Large N_c gauge theory and Chiral Random Matrix Theory

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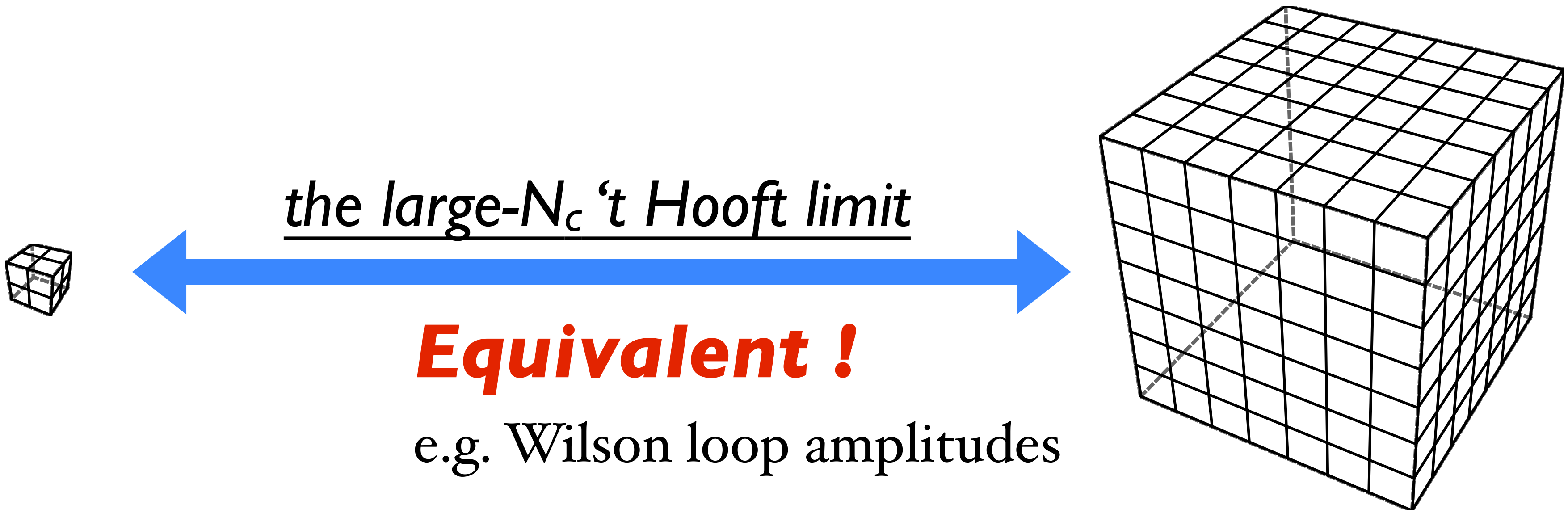
In the large-N_c 't Hooft limit of QCD, the theory is greatly simplified (planarity) while sharing the same properties (asymptotic freedom, confinement, chiral symmetry breaking, ...) with QCD.

Consider $SU(N_c)$ gauge theory



This is so called *the Large-N_c volume equivalence. (Eguchi-Kawai 82')*
(Eguchi-Kawai equivalence)

***Question:* In SU(N_c) gauge theory with N_f=2 adj. fermions at infinite V, the chiral symmetry is broken or not?**

✤ SU(2) with N_f=2 adjoint fermions - candidate of the Minimal Walking Technicolor model

Instead of considering the theory at infinite volume, *in the 't Hooft large-N_c limit (thanks to Large-N_c volume equivalence) we study*

☛ SU(N_c) gauge theory with N_f=2 adj. fermions at small V (=2^4 in our simulation)

The breakdown of the chiral symmetry can be detected by using *the Chiral Random Matrix Theory (chRMT).*

Large N_c Volume Independence (80's)

In 1982, Eguchi and Kawai proposed that large volume SU(N_c) gauge theory is identical to a single-site reduced matrix model in the large N_c limit if certain conditions are satisfied. The most crucial condition is the <u>unbroken center symmetry</u>.

However, single-site Eguchi-Kawai volume reduction Fails $\mathrm{tr}\, U_\mu \neq 0$

Bhanot, Heller & Neuberger 82

<u>Prescriptions in early 80's</u>

Quenched EK

Bhanot, Heller & Neuberger 82

Fails Brigoltz & Sharpe 2008

Twisted EK

Gonzales-Arroyo & Okawa 83

Teper & Vairinhos 2007

Azeyanagi et al 2008

Bietenholtz et al 2007

<u>Revival (2007 ~) - center symmetry restoration</u>

Partial reduction (aL>1fm)	Kiskis, Narayanan, Neuberger (2003)
Adjoint EK (AEK)	Kovtun, Unsal, & Yaffe (2007)
Deformed EK	Unsal & Yaffe (2008)
Modified TEK	Gonzales-Arryo & Okawa (2010)

Adjoint Eguchi-Kawai model (AEK)

- **Perturbative analysis**

$N_f \geq 1$ **fermions in adjoint representation with periodic boundary conditions**

1-loop effective potential for Wilson lines is repulsive → **stabilized center symmetry**

