構造を仮定しない計算による
低密度原子核物質の非一様構造

Non-uniform structure of low-density nuclear matter by three-dimensional calculation without any assumption of the structures

Minoru Okamoto[1,2] , Toshiki Maruyama[2]
Kazuhiro Yabana[1,3] , Toshitaka Tatsumi[4]

Graduate School of Pure and Applied Science, University of Tsukuba[1]
Advanced Science Research Center, Japan Atomic Energy Agency[2]
Center of Computational Sciences, University of Tsukuba[3]
Graduate School of Science, Kyoto Universyty[4]

▪ *Menu*

Ⅰ. Pasta structure

Ⅱ. Relativistic mean field theory

Ⅲ. Results

ⅰ) Fixed proton fraction

ⅱ) β-equilibrium

Ⅳ. Conclusion

Pasta Structure : "Mixed phase with structure" in first phase transition

Suggested by D.G.Ravenhall, C.J.Pethick, J.R.Wilson in 1983

"meat ball"

$\sim 0.2\rho_0$

"spaghetti"

$\sim 0.4\rho_0$

"lasagna"

$\sim 0.5, 0.6\rho_0$

"macaroni"

$\sim 0.7\rho_0$

"cheese"

Blue : nuclear matter
cyan : gas nuclei

ρ_0 : normal nucleus density
$\sim 0.16 \mathrm{fm}^{-3}$

K.Oyamatsu, NPA 561 (1993) 431

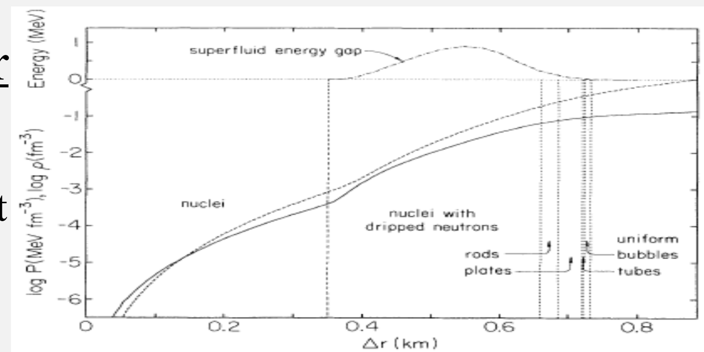
Balance with Surface tension and Coulomb repulsion

$$\boxed{2E_C = E_S \quad \text{(Equilibrium condition)}}$$

Pasta structure in Neutron star

- Half mass of Crust region

  → Influence on EOS of Crust

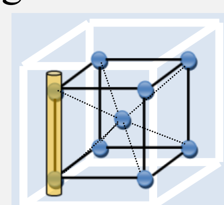


C.P.Lorenz et al, PRL 70 (1993) 4

- Glitch

  Rapid increasing of rotational speed of Neutron Star

Whirlpool of super fluidity of neutron pinned to nuclei



Nuclei

Super fluidity (Neutron)

It may occur from the collapse of whirlpool of neutron super fluidity on lattice points?

→ It may depend on the crystalline of nuclei and more exotic structure like "pasta"

- Neutrino cross-section in supernova and collapsing stellar cores

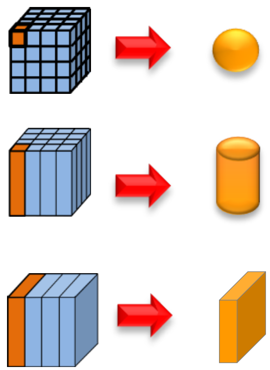
H.Sonoda, G.Watanabe et al PRC 75 042801(2007)

Conventional Studies

Wigner-Seitz(WS) cell approximation

Whole space is divided into equivalent cells.
These are imposed geometrical symmetry as follow

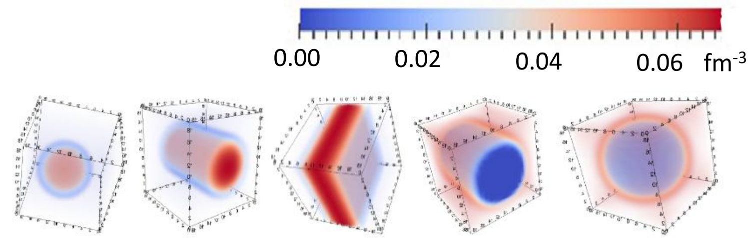
3D → sphere
2D → cylinder
1D → plate



Merit : All calculations are 1D's
↓
Fast calculation & low cost

But …
Only simple structure appear

Calculation in small cell



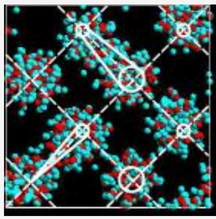
"droplet" "rod" "slab" "tube" "bubble"

