Coupled channel approach to $S=-2$ baryon-baryon system in lattice QCD

Kenji Sasaki (CCS, University of Tsukuba)

for HAL QCD collaboration

HAL (Hadrons to Atomic nuclei from Lattice) QCD Collaboration

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Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

One of the most important subject in the (hyper-) nuclear physics

- key to understand atomic nuclei,
- structure of neutron stars
- supernova explosions, etc

We need the way from quarks to hadrons

Three flavor (u,d,s) world

H-dibaryon state is expected

Wide variety of BB interaction

Coupled channel treatment is indispensable!
“H-dibaryon”

Study of baryon-baryon interactions with strangeness $S=-2$

- Structures of double-$\Lambda$ hypernuclei and $\Xi$-hypernuclei.
- Fate of “H-dibaryon” at physical point.

Recent Lattice QCD studies

- HAL QCD: SU(3) limit
  - $BE = 26\text{MeV}$ $m_\pi = 470\text{MeV}$
- NPLQCD: SU(3) breaking
  - $BE = 13\text{MeV}$ $m_\pi = 390\text{MeV}$

Conclusions of the “NAGARA Event”

- $\Lambda-N$ attraction
- $\Lambda-\Lambda$ weak attraction
  - $m_H \geq 2m_\Lambda - 6.9\text{MeV}$

What happens on the physical point?

K. Nakazawa and KEK-E176 & E373 collaborators
Quarks to hadrons

\[ L_{QCD} = \bar{q} \left( i \gamma_\mu D^\mu - m \right) q + \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} \]

Due to the nonperturbative nature of low energy QCD

Constituent quark model
Quark Pauli effects are taking into account
Meson exchange model
Described by hadron dof with phenomenological repulsive core

BB interaction (potential)
Quarks to hadrons

\[ L_{QCD} = \bar{q} \left( i \gamma_\mu D^\mu - m \right) q + \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} \]

Lattice QCD simulation can connect the fundamental QCD with nuclear physics

The potential through our method is faithful to the phase shift by QCD


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Nambu-Bethe-Salpeter wave function

**Definition**: equal time NBS w.f.

\[
\Psi_{\nu}(E, t-t_0, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_i(t, \vec{x} + \vec{r}) B_j(t, \vec{x}) | E, \nu, t_0 \rangle 
\]

\[
B = \epsilon^{abc} (q_a^T C \gamma_5 q_b) q_c
\]

The ket stands for the eigenstate of the complete set of observables

E : Total energy of system
\nu : other observables which needs to form the complete set

Local composite interpolating operators

\[
p_{\alpha} = \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3)
\]

\[
\Sigma_{\alpha}^0 = - \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \frac{1}{2} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3) \right]
\]

\[
\Lambda_{\alpha} = - \epsilon_{c_1 c_2 c_3} (C \gamma_5)_{d_1 d_2} \delta_{d_3 \alpha} \frac{1}{6} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3) \right]
\]

NBS wave function has a same asymptotic form with quantum mechanics.
(NBS wave function is characterized from phase shift)

\[
\Psi(t-t_0, \vec{r}) \approx A \frac{\sin(pr + \delta(E))}{pr}
\]
Define the energy-independent potential in Schrödinger equation (most general form)

\[
\left(\frac{k^2}{2\mu} - H_0\right)\psi(\vec{x}) = \int U(\vec{x}, \vec{y})\psi(\vec{y})d^3y
\]

Recent development: Time dependent method.

We replace \(\psi\) to \(R\) defined below

\[
\partial_t R_\alpha(\vec{x}, E) \equiv \partial_t \left( \frac{A\Psi_\alpha(\vec{x}, E)e^{-Et}}{e^{-m_\alpha t}e^{-m_B t}} \right) \propto -\frac{p_\alpha^2}{2\mu_\alpha} R_\alpha(\vec{x}, E)
\]

Performing the derivative expansion for the interaction kernel

\[
\left(\frac{-\partial}{\partial t} - H_0\right)R(\vec{x}) = \int U(\vec{x}, \vec{y}) R(\vec{y})d^3y
\]

Taking the leading order of derivative expansion of non-local potential

\[
U(\vec{x}, \vec{y}) \simeq V_0(\vec{x})\delta(\vec{x} - \vec{y}) + V_1(\vec{x}, \nabla)\delta(\vec{x} - \vec{y}) \cdots
\]

Finally local potential was obtained as

\[
V(\vec{x}) = -\frac{\partial_t R(\vec{r})}{R(\vec{v})} + \frac{1}{2\mu} \frac{\nabla^2 R(\vec{x})}{R(\vec{x})}
\]
**Coupled channel Schrödinger equation**

