# Evolution of Rotating Massive Stars

**Takashi Yoshida & Hideyuki Umeda**

吉田 敬, 梅田秀之

*Department of Astronomy, University of Tokyo*

素核宇融合による計算基礎物理学の進展

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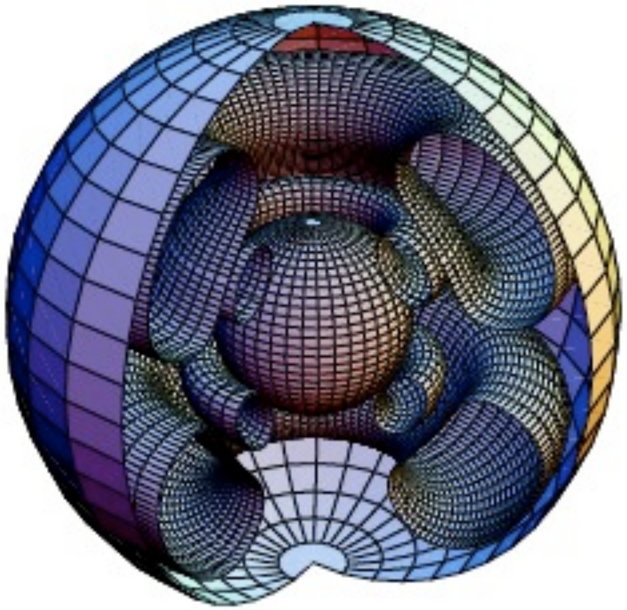
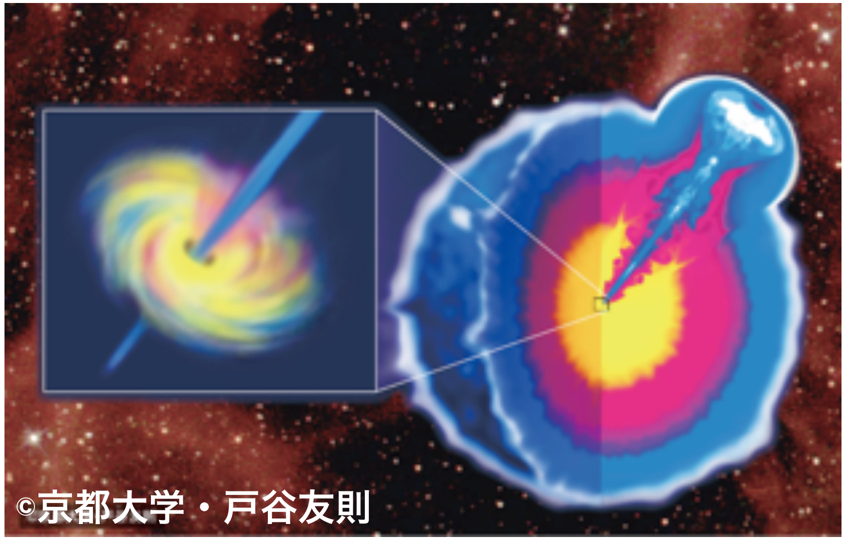
# Rotating Massive Stars

- Effects of rotation in stellar evolution
  - → Rotational mixing
  - Mass loss
  - Angular momentum distribution



(Meynet & Maeder 2002)




- Rotating massive stars
  - → Aspherical supernovae
  - *Collapsars* → Long GRBs

**Development of rotating massive star models**

# Massive Star Evolution Code

- **Code updated from Saio code**

  (Saio, Nomoto, & Kato 1988; Umeda & Nomoto 2008)

  - **Nuclear reaction network and energy generation**

    ➡ **282 species of nuclei from *n*, *p*, to Br**

    **NSE approximation is NOT used.**

  - **Mass loss rates for OB stars, Red giants, and WR stars**
  - **Schwarzshild criterion for convection**

➡ **Wide range of mass and metallicity**

$M_{MS} \geqq 9\ M_\odot$ , $Z \geqq 0$

**Umeda-san's talk for $9 \leqq M_{MS} \leqq 11\ M_\odot$ stars**

**Okita-san's talk for $M_{MS} = 110\ M_\odot$ ($Z$=0.004) stars**

# Rotating Star Model

- **Mass coordinate as isobar $M_r \rightarrow M_P$**

  ➡ **Radius is determined from the volume enclosed by isobar surface**

  $$r(r_0,\theta) = r_0(1-AP_2(\cos\theta))$$

$$\frac{\partial P}{\partial M_P} = -\frac{GM_P}{4\pi r_P^4} f_P$$

$$\frac{\partial r_P}{\partial M_P} = \frac{1}{4\pi r_P^2 \bar{\rho}}$$

$$\frac{\partial \ln \bar{T}}{\partial \ln P} = \min(\nabla_{ad}, \nabla_{rad}\frac{f_T}{f_P})$$

$$\frac{\partial L_P}{\partial M_P} = \varepsilon_{nucl} - \varepsilon_\nu + \varepsilon_{grav}$$

$$\dot{M}(\omega) = \dot{M}(\omega=0)\ \left(\frac{1}{1-v/v_{crit}}\right)^{0.43}$$

$$r_P = \left(\frac{3}{4\pi}V_P\right)^{1/3}$$

$$f_P = \frac{4\pi r_P^4}{GM_P S_P}\frac{1}{<g^{-1}>}$$

$$f_T = \left(\frac{4\pi r_P^2}{S_P}\right)^2 \frac{1}{<g^{-1}><g>}$$

**$<g>$: effective gravity averaged in angular direction**

**(e.g., Endal & Sofia 1976, Meynet & Maeder 1997, Heger, Langer, & Woosley 2000)**

# Mixing and Angular Momentum Transport

- *Advection or diffusion?*

**Advection: Geneva stellar evolution code**

(e.g. Hirschi, Meynet, & Maeder 2004)

$$\bar{\rho}\frac{d}{dt}(r_P{}^2\omega)_{Mr} = \frac{1}{5r_P{}^2}\frac{\partial}{\partial r_P}\{\bar{\rho}r_P{}^4U(r_P)\} + \frac{1}{r_P{}^2}\frac{\partial}{\partial r_P}\{\bar{\rho}\nu_{\rm shear}r_P{}^4\frac{\partial\omega}{\partial r_P}\}$$

$u(r_P,\theta) = U(r_P)P_2(\cos\theta)$ Vertical velocity of meridional circulation

Approximation form of $U$ is described in Maeder & Zahn (1998).

**Diffusion: Kepler & STERN** (e.g., Heger, Langer, & Woosley 2000)

$$\frac{\partial\omega}{\partial t} = \frac{1}{i}\frac{\partial}{\partial M_P}\left\{(4\pi r_P{}^2\bar{\rho})^2\,\nu\,\frac{\partial\omega}{\partial M_P}\right\} - \frac{2\omega}{r_P}\left(\frac{\partial r}{\partial t}\right)_{M_P}\frac{1}{2}\frac{\partial\ln i}{\partial\ln r_P}$$

$\nu$ : Diffusion coefficient by convection and rotational instabilities

- **Solid rotation in convective layer is still in debate.**

(Potter, Tout, and Eldridge 2011)

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# Mixing and Angular Momentum Transport

- **Angular momentum transport**

$$\frac{\partial \omega}{\partial t} = \frac{1}{i}\frac{\partial}{\partial M_P}\left\{(4\pi r_P^2 \bar{\rho})^2 \nu \frac{\partial \omega}{\partial M_P}\right\} - \frac{2\omega}{r_P}\left(\frac{\partial r}{\partial t}\right)_{M_P}\frac{1}{2}\frac{\partial \ln i}{\partial \ln r_P}$$

- **Rotational mixing**

$$\frac{\partial X_n}{\partial t} = \frac{\partial}{\partial M_P}\left\{(4\pi r_P^2 \bar{\rho})^2 D \frac{\partial X_n}{\partial M_P}\right\} + \left(\frac{\partial X_n}{\partial t}\right)_{\mathrm{nucl}}$$

$i$: specific angular moment, $\nu$: turbulent viscosity

$$D = D_{\mathrm{conv}} + D_{\mathrm{semi}} + f_c(D_{\mathrm{DSI}} + D_{\mathrm{SHI}} + D_{\mathrm{SSI}} + D_{\mathrm{ES}})$$

$$\nu = D_{\mathrm{conv}} + D_{\mathrm{semi}} + D_{\mathrm{DSI}} + D_{\mathrm{SHI}} + D_{\mathrm{SSI}} + D_{\mathrm{ES}}$$

