Finite temperature study of axial symmetry on the lattice

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Summary

- ✔ Introduction – chiral symmetry at finite temperature
- ✔ Simulations with dynamical overlap fermions
  - ✔ Fixing topology
- ✔ Results:
  - ✔ (test case) Pure gauge
  - ✔ N_f=2 case
  - ✔ Other studies
- ✔ Discussion and conclusions

People involved in the collaboration:
JLQCD group: H. Fukaya, S. Hashimoto, S. Aoki, T. Kaneko, H. Matsufuru, J. Noaki

See previous Lattice proceedings (10-11-12), article in prep.

Introduction – chiral symmetry

Pattern of chiral symmetry breaking
at low temperature QCD

$$\underbrace{SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A}_{\text{Symmetry of the Lagrangian}} \to SU(N_f)_V \times U(1)_V$$

Symmetries in the real world ($N_f{=}3$) at zero temperature

- $U(1)_V$ the baryon number conservation
- $SU(3)_V$ intact (softly broken by quark masses) – 8 Goldstone bosons (GB)
- $SU(3)_A$ is broken spontaneously by the non zero e.v. of the quark condensate
- No opposite parity GB, $U(1)_A$ is broken, but no 9th GB is found in nature.

Axial symmetry is not a symmetry of the quantum theory ('t Hooft - instantons)

$$\partial_\mu j_5^\mu(x) = 2iN_f q(x)$$ topological charge density

Witten-Veneziano: mass splitting of the $\eta'(958)$ from topological charge at large N.

Introduction – finite temperature

At finite temperature,
in the chiral limit $m_q \to 0$, chiral symmetry is restored

- Phase transition at $N_f{=}2$
- Crossover with 2+1 flavors

What is the fate of the axial $U(1)_A$ symmetry at finite temperature $T \gtrsim T_c$?

Complete restoration is not possible since it is an anomaly effect.
Exact restoration is expected only at infinite T (see instanton-gas models)
At most we can observe strong suppression

On the lattice the overlap Dirac operator is the best way to answer this questions since it preserves the maximal amount of chiral symmetry.
An operator satisfying the Ginsparg-Wilson relation has several nice properties e.g.

- exact relations between eigenmodes (EM) and topological charge
- exact relation among hermitian-operator EM and the non-hermitian ones
- ...

Project intent

Check the effective restoration of axial $U(1)_A$ symmetry by measuring (spatial) meson correlators at finite temperature in full QCD with the Overlap operator

Degeneracy of the correlators is the signal that we are looking for

$$\begin{tikzcd}
\sigma(1_4 \otimes 1_2) \arrow[r, leftrightarrow, "SU(2)_A"] \arrow[d, leftrightarrow, "U(1)_A"'] & \pi(i\gamma_5 \otimes \tau^a) \arrow[d, leftrightarrow, "U(1)_A"] \\
\eta(i\gamma_5 \otimes 1_2) \arrow[r, leftrightarrow, "SU(2)_A"] & \delta(1_4 \otimes \tau^a)
\end{tikzcd}$$

Dirac operator eigenvalue density is also a relevant observable for chiral symmetry

First of all there are some issues to solve before dealing with the real problem...

Simulations with Overlap fermions

The sign function in the overlap operator gives a delta in the force when H_W modes cross the boundary (i.e. topology changes).

In order to avoid expensive tricks to handle the zero modes of the Hermitian Wilson operator JLQCD simulations use (JLQCD 2006):

- Iwasaki action (suppresses Wilson operator near zero modes)
- Extra Wilson fermions and twisted mass ghosts to rule out the zero modes

Topology is thus fixed throughout the HMC trajectory.

The effect of fixing topology is expected to be a Finite Size Effect (actually O(1/V)), next slides

Fixing Topology: zero temperature

Partition function at fixed topology

$$Z_Q = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta\, \exp(-VF(\theta)) \qquad F(\theta) \equiv E(\theta) - i\theta Q/V$$

where the ground state energy can be expanded (T=0)

$$E_0(\theta) = \sum_{n=1}^{\infty} \frac{c_{2n}}{(2n)!}\theta^{2n} = \frac{\chi_t}{2}\theta^2 + O(\theta^4)$$

Using saddle point expansion around $\theta_c = i\frac{Q}{\chi_t V}(1 + O(\delta^2))$

one obtains the Gaussian distribution

$$Z_Q = \frac{1}{\sqrt{2\pi\chi_t V}} \exp\left[-\frac{Q^2}{2\chi_t V}\right]\left[1 - \frac{c_4}{8V\chi_t^2} + O\left(\frac{1}{V^2}, \delta^2\right)\right].$$

Fixing Topology

From the previous partition function we can extract the relation between correlators at fixed θ and correlators at fixed Q

In particular for the topological susceptibility and using the Axial Ward Identity we obtain a relation involving fermionic quantities:

$$\lim_{|x|\to\text{large}} \langle mP(x)mP(0)\rangle_Q^{\text{disc}} \equiv \frac{1}{V}\left(\frac{Q^2}{V} - \chi_t - \frac{c_4}{2\chi_t V}\right) + O(e^{-m_\pi |x|})$$

P(x) is the flavor singlet pseudo scalar density operator

Aoki *et al.* PRD76,054508 (2007)

What is the effect of fixing Q at finite temperature?

Results

- ✔ Simulation details
- ✔ Finite temperature quenched $SU(3)$ at fixed topology (TEST CASE)
  - ✔ Eigenvalues density distribution
  - ✔ Topological susceptibility
- ✔ Finite temperature two flavors QCD at fixed topology
  - ✔ Eigenvalues density distribution
  - ✔ Meson correlators

BG/L

Hitachi SR16K

Simulation details

Pure gauge (16^3x6, 24^3x6):
Iwasaki action + top. fixing term

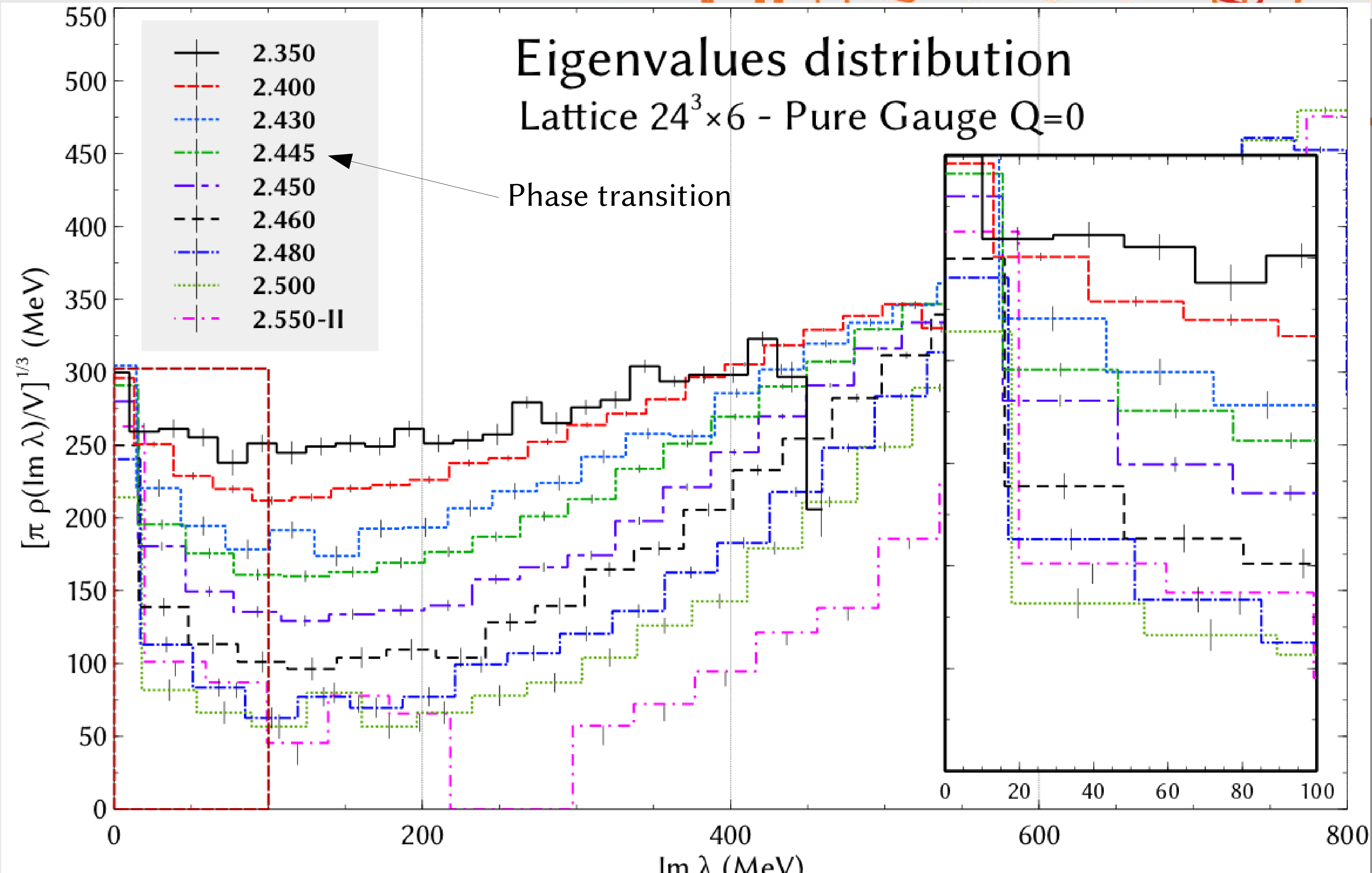
β	a(fm)	T (MeV)	T/Tc
2.35	0.132	249.1	0.86
2.40	0.123	268.1	0.93
2.43	0.117	280.9	0.97
2.44	0.115	285.7	0.992
2.445	0.114	288	1.0
2.45	0.1133	290.2	1.01
2.46	0.111	295.1	1.02
2.48	0.107	305.6	1.06
2.50	0.104	316.2	1.10
2.55	0.094	347.6	1.20

