格子量子色力学を用いた軽い原子核の計算
Calculation of small nuclei from lattice QCD

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素核宇融合による計算基礎物理学の進展 - ミクロとマクロのかけ橋の構築 -

@合歓の郷, Dec. 3–5

1. Introduction

Nuclei from first principle of strong interaction

$\longrightarrow$ lattice QCD

Recent studies of lattice QCD for bound state of multi-baryon systems

1. Three- and four-nucleon systems

   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV

2. H dibaryon in ΛΛ system (S=−2, I=0)

   ('88 Iwasaki et al. $N_f = 0$ $m_\pi > 0.7$ GeV)

   '11 NPLQCD $N_f = 2 + 1$ $m_\pi = 0.39$ GeV

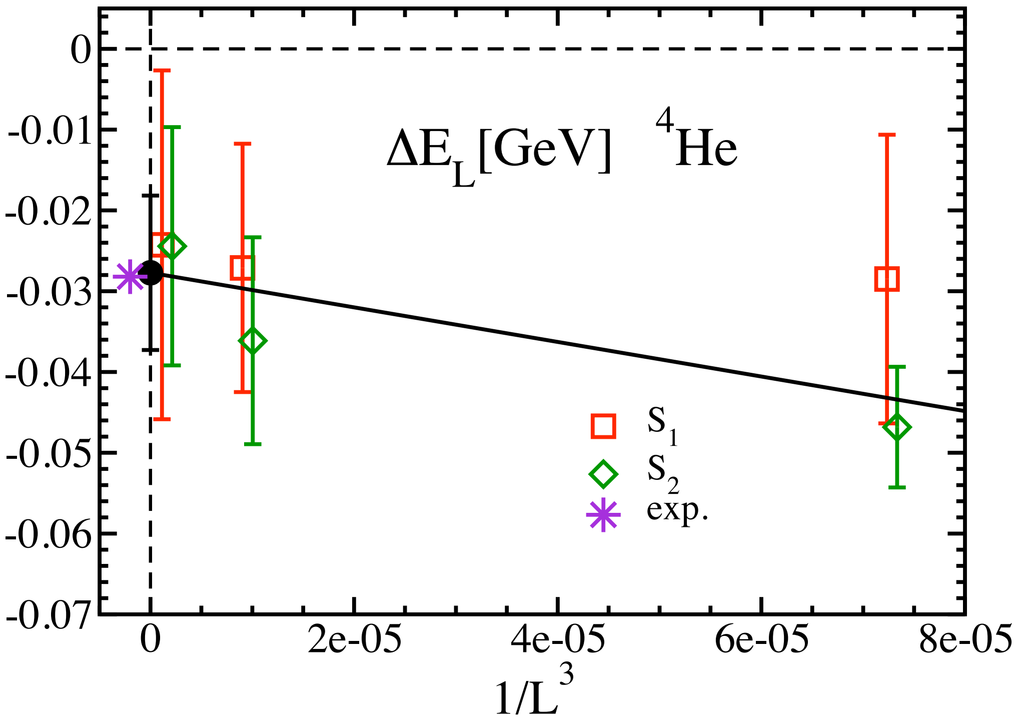
   '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV

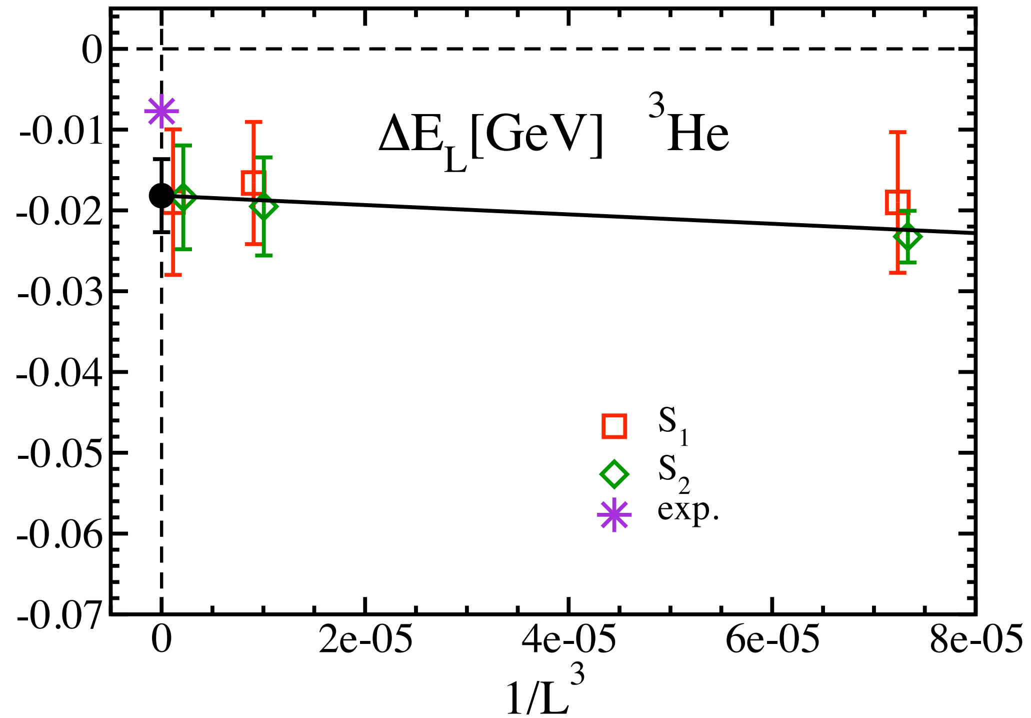
   '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV