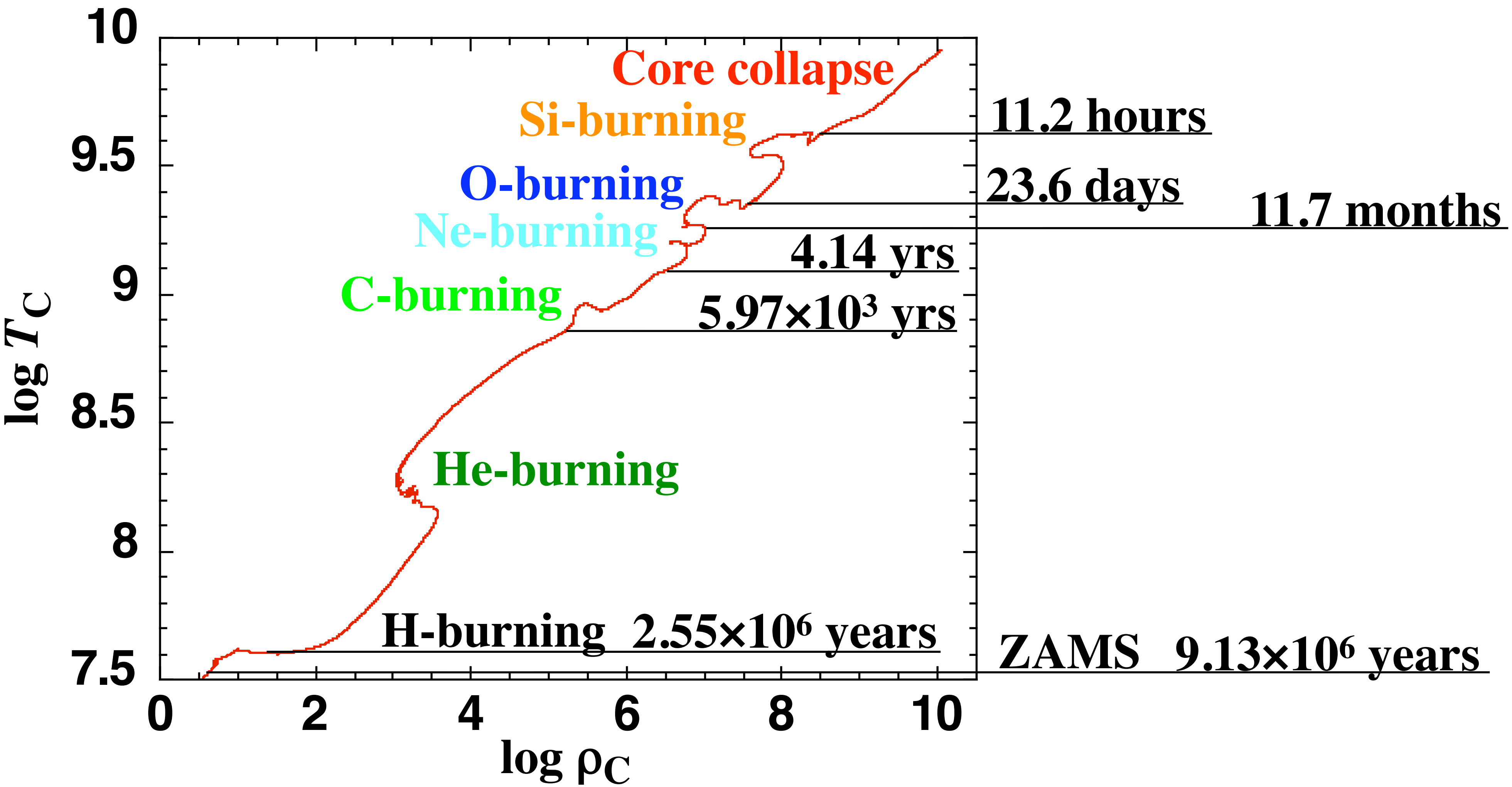
- Convection (Ledoux criterion)
- Semionvection
- Dynamical shear instability
- Solberg-Hoiland instability
- Secular shear instability
- Eddington-Sweet circulation

(e.g., Heger, Langer, & Woosley 2000)

# log $\rho_C$-log $T_C$ Diagram

- **Test calculations**

$M_{MS} = 20\ M_\odot$ , $Z = 0.02$, $V_{r0} = 200$ km s$^{-1}$



# $T$ and ρ Profiles

- **Test calculations**

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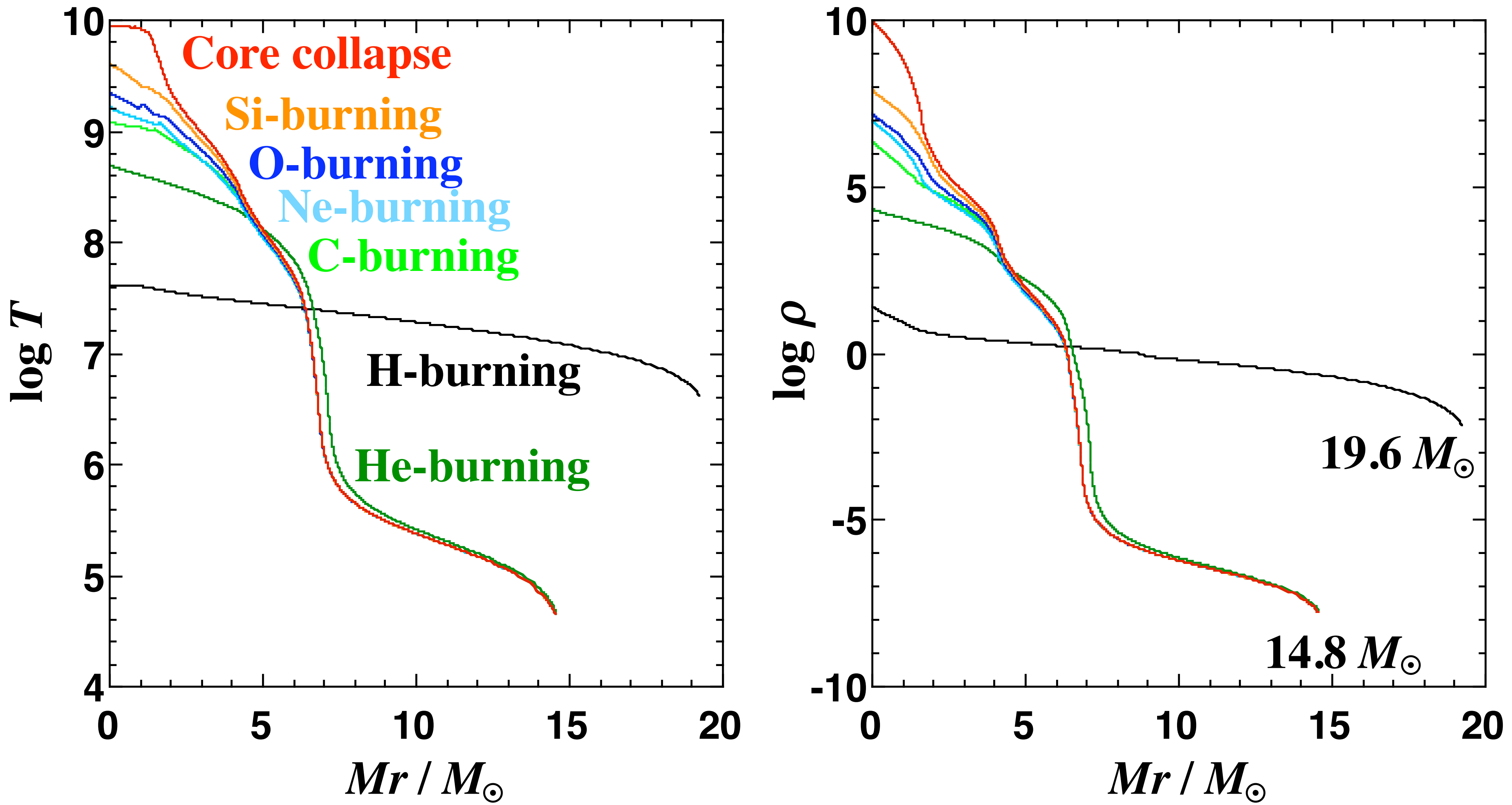
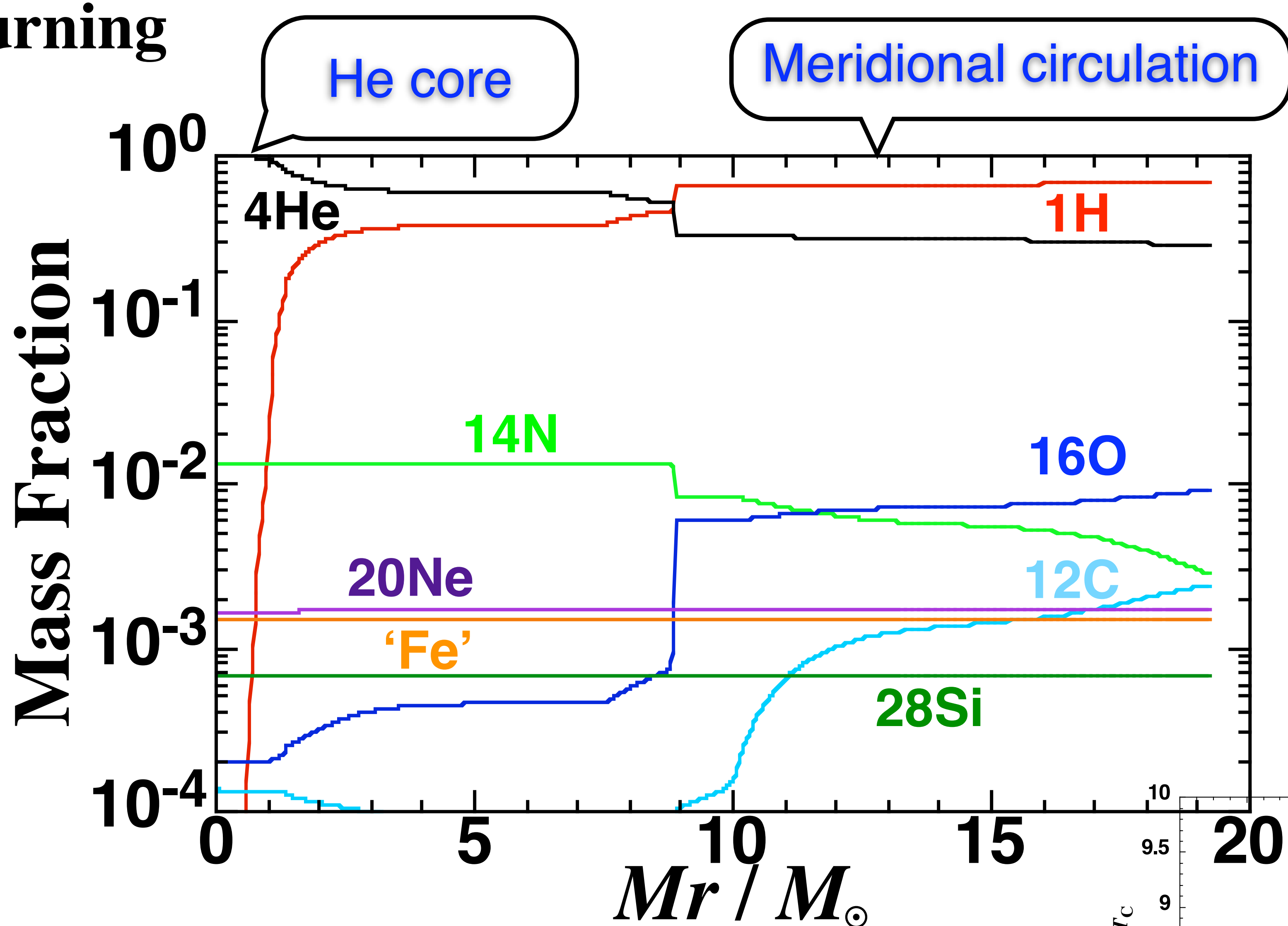
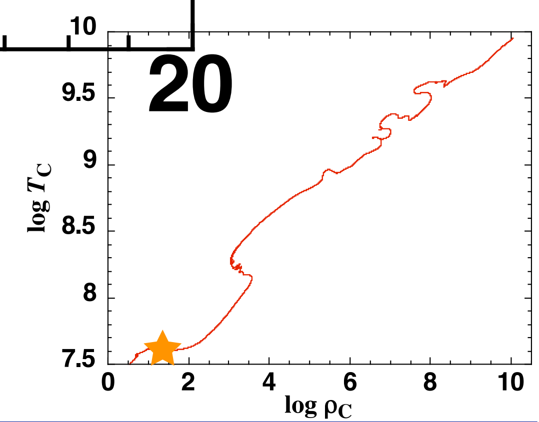


