Complex phase in QCD with finite chemical potential

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in collaboration with

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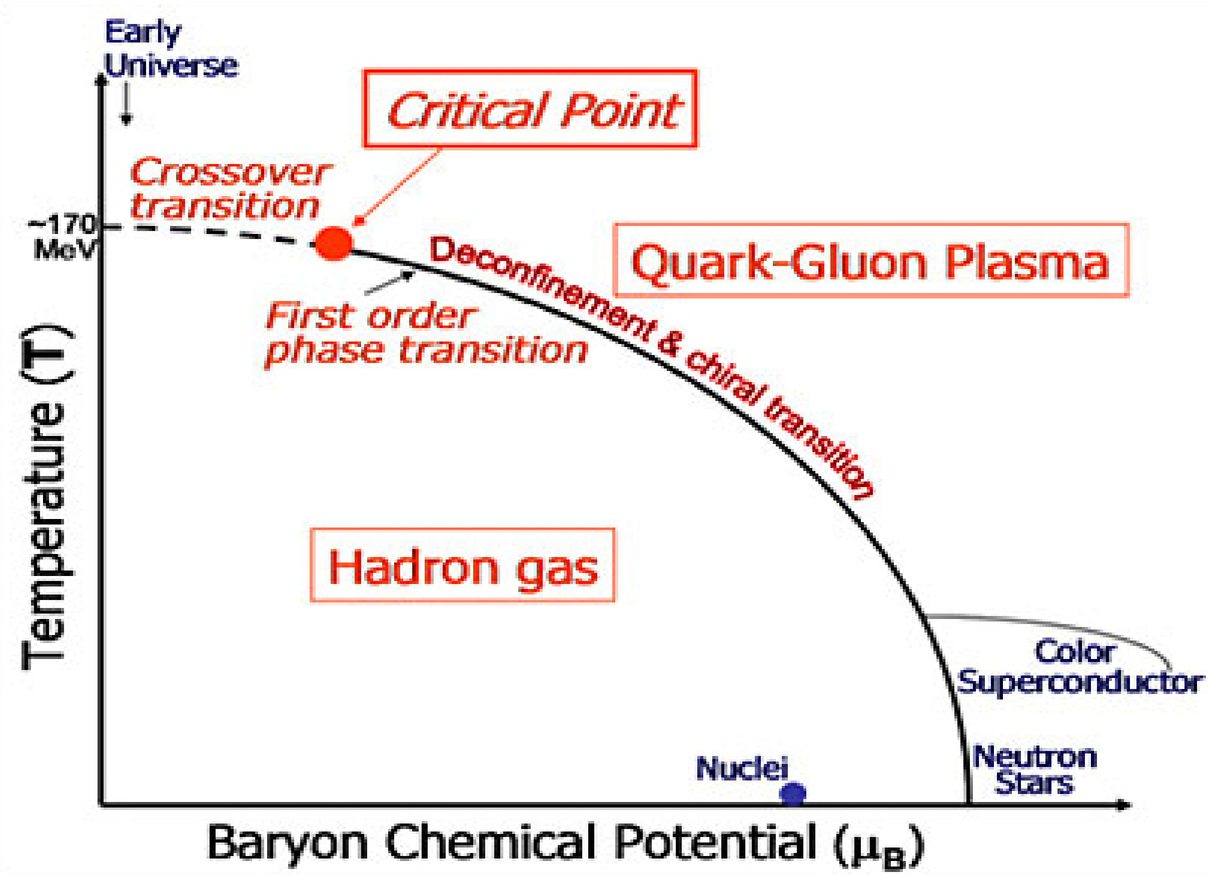
QCD phase structure



Difficulties

- The complex phase ruins an applicability of the Monte Carlo technique

$$Z_{\mathrm{QCD}}(\mu) = \int [dU] \det D(\mu) e^{-S_{\mathrm{G}}}$$

$$\det D(\mu) = |\det D(\mu)| e^{i\theta(\mu)} \in \mathbb{C}$$

- Reweighting $\langle O \rangle_{|\det D| e^{i\theta}} = \dfrac{\langle O e^{i\theta} \rangle_{|\det D|}}{\langle e^{i\theta} \rangle_{|\det D|}}$

- Large phase fluctuation

$$\langle e^{i\theta} \rangle \sim 0$$

⇨ Sign problem

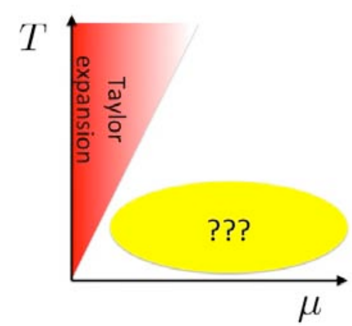
Phase (of determinant) study

So far

- Lattice, Taylor expansion
  Allton (2005) Ejiri (2008)
  - Not reliable when $\mu/T>1$
- Effective theory/model
  Splittorff & Verbaarschot (2008)
  Han & Stephanov (2008)
  - Not reliable in region near transition/systematic error

We try to do here

- We want to obtain/understand the phase more precisely & irrespective of the location of the phase diagram
- Especially we are interested in low T & high μ



Plan

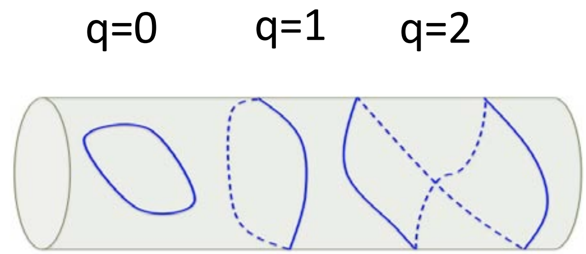
- Winding number expansion
  - Better convergence in low temperature region
- Property of the phase
  - Understanding the sign problem
  - Is there a parameter region where the sign problem can be controlled?
- Reweighting
  - 4 flavor QCD phase structure

Winding number expansion (WNE)

Danzer & Gattringer PRD78.114506 (2008)

Winding number

Gauge loops



$$\det D_{\mathrm{w}}(\mu)$$

$$= \quad A_0 \exp\left\{ \sum_{q\in\mathbb{Z}} L^{(q)} e^{q\mu/T} \right\}$$

$$\boxed{L^{(-q)} = L^{(q)*}}$$

$$= \quad A_0 e^{L^{(0)}} \exp\left\{ 2\sum_{q=1}^{\infty} \mathrm{Re}[L^{(q)}]\cosh(q\mu/T) + i2\sum_{q=1}^{\infty} \mathrm{Im}[L^{(q)}]\sinh(q\mu/T) \right\}$$

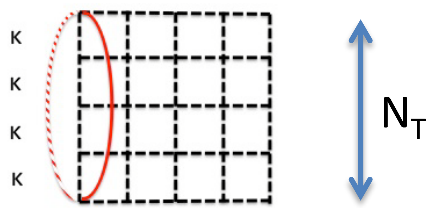
Phase

μ-independent & real

Phase in terms of WNE

$$\theta = 2\sum_{q=1}^{\infty} \mathrm{Im}[L^{(q)}] \sinh(q\mu/T) \longrightarrow \sum_{q=1}^{\infty} c^{(q)} \exp\{qN_{\mathrm{T}}(\ln(2\kappa)+a\mu)\}$$

$$O\left((2\kappa)^{qN_{\mathrm{T}}}\right)$$



$$\kappa = \frac{1}{8 + 2m_0}$$

- converges if $\ln(2\kappa) + a\mu < 0$

  $\ln(2\kappa) \approx -1$ for $\kappa = 0.17$ $\Rightarrow$ $a\mu < 1$

- better convergence range $\mu/T < N_{\mathrm{T}}$ than Taylor expansion

$$N_{\mathrm{T}} = 1/aT$$

Convergence is better for larger N_{T} !!!

Really?

Numerical check of the convergence

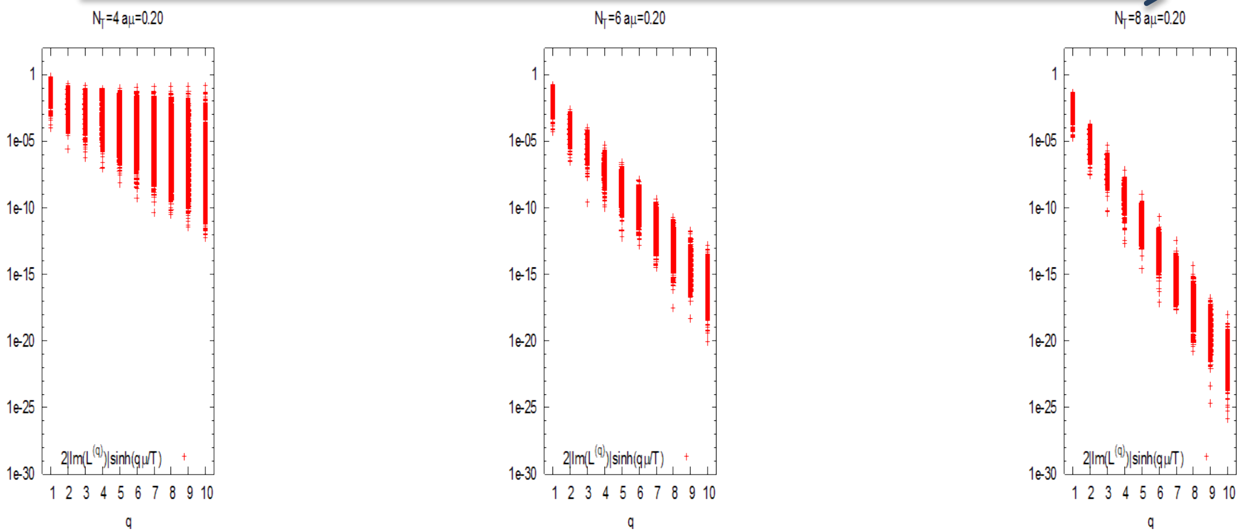
Kentucky group PRD82.054502(2010):

$N_{\mathrm{f}} = 4, N_{\mathrm{L}} = 6, a^{-1} = 610\mathrm{MeV}, \kappa = 0.1371 (m_\pi = 830\mathrm{MeV})$

$$\theta(\mu) = \sum_{q=1}^{\infty} 2\mathrm{Im}[L^{(q)}] \sinh(q\mu/T)$$

On phase quenched configurations O(1000)

large N_{T}



Convergence is better for larger N_{T} !!!