Expoloratory study of three- and four-nucleon systems

PACS-CS Collaboration, PRD81:111504(R)(2010)

Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\mathrm{He}} = 27.7(7.8)(5.5)$ MeV $\Delta E_{^3\mathrm{He}} = 18.2(3.5)(2.9)$ MeV

1. Observe bound state in both channels
2. Same order of ΔE to experiment

However, several systematic errors, e.g., $m_\pi = 0.8$ GeV

1. Introduction

Nuclei from first principle of strong interaction

→ lattice QCD

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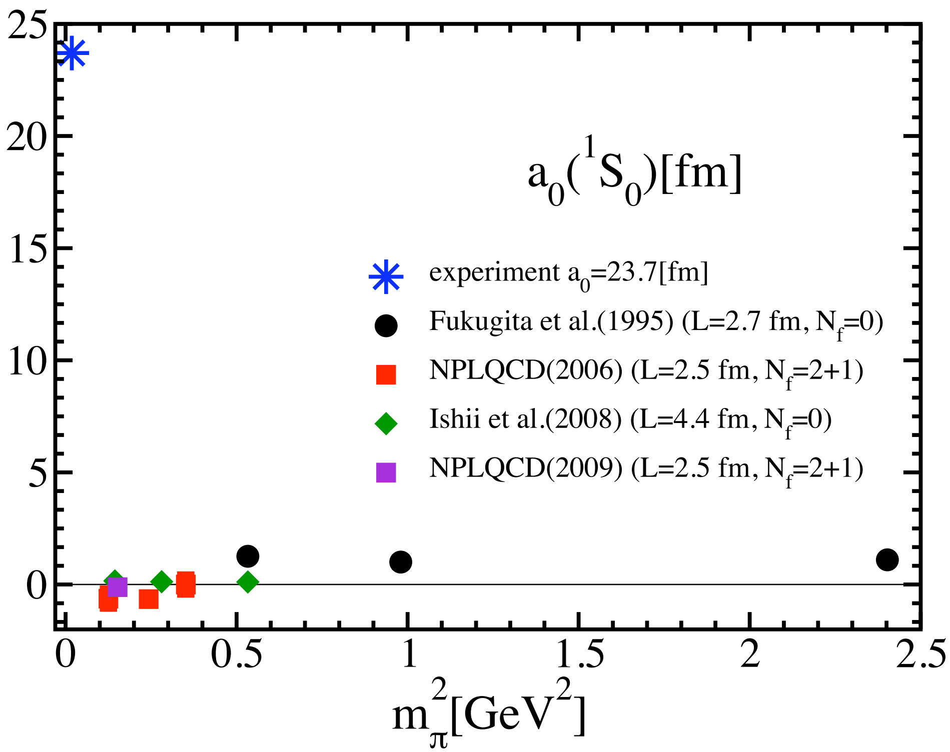
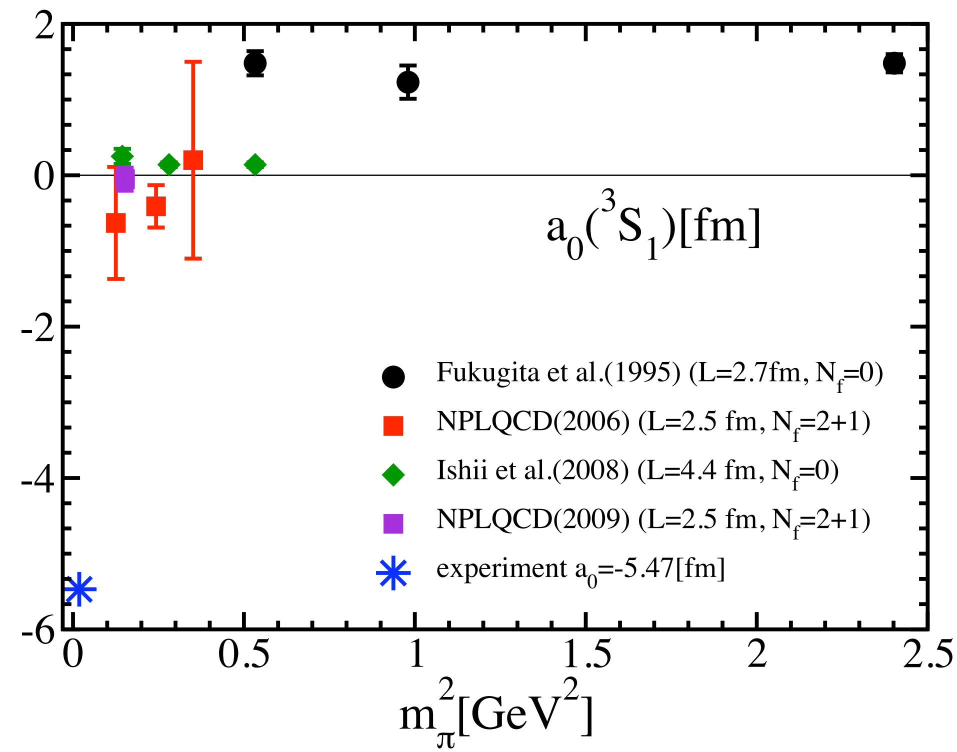
Bound state in simplest multi-nucleon system, NN system?