Proton density distribution (based on RMF)

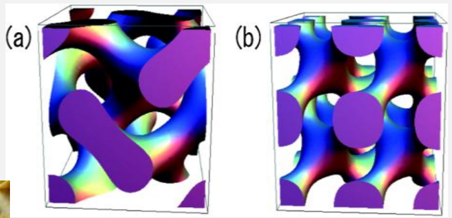
- Fast calculation & low cost
- Several complex structures appear as meta stable state
- Restriction of periodic boundary condition

▪Existence of Non-typical pasta structure

M.Matsuzaki PRC **73, 028801 (2006)**
K.Nakazato,K.Oyamatsu PRL **103, 132501 (2009)**
G.Watanabe et.al PRC **68, 035806 (2003)**



dumbbell
(farfalle)

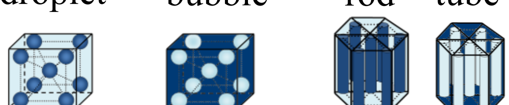


Gyroid double-diamond

▪Crystalline of Pasta structure

Conventional studies

"droplet" "bubble" "rod" "tube"



body-centered cubic honeycomb

K.Oyamatsu, NPA 561 (1993) 431

**Full real space
3D calculation**

Relativistic mean field theory

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m-g_{\sigma}\sigma-g_{\omega}\gamma^{\mu}\partial_{\mu}-g_{\rho}\gamma^{\mu}\tau^{a}\rho_{\mu}^{a}\right)\psi$$
$$+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-\frac{1}{2}m_{\sigma}^{2}\sigma^{2}+\frac{1}{3}bm_{\sigma}(g_{\sigma}\sigma)^{3}+\frac{1}{4}c(g_{\sigma}\sigma)^{3}$$
$$+\frac{1}{2}m_{\omega}^{2}\omega_{\mu}\omega^{\mu}+\frac{1}{2}\left[\frac{1}{2}\left(\partial^{\mu}\omega^{\nu}-\partial^{\nu}\omega^{\mu}\right)\left(\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}\right)\right]$$
$$+\frac{1}{2}m_{\rho}^{2}R_{\mu}R^{\mu}-\frac{1}{4}R_{\mu\nu}R^{\mu\nu}$$
$$-\frac{1}{2}\left[\frac{1}{2}\left(\partial^{\mu}A^{\nu}-\partial^{\nu}A^{\mu}\right)\left(\partial_{\mu}A_{\nu}-\partial_{\nu}A_{\mu}\right)\right]$$

Mean field approximation

$$\langle\sigma\rangle=\sigma$$
$$\langle\omega^{\mu}\rangle=\delta^{\mu,0}\omega^{\mu}$$
$$\langle R^{\mu\nu}\rangle=\delta^{\nu,0}R^{\mu\nu}$$

