Towards Supernova Simulations with Multi-Dimensional Boltzmann Transport

～多次元ニュートリノ輻射流体コードの開発～

Hiroki Nagakura（長倉　洋樹）
(HPCI researcher Kyoto Univ/Waseda Univ)

Collaborators:
Kohsuke Sumiyoshi (Numazu), Shoichi Yamada (Waseda)
Shun Furusawa (Waseda)

2011/12/3-5 素核宇 Workshop @ 合歓の郷Iseshima

Outline

1. Introduction

- Standard Scenario for Core Collapse Supernova

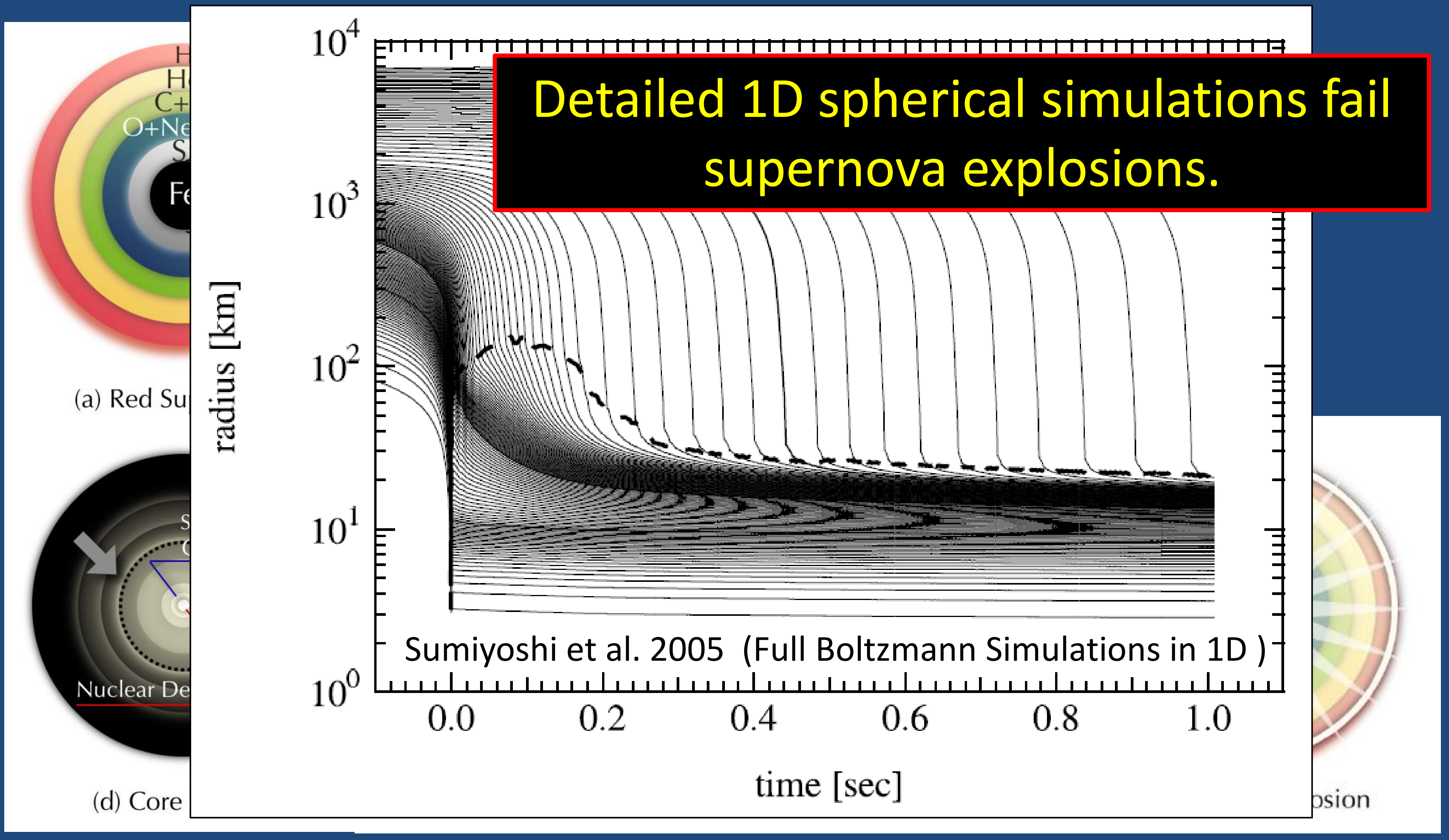
2. Flow chart for developing numerical code

3. What I have done so far ?

- Reconstruction of EOS table
- Check the spherical collapse (Adiabatic calculation)
- Sustaining the total energy conservation

4. Summary and Future Work

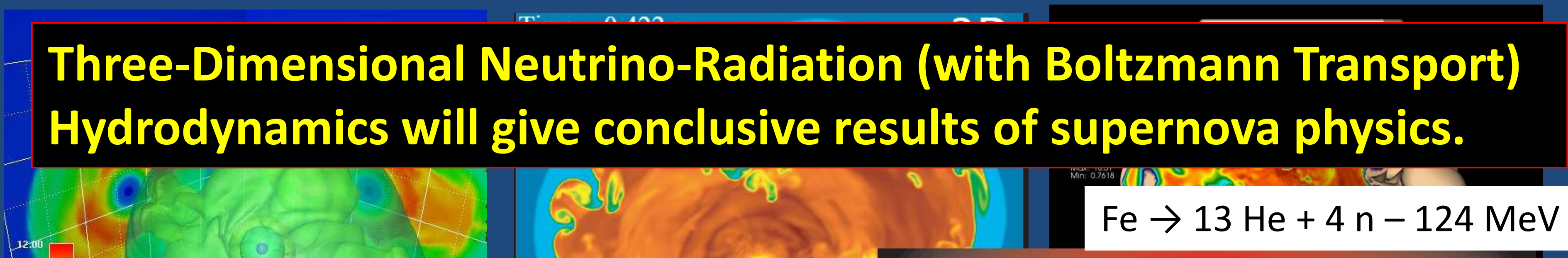
Standard Scenario for Core Collapse Supernovae（Delayed Explosion Scenario）

