格子上のベーテ･サルピータ波動関数に基づくクォーク間ポテンシャル

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Refs.) Y.Ikeda and H.Iida, arXiv:1102.2097.
Y.Ikeda and H.Iida, PoS LATTICE2010 (2010) 143.

「素核宇融合による計算基礎物理学の進展」
– ミクロとマクロのかけ橋の構築 –
2011年12月3日（土）–5日（月） @合歓の郷

Contents

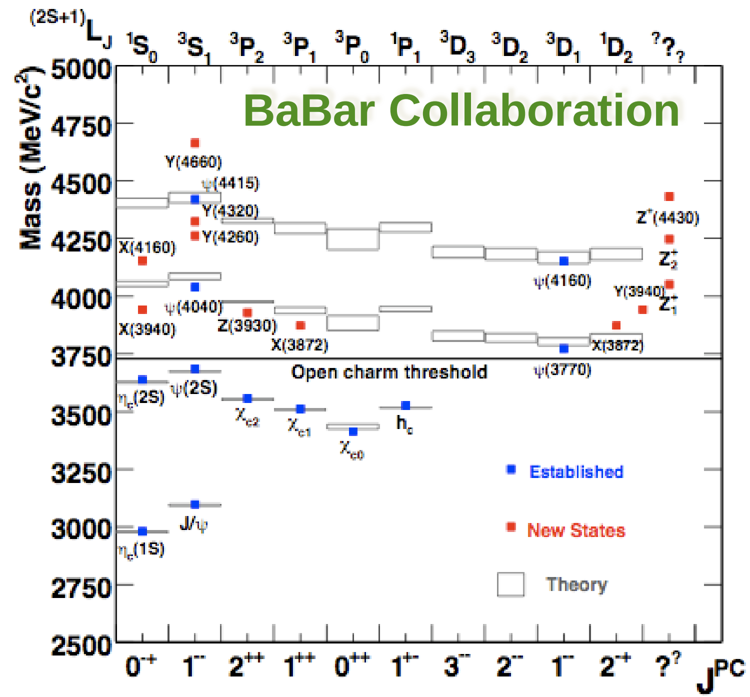
- Introduction
- Our approach to define interquark potentials for finite quark mass on lattice
- Results
- Summary and future plans

Heavy quark system

- Quark potential models well describe mass spectra below open charm threshold

  Godfrey, Isgur, PRD 32 (1985).
  Barnes, Godfrey, Swanson, PRD 72 (2005).

- Exotic states (X, Y, Z) can be expected as non-standard c^{bar}-c mesons



Important information is T-matrix elements based on QCD

$$T = V + VGT$$

Interaction part is determined from QCD

… First principle calculation is important

Q^{bar}-Q interquark potential

- Q^{bar}-Q potentials can be expected having the following form:

$$V_{\bar{Q}Q}(r) = \boxed{\sigma r - \frac{4}{3}\frac{\alpha_s}{r}} + \boxed{V_{\rm spin}(r)\vec{S}_{\bar{Q}}\cdot\vec{S}_{Q} + V_{T}(r)\hat{S}_{12} + V_{LS}(r)\vec{L}\cdot\vec{S}} + \cdots$$

Spin-independent Spin-dependent

☑ Effective field theory approach (pNRQCD) for charmonium spectra :
Wilson loop + relativistic correction ($1/m_Q$, v (velocity), $1/m_Q v$ expansion)

Bali, Phys. Rept. 343 (2001).
Brambilla, Pineda, Soto, Vairo, NPB 566 (2000); Rev. Mod. Phys. 77 (2005).
Koma et al., PRL 97 (2006).
Koma et al., NPB 769 (2007).

☑ Our approach through Nambu-Bethe-Salpeter (NBS) amplitude :
We define effective inter-quark potential with finite quark mass.

Lin et al., NPB 619 (2001).
Aoki, Hatsuda, Ishii, PTP 123 (2010).

Reliable input based on QCD for quark potential models can be extracted

How to define quark model on the lattice

Y.Ikeda., H.I., arXiv:1102.2097[hep-lat](2011).