# Evolution of 20 $M_\odot$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

- H-burning



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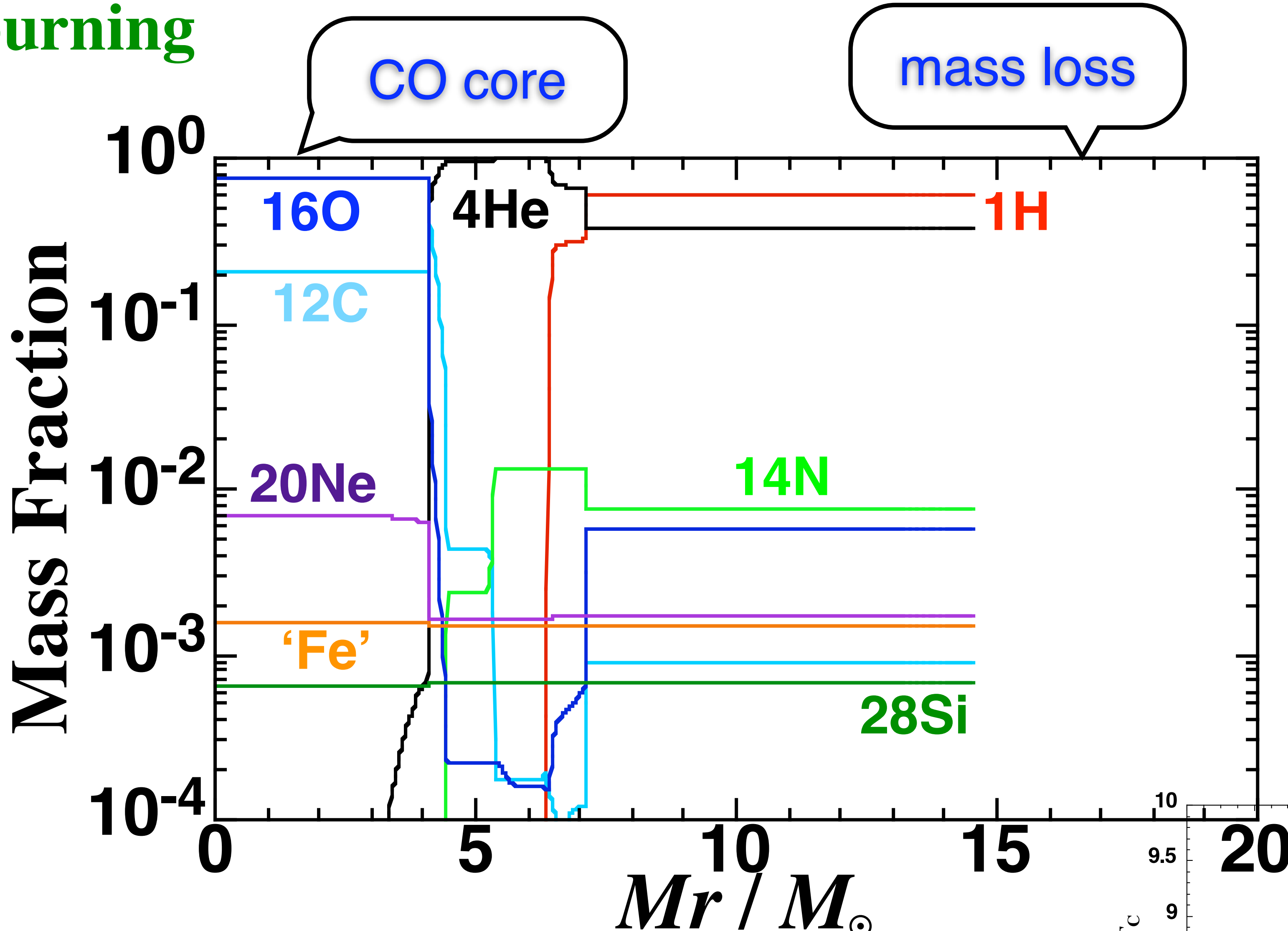
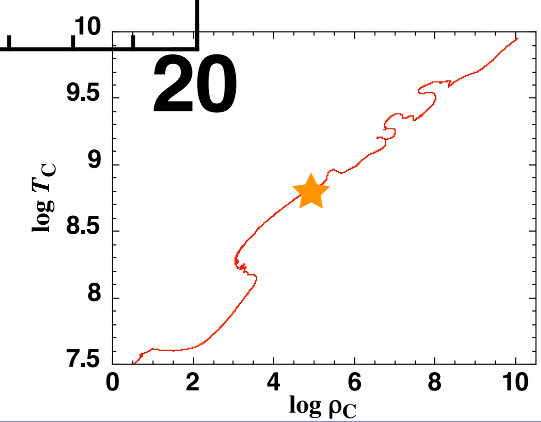
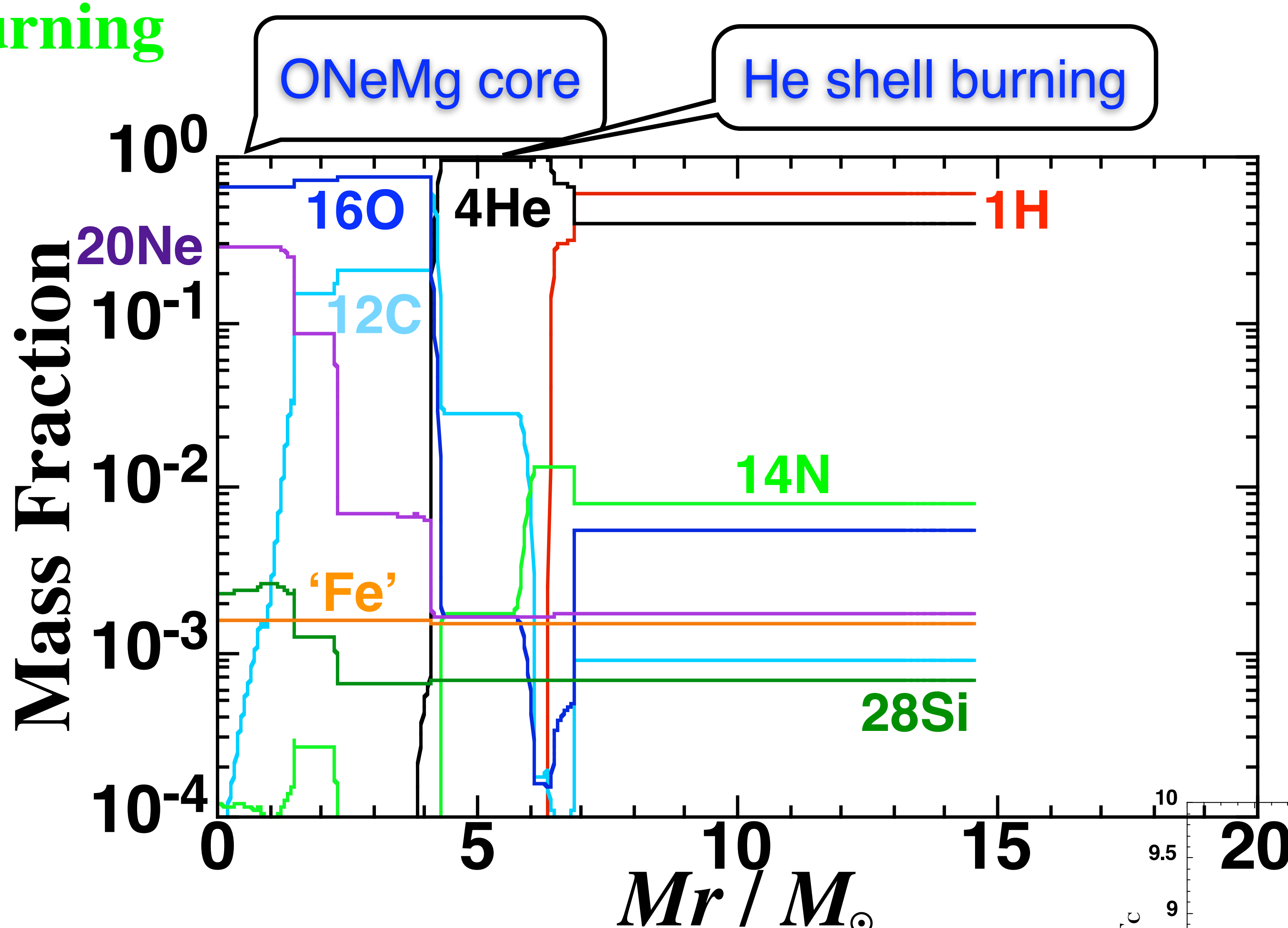
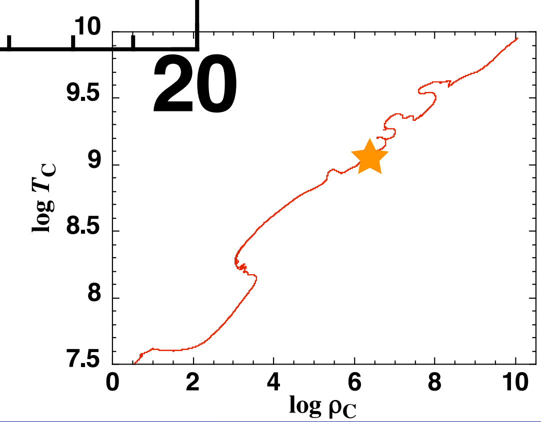


