Refinement of quark potential models from lattice QCD

格子QCDによるクォーク間ポテンシャルの精密化

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T. Kawanai, SS, PRL 107 (2011) 091601
T. Kawanai, SS, arXiv: 1110.0888
Renaissance of Hadron Spectroscopy

**X(3872)**

**Y(4260)**

**Y(3940)**

**Z(3930)**

**Z(3900)**

**D_{sJ}(2317)**

**D_{sJ}(2457)**

**Z^+(4430)**
Charmonium-like XYZ mesons are discovered.

“Exotic” = “Non-standard”?

XYZ mesons could not be simply explained by a constituent quark description as quark and antiquark bound states.

“Standard” states can be defined in potential models.

→ Does it sound reliable?
Why back to quark potential models?

* Interquark potential in non-relativistic quark potential models

\[
V_{cc} = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r + \frac{32\pi\alpha_s}{9m_q^2} \delta(r) \mathbf{S}_q \cdot \mathbf{S}_\bar{q} + \frac{1}{m_q^2} \left[ \left( \frac{2\alpha_s}{r^3} - \frac{b}{2r} \right) \mathbf{L} \cdot \mathbf{S} + \frac{4\alpha_s}{r^3} T \right]
\]

Cornell potential  
spin-dependent potential

- Spin-spin, tensor, LS terms appear as **corrections in powers of** \(1/m_q\)
- Spin-dependent potentials determined by **one-gluon exchange at tree level**

→ **There are large theoretical ambiguities for higher-mass charmonia**

The reliable interquark potential derived from lattice QCD is hence desired at the charm quark mass
Status of lattice QCD spectroscopy

Nf=2 Clover

lightest pion mass
\[ m_\pi = 0.28 \text{ GeV} \]
lattice cut off
\[ 1/a = 2.6 \text{ GeV} \]

Potential from BS amplitude

- Equal-time BS wave function
  \[ \phi_\Gamma(\mathbf{r}) = \sum_\mathbf{x} \langle 0 | \bar{Q}(\mathbf{x}) \Gamma Q(\mathbf{x} + \mathbf{r}) | \bar{Q} \rangle \]

- Schrödinger eq. with non-local potential
  \[- \frac{\nabla^2}{2\mu} \phi_\Gamma(\mathbf{r}) + \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \phi_\Gamma(\mathbf{r}') = E_\Gamma \phi_\Gamma(\mathbf{r}) \]

- Velocity expansion
  \[ U(\mathbf{r}', \mathbf{r}) = \left\{ V(\mathbf{r}) + V_S(\mathbf{r}) \mathbf{S}_Q \cdot \mathbf{S}_\bar{Q} + V_T(\mathbf{r}) S_{12} + V_{LS}(\mathbf{r}) \mathbf{L} \cdot \mathbf{S} + O(\nabla^2) \right\} \delta(\mathbf{r}' - \mathbf{r}) \]

\(Q\bar{Q}\) potential from BS wave func.

- Ikeda-lida, arXiv:1011.2866 & 1102.2097

\[
\frac{\nabla^2 \phi_{Q\bar{Q}}(r)}{\phi_{Q\bar{Q}}(r)} = m_Q \left[ V(r) - E \right]
\]

Cornell-like behavior!

(a) PS

Inconsistent with the Wilson loops in the \(m_Q \to \infty\) limit
Novel determination of quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[
\left\{-\frac{\nabla^2}{m_Q} + V_\bar{q}q(r) + \mathbf{s}_Q \cdot \mathbf{s}_\bar{q} V_{\text{spin}}(r)\right\} \phi_\Gamma(r) = E_\Gamma \phi_\Gamma(r) \quad \text{for } \Gamma = PS, V
\]

Q. How can we determine a *quark mass* in the Schrödinger equation?

A. Look into asymptotic behavior of wave functions at long distances

\[
V_{\text{spin}}(r) - \Delta E_{\text{hyp}} = \frac{1}{m_Q} \left( \frac{\nabla^2 \phi_V(r)}{\phi_V(r)} - \frac{\nabla^2 \phi_{PS}(r)}{\phi_{PS}(r)} \right)
\]

