Numerical approach to QED contribution in the lepton $g-2$

Masashi Hayakawa

Department of Physics, Nagoya University

@Nara, 13-16 December, 2012

Purpose

I report the updated result for $a_\mu(\mathrm{QED})$ (T.Aoyama, M.H T.Kinoshita and M.Nio, Phys. Rev. Lett. **109**, 111808 (2012))

$$10^{11} \times a_\mu(\mathrm{QED}) = 116\ 584\ 718.951\ (80)\,.$$

The new ingredients are summarized as follows;

(1) compelete calculation of 5-loop (10th-order) coefficient, $a_\mu^{(10)}$

(2) improvement of precision of 4-loop (8th-order) calculation, $a_\mu^{(8)}$

(3) recalculation of lower-order mass-dependent terms using the latest values of m_μ/m_e, m_μ/m_τ found in P. J. Mohr, B. N. Taylor and D. B. Newell, arXiv:1203.5425 [physics.atom-ph]

In particular, (1) plays the essential role in the substantial reduction of its uncertainty;

$$\delta a_\mu(\mathrm{QED})[\mathit{ignorance\ on}\ a_\mu^{(10)}] = O(1) \times 10^{-11}$$

$$\Rightarrow \delta a_\mu(\mathrm{QED}) = 0.08 \times 10^{-11} \ll \delta a_\mu(\mathrm{next\ expr.}) = O(1) \times 10^{-11}\,.$$

Content

- I introduce the basic features of the muon $g-2$,
- I overview the current status of the muon $g-2$, in particular of the QED contribution $a_\mu(\mathrm{QED})$,
- I explain a particular numerical approach to the high-order perturbative QED calculation.
- The brief overview is given in

  T. Aoyama, M. H., T. Kinoshita and M. Nio,

  Prog. Theor. Exp. Phys. **2012**, 01A107 (2012) .

What is $g-2$?

- A charged massive particle ψ becomes magnetized in the form of magnetic dipole moment, if it has non-zero spin

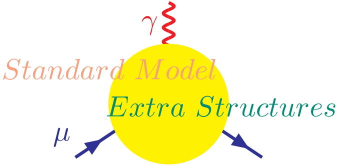
$$\boldsymbol{\mu}_\psi = g_\psi \frac{e\hbar}{2m_\psi c}\, \boldsymbol{s}\,,$$

  where g_ψ is g-factor.
- If ψ has spin $s=\frac{1}{2}$, $g_\psi|_{\rm classically} = 2$.
- The deviation $a_\psi \equiv (g_\psi - 2)/2$, called anomalous magnetic dipole moment or "$g-2$", is a quantity predictable in any renormalizable quantum field theory.
- Here, we focus on the muon $g-2$ (a_μ).
- The status for a_e is described in backup slides.

Why is muon $g-2$?

- The experimentally measured value, $a_\mu(\mathrm{exp})$, of a_μ consists of the quantum-mechanical dynamics of the standard model, $a_\mu(\mathrm{SM})$, and possibly those from extra structures, $a_\mu(\mathrm{new})$;

$$a_\mu(\mathrm{exp}) = a_\mu(\mathrm{SM}) + a_\mu(\mathrm{new})\,.$$



- In order to explore the existence of $a_\mu(\mathrm{new})$, we ask if $a_\mu(\mathrm{SM})$ differs from $a_\mu(\mathrm{exp})$;

$$a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = a_\mu(\mathrm{new}) \neq 0 \;?$$

What is QED contribution ?

- Standard model contribution $a_\psi(\mathrm{SM})$ ($\psi = \mu$ or e) consists of three types of contributions;

$$a_\psi(\mathrm{SM}) = a_\psi(\mathrm{QED}) + a_\psi(\mathrm{QCD}) + a_\psi(\mathrm{weak})\,.$$

- QED contribution, $a_\psi(\mathrm{QED})$, is calculated by QED with photons and charged leptons (e, μ, τ) only.
- QCD contribution $a_\psi(\mathrm{QCD})$ is calculated by QCD + QED with purely leptonic contribution ($\equiv a_\psi(\mathrm{QED})$) subtracted.
- $a_\psi(\mathrm{weak})$ consists of all the others, i.e., the diagrams with at least one W boson, Z boson or Higgs boson.