# Evolution of 20 $M_\odot$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

- C-burning



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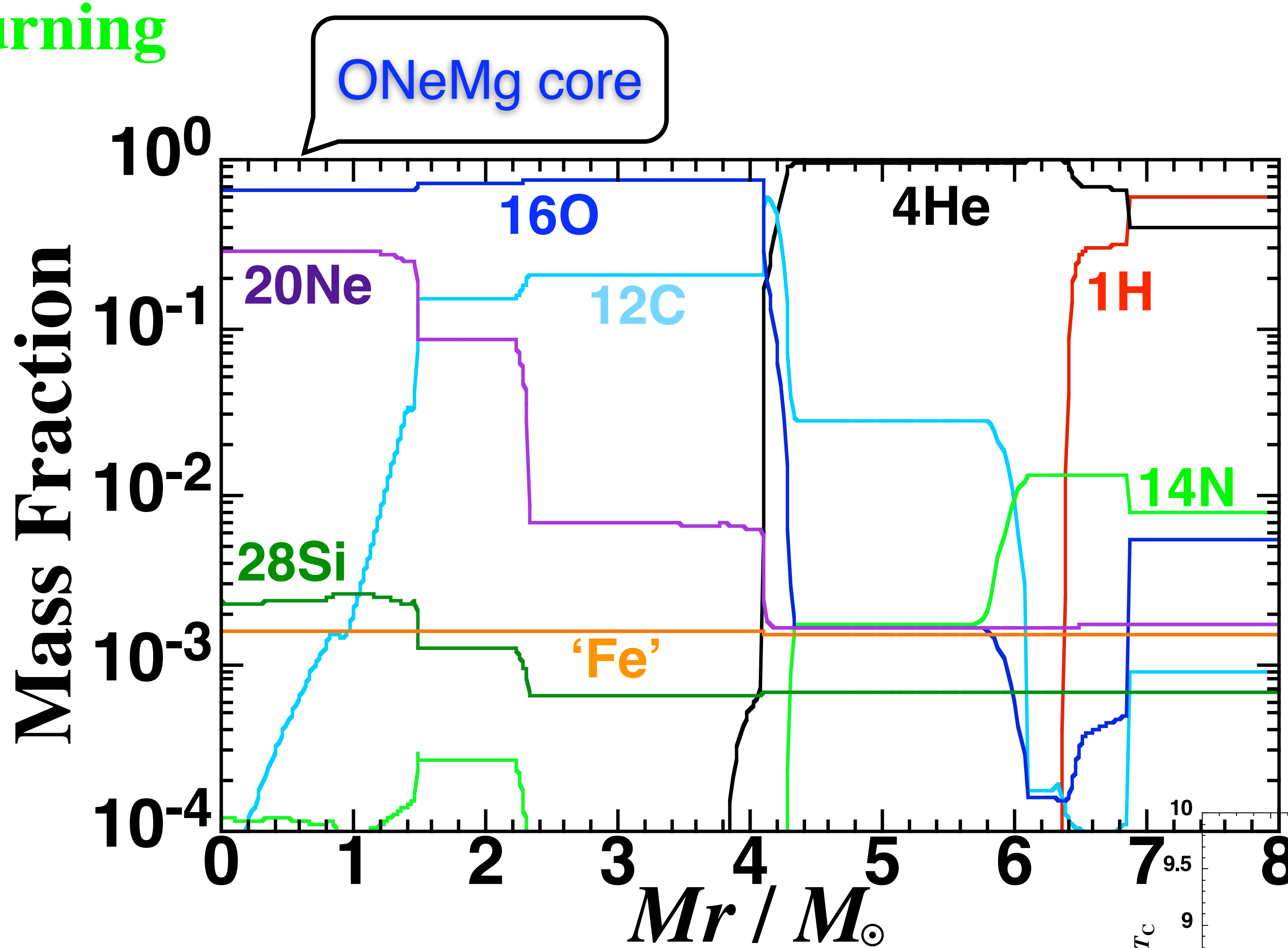
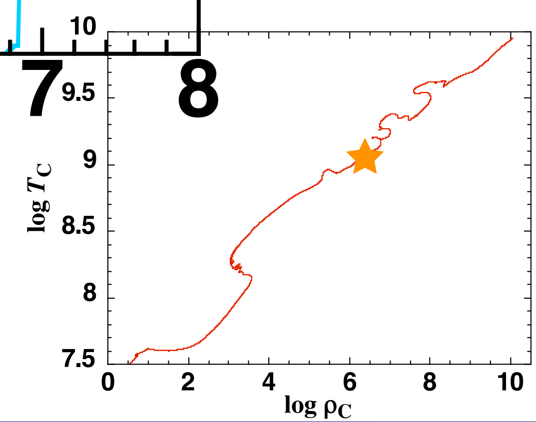
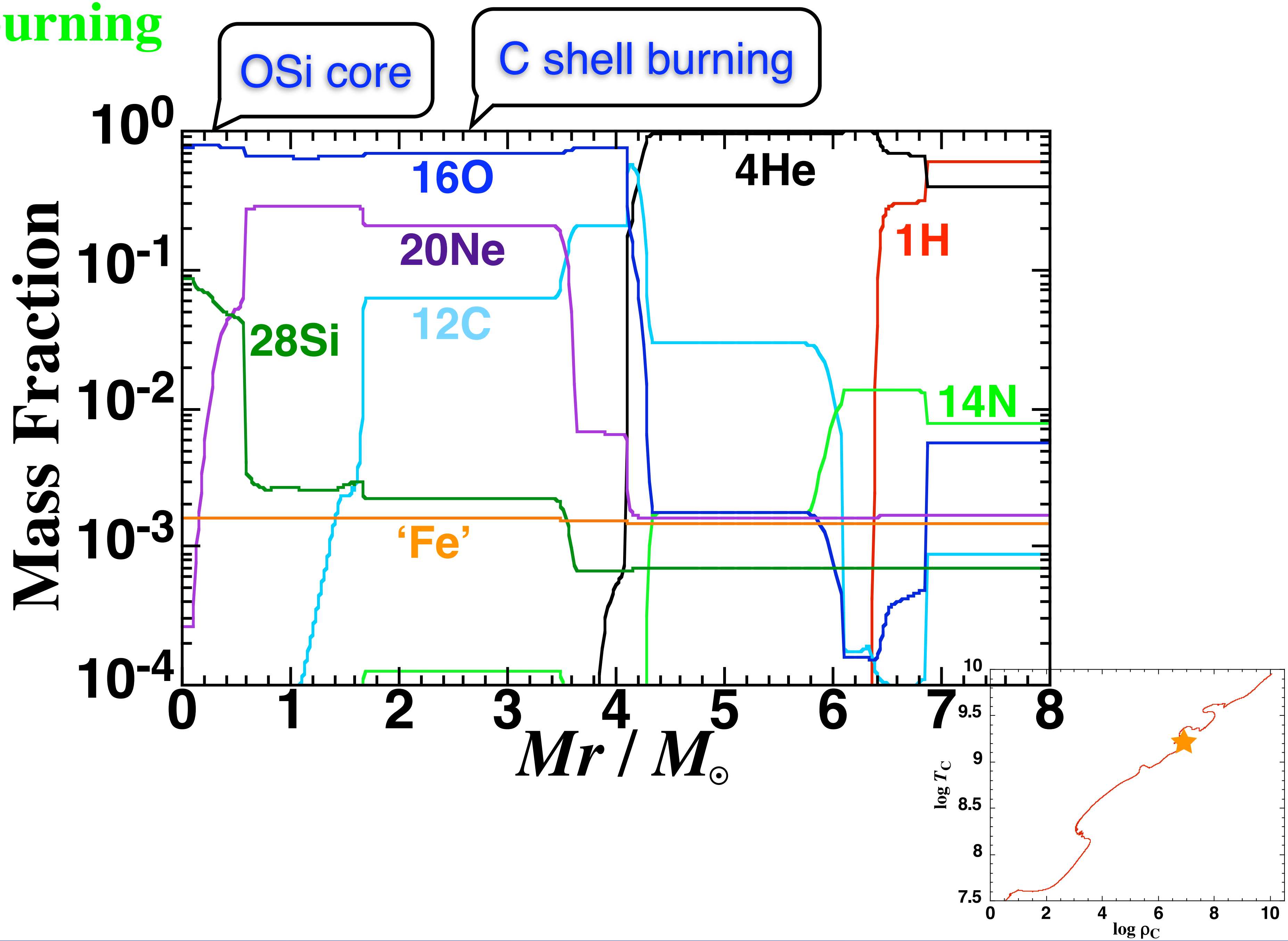