Scattering length a_0 in NN system from lattice QCD at ∼'09





a_0: Far from experiments due to $m_\pi \gtrsim$ 0.3 GeV

Deuteron: $^3S_1$ channel $\Delta E_d = 2.2$ MeV

Assumption: unbound due to $m_\pi \gtrsim$ 0.3 GeV

Aim of this work : check assumption with simpler method

c.f. using nuclear force '09 HALQCD

Existence of bound state for a_0

System	w/ bound state	w/o bound state
0th	bound state	scattering state
1st	scattering state	scattering state
a_0	< 0 from 1st	> 0 from 0th

Bound state exists $\rightarrow$ a_0 never obtained from 0th state
by Lüscher's finite volume method

$$\Delta E_L = E^0_{NN} - 2M = -\frac{4\pi a_0}{ML^3} + \cdots \qquad \text{('86, '91 Lüsher)}$$

Need to check existence of bound state to calculate a_0

Two methods

1. Volume dependence of 0th state
2. Properties of 1st state energy

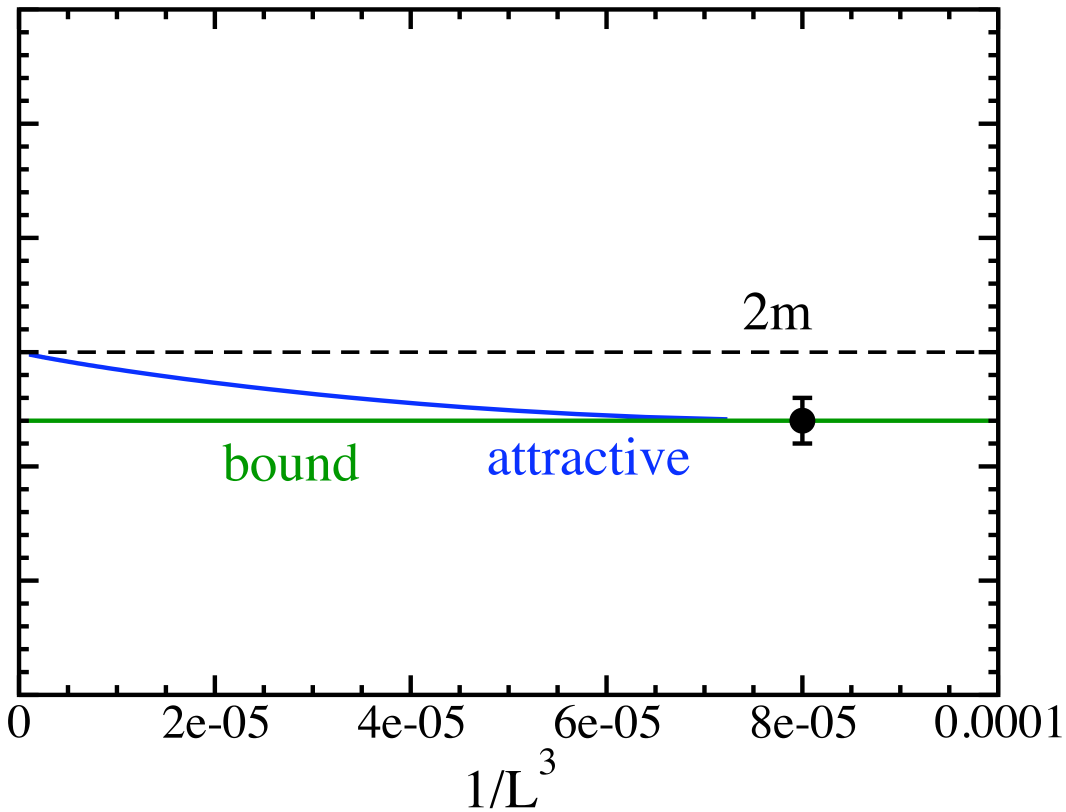
Outline

1. ~~Introduction~~
2. Methods
    1. Identification of bound state
    2. Properties of 1st excited state
3. Simulation parameters
4. Results
    1. Single state analysis
    2. Two-state analysis
5. Summary and future work

2. Methods

1. Identification of bound state in finite volume

observe small $\Delta E_L = E - 2m < 0$ at one L is not enough



Bound state : $\Delta E_L = \Delta E_\infty + O\left(e^{-CL}\right) < 0$

'04 Beane *et al.*, '06 Sasaki & TY

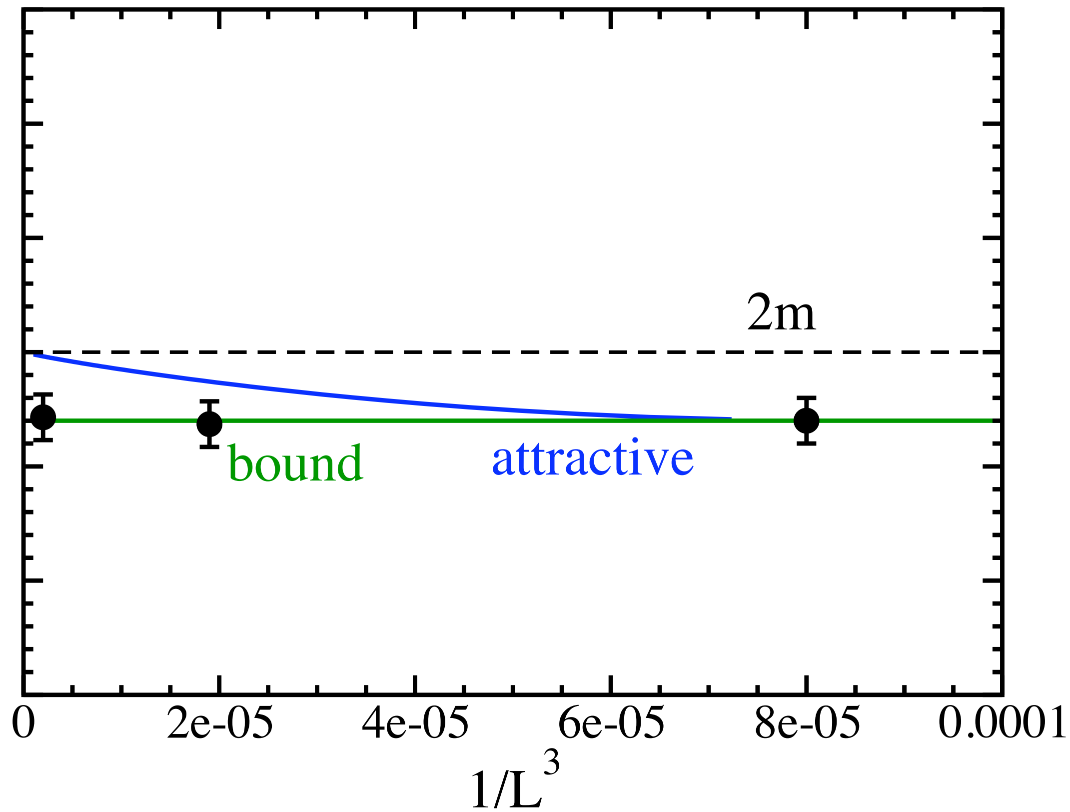
Attractive scattering state : $\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