Current status of muon $g-2$

We have 2.8σ discrepancy between the measured value ($a_\mu(\mathrm{exp})$) and the theory of the muon $g-2$ ($a_\mu(\mathrm{SM})$);

$$10^{11} \times a_\mu(\mathrm{exp}) = 116\ 592\ 089\ (63)$$
$$10^{11} \times a_\mu(\mathrm{SM}) = 116\ 591\ 840\ (59)$$
$$10^{11} \times \{a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM})\} = 249\ (87)$$

a_μ is more sensitive to the still-unknown particle(s)/interaction(s) than a_e, by $(m_\mu/m_e)^2 \simeq 40000$, but *also to QCD*;

$$10^{11} \times a_\mu(\mathrm{QCD}) = 6\ 967\ (59)\ .$$

The uncertainty of $a_\mu(\mathrm{SM})$ is *now* saturated by that of $a_\mu(\mathrm{QCD})$, due to the *significant reduction* of the uncertainty of $a_\mu(\mathrm{QED})$ this year: $10^{11} \times \delta a_\mu(\mathrm{QED}) = 0.080$!

Current status of muon $g-2$

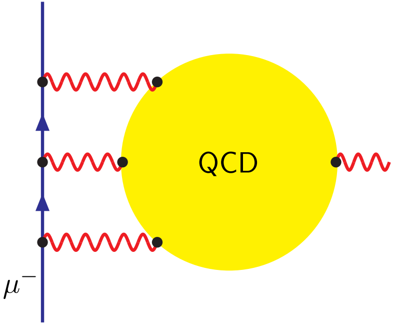
- New experiments (Fermilab(E989), J-PARC(E34)) are planned to achieve the precision $10^{11} \times \delta a_\mu(\text{next exp}) = O(1)$;

$10^{11} \times a_\mu(\text{LO. had. v.p.})$	$=$	$6\,949.1\ (42.8)$
$10^{11} \times a_\mu(\text{NLO. had. v.p.})$	$=$	$-98.4\ (0.8)$
$10^{11} \times a_\mu(\text{had. lbyl})$	$=$	$116\ (40)$
$10^{11} \times a_\mu(\text{QCD})$	$=$	$6\,967\ (59)$
$10^{11} \times a_\mu(\text{SM})$	$=$	$116\,591\,840\ (59)$
$10^{11} \times \{a_\mu(\text{exp}) - a_\mu(\text{SM})\}$	$=$	$249\ (87)$
$10^{11} \times \delta a_\mu(\text{next exp})$	$=$	$O(1)$

- $a_\mu(\text{LO. had. v.p.})$ requires knowledge on QCD with precision of $O(0.1)\,\%$.
  Its evaluation mostly relies on the high precision experiment of $\sigma(e^+e^-(s) \to \text{hadrons})$.

Current status on muon $g-2$

- It is indispensable to compute $a_\mu(\text{had. lbyl}) \sim O(100) \times 10^{-11} \sim O(a_\mu(\text{exp}) - a_\mu(\text{SM}))$ by means of lattice QCD simulation .



- Improvement of precision of $a_\mu(\text{QED})$ and $a_\mu(\text{exp})$ assumes development in $a_\mu(\text{QCD})$.