- **Homogenous Nambu-Bethe-Salpeter equation :**

$$\boxed{\tilde{\chi}_E(p^0,\mathrm{p}) = G(p;P)\int d^4p' K(p,p';P)\tilde{\chi}_E(p'^0,\mathrm{p}')}$$

P : meson 4-momentum P = (E, 0) = (M_{meson}, 0) at meson-rest frame
p, p' : relative 4-momentum of Q^{bar}-Q system
K(p,p';P) : irreducible kernel
G(p;P) : product of quark propagator w/ assumption of constant quark mass m_Q

- **Non-relativistic reduction through Levy-Klein-Macke (LKM) method :**

Reviewed in Klein, Lee, PRD 10 (1974).

$$\boxed{\tilde{\psi}_E(\mathrm{p}) = \frac{1}{2\pi i}\int dp^0\left[(p^0 - P^0/2 + \epsilon(\mathrm{p}) - i\delta)^{-1} + (p^0 \to -p^0)^{-1}\right]\tilde{\chi}_E(p^0,\mathrm{p})}$$

-> Equal-time Nambu-Bethe-Salpeter amplitude = Nambu-Bethe-Salpeter wave function

- **Nambu-Bethe-Salpeter wave functions satisfy Schrödinger-type equation :**

$$\boxed{(E - 2\epsilon(\mathrm{p}))\tilde{\psi}_E(\mathrm{p}) = \int d\mathrm{p}'\tilde{U}(\mathrm{p},\mathrm{p}')\tilde{\psi}_E(\mathrm{p}')}$$

$$\epsilon(\mathrm{p}) = \sqrt{m_Q^2 + \mathrm{p}^2}$$

Note :
Potential is non-local but energy-independent below open charm threshold because of Krolikowski-Rsewuski relation

Krolikowski, Rzewuski, Nuovo Cimento, 4 (1956).

q^{bar}-q interquark potential on the lattice

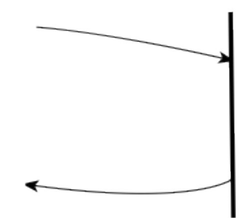
Aoki, Hatsuda, Ishii, PTP 123 (2010).
Ikeda, Iida, arXiv:1102.2097[hep-lat](2011).

1. Measure equal-time Nambu-Bethe-Salpeter wave function

$$\boxed{\phi_E(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E; J^{PC}\rangle}$$

Spatial correlation of 4-point function

$$\begin{aligned} G^{(2)}(\mathbf{r}, t - t_{\mathrm{src}}) &= \sum_{\mathbf{x},\mathbf{X},\mathbf{Y}} \langle 0|\bar{q}(\mathbf{x},t)\Gamma q(\mathbf{x}+\mathbf{r},t)\left(\bar{q}(\mathbf{X},t_{\mathrm{src}})\Gamma q(\mathbf{Y},t_{\mathrm{src}})\right)^{\dagger}|0\rangle \\ &= \sum_{\mathbf{x}} \sum_{E_n} A_{E_n} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E_n\rangle e^{-E_n(t-t_{\mathrm{src}})} \\ &\to A_{E_0}\phi_{E_0}(\mathbf{r})e^{-E_0(t-t_{\mathrm{src}})} \qquad (E_0 = M, \ \ t \gg t_{\mathrm{src}}) \end{aligned}$$



2. Define potential through Schrödinger-type equation

$$\boxed{(E - H_0)\phi_E(\mathbf{r}) = \int d^3r' U(\mathbf{r},\mathbf{r}')\phi_E(\mathbf{r}')}$$

3. Velocity expansion of non-local potential

$$U(\mathbf{r},\mathbf{r}') = (\underbrace{V_{\mathrm{C}}(r) + V_{\mathrm{spin}}(r)\vec{S}_{\bar{\mathrm{Q}}}\cdot\vec{S}_{\mathrm{Q}} + V_{\mathrm{T}}(r)\hat{S}_{12}}_{\text{Leading order}} + \underbrace{V_{\mathrm{LS}}(r)\vec{L}\cdot\vec{S}}_{\text{NLO}} + \cdots)\delta(\mathbf{r}-\mathbf{r}')$$

We examine s-wave "effective LO potentials" in pseudo-scalar and vector channels

LQCD setup

Y.I., Iida, arXiv:1102.2097[hep-lat](2011).