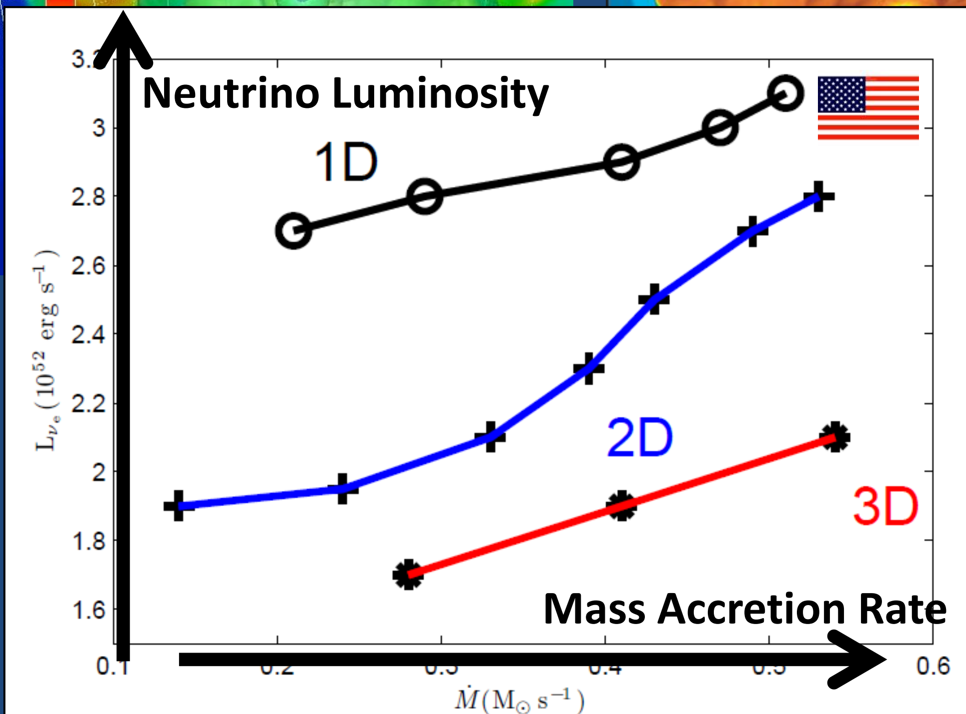


Catoon From Iwakami D thesis

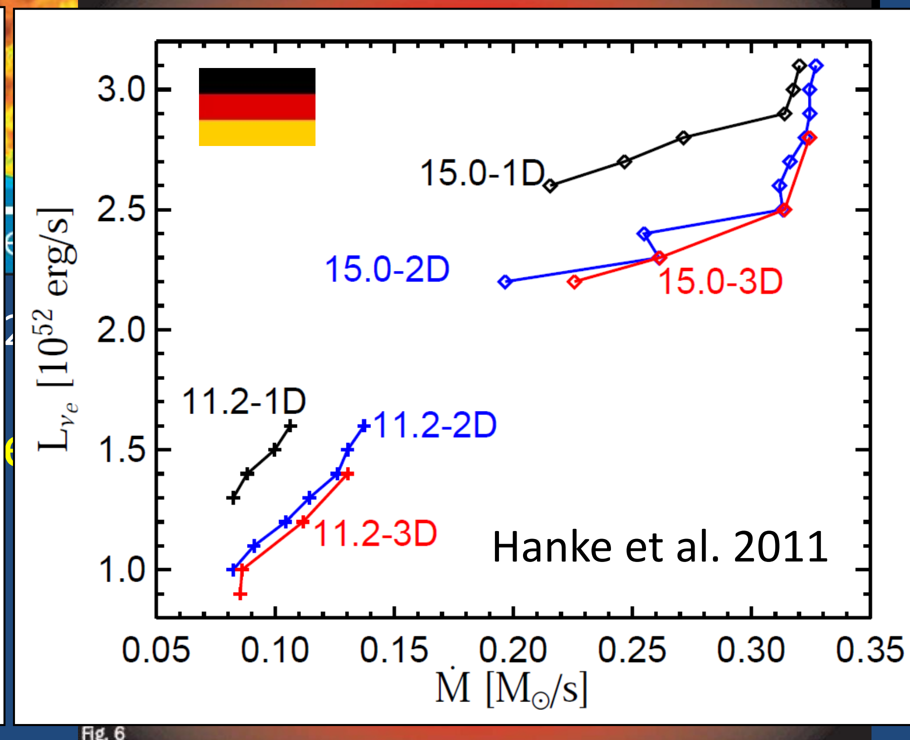
Three-Dimensional Effect to Supernova is still matter of debate.

Three-Dimensional Neutrino-Radiation (with Boltzmann Transport) Hydrodynamics will give conclusive results of supernova physics.





Nordhaus et al. 2010



Cartoon from Janka

2. Flow chart for developing numerical code

Today's talk

Multi-Dimensional Hydrodynamic Code

- High resolution central scheme
- MICCG (Poisson solver for gravity)

Multi-Dimensional Neutrino Transfer Code

Developed by K. Sumiyoshi

→ **Multi-Dimensional Neutrino Radiation-Hydrodynamic Code**

Some Important Assumptions

- Neglect General Relativistic Effects
- Neglect Magnetic Field Effects
- Neglect Incoherent Neutrino Scattering Processes
- Neglect Neutrino-Oscillations

Basic Equations

Rad-Hydro Equations

V $$\partial_t(r^2 \sin\theta\rho) + \partial_j\left(\rho_* v^j\right) = 0, \quad (1)$$

V $$\partial_t(r^2 \sin\theta\rho u_r) + \partial_j\left(r^2 \sin\theta\, T^j_{\ r}\right)$$
$$= r^2 \sin\theta\left\{-\underline{\rho\,\psi_{,r}} + r\rho(u^\theta)^2 + r\sin^2\theta\rho(u^\phi)^2 + \frac{2p}{r}\right\} + \underline{M_r}, \quad (2)$$