Kovtun, Unsal & Yaffe 2007/2010
Azeyanagi, Hanada, Unsal, Yacoby. 2010

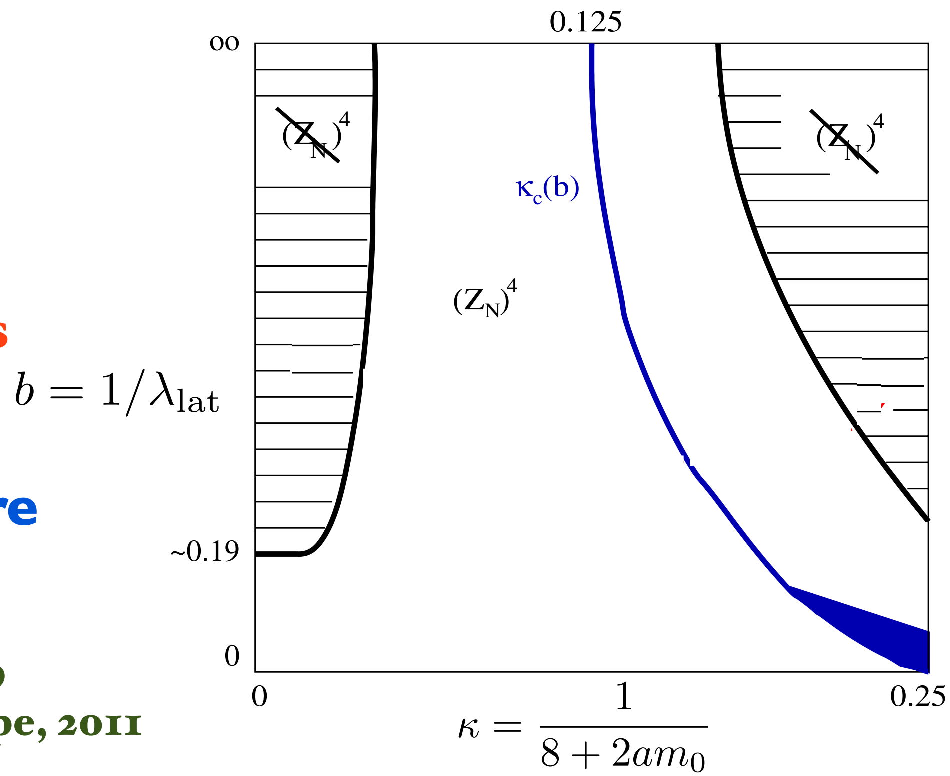
- **Numerical simulations**

single-site EK with N_f=1,2 adj. fermions

$(Z_{N_c})^4$ center symmetry is unbroken even at weak coupling & heavy fermions

$$m_0 \sim \mathcal{O}(1/a)$$

→ **non-perturbative study of pure $SU(N_c)$ gauge theory**



Bringoltz, Sharpe, 2009
Bringoltz, Koren, Sharpe, 2011

Chiral Random Matrix Theory (chRMT)

chRMT provides equivalent descriptions of the low-energy limit of QCD-like theories if the chiral symmetry is spontaneously broken.

$$Z = \int DW \prod_{f=1}^{N_f} \det(\mathcal{D} + m_f) e^{-\frac{N\beta}{4}\mathrm{Tr}v(W^\dagger W)}$$

$$\mathcal{D} = \begin{pmatrix} 0 & iW \\ iW^\dagger & 0 \end{pmatrix}$$ **anti-Hermitian**

W is $N \times N$ complex matrix

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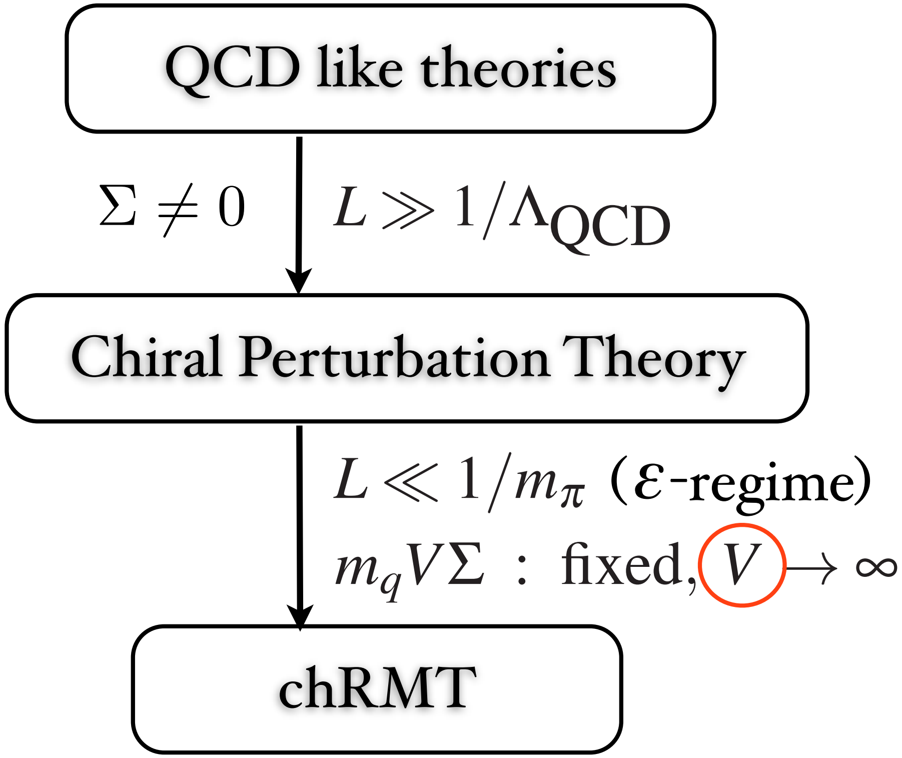


spectral density of low-lying Dirac eigenvalues

$$\rho(z), \quad z = \lambda V \Sigma \sim \mathcal{O}(1)$$

- **Order parameter of the spontaneous symmetry breaking**

$$\Sigma = |\langle \bar{\psi}\psi \rangle| = \lim_{\epsilon \to 0} \lim_{m \to 0} \lim_{V \to \infty} \frac{\pi \rho(\epsilon)}{V}$$ **Banks-Casher relation**

The chRMT & large-N_c gauge theories

In large-N_c gauge theories, colors are also relevant degrees of freedom. Thus,

$$V(N_c)^\alpha \longleftrightarrow N$$

To compare with chRMT

'chRMT limit': $m_q V(N_c)^\alpha \tilde{\Sigma}$: fixed, $N_c \to \infty$

spectral density: $\rho(z)\,,\quad z = \lambda V(N_c)^\alpha \tilde{\Sigma} \sim \mathcal{O}(1)$

- **Order parameter of the spontaneous symmetry breaking**

$$\tilde{\Sigma} = |\langle \bar{\psi}\psi \rangle| = \lim_{\epsilon\to 0} \lim_{m\to 0} \lim_{N_c\to\infty} \frac{\pi\rho(\epsilon)}{V(N_c)^\alpha}$$

The chRMT limit vs 't Hooft limit

'chRMT limit': $m_q V (N_c)^\alpha \tilde{\Sigma}\ :\ \text{fixed},\ N_c \to \infty$

't Hooft limit': $m_q, V\ :\ \text{fixed},\ N_c \to \infty$

Different!

The EK volume equivalence does not hold in the chRMT limit.