Requirement for $a_\mu(\mathrm{QED})$ from $\delta a_\mu(\text{next exp})$

- What should we do to realize $\delta a_\mu(\mathrm{QED}) \lesssim \delta a_\mu(\text{next exp}) = O(1) \times 10^{-11}$?
- To what order $2n$ of perturbative expansion is necessary to know,

$$a_\mu(\mathrm{QED}) = \sum_{n=1}^{\infty} a_\mu^{(2n)} \left(\frac{\alpha}{\pi}\right)^n ,$$

  where QED predicts $a_\mu^{(2n)}$?
- Perturbative aspects of QED in $a_\mu(\mathrm{QED})$ is quite different from those in $a_e(\mathrm{QED})$;

$$a_e(\mathrm{QED}) = 0.5 \times \frac{\alpha}{\pi} + O(1) \times \left(\frac{\alpha}{\pi}\right)^2 + O(1) \times \left(\frac{\alpha}{\pi}\right)^3 + O(1) \times \left(\frac{\alpha}{\pi}\right)^4 + O(1) \times \left(\frac{\alpha}{\pi}\right)^5 + \cdots .$$

$$a_\mu(\mathrm{QED}) = 0.5 \times \frac{\alpha}{\pi} + O(1) \times \left(\frac{\alpha}{\pi}\right)^2 + O(10) \times \left(\frac{\alpha}{\pi}\right)^3 + O(100) \times \left(\frac{\alpha}{\pi}\right)^4 + O(1,000) \times \left(\frac{\alpha}{\pi}\right)^5 + \cdots .$$

Requirement for $a_\mu(\mathrm{QED})$ from $\delta a_\mu(\text{next exp})$

- The rough estimate of orders of magnitude will show that the 10th order may also be relevant;

$$a_\mu^{(8)} \times \left(\frac{\alpha}{\pi}\right)^4 = O(100) \times 10^{-11} \sim a_\mu(\exp) - a_\mu(\mathrm{SM}),$$
$$a_\mu^{(10)} \times \left(\frac{\alpha}{\pi}\right)^5 = O(1) \times 10^{-11} \sim \delta a_\mu(\text{next exp}).$$

- These are exactly the reasons why
  - we improve the numerical precision of $a_\mu^{(8)}$ ($\Rightarrow O(0.01)\,\%$!!!),
  - we try to compute $a_\mu^{(10)}$,

over about 8 years !

2012 Update of $a_\mu(\mathrm{QED})$

Table: $a_\mu(\mathrm{QED})$ at each order $2n$, scaled by 10^{11} (T. Aoyama, M. H., T. Kinoshita and M. Nio, Phys. Rev. Lett. **109**, 111808 (2012))

order $2n$	using $\alpha(\mathrm{Rb})$	using $\alpha(a_e)$
2	$116\ 140\ 973.318\ (77)$	$116\ 140\ 973.213\ (30)$
4	$413\ 217.6291\ (90)$	$413\ 217.6284\ (89)$
6	$30\ 141.902\ 48\ (41)$	$30\ 141.902\ 39\ (40)$
8	$381.008\ (19)$	$381.008\ (19)$
10	$5.0938\ (70)$	$5.0938\ (70)$
sum	$116\ 584\ 718.951\ (80)$	$116\ 584\ 718.846\ (37)$

The complete calculation of $a_\mu^{(10)}$ *eliminates the uncertainty* $\sim O(1)\times 10^{-11} \sim \delta a_\mu(\mathrm{next\ exp})$, *which has been present unless it is done.*
Now, the uncertainties in $a_\mu(\mathrm{QED})$ come mostly from

1. statistical uncertainty in the Monte Carlo integration of the 8th-order terms,
2. uncertainty in the fine structure constant α $(2n = 2)$.

Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

I next

- estimate rough orders of magnitude of $a_\mu^{(2n)}$ for $2n \geq 6$,

$$a_\mu(\mathrm{QED}) = 0.5 \times \frac{\alpha}{\pi} + O(1) \times \left(\frac{\alpha}{\pi}\right)^2 + O(10) \times \left(\frac{\alpha}{\pi}\right)^3 + O(100) \times \left(\frac{\alpha}{\pi}\right)^4 + O(1,000) \times \left(\frac{\alpha}{\pi}\right)^5 + \cdots .$$