Thomas-Fermi approximation

&

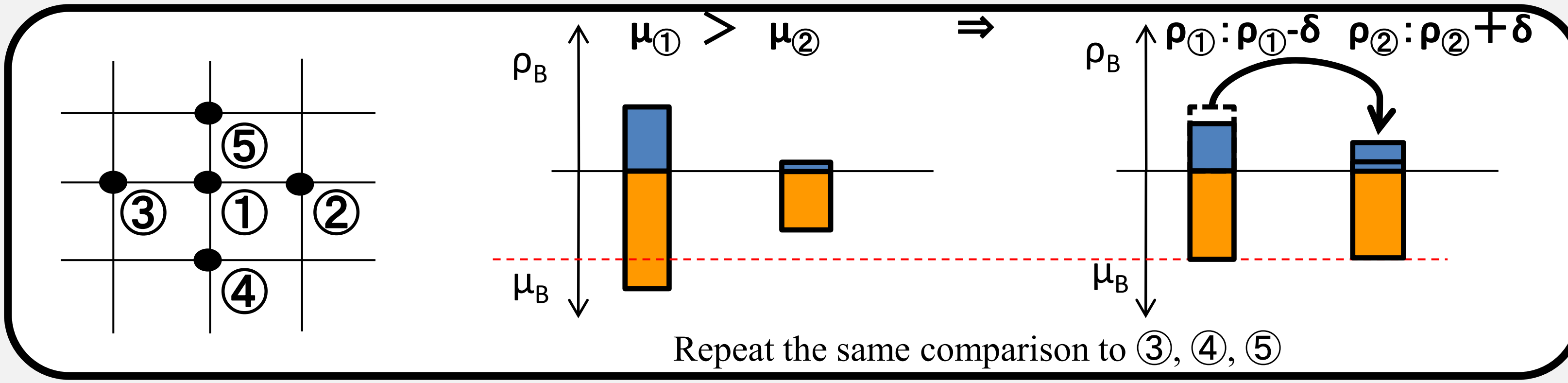
Zero temperature

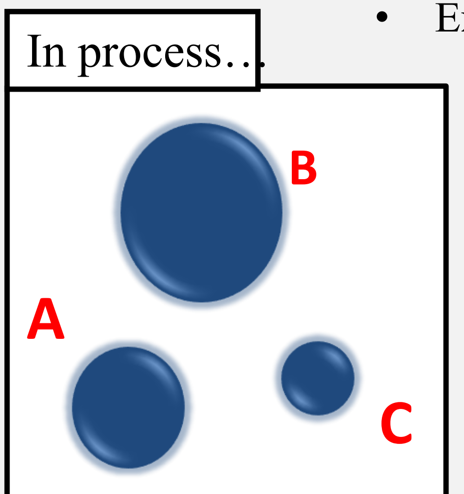
$$-\nabla^{2}\sigma(\vec{r})+m_{\sigma}^{2}\sigma^{2}(\vec{r})=-\frac{dU(\sigma)}{d\sigma}+g_{\sigma N}\left(\rho_{n}^{s}(\vec{r})+\rho_{p}^{s}(\vec{r})\right)$$
$$-\nabla^{2}\omega_{0}(\vec{r})+m_{\omega}^{2}\omega_{0}(\vec{r})=g_{\omega N}\left(\rho_{p}(\vec{r})+\rho_{n}(\vec{r})\right)$$
$$-\nabla^{2}R_{0}(\vec{r})+m_{\rho}^{2}R_{0}(\vec{r})=g_{\rho N}\left(\rho_{p}(\vec{r})-\rho_{n}(\vec{r})\right)$$
$$\nabla^{2}V(\vec{r})=4\pi e^{2}(\rho_{p}(\vec{r})+\rho_{e}(\vec{r}))$$
$$\mu_{p}=\mu_{B}-\mu_{e}=\sqrt{k_{F,p}^{2}+m_{N}^{*2}}+g_{\omega N}\omega_{0}+g_{\rho N}R_{0}-V$$
$$\mu_{n}=\mu_{B}=\sqrt{k_{F,n}^{2}+m_{N}^{*2}}+g_{\omega N}\omega_{0}-g_{\rho N}R_{0}$$
$$\rho_{e}(\vec{r})=-\frac{(V(\vec{r})-\mu_{e})^{3}}{3\pi^{2}}$$

Solutions

- As an initial condition, randomly distribute fermions (n, p, e) over the grid
- We solve coupled differential equations, and simultaneously relax fermions density distributions to attain the uniformity of their chemical potential

$$\mu_p(\vec{r}) = \sqrt{k_{F,p}^2(\vec{r}) + m_N^{*\,2}(\vec{r})} + g_{\omega N}\omega_0(\vec{r}) + g_{\rho N}\rho_0(\vec{r}) - V(\vec{r})$$
$$\mu_n(\vec{r}) = \sqrt{k_{F,n}^2(\vec{r}) + m_N^{*\,2}(\vec{r})} + g_{\omega N}\omega_0(\vec{r}) - g_{\rho N}\rho_0(\vec{r})$$