- Quench QCD simulation

- Plaquette gauge action & Standard Wilson quark action
- β=6.0 (a=0.104 fm, a^{-1}=1.9GeV)
- Box size : 32^3 x 48 -> L=3.3 (fm)
- Four different hopping parameters (κ=0.1320, 0.1420, 0.1480, 0.1520)

  -> M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)
- N_{conf}=100

- Wall source
- Coulomb gauge fixing

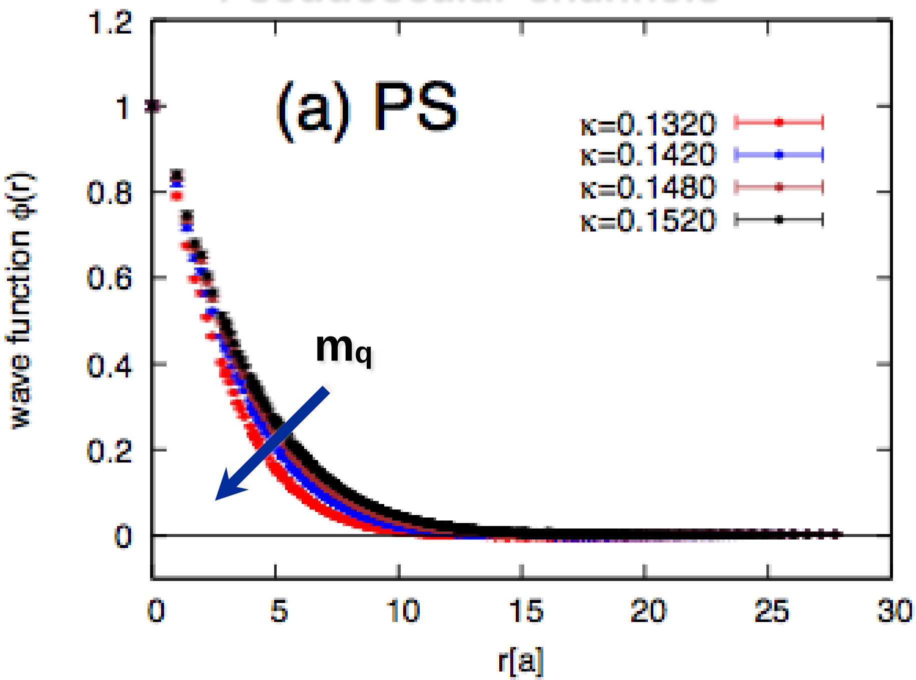
q^{bar}-q wave function

NBS wave functions (channel & quark mass dependence)

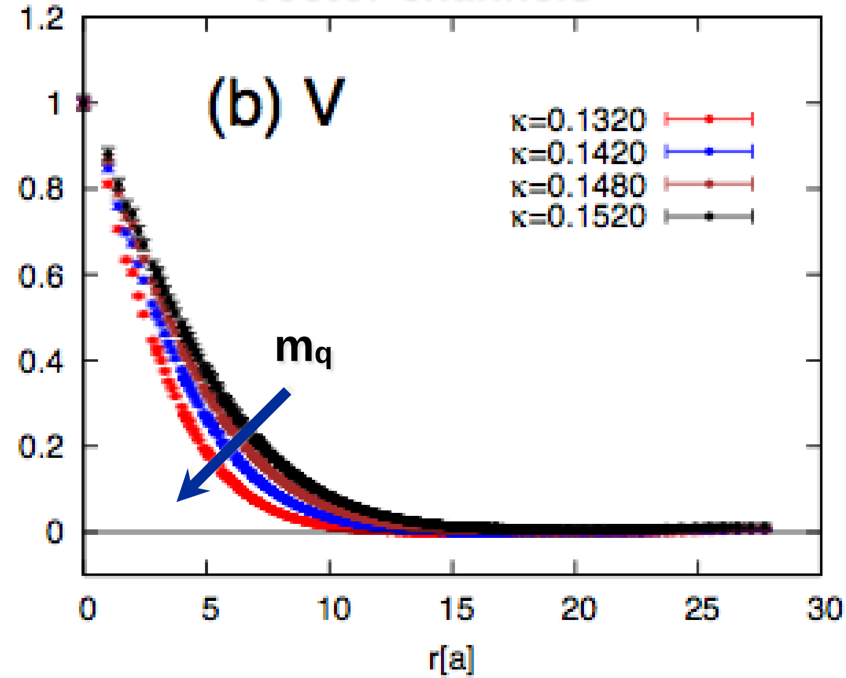
M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)

$$\phi_E(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})\Gamma q(\mathbf{x}+\mathbf{r})|E; J^{PC}\rangle$$