- show the validity of our numerical result for the 8th and 10th order terms;

$$a_\mu^{(8)} = 130.879\ 6\ (63)\,,$$
$$a_\mu^{(10)} = 753.29\ (1.04)\,.$$

Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

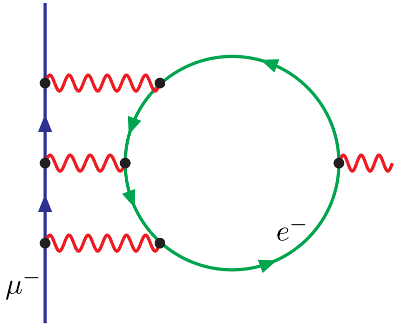
- Since $m_\mu > m_e$ while $m_\mu < m_\tau$, $a_\psi^{(2n)}$ is dominated by $A_2^{(2n)}(m_\mu/m_e)$ in the following decomposition:

$$a_\mu^{(2n)} = A_1^{(2n)} + A_2^{(2n)}\left(\frac{m_\mu}{m_e}\right) + A_2^{(2n)}\left(\frac{m_\mu}{m_\tau}\right) + A_3^{(2n)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right),$$

  - The term $A_1^{(2n)}$ is a pure number, called mass-independent term. $A_1^{(2n)}$ universally contributes to all $a_l^{(2n)}$, and is calculated by QED with electron only.
  - The term $A_2^{(2n)}\left(\frac{m_\mu}{m_e}\right)$ represents the contribution of all Feynman diagrams with at least one e-loop but *with no τ-loop*. Similarly for $A_2^{(2n)}\left(\frac{m_\mu}{m_\tau}\right)$.
  - $A_3^{(2n)}\left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$ represents the contribution of all Feynman diagrams with both e-loop(s) and τ-loop(s).

Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

At the sixth order $(n = 3)$, a_μ receives large contribution through the light-by-light scattering due to virtual e^-e^+, $a_\mu(\text{lbyl}_{6e})$.



Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

- The sixth-order light-by-light scattering contribution $a_\mu(\text{lbyl}_{6e})$ is given as follows;

$$a_\mu(\text{lbyl}_{6e}) \simeq \left(\frac{\alpha}{\pi}\right)^3 \times \left\{ H \ln\left(\frac{m_\mu}{m_e}\right) + c \right\},$$

$$\ln\left(\frac{m_\mu}{m_e}\right) \sim 5.33\,,\ H = \frac{2}{3}\pi^2 = 6.57973627\ldots\,,$$

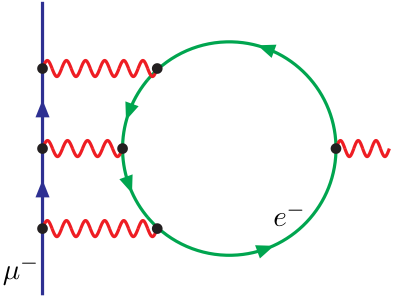
$$c \simeq -2H\ (\text{numerically}) \ \Rightarrow\ \{\ldots\} \simeq H \times (5.33 - 2) \simeq 15\,.$$

(The leading logarithmic approximation, $\simeq 35$, is valid up to a factor.)
We thus have

$$\left(\frac{\alpha}{\pi}\right)^3 \times a_\mu^{(6)} \simeq a_\mu(\text{lbyl}_{6e}) = O(10) \times \left(\frac{\alpha}{\pi}\right)^3\,.$$

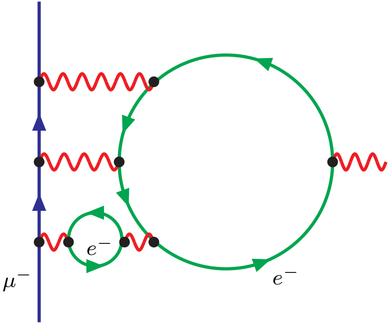
Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

- π^2 arises in the following manner;
  - For $g-2$, $O(\alpha^3)$ interactions must consist of two Coulombic interactions (γ_μ) and one hyperfine interaction $(\sigma_{\mu\nu}\, q^\nu)$.
  - These Coulombic interactions act statically, yielding $i\pi\delta(k^0) \times i\pi\delta(q^0)$.
- The enhancement of $a_\mu(\mathrm{lbyl}_{6e})$ is caused by the dynamics forming a bound state of a μ^- and a e^+, called muonium (A.S.Yelkhovsky, Sov.J.Phys.49, 654 (1989)).



Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

- At the 8th-order,



will be dominant and is estimated as

$$A_2^{(8)}\left(\frac{m_\mu}{m_e}\right) \simeq A_2^{(6)}\left(\frac{m_\mu}{m_e}; l\text{-}l\right) \times \left\{\frac{2}{3}\ln\left(\frac{m_\mu}{m_e}\right) - \frac{5}{9}\right\} \times 3$$
$$\simeq 180\,.$$

Perturbative features of $a_\mu(\mathrm{QED})$ and rough estimate

- Our computed result $A_2^{(8)}\left(\frac{m_\mu}{m_e}\right) = 132.685\ 2\ (60)$ is roughly identical with the estimation ~ 180.
- At the 10th order, the contribution of the diagrams (Set VI(a)) obtained by inserting two second-order electron-loop-induced vacuum polarizations in the six-order light-by-light scattering diagrams will be dominant :

$$A_2^{(10)}\left(\frac{m_\mu}{m_e}\right) \simeq A_2^{(6)}\left(\frac{m_\mu}{m_e}; l\text{-}l\right) \times \left\{\frac{2}{3}\ln\left(\frac{m_\mu}{m_e}\right) - \frac{5}{9}\right\}^2 \times 6$$
$$\simeq 1000\,,$$

which gives

$$a_\mu^{(10)} \times \left(\frac{\alpha}{\pi}\right)^5 \simeq 1000 \times (6.76 \times 10^{-14})$$
$$\simeq 6.8 \times 10^{-11} \quad (\delta a_\mu(\mathrm{exp}) = 63 \times 10^{-11})\ .$$

- Our result obtained by complete calculation is consistent with rough estimate

$$A_2^{(10)}\left(\frac{m_\mu}{m_e}\right)[\text{Set VI(a)}] = 629.141\ (12)\,.$$

$$A_2^{(10)}\left(\frac{m_\mu}{m_e}\right) = 742.18\ (87)\,.$$

Numerical Approach to QED contribution

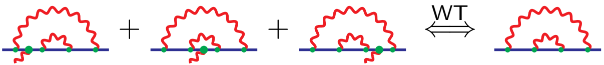
- We employ the parametric integral formulation, whose basic part was described in P.Cvitanovic and T.Kinoshita, Phys. Rev. D **10**, 3978 (1974).
- It deals with the integral on the Feynman parameter space.
- It intends to subtract UV divergence at the numerical level.
- The numerical subtraction is possible only if divergences are subtracted in a pointwise way by the terms that are expressed as the integrands on the same Feynman parameter space;

$$\int [dz] \left\{ f_{\mathcal{G}}^{\mathrm{bare}}(z) - \sum_{\mathfrak{F}} f_{\mathfrak{F}}^{\mathrm{UV}}(z) \right\} ,$$

where $z = \{z_i\}$ are Feynman parameters, and the sum is taken over all normal forests $\mathfrak{F}$ of $\mathcal{G}$.

