Neutron Stars from Lattice QCD

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for

HAL QCD Collaboration

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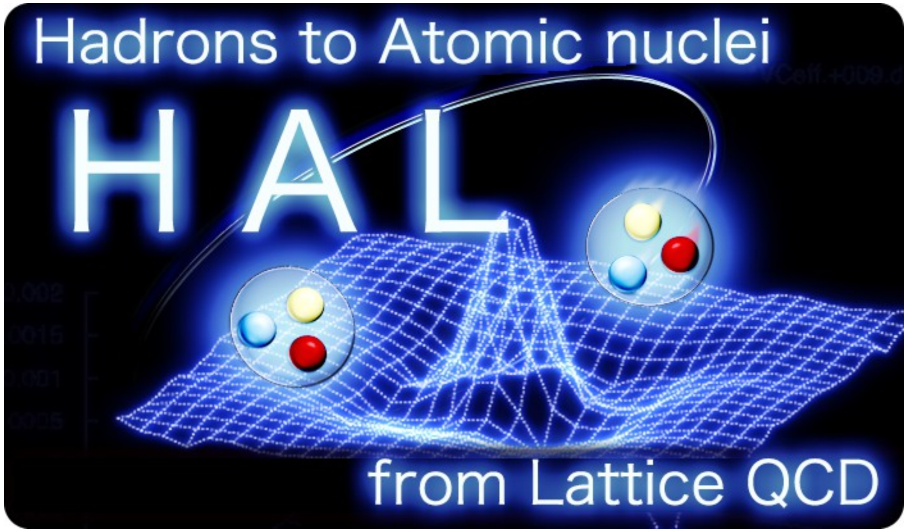
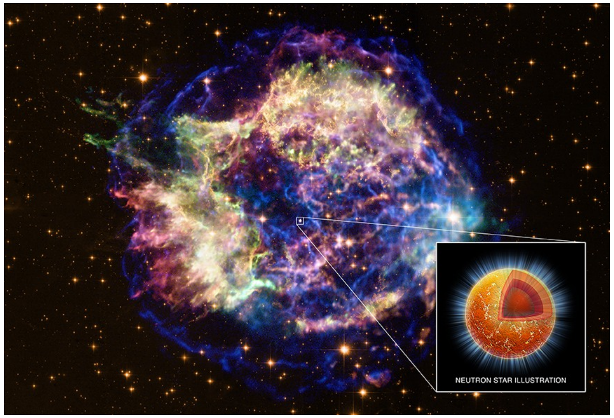


Quarks to Universe, Dec 13 to 16, Nara

Introduction

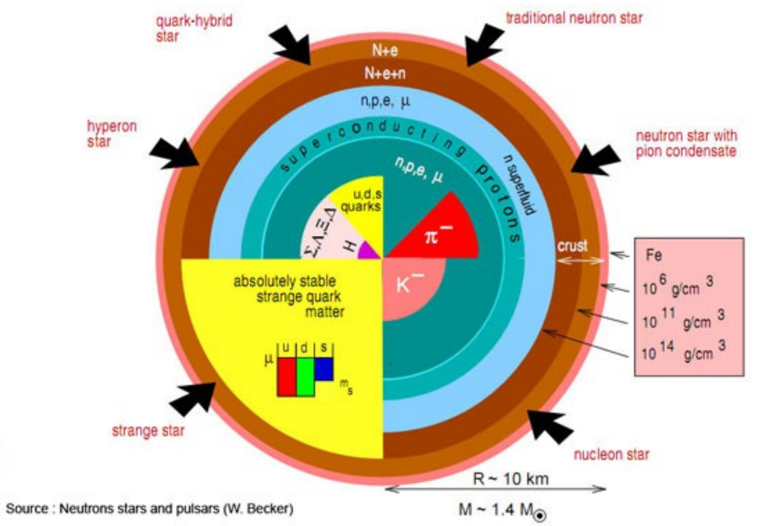
✯ Neutron Star



- is a compact star formed after supernova explosion of massive star.
- Remnant core exceeding $M_{Cha.}=1.4\ M_{\odot}$ goes further gravitational collapse.
- Electrons(e^{-}) and protons(p) are combined to neutrons(n).
- Finally, it is supported by neutron degeneracy pressure + ...
- NS is predicted by Baade and Zwiky in 1934, and first observed by Hewish and Bell in 1967 as a pulsar.
- Typically, $M = 1.5\ M_{\odot}$, $R = 10$ km.
- Mass-energy density at the center is about 10^{15} [g/cm^3] !

Most dense matter in the Universe

Introduction



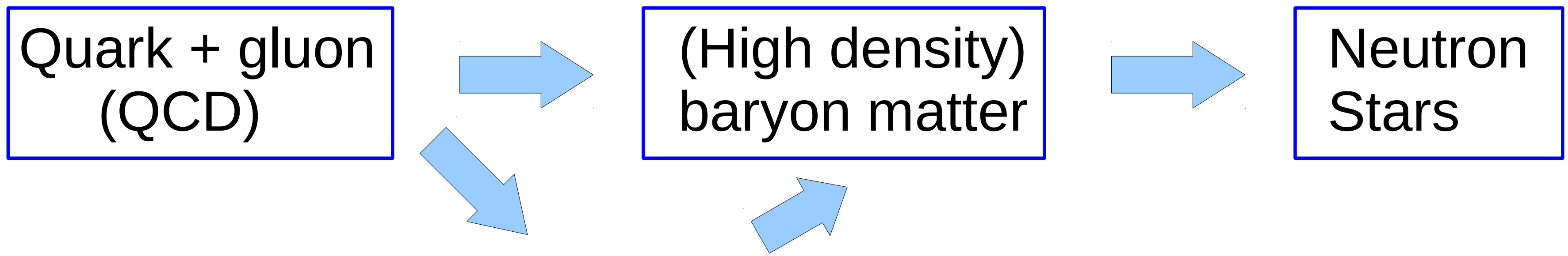
- Many questions
  - How formed (cooled) ?
  - How heavy at most ?
  - How consist ?

- These questions are connected to
  - Neutrino matter reaction rate
  - EoS of high density baryonic matter
  - QCD phase diagram
  - General relativity etc.

- NS is frontiers for particle-, nuclear & astro-physicists.

Introduction

- Today, I try to



- Our approach
  - We put one step [General BB interactions], and we extract them from QCD by numerical simulations on the lattice.
  - At the later stages, we follow standard frameworks.

We can answer these questions in principle.
But, we have many limitations at present.
So, my talk is just a first step.

- We want to answer:
  - Can QCD reproduce know features of nuclear matter ?
  - What does QCD predict about hyperons in medium ?
  - How about EoS of NS matter & $M^{\rm max}$ of neutron star ?

Outline

1. Introduction
2. General BB Interaction from QCD
3. Nuclear Matter EoS from QCD
4. Neutron Stars from QCD
5. Hyperon in medium from QCD
6. Summary and Plan

General BB Interaction from QCD

LQCD simulation at $SU(3)_F$ Limit

- I've carried out LQCD simulations at flavor SU(3) limits, in order to capture essential feature of BB interaction.

$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Irreducible multiplet = Convenient basis to describe interaction

In this limit,

- Any BB interaction (e.g. NN and ΛN) can be reconstructed from interactions in these basis and C.G. coefficients.

saving computation time

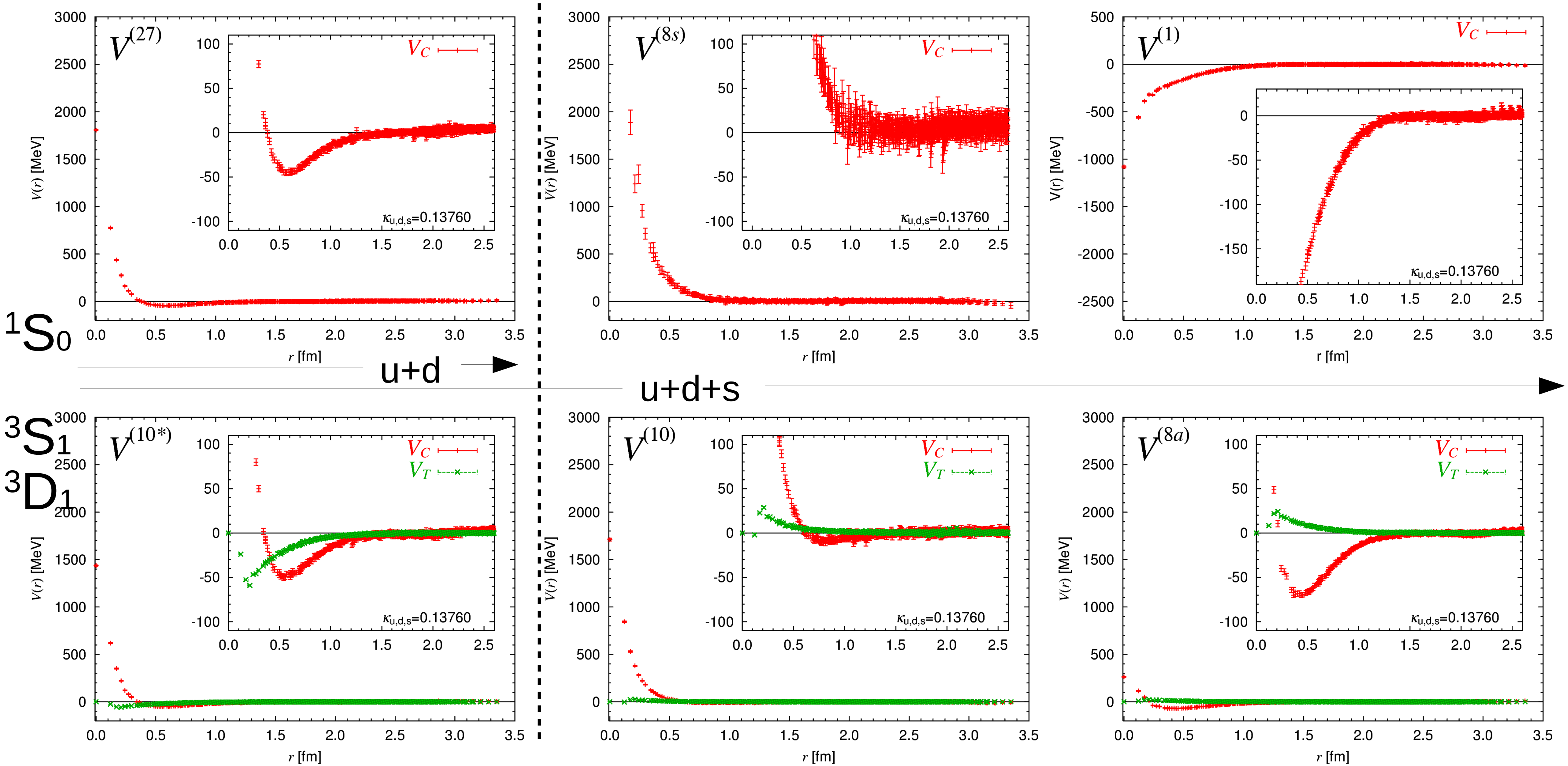
size	β	C_{SW}	a [fm]	L [fm]
32^3 x 32	1.83	1.761	0.121(2)	3.87

- Iwasaki gauge & Wilson quark.
- Thanks to PACS-CS collaboration for their DDHMC/PHMC code.