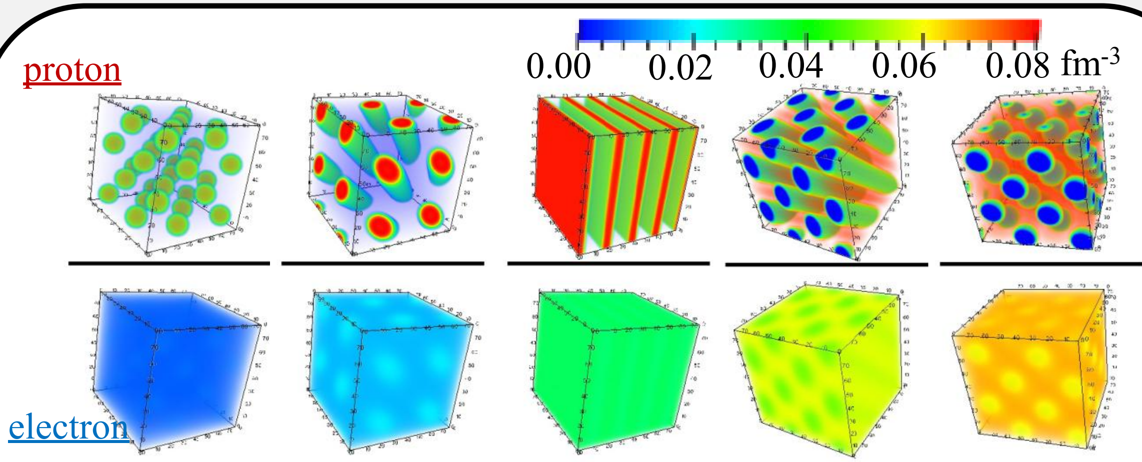
- Exchanging the baryon density, we solve coupled differential equation again.

- We adjust the particle densities between distant grid points chosen randomly so as to avoid making separate droplets with different chemical potentials

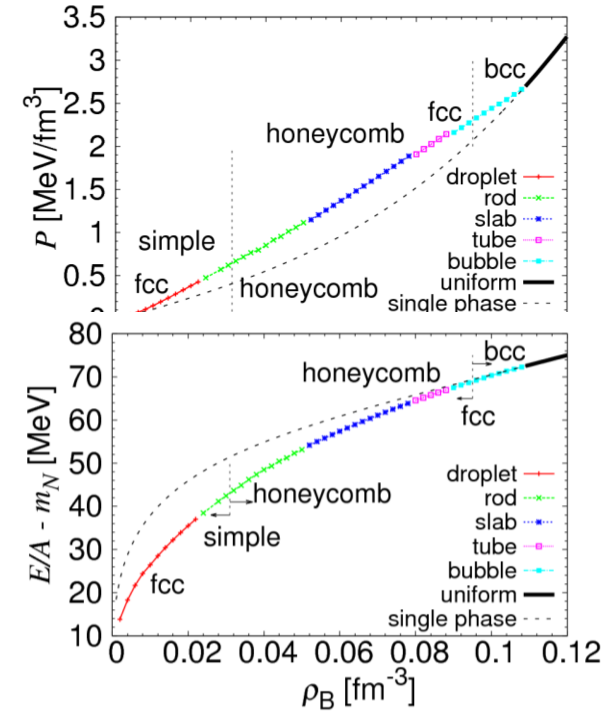
- To save the calculation cost, we did not compare the chemical potential with all grids point and performed the comparison of the chemical potentials with the small regions frequently.

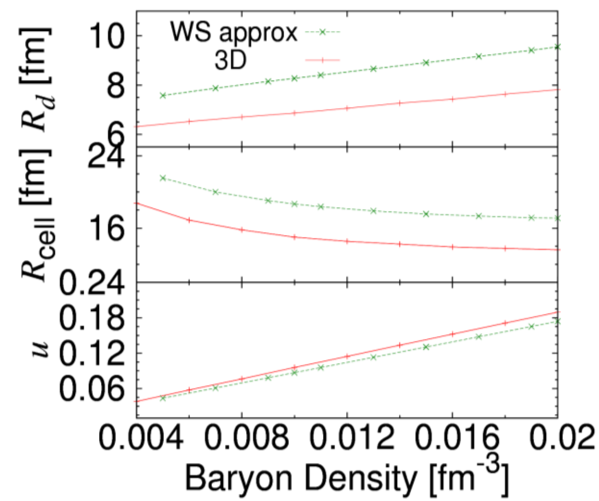
Results : Fixed proton ratio Yp=Z/A (A: total mass , Z: proton number)

Yp=N/A=0.5 (A: total mass, N: proton number)



"droplet"	"rod"	"slab"	"tube"	"bubble"
[fcc]	[honeycomb]		[honeycomb]	[fcc]
ρ_B=0.012 fm^{-3}	0.024 fm^{-3}	0.05 fm^{-3}	0.08 fm^{-3}	0.094 fm^{-3}





- We obtained the typical pasta structures as ground states.
- Crystalline of Pasta structure