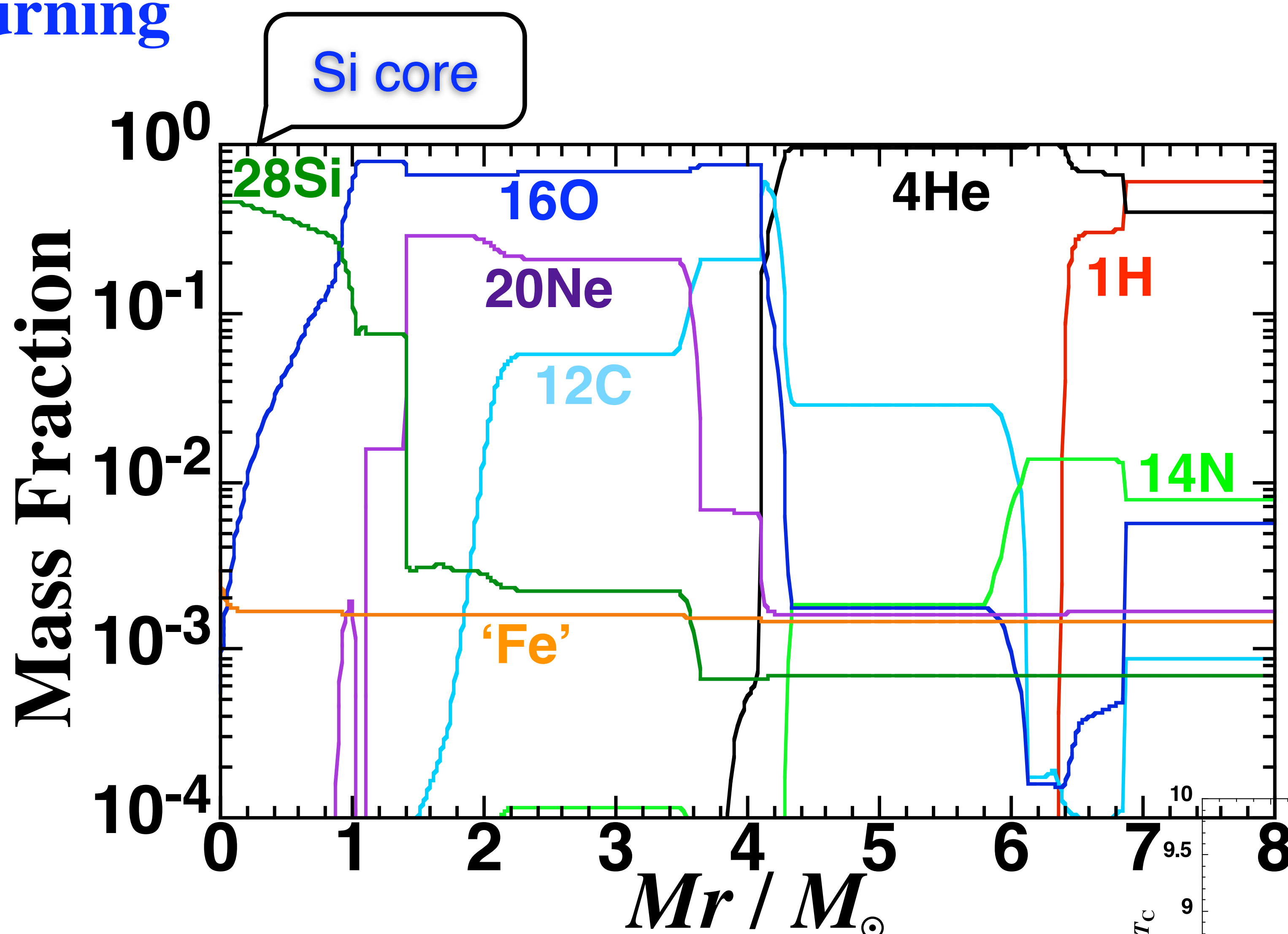
# Evolution of 20 $M_\odot$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

