 : $m_\pi$= 701 MeV***



***Esb2 : $m_\pi$= 570 MeV***



***Esb3 : $m_\pi$= 411 MeV***



**Preliminary!**

- Esb1:
  - Bound H-dibaryon
- Esb2:
  - H-dibaryon is near the $\Lambda\Lambda$ threshold
- Esb3:
  - The H-dibaryon resonance energy is close to $N\Xi$ threshold..

- We can see the clear resonance shape in $\Lambda\Lambda$ phase shifts for Esb2 and 3.
- The “binding energy” of H-dibaryon from $N\Xi$ threshold becomes smaller as decreasing of quark masses.

# *Summary and outlook*

- We have investigated the BB system with strangeness from lattice QCD.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
- Potentials are derived from NBS wave functions calculated with PACS-CS configurations
- Quark mass dependence of potentials can be seen as an enhancement of repulsive core.
- H-dibaryon tends to be resonance as decreasing light quark masses
- SU(3) breaking effects are still small even in $m\pi/mK$=0.65 situation but it would be change drastically at physical situation $m\pi/mK$=0.28.

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# *Backup slides*

# Energy independent potential in coupled channel S.E.

Inside the interaction range, we can define the interaction kernel

$$\begin{pmatrix} p^2+\nabla & q^2+\nabla \\ p^2+\nabla & q^2+\nabla \end{pmatrix}\begin{pmatrix} \psi^a_a(\vec{x},E) & \psi^b_a(\vec{x},E) \\ \psi^b_a(\vec{x},E) & \psi^b_a(\vec{x},E) \end{pmatrix} = \begin{pmatrix} K^a_a(\vec{x},E) & K^a_b(\vec{x},E) \\ K^b_a(\vec{x},E) & K^b_b(\vec{x},E) \end{pmatrix}$$

Factorization of the interaction kernel

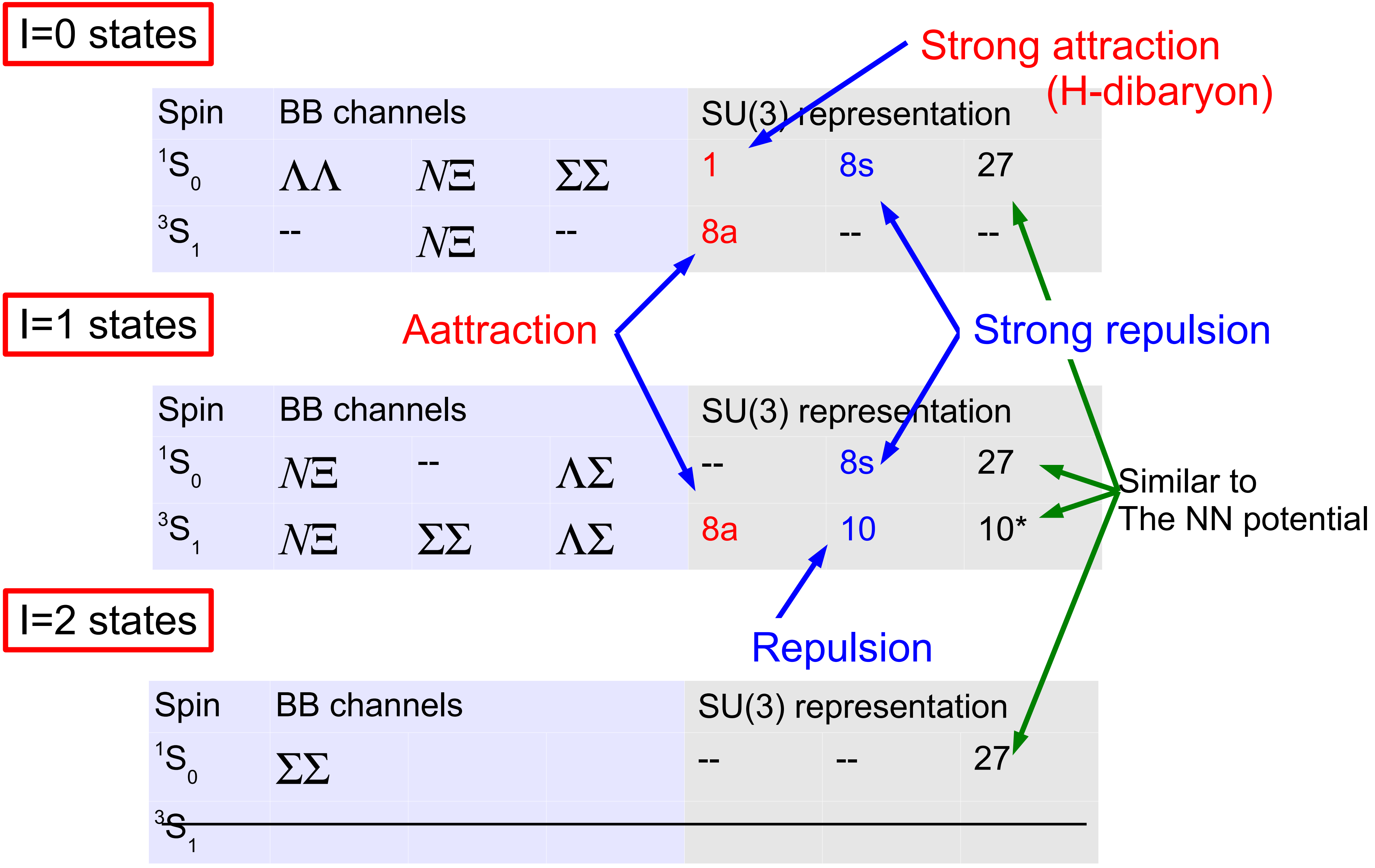
$$\begin{pmatrix} K^a_a(\vec{x},E) & K^a_b(\vec{x},E) \\ K^b_a(\vec{x},E) & K^b_b(\vec{x},E) \end{pmatrix} = \int dy \begin{pmatrix} U^a_a(\vec{x},\vec{y}) & U^a_b(\vec{x},\vec{y}) \\ U^b_a(\vec{x},\vec{y}) & U^b_b(\vec{x},\vec{y}) \end{pmatrix}\begin{pmatrix} \psi^a_a(\vec{y},E) & \psi^a_b(\vec{y},E) \\ \psi^b_a(\vec{y},E) & \psi^b_b(\vec{y},E) \end{pmatrix}$$