'86, '91 Lüscher

2. Methods

1. Identification of bound state in finite volume

observe small $\Delta E_L = E - 2m < 0$ at several L



Bound state : $\Delta E_L = \Delta E_\infty + O\left(e^{-CL}\right) < 0$

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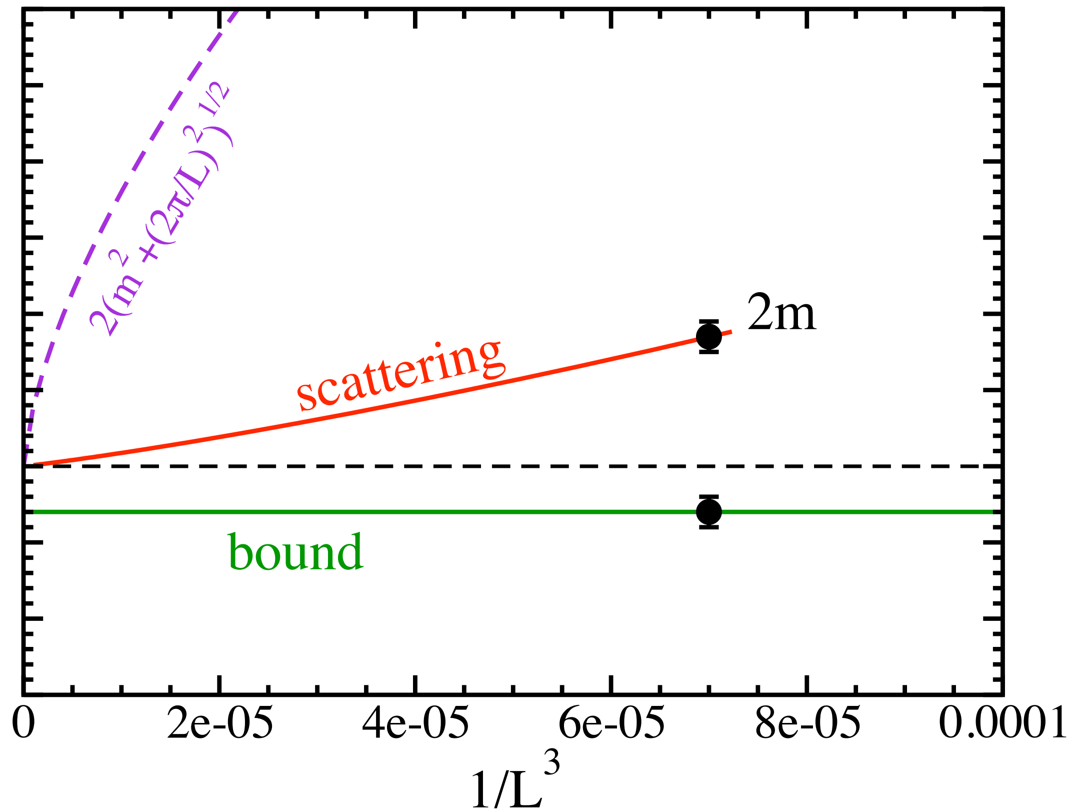
Attractive scattering state : $\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) < 0 \quad (a_0 > 0)$

'86, '91 Lüscher

2. Methods
2. Properties of 1st excited state in finite volume

'06 Sasaki & TY

1st excited state $\Delta E_L = E - 2m > 0$ at finite L



Scattering state : $\Delta E_L = O\left(-\frac{a_0}{ML^3}\right) > 0 \quad (a_0 < 0)$ '86, '91 Lüscher

1st excited state ← diagonalization method '90 Lüscher & Wolff

3. Simulation parameters

- Quenched Iwasaki gauge action at $\beta = 2.416$

  $a^{-1} = 1.54$ GeV with $r_0 = 0.49$ fm
- Tad-pole improved Wilson fermion action

  $m_\pi = 0.8$ GeV and $m_N = 1.62$ GeV

  reduce large statistical fluctuation

1. Finite volume dependepce of 0th state (Single state analysis)

- Three volumes: $L = 3.1, 6.1, 12.3$ fm
- Two smearing sources: for consistency check

2. Property of 1st excited state (Two-state analysis)

- Two volumes: $L = 3.1, 6.1$ fm
- Wavefunction smearing sink operators

  assumption: 0th energy = one of single state analysis

Simulations:

