Pion form factors in the ε regime

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1. Introduction

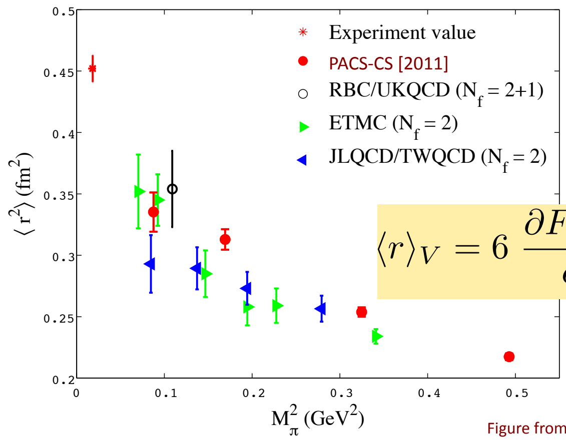
For calculation of pion form factors, we need

1. Very small pion mass
2. Good control of chiral symmetry
   -> large chiral logarithm $\sim \ln m_\pi^2$

We simulate (exactly chiral symmetric) overlap quarks with $m_{ud} \sim$ 3 MeV.

1. Introduction

Eletctro-magnetic form factor of pion



$$\langle r \rangle_V = 6 \left. \frac{\partial F_V(q^2)}{\partial q^2} \right|_{q^2=0}$$

Figure from PACS-CS [2011]

1. Introduction

JLQCD (& TWQCD) project [2006-2012]

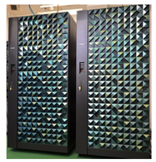
= QCD with overlap quarks (exact chiral sym.).

1/a ~ 1.8 GeV, L ~ 1.8 fm

p regime lattices : m_π = 290-780 MeV

ε regime lattice : m_π ~ 100 MeV

$m_\pi L \sim 0.9$ → Large finite V effects



(New project with Hitachi SR16K & IBM BG/Q started.)

1. Introduction

Good chirality = high numerical cost

quark action	Chiral symmetry	Discretization error	Numerical cost	Volume size
Overlap	**Exact**	**O(a²)**	**Very expensive**	**~2fm**
Domain-wall	Weakly broken	O(a²)	Expensive	~4fm
Wilson	Broken	O(a)	Marginal	~5fm
Staggered	Broken (U(1) remains)	O(a²)	Cheap	~6fm

JLQCD = Small volume QCD...

1. Introduction

Finite volume = Pion physics.

Correlation length (1/M) of QCD particles

Pions(∼140MeV) ∼ 1.4fm
Kaons(∼500MeV)∼0.4fm
Rho (∼800MeV)∼0.26fm
Proton (∼1GeV) ∼0.2fm

$$e^{-M_\pi L} = 0.03\text{–}0.25$$
$$e^{-M_\rho L} < 0.0005$$
$$(\text{for } \ 2\text{fm} < L < 4\text{fm})$$

At $E \sim p \lesssim 200$ MeV, QCD = pion (+ kaon) theory.

Namely, finite volume correction in QCD =
pion theory (chiral perturbation theory)

weakly coupled = analytically calculable.

1. Introduction

Our strategy

Numerical calculation — Analytic calculation

Small lattice QCD with exact chiral symmetry + Finite volume correction by pion theory

QCD at V=∞

1. Introduction

What's new in this talk ?
Automatic cancellation of finite V effects

1. non-zero momenta
2. take an appropriate ratio

-> We can eliminate LO finite V effects
(= we can forget about Bessel functions)
even in the ε regime.

=> Pion form factors

CONTENTS

2. Pion effective theory

What is the ε regime ?

In the p regime, the vacuum is fixed:

$$U(x) = 1 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right), \quad \in SU(N_f)$$

but near the chiral limit at finite V, $M_\pi L < 1$

vacuum= zero-mode = dynamical variable

$$U(x) = U_0 \exp\left(i\frac{\sqrt{2}\pi(x)}{F}\right),$$

U_0 should be non-perturbatively treated.

