素核宇融合による計算基礎物理学の進展
- ミクロとマクロのかけ橋の構築 -

Refinement of quark potential models from lattice QCD

格子QCDによるクォーク間ポテンシャルの精密化

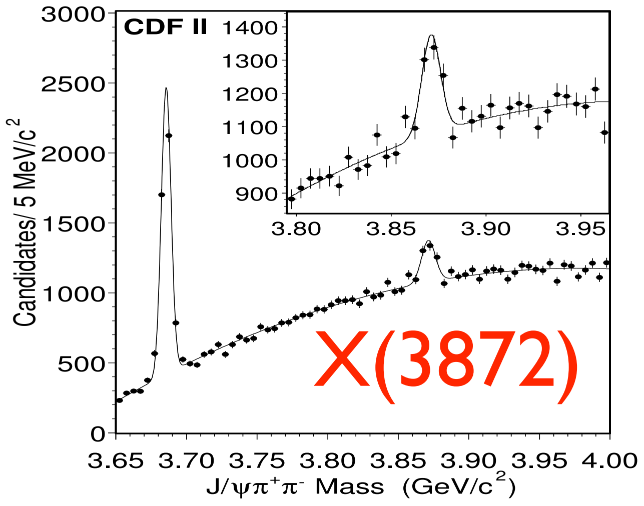
Shoichi Sasaki (Univ. of Tokyo)

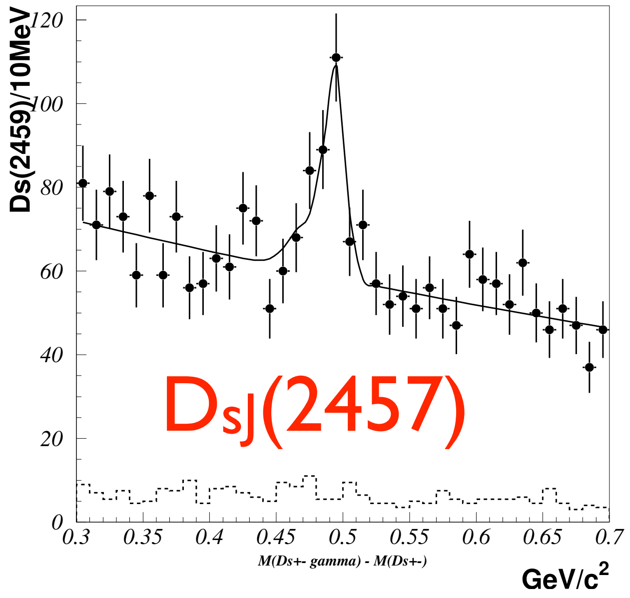
佐々木　勝一　(東大理)

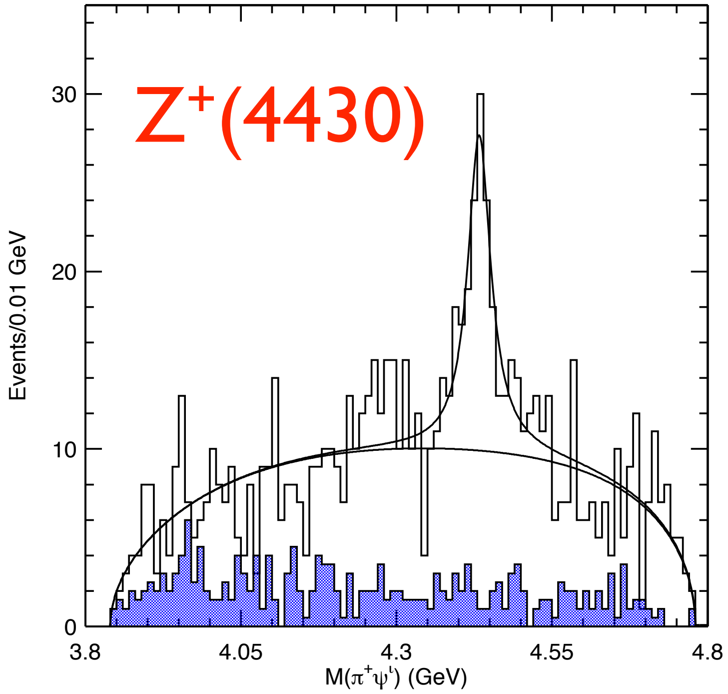
T. Kawanai, SS, PRL 107 (2011) 091601
T. Kawanai, SS, arXiv: 1110.0888

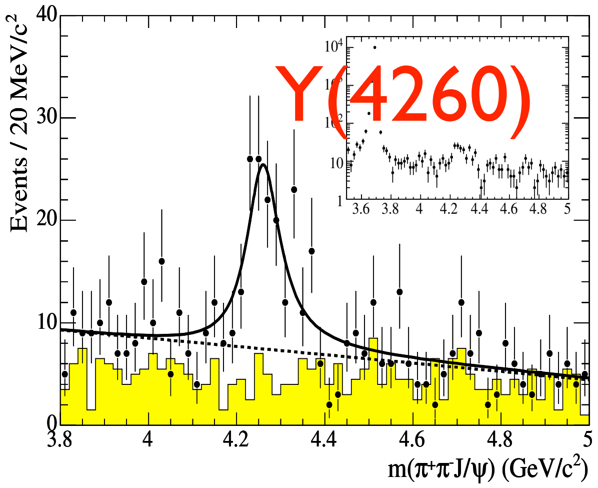
SCHOOL OF SCIENCE
THE UNIVERSITY OF TOKYO

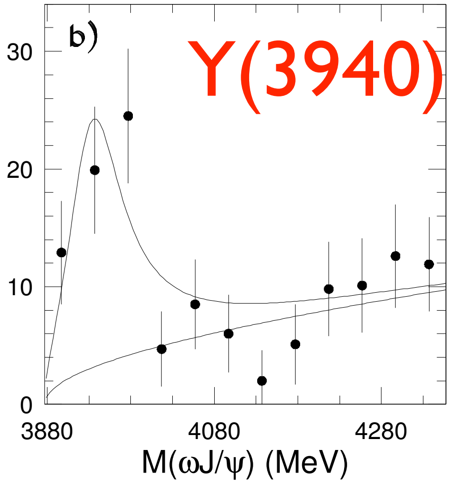
Renaissance of Hadron Spectroscopy

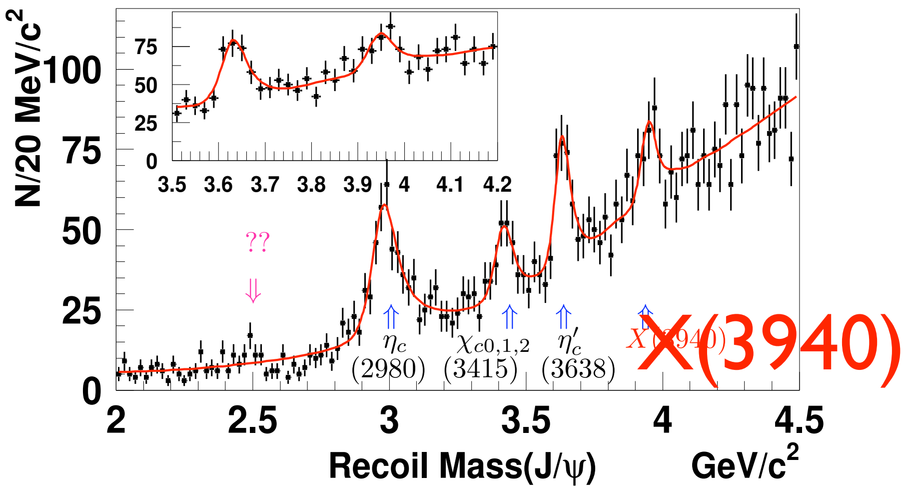


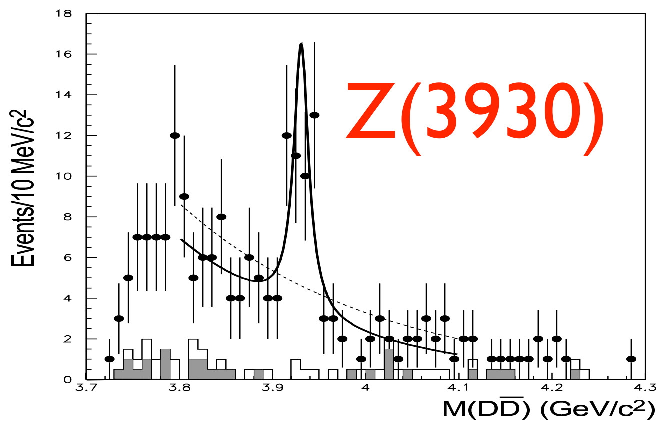


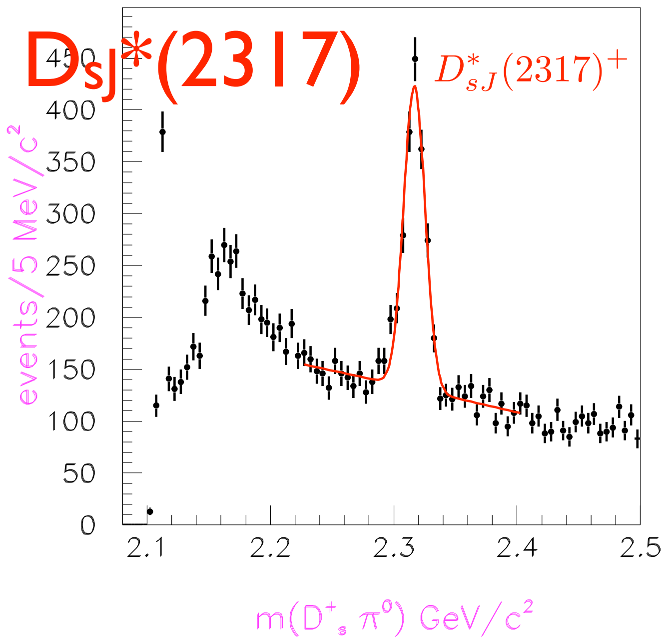






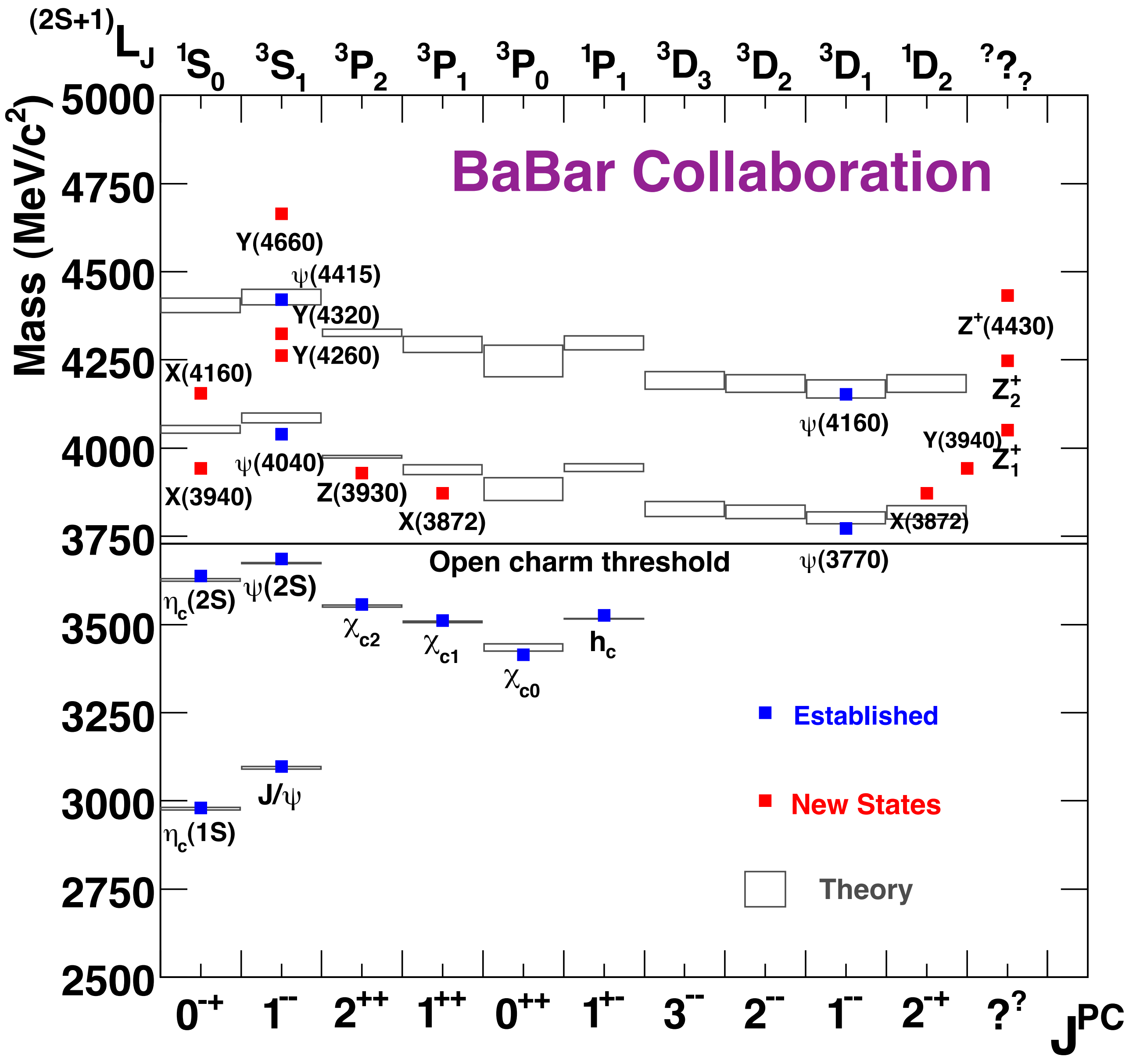






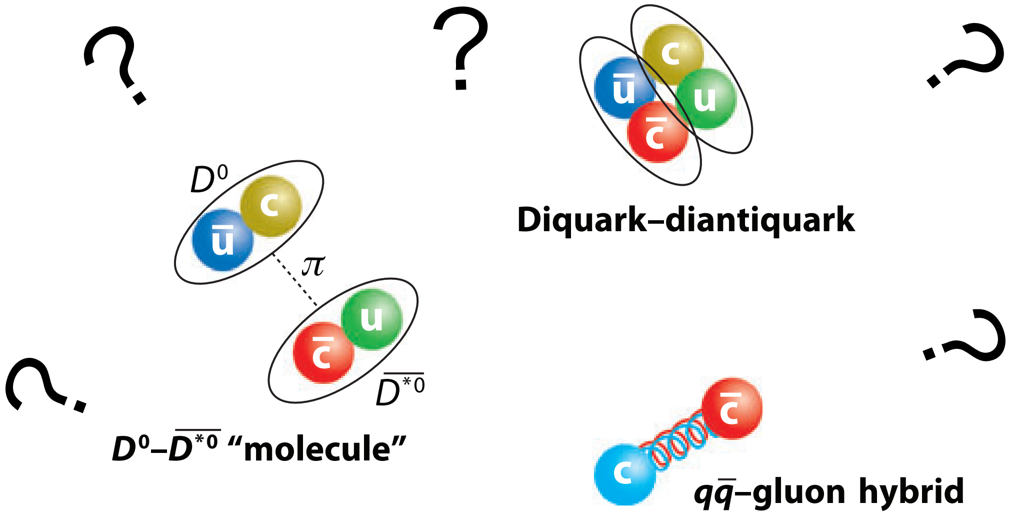
Why back to quark potential models ?