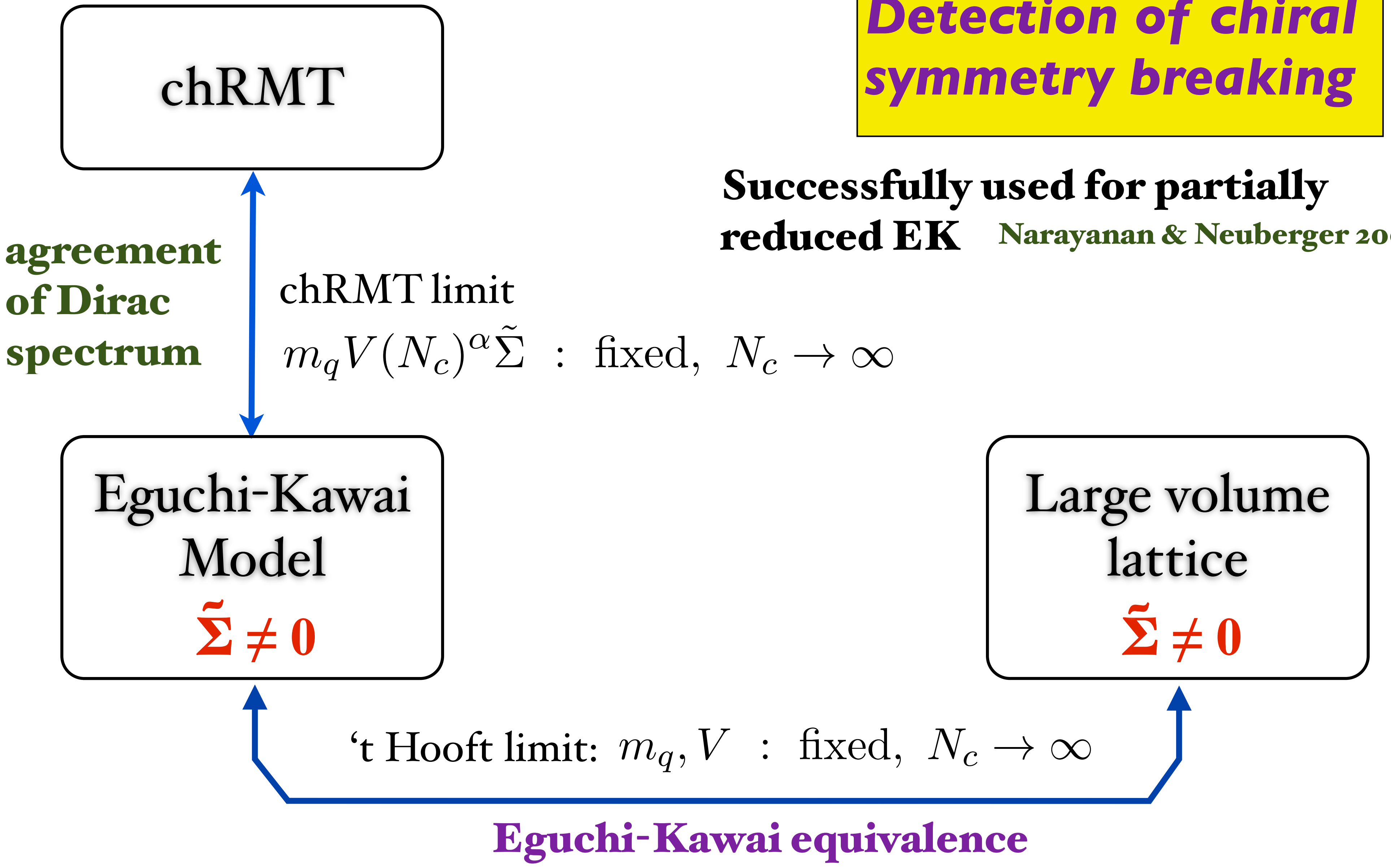
Can we still study the chiral symmetry breaking using the EK volume equivalence and chRMT in large-N_c gauge theories?

Yes!

EK model, large V LGT and chRMT

Detection of chiral symmetry breaking

Successfully used for partially reduced EK Narayanan & Neuberger 2006



To establish the method, we numerically study *the quenched approximation*, where *the answer is known*.

In the large-N_c 't Hooft limit,

SU(N_c) EK model with N_f=2 heavy adjoint fermions (m_0~1/a) on a 2^4 lattice

Center symmetry is unbroken and thus the EK equivalence holds.

Adjoint fermions are not dynamical.

pure SU(N_c) lattice gauge theory

Chiral symmetry is spontaneously broken?

Action and Symmetries

- **Action for reduced model on a 2^4 lattice**

$$Z_{reduced} = \int DUD\overline{\psi}D\psi\, \exp\left(S_g + S_f\right), \quad n \in 2^4$$

Periodic B.C.

$$S_g = 2N^2 b \sum_n \sum_{\mu<\nu} \left(1 - \frac{1}{N_c}\mathrm{ReTr}P_{\mu\nu}(n)\right), \qquad b = \frac{1}{g^2 N_c}$$

$$S_f = \sum_{j=1}^{2} \sum_n \overline{\psi}_{n,j} \left(1 - \kappa \left[\sum_{\mu=1}^{4} (1-\gamma_\mu) U^{adj}_{n,\mu} \psi_{n+\mu,j} + (1+\gamma_\mu) U^{\dagger,adj}_{n-\mu,\mu} \psi_{n-\mu,j}\right]\right).$$

$$\kappa = \frac{1}{2(m_0 a + 4)}, \quad m_0 \sim \mathcal{O}\left(\frac{1}{a}\right)$$

- **Symmetries**

$SU(N_c)$ **gauge symmetry:** $\quad U_{n,\mu} \to \Omega_n U_{n,\mu} \Omega^\dagger_{n+\mu}, \quad \Omega_n \in SU(N_c)$

$Z^4_{N_c}$ **center symmetry:** $\quad U_{n,\mu} \to e^{2\pi i n_\mu / N_c} U_{n,\mu}, \quad n_\mu \in Z_{N_c}$

Simulation details

- **fermions in action**

  quenched simulation (kappa=0)

  heavy Wilson fermions in adjoint rep. (kappa=0.09) ⟷ unbroken center symmetry

- **probe fermions**

  massless overlap fermions in adjoint rep.

  closer to RMT limit

  good chiral symmetry

- **number of colors up to N_c=16**

- **bare coupling b = 0.1, 0.2, 0.3, 0.4, 0.5**

  strong … weak

- **Hybrid Monte Carlo (HMC) algorithm**

  500 configs (10 trajectories) for each ensemble

  200 trajectories for thermalization

Tests of the Center Symmetry

If the $(Z_{N_c})^4$ center symmetry is unbroken,

(1) Polyakov loop scatters radially in the vicinity of the origin.

$$P_\mu = \frac{1}{N_c} \mathrm{tr} U_{n,\mu} U_{n+\mu,\mu} \,, \quad n \in 2^4$$

(2) Magnitude of the Polyakov loop goes to zero in the large N_c limit.

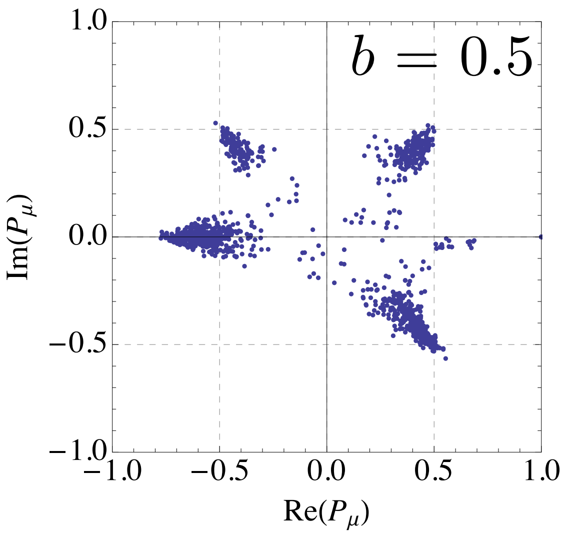
$$|P_\mu| \longrightarrow 0 \quad \text{as} \quad N_c \longrightarrow \infty$$