$\rightarrow$ ε regime

2. Pion effective theory

Pion theory at finite V

$$\mathcal{L} = -\frac{\Sigma}{2}\mathrm{Tr}\left[\mathcal{M}^\dagger U_0 + U_0^\dagger \mathcal{M}\right]$$

Zero-mode = SU(N) (or U(N)) matrix model

$$+\frac{1}{2}\mathrm{Tr}(\partial_\mu \xi)^2$$

Non-zero-mode = massless bosons

$$+\frac{\Sigma}{2F^2}\mathrm{Tr}[\mathcal{M}^\dagger U_0 \xi^2 + \xi^2 U_0^\dagger \mathcal{M}] + \cdots,$$

(perturbative) interactions

= hybrid system of
matrix model and massless bosonic fields

2. Pion effective theory

2pt functions in ε expansion of ChPT

Before d^3x integral

$$\langle P(x)P(0)\rangle = A + B\sum_{p'\neq 0}\frac{1}{V}\frac{e^{ip'x}}{p'^2} + C\sum_{p'\neq 0}\frac{e^{ip'x}}{p'^4} + \cdots$$

$A, B, C \cdots$: matrix integrals (Bessel functions of $m\Sigma V$)

$\sum_{p'\neq 0}(\cdots)$: non-zero mode's

2. Pion effective theory

2pt functions in ε expansion of ChPT

After d³x integral → power function of t.

$$\frac{1}{L^3}\int d^3x\langle P(x)P(0)\rangle = A + B\frac{1}{V}\left[\frac{1}{2}\left(t-\frac{T}{2}\right)^2 - \frac{1}{24}\right] + C[\cdots] + \cdots$$

(with periodic boundary conditions)

cf. large V result:

$$B'\cosh(m_\pi(t-T/2))$$

3. 3pt functions at finite V

2pt functions in ε expansion of ChPT

Ratio of 2pt functions with $\mathbf{p}\neq 0$

$$\frac{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle}{\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}'x}\langle P(x)P(0)\rangle} = \frac{\cosh(E(\mathbf{p})(t-T/2))}{\cosh(E(\mathbf{p}')(t-T/2))} + \cdots,$$

LO finite V effect is eliminated !

NLO $\sim \dfrac{1}{4\pi F^2 L^2}$

3. 3pt functions at finite V

2pt functions in ε expansion of ChPT

After d³x integral with $\mathbf{p}\neq 0$ (w/ periodic b.c.)

$$\frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x}\langle P(x)P(0)\rangle = \frac{1}{L^3}\int d^3x e^{-i\mathbf{p}x} A + B\sum_{p_0'}\frac{1}{V}\frac{e^{-ip_0't}}{p_0'^2+\mathbf{p}^2} + \cdots$$

$$= 0 + \frac{B}{2E(\mathbf{p})L^3\sinh(E(\mathbf{p})T/2)}\cosh(E(\mathbf{p})(t-T/2)) + \cdots,$$

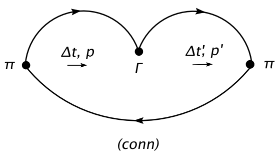
where $E(\mathbf{p}) = |\mathbf{p}|$. the same form as the p-regime :

$$B'\cosh(\sqrt{|\mathbf{p}|^2+m_\pi^2}(t-T/2))$$

But Bessel functions are still contained in B .

3. 3pt functions at finite V

3pt functions in ε regime



$$C^{3\mathrm{pt}}_{PV_0P}(\Delta t, \Delta t'; \mathbf{p}_i, \mathbf{p}_f)$$
$$= B^{3\mathrm{pt}}(m\Sigma V)\left[E(\mathbf{p}_i)+E(\mathbf{p}_f)\right]F_V(q^2)$$
$$\times\cosh(E(\mathbf{p}_i)(\Delta t - T/2))\cosh(E(\mathbf{p}_f)(\Delta t' - T/2)) + \cdots$$

$$R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2) \equiv \frac{\frac{1}{N^{3\mathrm{pt}}_{|\mathbf{p}_i|,|\mathbf{p}_f|}}\sum_{\mathrm{fixed}|\mathbf{p}_i|,|\mathbf{p}_f|,q^2}\frac{C^{3\mathrm{pt}}_{PVP}(t,t';\mathbf{p}_i,\mathbf{p}_f)}{E(\mathbf{p}_i)+E(\mathbf{p}_f)}}{\left(\frac{1}{N^{2\mathrm{pt}}_{|\mathbf{p}_i|}}\sum_{\mathrm{fixed}|\mathbf{p}_i|}C^{2\mathrm{pt}}_{PP}(t,\mathbf{p}_i)\right)\left(\frac{1}{N^{2\mathrm{pt}}_{|\mathbf{p}_f|}}\sum_{\mathrm{fixed}|\mathbf{p}_f|}C^{2\mathrm{pt}}_{PP}(t',\mathbf{p}_f)\right)}$$

$$= B^{3\mathrm{pt}/2\mathrm{pt}}(m\Sigma V)F_V(q^2) + \cdots$$

3. 3pt functions at finite V

Correlator with zero momentum

The constant term can be cancelled by

$$\Delta_{t'} C^{\rm 3pt}_{PVP}(t,t';\mathbf{p}_i,0) \equiv C^{\rm 3pt}_{PVP}(t,t';\mathbf{p}_i,0) - C^{\rm 3pt}_{PVP}(t,t_{\rm ref};\mathbf{p}_i,0)$$