✳ Charmonium-like XYZ mesons are discovered



“Exotic” = “Non-standard”?

XYZ mesons could not be simply explained by **a constituent quark description** as quark and anti-quark bound states



“Standard” states can be defined in potential models

S. Godfrey and S. L. Olsen,
Ann. Rev. Nucl. Part. Sci. 58, 51 (2008)

→ **Does it sound reliable?**

Why back to quark potential models？

✳ Interquark potential in non-relativistic quark potential models

S. Godfrey and N. Isgur, PRD 32, 189 (1985).
T. Barnes, S. Godfrey and E. S. Swanson, PRD 72, 054026 (2005)

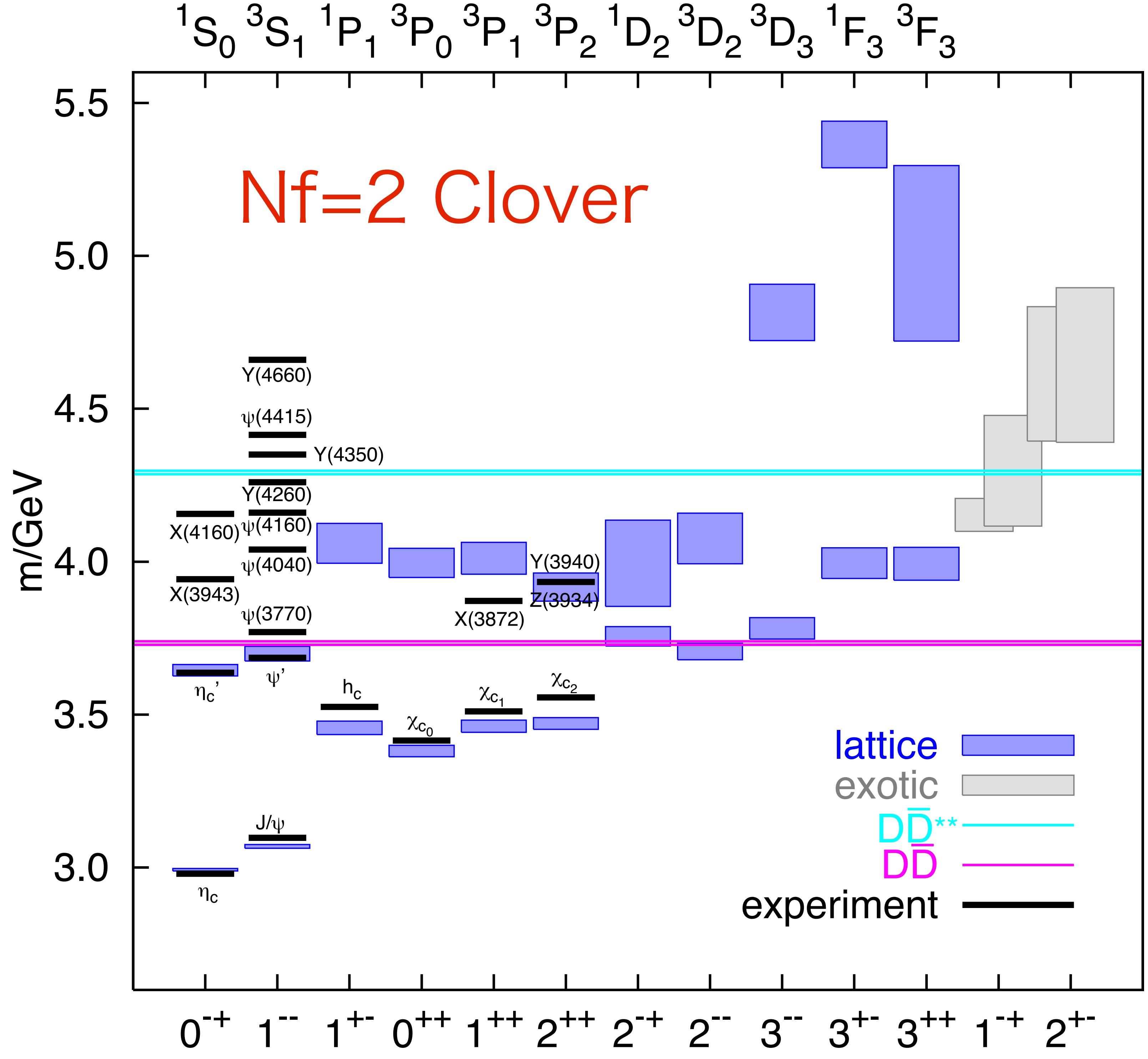
$$V_{c\bar{c}} = \underbrace{-\frac{4}{3}\frac{\alpha_s}{r} + \sigma r}_{\text{Cornell potential}} + \underbrace{\frac{32\pi\alpha_s}{9m_q^2}\delta(r)\mathbf{S}_q\cdot\mathbf{S}_{\bar{q}} + \frac{1}{m_q^2}\left[\left(\frac{2\alpha_s}{r^3} - \frac{b}{2r}\right)\mathbf{L}\cdot\mathbf{S} + \frac{4\alpha_s}{r^3}T\right]}_{\text{spin-dependent potential}}$$

- Spin-spin, tensor, LS terms appear as **corrections in powers of $1/m_q$**
- Spin-dependent potentials determined by **one-gluon exchange at tree level**

→ **There are large theoretical ambiguities for higher-mass charmonia**

The reliable interquark potential derived from lattice QCD is hence desired at the charm quark mass

Status of lattice QCD spectroscopy



lightest pion mass

$m_\pi = 0.28$ GeV

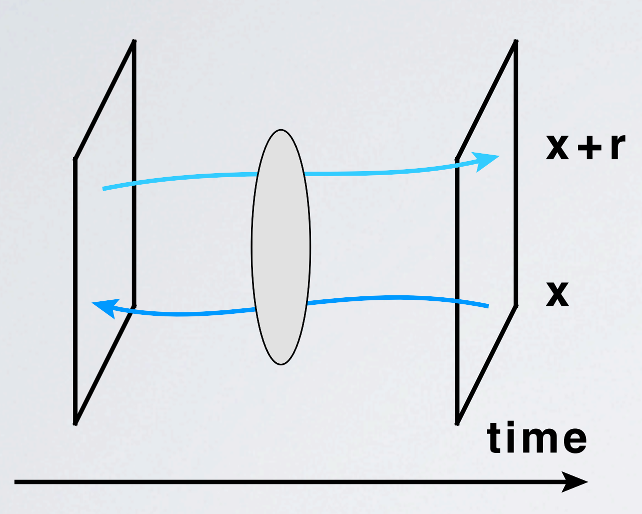
lattice cut off

$1/a = 2.6$ GeV

G. Bali, S. Collins, C. Ehmann, arXiv:1110.2381

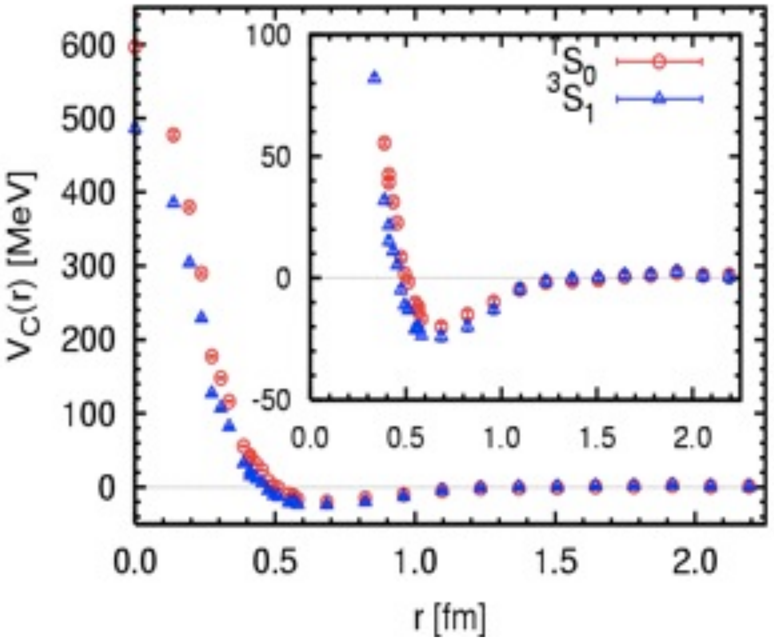
Potential from BS amplitude

- Equal-time BS wave function $\phi_\Gamma(\mathbf{r}) = \sum_{\mathbf{x}} \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|Q\bar{Q}\rangle$



$$
\begin{aligned}
G_{4\mathrm{pt}} &= \sum_{\mathbf{x},\mathbf{x}',\mathbf{y}'} \langle 0|\bar{Q}(\mathbf{x},t)\Gamma Q(\mathbf{x}+r,t)(\bar{Q}(\mathbf{x}',t_{\mathrm{src}})\Gamma Q(\mathbf{y}',t_{\mathrm{src}}))^\dagger|0\rangle \\
&= \sum_{\mathbf{x}}\sum_{n} A_n \langle 0|\bar{Q}(\mathbf{x})\Gamma Q(\mathbf{x}+\mathbf{r})|n\rangle e^{-M_n^\Gamma(t-t_{\mathrm{src}})} \\
&\xrightarrow{t\gg t_{\mathrm{src}}} A_0\phi_\Gamma(\mathbf{r})e^{-M_0^\Gamma(t-t_{\mathrm{src}})}
\end{aligned}
$$

N-N potential



- Schrödinger eq. with non-local potential

$$
-\frac{\nabla^2}{2\mu}\phi_\Gamma(\mathbf{r}) + \int dr' U(\mathbf{r},\mathbf{r}')\phi_\Gamma(\mathbf{r}') = E_\Gamma\phi_\Gamma(\mathbf{r})
$$

- Velocity expansion

$$
U(\mathbf{r}',\mathbf{r}) = \left\{V(r) + V_{\mathrm{S}}(r)\mathbf{S}_Q\cdot\mathbf{S}_{\bar{Q}} + V_{\mathrm{T}}(r)S_{12} + V_{\mathrm{LS}}(r)\mathbf{L}\cdot\mathbf{S} + \mathcal{O}(\nabla^2)\right\}\delta(\mathbf{r}'-\mathbf{r})
$$

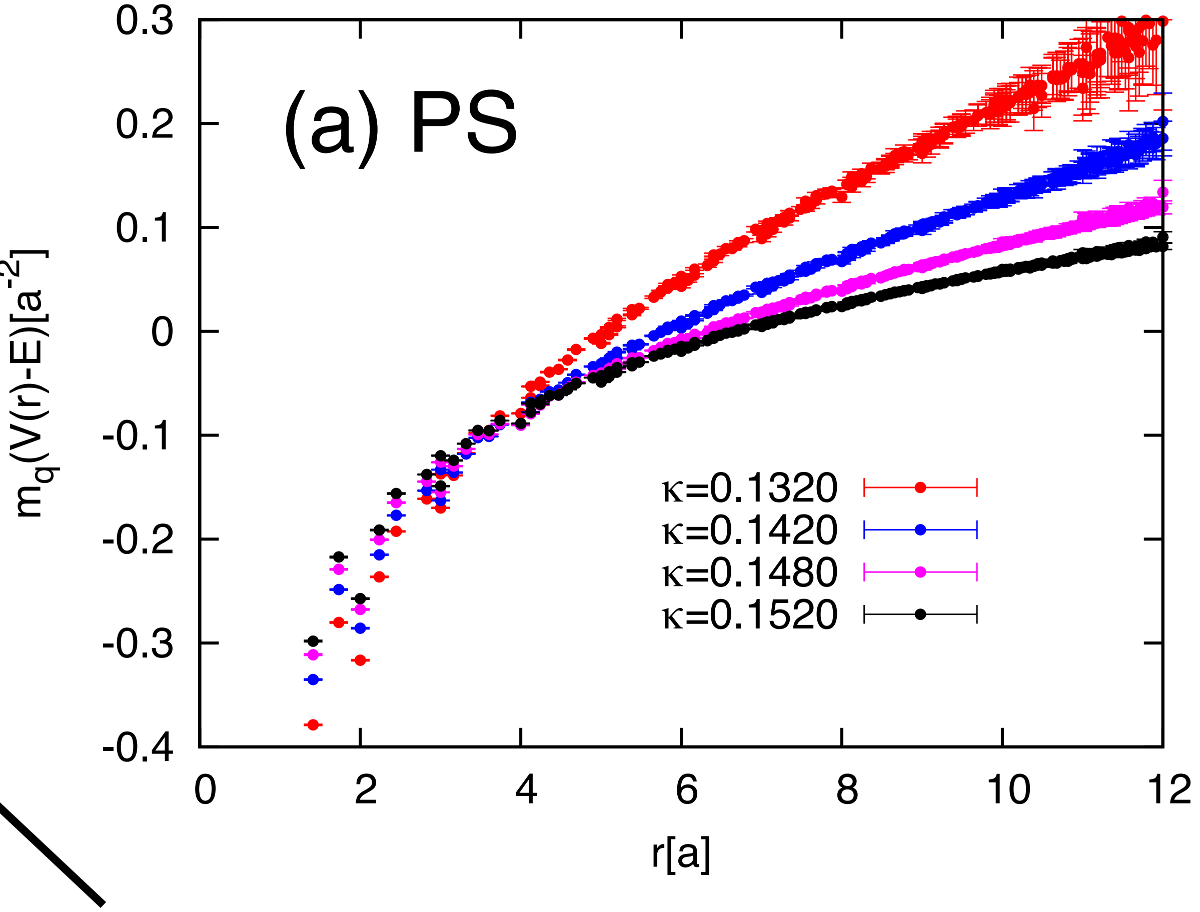
N. Ishii, S. Aoki, and T. Hatsuda, Phys. Rev. Lett. 99 (2007) 022001.
S. Aoki, T. Hatsuda and N. Ishii, Prog. Theor. Phys. 123 (2010) 89