An upper bound for the phase

WNE

$$|\theta| \longrightarrow 2|\mathrm{Im}[L^{(1)}]|\sinh(\mu/T)$$

$$\kappa = \frac{1}{8 + 2m_0}$$

Hopping parameter expansion (heavy mass)

$$\boxed{\kappa \longrightarrow 0}$$

$$(2\kappa)^{N_\mathrm{T}}\mathrm{tr}[P_+]|\mathrm{Im}[\text{Polyakov loop}]| \qquad \leq 3N_\mathrm{L}^3$$

$$\Rightarrow \quad \boxed{|\theta| \leq 12N_\mathrm{L}^3(2\kappa)^{N_\mathrm{T}}\sinh(\mu/T)}$$

if $\ln(2\kappa) + a\mu < 0$

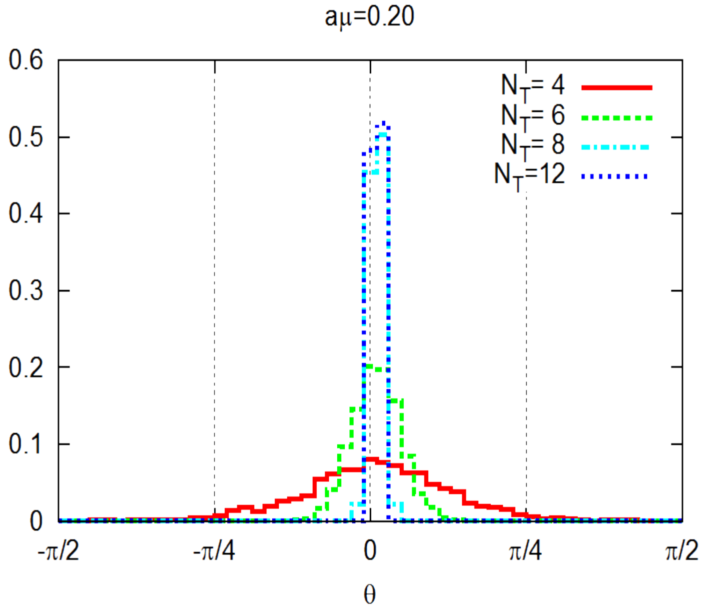
The phase gets exponentially suppressed for larger N_T !!!

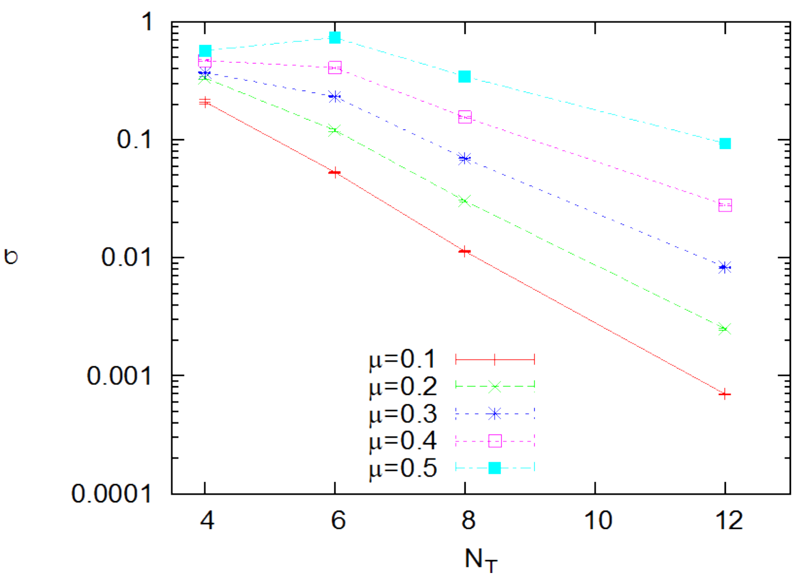
Distribution of the exact phase

On the same configurations as before

$N_f = 4, N_L = 6, a^{-1} = 610\text{MeV}, \kappa = 0.1371 (m_\pi = 830\text{MeV})$

$$\text{distribution} \propto \exp\left(-\frac{\theta^2}{2\sigma^2}\right)$$



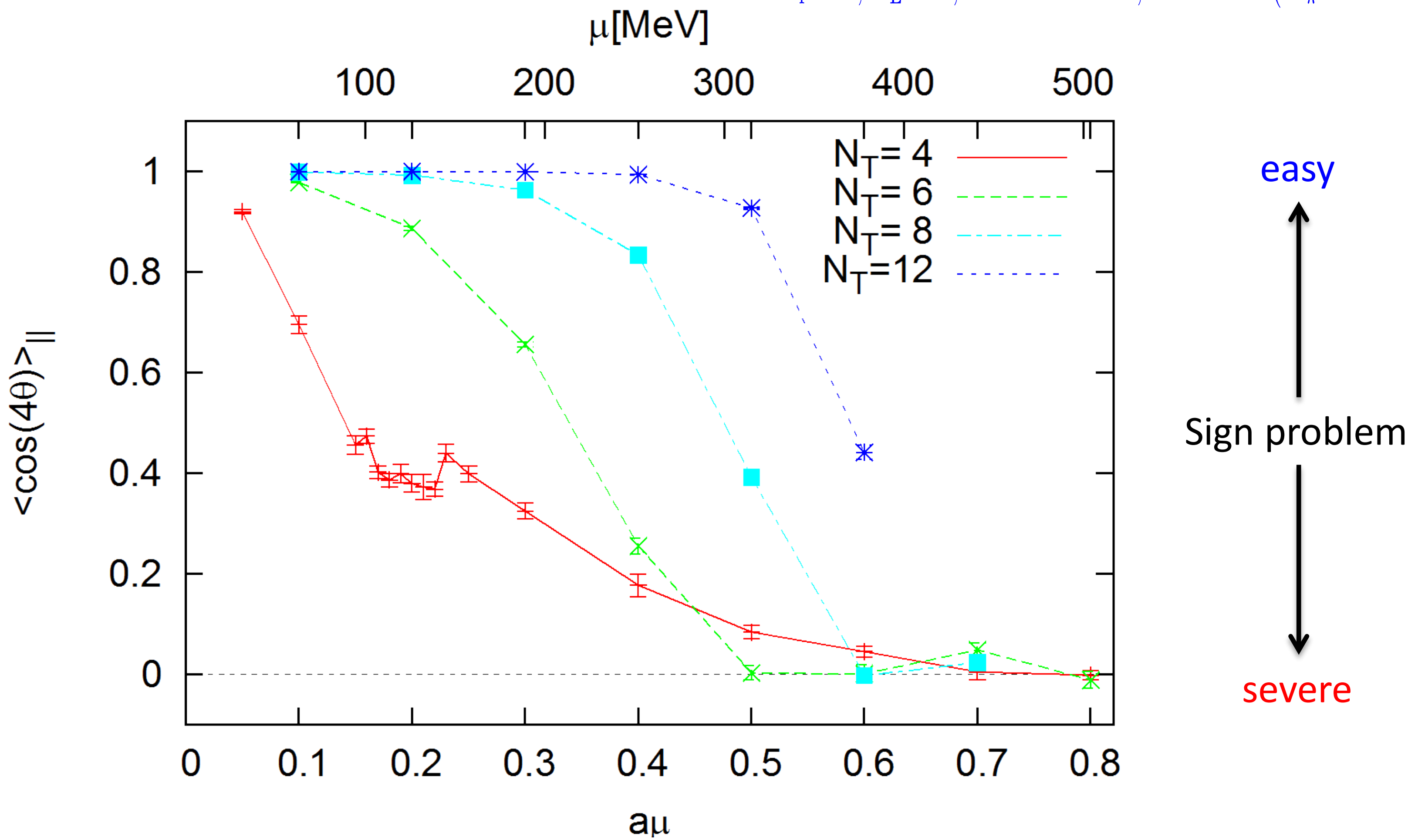


Fluctuation of the phase gets small for large N_T

Reweighting factor

On the phase quenched configurations

$N_f = 4, N_L = 6, a^{-1} = 610\text{MeV}, \kappa = 0.1371(m_\pi = 830\text{MeV})$