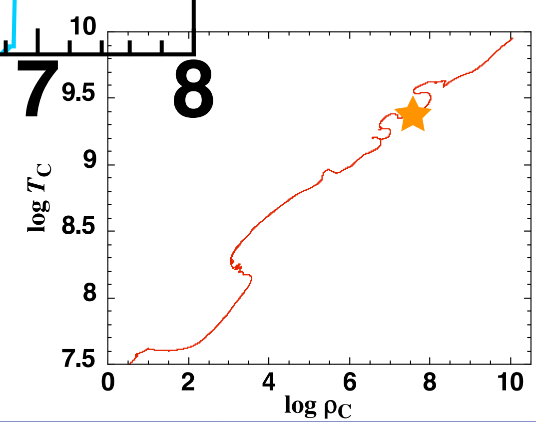
- Ne-burning



# Evolution of 20 $M_{\odot}$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

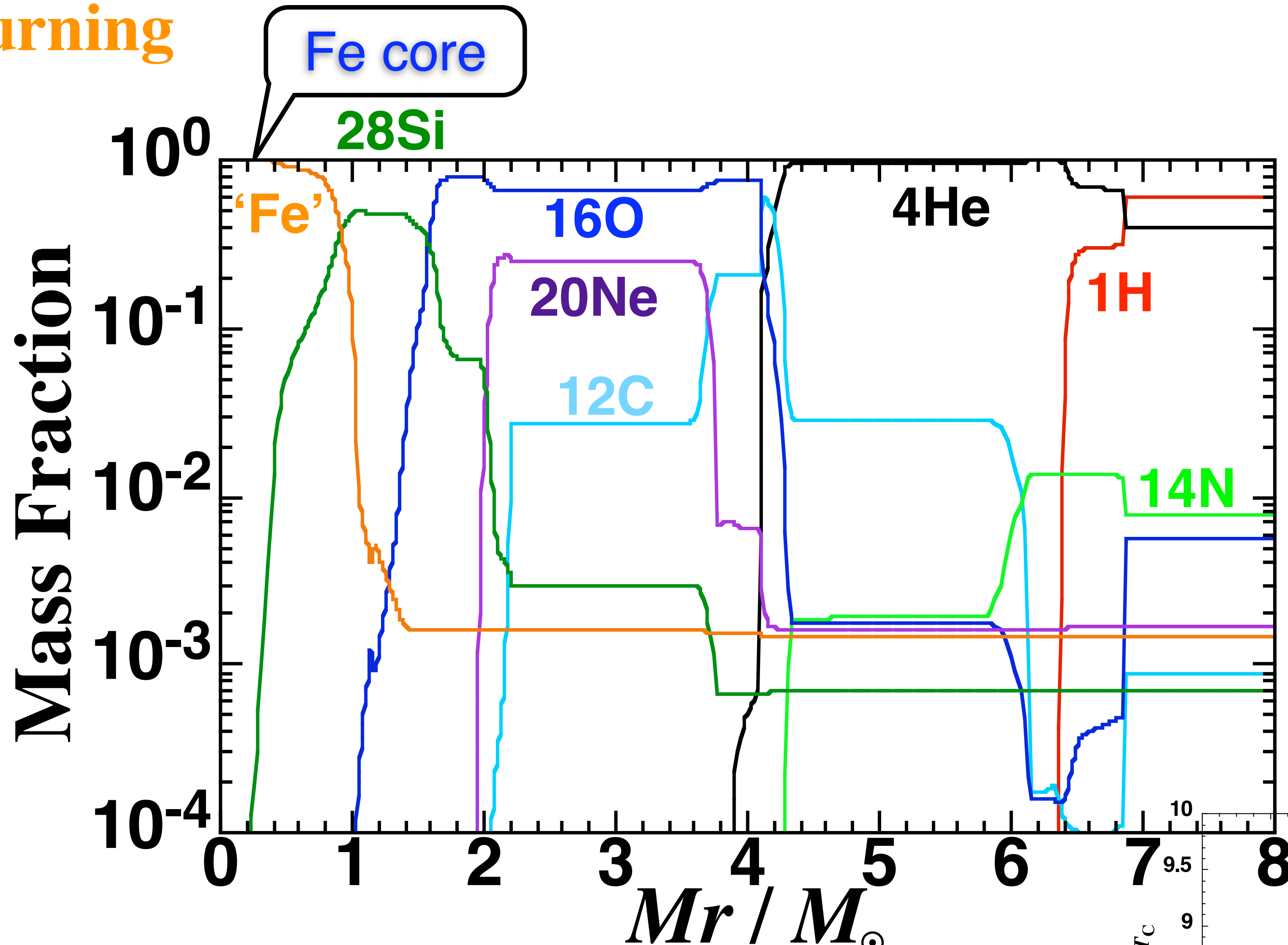
- **O-burning**

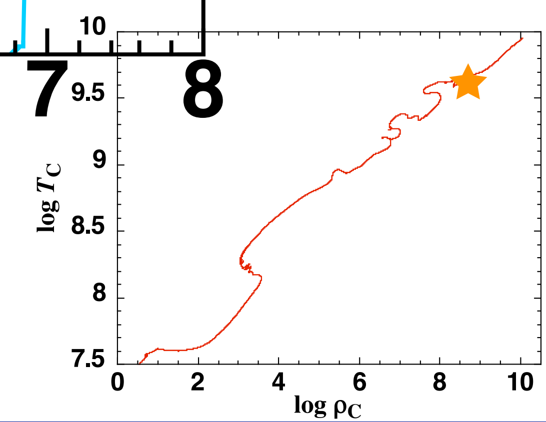


# Evolution of 20 $M_\odot$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

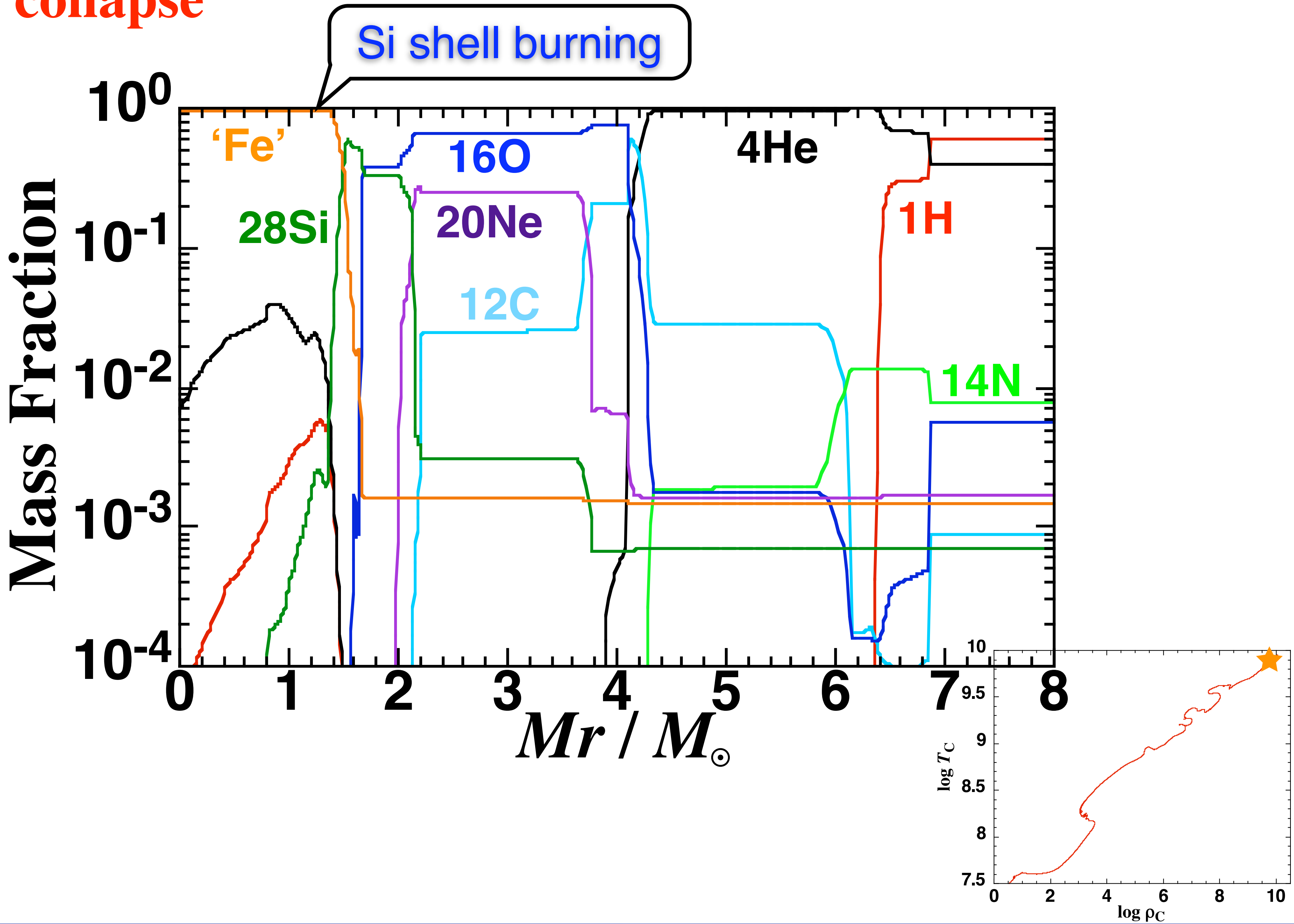
- Si-burning



# Evolution of 20 $M_\odot$, $Z$=0.02 $V_{r0}$=200km s$^{-1}$ Star

- Core collapse