V $$\partial_t(r^2 \sin\theta\rho u_\theta) + \partial_j\left(r^2 \sin\theta\, T^j_{\ \theta}\right)$$
$$= r^2 \sin\theta\left\{\underline{-\rho r^2 \sin\theta\,\psi_{,\theta}} + \rho r^4 \sin^2\theta\cos^\theta(v^\phi)^2 + r^2\cos\theta\, p\right\} + \underline{M_\theta}, \quad (3)$$

V $$\partial_t(r^2 \sin\theta\rho u_\phi) + \partial_j\left(r^2 \sin\theta\, T^j_{\ \phi}\right) = \underline{-\rho r^2 \sin\theta\,\psi_{,\phi}} + \underline{M_\phi}, \quad (4)$$

V $$\partial_t\left\{r^2 \sin\theta\rho(\epsilon c^2 + \frac{1}{2}u^2)\right\} + \partial_j\left(r^2\sin\theta\, T^{0j}\right) = \underline{-r^2\sin\theta\,\rho u^i\psi_{,i}} + \underline{Q}, \quad (5)$$

V $$\partial_t\left(r^2\sin\theta\rho Y_e\right) + \partial_j\left(r^2\sin\theta\rho Y_e v^j\right) = \underline{L}, \quad (6)$$

V $$\Delta\psi = 4\pi G\rho \quad (7)$$

V $$\frac{1}{c}\frac{\partial f^{in}}{\partial t} + \cos\,\theta_\nu\frac{\partial f^{in}}{\partial r} + \frac{\sin\,\theta_\nu\,\cos\,\phi_\nu}{r}\frac{\partial f^{in}}{\partial\theta} + \frac{\sin\,\theta_\nu\,\sin\,\phi_\nu}{r\sin\,\theta}\frac{\partial f^{in}}{\partial\phi}$$
$$+ \frac{\sin^2\,\theta_\nu}{r}\frac{\partial f^{in}}{\partial\cos\,\theta_\nu} - \frac{\sin\,\theta_\nu\,\sin\,\phi_\nu}{r}\frac{\cos\,\theta}{\sin\,\theta}\frac{\partial f^{in}}{\partial\phi_\nu} = \left[\frac{1}{c}\frac{\delta f^{in}}{\delta t}\right]_{collision}$$

Boltzmann Equation

Outline

1. Introduction

2. Flow chart for developing numerical code

3. What I have done so far ?

4. Summary and Future Work

3. What I have done so far ?

1. Reconstruction of EOS table

2. Check the spherical collapse
(adiabatic calculation)

3. Sustaining the total energy conservation

Reconstruction of EOS Table

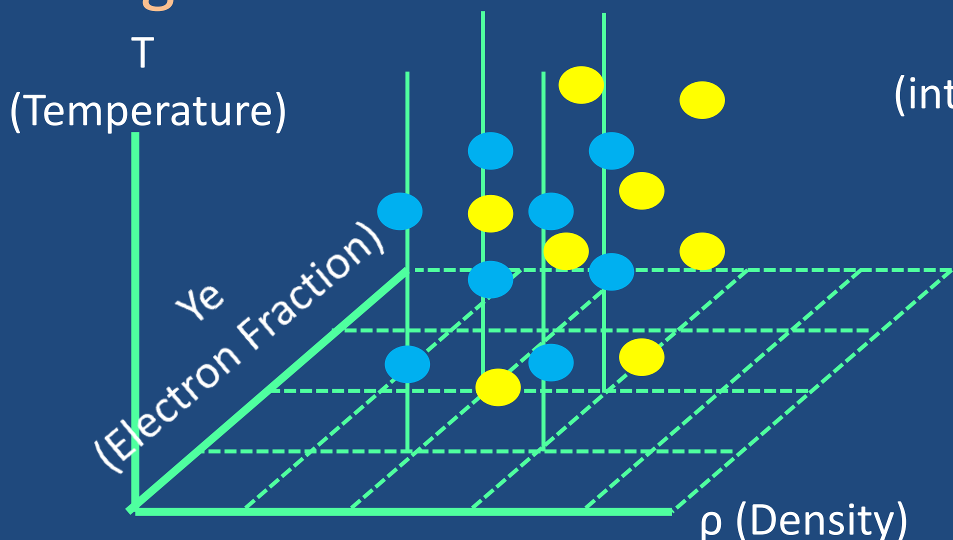
We employ Shen EOS in our simulations.

Low resolutions for direct tri-linear interpolations in simulations.

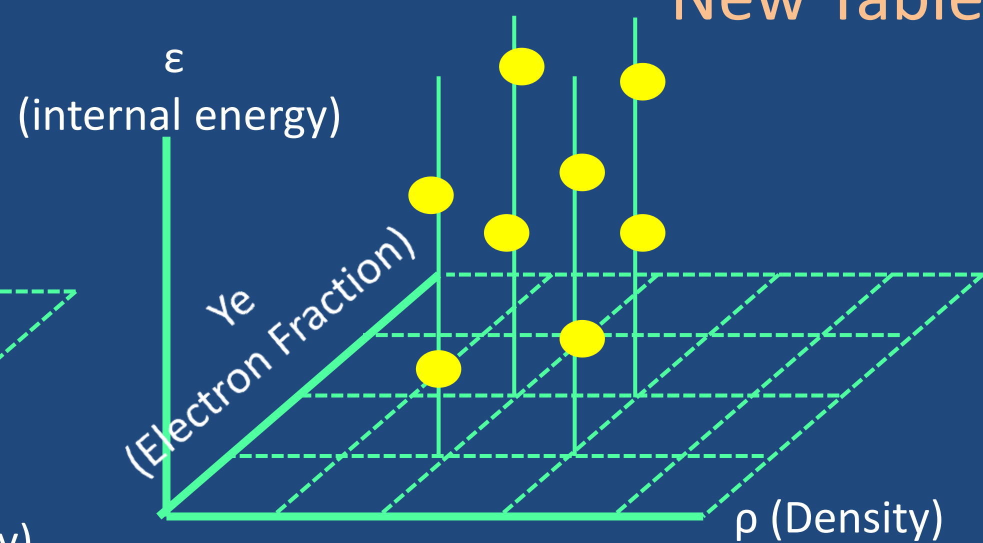
Reconstruction of EOS Table

(Tri-Cubic Hermite Polynominal Interpolations)