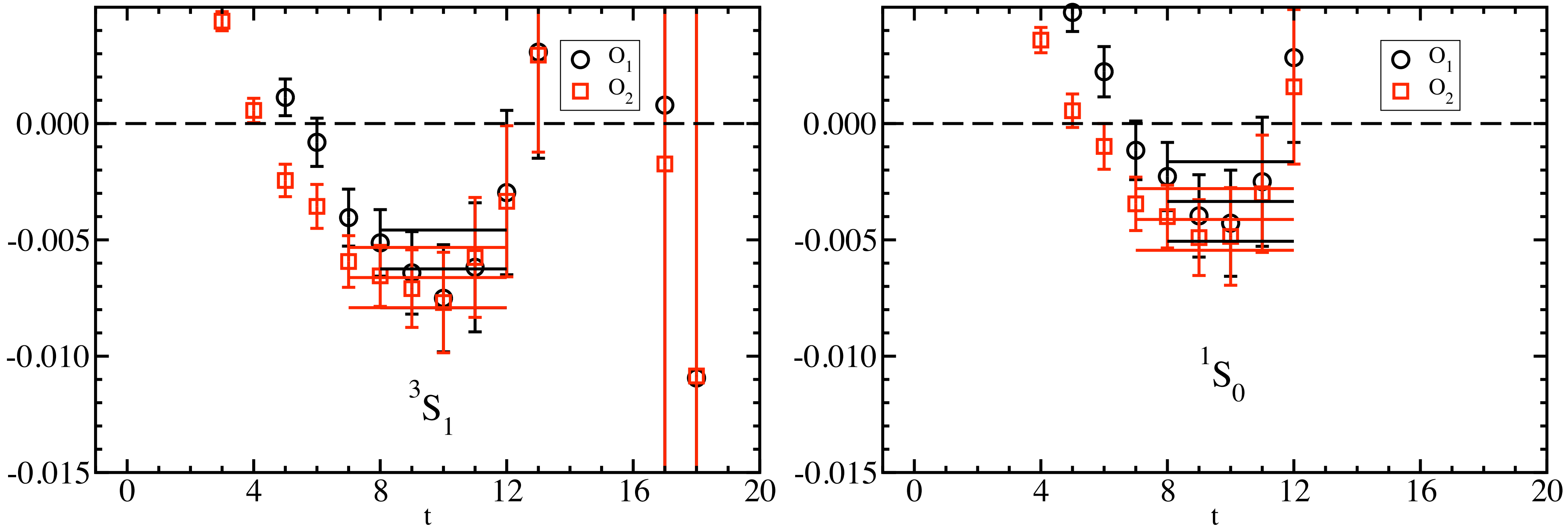
PACS-CS, T2K-Tsukuba at Univ. of Tsukuba, HA8000 at Univ. of Tokyo

4. Results

1. Single state analysis

Effective energy shift $\Delta E_L = E_{\mathrm{NN}} - 2m_N$ at $L = 6.1$ fm

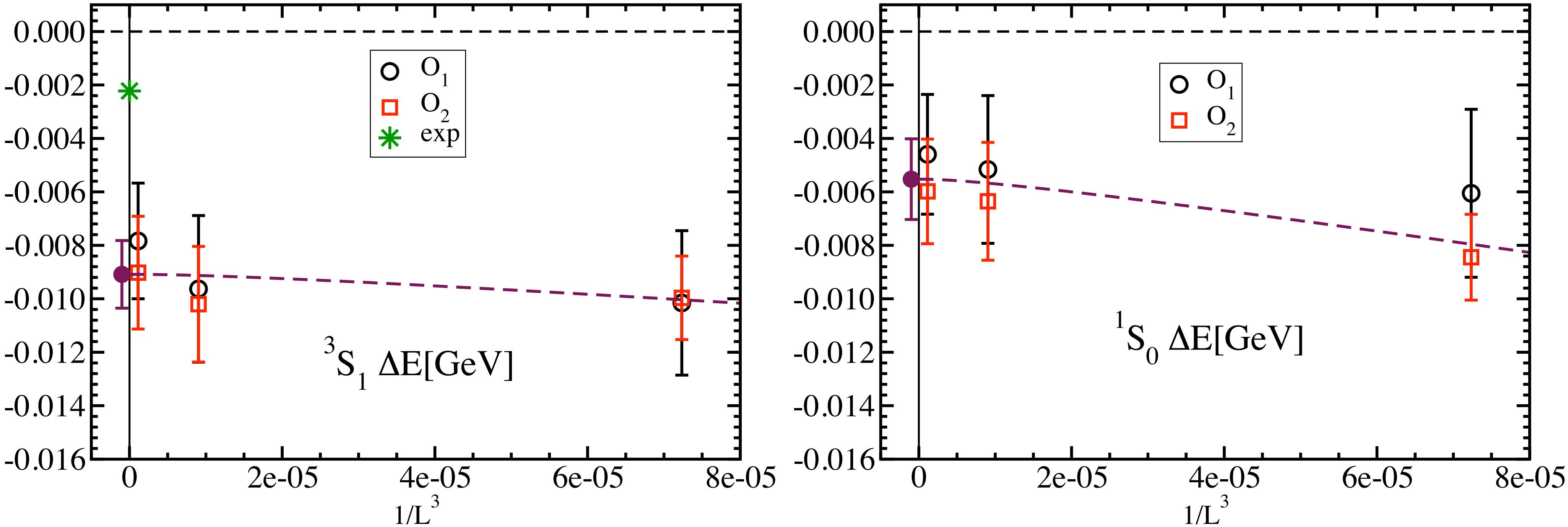
$$\Delta E_L = \log\left(\frac{R(t)}{R(t+1)}\right), \quad R(t) = \frac{C_{\mathrm{NN}}(t)}{(C_N(t))^2}$$