Two flavors QCD (**16^3x8**)
Iwasaki + Overlap + top. Fix
O(300) trajectories per T
am=0.05, 0.025, 0.01

β	a(fm)	T (MeV)	T/T_c
2.18	0.1438	171.5	0.95
2.20	0.1391	177.3	0.985
2.25	0.12818	192.2	1.06
2.30	0.1183	208.5	1.15
2.40	0.1013	243.5	1.35
2.45	0.0940	262.4	1.45

Pion mass: ~290 MeV @ am=0.015 , β=2.30
T_c was conventionally fixed to 180, not relevant for the results (but supported by Borsanyi et al. results)

Eigenvalues distribution SU(3)

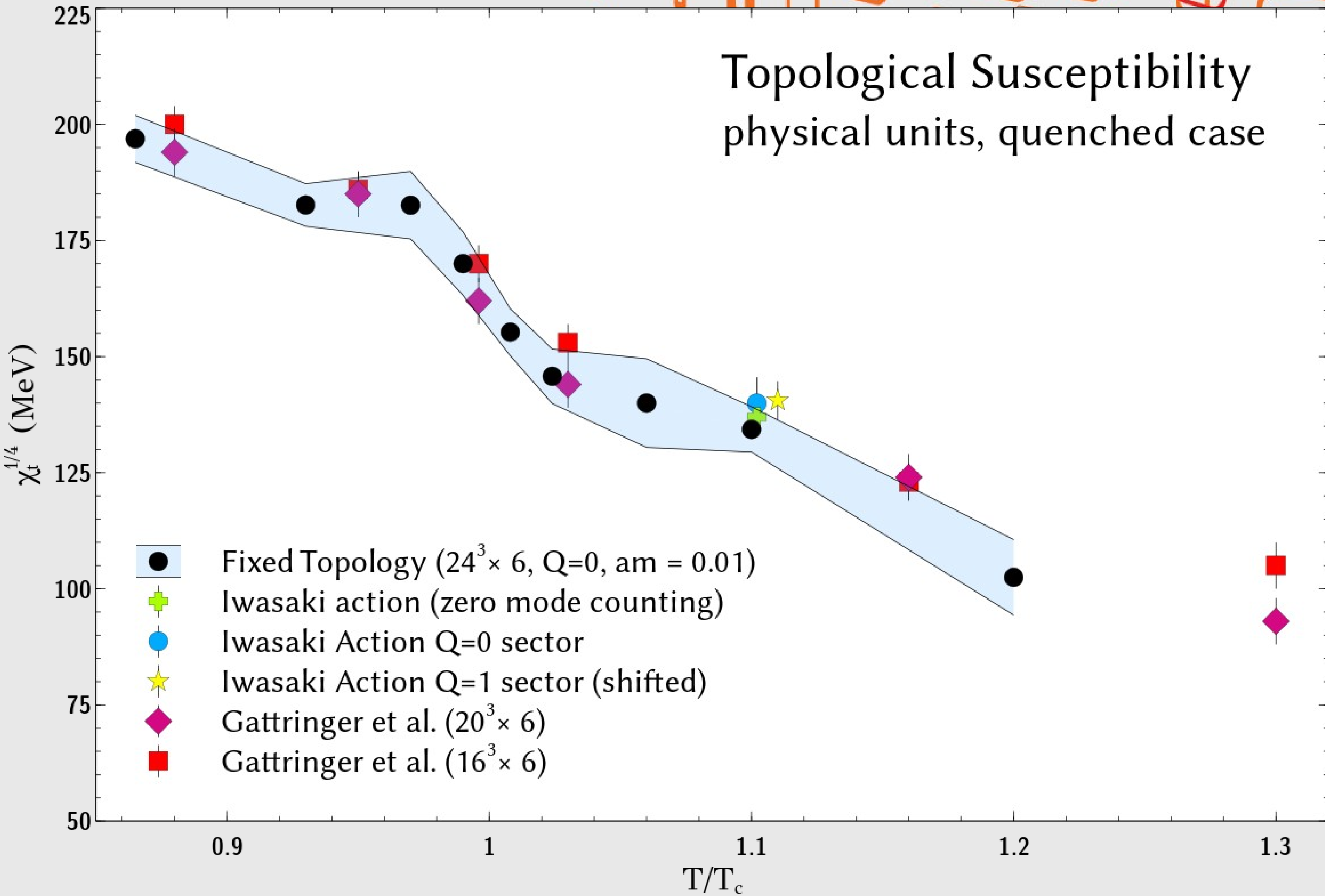


Topological susceptibility

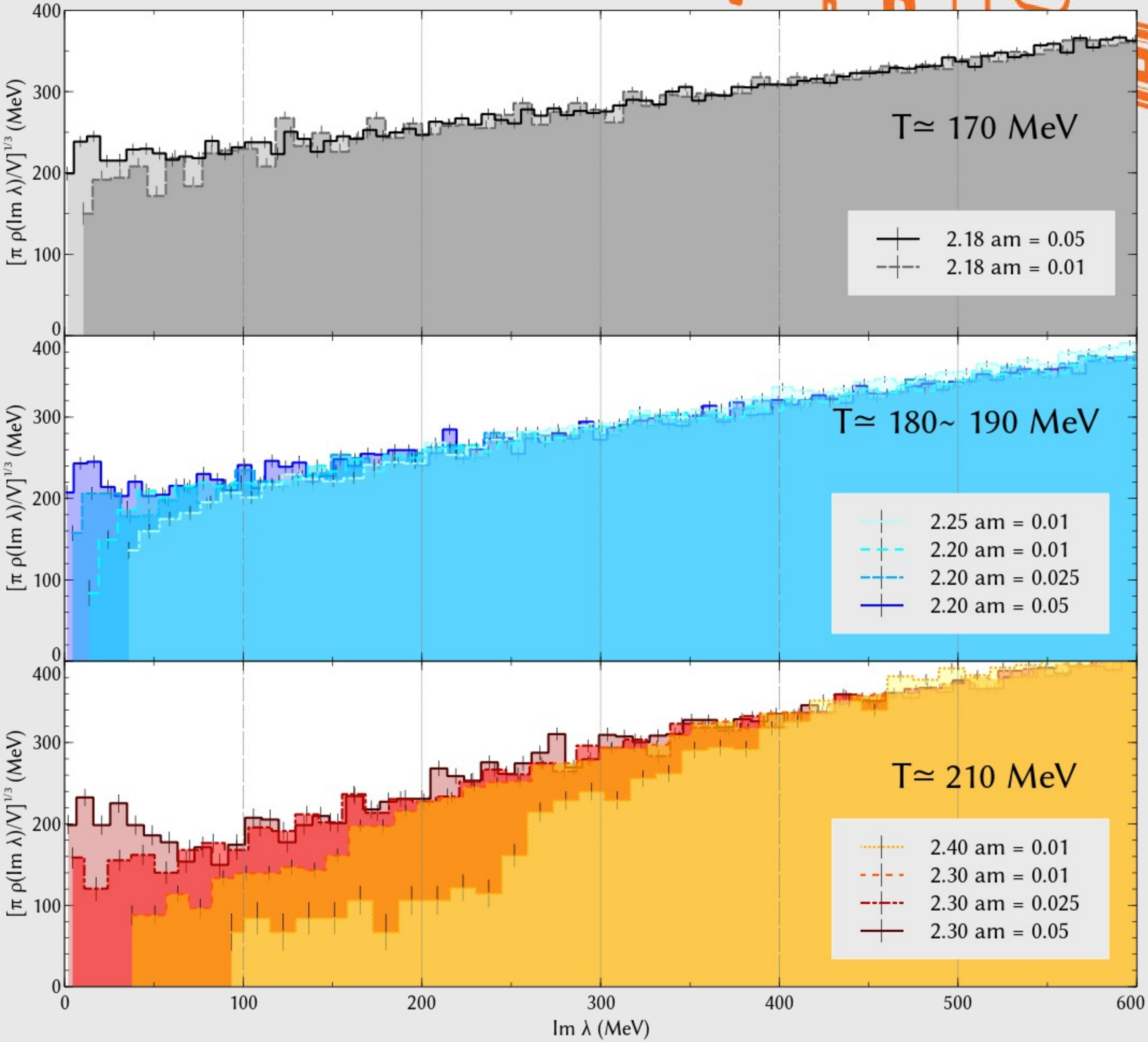
$$\lim_{|x|\to\text{large}} \langle mP(x)mP(0)\rangle_Q^{\text{disc}} \equiv \frac{1}{V}\left(\frac{Q^2}{V} - \chi_t - \frac{c_4}{2\chi_t V}\right) + O(e^{-m_\pi |x|})$$

- (Spatial) Correlators are always approximated by the first 50 eigenvalues
- Pure gauge: double pole formula for disconnected diagram
- Q=0, assume c_4 term is negligible, then check consistency
- Topological susceptibility estimated by a joint fit of connected and disconnected contribution to maximize info from data