New results

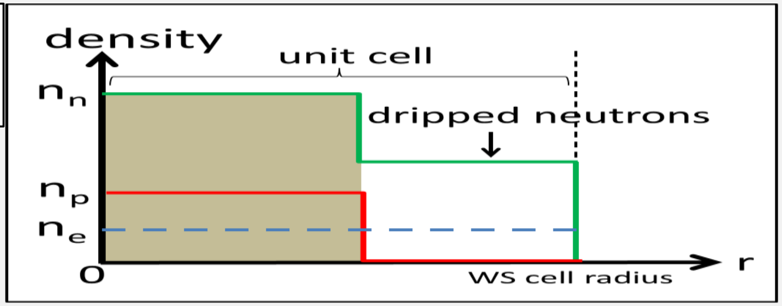
- Crystalline of droplet :
  "fcc" is energetically more favorable than "bcc"
- Size of droplet and lattice constant :
  Smaller than WS cell approximation

Effect of the different size of droplet

In the previous study of "Pasta structure" …

D.G. Ravenhall et.al, PRL. 50, 2069 (1983)
K.Oyamatsu et.al, PTP. 72, 2 (1984)

Liquid drop model



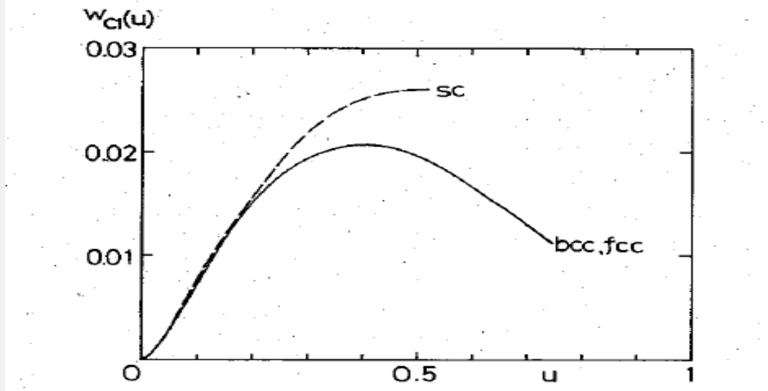


Fig. 1. Relative Coulomb energy $w_{c1}(u)$ for the matter with spherical nuclei. The curves correspond to three lattice types. The curve for bcc is actually lower than the one for fcc only by an indistinguishable amount. The range of u for bcc is $0 \leq u \leq 0.6802$, which is narrower than that for fcc.

$$\text{Total Energy} = (\text{bulk}) + (\text{Surface}) + (\text{Coulomb})$$

Assuming the same size of droplet for "bcc" and "fcc"

All the energy difference is the Coulomb Energy of "bcc" and "fcc"