Pseudoscalar channels



Vector channels



Channel dependence appears in light quark sector

-> spin-spin interaction is enhanced

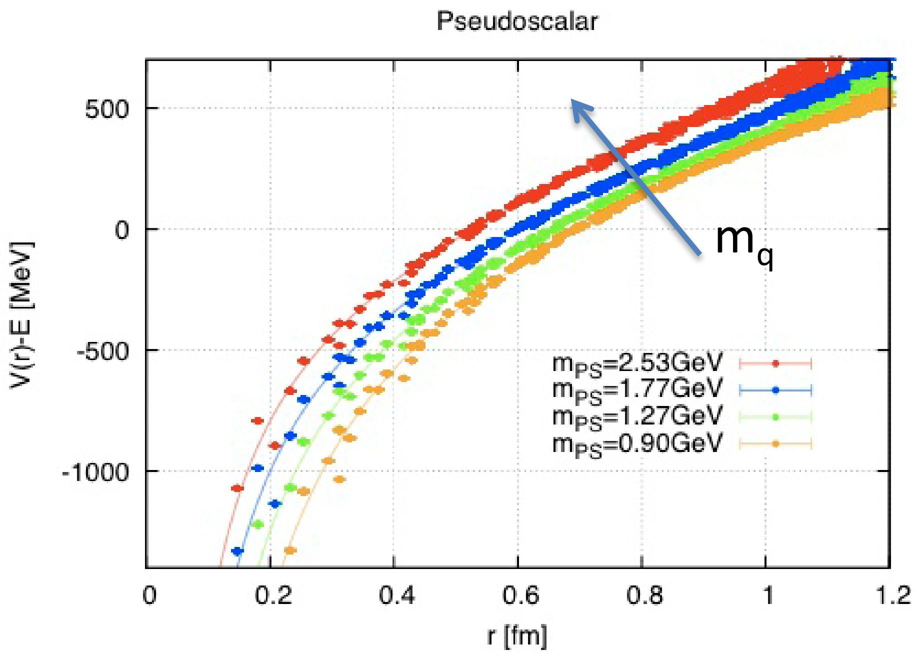
Size of wave function becomes smaller as increasing m_q

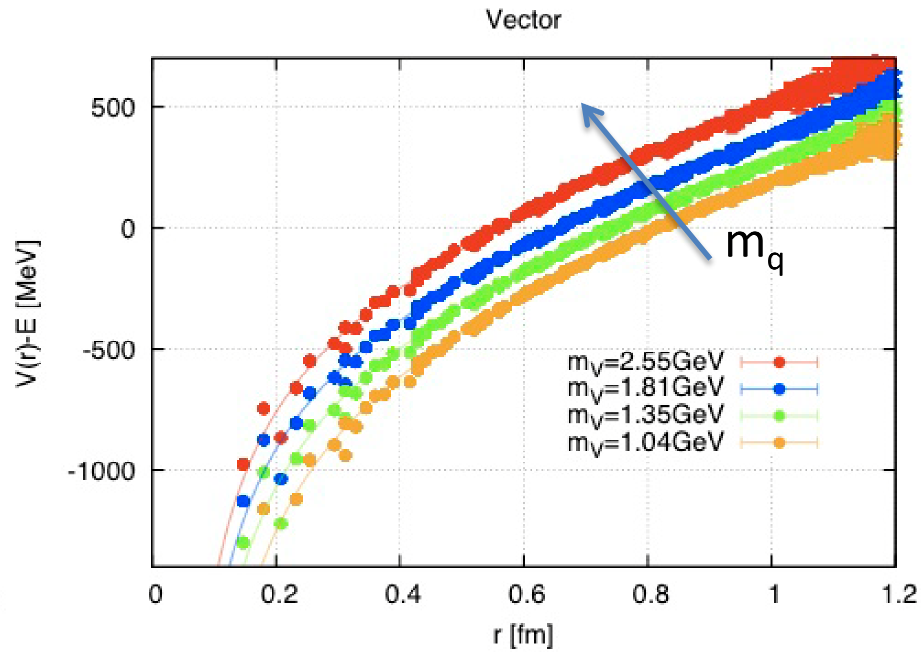
q^{bar}-q potential from NBS wave functions