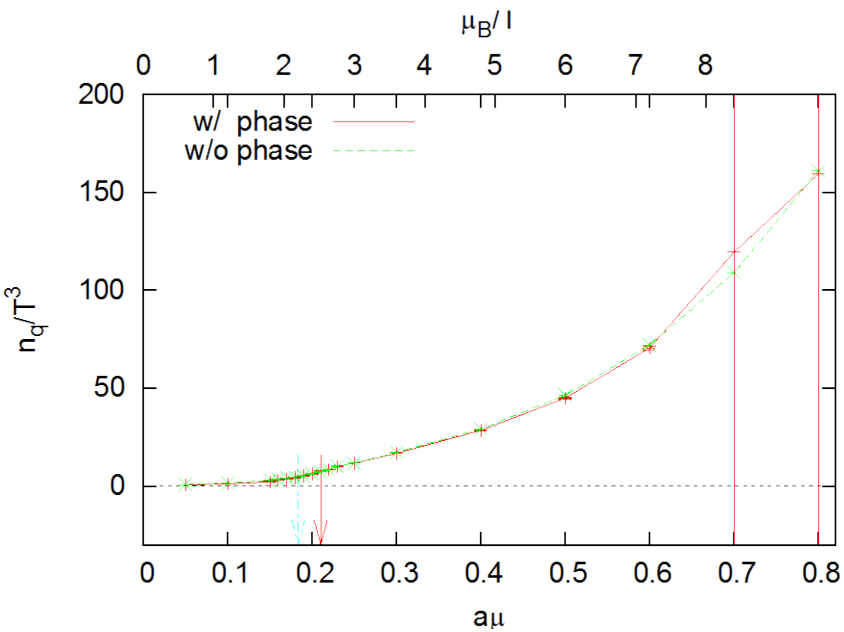


Phase reweighting

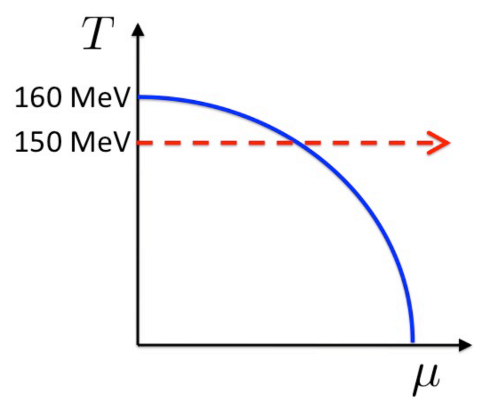
$$\langle O \rangle = \frac{\langle O e^{i N_f \theta} \rangle_{||}}{\langle e^{i N_f \theta} \rangle_{||}}$$

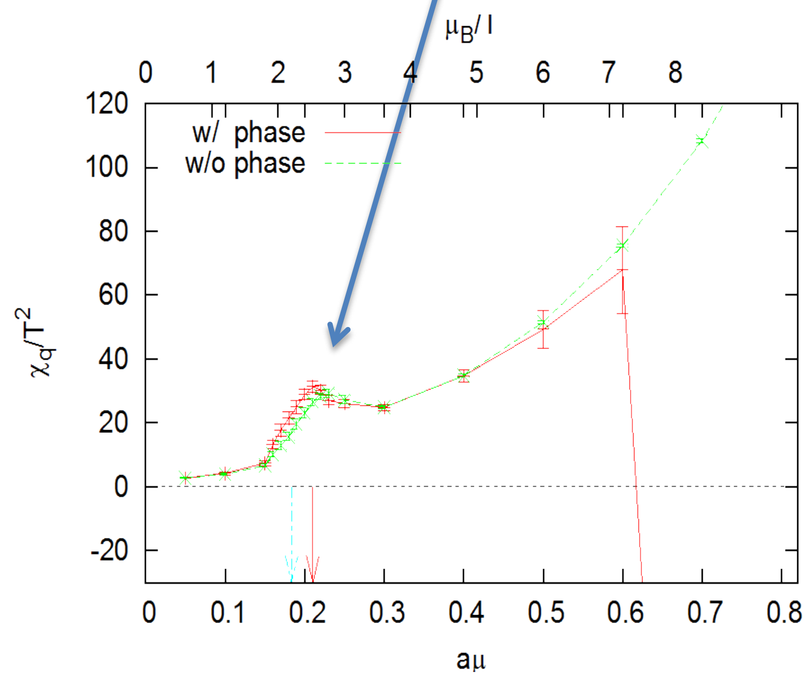
By using reduction technique we can compute phase, Quark Number & Susceptibility exactly

Quark Number and its susceptibility

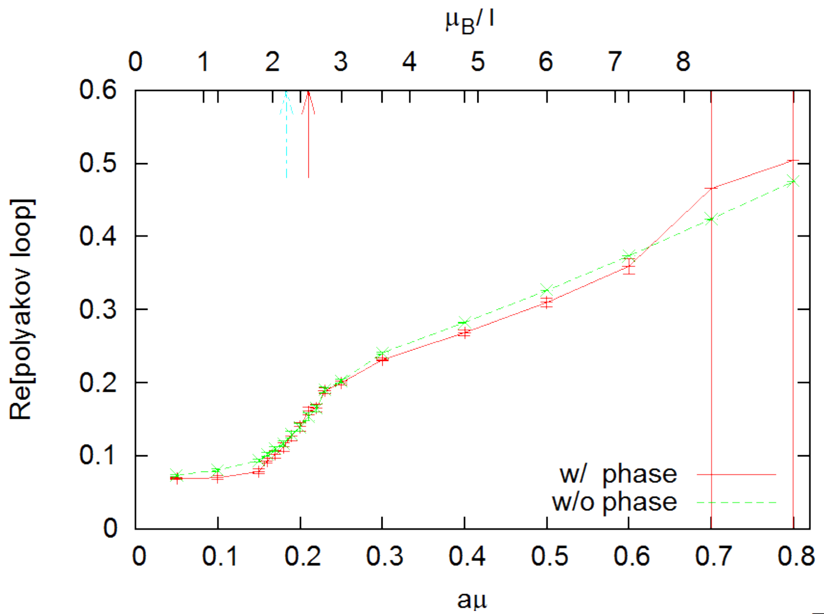


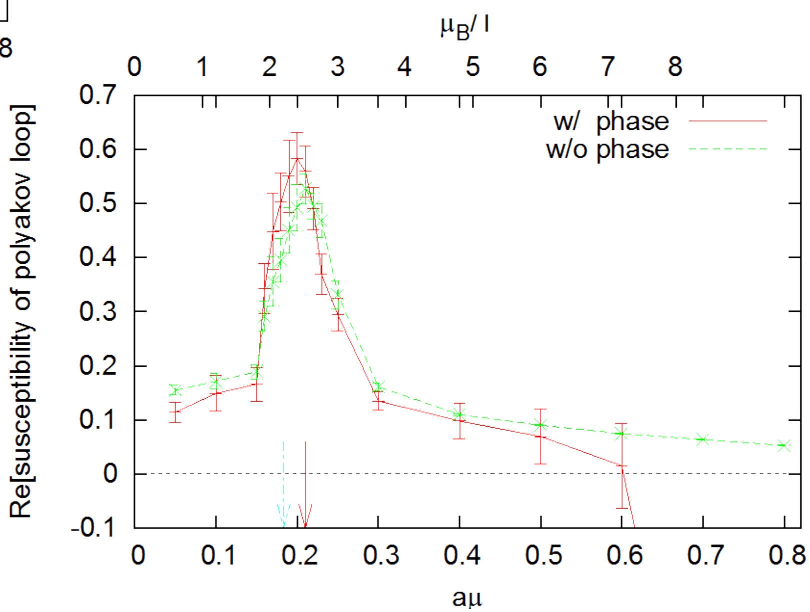
transition point at T=150MeV (N_T=4)
determined by the canonical
approach where T_c=160MeV (μ=0)
Kentucky group (2010)