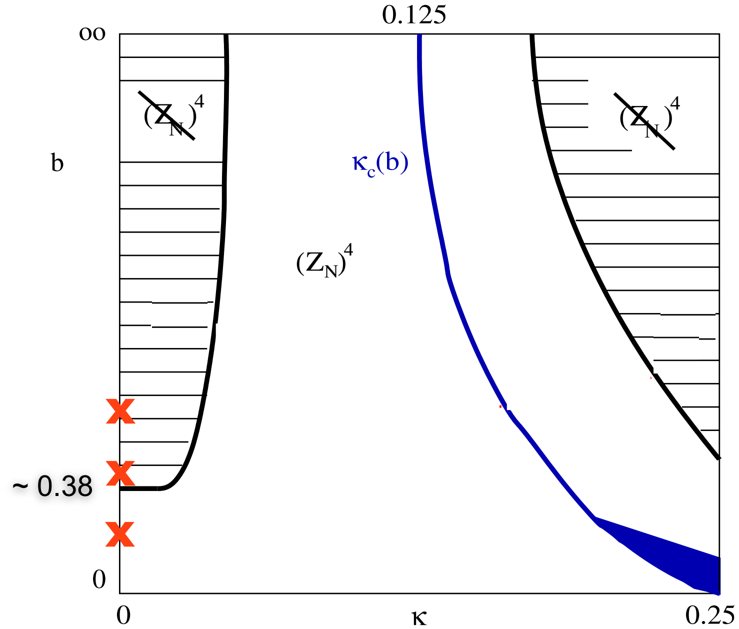
(3) Average plaquette value agrees with that of the large volume theory in the large N_c limit.

$$\langle \text{plaquette} \rangle = \frac{1}{N_c} \left\langle \sum_{\mu<\nu} \mathrm{Tr} U_{n,\mu} U_{n+\mu,\nu} U^\dagger_{n+\nu,\mu} U^\dagger_{n,\nu} \right\rangle_n \,, \quad n \in 2^4$$

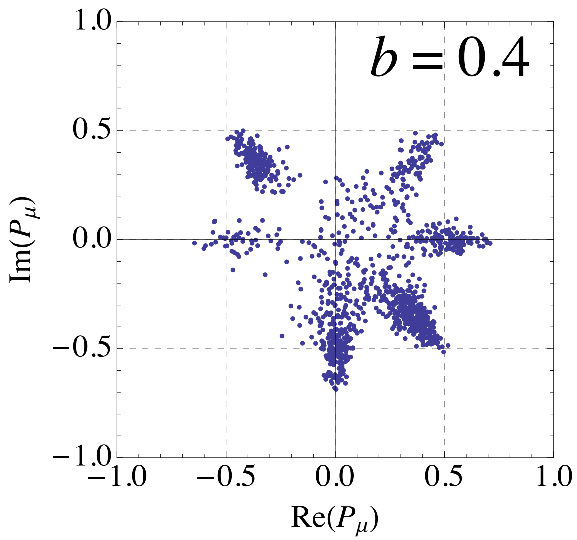
Polyakov Loops - N_f =0(Quenched)

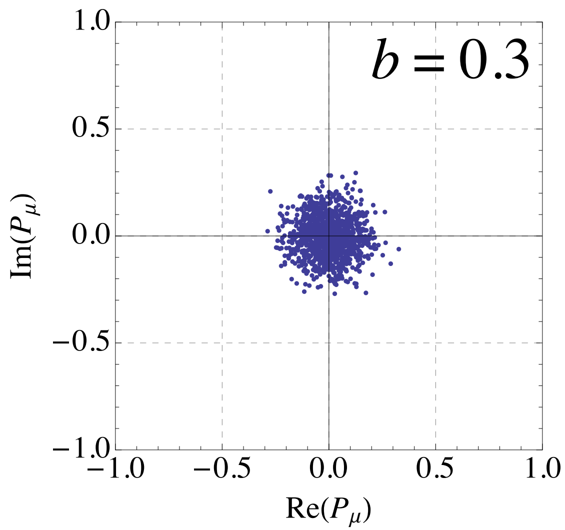
For $N_c = 8$,





Center Symmetry is broken at weak coupling limit.

