J/ψ-Φ interaction and Υ(4140) on the lattice QCD

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Recently many charmonium($c\bar{c}$) like particles XYZ are observed in several big facilities in the world.

Among them, some Y resonances have interesting features.

1) Although these resonances are heavy, these are very stable. Widths are quite narrow as compared to typical hadron resonances.

2) Open charm channel decays seem to be suppressed.
Recently many charmonium($c\bar{c}$) like particles $XYZ$ are observed in several big facilities in the world.

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2) Open charm channel decays seem to be suppressed.
Example 1) Initial State Radiation (ISR)-produced \( \Upsilon \) families (including \( \Upsilon(4260) \)):
Example 2) $Y(4140)$.  

$$B \rightarrow J/\psi\phi K$$  

$Y(4140)$  

$M_Y = 4143.0 \pm 2.9 \pm 1.2\text{MeV}$  

$\Gamma_Y = 11.6^{+8.3}_{-5.0} \pm 3.7\text{MeV}$ quite narrow width

$23\text{MeV} \sim$  

$60\text{MeV} \sim$  

$200\text{MeV} \sim$  

$Y(4140)$  

$J/\psi\phi$  

$D_s^+ D_s^*$  

$D_s^+ D_s^-$  

It seems that some $Y$ states do not couple to open charm channels.  

Is there a specific selection rule? 

These features should be related to the structure of $Y$ states, and charmonium($J/\psi$)-hadron interactions.
Example 2) Y(4140).

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\[ Y(4140) \]

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? \[ D_s^+ D_s^- \]

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These features should be related to the structure of Y states, and charmonium (J/Ψ)-hadron interactions.
Charmonium-hadron interactions

\[ J/\psi \] (hadron) \rightarrow \pi, \rho \text{ exchanges are forbidden!}

vacuum quantum number
Charmonium-hadron interactions

So if there exist a $(\Upsilon)$ resonance in charmonium-hadron system, gluons would play very interesting role!

Non-perturbative method such as lattice QCD is really needed to study this system.
$J/\psi$-Φ scattering and $Y(4140)$

$E \sim 20 \text{ MeV}$

low energy scattering

with

$J^P = (0^\pm, 1^\pm, 2^\pm)$ channels

Today, we focus on s-wave: $J^P = (0^+, 1^+, 2^+)$
Finite size method

Finite size formula

\[ \tan \delta_0(k) = \frac{\pi^{3/2}q}{Z_{00}(1, q^2)} \quad \text{where} \quad q = \frac{Lk}{2\pi} \]

Generalized zeta-function

\[ Z_{00}(s, q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbb{Z}^3} (\vec{n}^2 - q^2)^{-s} \]

Finite size formula is the relation which connects energy eigenvalue in a finite volume with scattering phase shift in an infinite volume.

This method successfully describe \( \rho \) meson from \( \pi-\pi \) scattering.

S. Aoki et al (CP-PACS) PRD 76, 094506 (07)
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- Finite size formula is the relation which connects energy eigenvalue in a finite volume with scattering phase shift in an infinite volume.

- We would like to successfully describe \( Y(4140) \) from \( J/\psi-\Phi \) scattering in terms of the finite size method.
\[ k = \sqrt{2\mu \Delta E} \]

\[
\Delta E = E_{J/\psi} - (M_{J/\psi} + M_\phi)
= [E_{J/\psi} - (E_{nJ/\psi} + E_{n\phi})] + [E_{nJ/\psi} - M_{J/\psi}] + [E_{n\phi} - M_\phi]
\]

\[
\Delta E = \delta E_n + \epsilon_{nJ/\psi} + \epsilon_{n\phi}
\]

Interaction strength \hspace{1cm} Energy of free 2 particles

\[
E_{nV} = \sqrt{\left(\frac{2\pi}{L} n\right)^2 + M_V^2}
\]

\[
\begin{align*}
n = 2 & \quad \delta E_2 = \epsilon_{2J/\psi} + \epsilon_{2\phi} \\
n = 1 & \quad \delta E_1 = \epsilon_{1J/\psi} + \epsilon_{1\phi} \\
n = 0 & \quad \delta E_0 = 0 : J/\psi - \phi \hspace{1cm} \text{threshold}
\end{align*}
\]
Measurement of $\delta E_n$

Two-point function

\[ G^\phi(t, t_{src}) = \langle \hat{O}_\phi(t) \hat{O}_\phi^\dagger(t_{src}) \rangle \]
\[ G^{J/\psi}(t, t_{src}) = \langle \hat{O}_{J/\psi}(t) \hat{O}_{J/\psi}^\dagger(t_{src}) \rangle \]

with \[ \hat{O}_\phi(t) = \bar{s}(t) \gamma_i \bar{s}(t) \]
\[ \hat{O}_{J/\psi}(t) = \bar{c}(t) \gamma_i \bar{c}(t) \]

Four-point function

\[ G^{J/\psi-\phi}(t, t_{src}) = \langle \hat{O}_\phi(t) \hat{O}_{J/\psi}(t) [\hat{O}_\phi(t_{src}) \hat{O}_{J/\psi}(t_{src})] \dagger \rangle \]

\[
\frac{G^{J/\psi-\phi}(t, t_{src})}{G^{J/\psi}(t, t_{src}) G^\phi(t, t_{src})} \sim e^{-\delta E_n t}
\]

\[
\delta E_n = \begin{pmatrix}
\phi \\
\hline
\end{pmatrix}
\begin{pmatrix}
\hline
J/\psi
\end{pmatrix}
- \begin{pmatrix}
J/\psi
\hline
\end{pmatrix}
+ \begin{pmatrix}
\phi
\hline
\end{pmatrix}
\]
Twisted Boundary Condition