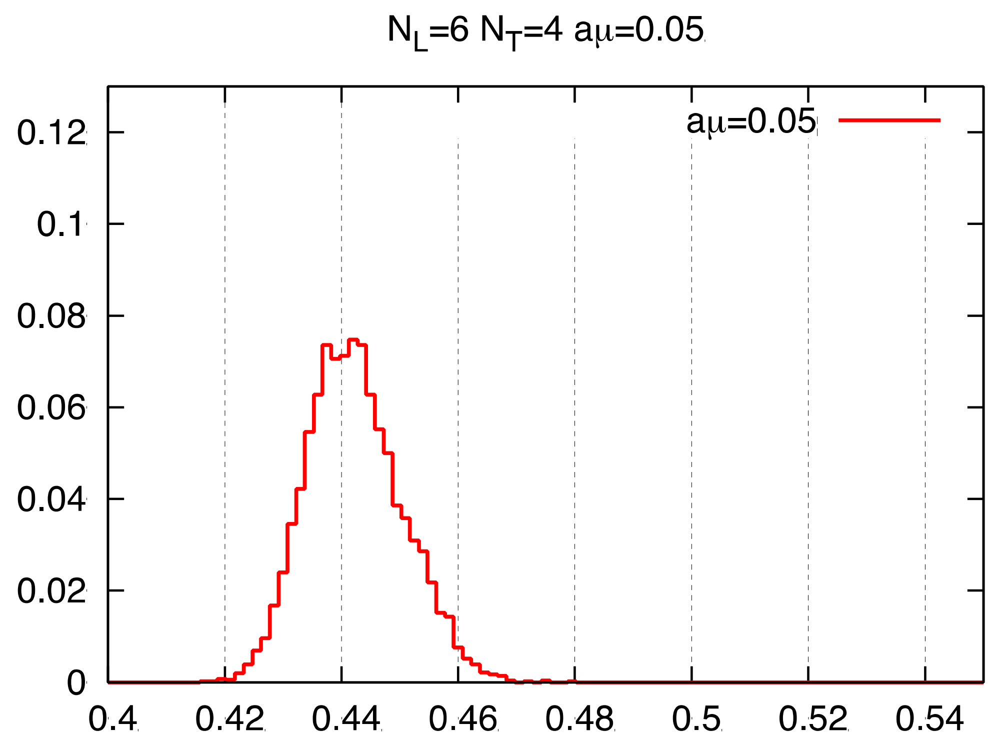


Polyakov loop and its susceptibility





Histogram of plaquette value on phase-quenched configuration



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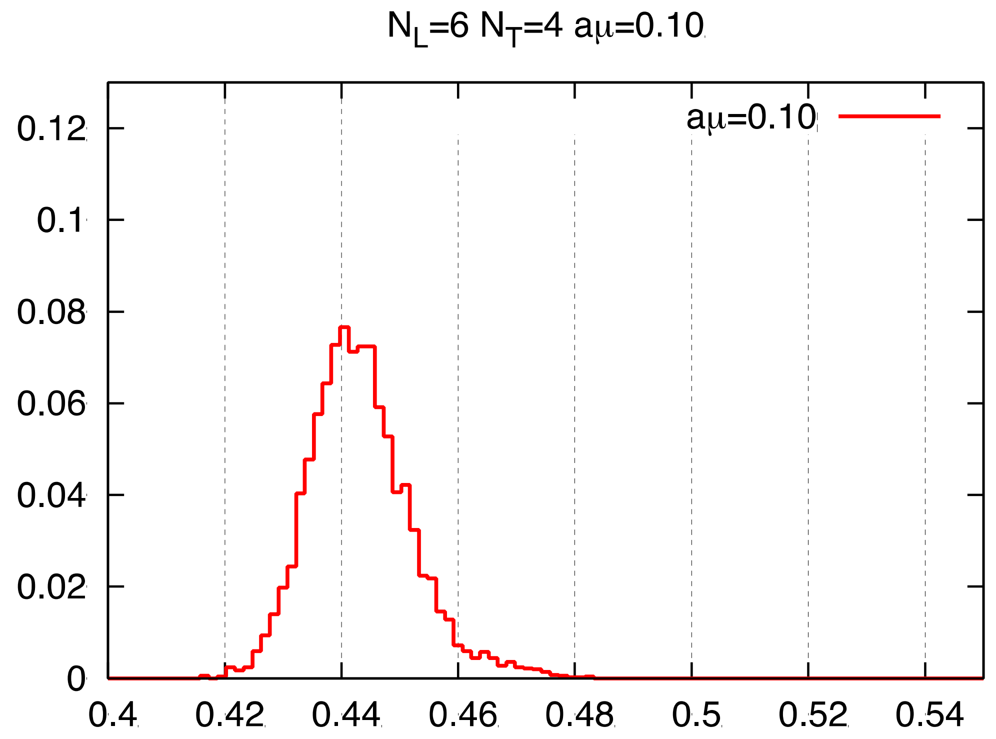

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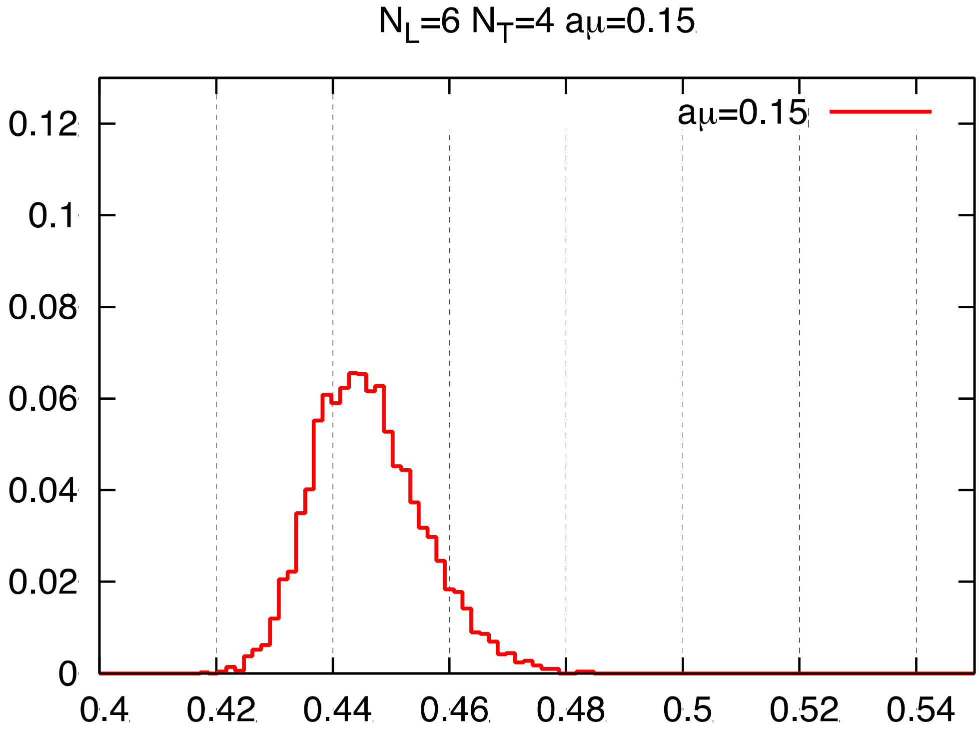

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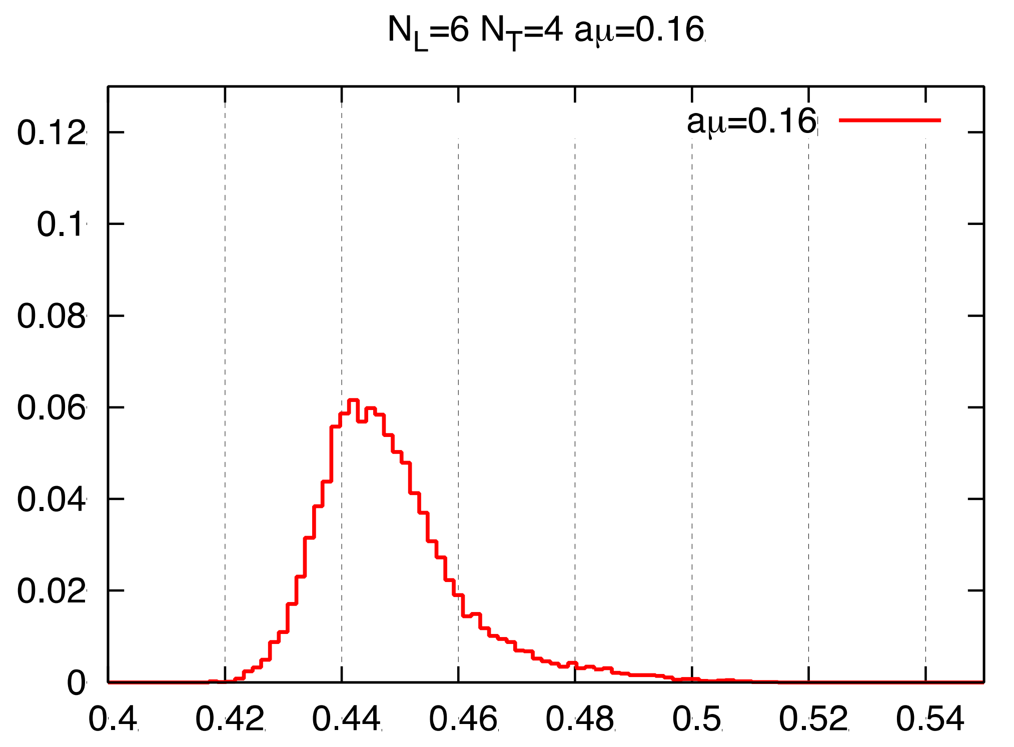