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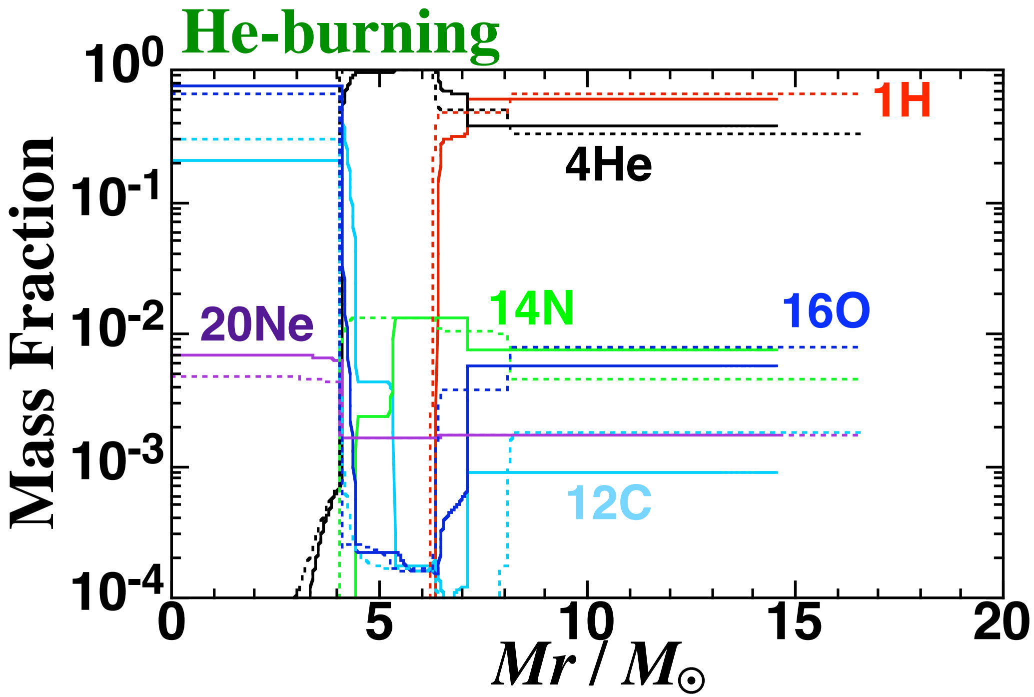
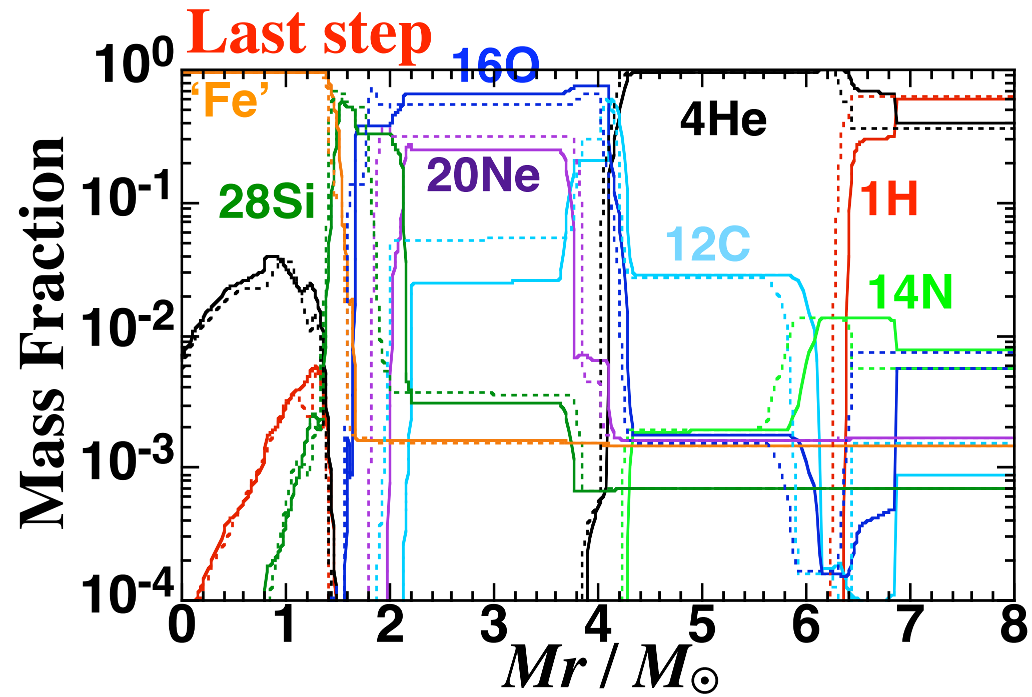


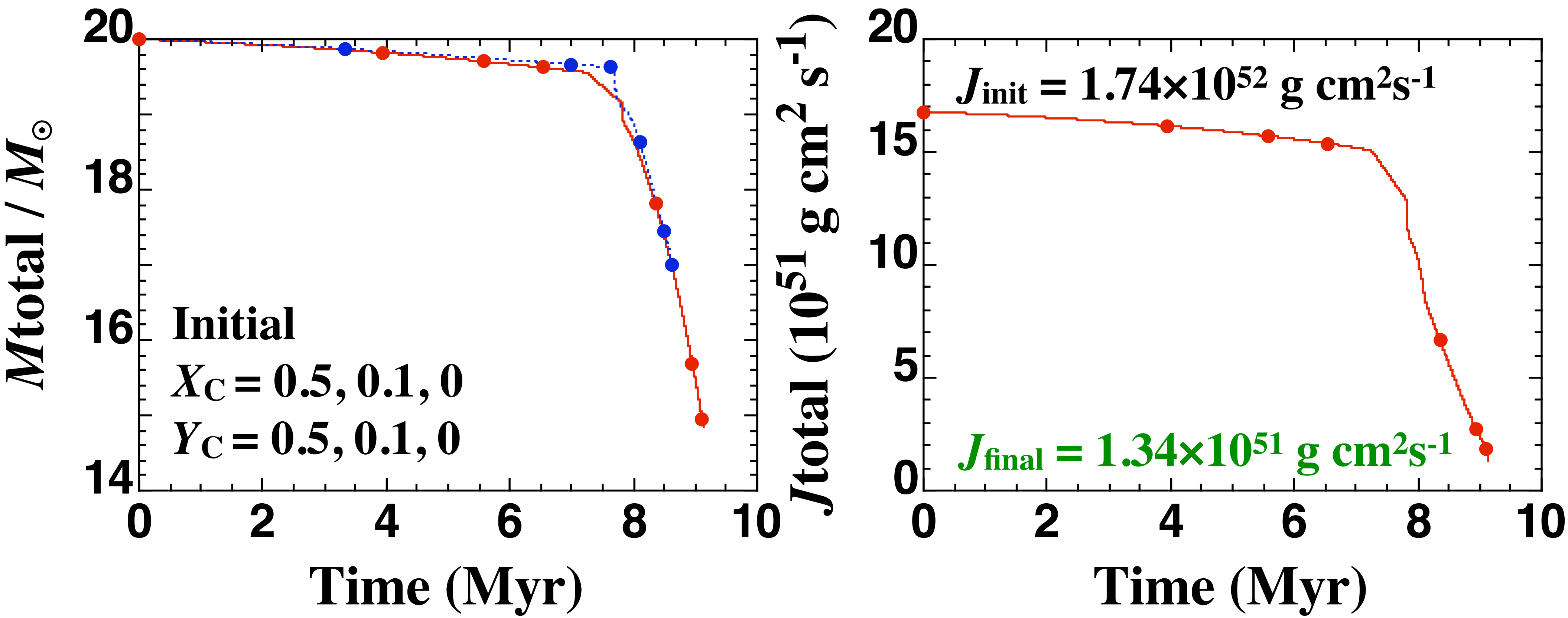




**Solid lines: $V_{r0}$ = 200 km s$^{-1}$; Dashed lines: $V_{r0}$ = 0 km s$^{-1}$**

- **Enhancement of surface N abundance**
- $M_f$ = 14.8 (16.9) $M_\odot$
- $M_{\mathrm{He\ core}}$ = 6.35 (6.25) $M_\odot$
- $M_{\mathrm{CO\ core}}$ = 4.07 (4.02) $M_\odot$
- $M_{\mathrm{Fe\ core}}$ = 1.47 (1.41) $M_\odot$