K_uds	M_P.S. [MeV]	M_Bar [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1161(2)

BB pot. in the flavor irr. basis

@SU3 limit
Mps = 837



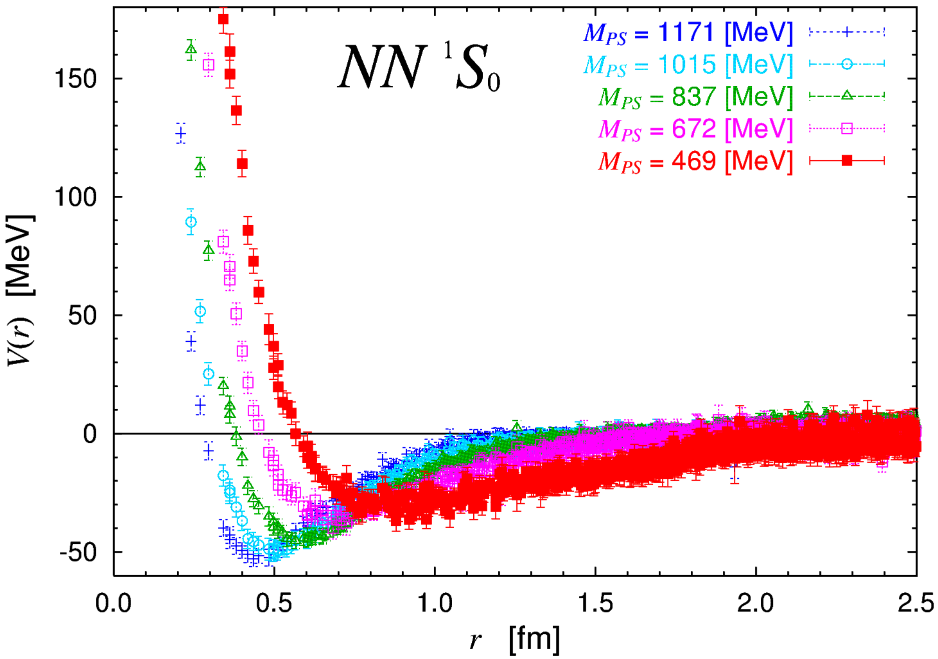
- Extracted by

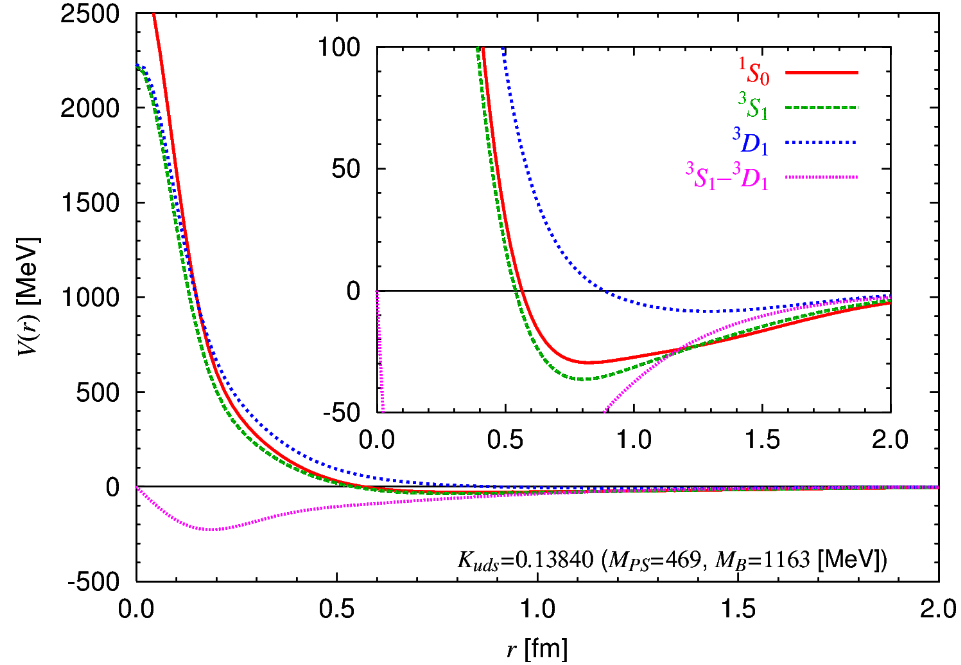
$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

$\psi(\vec{r},t)$: NBS W.F.

Talk by T. Doi

LQCD NN potentials



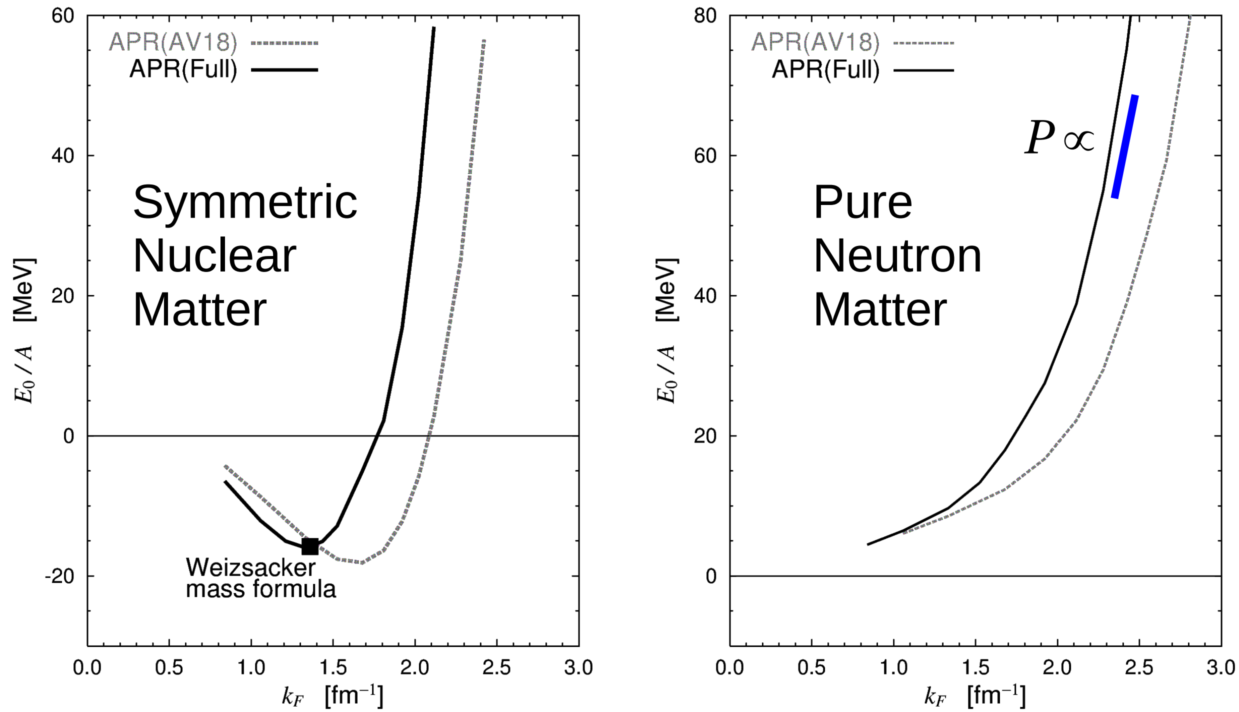


- Left: NN $^1S_0$ potential at five quark mass. (27-plet)
  - Repulsive core & attractive pocket grow as m_q decrease.
- Right: NN potential in partial waves at the lightest m_q.

Nuclear Matter
from QCD

Equation of State

- Ground state of interacting infinite nucleon system
  - Relativistic Mean Field — J. D. Walecka, Ann. Phys. 83 (1974) 491
  - Fermi Hyper-Netted Chain — A. Akmal, V.R. Phandharipande, D.G. Ravenhall Phys. Rev. C 58 (1998) 1804



They used phenomenological Argonne NN force & “Urbana” NNN force.

- For SNM, most important feature is the saturation.
- For PNM or NS, the slope at large k_F is important.