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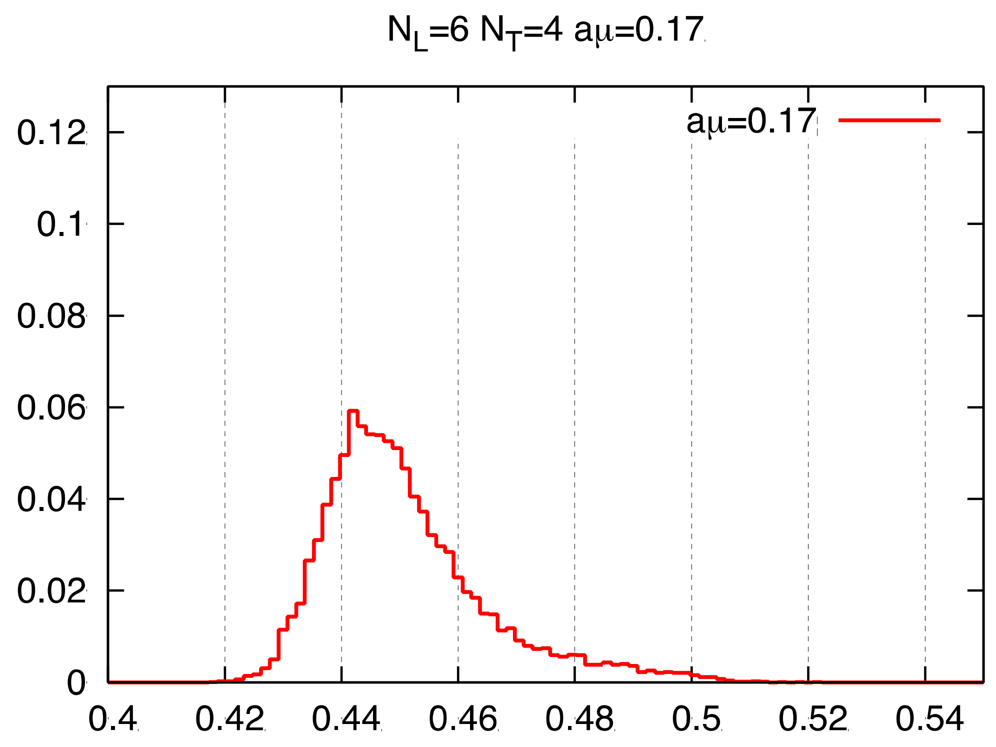

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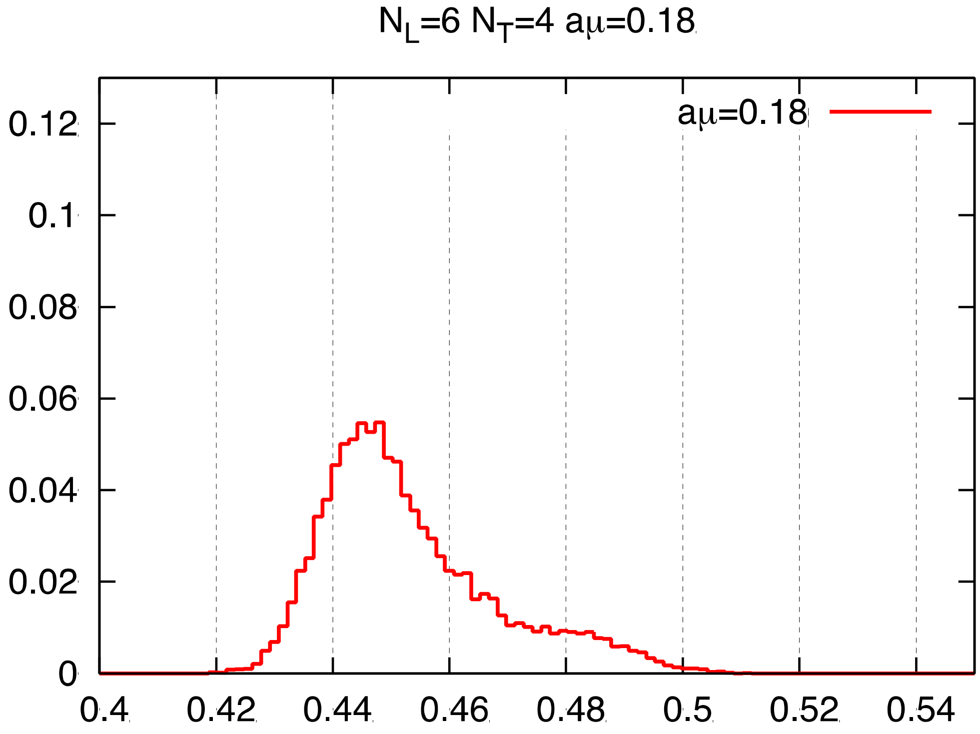

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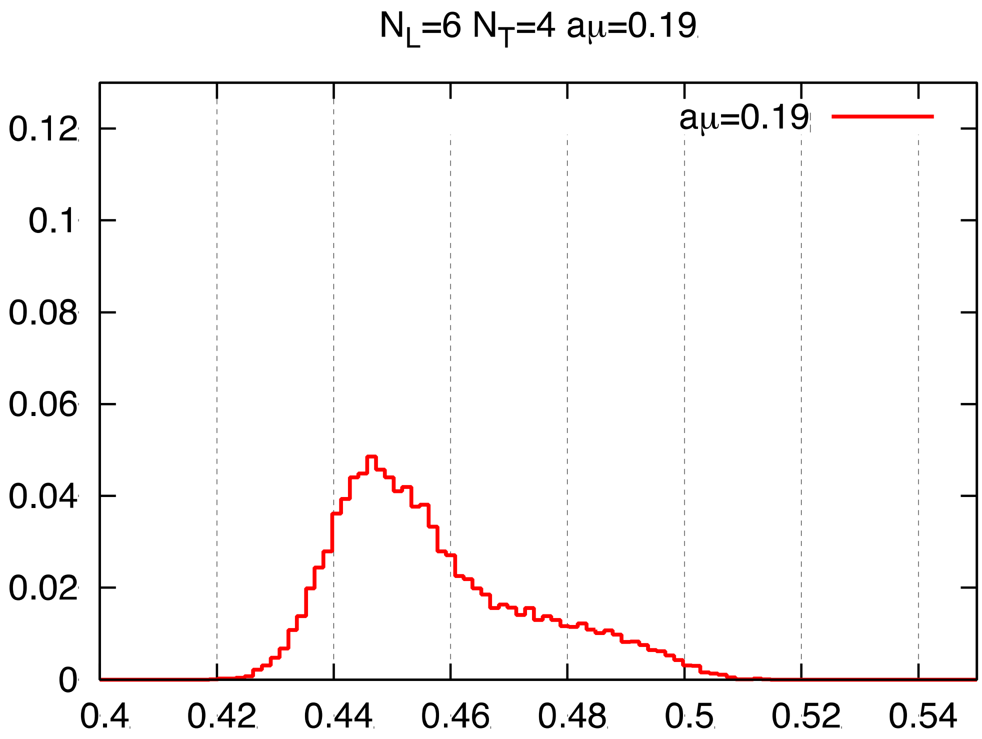

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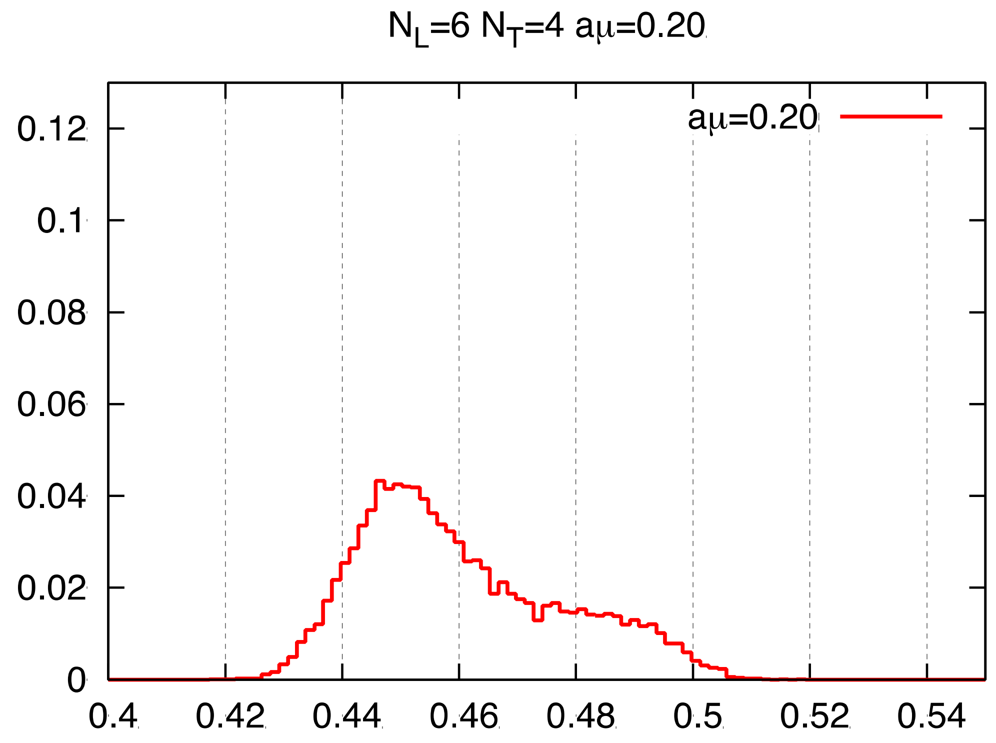