Topological susceptibility



Eigenvalues distribution - QCD $N_f\!=\!2$



Effect of axial symmetry on the Dirac spectrum

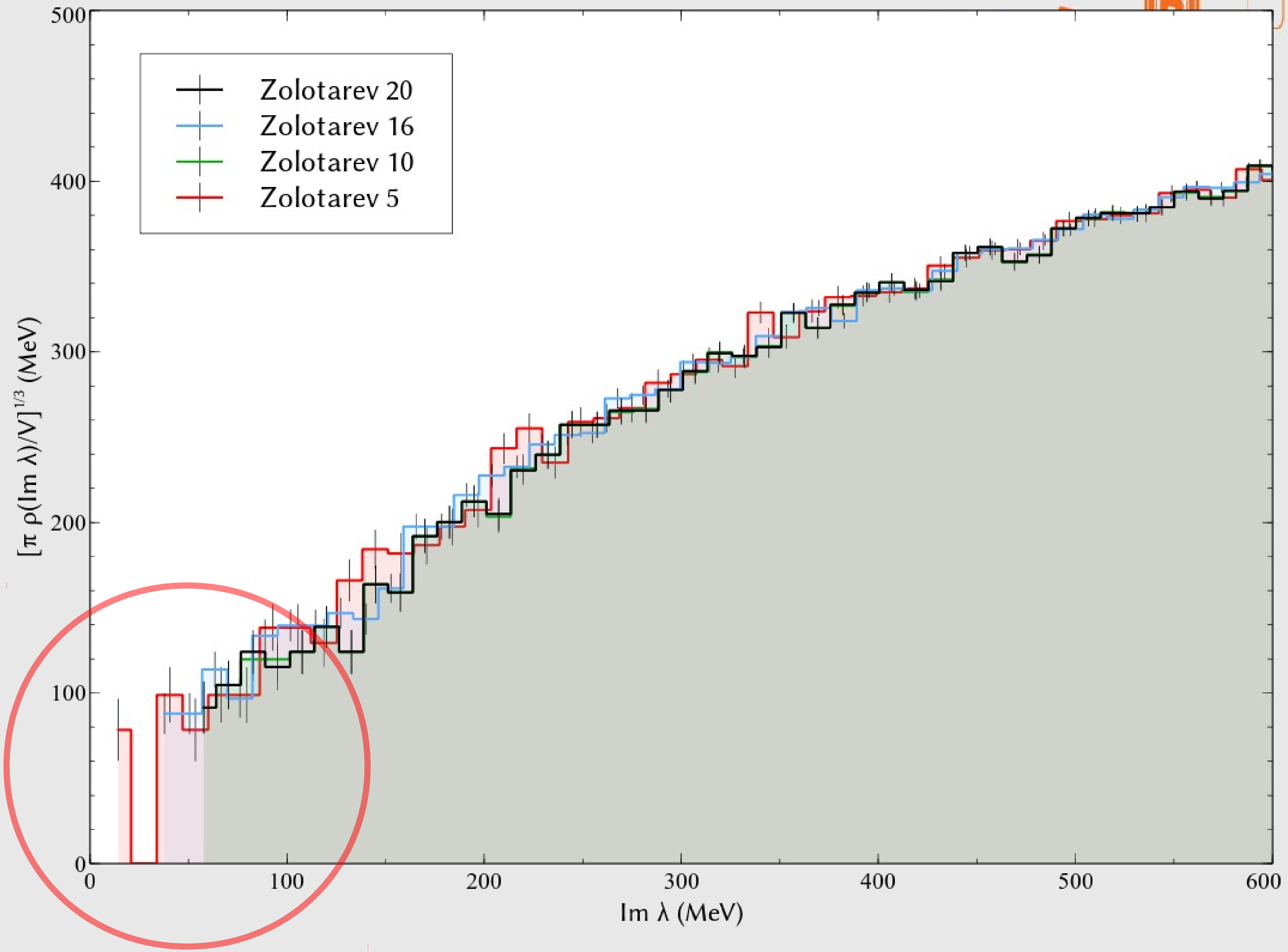
$$\chi^{\pi-\delta} = \int_0^\infty d\lambda \rho_m(\lambda) \frac{4m^2}{\lambda^2 + m^2}$$

If axial symmetry is restored we can obtain constraints on the spectral density

$$\lim_{\lambda\to 0}\lim_{m\to 0} \frac{\rho_m(\lambda)}{\lambda^2} = 0$$

Ref: S. Aoki, H. Fukaya, Y. Taniguchi arXiv:1209.2061

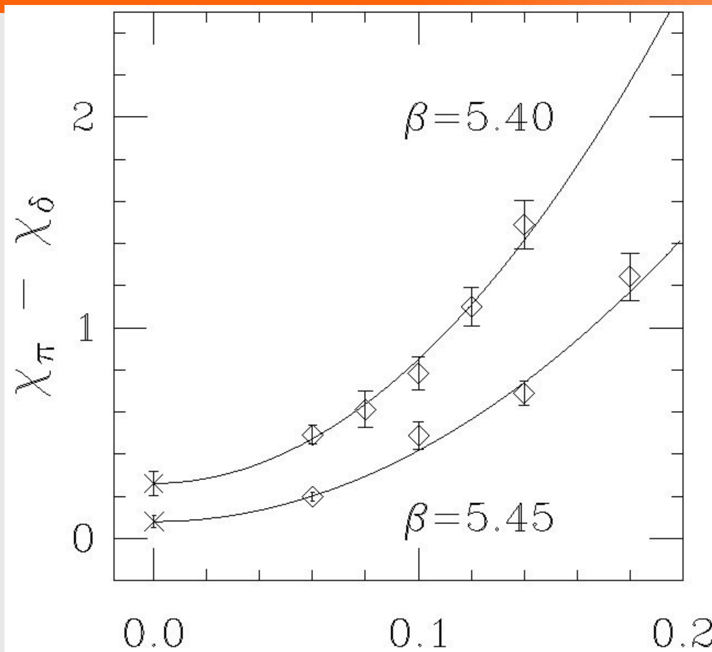
Eigenvalues distribution - Approximation



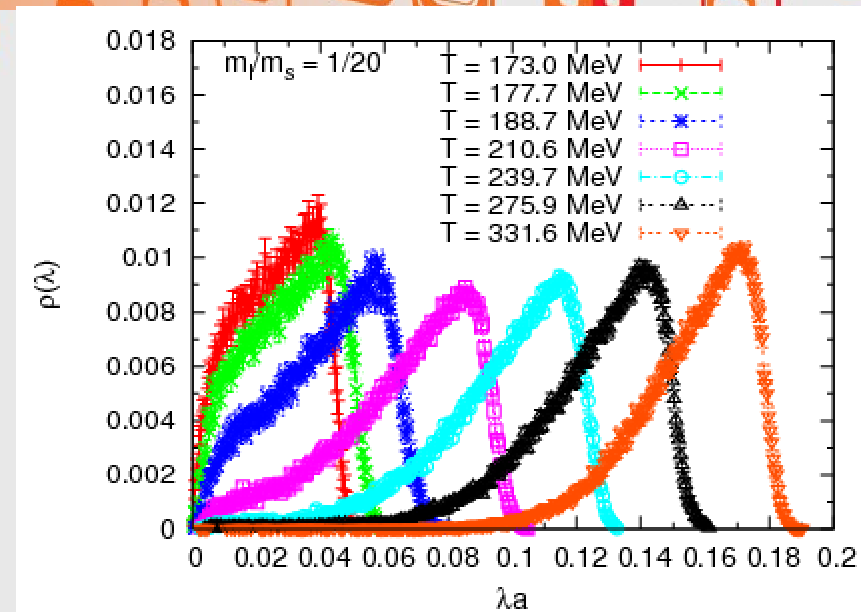
Decreasing Zolotarev poles in the approx. of the sign function