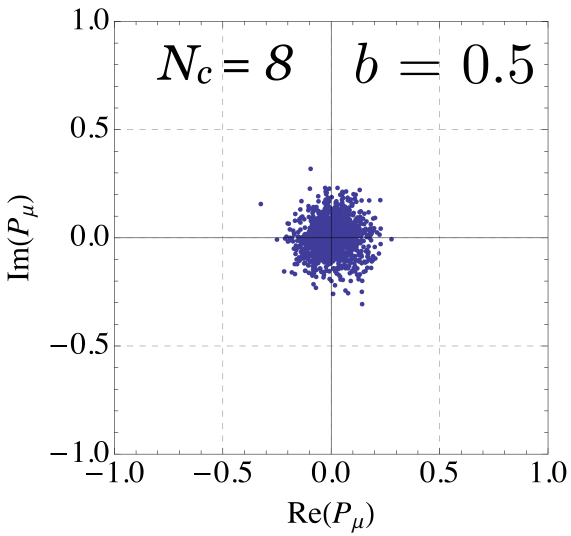


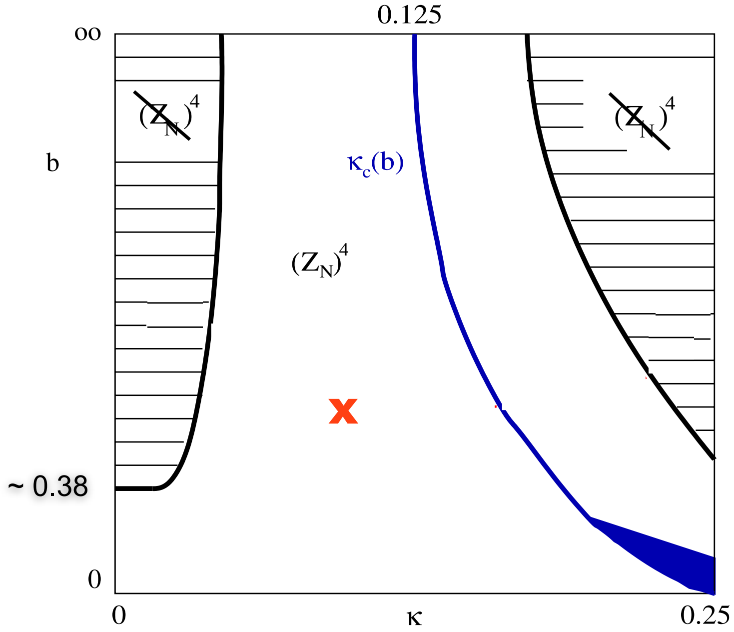


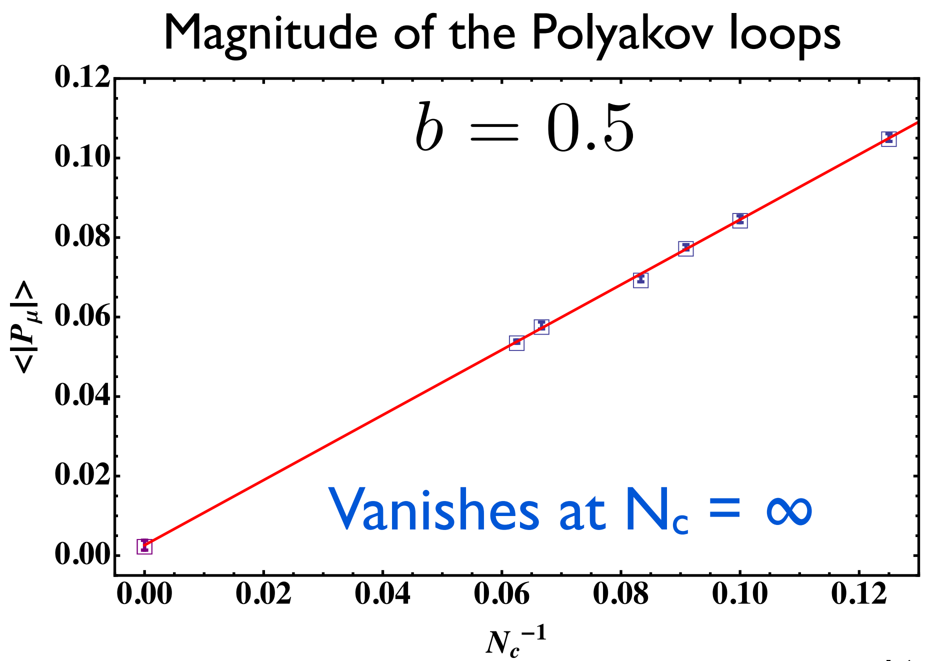
$$b = 1/(g^2 N_c)$$

Center Symmetry is restored in the strong coupling limit.

Polyakov Loops - N_f=2 Heavy Adjoints Fermions

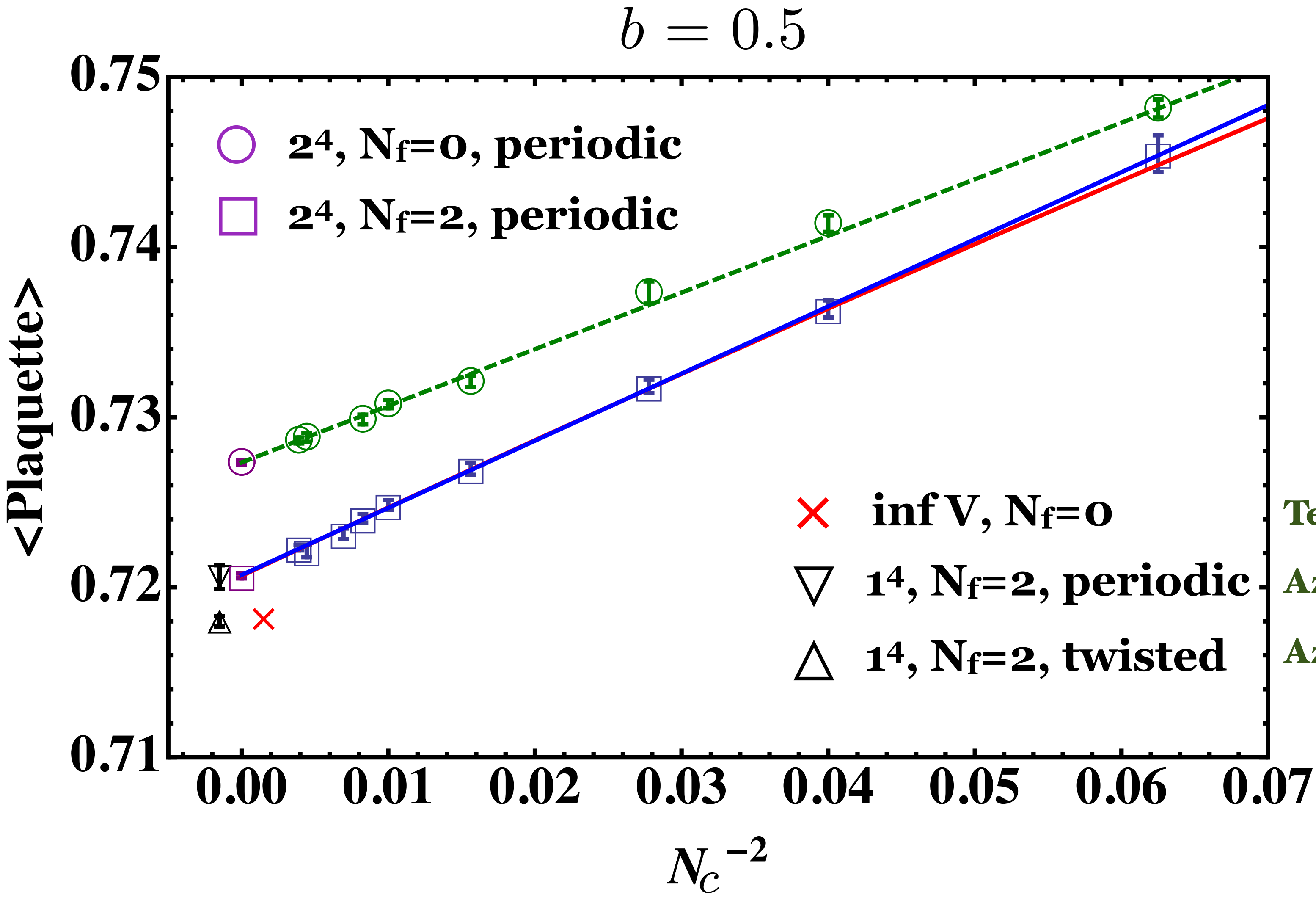






Center Symmetry is unbroken even in the weak coupling limit !

Plaquette values (large N_c limit)



Teper, Vairinhos 2009

Azeyanagi et al 2010

Azeyanagi et al 2010

Plaquette value for N_f=2 is consistent with that from large volume calculation, while N_f=0 value is far above all others.