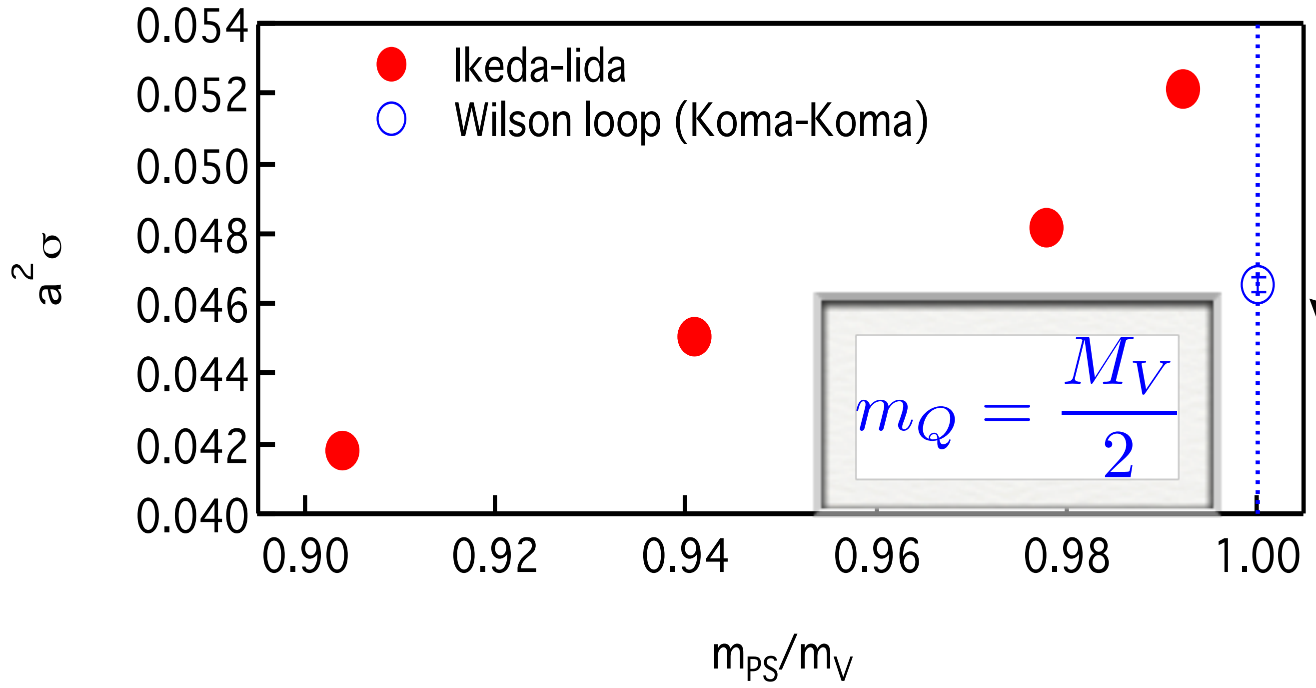
$Q\overline{Q}$ potential from BS wave func.

- Ikeda-Iida, arXiv:1011.2866 & 1102.2097

Cornell-like behavior!

$$\frac{\nabla^2 \phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)} = m_Q \left[V(r) - E \right]$$





$$m_Q = \frac{M_V}{2}$$

Wilson loops

Inconsistent with the Wilson loops in the $m_Q \rightarrow \infty$ limit

Novel determination of quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

$$\left\{-\frac{\nabla^2}{m_Q}+V_{Q\overline{Q}}(r)+\mathbf{S}_Q\cdot\mathbf{S}_{\overline{Q}}V_{\mathrm{spin}}(r)\right\}\phi_\Gamma(r)=E_\Gamma\phi_\Gamma(r)\quad\text{for}\quad\Gamma=\mathrm{PS},\mathrm{V}$$

Q. How can we determine a quark mass in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

$$V_{\mathrm{spin}}(r)-\Delta E_{\mathrm{hyp}}=\frac{1}{m_Q}\left(\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}-\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}\right)$$

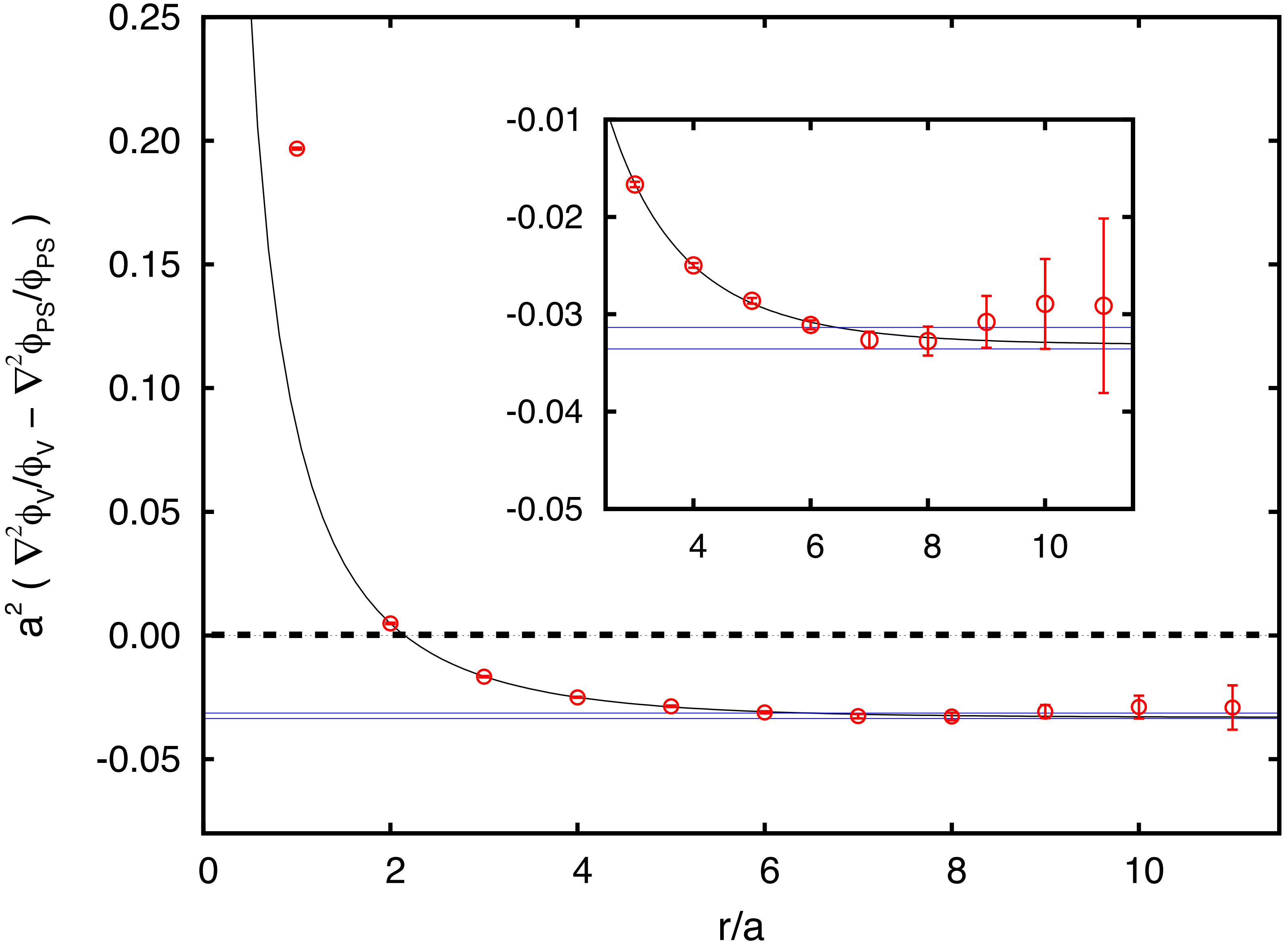
Under a simple, but reasonable assumption of $\lim_{r\to\infty}V_{\mathrm{spin}}(r)=0$

$$\boxed{m_Q=\lim_{r\to\infty}\frac{1}{\Delta E_{\mathrm{hyp}}}\left(\frac{\nabla^2\phi_{\mathrm{PS}}(r)}{\phi_{\mathrm{PS}}(r)}-\frac{\nabla^2\phi_{\mathrm{V}}(r)}{\phi_{\mathrm{V}}(r)}\right)}$$

Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

$$m_Q = \lim_{r\to\infty} \frac{1}{\Delta E_{\rm hyp}} \left(\frac{\nabla^2 \phi_{\rm PS}(r)}{\phi_{\rm PS}(r)} - \frac{\nabla^2 \phi_{\rm V}(r)}{\phi_{\rm V}(r)} \right)$$



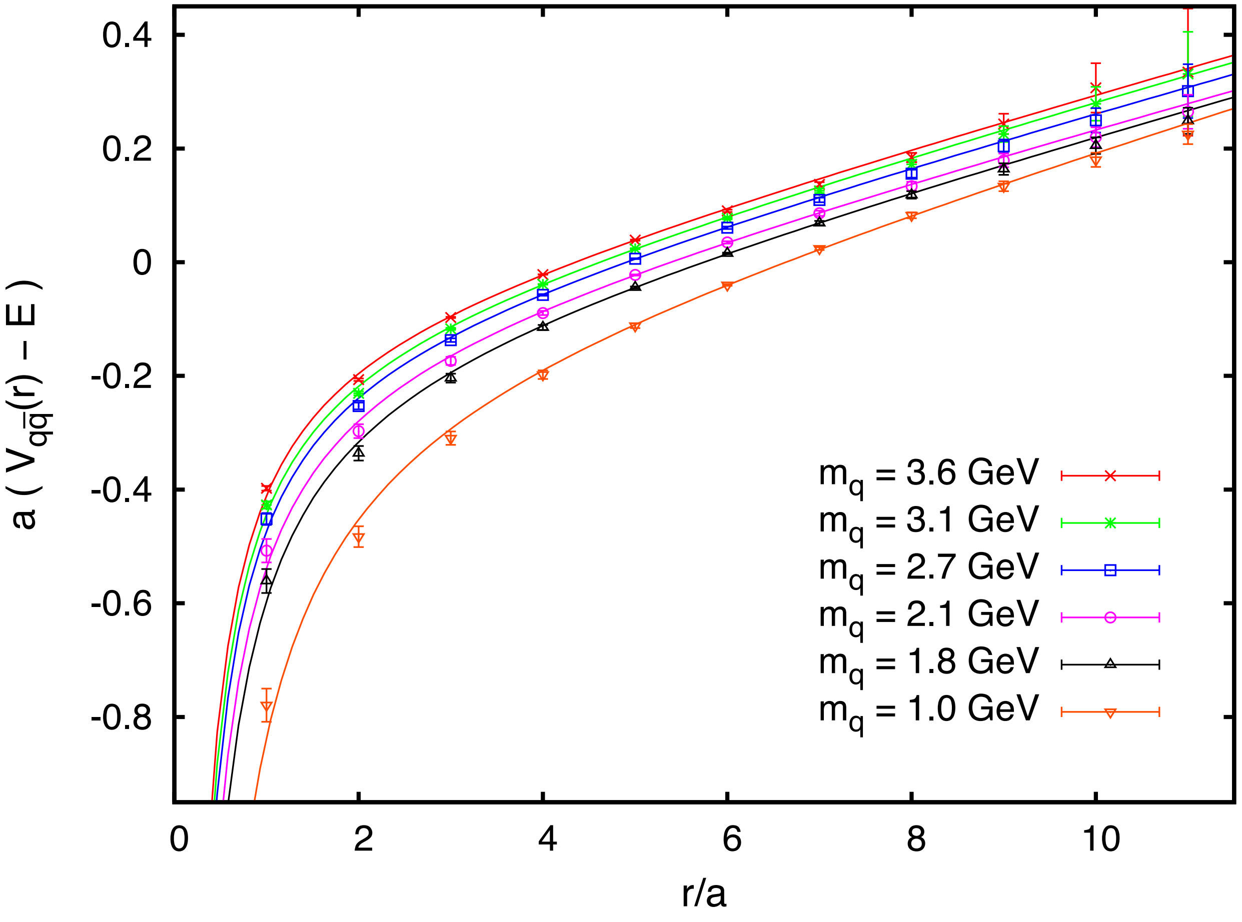
$$\lim_{r\to\infty} V_{\rm spin}(r) = 0$$

$$\updownarrow \quad -m_Q \Delta E_{\rm hyp}$$

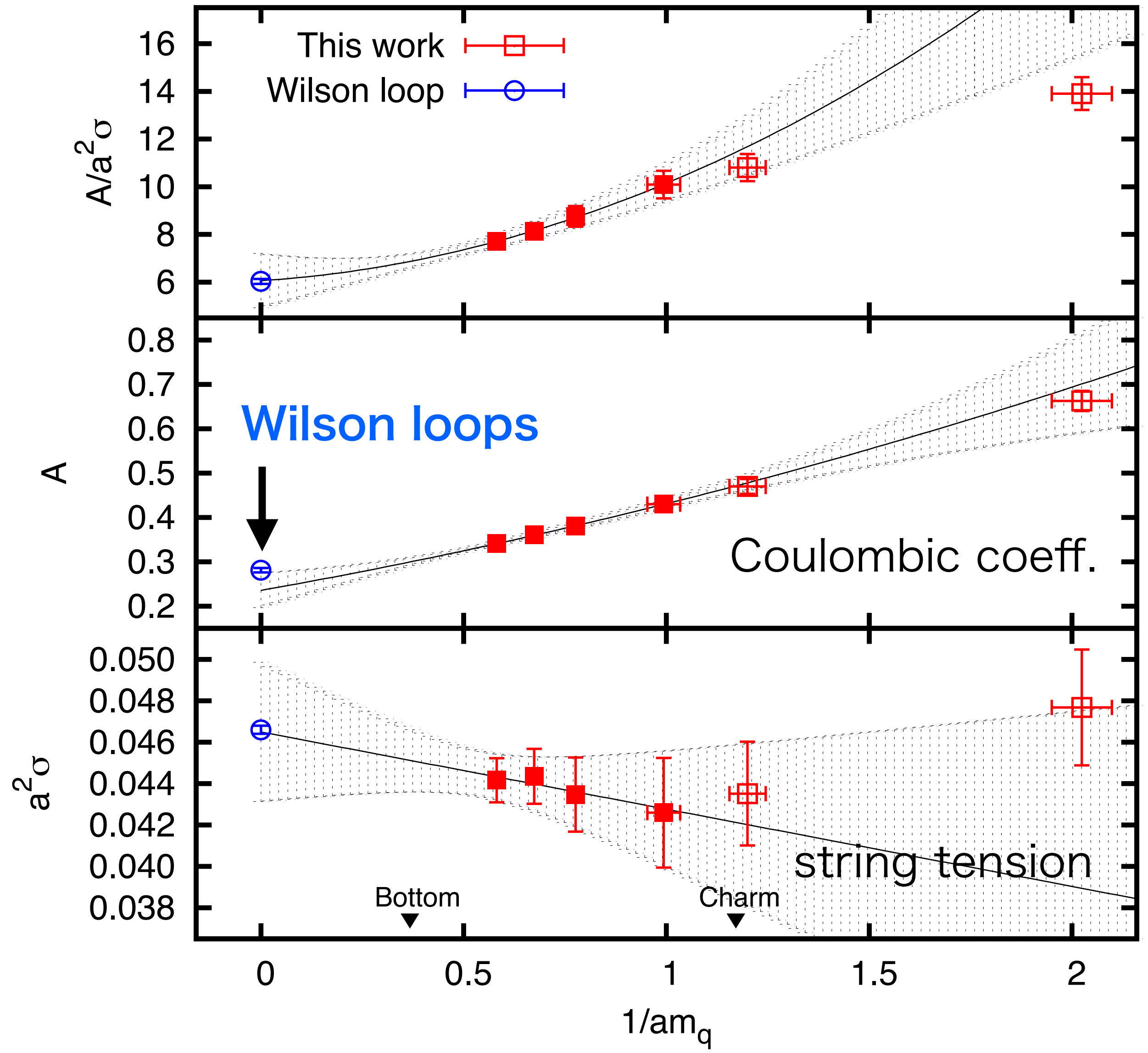
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

$$V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0$$



Quench + RHQ



Consistent with the Wilson loops in the $m_q \to \infty$ limit

How to treat heavy quarks

- Heavy quark mass introduces discretization errors of **O((ma)^n)**

✓ **At charm quark, it becomes severe:**

m_c ∼1.5 GeV and 1/a ∼2 GeV, then $m_c a$ ∼ O(1)