Numerical Approach to QED contribution

- The integral must be constructed *separately* for the individual vertex diagrams, or for a set of vertex diagrams (, which share similar UV-divergent structure) which are related via a Ward-Takahashi (WT) identity to a single *self-energy-like diagram* $\mathcal{G}$;



$$\equiv \mathcal{G}\,.$$

- The individual integral in general contains infrared (IR) divergence. The numerical calculation can be done for the quantity which are free from both UV and IR divergences;

$$\Delta M_{\mathcal{G}} = \int [dz] \left\{ f_{\mathcal{G}}^{\mathrm{bare}}(z) - \sum_{\mathfrak{F}} f_{\mathfrak{F}}^{\mathrm{UV}}(z) - \sum_{\mathfrak{E}} f_{\mathfrak{E}}^{\mathrm{IR}}(z) \right\} .$$

Numerical Approach to QED contribution

- The UV subtraction terms are *not* the ones required from the on-shell renormalization condition. We thus have to add the residual renormalization terms $\{R_k\}$ to get the contribution to $a_l^{(2n)}$ from the gauge-invariant subset S of diagrams (IR-divergence cancels among them)

$$a_l^{(2n)}[\text{Set } S] = \sum_{\mathcal{G}\in S} \Delta M_{\mathcal{G}} + \sum_k R_k \,.$$

  Our construction of $f_{\mathfrak{F}}^{\mathrm{UV}}(z)$ and $f_{\mathfrak{E}}^{\mathrm{IR}}(z)$ guarantees that every R_k is given in terms of finite quantities *at lower-order*.

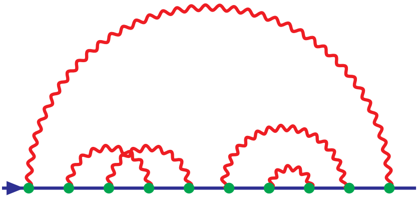
- The integration has been carried out for $\Delta M_{\mathcal{G}}$ and the constituents of R_k with help of adaptive Monte Carlo integration routine, VEGAS, on RIKEN supercomputer systems, RSCC and RICC.

Strategy for calculation of 10-th order QED contribution

- The calculation of $a_l^{(8)}$ $(l = e,\, \mu)$ required about 20 years (although it is corrected later on).
- The number (12678) of Feynman diagrams at the 10th order is 14 times larger than that (891) at the 8th order.
- Writing the numerical program for a 10th-order Feynman diagram *correctly* is much harder than for the 8th-order.
- Thus, the 10th-order calculation was considered to need $500 \sim 1000$ years to complete.
- We have completed the 10th-order calculation in less than 10 years.

Strategy for calculation of 10-th order QED contribution

- The most difficult gauge-invariant subset is Set V consisting of quenched-type (*q-type*) diagrams:



1. The number of q-type diagram at the 10th order is 6354.
2. A q-type diagram has complicated structure of UV and IR singularities (, and thus requires many subtraction terms).
3. There are tenth-order q-type diagrams having *linear* IR subdivergence. (The corresponding subtraction terms can no longer be constructed just in the power-counting scheme.)

Strategy for calculation of 10-th order QED contribution

- If we write 6354 programs manually, mistakes will be scattered over those programs randomly.
- If we write a code generator which produces 6354 programs (to calculate $\Delta M_{\mathcal{G}}$ for $\mathcal{G} \in$ Set V), mistakes will be strongly correlated, and it is only necessary to manage the code generator itself.
- The crucial point to successfully implement the code generator is the invention of a *systematic* scheme, which enables to subtract linear and higher IR singularities (T.Aoyama, M.H., T.Kinoshita and M.Nio, Nucl. Phys. B **796** (2008) 184.)
- Quadruple precision is needed
  - to realize subtraction of linear IR divergence,
  - to achieve the precision $O(0.01)$ % for $a_e^{(8)}$.

Strategy for calculation of 10-th order QED contribution

- A code generator for q-type diagrams was implemented for arbitrary order $2n$ of perturbation. We have tested its validity for $2n = 4, 6$ and 8.
- We found the incorrectness of the previous result for $A_1^{(8)}$ (T.Aoyama, M.H., T.Kinoshita and M.Nio, Phys.Rev.Lett. **99**, 110406 (2007));