Interquark potentials with various finite quark masses

M_{PS}=2.53, 1.77, 1.27, 0.94 (GeV), M_V=2.55, 1.81,1.35, 1.04 (GeV)

$$V(\mathbf{r}) - E = \frac{1}{m_q}\frac{\nabla^2\phi(\mathbf{r})}{\phi(\mathbf{r})} \qquad m_q = m_V/2$$



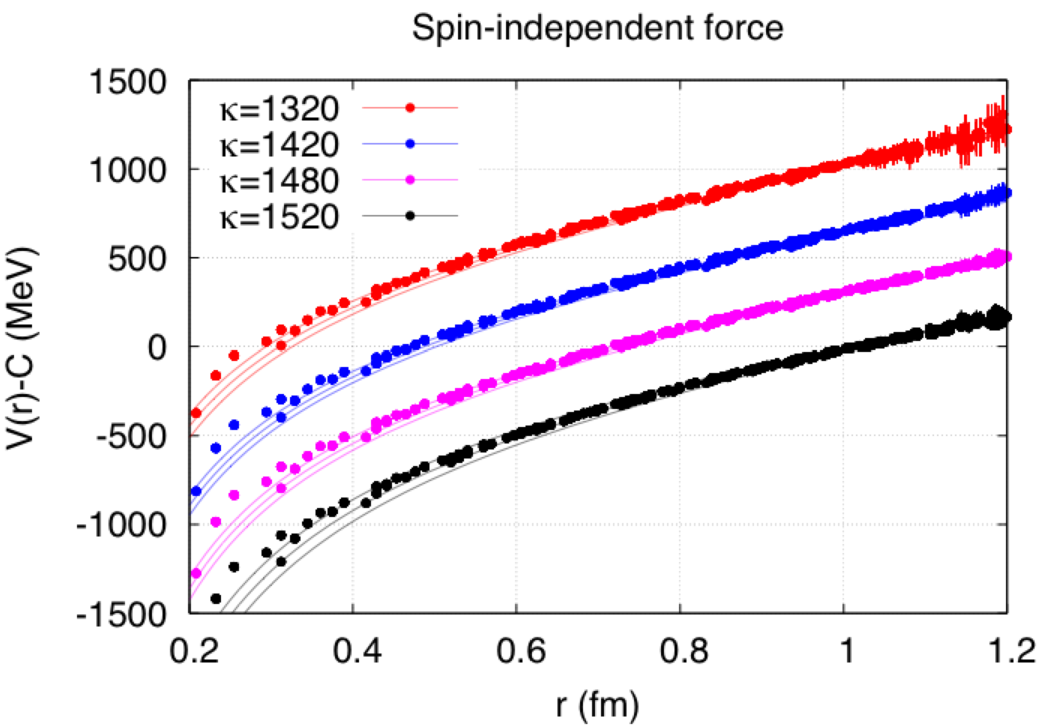


Coulomb + linear confinement forces are reproduced with finite quark masses

(solid curves representing Coulomb + linear functions)

Fitting results of q^{bar}-q potential

$$V^{\mathrm{eff}}_{\mathrm{spin-indep.}}(\mathbf{r}) - E = \frac{1}{m_q}\left[\frac{1}{4}\frac{\nabla^2\phi_{\mathrm{PS}}(\mathbf{r})}{\phi_{\mathrm{PS}}(\mathbf{r})} + \frac{3}{4}\frac{\nabla^2\phi_{\mathrm{V}}(\mathbf{r})}{\phi_{\mathrm{V}}(\mathbf{r})}\right]$$



fit function: $V(\mathbf{r}) = \sigma r - \frac{A}{r} + C$

M_V (GeV)	σ (MeV/fm)	A (MeV fm)
2.55	822 (49)	200 (7)
1.87	766 (38)	228 (6)
1.35	726 (39)	269 (7)
1.04	699 (57)	324 (12)

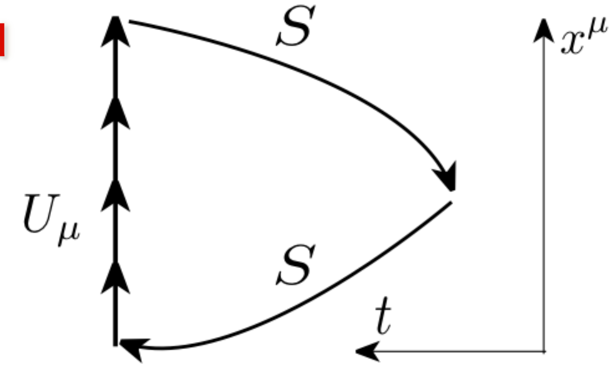
- String tension σ has moderate m_q dependences
- σ for the heaviest quark mass gives comparable value to that from Wilson loop
- Coulomb coefficients increase as decreasing m_q

see also, Kawanai and Sasaki, PRL 107 (2011).