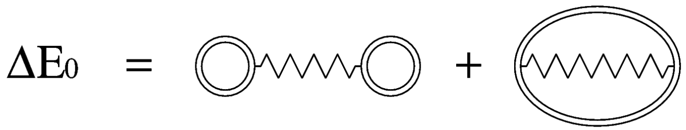
Brueckner-Hartree-Fock

K.A. Brueckner and J.L.Gammel
Phys. Rev. 109 (1958) 1023

M.I. Haftel and F. Tabakin,
Nucl. Phys. A158(1970) 1-42

- Ground state energy in BHF framework

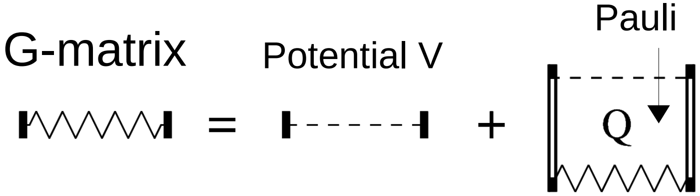
$$E_0 = \gamma \sum_{k}^{k_F} \frac{k^2}{2M} + \frac{1}{2} \sum_{i}^{N_{ch}} \sum_{k,k'}^{k_F} \langle\, G_i\big(e(k)+e(k')\big)\,\rangle_A$$



- G-matrix

LQCD V_{NN}

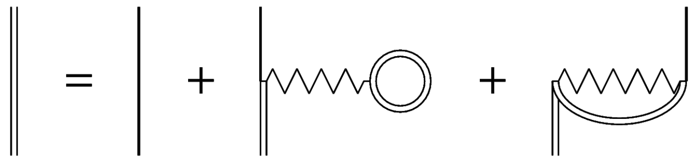
$$\langle k_1 k_2 | G(\omega) | k_3 k_4 \rangle = \langle k_1 k_2 | V | k_3 k_4 \rangle + \sum_{k_5, k_6} \frac{\langle k_1 k_2 | V | k_5 k_6 \rangle Q(k_5, k_6) \langle k_5 k_6 | G(\omega) | k_3 k_4 \rangle}{\omega - e(k_5) - e(k_6)}$$



- Single particle spectrum & potential

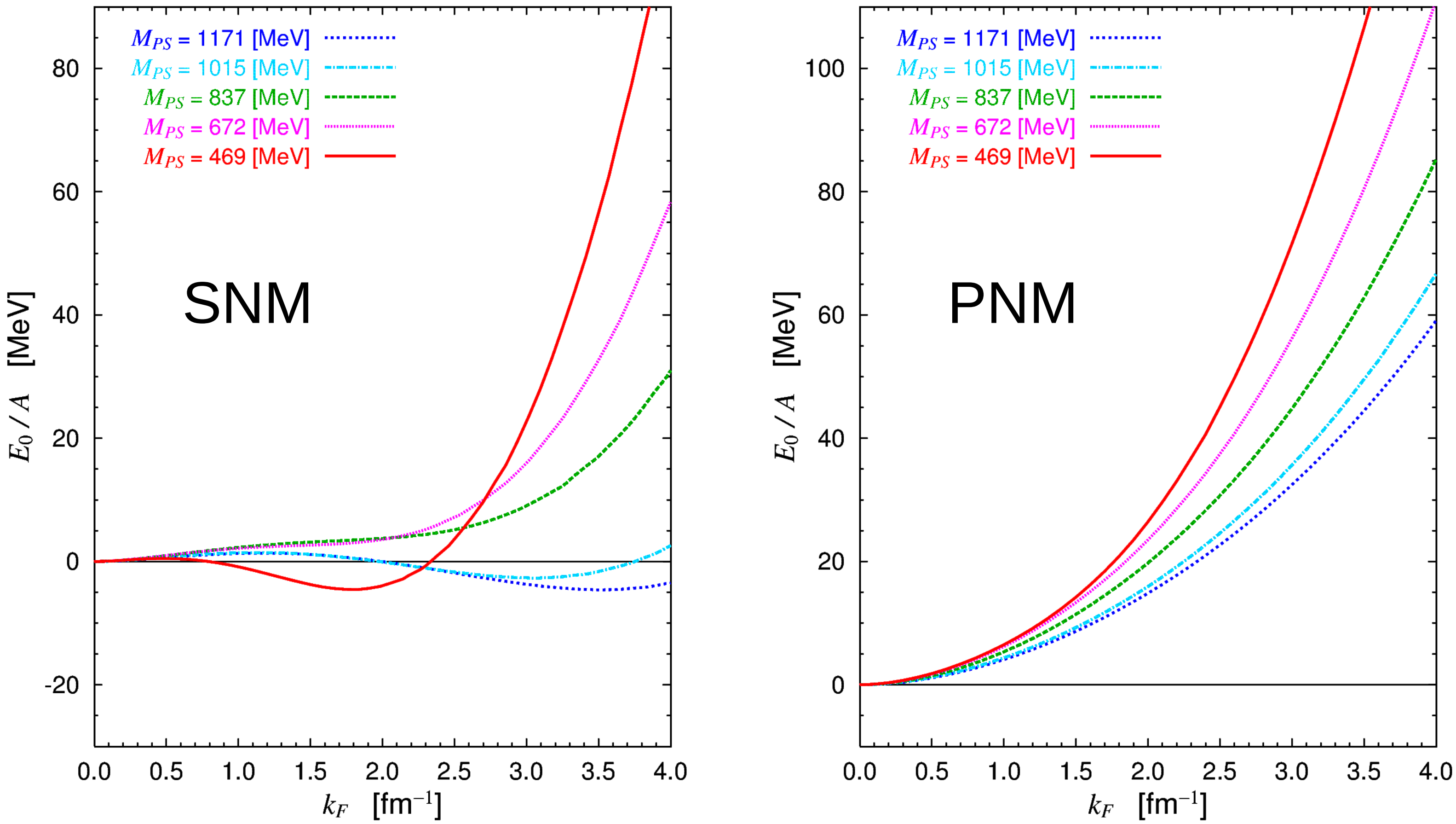
$$e(k) = \frac{k^2}{2M} + U(k)$$

$$U(k) = \sum_i \sum_{k' \leq k_F} \langle k\, k' | G_i\big(e(k)+e(k')\big) | k\, k' \rangle_A$$



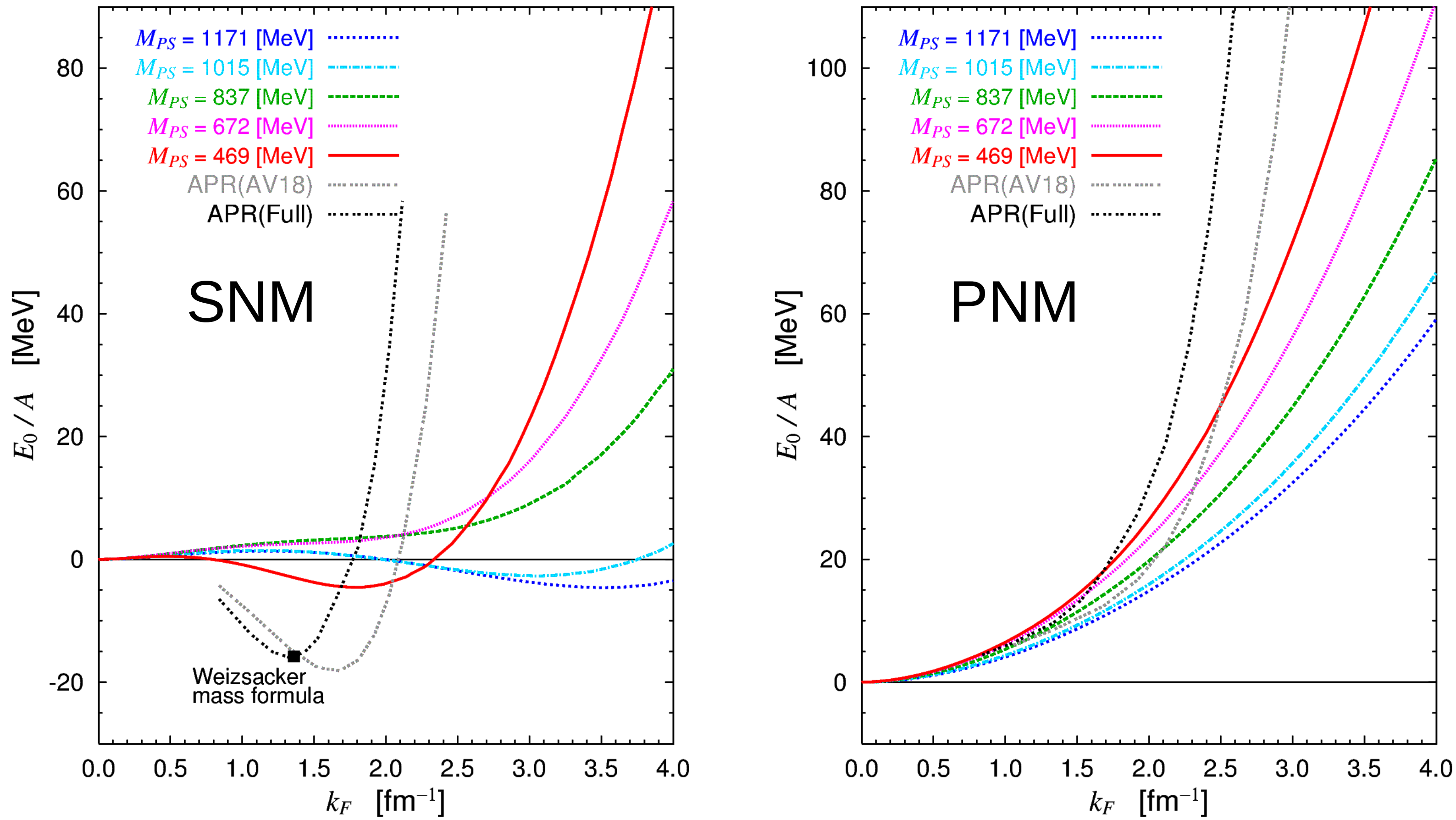
- Angle averaged Q-operator, Continuous choice w/ parabolic approximation

Matter EoS from QCD



- SNM is bound and the saturation occurs at M_{PS} = 469 MeV.
  - Saturation is very delicate against change of quark mass.
- PNM is unbound as normal.
  - PNM become stiff at high density as quark mass decrease.