Periodic Boundary Condition

\[ \phi(\vec{x} + L\vec{e}_i) = \phi(\vec{x}) , \quad i = x, y, z \]

\[ \vec{k} = \frac{2\pi}{L} \vec{n} \]

\[ E_1 = k_1^2 / 2\mu = 115 \text{ MeV} \quad \rightarrow \quad \text{Bad resolution} \]

Twisted Boundary Condition

\[ \phi(\vec{x} + L\vec{e}_i) = e^{i\theta_i} \phi(\vec{x}) \]

\[ \vec{k} = \frac{2\pi}{L} (\vec{n} + \vec{d}) , \quad \vec{d} = \left( \frac{\theta_x}{2\pi}, \frac{\theta_y}{2\pi}, \frac{\theta_z}{2\pi} \right) \]
Lattice set up

- PACS-CS 2+1 flavor dynamical gauge configurations at \( m_\pi = 156 \text{ MeV} \)  
  - 32\(^3\) x 64 lattice
  - \( a = 0.0907(13) \text{ fm} \)
  - \( L_a \sim 2.9 \text{ fm} \)
  - 198 configs
  - \( \kappa_s = 0.13640 \)
  - Wall source

- Relativistic Heavy Quark (RHQ) action for charm
  - Tsukuba type RHQ action (5 parameters)
    
    \[
    \begin{array}{|c|c|c|c|c|}
    \hline
    \kappa_{\text{charm}} & \nu & r_s & C_B & C_E \\
    \hline
    0.1082 & 1.2153 & 1.2131 & 2.0268 & 1.7911 \\
    \hline
    \end{array}
    \]
\[
\Delta E = E - (M_{J/\psi} + M_{\phi}) \text{ MeV}
\]

\(\delta E\) vs \(\Delta E\):

- The J/\(\psi\)-\(\Phi\) interaction is attractive.
- The strength of the interaction is \(E\)-independent.
Scattering phase shift

$J = 0$

$\Delta E = E - (M_{J/\psi} + M_{\phi})$ MeV
No structure in $s$-wave $J/\psi$ system.

These seem typical $s$-wave behaviors: $\delta_l \sim (\Delta E)^{l+\frac{1}{2}}$

No structure in $s$-wave $J/\psi$-$\Phi$ system.
$J = 1$

$J = 2$

\[ \Delta E = E - (M_{J/\psi} + M_\phi) \text{ MeV} \]

These seem typical s-wave behaviors: \( \delta_l \sim (\Delta E)^{l+\frac{1}{2}} \)

No structure in s-wave J/\( \psi \)-\( \Phi \) system.

We will study P-wave J/\( \psi \)-\( \Phi \) scattering as a next step. [ Our homework ]
Scattering lengths

\[
\begin{align*}
    a^{J=0}_{J/\psi-\phi} &= -0.151(20) \text{ fm} \\
    a^{J=1}_{J/\psi-\phi} &= -0.130(18) \text{ fm} \\
    a^{J=2}_{J/\psi-\phi} &= -0.109(18) \text{ fm}
\end{align*}
\]

Quenched results

\[
\begin{align*}
    a^{J=0}_{J/\psi-\phi} &= -0.178(21) \text{ fm} \\
    a^{J=1}_{J/\psi-\phi} &= -0.152(23) \text{ fm} \\
    a^{J=2}_{J/\psi-\phi} &= -0.123(16) \text{ fm}
\end{align*}
\]

Quenched results and full QCD results are quite close within 1\sigma.

\[\tan\delta_0 \bigg|_{k\to 0} = a_{J/\psi-\phi} \]

Our definition

\[a_{J/\psi-\phi} < 0: \text{ attractive}\]

This reflects that J/\psi-\Phi system is indeed governed by gluon-dynamics.
Charmonium

Φ meson

$ss$

$cc\bar{c}$
Charmonium

$\Phi$ meson

$s\bar{s}$

Bottomonium

$cc$

$bb$
Results are preliminary.
Result of $\delta E$

$J = 0$

$J/\psi - \gamma$

$\phi - \gamma$

$\delta E$

$\Delta E$ MeV

Preliminary
Summary

We investigate the low energy s-wave $J/\psi-\Phi$ scattering with PACS-CS 2+1 dynamical gauge configurations ($m_\pi=156$ MeV).

Their interactions are attractive, but no E-dependence.

In terms of the finite size method, we calculate scattering phase shifts, but there is no resonance in s-wave systems.

We also calculate scattering lengths, and compare with quenched results.

Gluon-dynamics seem important in this system.

Bottomonium scatterings are now ongoing.
Prospects

- As a next step we will perform P-wave calculations \((J^P = (0^-, 1^-, 2^-))\) and search \(Y(4140)\) resonance.

  If \(Y(4140)\) resonance exist on the lattice...

- Determine parity and spin of the resonance.

- Construct \(J/\psi-\Phi\) potential, and investigate the structure of \(Y(4140)\) resonance in term of its BS wave function.