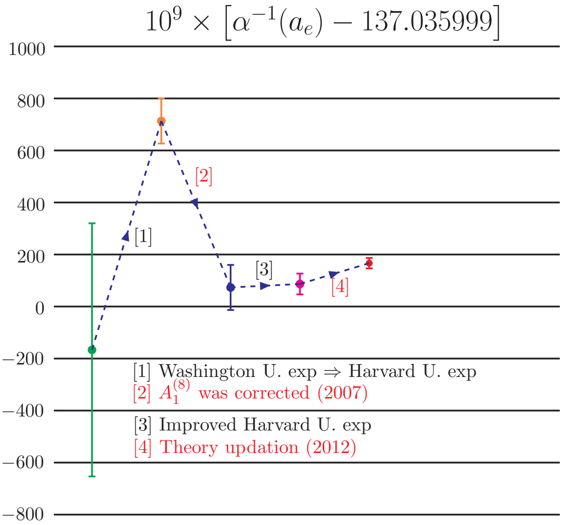
$$A_1^{(8)}(\mathrm{old}) = -1.7203\,,$$
$$A_1^{(8)}(\mathrm{new}) = -1.9106\,,$$

which affects to a_l as

$$a_l\left[A_1^{(8)}(\mathrm{old})\right] = -5.0313 \times 10^{-11}\,,$$
$$a_l\left[A_1^{(8)}(\mathrm{new})\right] = -5.5620 \times 10^{-11}\,.$$

This causes a *significant change to a_e* in light of its experimental precision, $\delta a_e(\mathrm{exp}) = 0.028 \times 10^{-11}$.

"Modern history" of $\alpha^{-1}(a_e)$



Strategy for calculation of 10-th order QED contribution

- We have developed code-generators for several subsets of diagrams other than quenched-type diagrams.
- Some tenth-order diagrams were calculated analytically;
  - A. L. Kataev, Phys. Lett. B **284**, 401 (1992) [Erratum-ibid. B **710**, 710 (2012)].
  - S. Laporta, Phys. Lett. B **328**, 522 (1994).
  - J. -P. Aguilar, D. Greynat and E. De Rafael, Phys. Rev. D **77**, 093010 (2008).
  - P. A. Baikov, K. G. Chetyrkin and C. Sturm, Nucl. Phys. Proc. Suppl. **183**, 8 (2008).

  These works provided us to test the validity of our results for the corresponding contributions.

Summary and future perspective

- The calculation of $12,672$ number of Feynman diagrams at the 10th order substantially reduced the uncertainty in $a_\mu(\mathrm{QED})$, which is now well below the expected uncertainty in the next-generation experiment.
- However, it is a reasonable question: Is our result correct ?
  - Even the eight-order term ($\sim a_\mu(\mathrm{SM}) - a_\mu(\exp)$), has been computed *only by us*.
  - We have seen the consistency with rough orders of estimate for $a_\mu^{(8)}$ and $a_\mu^{(10)}$.
  - *Check for $a_\mu^{(8)}$ by third persons* is an important subject.
- The Harvard group is now preparing the new measurement of a_e. Accordingly, the reduction of the uncertainty of $a_e(\mathrm{QED})$ is strongly requested. We need
  - optimized quadruple precision arithmetics,
  - more sophisticated adaptive Monte Carlo integrator (c.f. R.Arthur and A.D.Kennedy, arXiv:1209.0650 [physics.comp-ph]).

Standard model prediction of electron $g-2$

The brand new value of the electron $g-2$ is (Aoyama, M. H., Kinoshita, Nio, Phys. Rev. Lett. **109**, 111807 (2012) for $a_e(\mathrm{QED})$)

$$10^{12}\times a_e(\mathrm{QED}) = 1159\ 652\ 180.07\ (6)_{\mathrm{8th}}\ (8)_{\mathrm{10th}}\ (77)_{\alpha(\mathrm{Rb})}$$

$$10^{12}\times a_e(\mathrm{QCD}) = 1.68\ (3)$$

$$10^{12}\times a_e(\mathrm{weak}) = 0.0297\ (5)$$

$$10^{12}\times a_e(\mathrm{SM}) = 1159\ 652\ 181.78\ (6)_{\mathrm{8th}}\ (8)_{\mathrm{10th}}\ (77)_{\alpha(\mathrm{Rb})}(3)_{\mathrm{QCD}}$$