Comparison to APR



- Deviation from empirical ones due to heavy u,d quark.
- LQCD curve approaches to empirical one as mq decrease.

optimistically?

Neutron Stars from QCD

Stable Neutron Stars (spherical & non-rotating)

- Tolman-Oppenheimer-Volkoff equation

$$\frac{dP(r)}{dr} = -\frac{G\left(E(r)+P(r)\right)\left(M(r)+4\pi r^3 P(r)\right)}{r\left(r-2GM(r)\right)}$$

R. C. Tolman, Phys Rev. 55(1934) 364

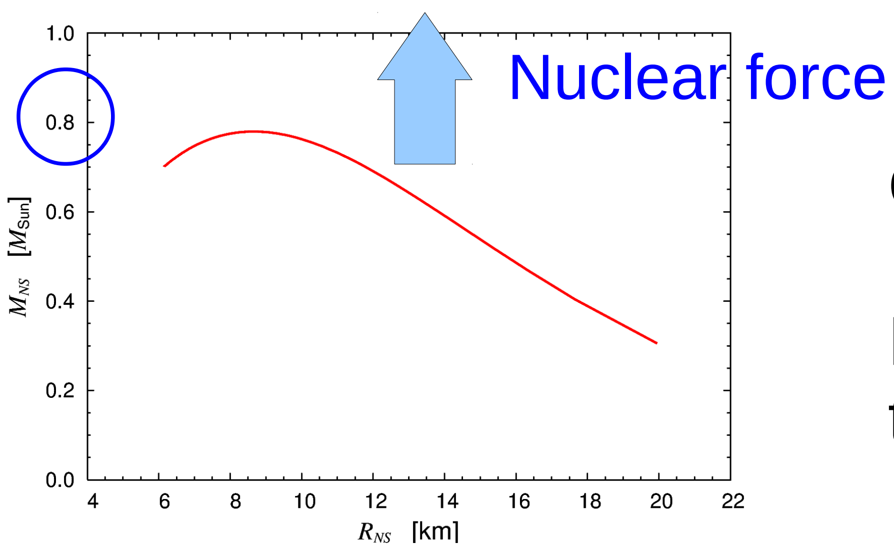
J. R. Oppenheimer and G. M. Volkoff
Phys. Rev. 55 (1939) 374

$$\frac{dM(r)}{dr} = 4\pi r^2 E(r)$$

$P(r)$: Pressure
$E(r)$: Mass-energy density
$M(r)$: Enclosed mass

Gravitational constant
G = 6.6743 [m^3 kg^{-1} s^{-2}]

- with *P(E)* (EoS) for a cold Fermi gas of neutrons.



QCD is essential for NS!

Let us apply LQCD EoS
to neutron stars.

Neutron Star Matter

We consider hyperons later.

- For the moment, I restrict component to n, p, e^- and μ^-.

- Asymmetric nuclear matter with the parabolic approx.

$$\frac{E_0}{A}(\rho, x) = \frac{E_0^{\mathrm{SNM}}}{A}(\rho) + \beta^2 E_{\mathrm{sym}}(\rho)$$

$$[\mu_n - \mu_p](\rho, x) = 4\beta E_{\mathrm{sym}}(\rho)$$

$$\rho = \rho_n + \rho_p, \quad x = \rho_p / \rho$$

$$\beta = 1 - 2x$$

$$E_{\mathrm{sym}}(\rho) = \frac{E_0^{\mathrm{PNM}}}{A}(\rho) - \frac{E_0^{\mathrm{SNM}}}{A}(\rho)$$

- Cold Fermi gas for lepton

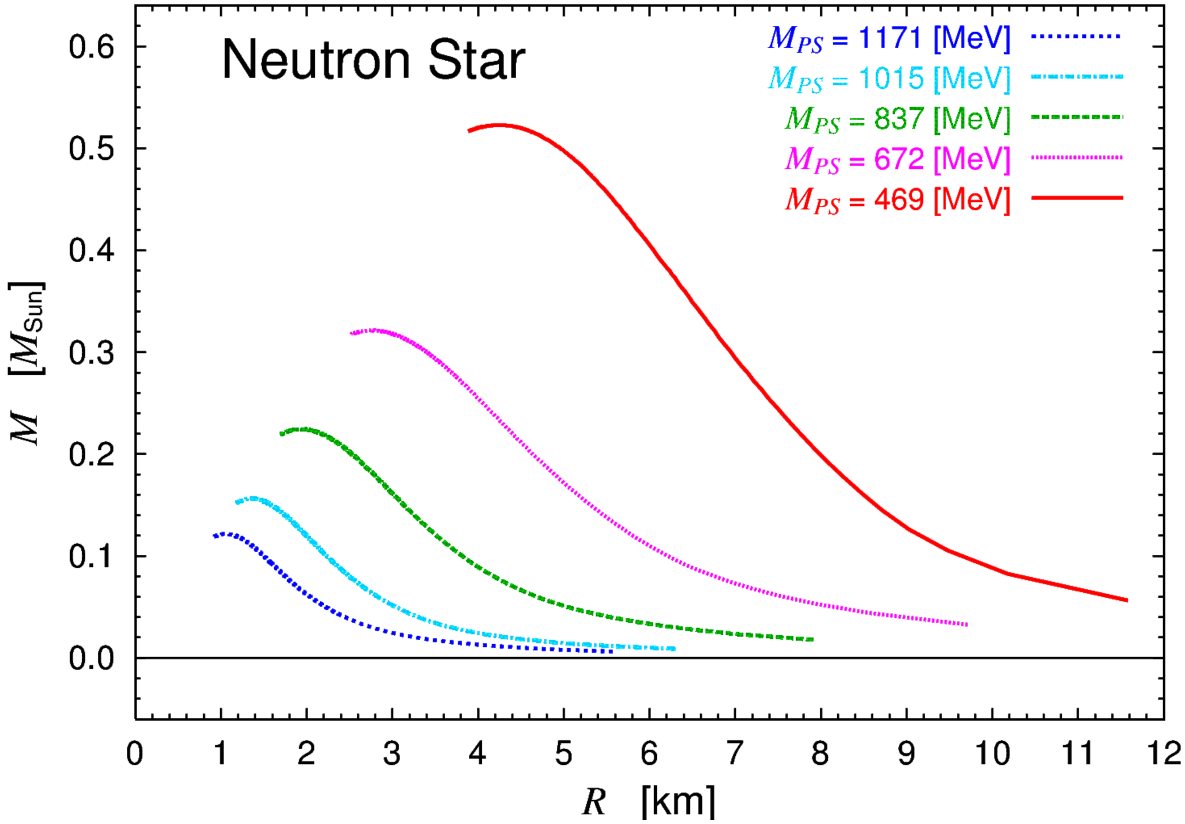
$$\mu_e = \varepsilon_F^e = k_F^e = (3\pi^2 \rho_e)^{(1/3)} \qquad \mu_\mu = \varepsilon_F^\mu = \sqrt{m_\mu^2 + (k_F^\mu)^2}$$

- Chemical equilibrium and charge neutrality

$$\mu_n - \mu_p = \mu_e = \mu_\mu \qquad \rho_p = \rho_e + \rho_\mu$$

- Particle fractions x_i and $P(E)$ are determined.

Neutron Star M-R relation



M_N = 2274 MeV
M_N = 2031 MeV
M_N = 1749 MeV
M_N = 1484 MeV
M_N = 1161 MeV

Crust part is ignored.
Uniform matter only.

- Mass-radius curve of neutron stars at five value of m_q.
  - M^{max} = 0.12 – 0.52 [M_{Sun}] for M_{ps} = 1171 – 469 [MeV].
  - due to heavy nucleon and weaker repulsive core.
  - M^{max} will be much bigger with lighter u,d quark.

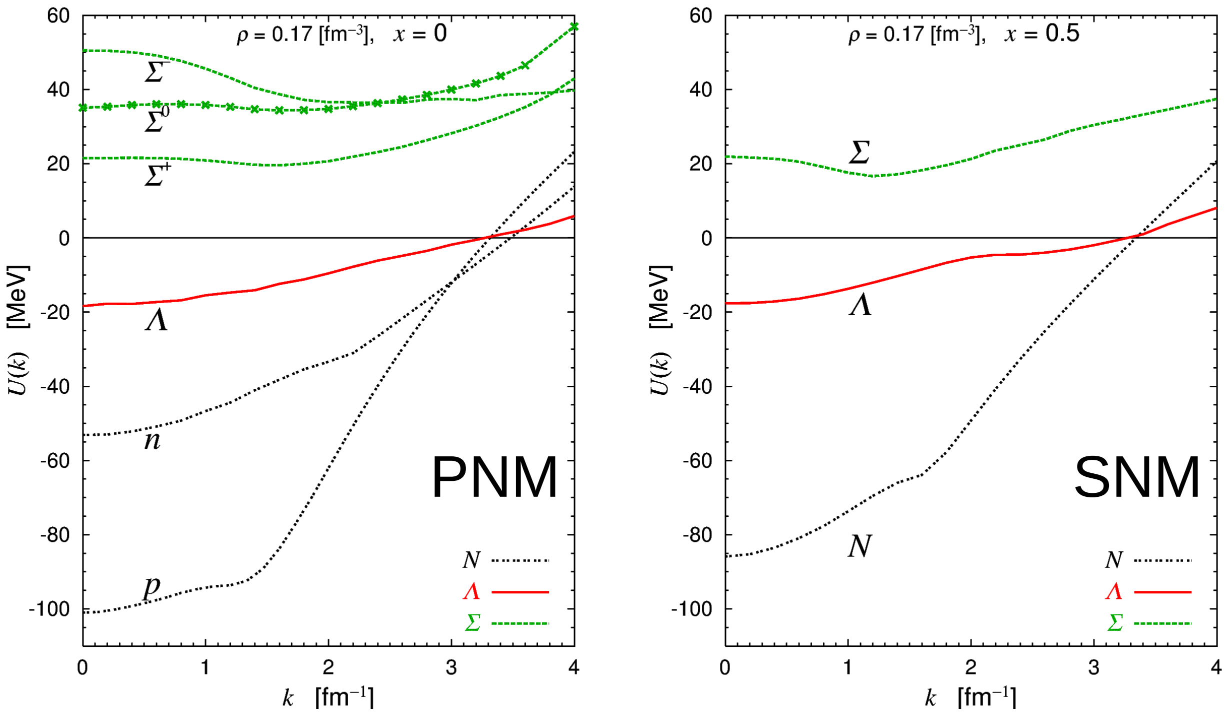
Hyperon in medium from QCD

Hyperon single particle potential

- Hyperon potential in the BHF framework

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \le k_F} \langle k\, k' |\, G^{SLJ}_{(YN)(YN)}\big(e_Y(k) + e_N(k')\big) \,|\, k\, k' \rangle$$