Spectrum of the Dirac operator

overlap-Dirac operator of fermions in adj. representation

$$H = \gamma_5 D \quad = \gamma_5 \frac{1}{2}\left[1 + \gamma_5 \,\mathrm{sgn}[H_{\mathrm{w}}(-m_0)]\right] \qquad \gamma_5 H_{\mathrm{w}}(-m_0)$$

Wilson-Dirac op.

fund. fermions **N** ⟶ **adj. fermions** **N^2**

calculate the low lying eigenvalues λ_i $\qquad z_i = \lambda_i V (N_c)^{\alpha} \tilde{\Sigma}$

chRMT predictions:

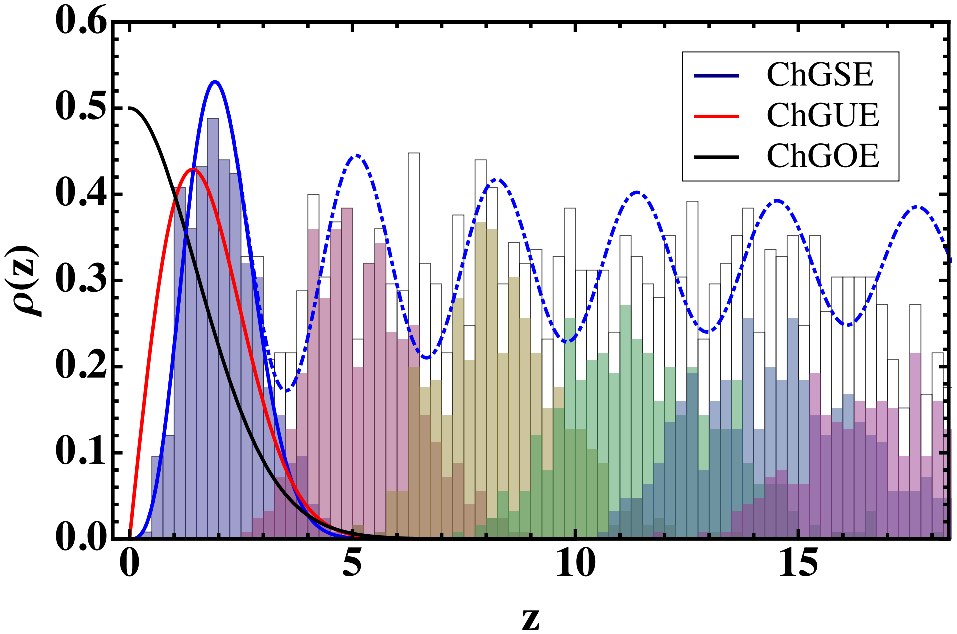
distribution of the smallest eigenvalue for quenched theory

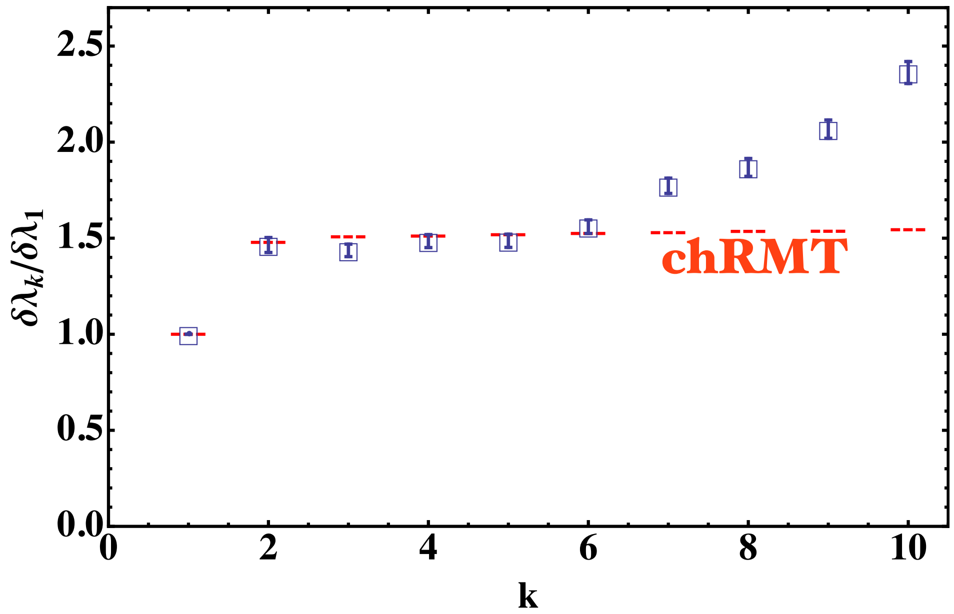
$$P_{\min}(z) = \begin{cases} \dfrac{2+z}{4}\, e^{-(z/2)-(z^2/8)} & \text{ChGOE} \\ \dfrac{z}{2}\, e^{-z^2/4} & \text{ChGUE} \\ \sqrt{\dfrac{\pi}{2}} z^{3/2} I_{3/2}(z)\, e^{-z^2/2} & \text{ChGSE} \end{cases}$$

QCD (adj)

Comparison with chRMT prediction

N_c = 16, b=0.5, N_f=2, V=2^4





(reduced model)

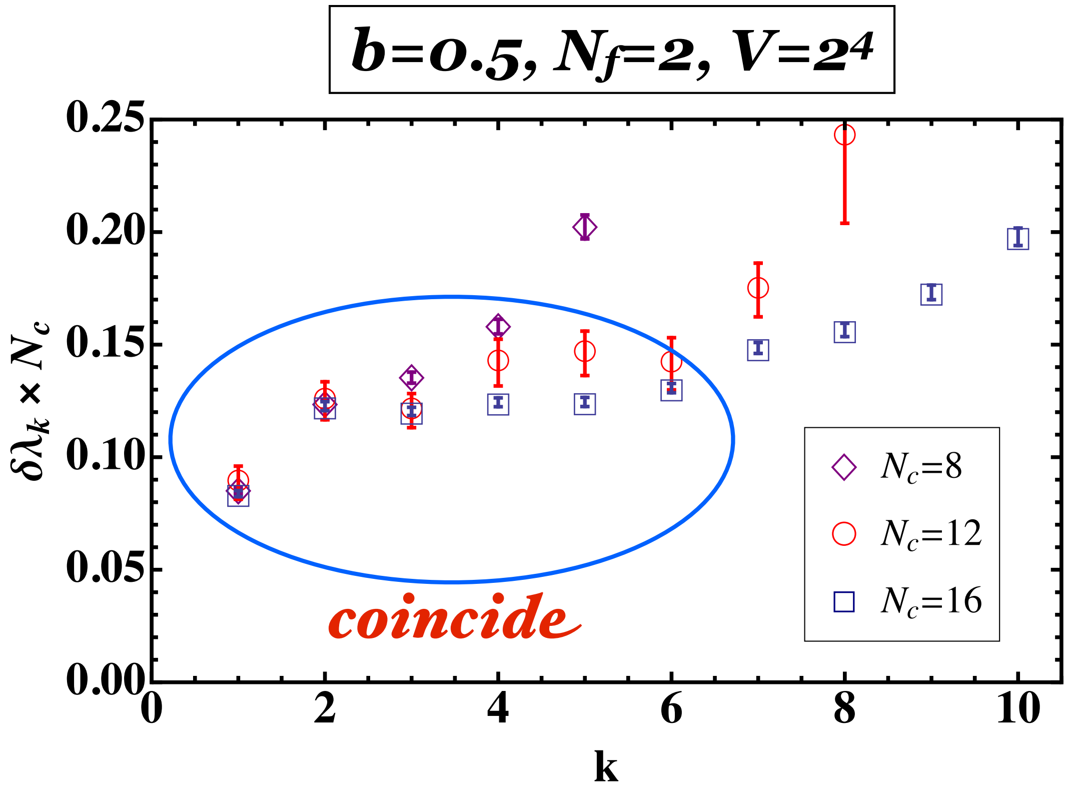
chRMT prediction *perfectly* agrees with the numerical data -
chiral symmetry is broken.