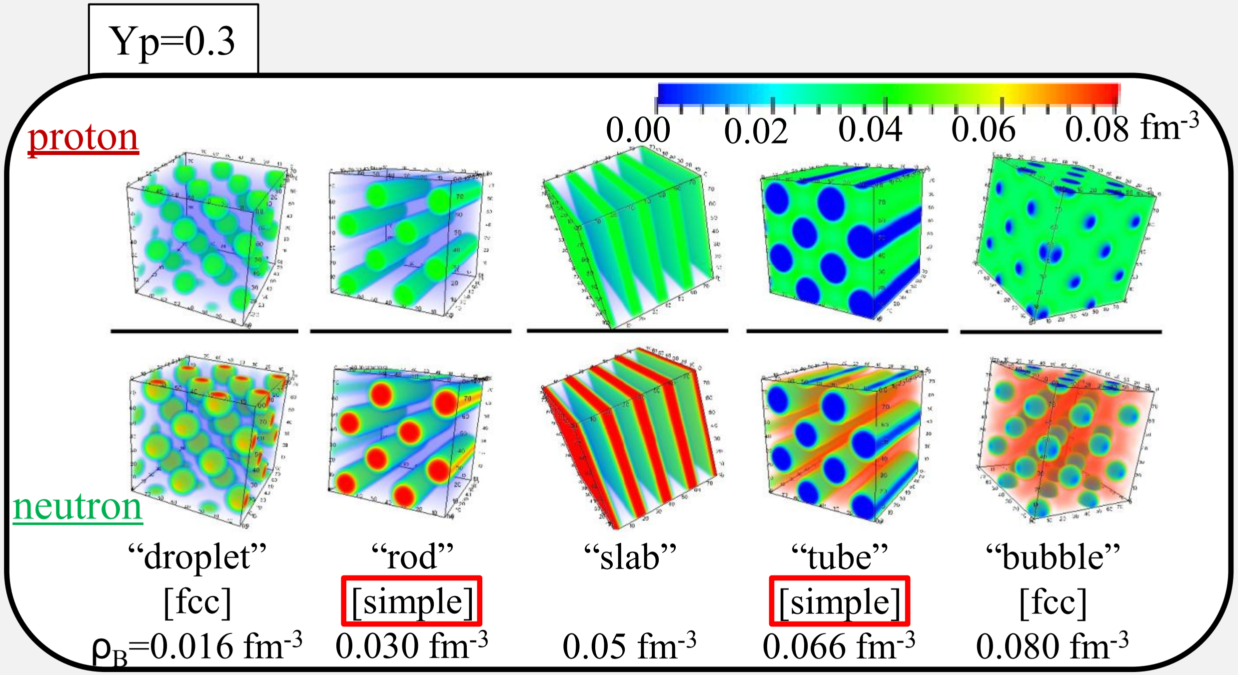
In our calculation …

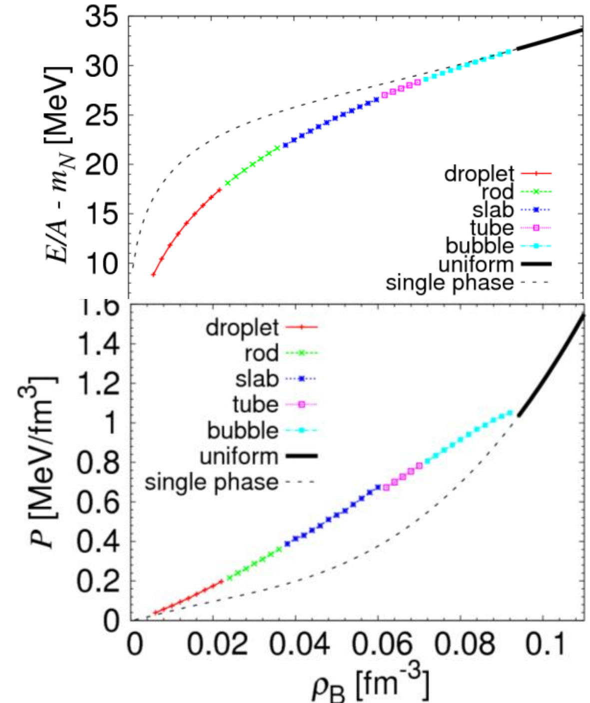
ρ_B [fm^{-3}]	0.010	0.012	0.014	0.016	0.018
R(fcc) [fm]	6.33	6.67	6.79	7.02	7.31
R(bcc) [fm]	6.88	6.95	7.17	7.45	7.62