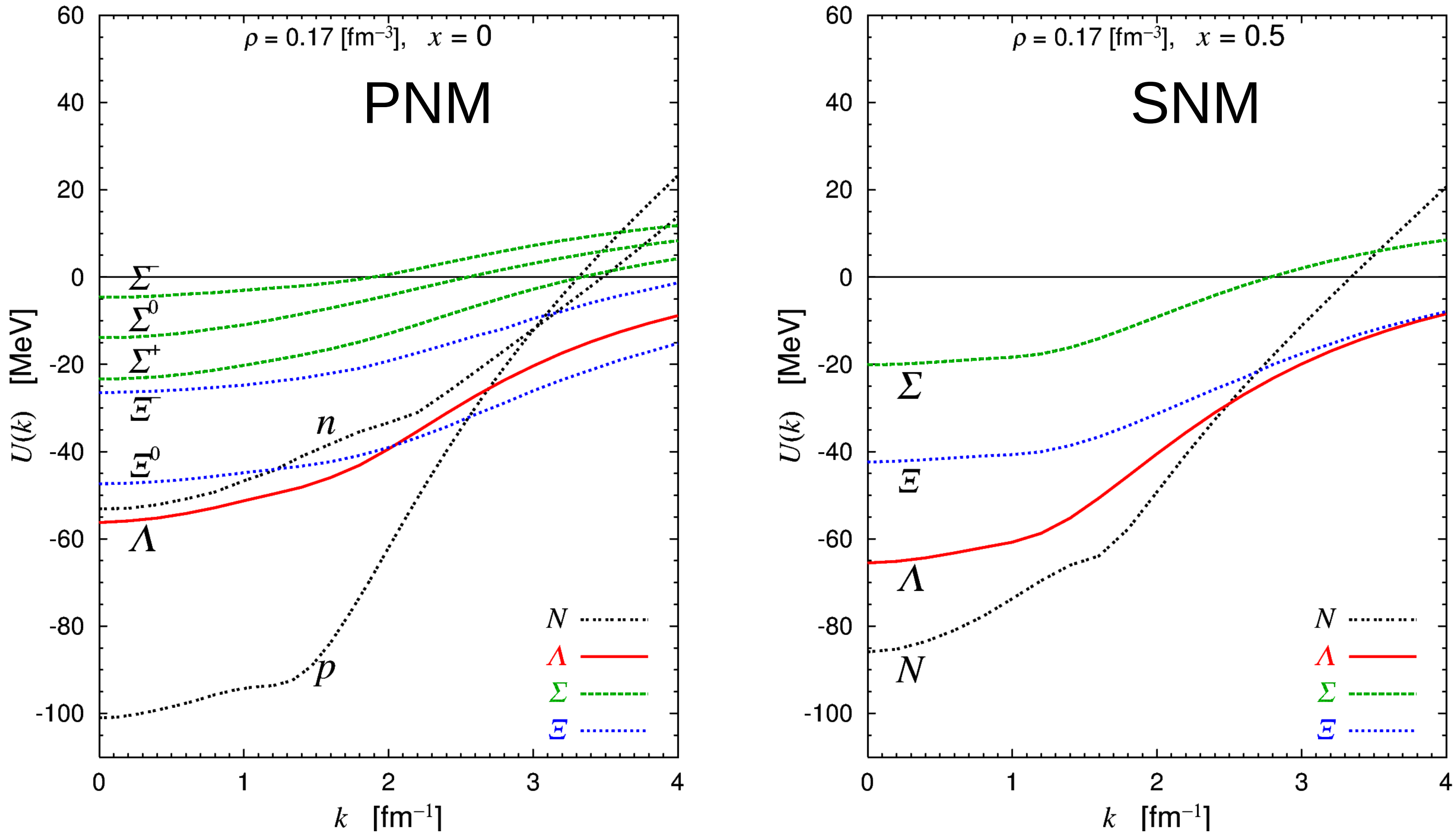
- AV18 NN + Nijmegen YN (ESC08c)



Interactions up to $^1S_0$, $^3SD_1$ channels.

- $U_\Lambda(0) \approx -20$ [MeV], $U_\Sigma > 0$ in both the nuclear matter.

M_B^{Phys} + AV18 NN + LQCD YN,YY



- with YN & YY potentials from LQCD at a $SU(3)_F$ limit.
- We see that $U_\Lambda(k) < U_\Xi(k) < U_\Sigma(k)$.
- LQCD $U_Y(k)$ are deeper than model predictions and data.

due to heavy u,d quark? Models are wrong?

Summary and Plan

- We've tried to reach Neutron Stars from QCD.
  - We studied BB interaction from LQCD.
    - We extracted BB potentials in 6 flavor irreducible basis.
  - We studied nuclear matter in the BHF theory.
    - We could reproduce the saturation feature of SNM.
  - We studied neutron stars solving TOV eq.
    - We obtained mq dependence of NS M-R relation.
  - We studied hyperons in nuclear medium.

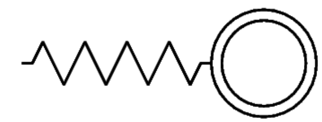
- Plan
  - Study NS with hyperons according to QCD.
  - Inclusion of P-wave BB and BBB interactions.
  - LQCD simulations with more realistic quark at 京.

Thank you!!

Hyperon single particle potential

- Hyperon potential in the BHF framework

$$U_Y(k) = \sum_{N=n,p} \sum_{SLJ} \sum_{k' \leq k_F} \langle k k' | G^{SLJ}_{(YN)(YN)}\left(e_Y(k) + e_N(k')\right) | k k' \rangle$$



- YN G-matrix

Q = 0

$$\begin{pmatrix} G^{SLJ}_{(\Lambda n)(\Lambda n)} & G_{(\Lambda n)(\Sigma^0 n)} & G_{(\Lambda n)(\Sigma^- p)} \\ G_{(\Sigma^0 n)(\Lambda n)} & G_{(\Sigma^0 n)(\Sigma^0 n)} & G_{(\Sigma^0 n)(\Sigma^- p)} \\ G_{(\Sigma^- p)(\Lambda n)} & G_{(\Sigma^- p)(\Sigma^0 n)} & G_{(\Sigma^- p)(\Sigma^- p)} \end{pmatrix}$$

Q = +1

$$\begin{pmatrix} G^{SLJ}_{(\Lambda p)(\Lambda p)} & G_{(\Lambda p)(\Sigma^0 p)} & G_{(\Lambda p)(\Sigma^+ n)} \\ G_{(\Sigma^0 p)(\Lambda p)} & G_{(\Sigma^0 p)(\Sigma^0 p)} & G_{(\Sigma^0 p)(\Sigma^+ n)} \\ G_{(\Sigma^+ n)(\Lambda p)} & G_{(\Sigma^+ n)(\Sigma^0 p)} & G_{(\Sigma^+ n)(\Sigma^+ n)} \end{pmatrix}$$

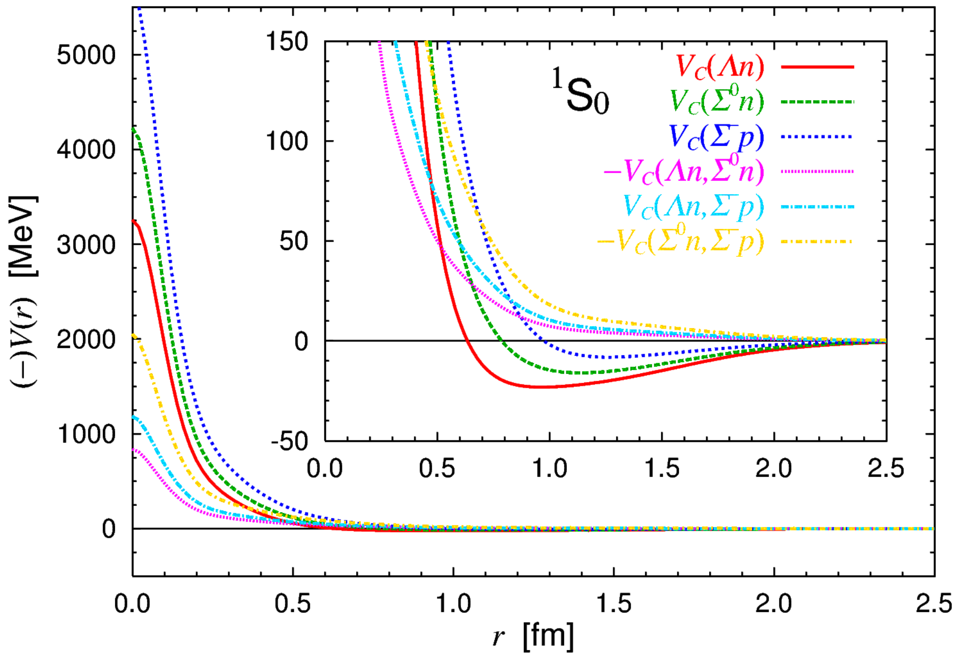
$G^{SLJ}_{(\Sigma^- n)(\Sigma^- n)}$ Q = −1