$$\int dx \begin{pmatrix} \tilde{\psi}^a_a(\vec{x},E') & \tilde{\psi}^a_b(\vec{x},E') \\ \tilde{\psi}^b_a(\vec{x},E') & \tilde{\psi}^b_b(\vec{x},E') \end{pmatrix}\begin{pmatrix} \psi^a_a(\vec{x},E) & \psi^a_b(\vec{x},E) \\ \psi^b_a(\vec{x},E) & \psi^b_b(\vec{x},E) \end{pmatrix} = 2\pi\delta(E-E')$$

$$\begin{pmatrix} U^a_a(\vec{x},\vec{y}) & U^a_b(\vec{x},\vec{y}) \\ U^b_a(\vec{x},\vec{y}) & U^b_b(\vec{x},\vec{y}) \end{pmatrix} = \int \frac{dE}{2\pi}\begin{pmatrix} K^a_a(\vec{x},E) & K^a_b(\vec{x},E) \\ K^b_a(\vec{x},E) & K^b_b(\vec{x},E) \end{pmatrix}\begin{pmatrix} \tilde{\psi}^a_a(\vec{y},E) & \tilde{\psi}^a_b(\vec{y},E) \\ \tilde{\psi}^b_a(\vec{y},E) & \tilde{\psi}^b_b(\vec{y},E) \end{pmatrix}$$

*Energy independent potential in Schrödinger equation.*

# *Lists of channels*



**I=0 states**

| Spin | BB channels | | | SU(3) representation | | |
|---|---|---|---|---|---|---|
| $^1S_0$ | $\Lambda\Lambda$ | $N\Xi$ | $\Sigma\Sigma$ | 1 | 8s | 27 |
| $^3S_1$ | -- | $N\Xi$ | -- | 8a | -- | -- |

**I=1 states**

| Spin | BB channels | | | SU(3) representation | | |
|---|---|---|---|---|---|---|
| $^1S_0$ | $N\Xi$ | -- | $\Lambda\Sigma$ | -- | 8s | 27 |
| $^3S_1$ | $N\Xi$ | $\Sigma\Sigma$ | $\Lambda\Sigma$ | 8a | 10 | 10* |

**I=2 states**

| Spin | BB channels | | | SU(3) representation | | |
|---|---|---|---|---|---|---|
| $^1S_0$ | $\Sigma\Sigma$ | | | -- | -- | 27 |
| ~~$^3S_1$~~ | | | | | | |