- Relativistic heavy quark (RHQ) approach:

A.X. El-Khadra, A.S. Kronfeld, P.B. Mackenzie (1997)

✓ **All O((ma)^n) and O(aΛ) errors are removed by the appropriate choice of six canonical parameters {m_0, ζ, r_t, r_s, C_B, C_E}**

$$S_{\rm lat} = \sum_{n,n'} \bar{\psi}_n \mathcal{K}_{n,n'} \psi_{n'}$$

explicit breaking of axis-interchange symmetry

$$\mathcal{K} = m_0 + \gamma_0 D_0 + \zeta \gamma_i D_i - \frac{r_t}{2} D_0^2 - \frac{r_s}{2} D_i^2 + C_B \frac{i}{4} \sigma_{ij} F_{ij} + C_E \frac{i}{2} \sigma_{0i} F_{0i}$$

✓ **We follow the Tsukuba procedure to determine parameters**

S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)

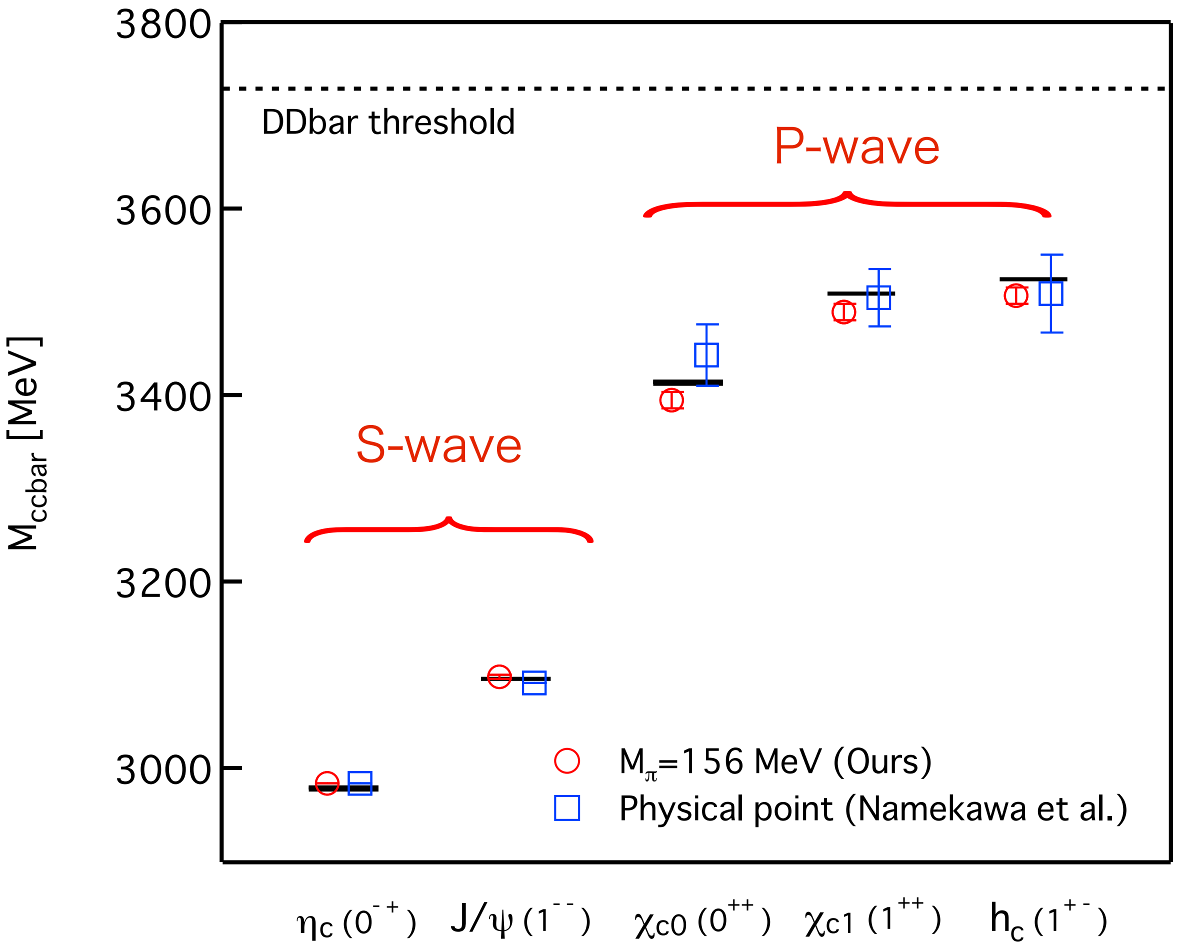
Tuning RHQ parameters for full QCD

- RHQ action (Tsukuba-type) with 5 parameters
  - ✳ PACS-CS configurations at m_π=156 MeV
  - ✳ Relativistic Heavy Quark (RHQ) action for charm
    - ✓ 32^3 x 64 lattice
    - ✓ a = 0.0907(13) fm
    - ✓ La ~ 2.9 fm
    - ✓ 198 configs

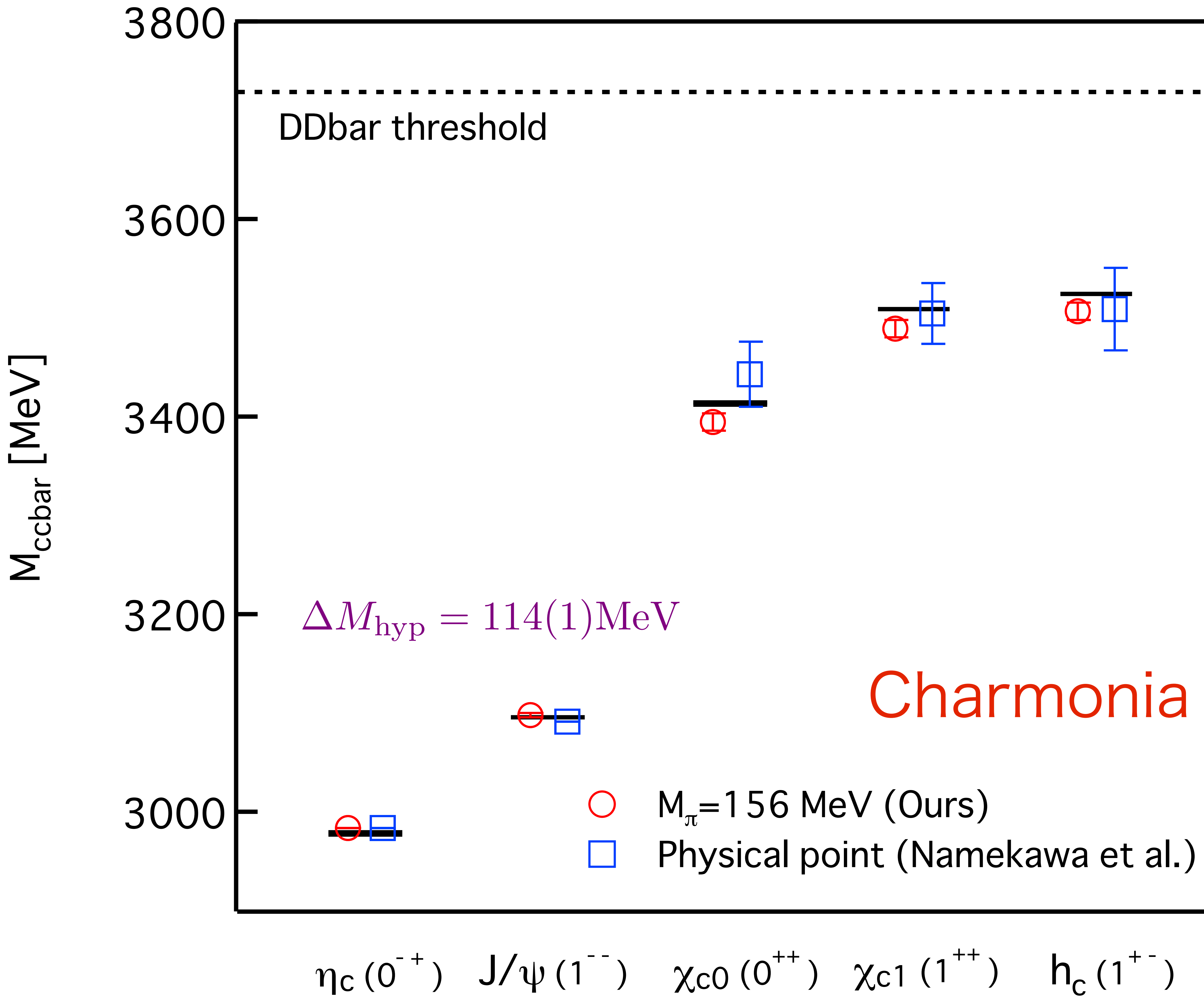
➡ $$\frac{1}{4}(M_{\eta_c} + 3M_{J/\psi}) = 3.070(1)\ \mathrm{GeV}$$

➡ $$\Delta M_{\mathrm{hyp}} = 114(1)\mathrm{MeV}$$

✓ $$c_{\mathrm{eff}}^2 = 1.04(5)$$



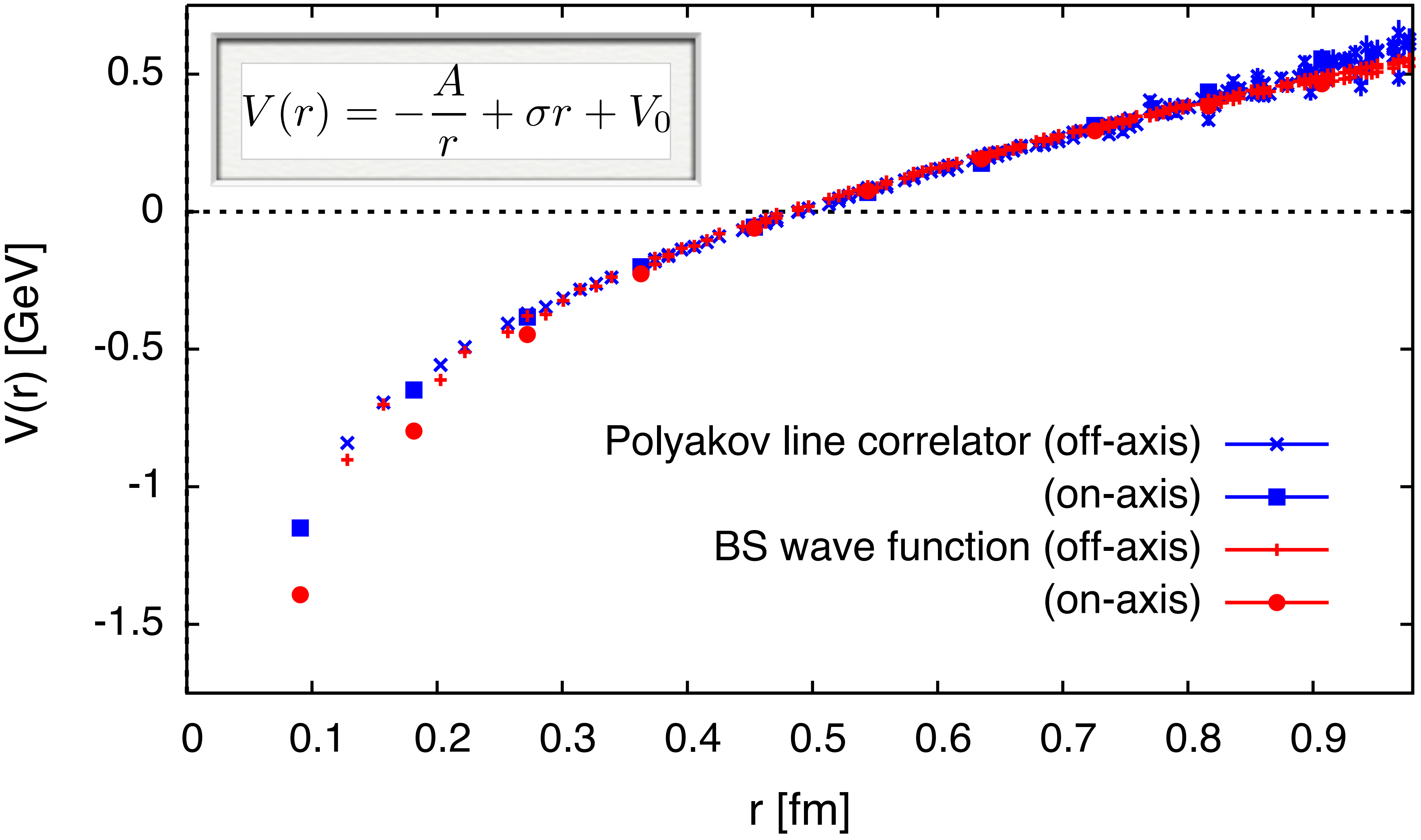
Namekawa et al., (PACS-CS), arXiv:1104.4600