Under a simple, but reasonable assumption of \(\lim_{r \to \infty} V_{\text{spin}}(r) = 0\)

\[
m_Q = \lim_{r \to \infty} \frac{1}{\Delta E_{\text{hyp}}} \left( \frac{\nabla^2 \phi_{PS}(r)}{\phi_{PS}(r)} - \frac{\nabla^2 \phi_V(r)}{\phi_V(r)} \right)
\]
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[ m_Q = \lim_{r \to \infty} \frac{1}{\Delta E_{\text{hyp}}} \left( \frac{\nabla^2 \phi_{\text{PS}}(r)}{\phi_{\text{PS}}(r)} - \frac{\nabla^2 \phi_{V}(r)}{\phi_{V}(r)} \right) \]

\[ \lim_{r \to \infty} V_{\text{spin}}(r) = 0 \]

\[ -m_Q \Delta E_{\text{hyp}} \]
Interquark potential at finite quark mass

- Kawanai-Sasaki, PRL 107 (2011) 091601

\[ V_{Q\bar{Q}}(r) = -\frac{A}{r} + \sigma r + V_0 \]

Quench + RHQ

Consistent with the Wilson loops in the \( m_q \to \infty \) limit
How to treat heavy quarks

- Heavy quark mass introduces discretization errors of $O((ma)^n)$

- At charm quark, it becomes severe:
  
  $$m_c \sim 1.5 \text{ GeV and } 1/a \sim 2 \text{ GeV, then } m_c a \sim O(1)$$

- Relativistic heavy quark (RHQ) approach:


- All $O((ma)^n)$ and $O(a^\Lambda)$ errors are removed by the appropriate choice of six canonical parameters \{$m_0, \zeta, r_t, r_s, C_B, C_E$\}

  $S_{\text{lat}} = \sum_{n,n'} \bar{\psi}_n K_{n,n'} \psi_{n'}$

  $K = m_0 + \gamma_0 D_0 + \zeta \gamma_i D_i - \frac{r_t}{2} D_0^2 - \frac{r_s}{2} D_i^2 + C_B \frac{i}{4} \sigma_{ij} F_{ij} + C_E \frac{i}{2} \sigma_{0i} F_{0i}$

  explicit breaking of axis-interchange symmetry

- We follow the Tsukuba procedure to determine parameters

  S. Aoki, Y. Kuramashi, S.-I. Tominaga (1999)
Tuning RHQ parameters for full QCD

- RHQ action (Tsukuba-type) with 5 parameters
  - PACS-CS configurations at $m_\pi=156$ MeV
  - Relativistic Heavy Quark (RHQ) action for charm
    - $32^3 \times 64$ lattice
    - $a = 0.0907(13)$ fm
    - $L_a \sim 2.9$ fm
    - 198 configs
    - $\frac{1}{4} (M_{\eta_c} + 3M_{J/\psi}) = 3.070(1)$ GeV
    - $\Delta M_{hyp} = 114(1)$ MeV
    - $c_{\text{eff}}^2 = 1.04(5)$

Namekawa et al., (PACS-CS), arXiv:1104.4600
Charmonium potential from full QCD

- Kawanai-Sasaki, arXiv:1110.0888

* PACS-CS configurations at $m_\pi=156$ MeV

\[ V(r) = -\frac{A}{r} + \sigma r + V_0 \]

- $A_{c\bar{c}} = 0.813(22)$
- $\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV
- $A_\infty = 0.403(10)$
- $\sqrt{\sigma_\infty} = 0.462(2)$ GeV

Polyakov line correlator (off-axis) - BS wave function (off-axis)

Polyakov loop correlator
Charmonium potential from full QCD

Kawanai-Sasaki, arXiv:1110.0888

### Spin-independent $c\bar{c}$ potential

- **lattice results**
  - $A_{c\bar{c}} = 0.813(22)$
  - $\sqrt{\sigma_{c\bar{c}}} = 0.394(7)$ GeV

- **NR quark model**
  - $A_{\text{NRp}} = 0.7281$
  - $\sqrt{\sigma_{\text{NRp}}} = 0.3775$ GeV

### spin-spin $c\bar{c}$ potential

- $V_S(r) = \begin{cases} \alpha \exp(-\beta r)/r & : \text{Yukawa form} \\ \alpha \exp(-\beta r) & : \text{Exponential form} \\ \alpha \exp(-\beta r^2) & : \text{Gaussian form.} \end{cases}$