$$10^{12}\times a_e(\mathrm{exp}) = 1159\ 652\ 180.73\ (28)$$

Here

- The numerals in a parenthesis denote the uncertainty in the final few digits.
- The above uses $\alpha(\mathrm{Rb})$, which was obtained by the recent determination of h/m_{Rb} via optical lattice technique (R. Bouchendira, P. Clade, S. Guellati-Khelifa, F. Nez and F. Biraben, Phys. Rev. Lett. **106**, 080801 (2011)).

Physical implication of electron $g-2$

- Since $a_e(\mathrm{QED})$ occupies $99.999\,999\,85$ % of a_e, the comparison of $a_e(\mathrm{SM})$ with $a_e(\mathrm{exp})$ provides us with a precision test of QED, at present.
- The precision of $a_e(\mathrm{QED})$ can be systematically improved in perturbation theory of QED;

$$a_e(\mathrm{QED}) = \sum_{n=1}^{\infty} a_e^{(2n)} \times \left(\frac{\alpha}{\pi}\right)^n ,$$

where QED predicts $a_e^{(2n)}$ (with help of lepton mass ratios for $2n \geq 4$).

8th-order QED contribution

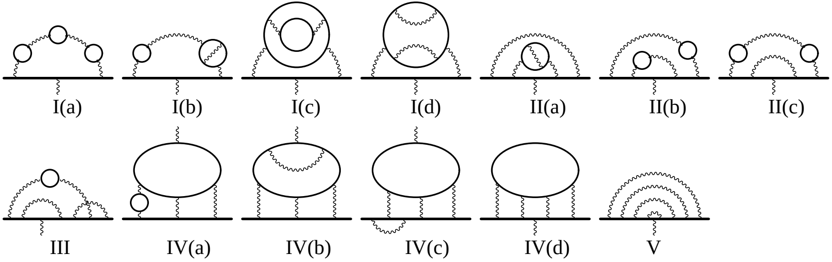


Figure: Typical vertex diagrams representing 13 gauge-invariant subsets contributing to the eighth-order lepton $g-2$ (891 diagrams in total).

10th-order QED contribution

The new ingredients in the update of $a_e(\mathrm{QED})$ in 2012 are

1. completion of computation of $12,672$ number of *5-loop* Feynman diagrams to get $a_e^{(10)}$,
2. improvement of numerical precision of $a_e^{(8)}$,
3. improvement of mass-dependent terms at the lower orders using the latest values of m_e/m_μ, m_e/m_τ found in P. J. Mohr, B. N. Taylor and D. B. Newell, arXiv:1203.5425 [physics.atom-ph] .

Physical implication of electron $g-2$

- Due to the above efforts, the uncertainty in $a_e(\mathrm{SM})$ is now dominated by that of $\alpha(\mathrm{Rb})$;

$$10^{12} \cdot a_e(\mathrm{SM}) = 1159\ 652\ 181.78\ (6)_{\mathrm{8th}}\ (8)_{\mathrm{10th}}\ (77)_{\alpha(\mathrm{Rb})}(3)_{\mathrm{QCD}}\ ,$$
$$10^{12} \cdot a_e(\mathrm{exp}) = 1159\ 652\ 180.73\ (28)\ .$$

- We suppose that no extra contribution exists in $a_e(\mathrm{exp})$.
- We can get $\alpha(a_e)$ by solving $a_e(\mathrm{SM}, \alpha) = a_e(\mathrm{exp})$ with unknown α.
- Check of compatibility of $\alpha(a_e)$ with the others, such as $\alpha(\mathrm{Rb})$, $\alpha(\mathrm{Cs})$, $\alpha(\mathrm{q\text{-}Hall})$, $\alpha(\mathrm{JC})$, ..., provides us with a cross-sectional understanding on various physical phenomena.