Original Table

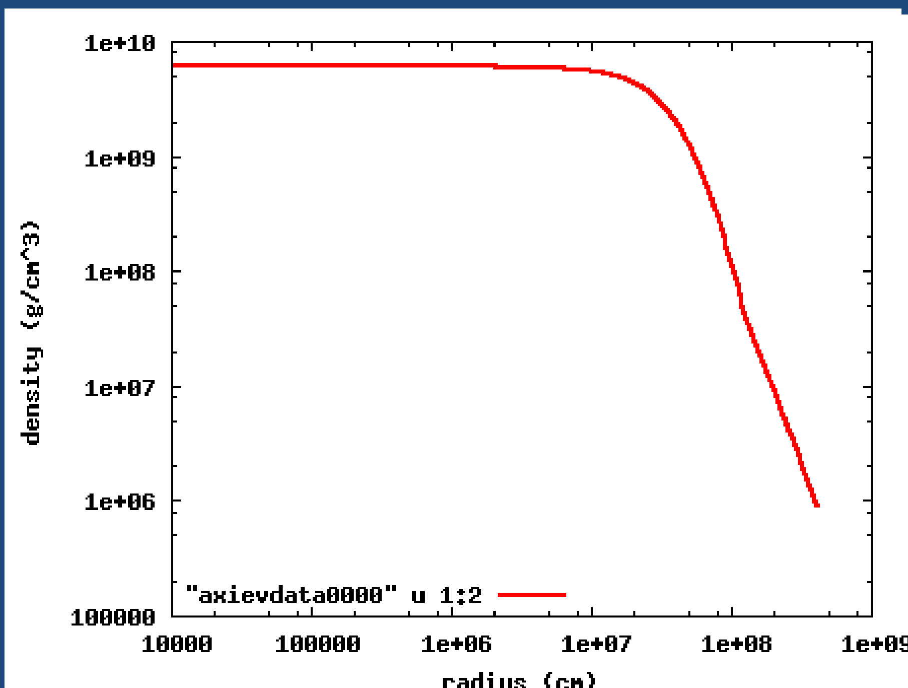


New Table

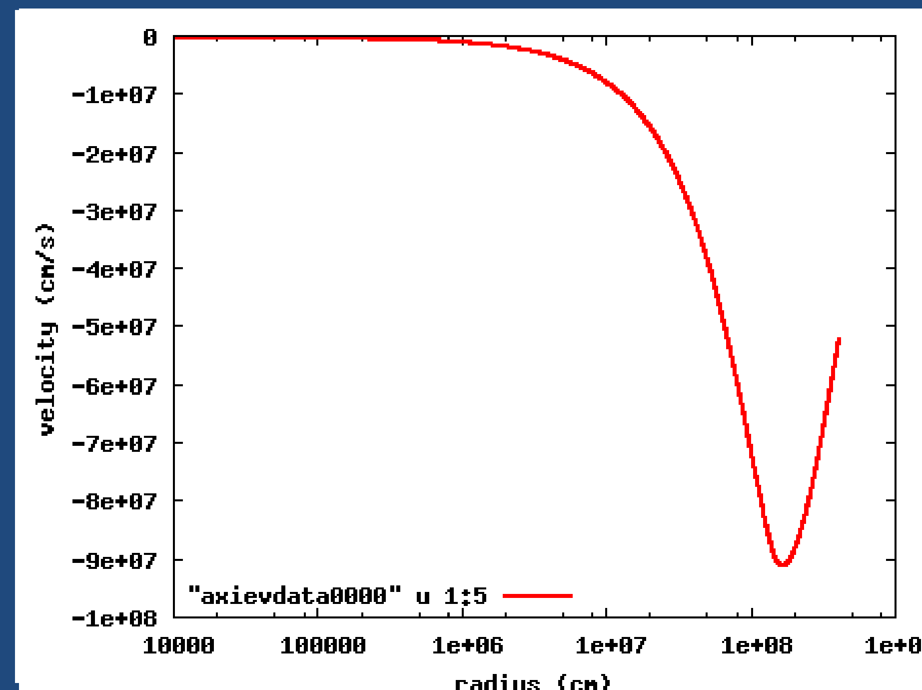


Check the spherical collapse (Adiabatic calculation)

Density



Velocity



Woosley 15 M_{sun}, M_{Fe} ～ 1.35 M_{sun}

Sustaining the total Energy Conservation

- Difficulty of energy conservation

The explosion energy of typical supernova is 10^{51} erg.