3800
3600
3400
3200
3000
M_{ccbar} [MeV]
DDbar threshold
$\Delta M_{\rm hyp} = 114(1)$MeV
Charmonia
M_π=156 MeV (Ours)
Physical point (Namekawa et al.)
η_c (0^{-+})
J/ψ (1^{--})
χ_{c0} (0^{++})
χ_{c1} (1^{++})
h_c (1^{+-})

Charmonium potential from full QCD

- Kawanai-Sasaki, arXiv:1110.0888

✳ PACS-CS configurations at m_π=156 MeV



$A_{c\bar{c}} = 0.813(22)$

$\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV

$A_\infty = 0.403(10)$

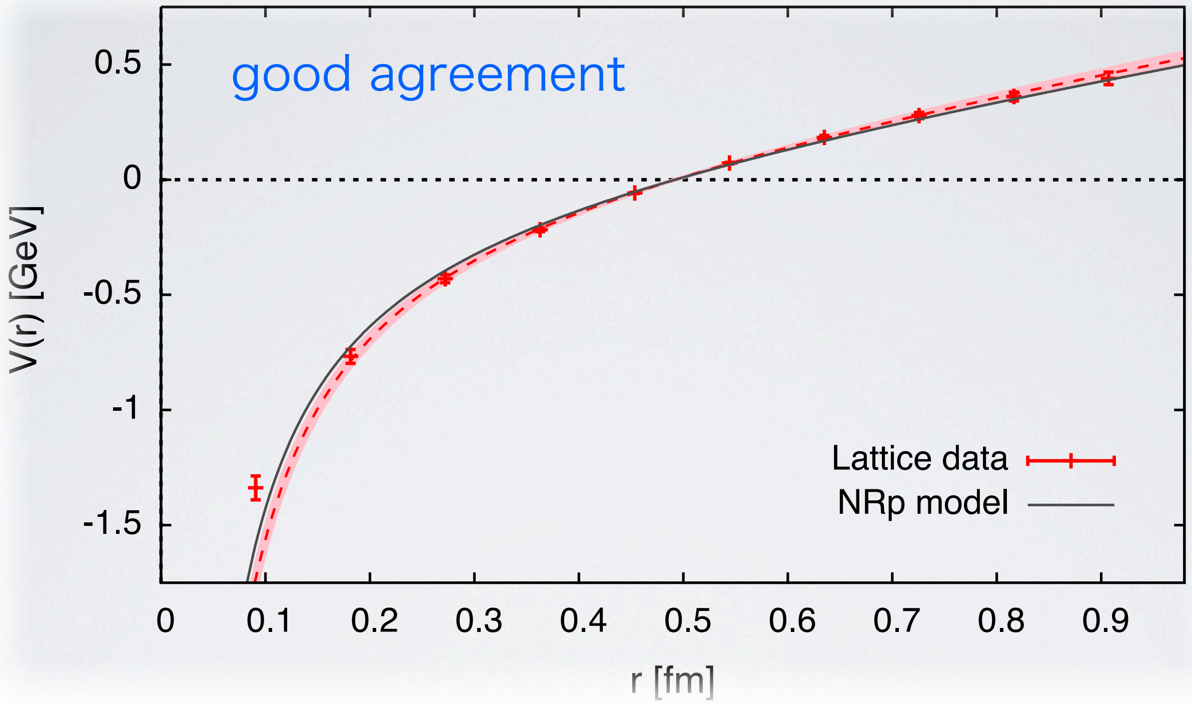
$\sqrt{\sigma_\infty} = 0.462(2)$ GeV

Polyakov loop correlator

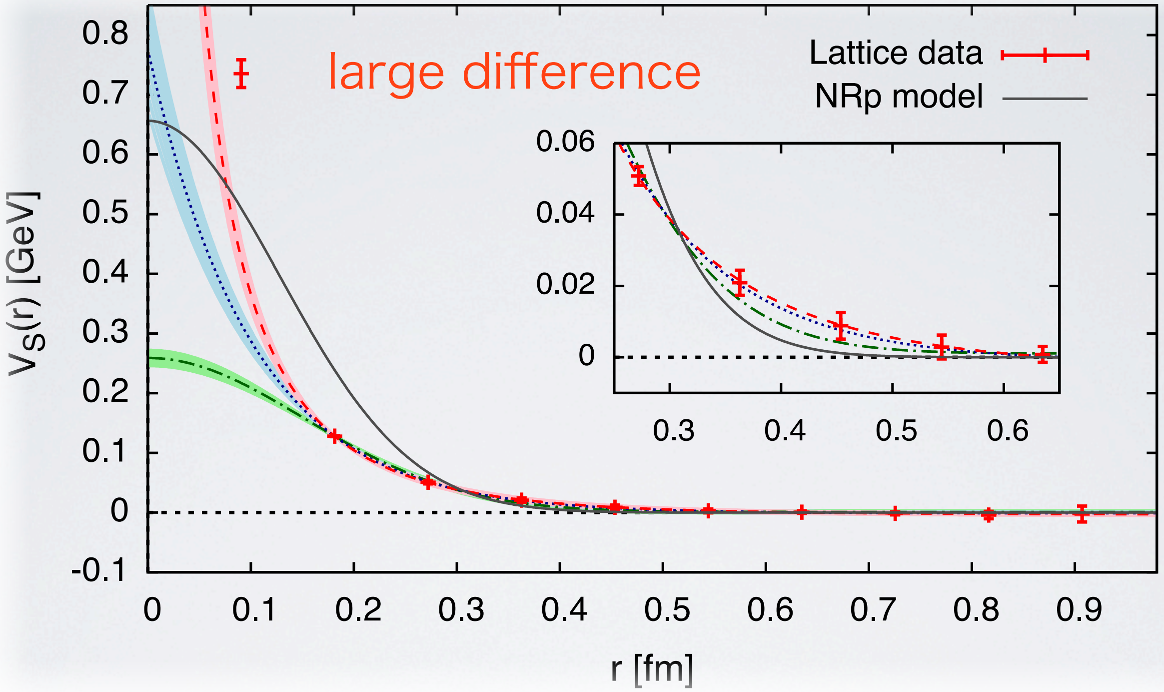
Charmonium potential from full QCD

Kawanai-Sasaki, arXiv:1110.0888

Spin-independent cc^{bar} potential



spin-spin cc^{bar} potential



lattice results $A_{c\bar{c}} = 0.813(22)$

$\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV

NR quark model $A_{\mathrm{NRp}} = 0.7281$

$\sqrt{\sigma_{\mathrm{NRp}}} = 0.3775$ GeV

$$V_{\mathrm{S}}(r) = \begin{cases} \alpha \exp(-\beta r)/r & : \ \text{Yukawa form} \\ \alpha \exp(-\beta r) & : \ \text{Exponential form} \\ \alpha \exp(-\beta r^2) & : \ \text{Gaussian form.} \end{cases}$$

functional form	α	β	χ^2/dof
Yukawa-type	0.287(8)	0.894(32) GeV	7.28
Exponential-type	0.825(19) GeV	1.982(24) GeV	1.46
Gaussian-type	0.314(4) GeV	1.020(11) GeV^2	22.79

Refinement of spin-dependent potentials

→ change the fine structure of charmonia

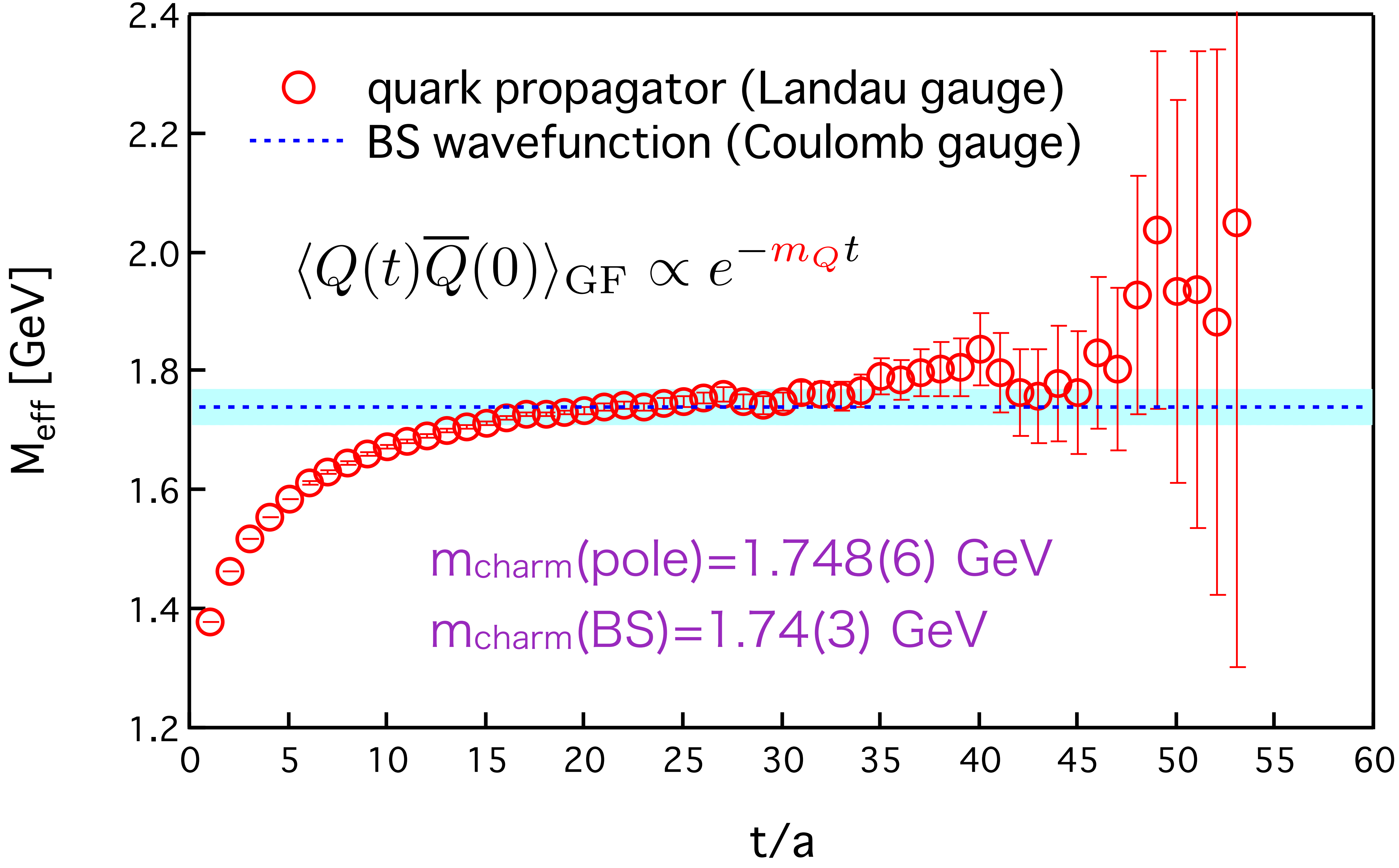
Non-relativistic potential model

T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Comment on two topics

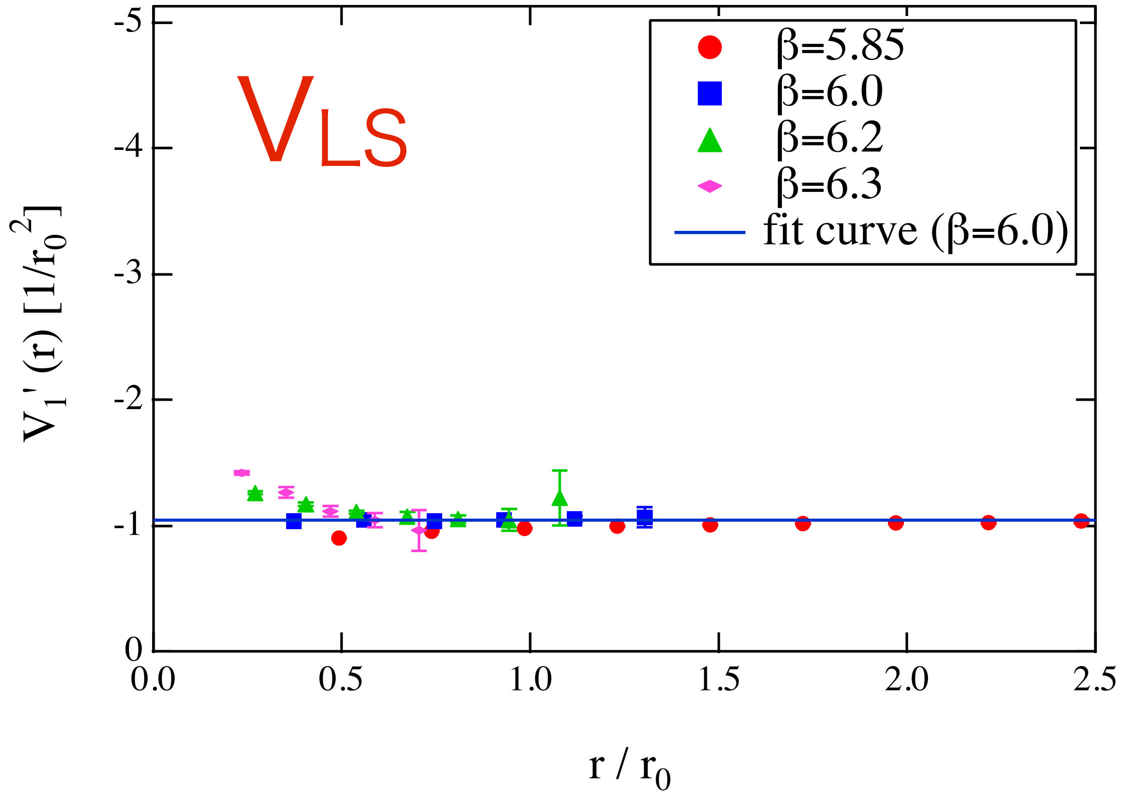
- Revisit of “quark mass”
- Spin-spin potential issue in the Wilson loop approach