➡ ***chiral symmetry is broken* in the large-N_c quenched QCD**

EK volume equivalence

Scaling behavior of the Dirac eigenvalues

chRMT limit: mVN_c^{α} is fixed, while N_c(and/or V) $\rightarrow \infty$



λVN_c^{α} is fixed in the same way.

☞ $\alpha = 1$

1/N_c scaling behavior has also been found in a single-site simulations for two heavy adjoint fermions. **Narayanan & Hietanen 2012**

Conclusion and Future work

- **Numerical study of QCD-like theories on a small lattice is possible by using EK volume equivalence and chiral RMT in the large N_c limit.**

Quenched QCD - chiral symmetry is spontaneously broken

- **Applying this method to**

$SU(N_c)$ gauge theory with $N_f = 2$ dynamical adjoint fermions

SU(2) two adjoint fermions - candidate of the Minimal Walking Technicolor model

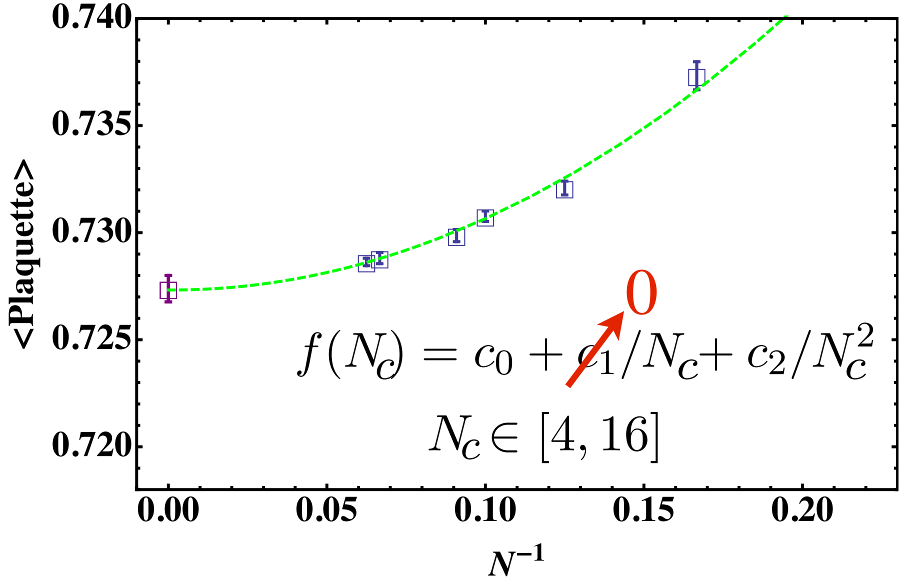
- **EK model is economical? Not really, because $V_{eff} \sim N_c$.**

e.g. twisted boundary condition $1/N_c \longrightarrow 1/N_c^2$

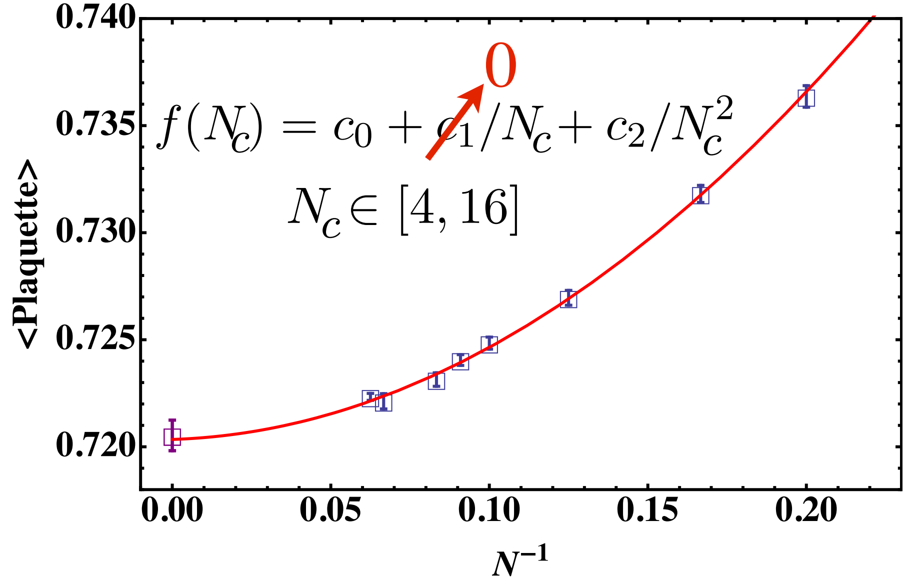
Thank you !

Backup slide 1 - Correction to plaquette values

$N_f = 0, \ b = 0.5$



$N_f = 2, \ b = 0.5$



plaquette value scales as $1/N_c^2$.

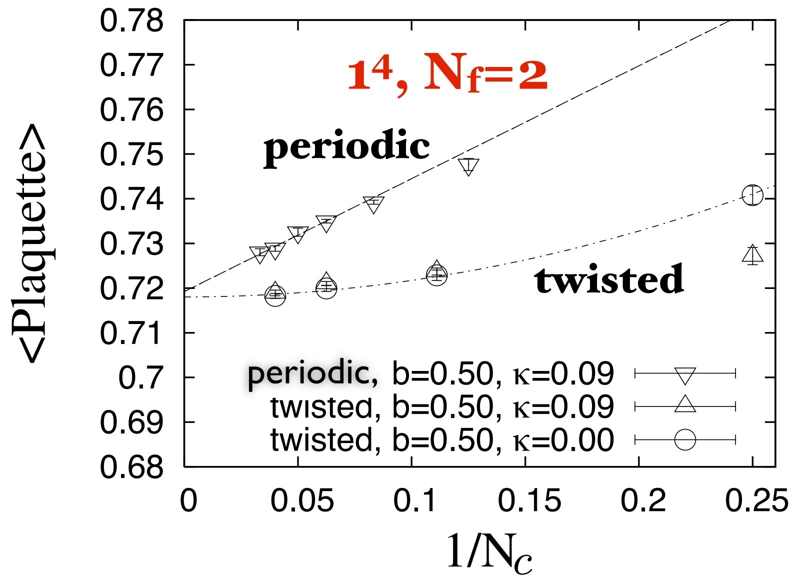
cf) Plaquette value scales as $1/N_c$ in single-site simulation with two heavy adjoint fermions.