$E_{gra} \approx -10^{51}(erg)$

$E_{\mathrm{int}} \approx 10^{51}(erg)$

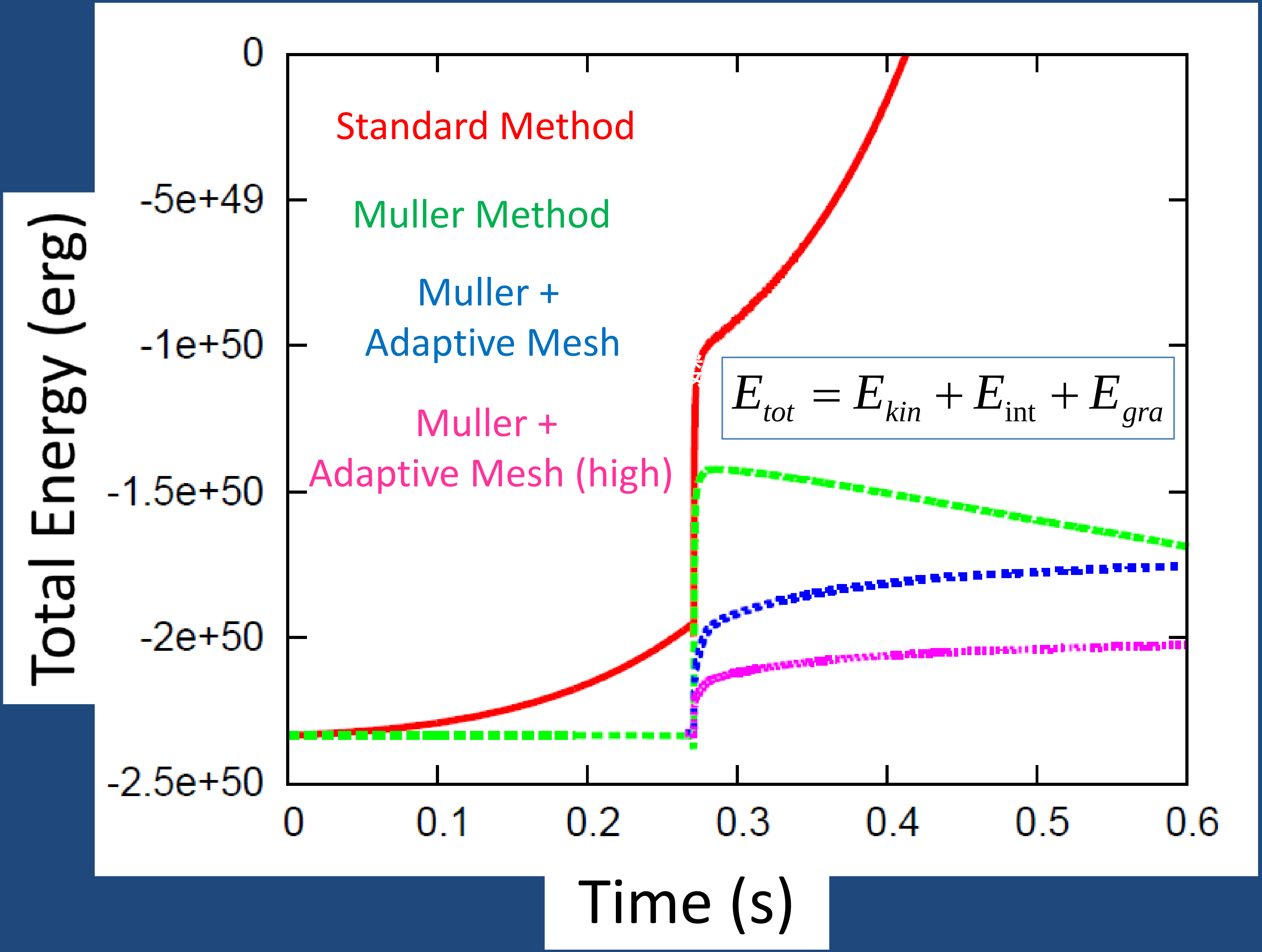
$$E_{tot} = E_{kin} + E_{\mathrm{int}} + E_{gra} \approx -10^{50}(erg)$$

$E_{gra} \approx -10^{53}(erg)$

$E_{\mathrm{int}} \approx 10^{53}(erg)$

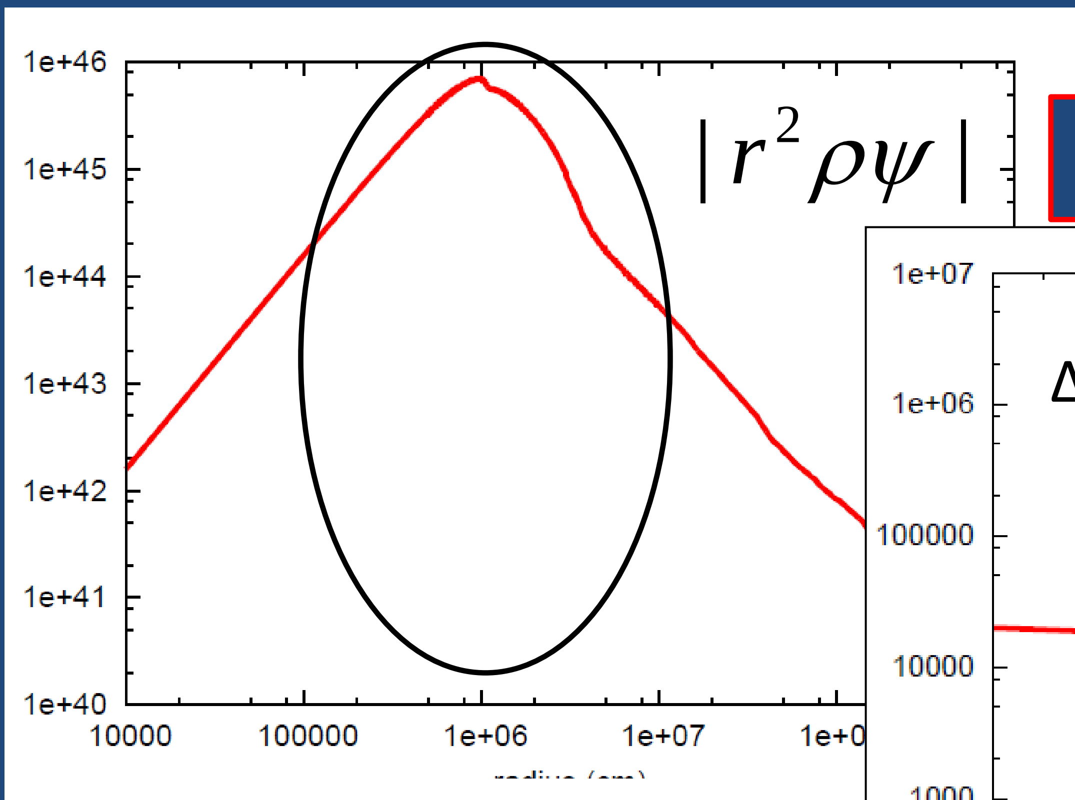
The order of total energy is **0.1 %** of gravitational (ineternal) energy of neutron stars.