### Table III: Fitting function to spin-spin charmonium potential employed in a NRp model

<table>
<thead>
<tr>
<th>functional form</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\chi^2$/dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yukawa-type</td>
<td>0.287(8)</td>
<td>0.894(32) GeV</td>
<td>7.28</td>
</tr>
<tr>
<td>Exponential-type</td>
<td>0.825(19) GeV</td>
<td>1.982(24) GeV</td>
<td>1.46</td>
</tr>
<tr>
<td>Gaussian-type</td>
<td>0.314(4) GeV</td>
<td>1.020(11) GeV$^2$</td>
<td>22.79</td>
</tr>
</tbody>
</table>

Non-relativistic potential model

T.Barnes, S. Godfrey, E.S. Swanson, PRD72 (2005) 054026

Refinement of spin-dependent potentials → change the fine structure of charmonia
Comment on two topics

- Revisit of “quark mass”
- Spin-spin potential issue in the Wilson loop approach
What does “quark mass” correspond to?

\[ \langle Q(t)\overline{Q}(0) \rangle_{\text{GF}} \propto e^{-m_Q t} \]

- Quark propagator (Landau gauge)
- BS wavefunction (Coulomb gauge)

\[
m_{\text{charm}}(\text{pole}) = 1.748(6) \text{ GeV}
\]
\[
m_{\text{charm}}(\text{BS}) = 1.74(3) \text{ GeV}
\]

Spatial information = Temporal information
Spin-dependent potentials

$V_{LS}$

$V_{TS}$

$V_{SS}$

Koma Koma Wittig 05, Koma Koma 06
Comment on spin-spin potential

\[ V(r) = V_{cc}(r) + S_Q \cdot S_{\bar{Q}} V_{\text{spin}}(r) \]

Our approach

\[ V_{\text{spin}}(r) \propto \nabla^2 V_{cc}(r) \]

Wilson loop approach

Note: \( M(0^-) < M(1^-) \)

Y. Koma and M. Koma, NPB769 (2007) 79
Wilson-loop approach may spoil $\delta$-type repulsive interaction

$$V_{\text{spin}}(r) \propto \nabla^2 V_{c\bar{c}}(r)$$

$$V_{c\bar{c}}(r) = \begin{cases} -\frac{1}{r} & \text{Coulomb} \\ -\frac{e^{-\alpha r}}{r} & \text{Yukawa} \end{cases}$$

\[\nabla^2 \left( \frac{e^{-\alpha r}}{r} \right) = -4\pi \delta(r) + \alpha^2 \frac{e^{-\alpha r}}{r}\]

\[\nabla^2 \left( \frac{1}{r} \right) = -4\pi \delta(r)\]

The origin of the repulsive interaction.

$$\nabla^2 V_{c\bar{c}} \rightarrow V''_{c\bar{c}} + \frac{2}{r} V'_{c\bar{c}}$$ in Wilson-loop approach
Quark mass dependence on spin-spin potential

\[ \frac{\nabla^2 \phi_V}{\phi_V} - \frac{\nabla^2 \phi_{PS}}{\phi_{PS}} \] [lattice unit]

\[ m_Q \to \infty \]

becomes very short-ranged toward a “\( \delta \)-like shape”

Note: not yet multiplied by a factor of \( 1/m_Q \)!
Our conjecture

![Graph showing the spin-spin potential $V_{\text{spin-spin}}(r)$ as a function of the distance $r$. The graph includes curves for BS (charm), BS, BS (bottom?), and Wilson loop result. The curves show the potential behavior as $m_Q \to \infty$.](image)
Towards the bottomonium system

Note:

\[ am_{\text{charm}} < 1 \]

\[ am_{\text{bottom}} > 1 \]

with \( 1/a \approx 2.2 \text{ GeV} \)
\[ \Delta M_{\text{hyp}}(S-wave) = 47.3(3) \text{ MeV} \]

\[ \Delta M_{\text{hyp}}(\text{Exp.}) = 69.3(2.8) \text{ MeV} \]
spin-spin $bb^{bar}$ potential from full QCD

very preliminary

needs a confirmation through lattice cutoff dependence studies
Summary

- New method to calculate $\text{QQ}^{\text{bar}}$ potential at finite quark mass
  - We propose a self-consistent determination of quark mass from the BS wave function
  - We confirm that spin-independent potential is consistent with the Wilson loop result in the $m_\text{Q} \to \infty$ limit

- Application to determine charmonium potential in full QCD
  - Central potential resembles the NRp model
  - Spin-spin potential properly exhibits the short range repulsive interaction
  - Bottomonium potential (now under way)

→ Improves interquark potentials from lattice QCD
→ Refines a guideline of “exotic” quarkonia XYZ