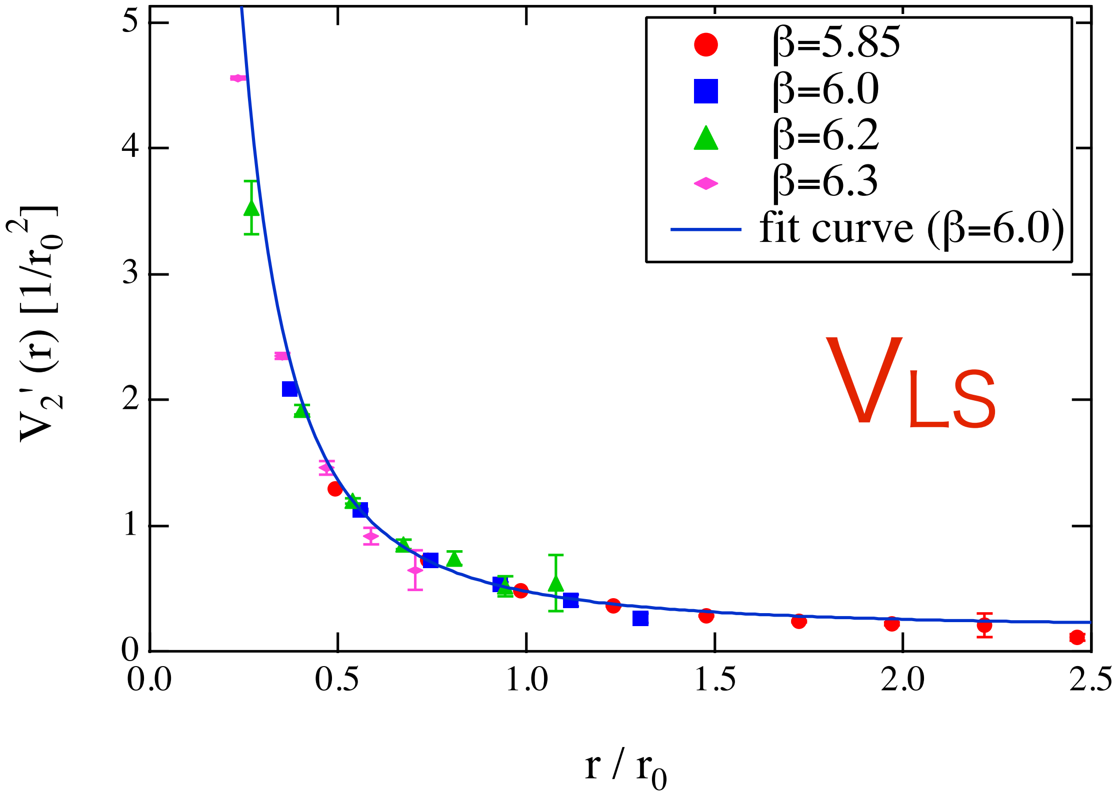
What does “quark mass” correspond to ?

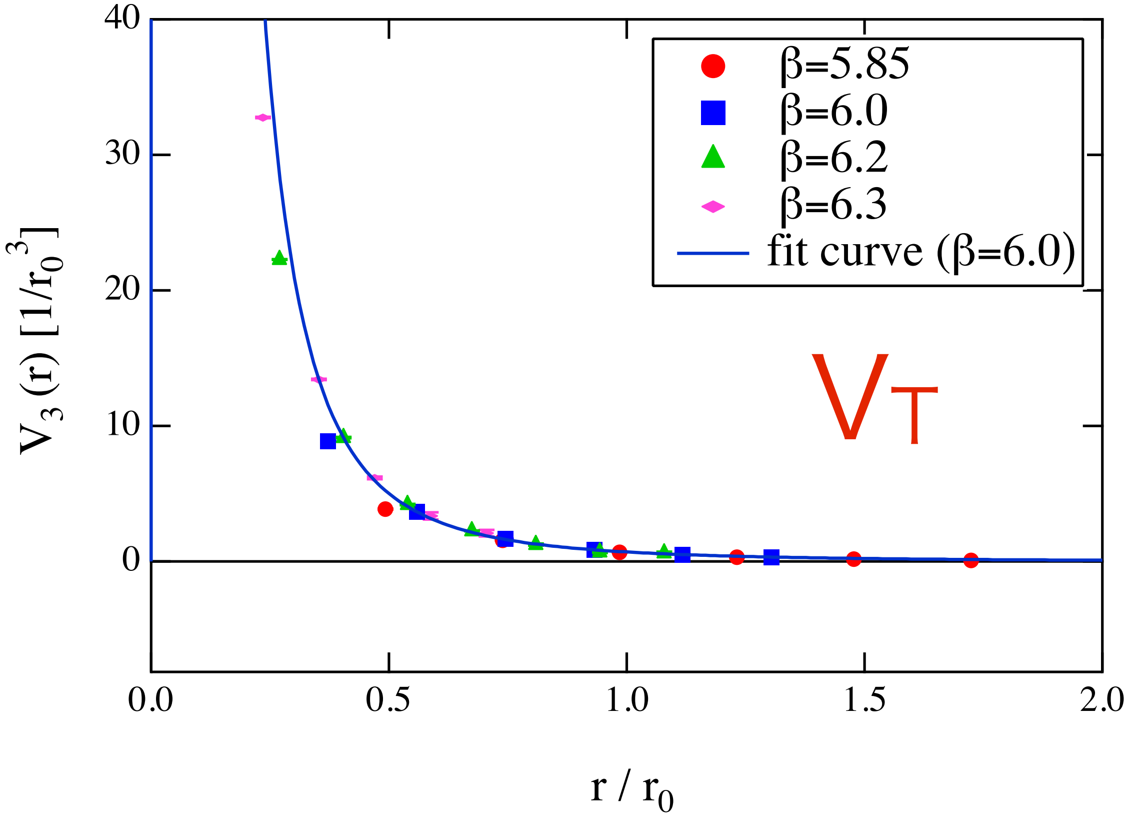


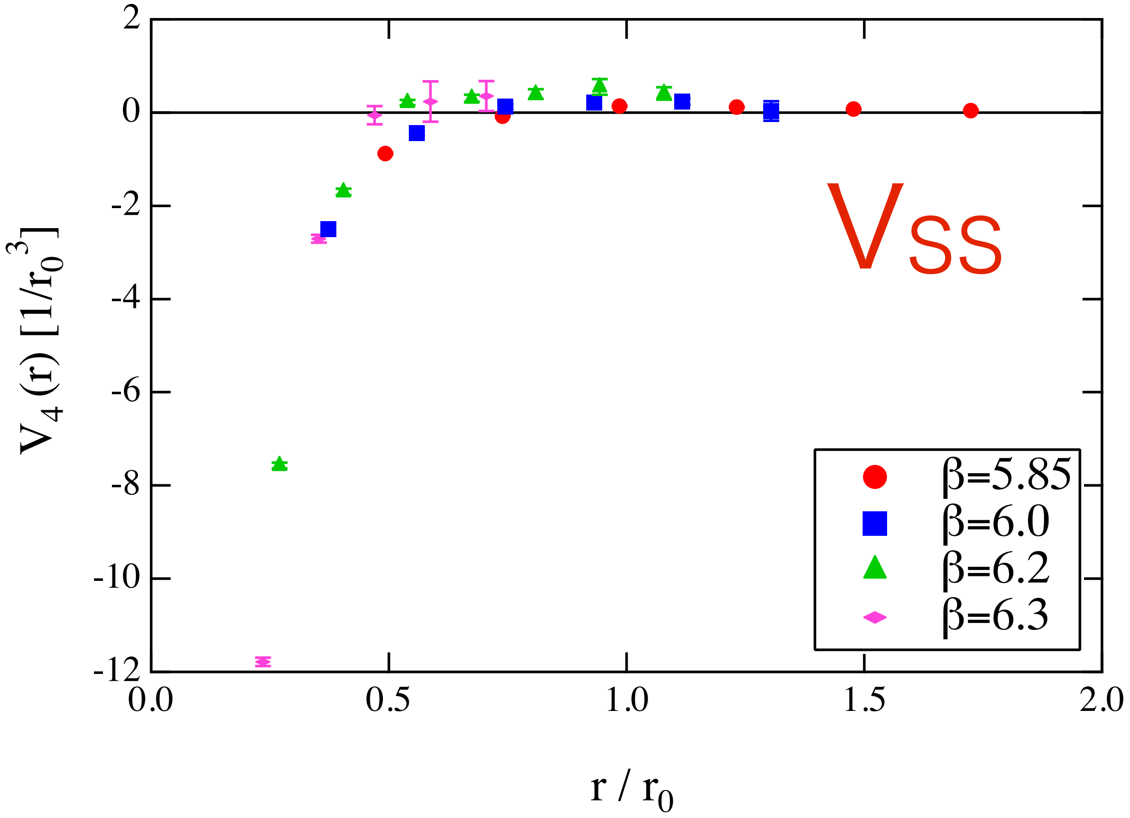
Spatial information = Temporal information

Spin-dependent potentials









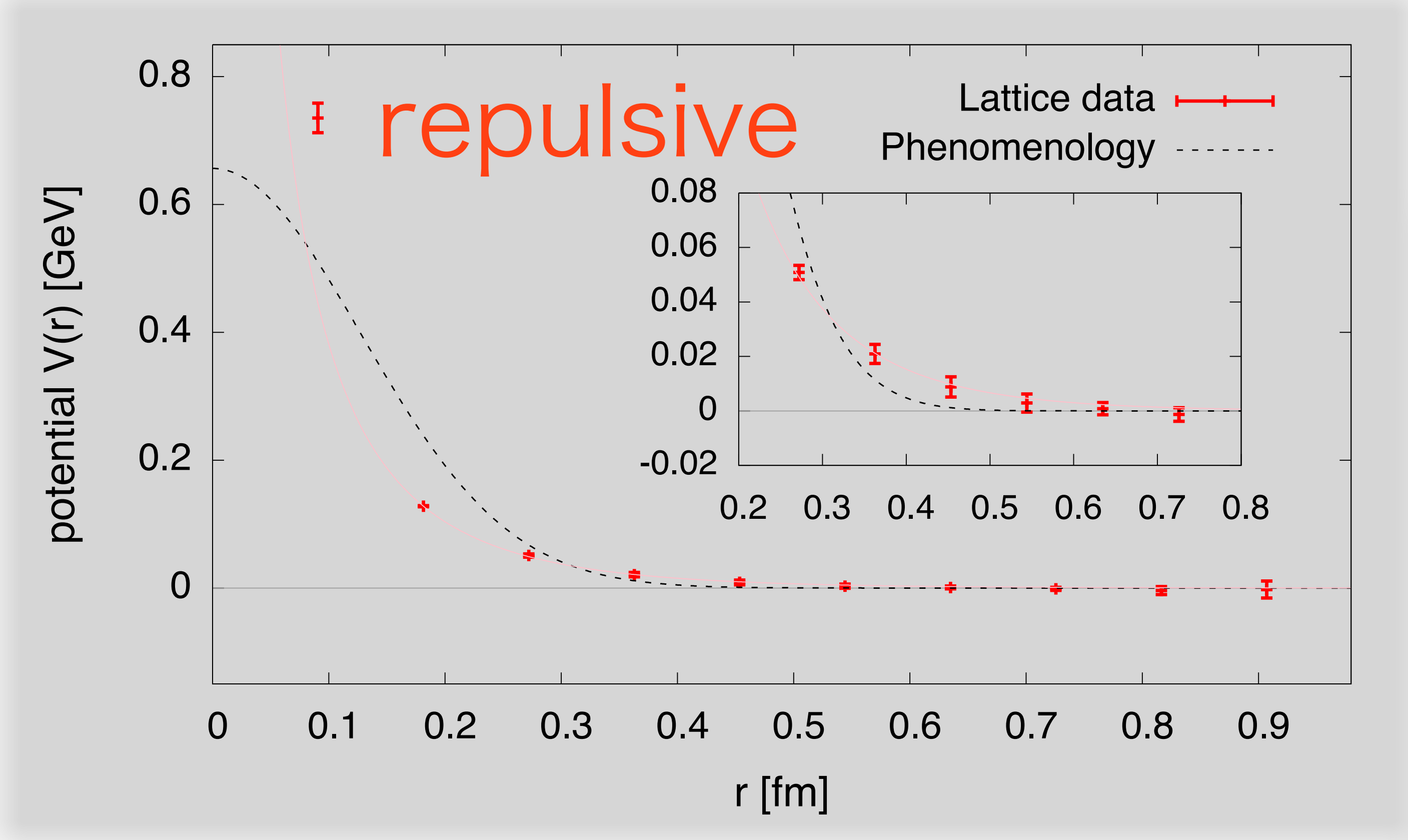
Koma Koma Wittig 05, Koma Koma 06

Comment on spin-spin potential

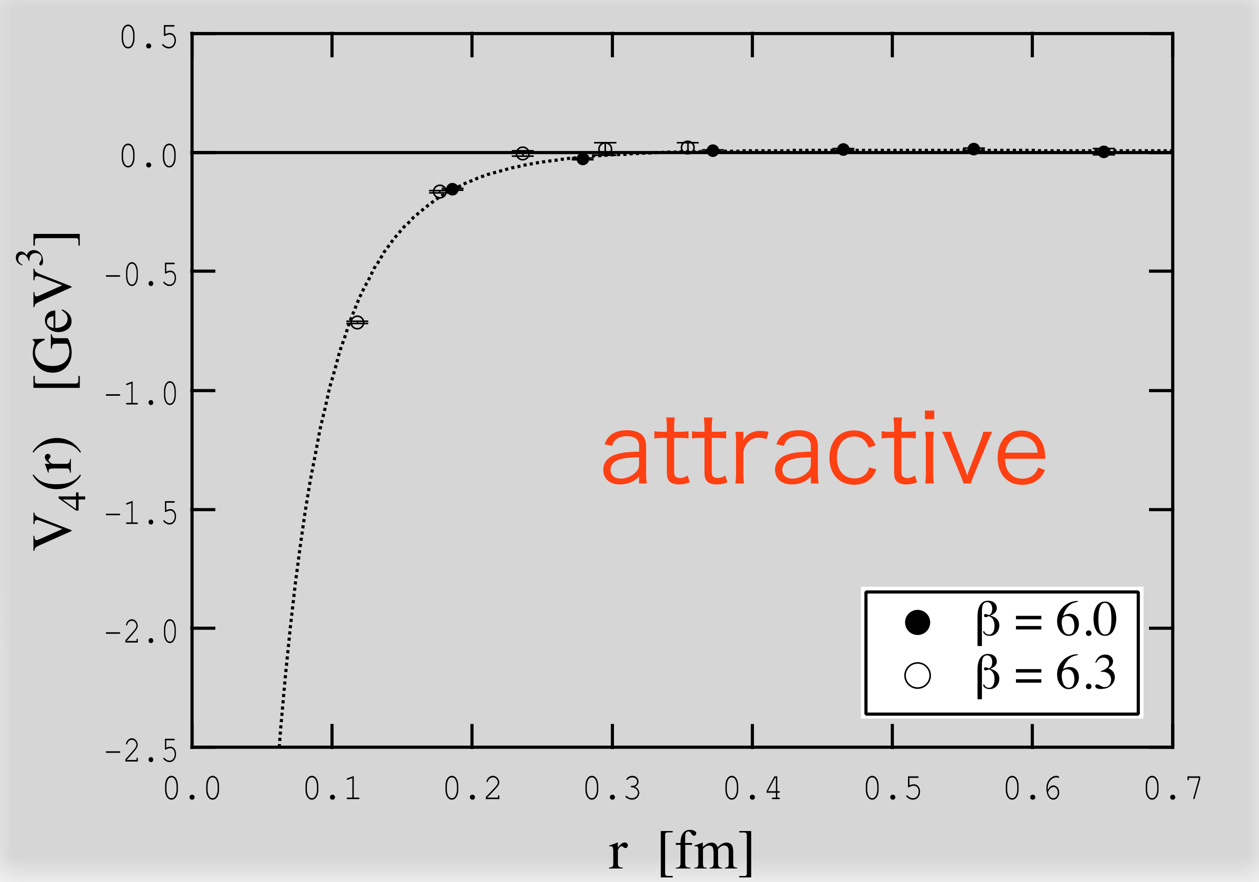
$$V(r) = V_{c\bar{c}}(r) + \mathbf{S}_Q \cdot \mathbf{S}_{\bar{Q}} V_{\text{spin}}(r)$$