Energy Conservation



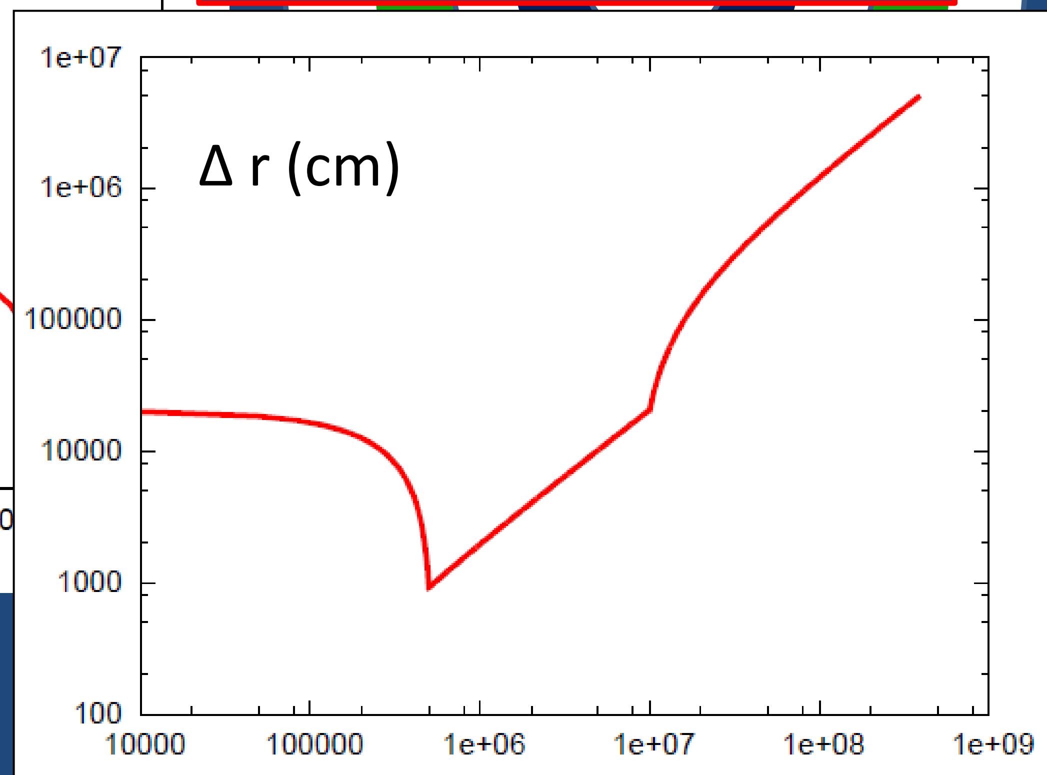
Proto Neutron Star

The ratio of Gravitational energy for each mass shell



Radius (cm)

The Adaptive Mesh



Radius (cm)

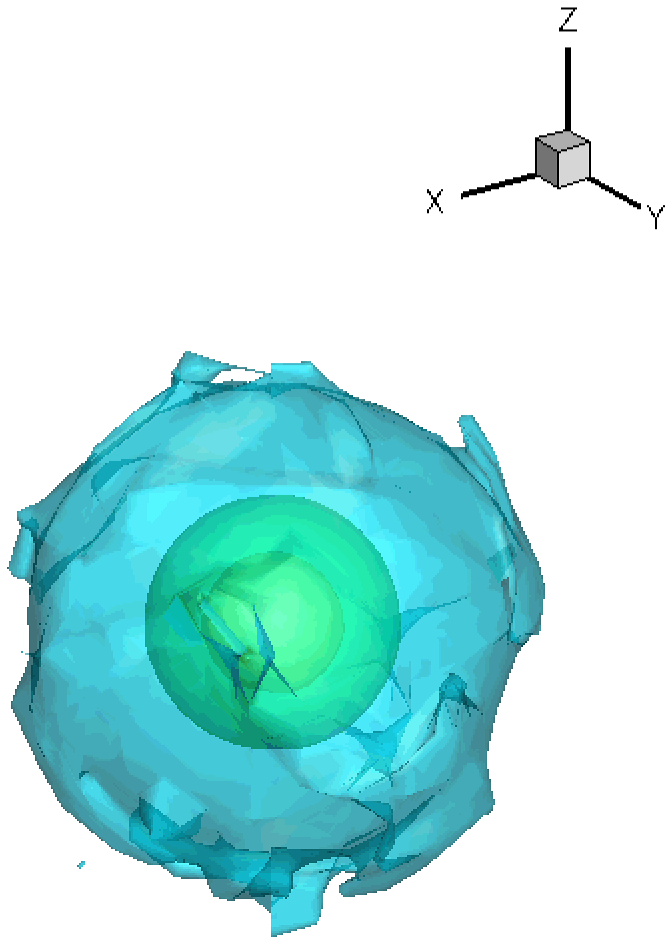
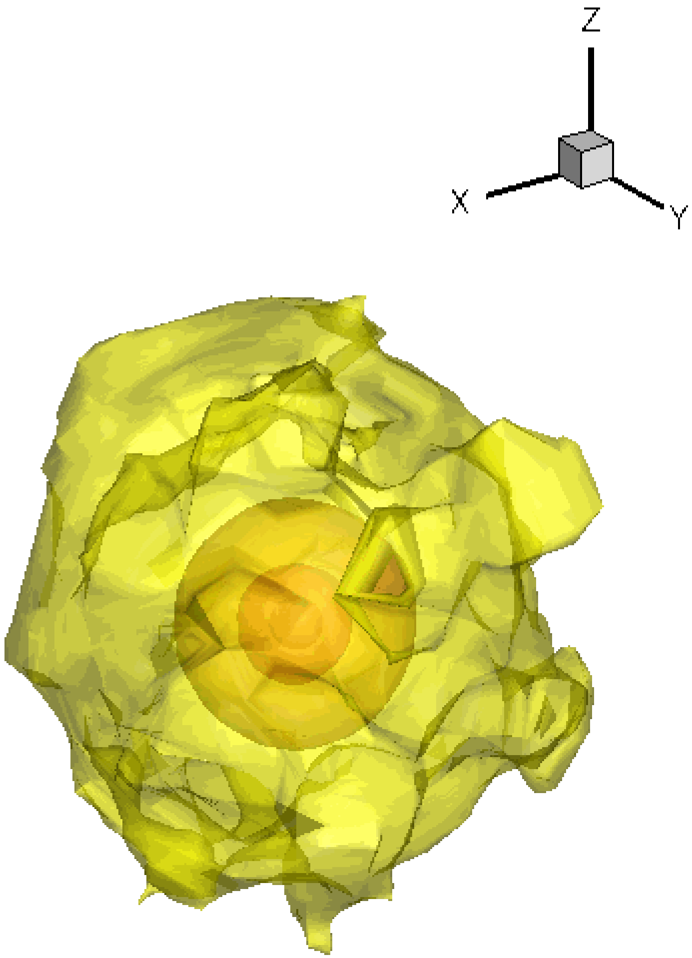
Summary and Future work