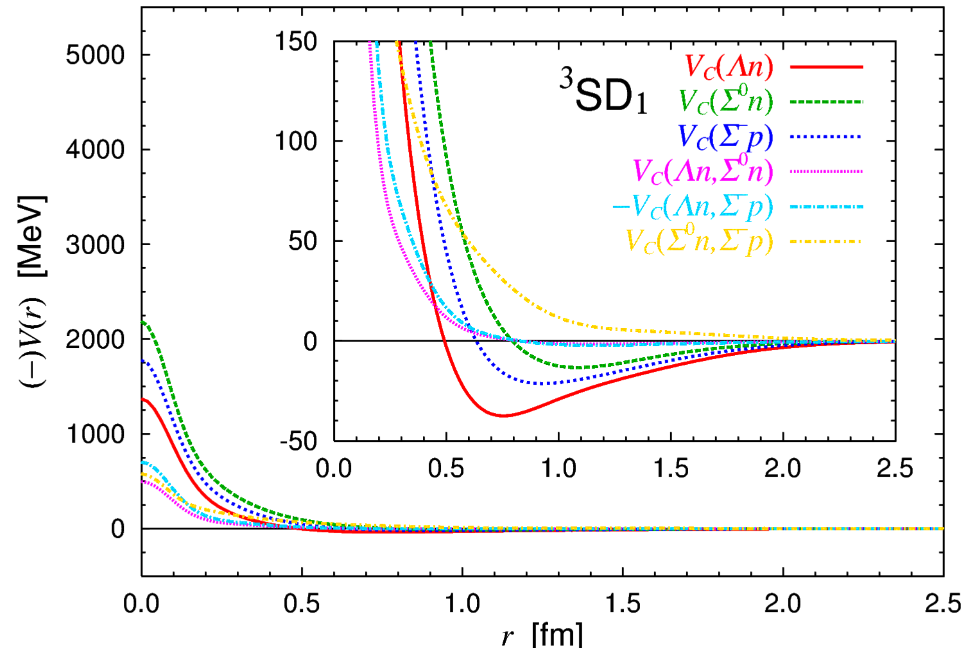
$G^{SLJ}_{(\Sigma^+ p)(\Sigma^+ p)}$ Q = +2

$$^{2S+1}L_J = {}^1S_0,\ {}^3S_1,\ {}^3D_1,\ {}^1P_1,\ {}^3P_J\ \cdots$$

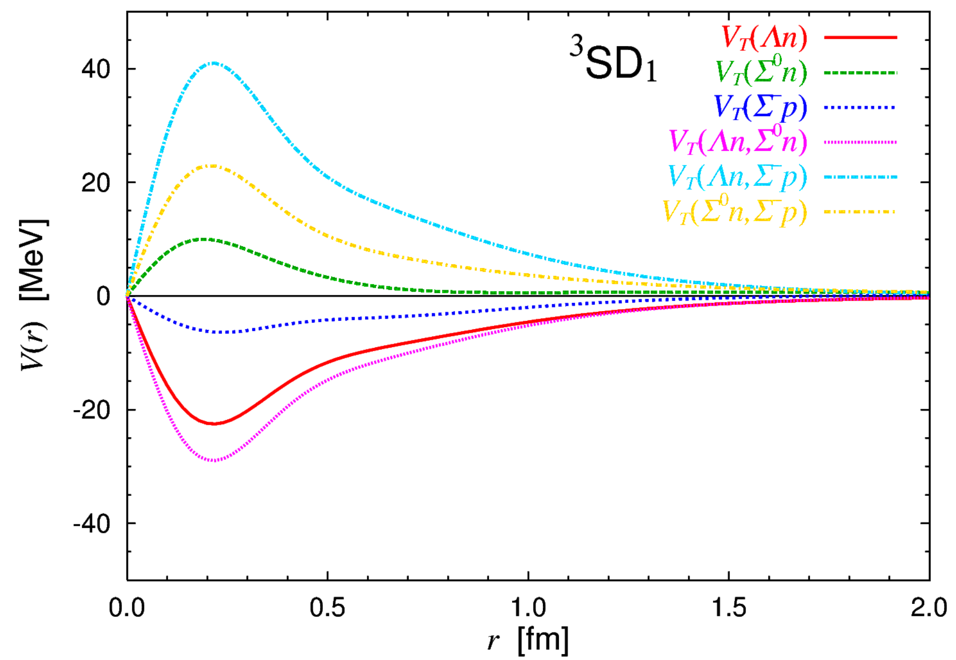
- Many elements!! Highly coupled!! Very challenging!!