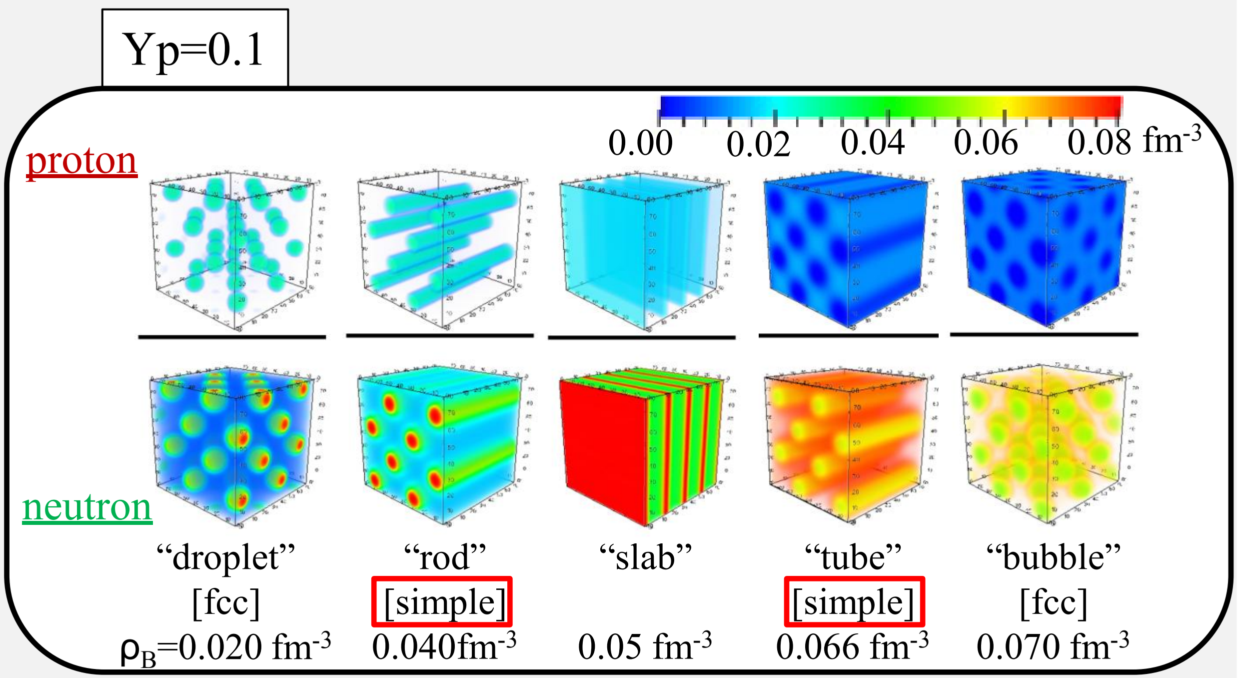
Radius of droplet

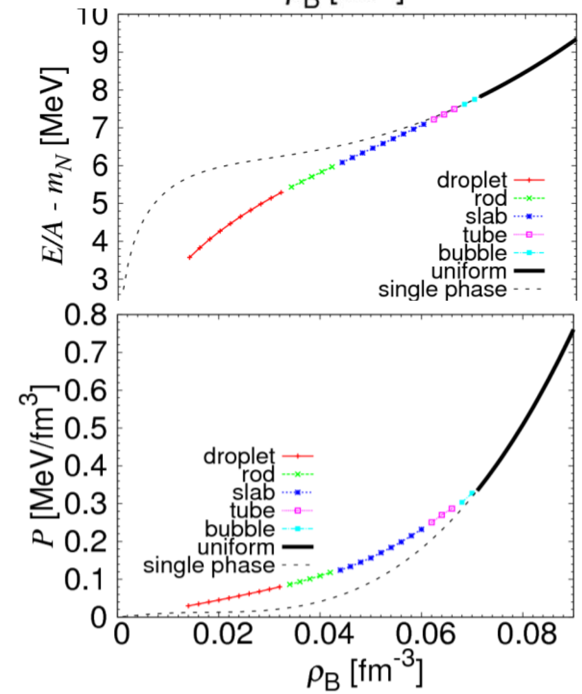
Energy difference between "fcc" and "bcc":
0.2〜0.8 MeV/A
The ratio of Coulomb energy difference is 20%

Need to calculate self-consistently including the size of "Pasta structure"