- The neutrino transport plays an crucial role for the delayed explosion model.
- I gave ad interim reports for developing Neutrino-Radiation Hydrodynamic code.
- The EOS table is reconstructed for satisfying thermodynamic relation.
- Our hydrodynamic code successfully simulates spherical core collapse with good energy conservations.

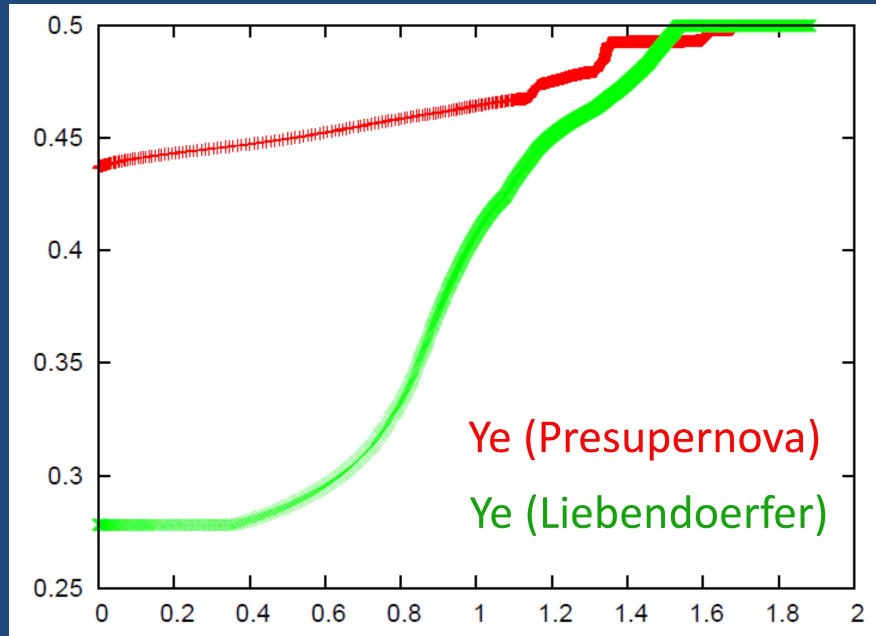
Next steps

- Multi-Dimensional Hydrodynamic Simulations.
- Coupling with neutrino Boltzmann transport.