# Mass and Angular Momentum Loss

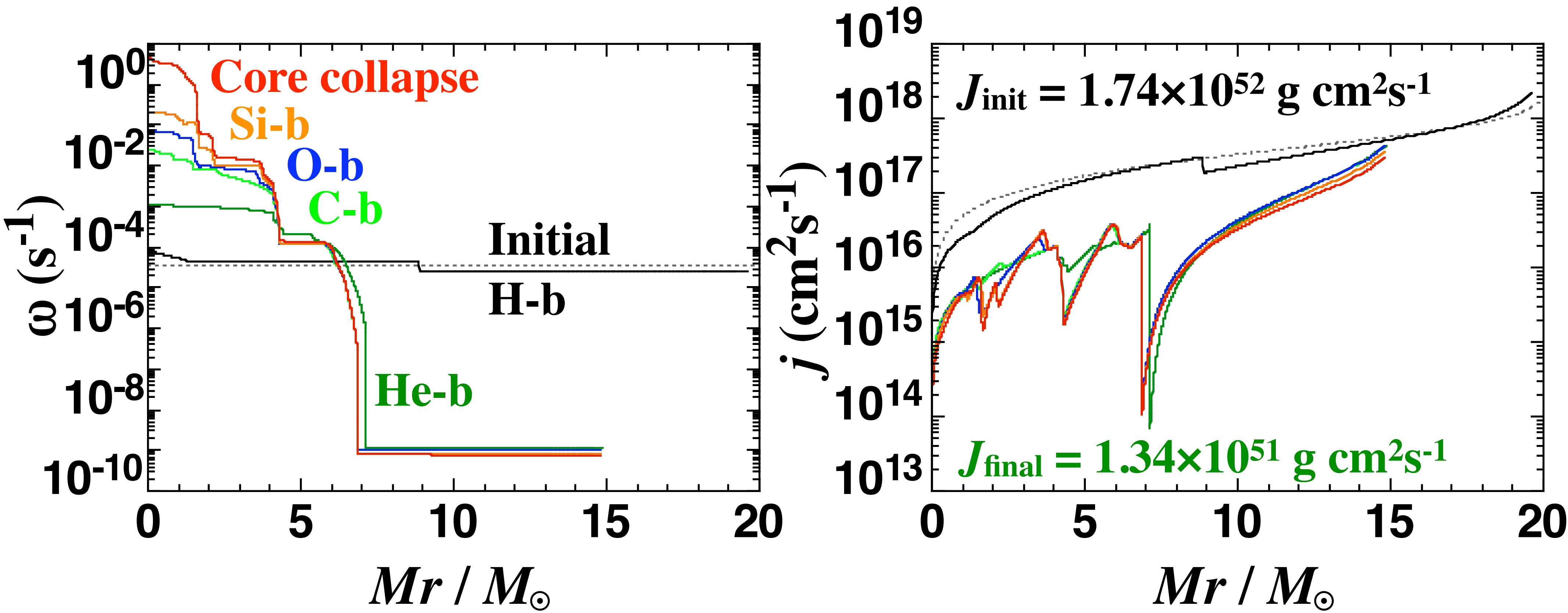
$M_{\rm MS} = 20\ M_\odot$, $Z = 0.02$, $V_{r0} =$ 0, 200 km s$^{-1}$




- He burning (red giant) ➡ Large loss of $M$ and $J$
- $J_{\rm final} \sim 10^{51}$ g cm$^2$ s$^{-1}$

# Angular Momentum Distribution

$M_{MS} = 20\ M_\odot$ , $Z = 0.02$, $V_{r0} = 200$ km s$^{-1}$




- **Rotational mixing**
  ➡ **Rigid rotation in convective layers**
  **Angular momentum moves outward**
- **Angular momentum loss by enhanced mass loss**

# Angular Momentum Distribution

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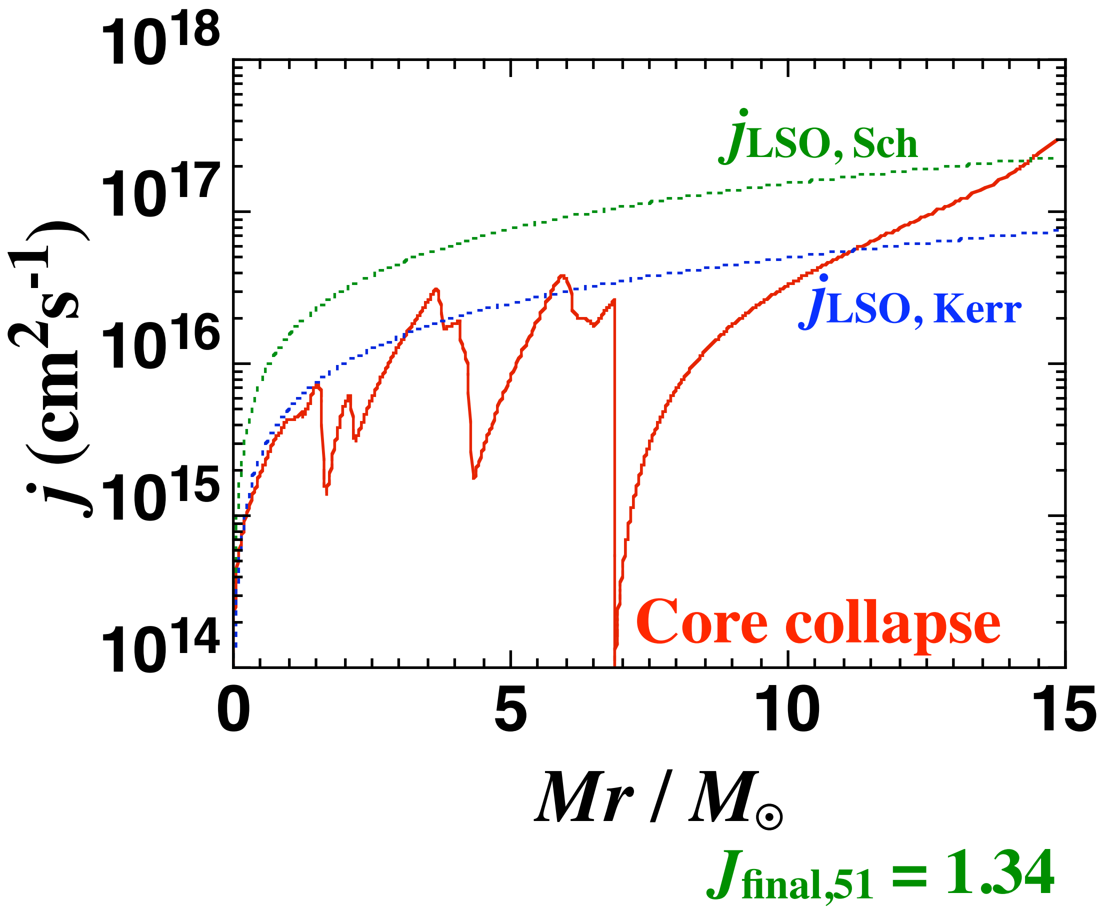


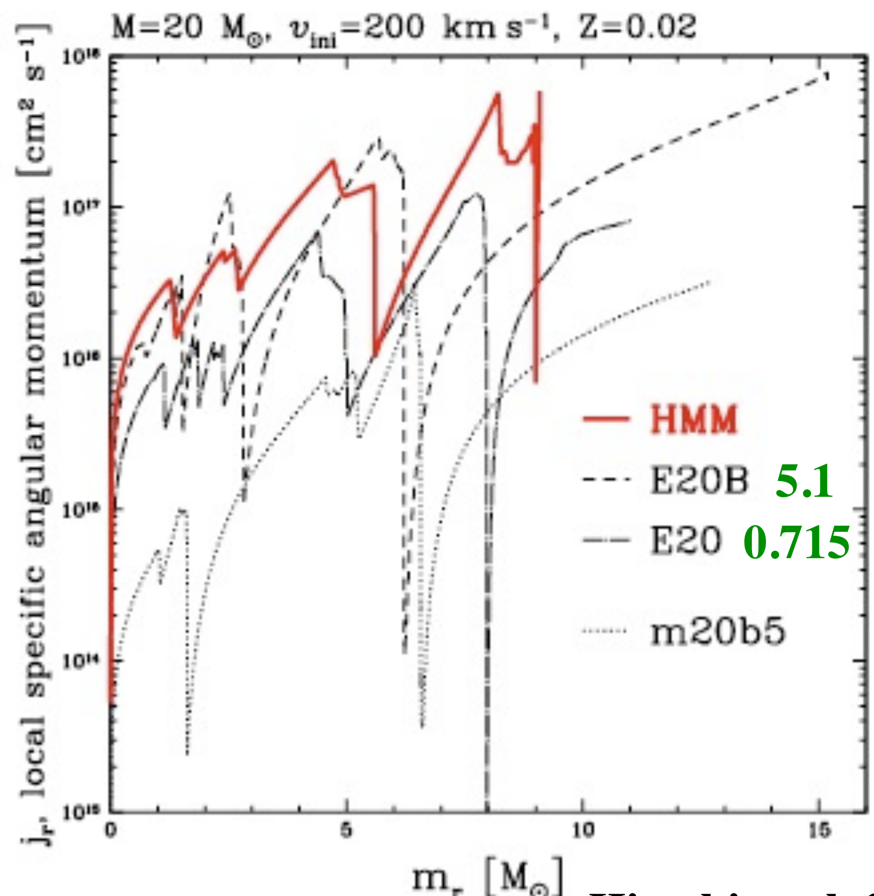


Hirschi et al. 2004

- Smaller angular momentum than other groups

# Concluding Remarks

**Test calculation of the evolution of rotating massive stars**

- $M_{\rm init} = 20\ M_\odot$ , $Z_0 = 0.02$, $V_{r0} = 200$ km s$^{-1}$

  **From H burning to the onset of the core collapse**

  ***Results of other groups are reproduced qualitatively.***

  - About 90% of angular momentum is lost.
  - Enhancement of surface N abundance
  - Masses of He, CO, Fe cores

***Current problems***

- Calculation stops in faster rotation
- Time step control & mass-coordinate resolution

  → Effects on mixing in shell burnings