Our approach

$$V_{\text{spin}}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$



Wilson loop approach



Note: $M(0^-) < M(1^-)$

Y. Koma and M. Koma, NPB769 (2007) 79

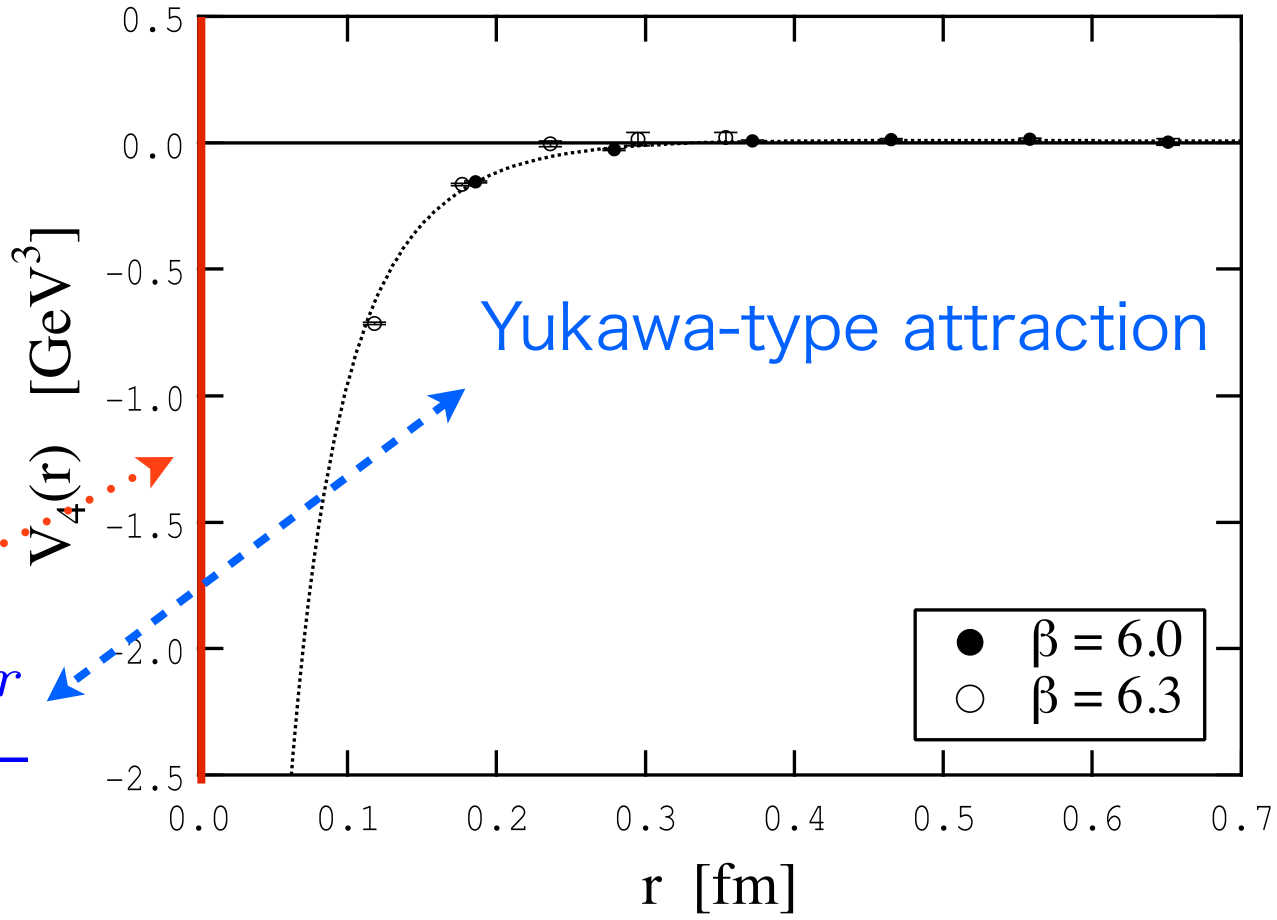
Wilson-loop approach may spoil δ-type repulsive interaction

$$V_{\rm spin}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$

$$V_{c\bar{c}}(r) = \begin{cases} -\dfrac{1}{r} & \text{Coulomb} \\ -\dfrac{e^{-\alpha r}}{r} & \text{Yukawa} \end{cases}$$

$$\nabla^2\left(\frac{e^{-\alpha r}}{r}\right) = -4\pi\delta(r) + \alpha^2\frac{e^{-\alpha r}}{r}$$

$$\nabla^2\left(\frac{1}{r}\right) = -4\pi\delta(r)$$

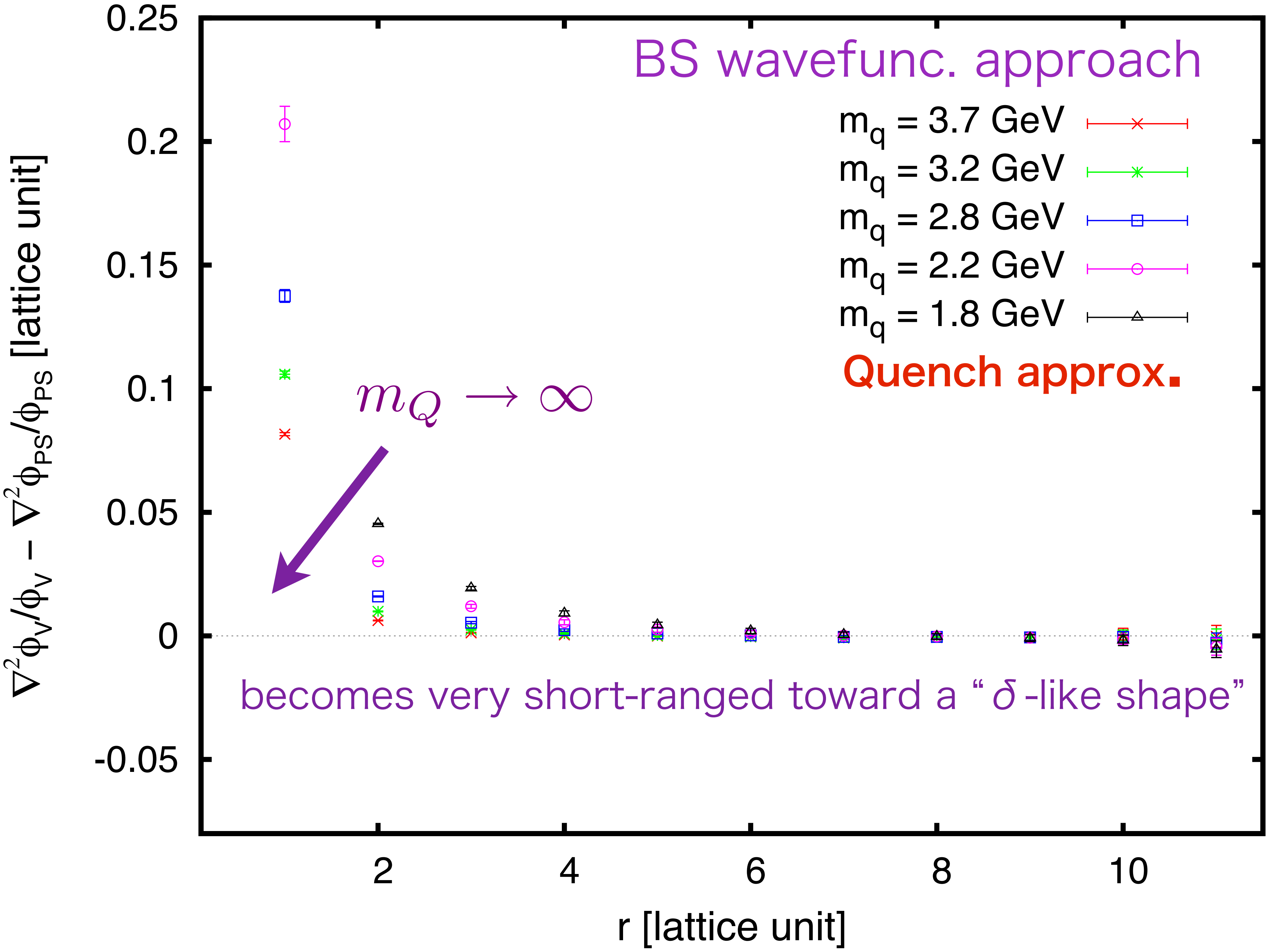


Yukawa-type attraction

origin of repulsive interaction

$$\nabla^2 V_{c\bar{c}} \to V''_{c\bar{c}} + \frac{2}{r}V'_{c\bar{c}}$$ in Wilson-loop approach

Quark mass dependence on spin-spin potential



BS wavefunc. approach

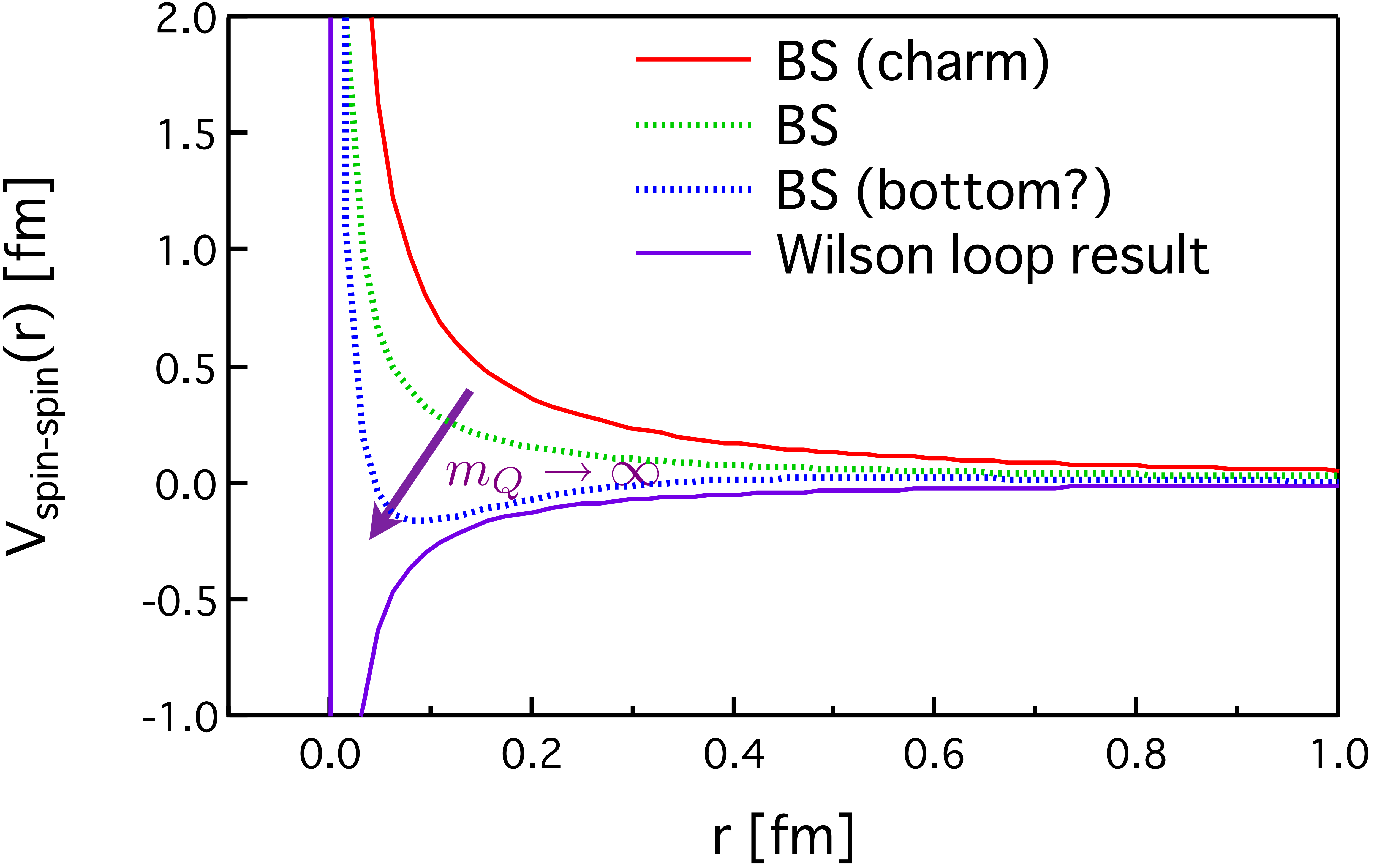
Quench approx.