At $N_{poles} = 5$ more near zero modes appear

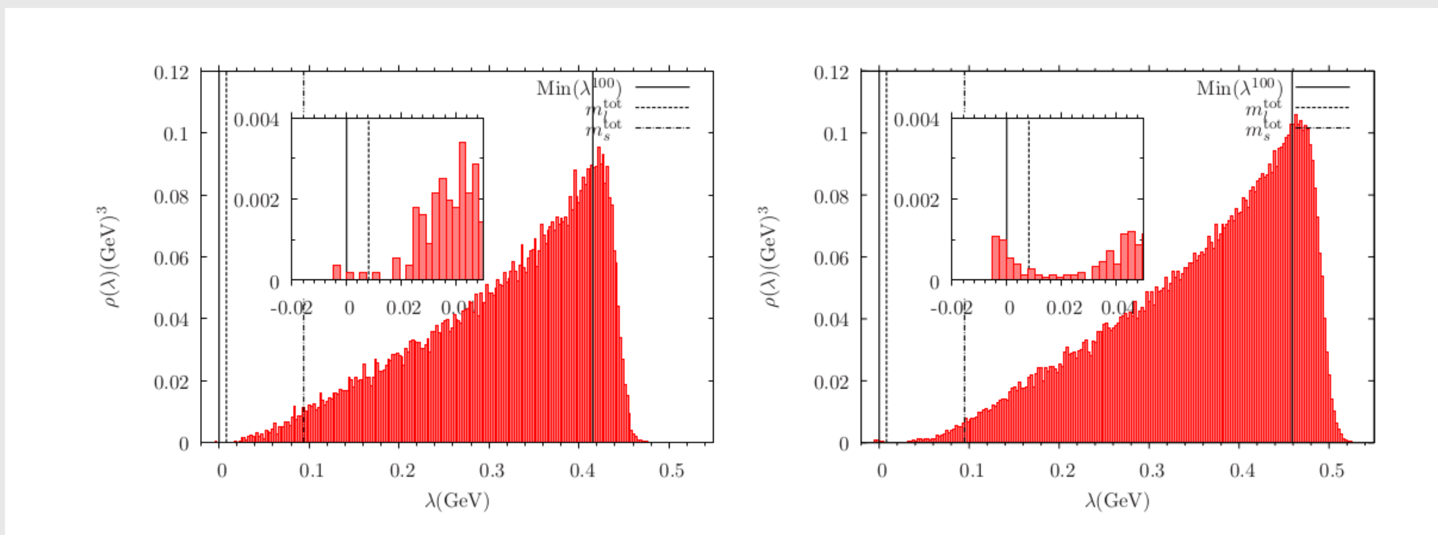
Other analyses



Vranas 2000
DWF
No restoration

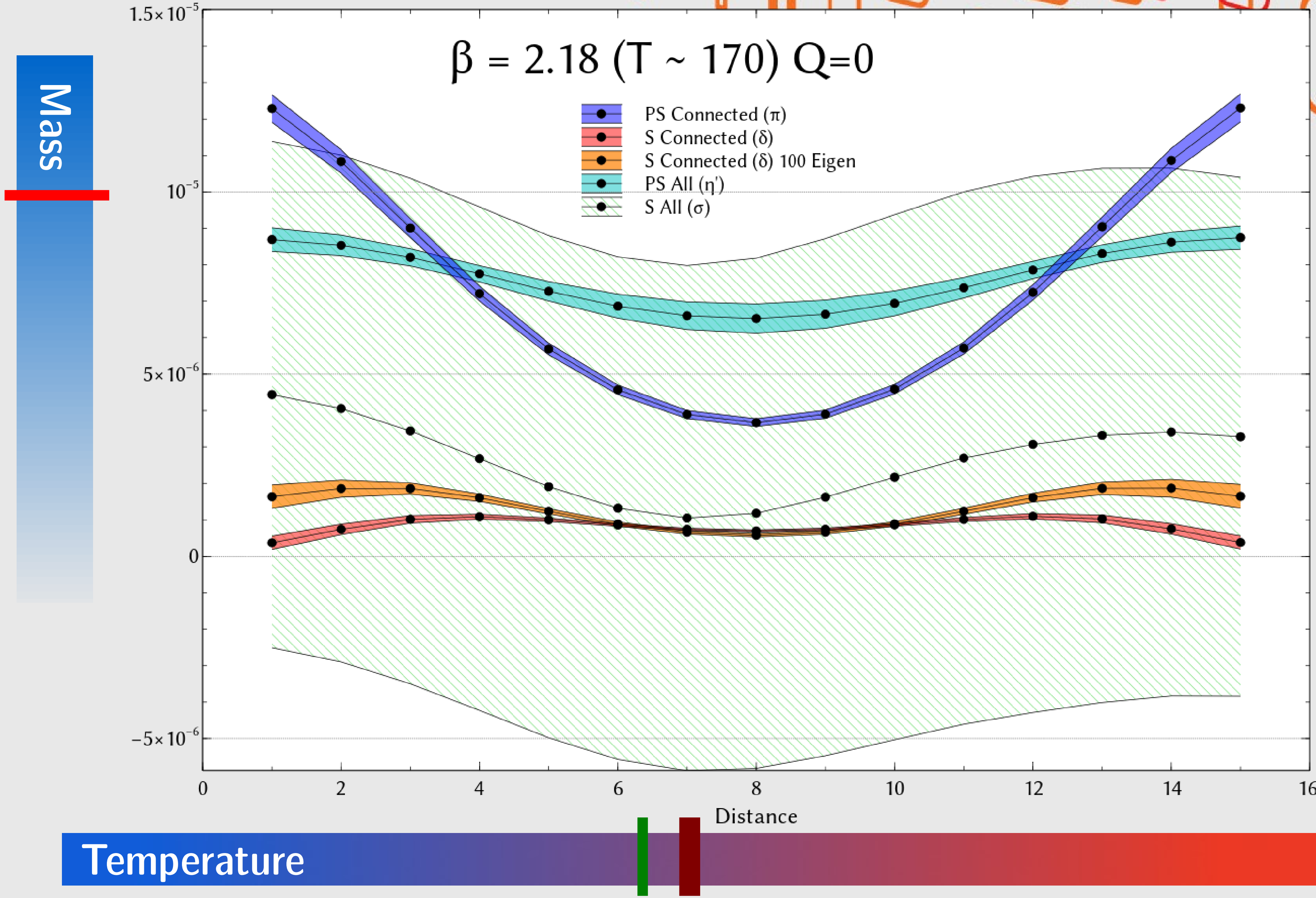


Ohno et al. 2011 – HISQ (2+1) tails



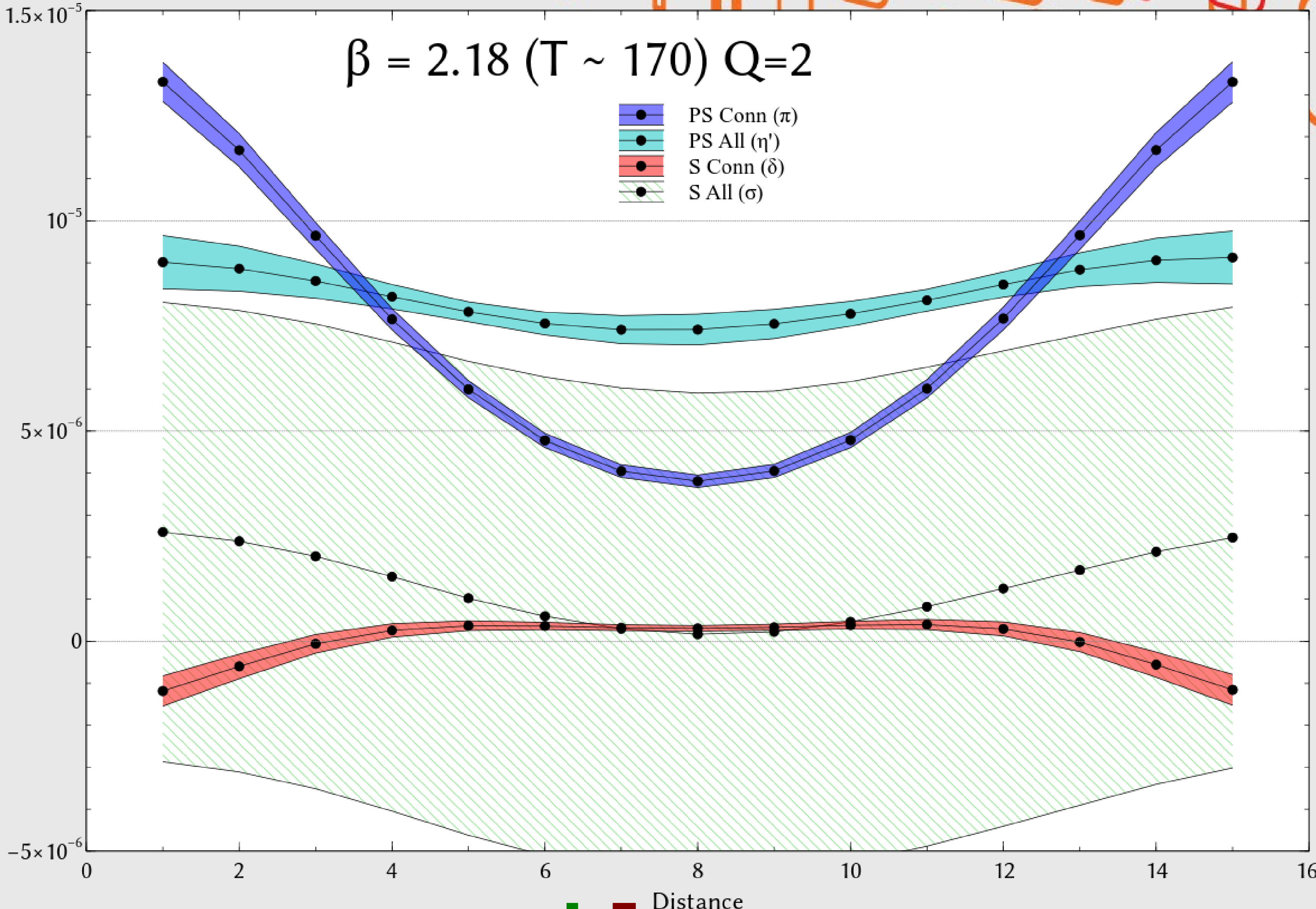
Bazavov et al. 2011
(HotQCD) DWF
Low modes