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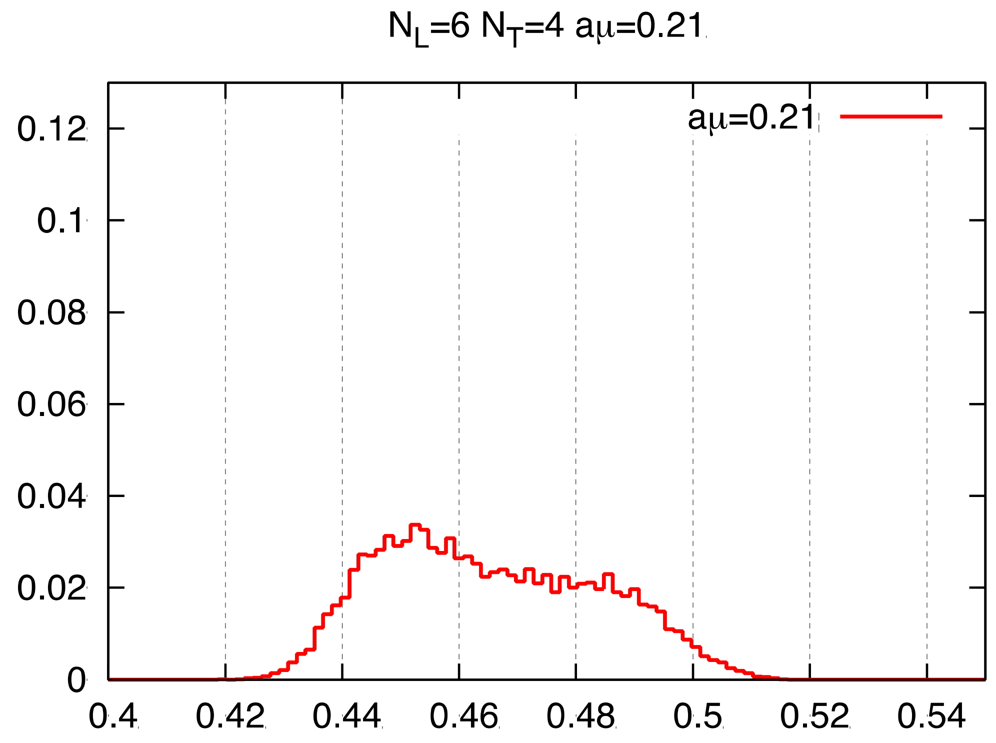

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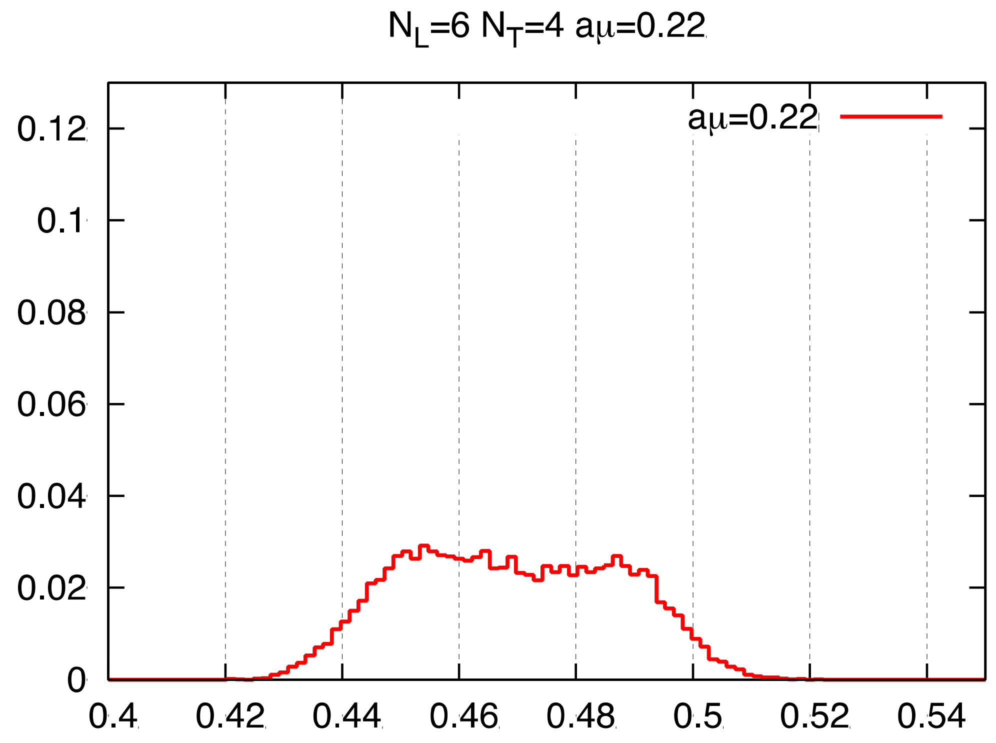

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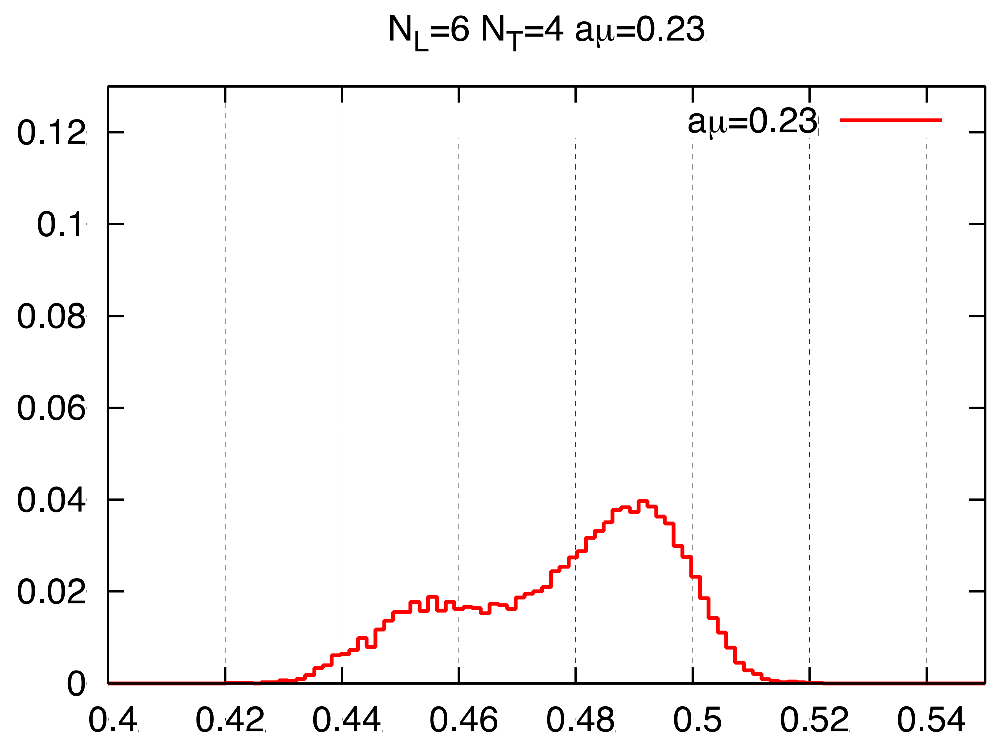

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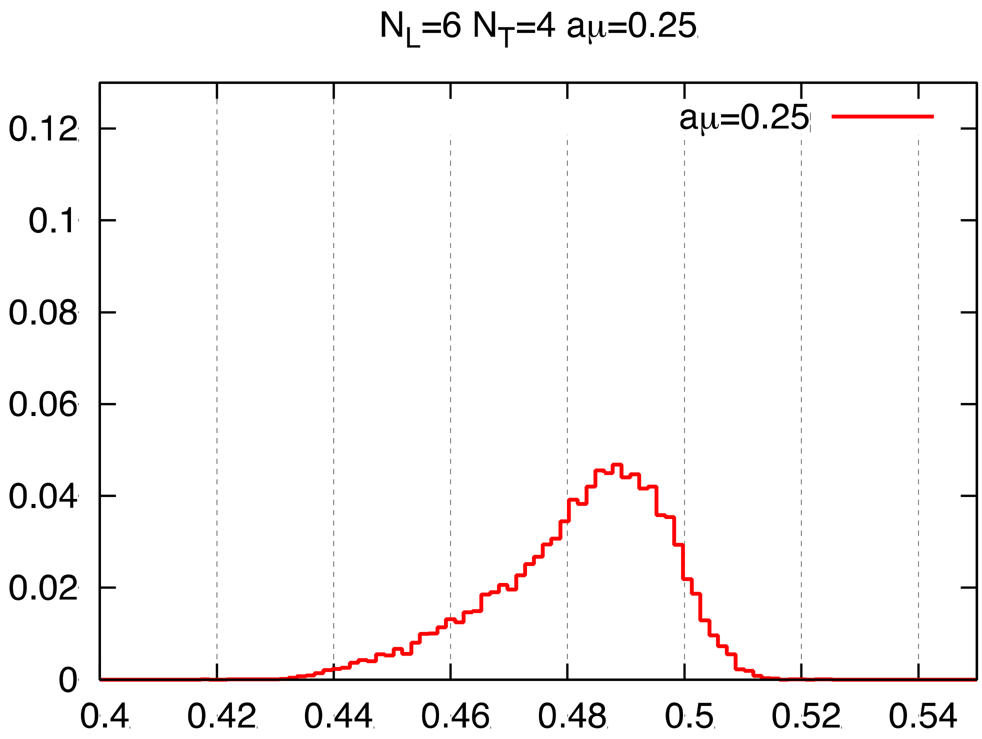