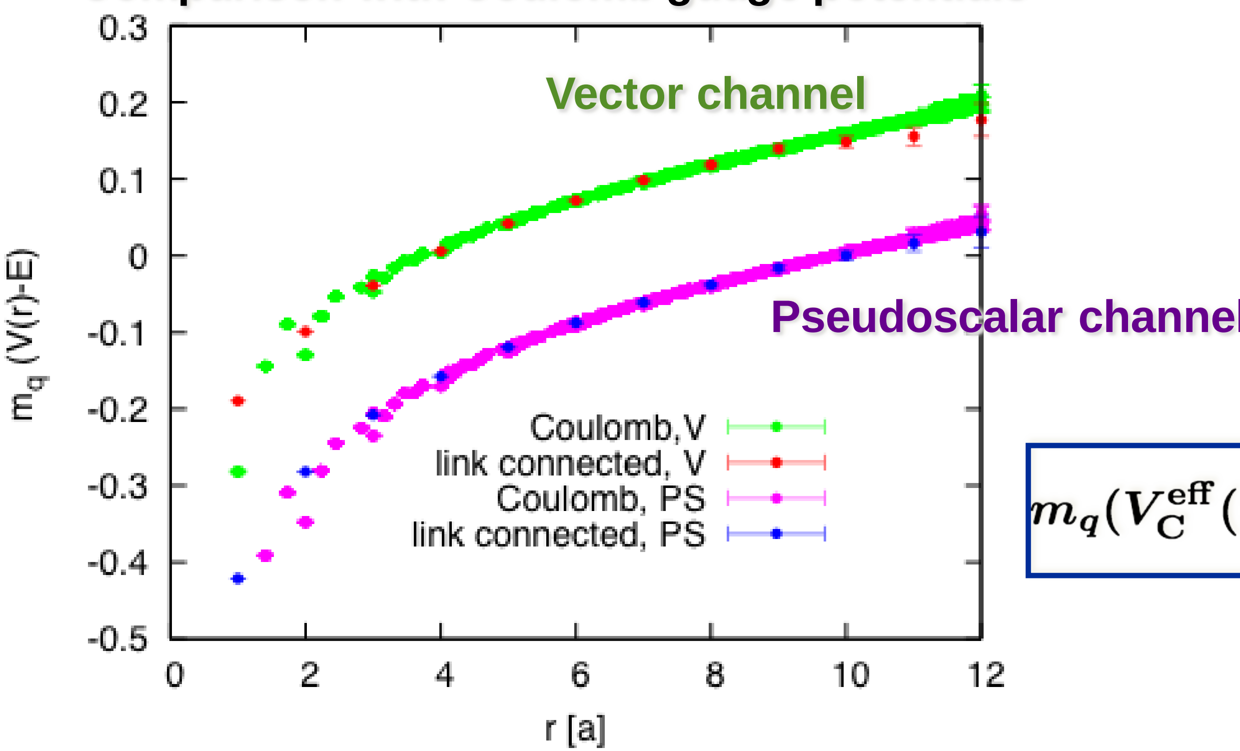
Operator dependence of q^{bar}-q potential

Operator dependence of inter-quark potential is studied by using gauge invariant smearing operator

$$\phi_E^{\text{smr.}}(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{q}(\mathbf{x})L(\mathbf{x},\mathbf{r})\Gamma q(\mathbf{x}+\mathbf{r})|E;J^{PC}\rangle$$