Meson Correlators



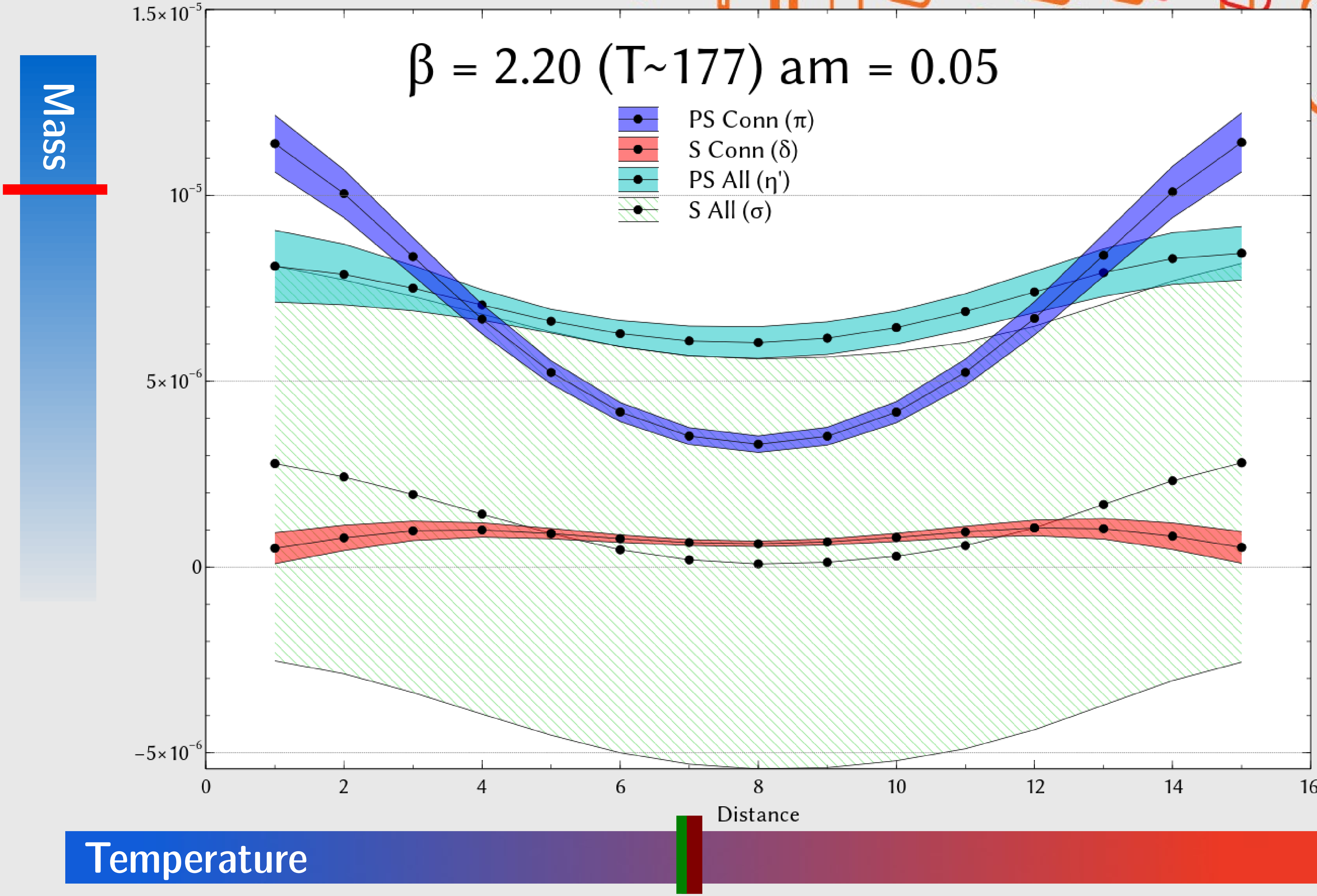
Meson Correlators

Mass



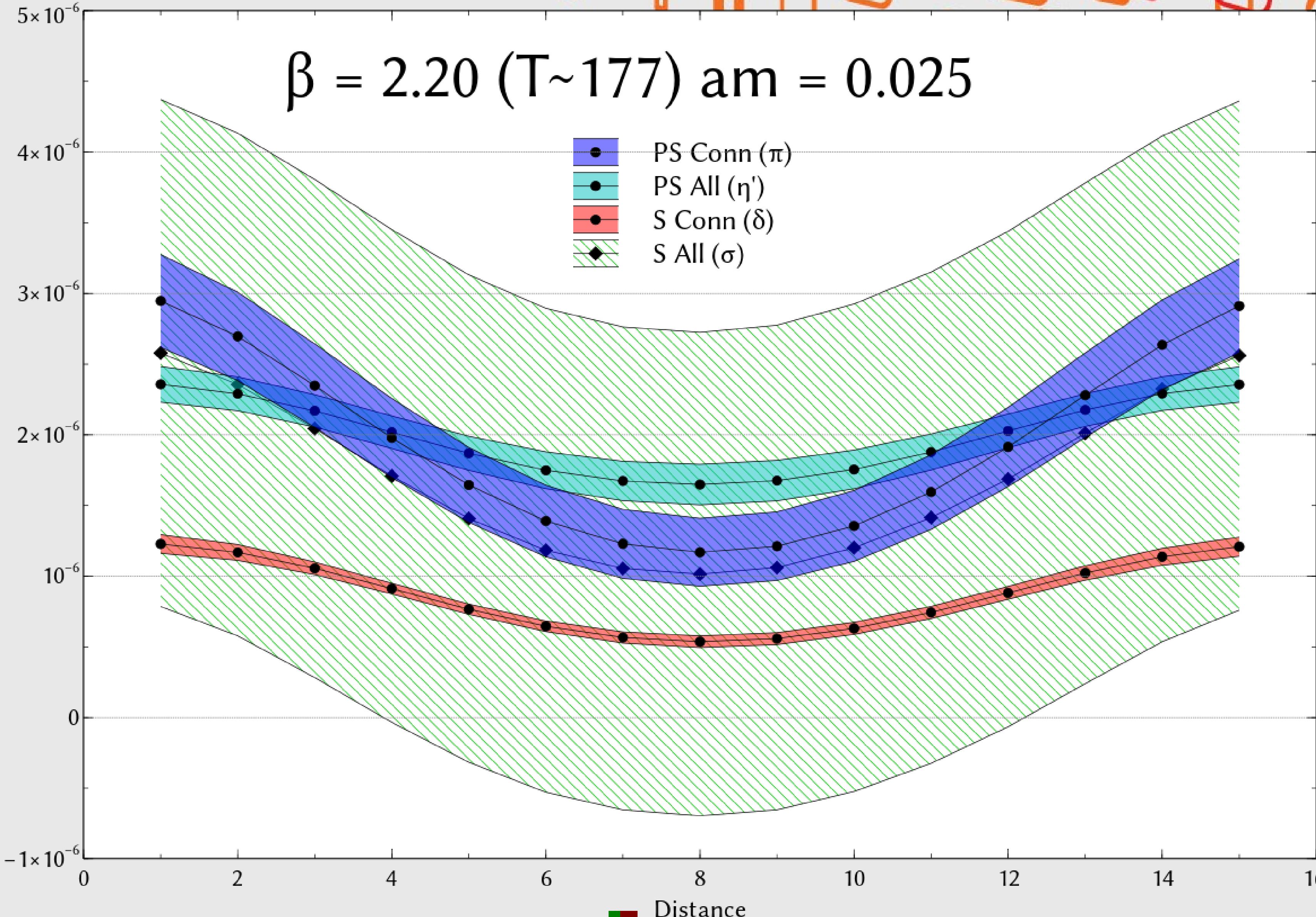
Temperature

Meson Correlators



Meson Correlators

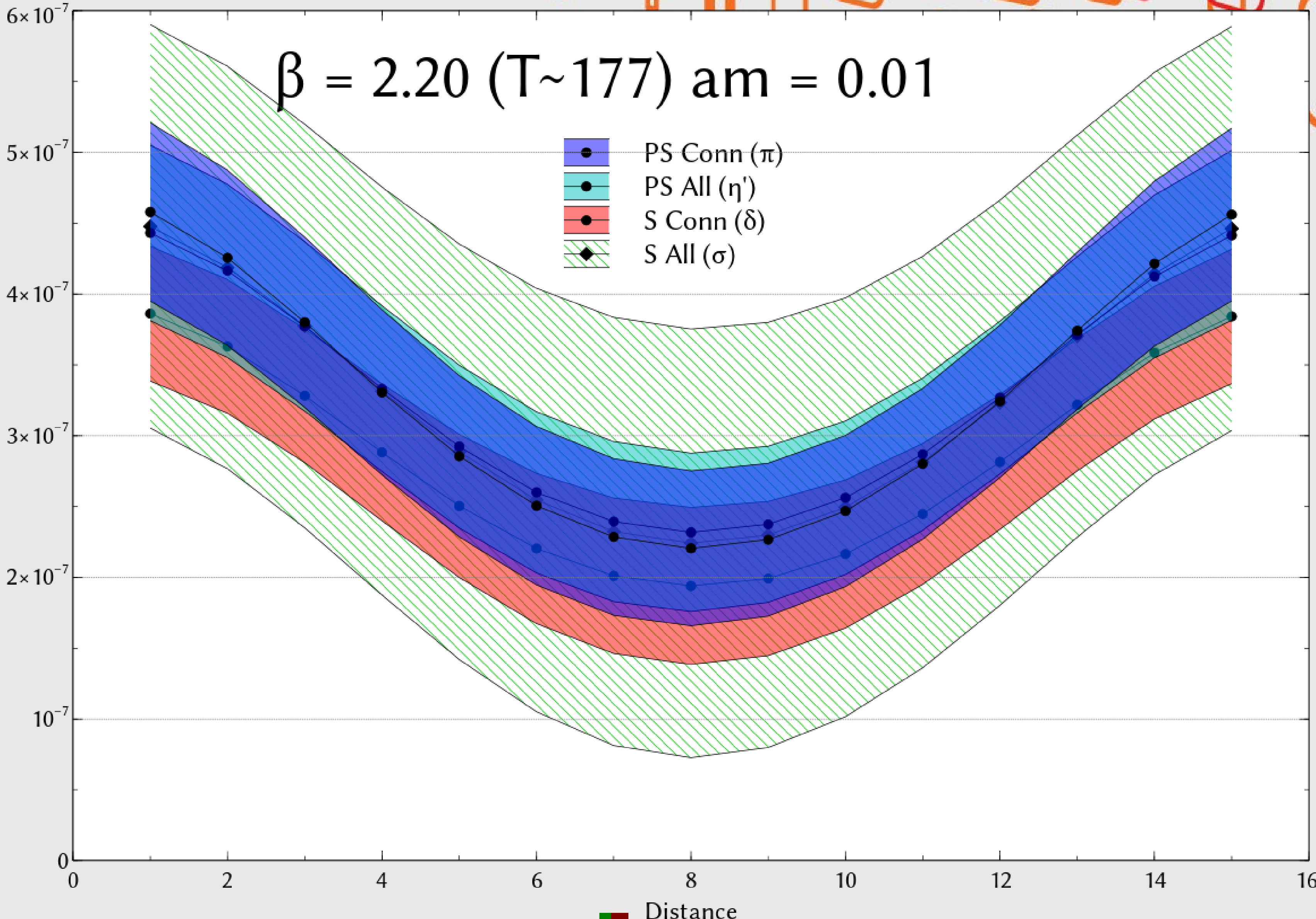
Mass



Temperature

Meson Correlators

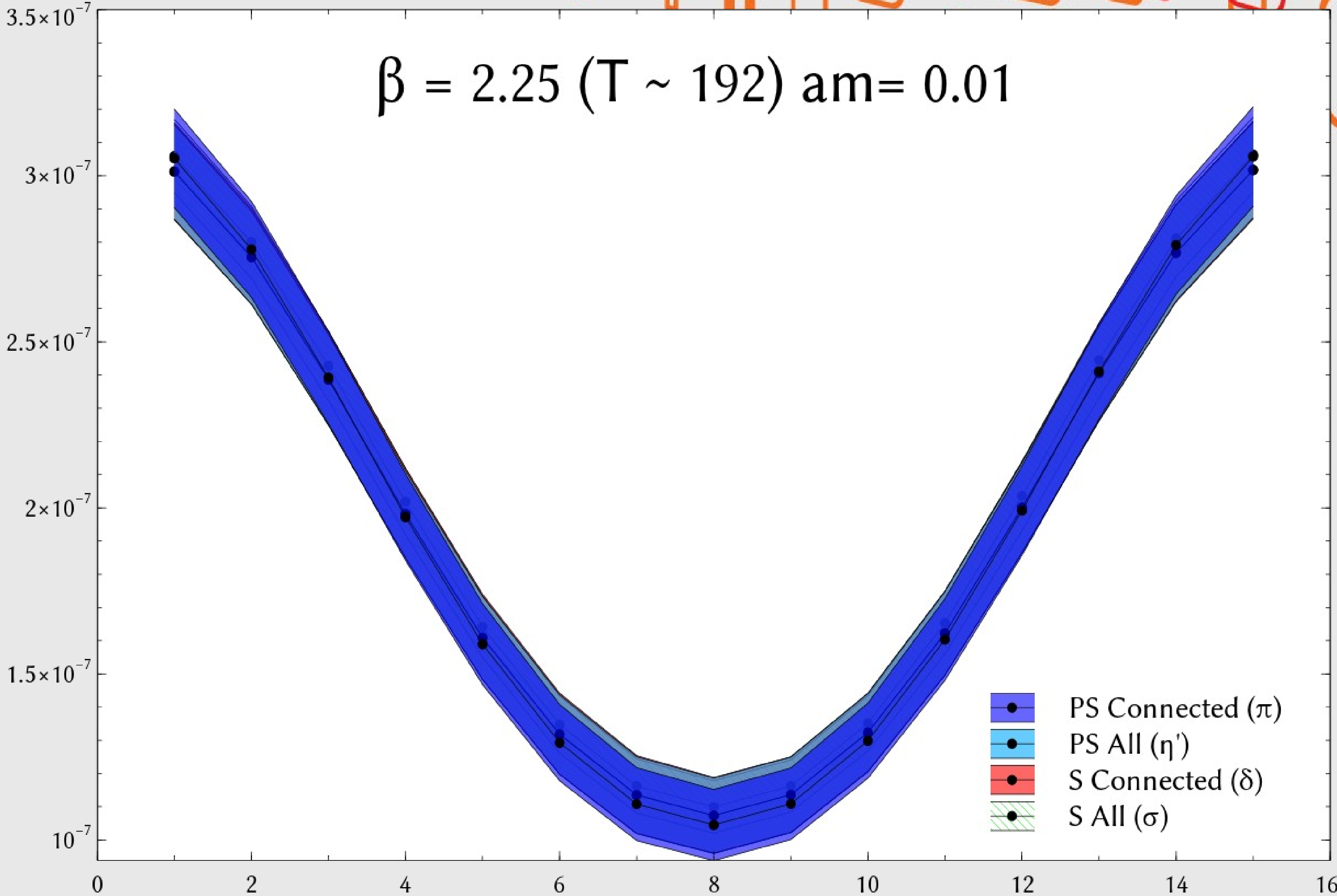
Mass



Temperature

Meson Correlators

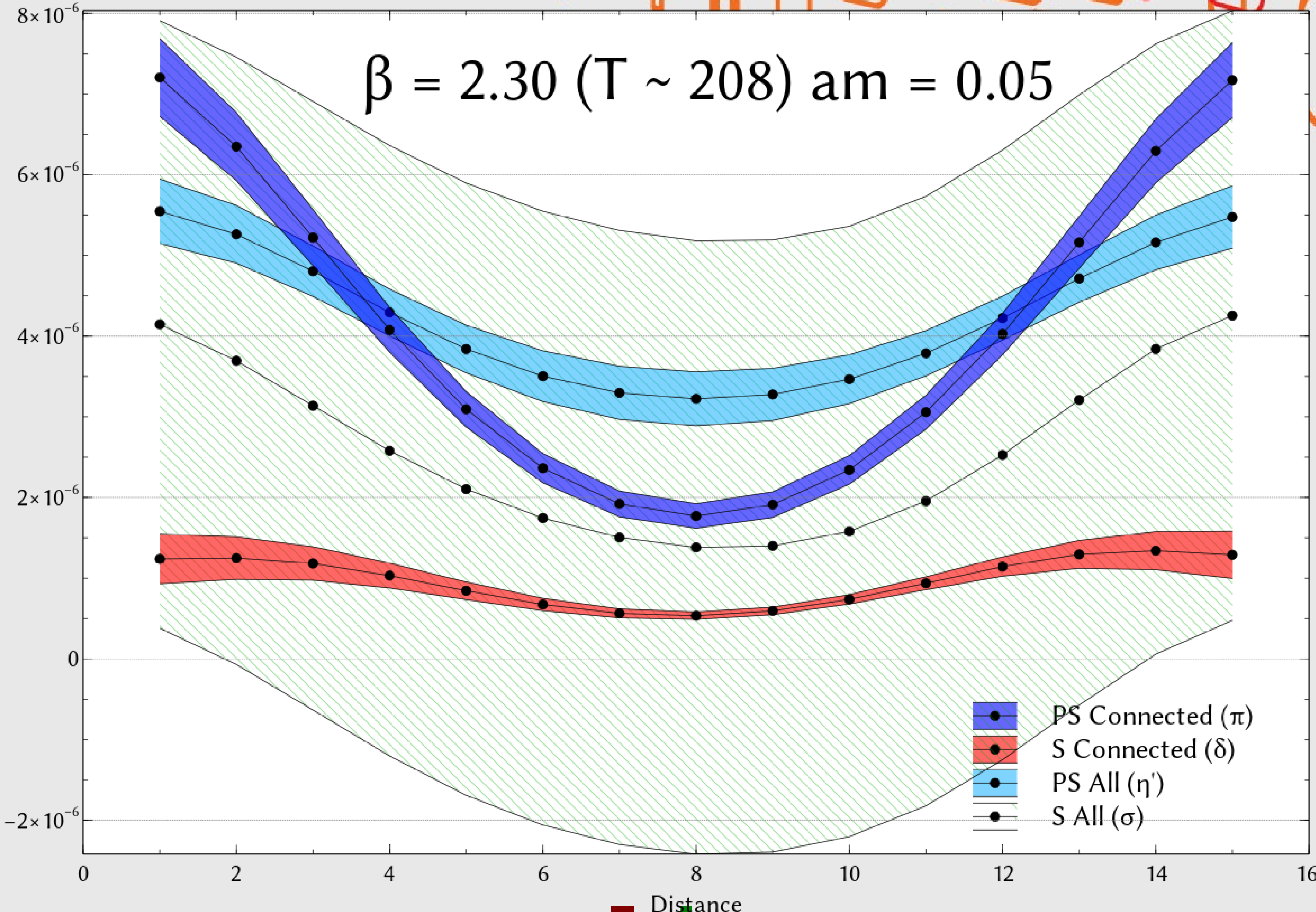
Mass



Temperature

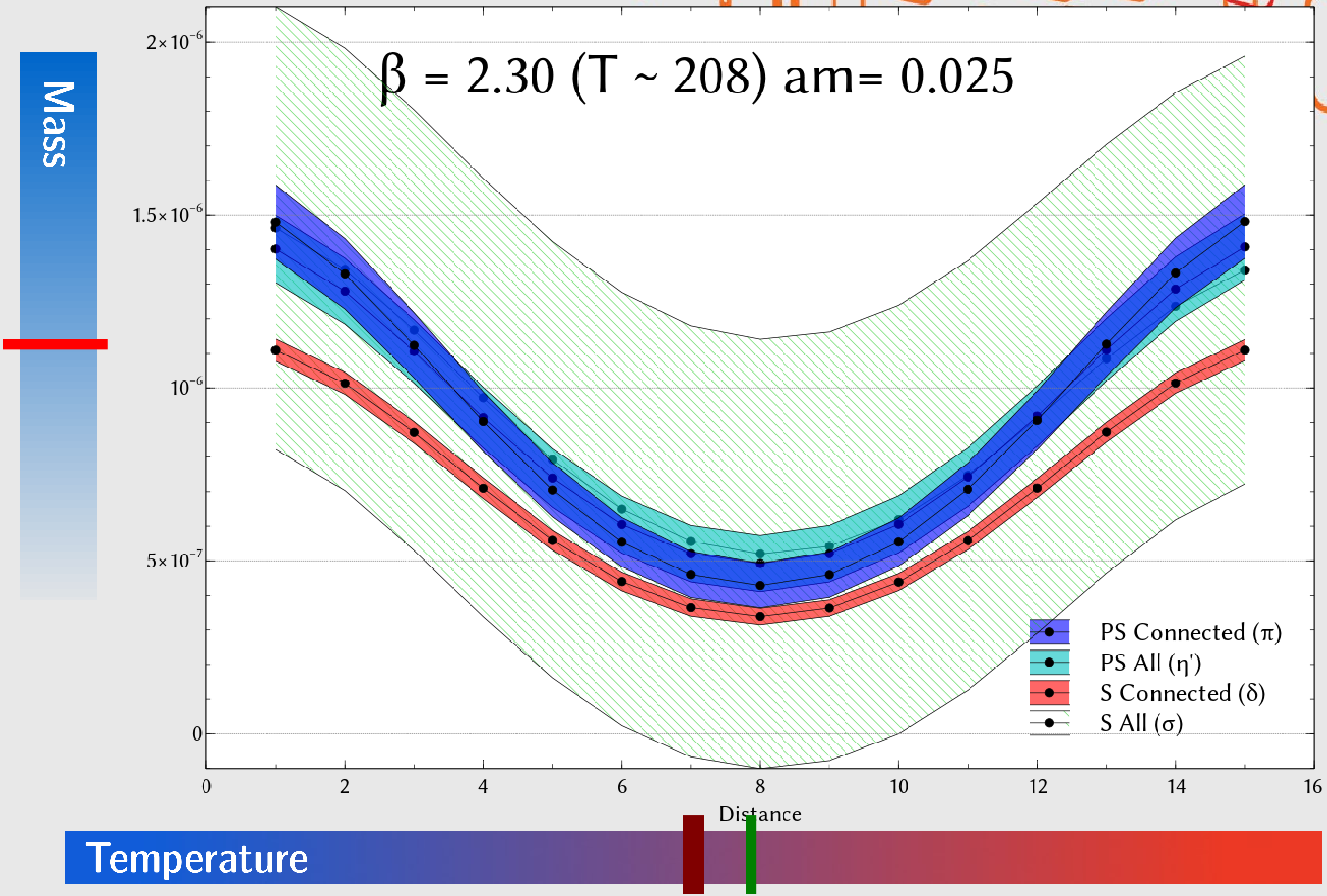