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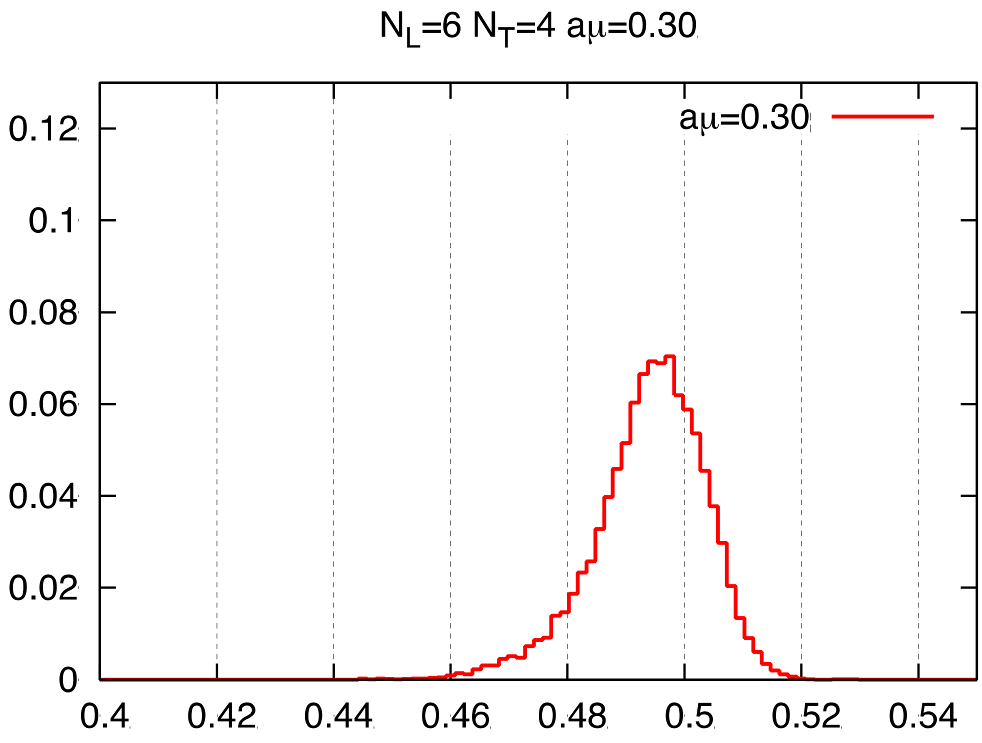

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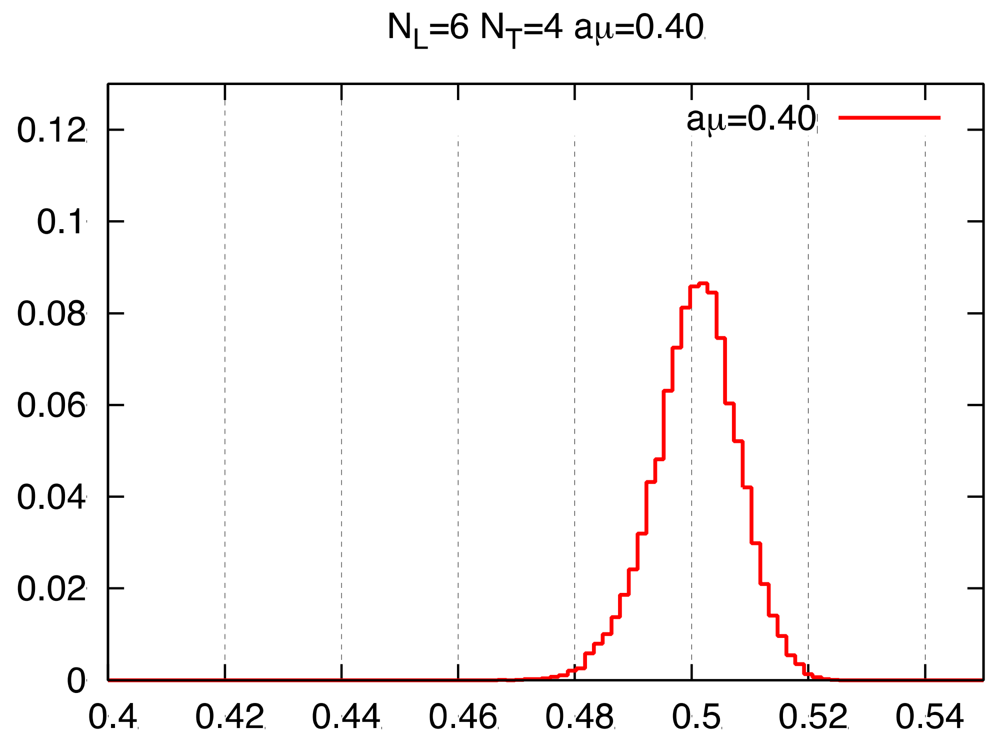

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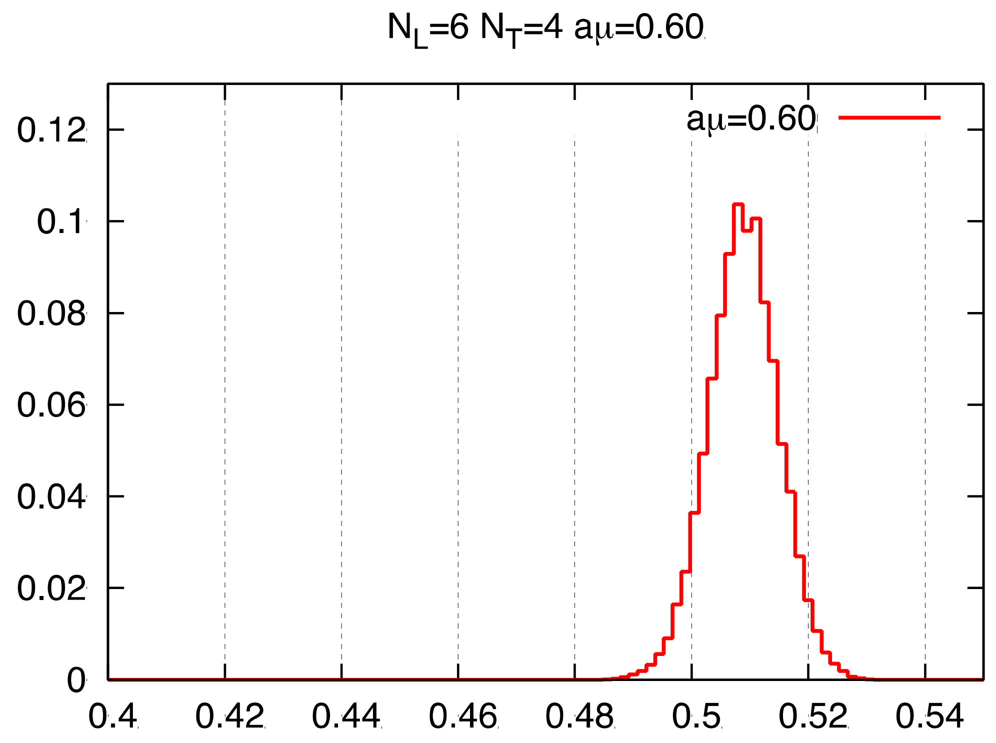

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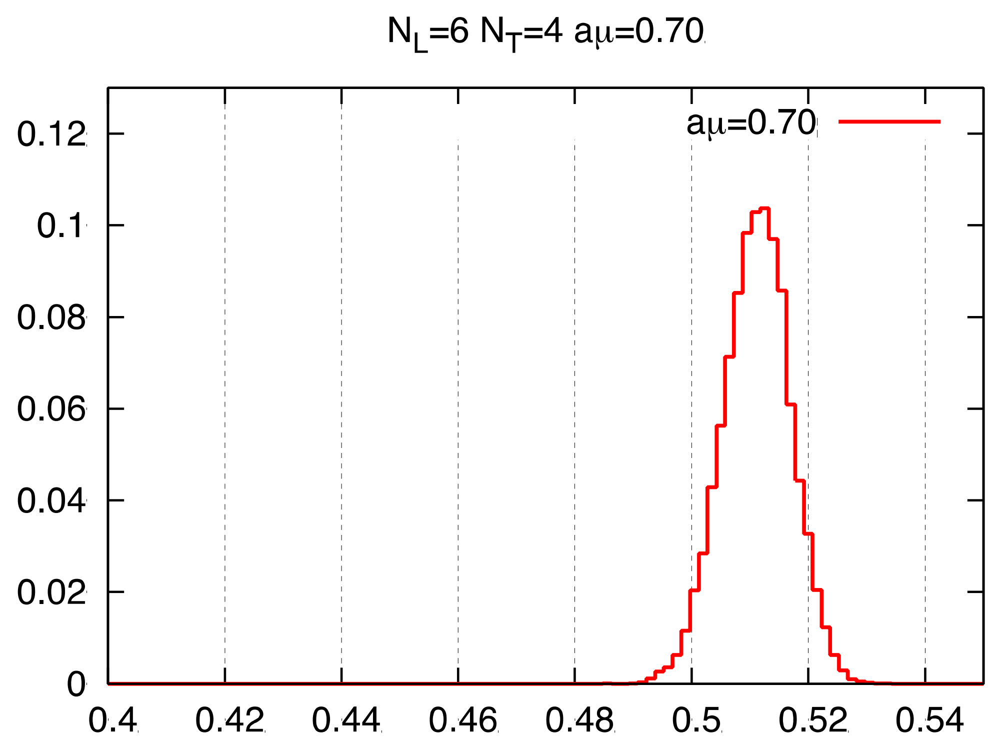