Work Table

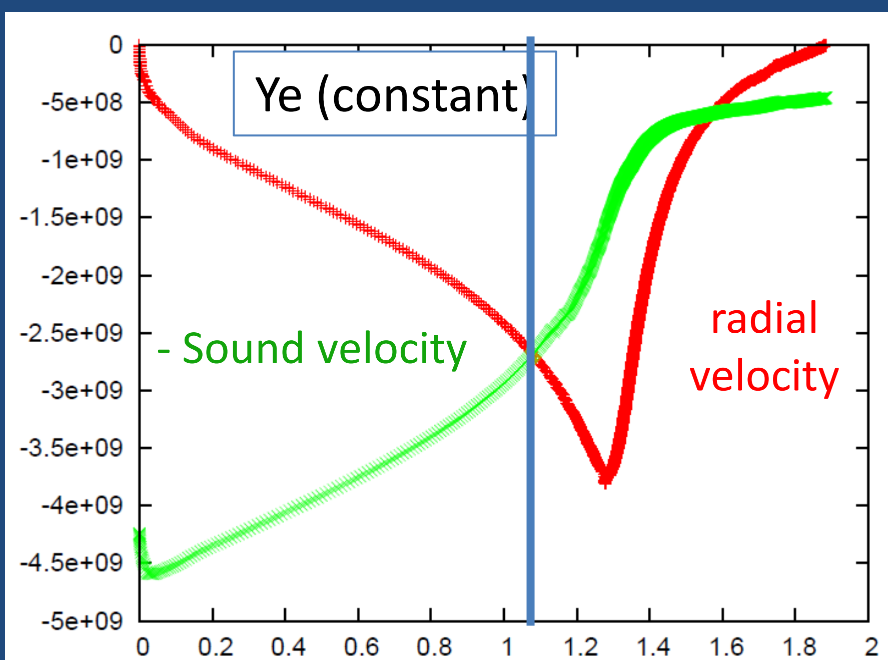

Z
X
Y

Z
X
Y

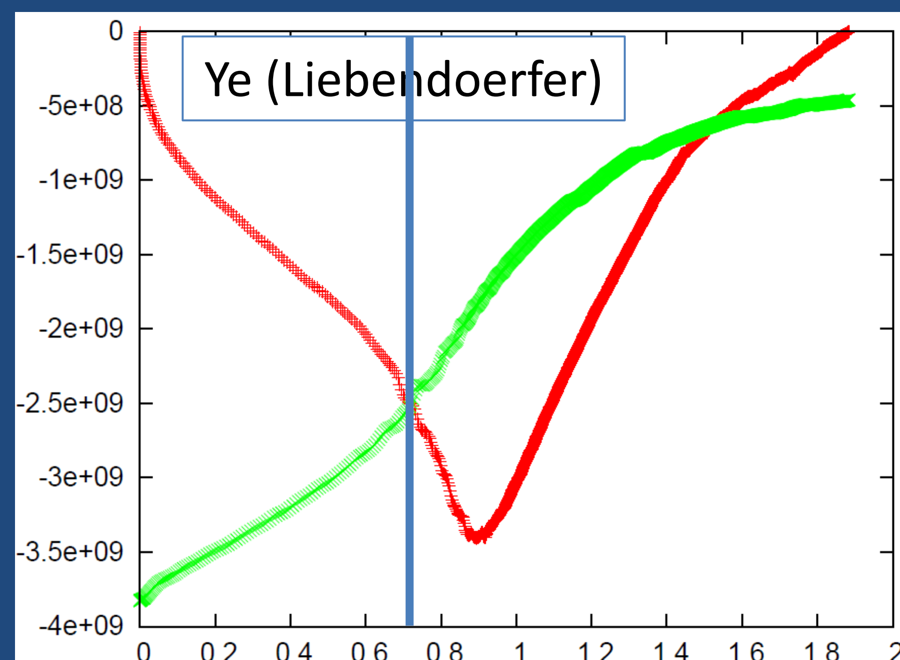
Ye (Electron Fraction)



$$M_{ch} \approx 1.4\left(\frac{Ye}{0.5}\right)^2$$

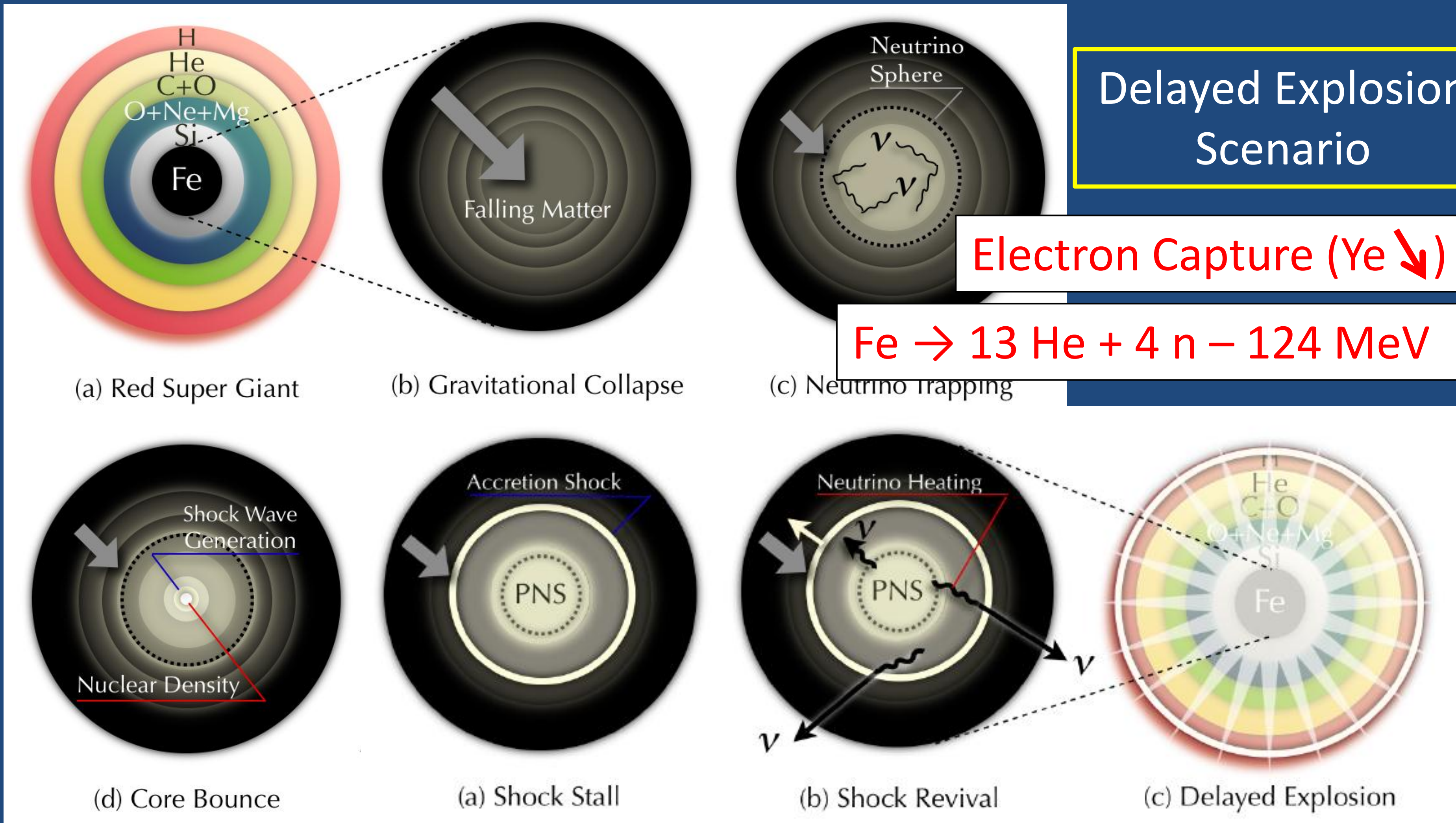
Enclosed mass (M_{sun})





Check the spherical collapse

Adiabatic Collapse (No coupling of neutrino)



Delayed Explosion Scenario

Electron Capture (Ye ↘)

Fe → 13 He + 4 n – 124 MeV

Catoon From Iwakami D thesis