$$\begin{aligned}\Delta_t\Delta_{t'} C^{\rm 3pt}_{PVP}(t,t';0,0) \equiv{}& C^{\rm 3pt}_{PVP}(t,t';0,0) - C^{\rm 3pt}_{PVP}(t,t_{\rm ref};0,0)\\ &- C^{\rm 3pt}_{PVP}(t_{\rm ref},t';0,0) + C^{\rm 3pt}_{PVP}(t_{\rm ref},t_{\rm ref};0,0)\end{aligned}$$

3. 3pt functions at finite V

Ratio of 3pt functions

$$F_V(t,t',q^2) \equiv \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R^2_V(t,t';0,0,0)} \left(\text{or } \frac{R^1_V(t,t';|\mathbf{p}_i|,0,q^2)}{R^2_V(t,t';0,0,0)}\right)$$

$$= F_V(q^2) + \mathcal{O}\left(\frac{1}{4\pi F^2 L^2}\right)$$

LO finite V effect is eliminated !

Cf. in the p regime, this ratio method is conventionally used for canceling the smearing effect, renormalization and so on.

$$F_V(t,t';q^2) = \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V(t,t';0,0,0)}$$

$$= F_V(q^2) + \mathcal{O}\left(e^{-m_\pi L}\right)$$

[Hashimoto et al. 2000]

3. 3pt functions at finite V

Correlator with zero momentum

The both ratios

$$R^1_V(t,t';|\mathbf{p}_i|,0,q^2) \equiv \frac{\frac{1}{N^{\rm 3pt}_{|\mathbf{p}_i|}} \sum_{\text{fixed}|\mathbf{p}_i|,q^2} \left(C^{\rm 3pt}_{PVP}(t,t';\mathbf{p}_i,0) - C^{\rm 3pt}_{PVP}(t,t_{\rm ref};\mathbf{p}_i,0)\right)}{\frac{1}{N^{\rm 2pt}_{|\mathbf{p}_i|}} \sum_{\text{fixed}|\mathbf{p}_i|} C^{\rm 2pt}_{PP}(t,\mathbf{p}_i)\left[-\Delta_{t'}\partial C^{\rm 2pt}_{PP}(t',0) + E(\mathbf{p}_i)\Delta_{t'}C^{\rm 2pt}_{PP}(t',0)\right]},$$

$$R^2_V(t,t';0,0,0) \equiv \frac{\Delta_t\Delta_{t'}C^{\rm 3pt}_{PVP}(t,t';0,0)}{-\Delta_t C^{\rm 2pt}_{PP}(t,0)\Delta_{t'}\partial C^{\rm 2pt}_{PP}(t',0) - \Delta_t\partial C^{\rm 2pt}_{PP}(t,0)\Delta_{t'}C^{\rm 2pt}_{PP}(t',0)},$$

$$= B^{\rm 3pt/2pt}(m\Sigma V) F_V(q^2) + \cdots$$

3. Preliminary lattice results

Summary of numerical simulations

Iwasaki gauge action with β=2.30

2+1 dynamical overlap quarks

1/a ~ 1.759 GeV, L=$16^3$48 (L~1.8 fm)

m_{ud} =0.002(~3 MeV), m_s=0.08

Q=0 fixed

Smearing is used for PS operators $\phi_s(\mathbf{r}) = e^{-0.4r}$

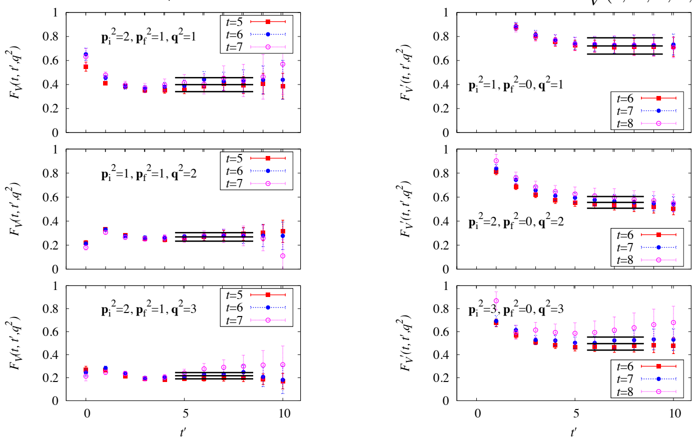
All-to-all propagators with 120 exact low-modes and noise method for the high modes.