YN potential in Q=0 sector

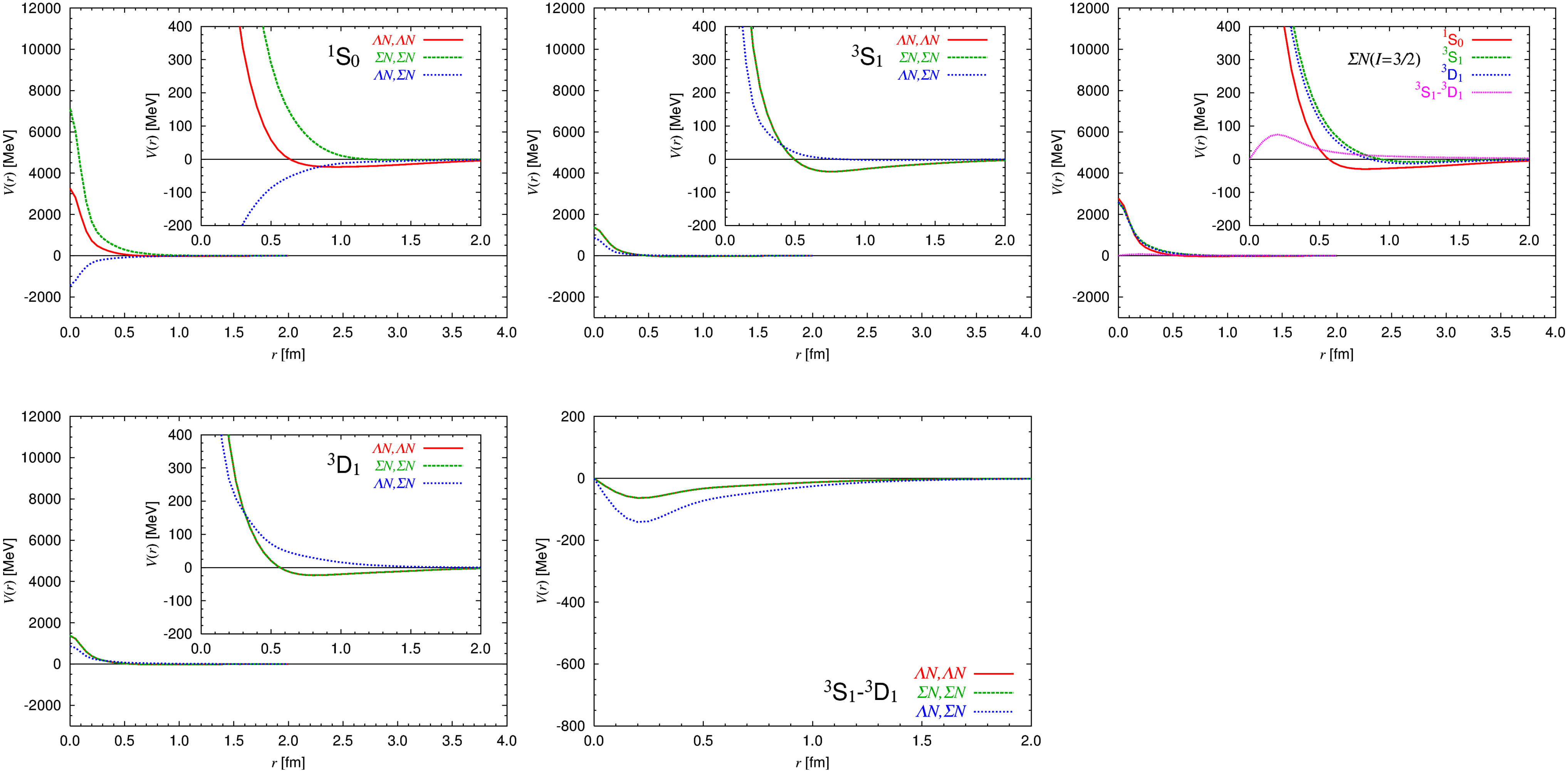




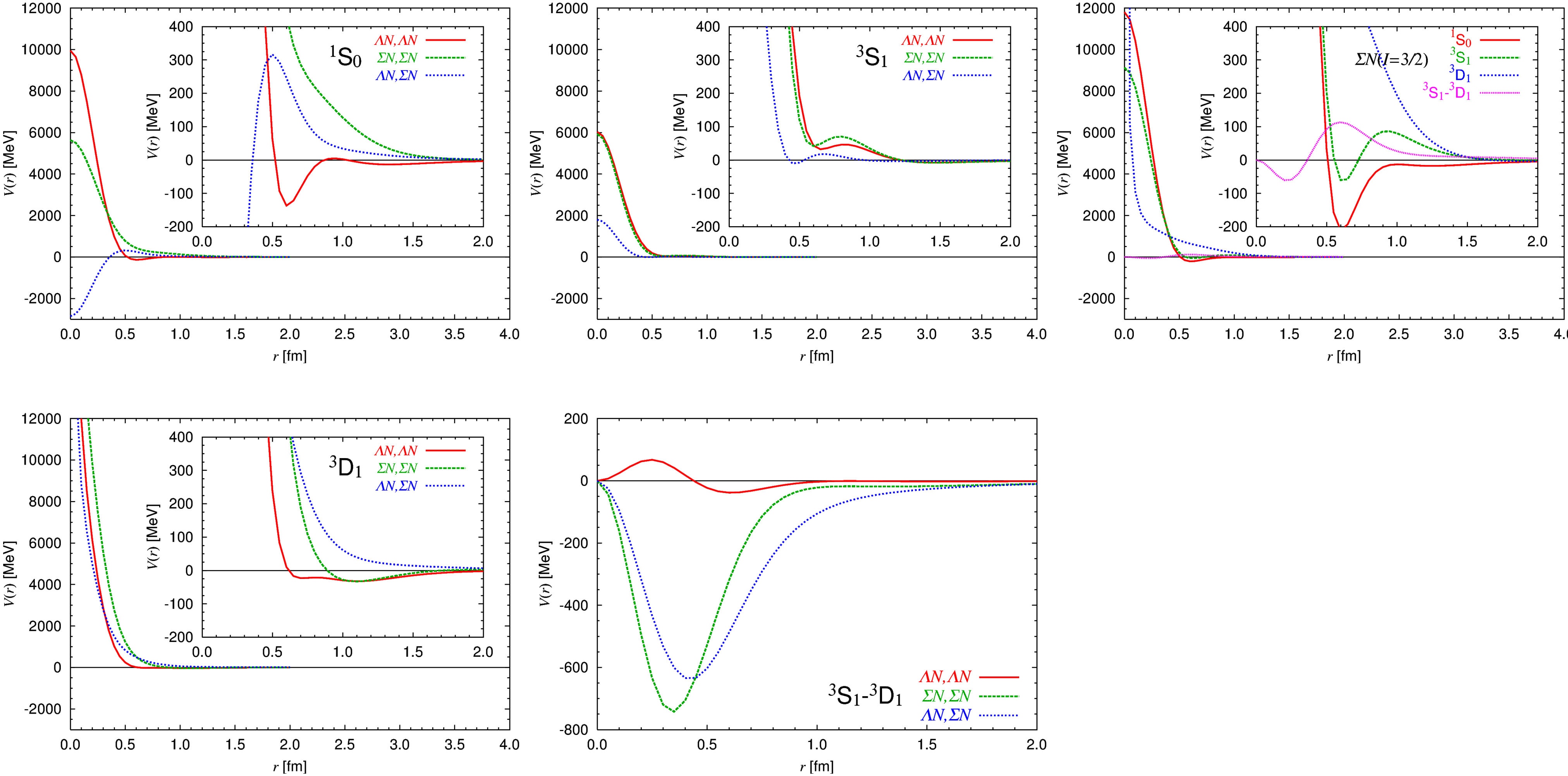
- @ SU(3)$_F$ limit w/ K=0.13840
  $M_N = M_\Lambda = M_\Sigma$ =1163 MeV,
  $M_\pi = M_K$ = 470 MeV
- Obtained by rotating $V^{(a)}(r)$.
- Concerning YN interaction, we have no ambiguity !!



LQCD YN @SU(3)F



Nijmegen YN ESC08c



ΞN G-matrix for U_{Ξ}

- Flavor symmetric $^1S_0$

$$\begin{pmatrix} G_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+ \Lambda)} \\ G_{(\Sigma^+ \Lambda)(\Xi^0 p)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)} \end{pmatrix} \quad Q = +1$$

$$\begin{pmatrix} G_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0 \Sigma^0)} & G_{(\Xi^0 n)(\Sigma^0 \Lambda)} & G_{(\Xi^0 n)(\Lambda \Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^- p)(\Sigma^0 \Sigma^0)} & G_{(\Xi^- p)(\Sigma^0 \Lambda)} & G_{(\Xi^- p)(\Lambda \Lambda)} \\ G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+ \Sigma^-)(\Xi^- p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)} & G_{(\Sigma^+ \Sigma^-)(\Lambda \Lambda)} \\ G_{(\Sigma^0 \Sigma^0)(\Xi^0 n)} & G_{(\Sigma^0 \Sigma^0)(\Xi^- p)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^0 \Sigma^0)(\Sigma^0 \Lambda)} & G_{(\Sigma^0 \Sigma^0)(\Lambda \Lambda)} \\ G_{(\Sigma^0 \Lambda)(\Xi^0 n)} & G_{(\Sigma^0 \Lambda)(\Xi^- p)} & G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Sigma^0)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)} & G_{(\Sigma^0 \Lambda)(\Lambda \Lambda)} \\ G_{(\Lambda \Lambda)(\Xi^0 n)} & G_{(\Lambda \Lambda)(\Xi^- p)} & G_{(\Lambda \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Lambda \Lambda)(\Sigma^0 \Sigma^0)} & G_{(\Lambda \Lambda)(\Sigma^0 \Lambda)} & G_{(\Lambda \Lambda)(\Lambda \Lambda)} \end{pmatrix} \quad Q = 0$$

$$\begin{pmatrix} G_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^- \Lambda)} \\ G_{(\Sigma^- \Lambda)(\Xi^- n)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Lambda)} \end{pmatrix} \quad Q = -1$$

ΞN G-matrix for U_{Ξ}

- Flavor anti-symmetric $^3SD_1$

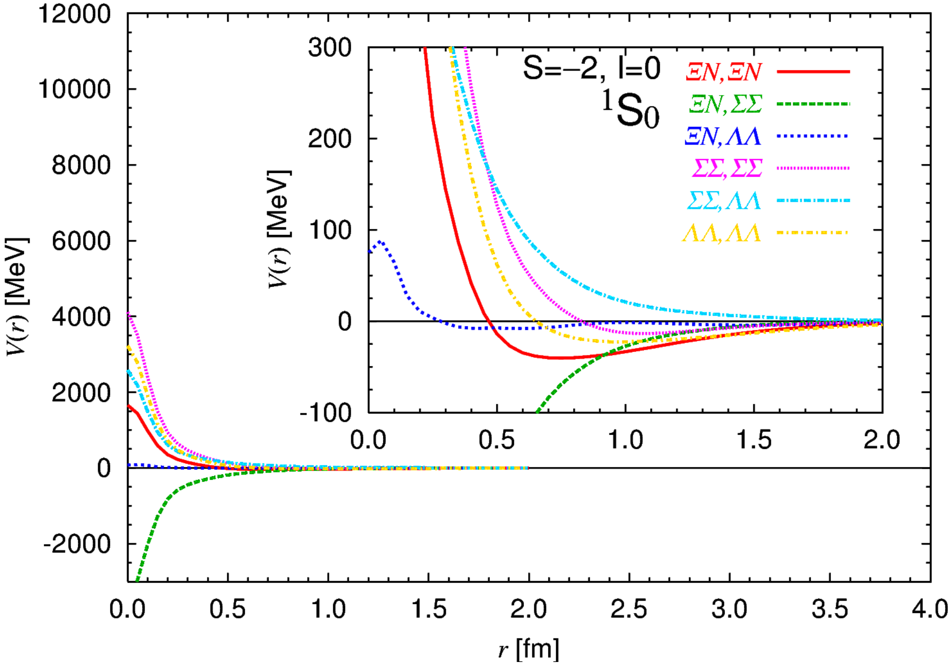
$$\begin{pmatrix} G_{(\Xi^0 p)(\Xi^0 p)} & G_{(\Xi^0 p)(\Sigma^+ \Sigma^0)} & G_{(\Xi^0 p)(\Sigma^+ \Lambda)} \\ G_{(\Sigma^+ \Sigma^0)(\Xi^0 p)} & G_{(\Sigma^+ \Sigma^0)(\Sigma^+ \Sigma^0)} & G_{(\Sigma^+ \Sigma^0)(\Sigma^+ \Lambda)} \\ G_{(\Sigma^+ \Lambda)(\Xi^0 p)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Sigma^0)} & G_{(\Sigma^+ \Lambda)(\Sigma^+ \Lambda)} \end{pmatrix} \quad Q = +1$$

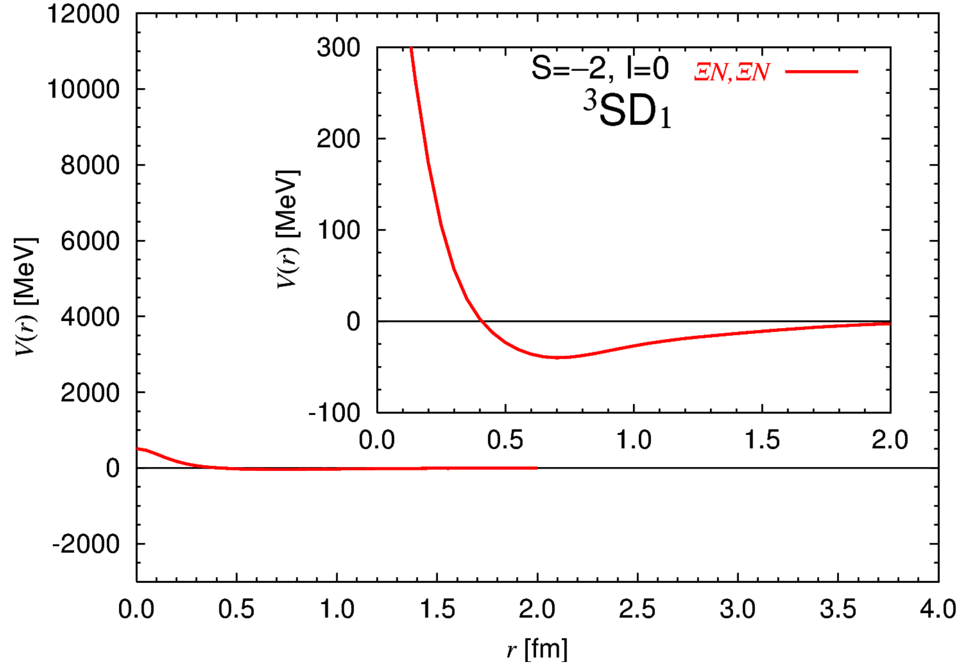
$$\begin{pmatrix} G_{(\Xi^0 n)(\Xi^0 n)} & G_{(\Xi^0 n)(\Xi^- p)} & G_{(\Xi^0 n)(\Sigma^+ \Sigma^-)} & G_{(\Xi^0 n)(\Sigma^0 \Lambda)} \\ G_{(\Xi^- p)(\Xi^0 n)} & G_{(\Xi^- p)(\Xi^- p)} & G_{(\Xi^- p)(\Sigma^+ \Sigma^-)} & G_{(\Xi^- p)(\Sigma^0 \Lambda)} \\ G_{(\Sigma^+ \Sigma^-)(\Xi^0 n)} & G_{(\Sigma^+ \Sigma^-)(\Xi^- p)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^+ \Sigma^-)(\Sigma^0 \Lambda)} \\ G_{(\Sigma^0 \Lambda)(\Xi^0 n)} & G_{(\Sigma^0 \Lambda)(\Xi^- p)} & G_{(\Sigma^0 \Lambda)(\Sigma^+ \Sigma^-)} & G_{(\Sigma^0 \Lambda)(\Sigma^0 \Lambda)} \end{pmatrix} \quad Q = 0$$