### Preparation for the NBS wave function

\[ \Psi^\alpha(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\alpha(t, \vec{x}) | E \rangle \]

\[ \Psi^\beta(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\beta(t, \vec{x}) | E \rangle \]

### Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

\[ \left( \frac{p_\alpha^2}{2 \mu_\alpha} + \frac{\nabla^2}{2 \mu_\alpha} \right) \psi^\alpha(\vec{x}, E) = V^\alpha_\alpha(\vec{x}) \psi^\alpha(\vec{x}, E) + V^\alpha_\beta(\vec{x}) \psi^\beta(\vec{x}, E) \]

\[ \mu_\alpha : \text{reduced mass} \]
\[ p_\alpha : \text{asymptotic momentum}. \]

### Two-channel coupling case

The same “in” state

Asymptotic momentum are replaced by the time-derivative of \( R \).

\[ R_{I_1B_2}^{B_1}(t, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{x}+\vec{r}) B_2(t, \vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1+m_2)t} \]

\[ \left( \begin{array}{cc} V^\alpha_\alpha(\vec{r}) & V^\alpha_\beta(\vec{r})x \vspace{1em} \\
 V^\beta_\alpha(\vec{r})x^{-1} & V^\beta_\beta(\vec{r}) \end{array} \right) = \left( \begin{array}{cc} \left( \frac{\nabla^2}{2 \mu_\alpha} - \frac{\partial}{\partial t} \right) R^\alpha_{II}(\vec{r}, E) & \left( \frac{\nabla^2}{2 \mu_\beta} - \frac{\partial}{\partial t} \right) R^\beta_{II}(\vec{r}, E) \\
 \left( \frac{\nabla^2}{2 \mu_\alpha} - \frac{\partial}{\partial t} \right) R^\alpha_{I\bar{I}}(\vec{r}, E) & \left( \frac{\nabla^2}{2 \mu_\beta} - \frac{\partial}{\partial t} \right) R^\beta_{I\bar{I}}(\vec{r}, E) \end{array} \right) \left( \begin{array}{cc} R^\alpha_{II}(\vec{r}, E) & R^\beta_{II}(\vec{r}, E) \vspace{1em} \\
 R^\alpha_{I\bar{I}}(\vec{r}, E) & R^\beta_{I\bar{I}}(\vec{r}, E) \end{array} \right)^{-1} \]

\[ x = \frac{\exp(-(-m_\alpha+m_\beta)t)}{\exp(-(-m_\beta+m_\alpha)t)} \]
2+1 flavor gauge configurations by PACS-CS collaboration.

- RG improved gauge action & O(a) improved clover quark action
- $\beta = 1.90$, $a^{-1} = 2.176 \text{ [GeV]}$, $32^3 \times 64$ lattice, $L = 2.902 \text{ [fm]}$.
- $\kappa_s = 0.13640$ is fixed, $\kappa_{ud} = 0.13700, 0.13727$ and $0.13754$ are chosen.

Flat wall source is considered to produce S-wave B-B state.

The KEK computer system A resources are used.

<table>
<thead>
<tr>
<th>In unit of MeV</th>
<th>Esb 1</th>
<th>Esb 2</th>
<th>Esb 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>701±1</td>
<td>570±2</td>
<td>411±2</td>
</tr>
<tr>
<td>$K$</td>
<td>789±1</td>
<td>713±2</td>
<td>635±2</td>
</tr>
<tr>
<td>$m_\pi/m_K$</td>
<td>0.89</td>
<td>0.80</td>
<td>0.65</td>
</tr>
<tr>
<td>$N$</td>
<td>1585±5</td>
<td>1411±12</td>
<td>1215±12</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>1644±5</td>
<td>1504±10</td>
<td>1351±8</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>1660±4</td>
<td>1531±11</td>
<td>1400±10</td>
</tr>
<tr>
<td>$\Xi$</td>
<td>1710±5</td>
<td>1610±9</td>
<td>1503±7</td>
</tr>
</tbody>
</table>

u,d quark masses lighter

SU(3) breaking effects becomes larger

$\Lambda \Lambda : 3288\text{MeV}$
$\Lambda \Xi : 3295\text{MeV}$
$\Sigma \Sigma : 3320\text{MeV}$

2702MeV
2718MeV
2800MeV
3008MeV
3021MeV
3062MeV
In this channel, our group found the “H-dibaryon” in the SU(3) limit.