Dispersion relation is used for E with $m_\pi = 98(5)\mathrm{MeV}$

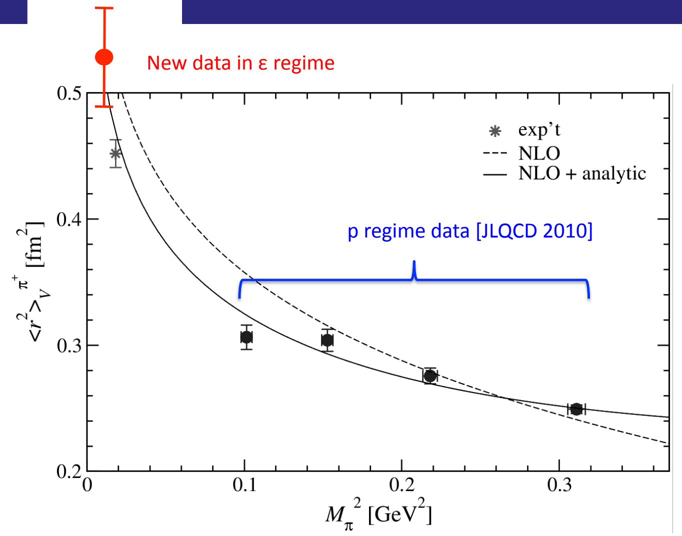
<u>148 samples from 2500 trj.</u>

3. Preliminary lattice results

$$F_V(t,t',q^2) \equiv \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V^2(t,t';0,0,0)} \qquad F_V(t,t',q^2) \equiv \frac{R_V^1(t,t';|\mathbf{p}_i|,0,q^2)}{R_V^2(t,t';0,0,0)}$$

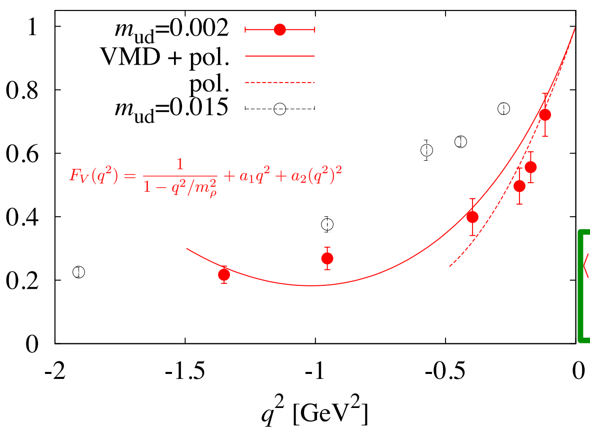


3. Preliminary lattice results



3. Preliminary lattice results

$F_V(q^2)$



$$\langle r^2\rangle_V = 6\left.\frac{\partial F_V(q^2)}{\partial q^2}\right|_{q^2=0} = 0.53(4)\ \mathrm{fm}^2.$$

3. Preliminary lattice results

NOTES

1. We DON'T need Bessel functions in the analysis.
2. The result still contain NLO $\sim \frac{1}{4\pi F^2 V^{1/2}} \sim 7\%$ finite V effects.

To do list :

1. Check the stability of q^2 fit.
2. Check the dispersion relation.
3. 1-loop ChPT corrections [HF & T. Suzuki, in progress]
4. Twisted boundary conditions.
5. Expand the volume in our new simulations.

4. Summary

Our strategy

Numerical calculation + Analytic calculation

Small lattice QCD with exact chiral symmetry + Finite volume correction by pion theory

↓

QCD at V=∞

4. Summary

What's new in this talk : Automatic cancellation of finite V effects

1. non-zero momenta (or subtraction)
2. take an appropriate ratio

-> We can eliminate LO finite V effects

(= we can forget about Bessel functions)

even in the ε regime.

=> Pion form factors

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Comarison with others

Physical point simulation by PACS-CS [2011]

The PACS-CS gauge configurations has one more set corresponding to $M_\pi \approx 156$ MeV. We tried to calculate the form factor on this set, and found that the pion two- and three-point correlators exhibit very large fluctuations, to the extent that taking a meaningful statistical average is difficult. This trend becomes more pronounced as the twist carried by quarks becomes larger. Since $LM_\pi \approx 2.3$ at this pion mass for $L = 32$, we suspect that this phenomenon is caused by a small size of the lattice relative to the pion mass, and consequent increase of large fluctuations.

Zero-mode subtracted correlators :

$$F_V(t,t',q^2) \equiv \frac{R_V(t,t';|\mathbf{p}_i|,|\mathbf{p}_f|,q^2)}{R_V^2(t,t';0,0,0)}$$ may be helpful ?