- $\Delta E_L < 0$ in $8 \lesssim t \leq 11$
- consistent plateaus in $8 \lesssim t \leq 11$

1. Single state analysis

Volume dependence of ΔE_L



- $\Delta E_L < 0$ in three volumes

$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\}, \ \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

'04 Beane *et al.*, '06 Sasaki & TY

- Bound state in both channels ← incosistent with experiment

$$\Delta E_{^3\text{S}_1} = 9.1(1.1)(0.5) \text{ MeV} \qquad \Delta E_{^1\text{S}_1} = 5.5(1.1)(1.0) \text{ MeV}$$

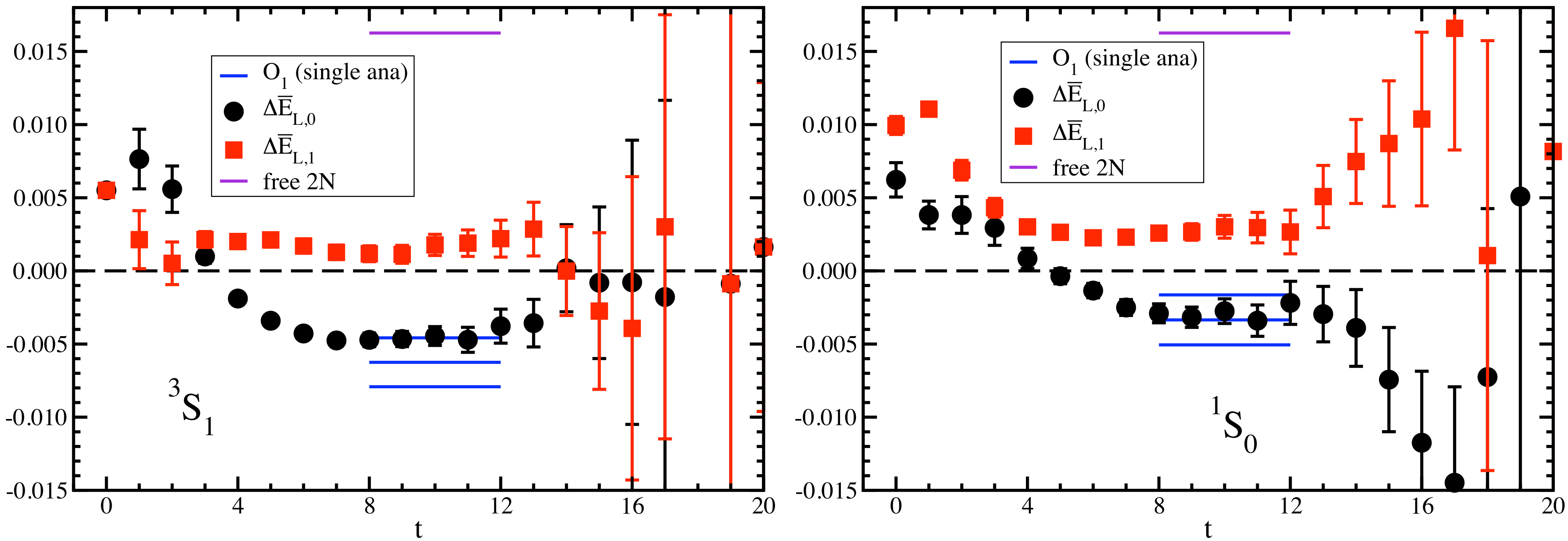
might be caused by heavy quark mass in calculation

2. Two-state analysis

Effective energy shift for ground and 1st excited states at $L = 6.1$ fm

$$\mathrm{Diag}\left[G^{-1}(t_0)G(t)\right] = \lambda(t) \text{ with } G_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle$$