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**ΛΛ and ΝΞ phase shifts**

- **Esb1**: Bound H-dibaryon
- **Esb2**: H-dibaryon is near the ΛΛ threshold
- **Esb3**: The H-dibaryon resonance energy is close to ΝΞ threshold.

- Preliminary!

- We can see the clear resonance shape in ΛΛ phase shifts for Esb2 and 3.
- The “binding energy” of H-dibaryon from ΝΞ threshold becomes smaller as decreasing of quark masses.

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Summary and outlook

- We have investigated the BB system with strangeness from lattice QCD.
- In order to deal with a variety of interactions, we extend our method to the coupled channel formalism.
- Potentials are derived from NBS wave functions calculated with PACS-CS configurations.
- Quark mass dependence of potentials can be seen as an enhancement of repulsive core.
- H-dibaryon tends to be resonance as decreasing light quark masses.
- SU(3) breaking effects are still small even in \(m_\pi/m_K=0.65\) situation but it would be change drastically at physical situation \(m_\pi/m_K=0.28\).
Backup slides
Inside the interaction range, we can define the interaction kernel

\[
\begin{pmatrix}
 p^2 + \nabla & q^2 + \nabla \\
 p^2 + \nabla & q^2 + \nabla
\end{pmatrix} \begin{pmatrix}
 \psi_a(x, E) & \psi_b(x, E) \\
 \psi_a(x, E) & \psi_b(x, E)
\end{pmatrix} = \begin{pmatrix}
 K_a(x, E) & K_a(x, E) \\
 K_b(x, E) & K_b(x, E)
\end{pmatrix}
\]

Factorization of the interaction kernel

\[
\begin{pmatrix}
 K_a(x, E) & K_a(x, E) \\
 K_b(x, E) & K_b(x, E)
\end{pmatrix} = \int dy \begin{pmatrix}
 U_a(x, \bar{y}) & U_b(x, \bar{y}) \\
 U_a(x, \bar{y}) & U_b(x, \bar{y})
\end{pmatrix} \begin{pmatrix}
 \psi_a(\bar{y}, E) & \psi_b(\bar{y}, E) \\
 \psi_a(\bar{y}, E) & \psi_b(\bar{y}, E)
\end{pmatrix}
\]

\[
\int dx \begin{pmatrix}
 \tilde{\psi}_a(x, E') & \tilde{\psi}_b(x, E') \\
 \tilde{\psi}_a(x, E') & \tilde{\psi}_b(x, E')
\end{pmatrix} \begin{pmatrix}
 \psi_a(x, E) & \psi_b(x, E) \\
 \psi_a(x, E) & \psi_b(x, E)
\end{pmatrix} = 2\pi \delta(E - E')
\]

\[
\begin{pmatrix}
 U_a(x, \bar{y}) & U_b(x, \bar{y}) \\
 U_a(x, \bar{y}) & U_b(x, \bar{y})
\end{pmatrix} = \int \frac{dE}{2\pi} \begin{pmatrix}
 K_a(x, E) & K_a(x, E) \\
 K_b(x, E) & K_b(x, E)
\end{pmatrix} \begin{pmatrix}
 \tilde{\psi}_a(\bar{y}, E) & \tilde{\psi}_b(\bar{y}, E) \\
 \tilde{\psi}_a(\bar{y}, E) & \tilde{\psi}_b(\bar{y}, E)
\end{pmatrix}
\]

Energy independent potential in Schrödinger equation.
### Lists of channels

#### I=0 states

<table>
<thead>
<tr>
<th>Spin</th>
<th>BB channels</th>
<th>SU(3) representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S_0$</td>
<td>$\Lambda\Lambda$ $\Xi\Xi$ $\Sigma\Sigma$</td>
<td>1 $8s$ 27</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>$\Xi\Xi$</td>
<td>8a -- --</td>
</tr>
</tbody>
</table>

#### I=1 states

<table>
<thead>
<tr>
<th>Spin</th>
<th>BB channels</th>
<th>SU(3) representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S_0$</td>
<td>$\Xi\Xi$ -- $\Lambda\Sigma$</td>
<td>-- $8s$ 27</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td>$\Xi\Xi$ $\Sigma\Sigma$ $\Lambda\Sigma$</td>
<td>8a 10 10*</td>
</tr>
</tbody>
</table>

#### I=2 states

<table>
<thead>
<tr>
<th>Spin</th>
<th>BB channels</th>
<th>SU(3) representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1S_0$</td>
<td>$\Sigma\Sigma$</td>
<td>-- -- 27</td>
</tr>
<tr>
<td>$^3S_1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Strong attraction (H-dibaryon)