Meson Correlators

Mass

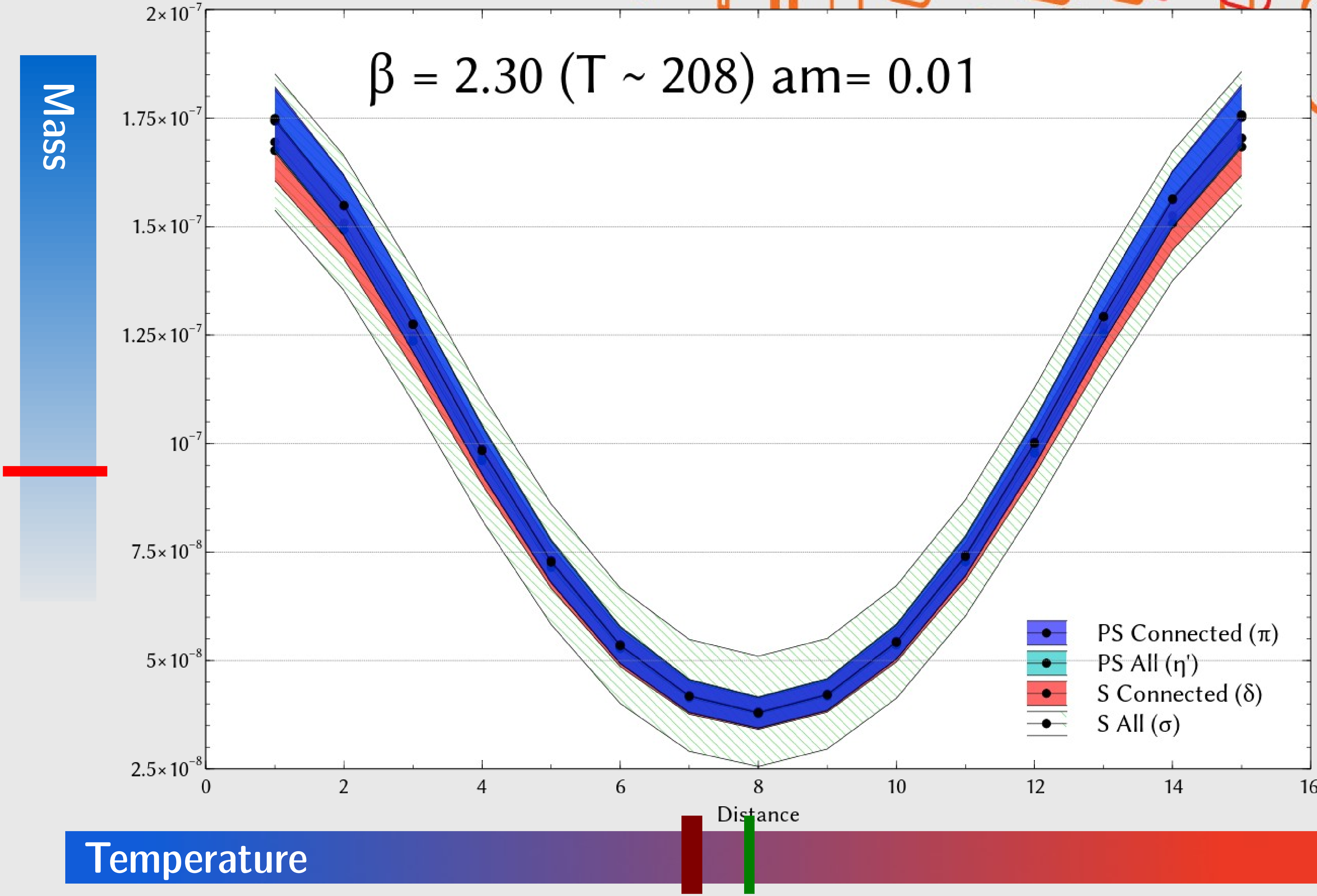


Temperature

Meson Correlators

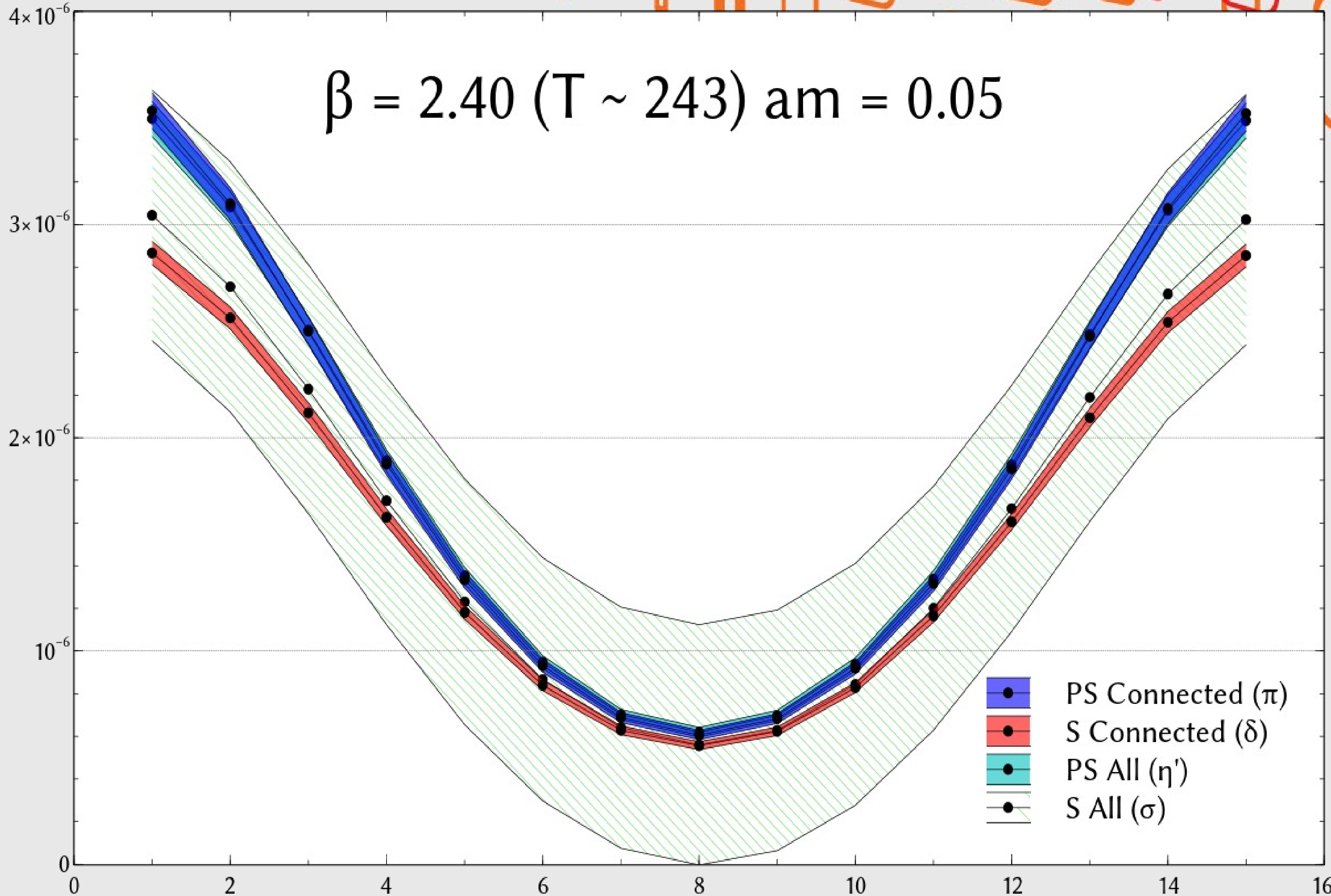


Meson Correlators



Meson Correlators

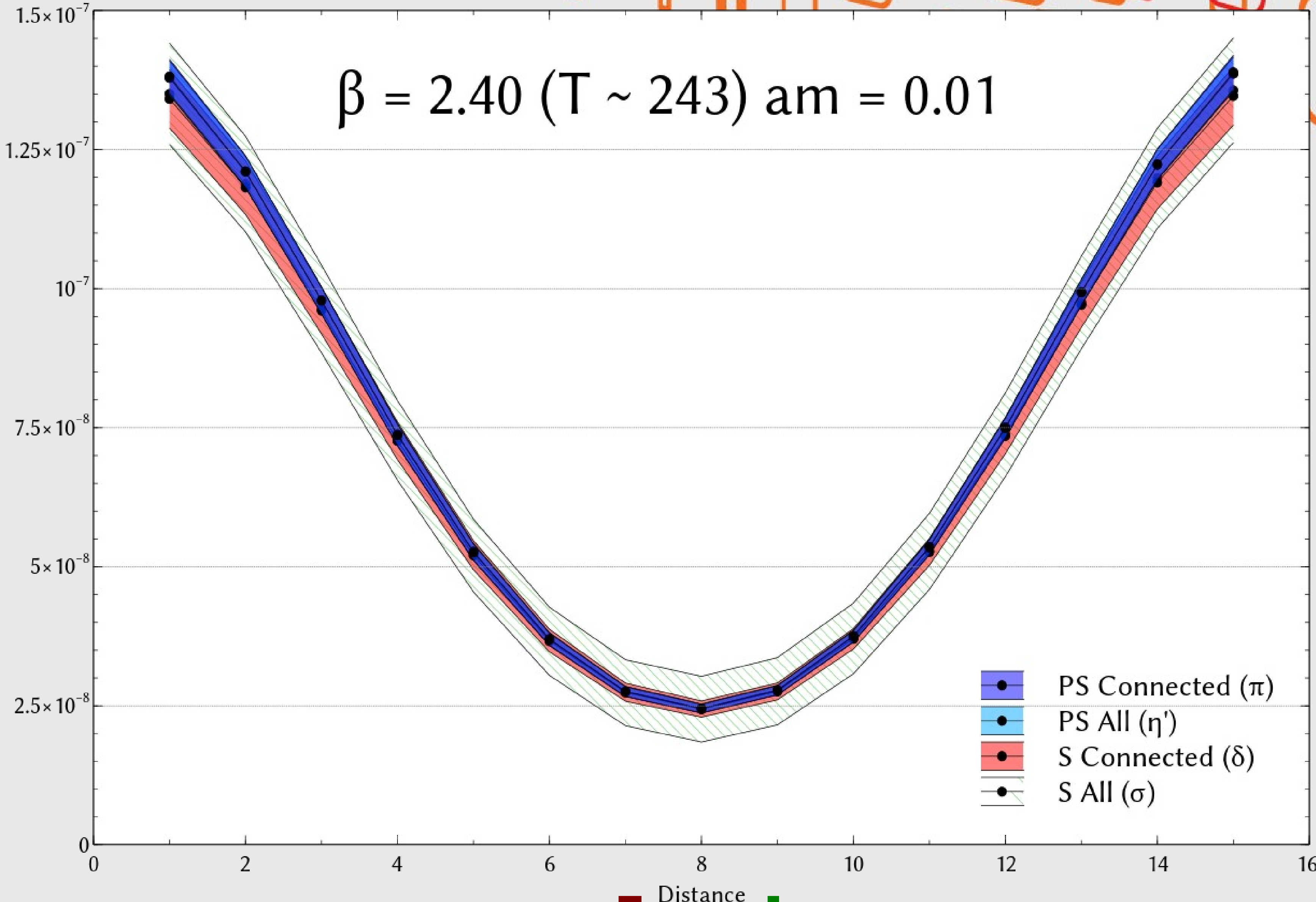
Mass



Temperature

Meson Correlators

Mass



Temperature

Conclusions

- With overlap fermions we have a clear theoretical setup for the analysis of spectral density and control on chirality violation terms.