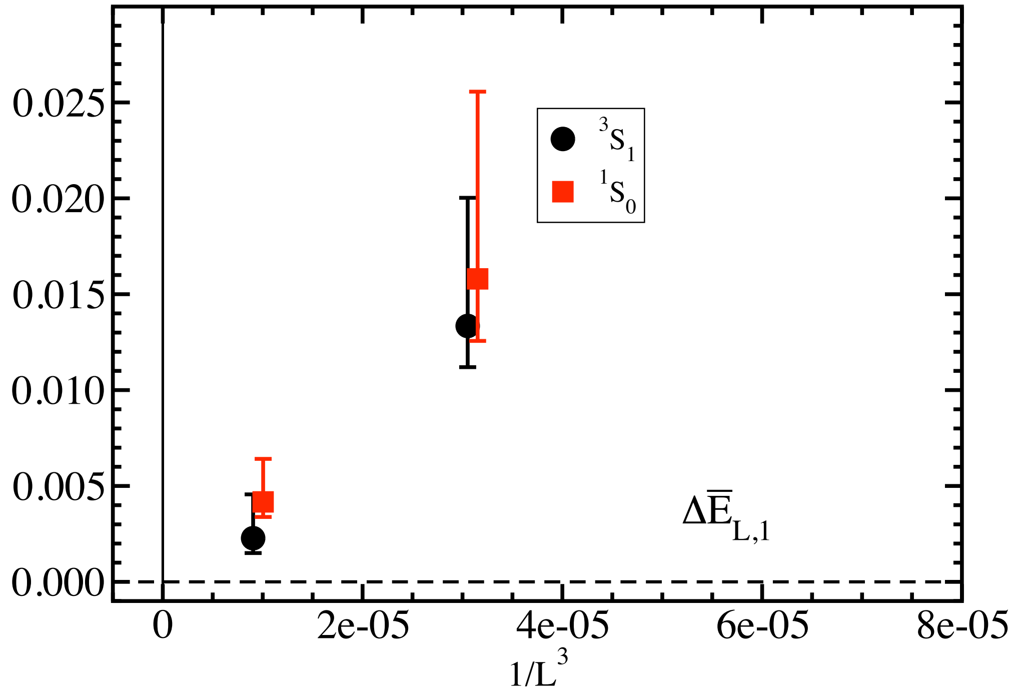
$$\Delta\overline{E}_{L,\alpha} = E_\alpha - 2m_N = \log\left(\frac{\overline{R}_\alpha(t)}{\overline{R}_\alpha(t+1)}\right), \quad \overline{R}_\alpha(t) = \frac{\lambda_\alpha(t)}{(C_N(t))^2}$$



- $\Delta\overline{E}_{L,0} < 0$ and consistent with ΔE_L
- small, but $\Delta\overline{E}_{L,1} > 0$ as expected

2. Two-state analysis

Volume dependence of $\Delta\overline{E}_{L,1}$



- $\Delta\overline{E}_{L,1} > 0$ and $1/L^3$ tendency
- Scattering length a_0 fm

L[fm]	3.1	6.1
$^3S_1$	$-1.5(0.2)\left(\begin{smallmatrix}+0.2\\-1.4\end{smallmatrix}\right)$	$-1.05(24)\left(\begin{smallmatrix}+0.05\\-0.65\end{smallmatrix}\right)$
$^1S_0$	$-1.8(0.3)\left(\begin{smallmatrix}+0.4\\-12.9\end{smallmatrix}\right)$	$-1.62(24)\left(\begin{smallmatrix}+0.01\\-0.75\end{smallmatrix}\right)$

Observe expected properties of 1st excited state

5. Summary and future work

Exploratory study of two-nucleon bound state
in quenched lattice QCD

- Unphysically heavy quark mass
- Volume dependence of energy shift of ground state
- Properties of 1st excited state energy

1. $\Delta E \neq 0$ of 0th state in infinite volume limit
2. Expected properties of 1st excited state

   $\rightarrow$ bound state in $^3S_1$ and $^1S_0$ at $m_\pi = 0.8$ GeV

Bound state in $^1S_0$: observed in $N_f = 2 + 1$ QCD at $m_\pi = 0.39$ GeV
(NPLQCD Collaboration, arXiv:1109.2889[hep-lat])

Not observed in experiment

Future work

Bound state in $^1S_0$ vanishes as quark mass decreases?

- Quark mass dependence of ΔE and a_0
- Reduce systematic errors

Heavy quark mass

$m_\pi = 0.8$ GeV

Quenched effect

Lattice spacing dependence

Future work

Bound state in $^1S_0$ vanishes as quark mass decreases?

- Quark mass dependence of ΔE and a_0
- Reduce systematic errors

Heavy quark mass

$m_\pi = 0.8$ GeV $\rightarrow$ 0.7 and 0.5 GeV

Quenched effect

$N_f = 2 + 1$ QCD

Lattice spacing dependence

Far future project

Calculation of NN, ^{3}He, ^{4}He channels

Very preliminary results at $m_\pi = 0.7$ GeV