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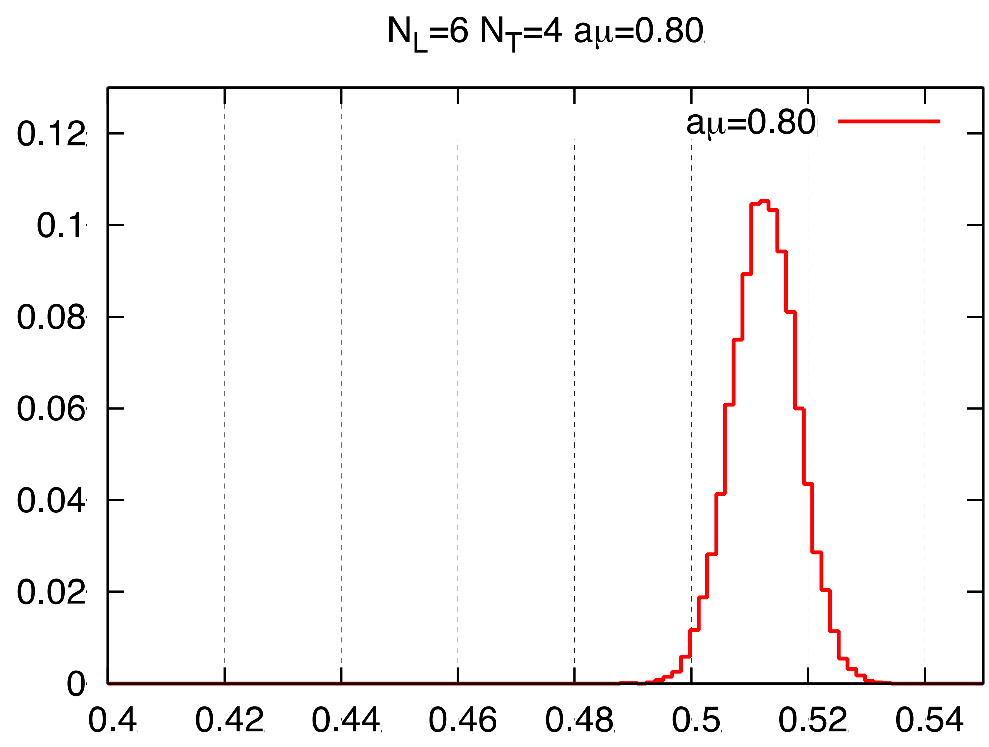


Technical remarks

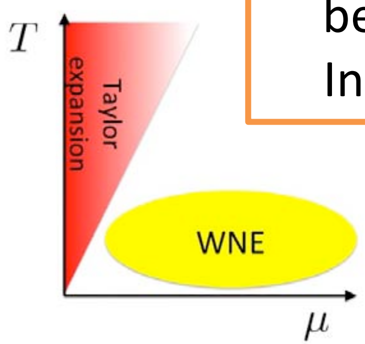
- Reduction technique [Danzer et al., (2008)] + Original further reduction technique
  - Inverse matrix (rank $\propto N_L{}^3$) & matrix-matrix product ➠ LAPACK
  - Cost $\propto (N_L{}^3)^3 \times N_T$
  - Memory size $\propto (N_L{}^3)^2$ independent of N_T
- Machine: T2K-Tsukuba

Summary

On the phase

- Winding number expansion is a good approximation for larger N_T
- The phase can be controlled for large N_T

We roughly observe such behavior numerically In heavy mass region



Reweighting

- N_f =4 at μ/T=0.8 (T=150MeV), we observe 1st order like behavior
- We plan N_f =3 or 2+1 with finite size scaling (Y.Nakamura and X-Y. Jin)

Other use of the WNE?

Far future plan

$$Z_{\mathrm{QCD}}(\mu) = \int [dU] \mathrm{Re}[\det D(\mu)] e^{-S_{\mathrm{G}}}$$

$$= \int [dU] \underbrace{\frac{\mathrm{Re}[\det D(\mu)]}{|\mathrm{Re}[\det D(\mu)]|}}_{\text{observable}} \underbrace{|\mathrm{Re}[\det D(\mu)]| e^{-S_{\mathrm{G}}}}_{\text{weight}}$$

- Real part
  => HMC hamiltonian
  => WNE to get analytic form of MD force
  => approximation can be corrected by computing determinant once per HMC trajectory
- Observable
  => sign problem
  => For large N_T, fluctuation should be small

Constant physics with rescaling cut off

Constant physics

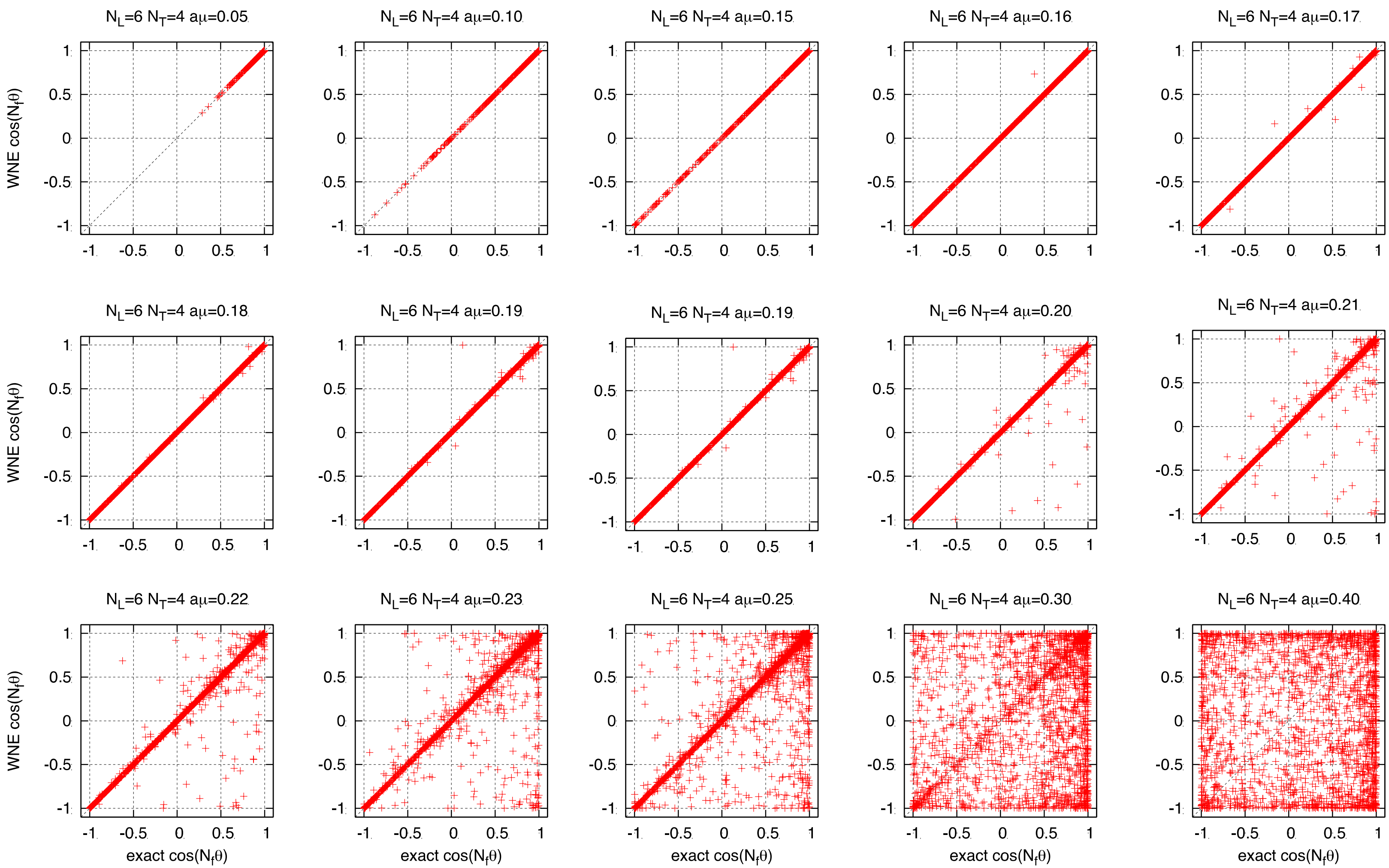
$$
\begin{aligned}
T &= 1/aN_{\mathrm{T}} \\
\mu & \\
V &= L^3 = (aN_{\mathrm{L}})^3 \\
m &
\end{aligned}
$$

Rescaling by factor b

$$
\begin{aligned}
a &\longrightarrow a/b \\
a\mu &\longrightarrow a\mu/b \\
N_{\mathrm{T}} &\longrightarrow bN_{\mathrm{T}} \\
N_{\mathrm{L}} &\longrightarrow bN_{\mathrm{L}} \\
\kappa &\longrightarrow \kappa' \approx \kappa(am \longrightarrow am/b)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\text{Bound of } |\theta|_{\text{after}}}{\text{Bound of } |\theta|_{\text{before}}} &= \frac{12(bN_{\mathrm{L}})^3(2\kappa')^{bN_{\mathrm{T}}}\sinh(\mu/T)}{12(N_{\mathrm{L}})^3(2\kappa)^{N_{\mathrm{T}}}\sinh(\mu/T)} \\
&= b^3(2\kappa)^{N_{\mathrm{T}}(b-1)}
\end{aligned}
$$

Exact vs. WNE for reweighting factor



WNE for physical quantities