Bringoltz, Koren, Sharpe, 2011 **Azeyanagi et al, 2010**

$1/N_c$ correction is expected to be suppressed by $V=1/2^4$

$$\langle \Box \rangle = \langle \Box_\infty \rangle + C_1 (N_c V)^{-1} + C_2 N_c^{-2} + \cdots$$

V_{eff}



Backup slide 2 - 1/N_c scaling, zero modes and V_{eff}

- **Spacing of the low-lying Dirac eigenvalues**

In the 't Hooft limit, $\langle\bar{\psi}\psi\rangle \sim N_c^2$ **then** $\Delta\lambda \sim \frac{1}{N_c^2}$ **Leutwlyer & Smilga 1992**

Our finding: $\Delta\lambda \sim \frac{1}{N_c}$

In compact space, momentum-zero modes are not gauged away and thus the low-lying Dirac spectrum can be determined by the zero modes, where their number scales as N_c. **Azeyanagi, Hanada, Unsal, Yacoby. 2010**

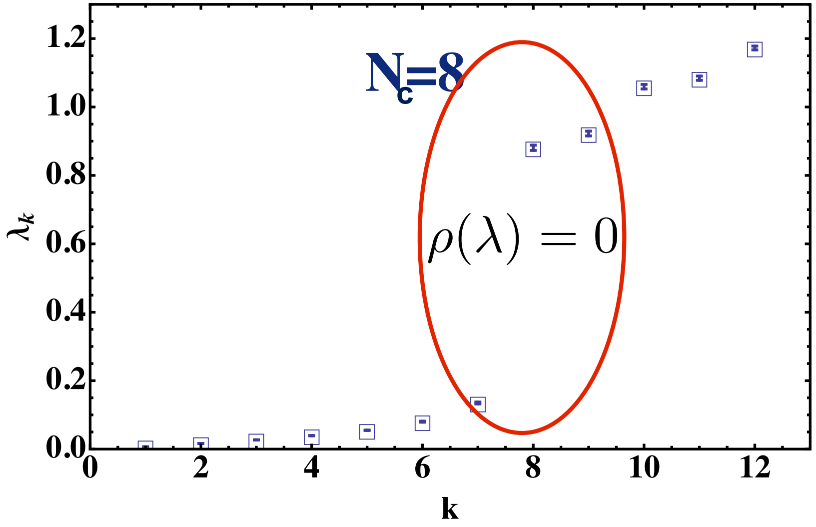
- **Effective volumes V_{eff}**

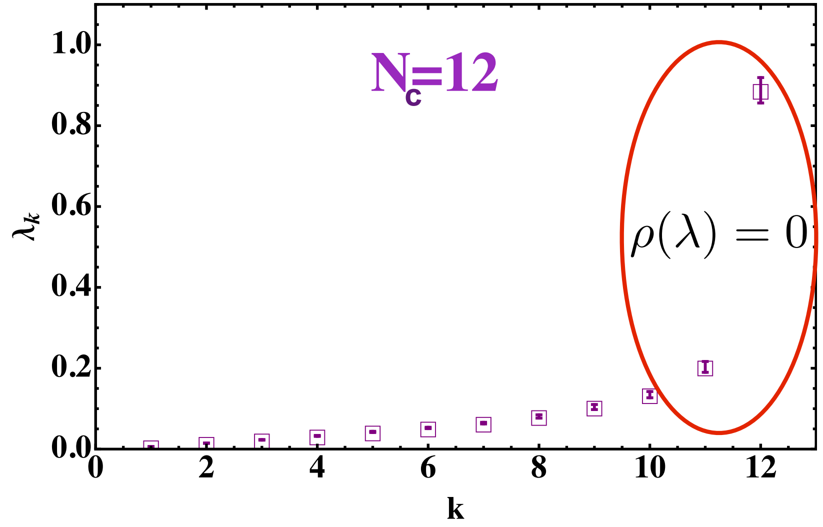
Finite N_c correction to the plaquette values and N_c dependence of the low-lying Dirac spectrum presumably indicate that the effective volume scales as N_c for AEK model with periodic b.c.

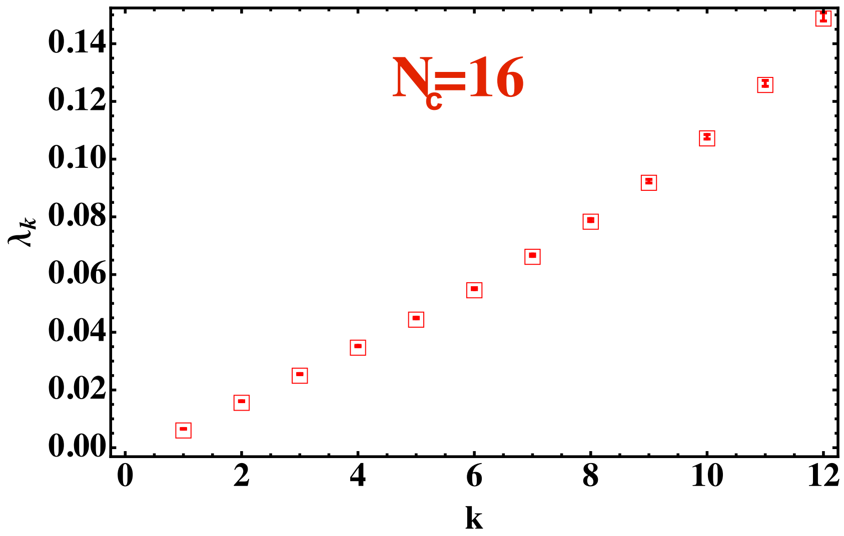
Volume expansion as an orbifold projection for adjoint QCD.

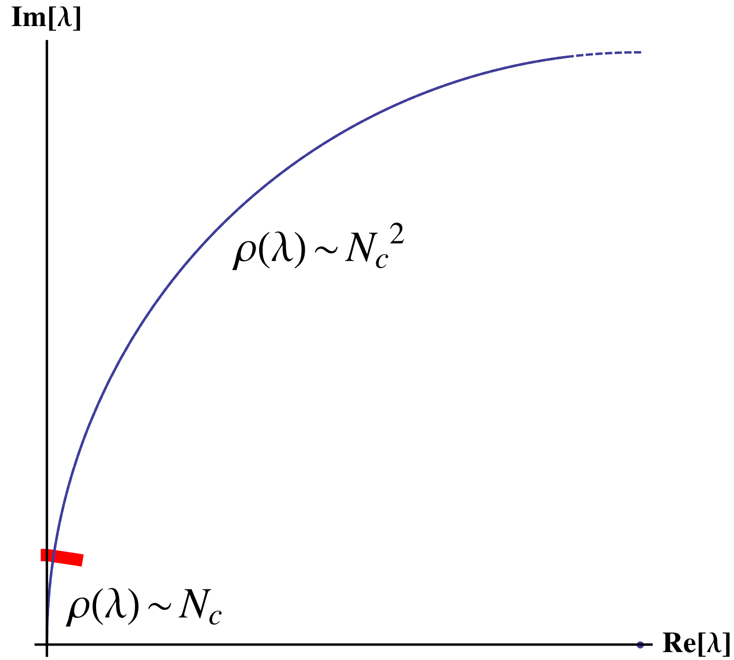
Kovtun, Unsal & Yaffe 2007

Backup slide 3 - *Bump* in the Dirac eigenvalues









# of zero modes of the link matrix = N_c- 1