Strong repulsion

Similar to The NN potential

Repulsion
Baryon operators

\[ p_\alpha = \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) d(\xi_2) u(\xi_3) \]

\[ n_\alpha = \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) d(\xi_2) d(\xi_3) \]

\[ \Sigma^+_{\alpha} = - \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} u(\xi_1) s(\xi_2) u(\xi_3) \]

\[ \Sigma^0_{\alpha} = - \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{2}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + u(\xi_1) s(\xi_2) d(\xi_3) \right] \]

\[ \Sigma^-_{\alpha} = - \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} d(\xi_1) s(\xi_2) d(\xi_3) \]

\[ \Lambda_{\alpha} = - \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} \sqrt{\frac{1}{6}} \left[ d(\xi_1) s(\xi_2) u(\xi_3) + s(\xi_1) u(\xi_2) d(\xi_3) - 2 u(\xi_1) d(\xi_2) s(\xi_3) \right] \]

\[ \Xi^0_{\alpha} = \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} s(\xi_1) u(\xi_2) s(\xi_3) \]

\[ \Xi^-_{\alpha} = \epsilon_{c_1 c_2 c_3} (C y_5)_{d_1 d_2} \delta_{d_3 \alpha} s(\xi_1) d(\xi_2) s(\xi_3) \]

- With corrected phase \[ \bar{1} = - \epsilon^{123} = -(ds - sd) = sd - ds \]
Irreducible BB source operator

\[
BB^{(27)} = + \sqrt{\frac{27}{40}} \Lambda \Lambda - \sqrt{\frac{1}{40}} \Sigma \Sigma + \sqrt{\frac{12}{40}} N \Xi
\]

\[
BB^{(8s)} = - \sqrt{\frac{1}{5}} \Lambda \Lambda - \sqrt{\frac{3}{5}} \Sigma \Sigma + \sqrt{\frac{1}{5}} N \Xi
\]

\[
BB^{(1)} = - \sqrt{\frac{1}{8}} \Lambda \Lambda + \sqrt{\frac{3}{8}} \Sigma \Sigma + \sqrt{\frac{4}{8}} N \Xi \quad \text{with}
\]

\[
\Sigma \Sigma = + \sqrt{\frac{1}{3}} \Sigma^+ \Sigma^- - \sqrt{\frac{1}{3}} \Sigma^0 \Sigma^0 + \sqrt{\frac{1}{3}} \Sigma^- \Sigma^+
\]

\[
N \Xi = + \sqrt{\frac{1}{4}} p \Xi^- + \sqrt{\frac{1}{4}} \Xi^- p - \sqrt{\frac{1}{4}} \bar{n} \Xi^0 - \sqrt{\frac{1}{4}} \Xi^0 \bar{n}
\]

\[
BB^{(10^*)} = + \sqrt{\frac{1}{2}} p \bar{n} - \sqrt{\frac{1}{2}} \bar{n} p
\]

\[
BB^{(10)} = + \sqrt{\frac{1}{2}} p \Sigma^+ - \sqrt{\frac{1}{2}} \Sigma^+ p
\]

\[
BB^{(8a)} = + \sqrt{\frac{1}{4}} \bar{p} \Xi^- - \sqrt{\frac{1}{4}} \Xi^- \bar{p} - \sqrt{\frac{1}{4}} \bar{n} \Xi^0 + \sqrt{\frac{1}{4}} \Xi^0 \bar{n}
\]
Isospin combinations of BB operator

\[
\Lambda\Lambda, \, p\Xi^-, \, n\Xi^0, \, \Sigma^+\Sigma^-, \, \Sigma^0\Sigma^0, \, \Lambda\Sigma^0
\]

**I=0 operators**

\[
\Sigma\Sigma = +\sqrt{\frac{1}{3}} \Sigma^+ \Sigma^- - \sqrt{\frac{1}{3}} \Sigma^0 \Sigma^0 + \sqrt{\frac{1}{3}} \Sigma^- \Sigma^+
\]

\[
N\Xi = +\frac{1}{2} p\Xi^{} - \frac{1}{2} n\Xi^0
\]

**I=1 operators**

\[
\Sigma\Sigma = +\frac{1}{2} \Sigma^+ \Sigma^- - \frac{1}{2} \Sigma^- \Sigma^+
\]

\[
N\Xi = +\frac{1}{2} p\Xi^- + \frac{1}{2} n\Xi^0
\]

**I=2 operators**

\[
\Sigma\Sigma = +\frac{1}{6} \Sigma^+ \Sigma^- + \frac{4}{6} \Sigma^0 \Sigma^0 + \frac{1}{6} \Sigma^- \Sigma^+
\]