Comparison with Coulomb gauge potentials



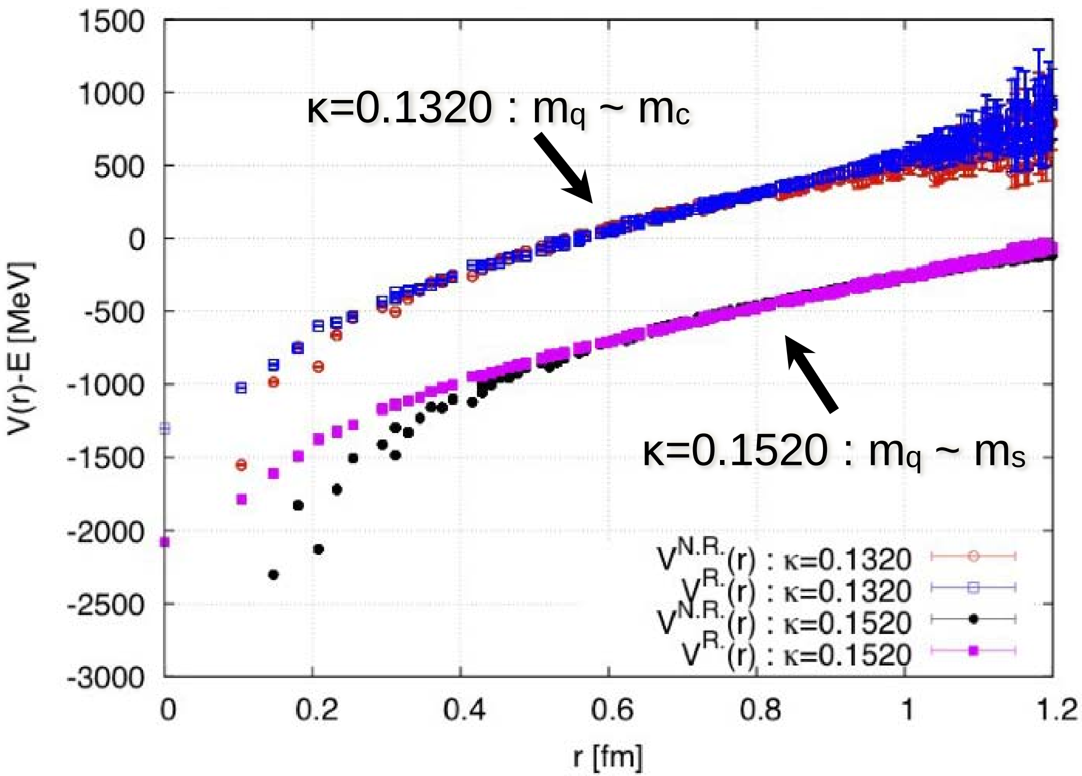
$$m_q(V_{\mathrm{C}}^{\mathrm{eff}}(\mathbf{r}) - E) = \frac{\nabla^2 \phi_E^{\text{smr.}}(\mathbf{r})}{\phi_E^{\text{smr.}}(\mathbf{r})}$$

The potentials obtained from gauge invariant smearing operators are comparable with Coulomb gauge potential

Relativistic kinematics

Inter-quark potentials with relativistic kinematics are studied

$$H_0\psi_E(\mathbf{r}) = \int d\mathbf{r}\Big(\int \frac{d\mathbf{p}}{(2\pi)^2} 2\sqrt{m_q^2 + \mathbf{p}^2} e^{-i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')}\Big)\psi_E(\mathbf{r}')$$



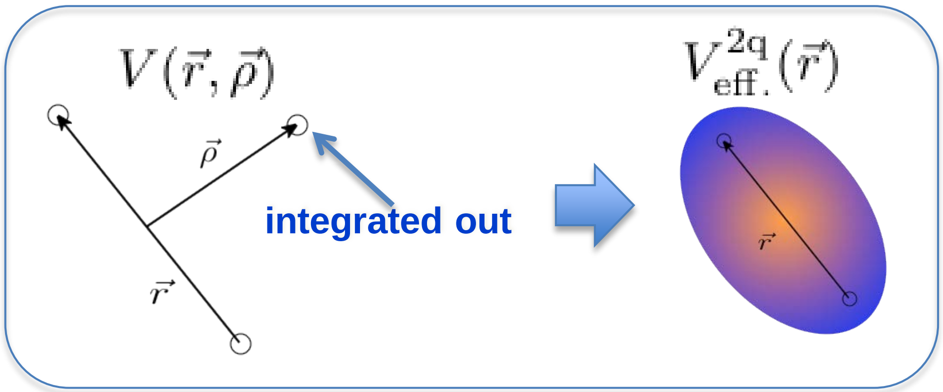
- Even for relativistic kinematics, Coulomb + linear potentials are obtained
- Long range parts of relativistic potentials are consistent with those of N.R. potentials
- For charmonium, non-relativistic kinematics is good enough
- In strangeness sector, non-locality of potentials gets to large, if one employs non-relativistic kinematics