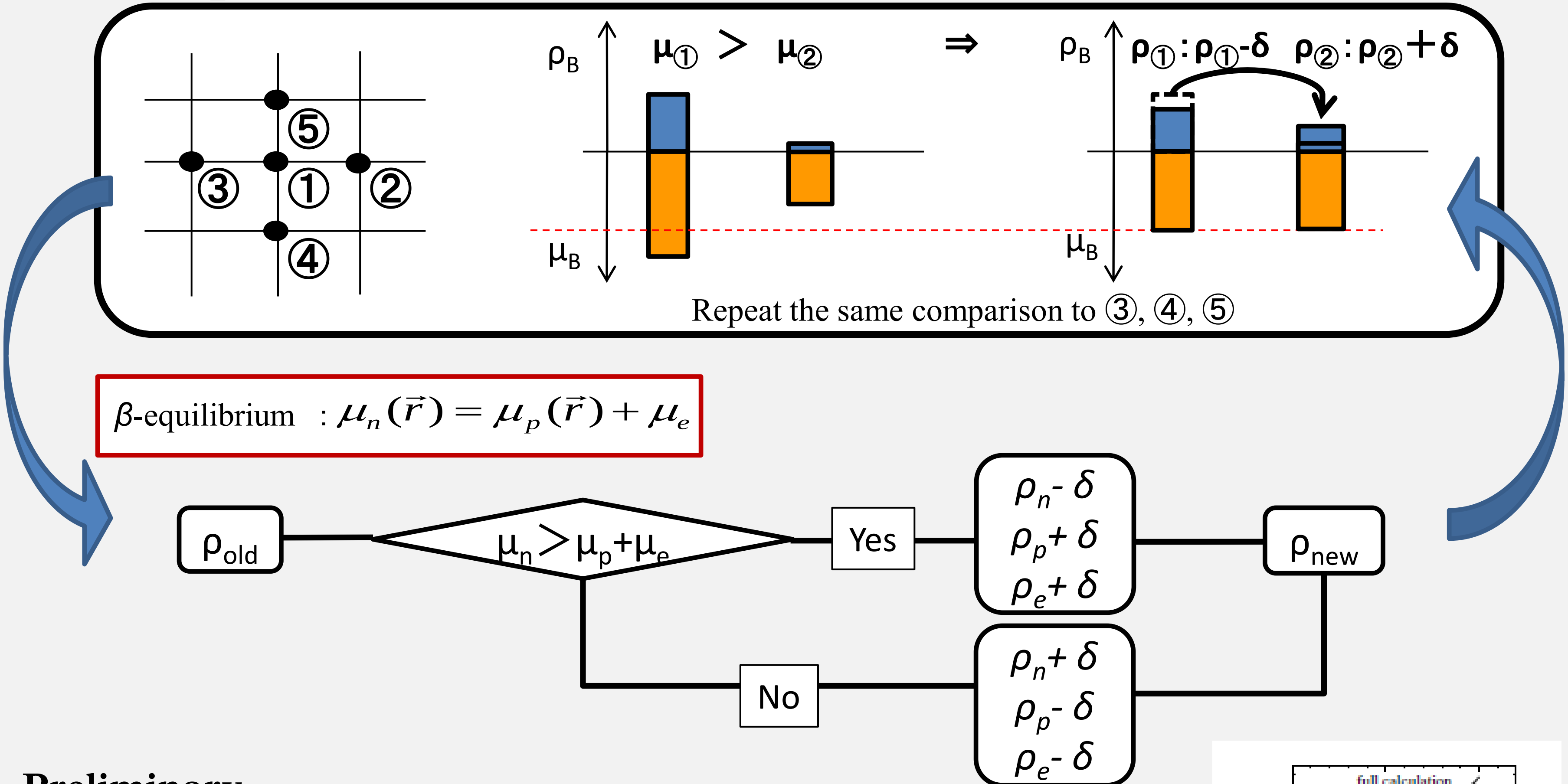






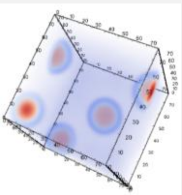
For any proton fraction, typical pasta structures appear as ground states

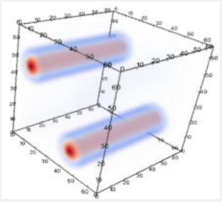
For *β*-equilibrium



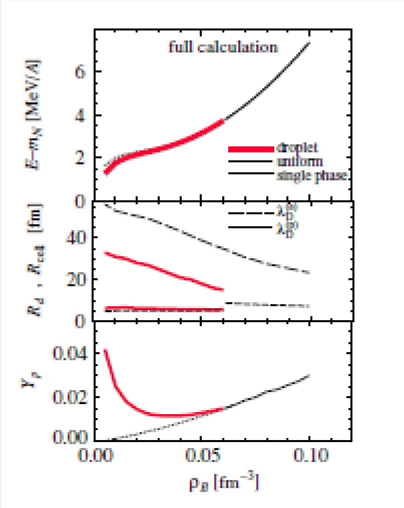
Preliminary

For WS cell approximation and small cell calculation, droplet only appear as ground state





- “droplet” and “rod” appear as ground states
- The comparison of energy between “droplet” and “rod” is not yet carried out.



▪Summary

We explored the pasta structures of low-density nuclear matter based on RMF without any assumption for the structures.

- In 3D calculation, we obtained the typical pasta structures as ground state
- We explored the crystalline structure of pasta
- ”fcc” is energetically more favorable than ”bcc” for droplet.
  ⇒ Difference of droplet’s size for “bcc” and ”fcc”
- In β-equilibrium, we obtained “droplet” & “rod” structure as ground state.

▪Future plans

- Explore of the EOS of β-equilibrium nuclear matter in detail
- Calculation of Shear modulus in the crust region of Neutron star
- Extension to the higher density and finite temperature nuclear matter
- Comparing with QMD calculation result for supernova compression process and local minimum states of our calculation