Strong attraction
(H-dibaryon)

Aattraction

Strong repulsion

Similar to
The NN potential

Repulsion

# Baryon operators

$$p_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) u(\xi_3)$$
$$n_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) d(\xi_2) d(\xi_3)$$

$$\Sigma^+_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, u(\xi_1) s(\xi_2) u(\xi_3)$$
$$\Sigma^0_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{2}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3) \right]$$
$$\Sigma^-_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, d(\xi_1) s(\xi_2) d(\xi_3)$$

$$\Lambda_\alpha = -\epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2u(\xi_1) d(\xi_2) s(\xi_3) \right]$$

$$\Xi^0_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, s(\xi_1) u(\xi_2) s(\xi_3)$$
$$\Xi^-_\alpha = \epsilon_{c_1 c_2 c_3} (C\gamma_5)_{d_1 d_2} \delta_{d_3 \alpha}\, s(\xi_1) d(\xi_2) s(\xi_3)$$

- With corrected phase $\bar{1} = -\epsilon^{123} = -(ds - sd) = sd - ds$

# Irreducible BB source operator

$$\overline{BB}^{(27)} = +\sqrt{\frac{27}{40}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{1}{40}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{12}{40}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB}^{(8s)} = -\sqrt{\frac{1}{5}}\,\overline{\Lambda}\,\overline{\Lambda} - \sqrt{\frac{3}{5}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{1}{5}}\,\overline{N}\,\overline{\Xi}$$

$$\overline{BB}^{(1)} = -\sqrt{\frac{1}{8}}\,\overline{\Lambda}\,\overline{\Lambda} + \sqrt{\frac{3}{8}}\,\overline{\Sigma}\,\overline{\Sigma} + \sqrt{\frac{4}{8}}\,\overline{N}\,\overline{\Xi}$$

with

$$\overline{\Sigma}\,\overline{\Sigma} = +\sqrt{\frac{1}{3}}\,\overline{\Sigma^+}\,\overline{\Sigma^-} - \sqrt{\frac{1}{3}}\,\overline{\Sigma^0}\,\overline{\Sigma^0} + \sqrt{\frac{1}{3}}\,\overline{\Sigma^-}\,\overline{\Sigma^+}$$

$$\overline{N}\,\overline{\Xi} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} + \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} - \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

$$\overline{BB}^{(10^*)} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{n} - \sqrt{\frac{1}{2}}\,\overline{n}\,\overline{p}$$

$$\overline{BB}^{(10)} = +\sqrt{\frac{1}{2}}\,\overline{p}\,\overline{\Sigma^+} - \sqrt{\frac{1}{2}}\,\overline{\Sigma^+}\,\overline{p}$$

$$\overline{BB}^{(8a)} = +\sqrt{\frac{1}{4}}\,\overline{p}\,\overline{\Xi^-} - \sqrt{\frac{1}{4}}\,\overline{\Xi^-}\,\overline{p} - \sqrt{\frac{1}{4}}\,\overline{n}\,\overline{\Xi^0} + \sqrt{\frac{1}{4}}\,\overline{\Xi^0}\,\overline{n}$$

# Isospin combinations of BB operator

$\Lambda\Lambda,\ p\Xi^-,\ n\Xi^0,\ \Sigma^+\Sigma^-,\ \Sigma^0\Sigma^0,\ \Lambda\Sigma^0$

I=0 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{3}}\Sigma^+\Sigma^- - \sqrt{\frac{1}{3}}\Sigma^0\Sigma^0 + \sqrt{\frac{1}{3}}\Sigma^-\Sigma^+$$

$$N\Xi = +\sqrt{\frac{1}{2}}\,p\Xi^- - \sqrt{\frac{1}{2}}\,n\Xi^0$$

I=1 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{2}}\Sigma^+\Sigma^- - \sqrt{\frac{1}{2}}\Sigma^-\Sigma^+$$

$$N\Xi = +\sqrt{\frac{1}{2}}\,p\Xi^- + \sqrt{\frac{1}{2}}\,n\Xi^0$$

I=2 operstors

$$\Sigma\Sigma = +\sqrt{\frac{1}{6}}\Sigma^+\Sigma^- + \sqrt{\frac{4}{6}}\Sigma^0\Sigma^0 + \sqrt{\frac{1}{6}}\Sigma^-\Sigma^+$$