$$\begin{pmatrix} G_{(\Xi^- n)(\Xi^- n)} & G_{(\Xi^- n)(\Sigma^- \Sigma^0)} & G_{(\Xi^- n)(\Sigma^- \Lambda)} \\ G_{(\Sigma^- \Sigma^0)(\Xi^- n)} & G_{(\Sigma^- \Sigma^0(\Sigma^- \Sigma^0))} & G_{(\Sigma^- \Sigma^0)(\Sigma^- \Lambda)} \\ G_{(\Sigma^- \Lambda)(\Xi^- n)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Sigma^0)} & G_{(\Sigma^- \Lambda)(\Sigma^- \Lambda)} \end{pmatrix} \quad Q = -1$$

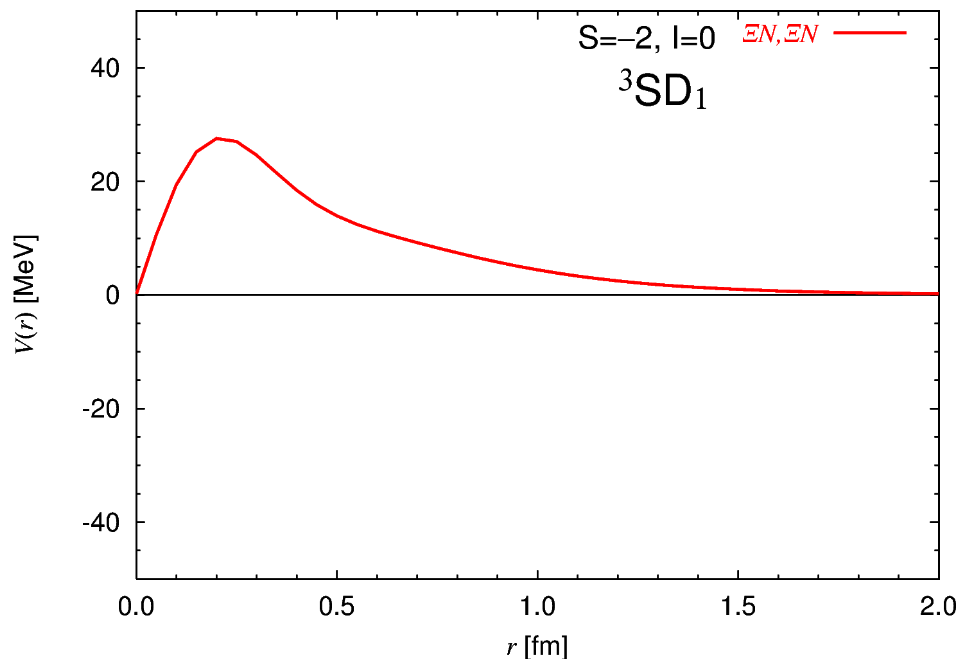
ΞN(I=0) pot. in LQCD @SU(3)$_F$

$M_N = M_\Lambda = M_\Sigma = 1161$,
$M_\pi = M_K = 469$ MeV



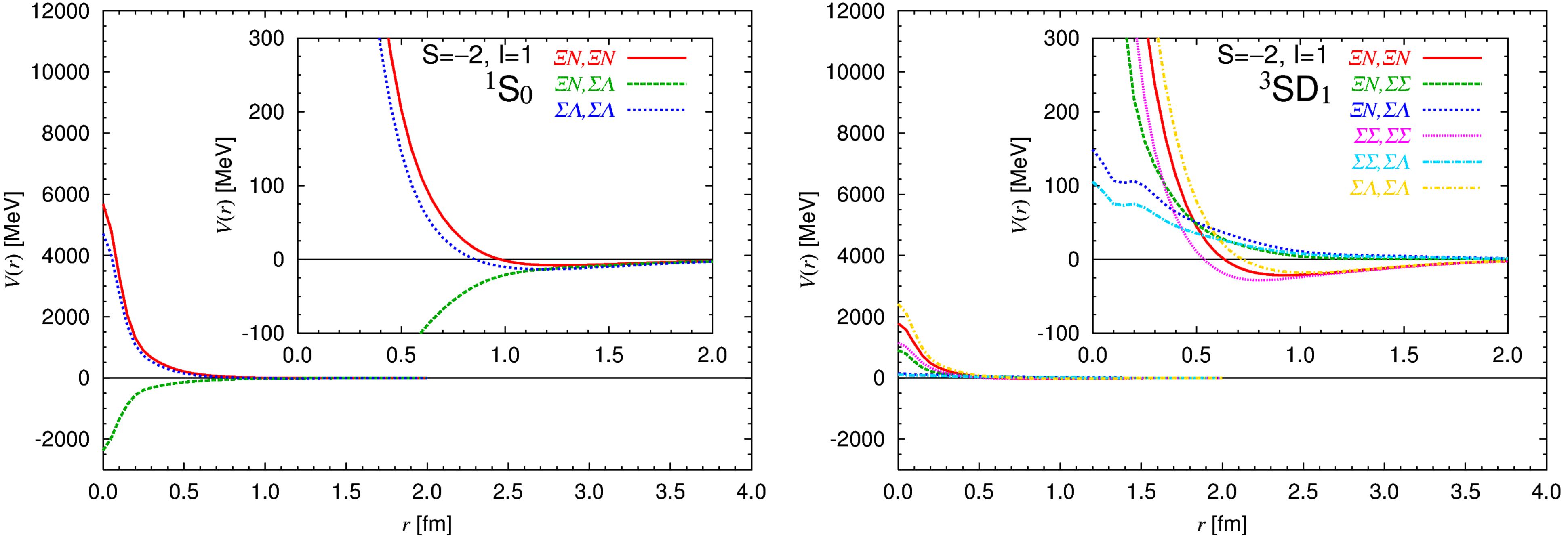


- By rotating $V^{(a)}(r)$ with C.G.

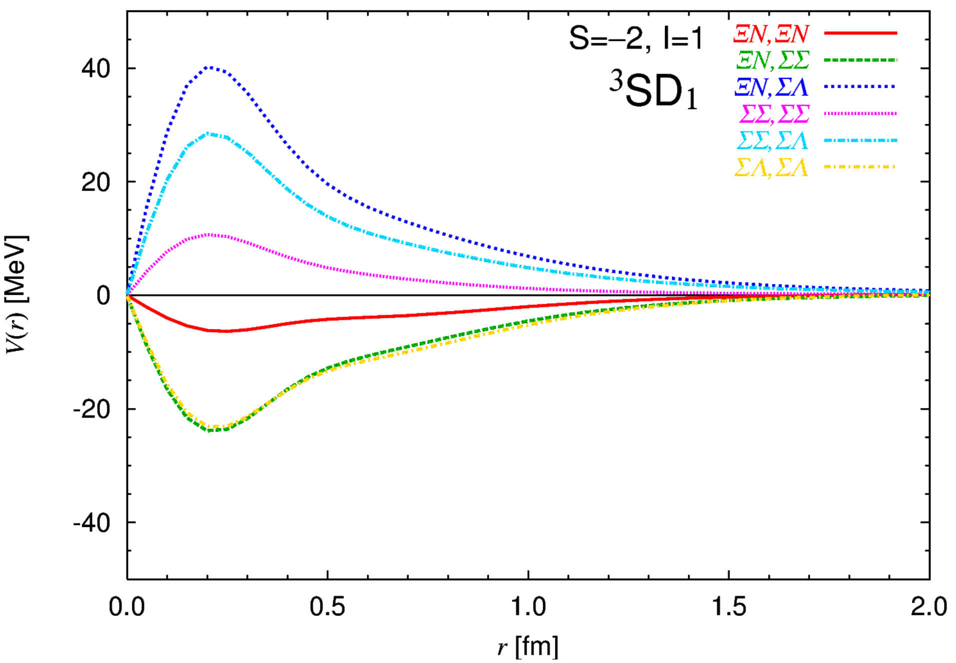


ΞN(I=1) pot. in LQCD @SU(3)$_F$

$M_N = M_\Lambda = M_\Sigma = 1161$, $M_\pi = M_K = 469$ MeV



- By rotating $V^{(a)}(r)$ with C.G.



Potential

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89(2010)
N. Ishii etal. [HAL QCD coll.] in preparation

NBS wave function $\psi(\vec{r},t) = \phi_{Gr}(\vec{r}) e^{-E_{Gr} t} + \phi_{1st}(\vec{r}) e^{-E_{1st} t} \cdots$

DEFINE a “potential” through the “Schrödinger eq.” for E-eigen-sates.

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{Gr}(\vec{r}\,')e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{1st}(\vec{r})e^{-E_{1st}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{1st}(\vec{r}\,')e^{-E_{1st}t} = E_{1st}\phi_{1st}(\vec{r})e^{-E_{1st}t}$$

Non-local but energy independent

By adding equations

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

∇ expansion & truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')[V(\vec{r}) + \cancel{\nabla + \nabla^2}\ldots]$$

Therefor, in the leading

$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$