- Realistic simulations are possible, but topology must be fixed
- A check of systematics due to topology fixing at finite temperature is necessary (finite volume corrections expected)
- Pure gauge test results show that we can control these errors as in the previous T=0 case.
  Finite volume effects are small in the SU(3) case

- Full QCD spectrum shows a gap at high temperature even at pion masses ~250 MeV
  - Statistics: high at T>200 MeV
  - Finite volume effects (beside topology fixing): to be checked
- Correlators show degeneracy of all channels when mass is decreased
- Results support effective restoration of $U(1)_A$ symmetry

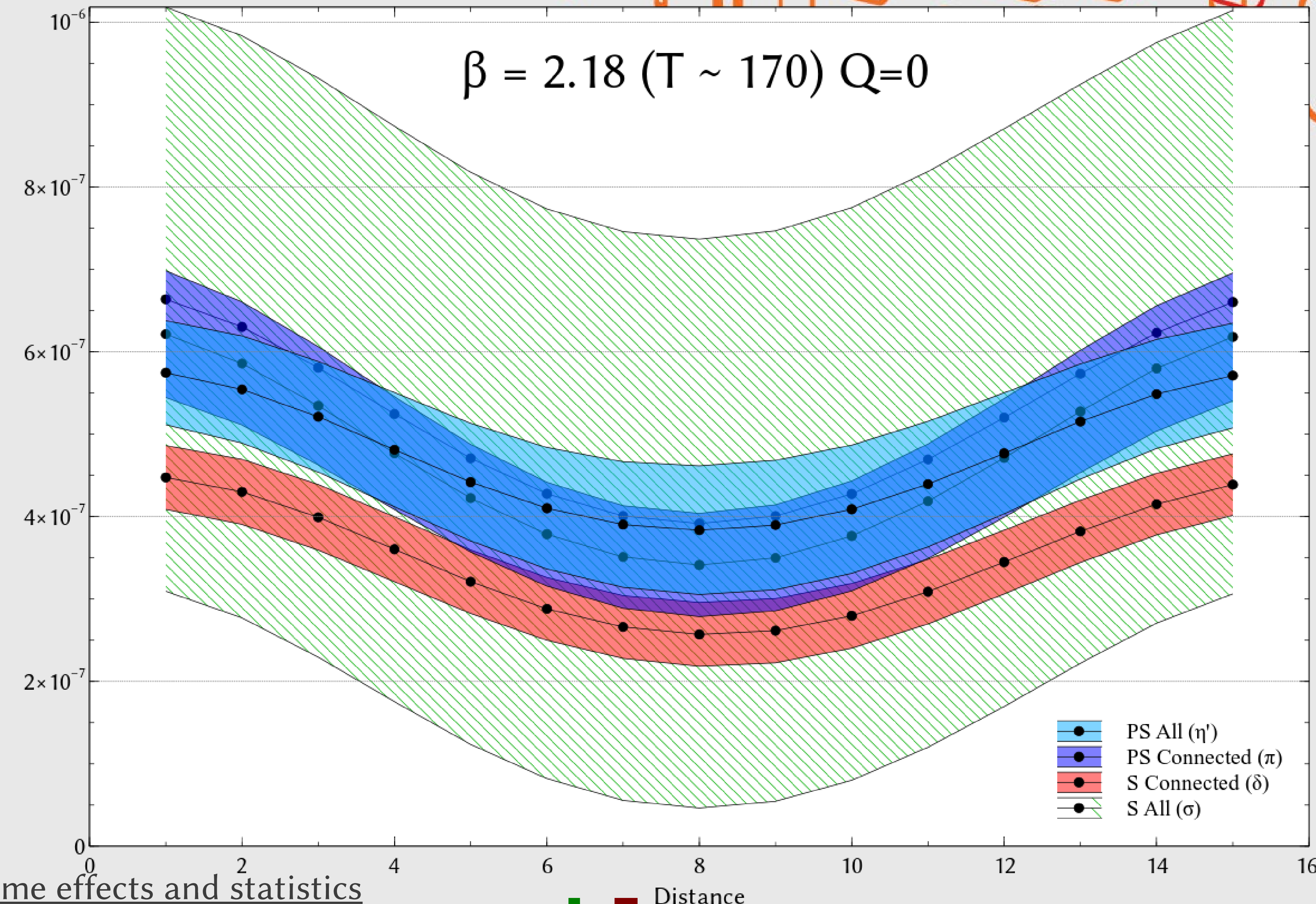
Thanks for your attention

LuxRay Artistic Rendering of Lowest Eigenmode

Backup slides

Meson Correlators

Mass



NB Volume effects and statistics

Temperature

Pure gauge theory