$m_Q \rightarrow \infty$

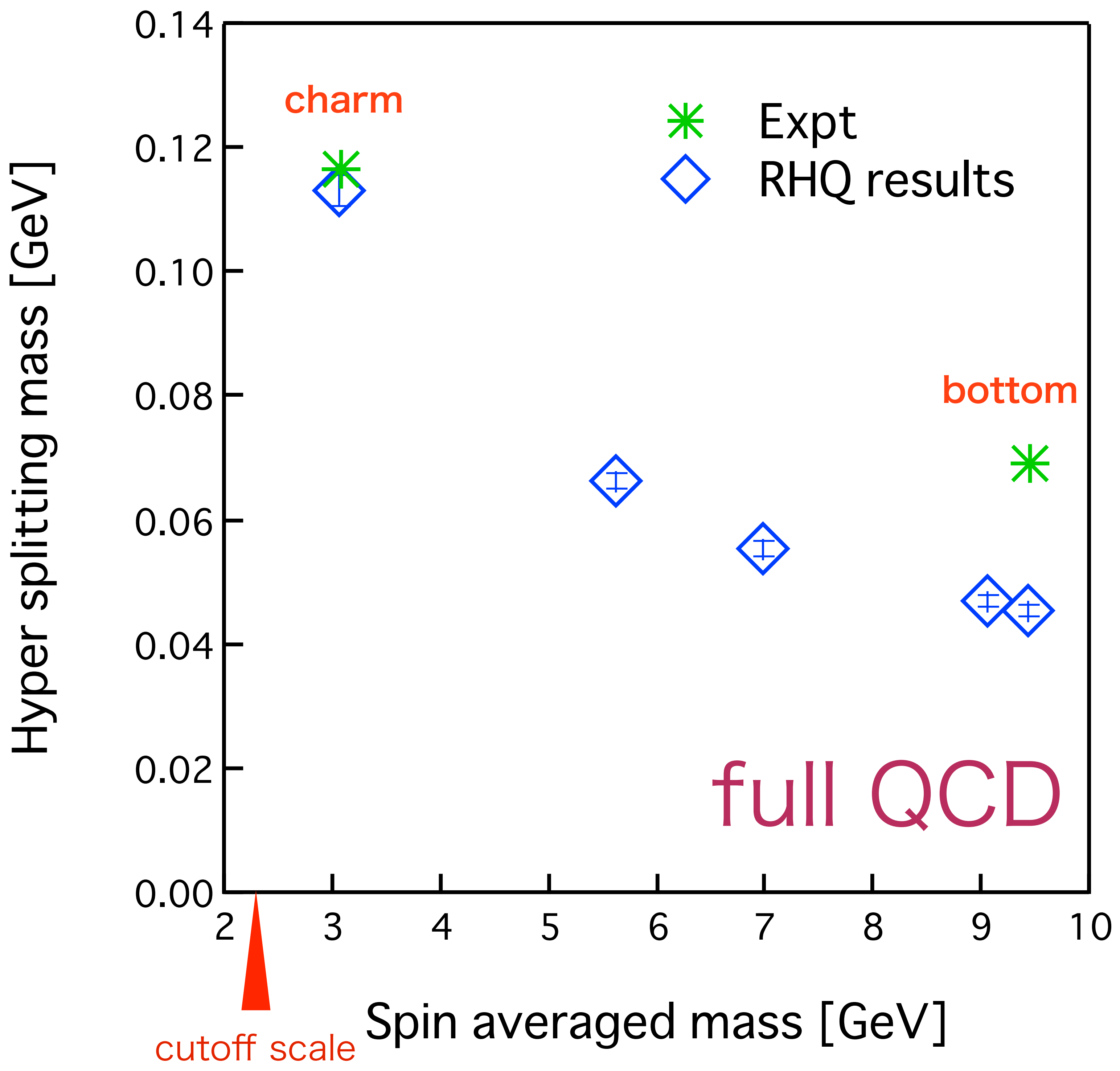
becomes very short-ranged toward a “δ-like shape”

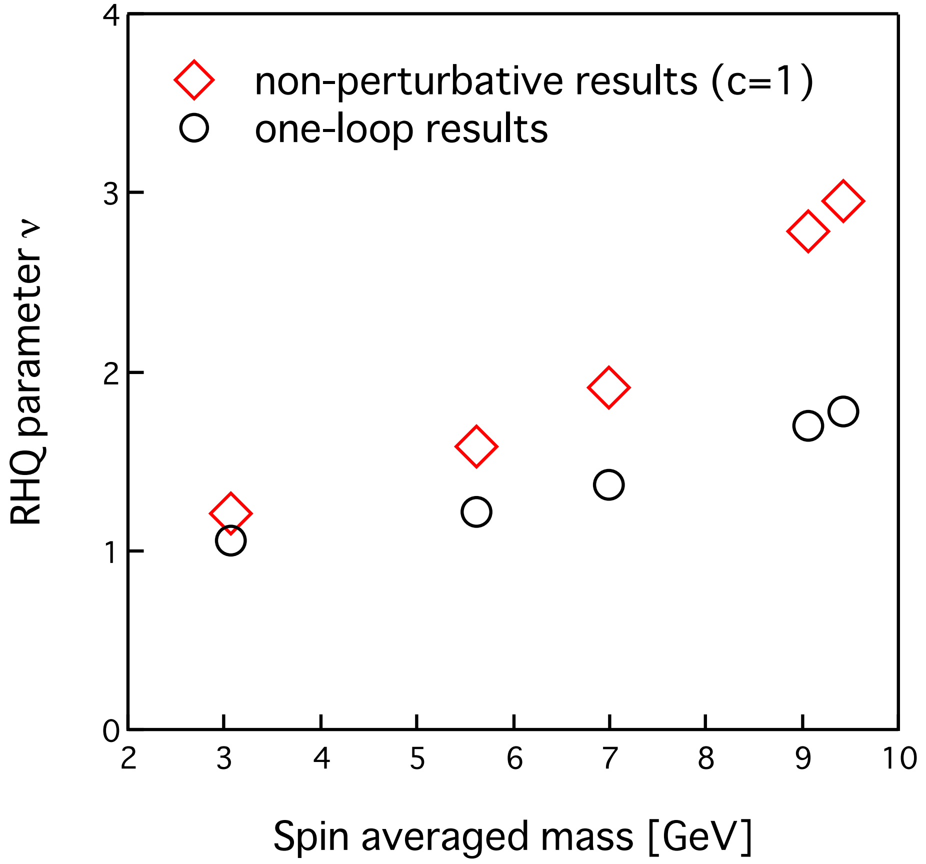
Note: not yet multiplied by a factor of $1/m_Q$!

Our conjecture



Towards the bottomonium system



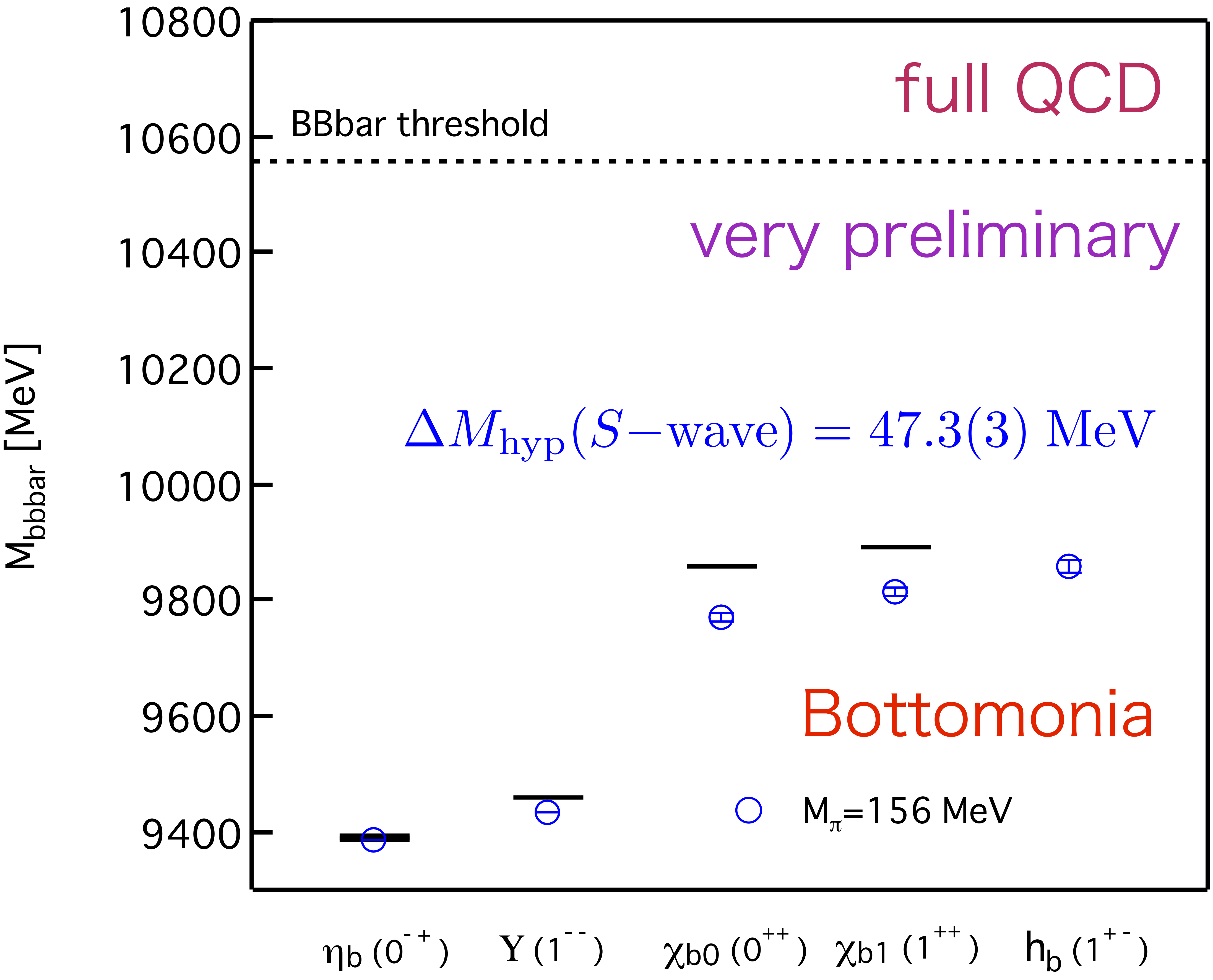


Note:

$$am_{\mathrm{charm}} < 1$$

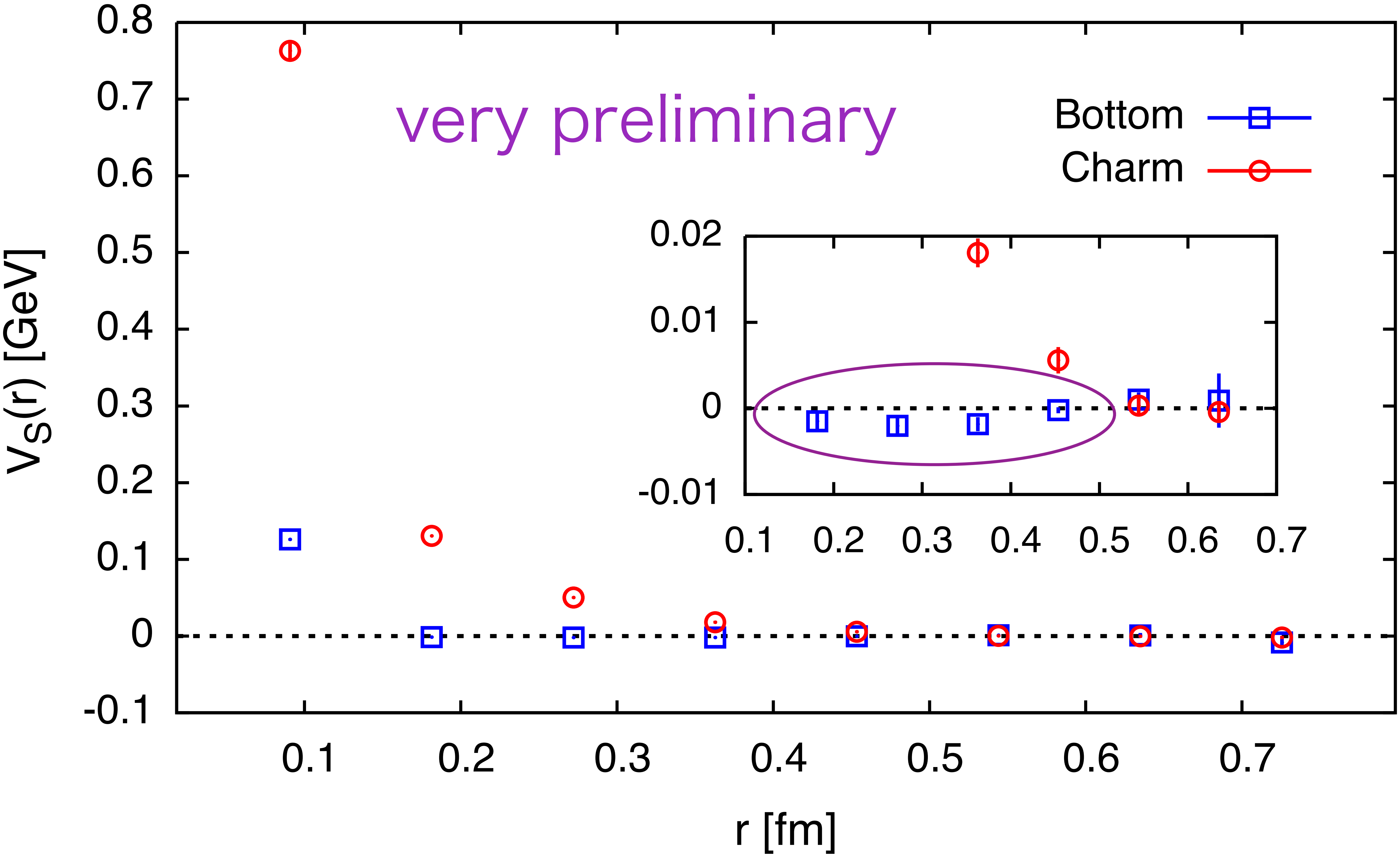
$$am_{\mathrm{bottom}} > 1$$

with $1/a \approx 2.2\ \mathrm{GeV}$



$$\Delta M_{\rm hyp}({\rm Exp.}) = 69.3(2.8)\ {\rm MeV}$$

spin-spin bb^{bar} potential from full QCD



➡ needs a confirmation through lattice cutoff dependence studies

Summary

- **New method to calculate QQ^{bar} potential at finite quark mass**
  - ✓ We propose a self-consistent determination of quark mass from the BS wave function
  - ✓ We confirm that spin-independent potential is consistent with the Wilson loop result in the $m_Q \to \infty$ limit
- **Application to determine charmonium potential in full QCD**
  - ✓ Central potential resembles the NRp model
  - ✓ Spin-spin potential properly exhibits the short range repulsive interaction
  - ✓ Bottomonium potential (now under way)

➡ Improves interquark potentials from lattice QCD

➡ Refines a guideline of “exotic” quarkonia XYZ