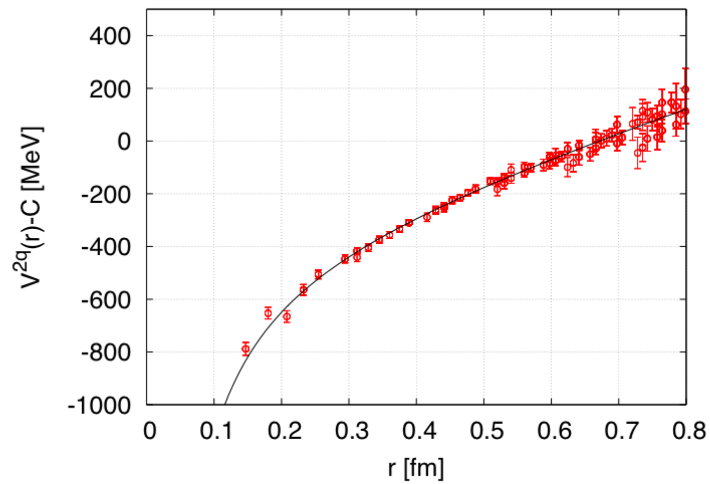
Some other systems (in progress)

- **Lattice QCD result of effective 2q potential…potential in baryons**

$$\boxed{V^{2q}_{\rm eff.}(\mathbf{r}) = \frac{1}{2\mu}\frac{\nabla^2_{\mathbf{r}}\psi^{2q}_E(\mathbf{r})}{\psi^{2q}_E(\mathbf{r})} + E}$$

$$\psi^{2q}_E(\mathbf{r}) = \sum_{\rho}\langle 0|\epsilon_{abc}(q^T_a(\mathbf{r}/2)C\gamma_5 q_b(-\mathbf{r}/2))q_{c,\alpha}(\rho)|B, J^P = 1/2^+\rangle$$



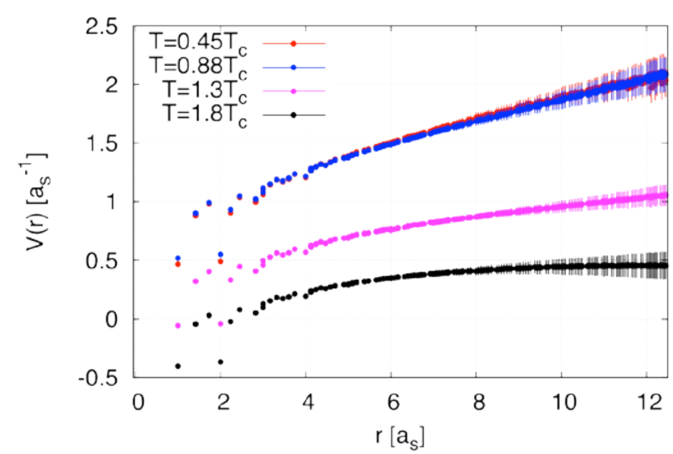


- **q^{bar}-q potential at finite temperature…related to charmonium dissociation**

Potential in Euclidean time is defined by

$$V(\tau,\vec{r}) = [-\partial_\tau + \Delta/2\mu - 2m]G_E(\tau,\vec{r})/G_E(\tau,\vec{r})$$

$$G_E(\tau,\vec{x}) \equiv \sum_{\vec{r}}\langle\bar{q}(\tau,\vec{r}+\vec{x})\Gamma q(\tau,\vec{r})\sum_{\vec{y},\vec{z}}\bar{q}(0,\vec{y})\Gamma q(0,\vec{z})\rangle$$



Summary

- ☑ We study inter-quark interactions with finite quark mass in quenched QCD simulation

- Effective central q^{bar}-q potentials from NBS amplitudes reveal Coulomb + linear forms
- Coulomb coefficients become smaller and smaller as increasing m_q
- String tension also has m_q dependence and is comparable with that of Wilson loop analysis with large m_q limit

Future plans

- ☑ Studies of tensor, LS, non-locality of inter-quark potential
- ☑ Three-quark potentials : ccc, ccs
- ☑ Coupled channel analysis toward above open charm threshold
- ☑ Investigation of exotic states (X, Y, Z)
- ☑ q^{bar}-q potential at finite temperatures … charmonium dissociation