有限密度格子QCDの研究
-高密度への挑戦-

K. Nagata,
Hiroshima Univ., RIISE
S. Motoki (KEK), A. Nakamura (Hiroshima)
in XQCD-J collaboration

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N. E. C. O.

Contents

- *Introduction*
- *Reduction formula for Wilson fermion determinant*

- *Application of reduction formula*
  1. *Taylor expansion*
  2. *Lee-Yang zeros*

  Imaginary chemical potential

- *Summary*

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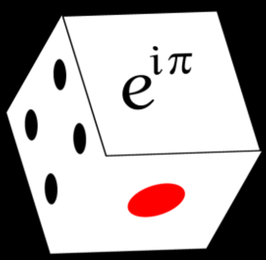
QCD at finite temperature and density

- *nature of matter in our universe*
  - *ordinary : hadron, nuclei, nuclear matter*
  - *dense : compact stars*
  - *hot : HIC, creation of matter in early universe*

- *QCD is non-perturbative.*

- *Finite density LQCD : sign problem*



To reach at low T & high mu

- *Several methods.*
  - *Most studies have been done for regions near Tc.*
  - *Each method has its own difficulty*
    - *Taylor : truncation error*
    - *MPR : overlap & sign problems*
    - *Imaginary : uncertainty of functional form*
    - *etc.....*
- *We have been studied those methods,*
  - *Wilson fermion, formula for determinant, real & imaginary*

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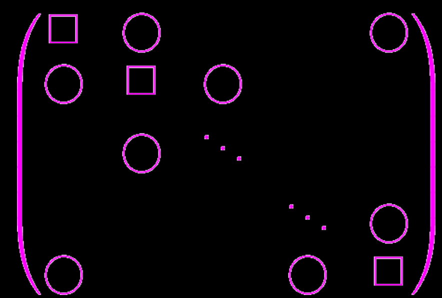
Fermion determinant

$$Z(\mu) = \int \mathcal{D}U (\det \Delta(\mu))^{N_f} e^{-S_G}$$

- *effects of chemical potential*
- *sign problem*
- *numerical hot spot*

A formula for determinant

$$Z(\mu) = \int \mathcal{D}U (\det \Delta(\mu))^{N_f} e^{-S_G}$$



- *Quark action as t-t matrix*
  - *temporal hop accompanies chemical potential*
  - *spatial (diag), temporal (n.n + b.c)*
- *Performing temporal determinant by hand*
  - *rank reduces to N/Nt (memory & CPU time ~1/Nt^2)*
  - *det D is an analytic function of mu.*
- *Reduction formula enables us*
  - *exact evaluation of determinant (useful for large mu)*
  - *for arbitrary values of chemical potential*
  - *suppress CPU time for low T (T=1/a Nt)*

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Reduction formula

$$\det \Delta(\mu) = C_0 e^{(N_r/2)\mu/T} \det(Q + e^{-\mu/T})$$

- *Q : rank = Nr = 4 Nc Ns^3 matrix (Nr = N/Nt)*
  - *function of link variables*
  - *independent of chemical potential*

Wilson type : KN&AN, PRD82,094027 ('10),

$$\det \Delta(\mu) = C_0 e^{(N_r/2)\mu/T} \prod_{n=1}^{N_r} (\lambda_n + e^{-\mu/T})$$

- *It can be used for many applications*

Taylor expansion

$$\frac{p\,(\mu, T)}{T^4} = c_0 + c_2(\mu/T)^2 + c_4(\mu/T)^4 + \cdots$$

- *Coefficients are calculated at mu=0 , and functions of T.*

$$c_n(T) = \frac{1}{n!} T^n \frac{\partial^n}{\partial \mu^n} \frac{p\,(\mu = 0)}{T^4}$$

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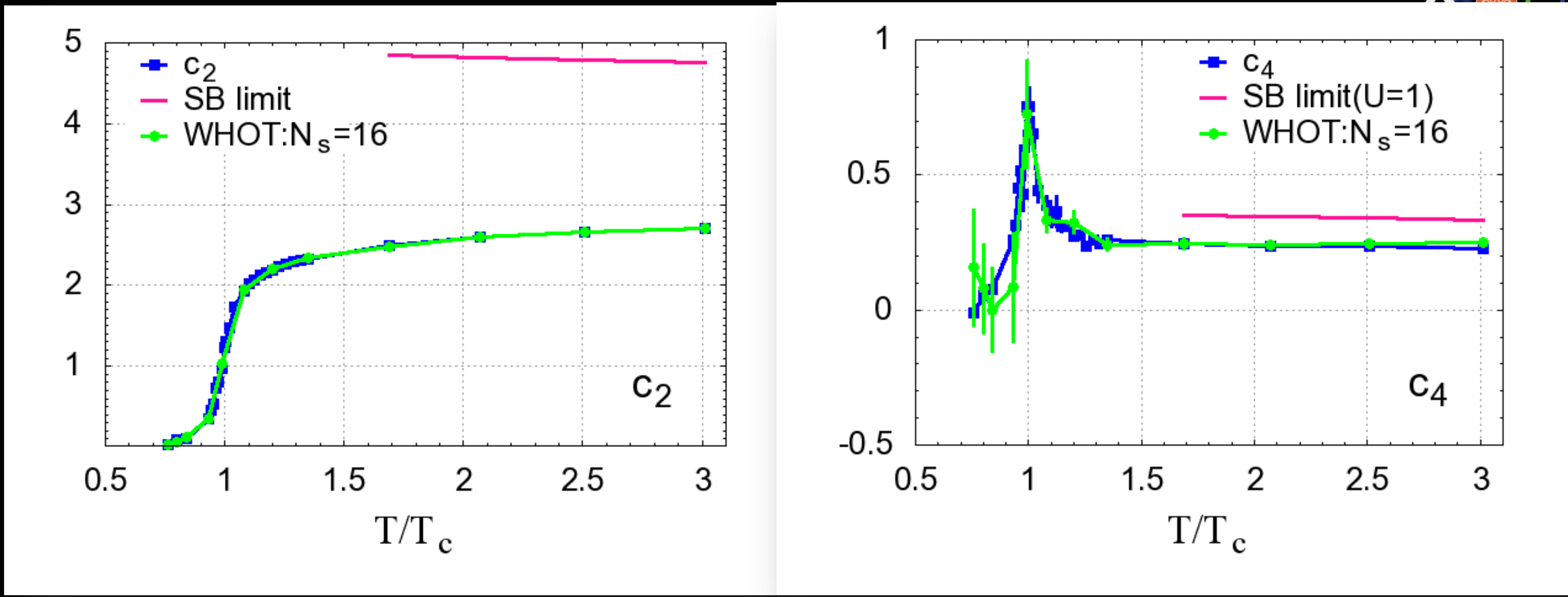
Taylor coefficients



Data of "WHOT" are taken from WHOT, Ejiri et al., PR**D82**, 014508 (2010), arXiv:0909.2121

clover-Wilson + RG-gauge(Nf=2)

Volume : 8^3x4

quark mass : mps/mV ~ 0.8

Configurations : HMC at mu=0

11 K steps including (therm. as 3-5K)

Eigen values : 400 configs.

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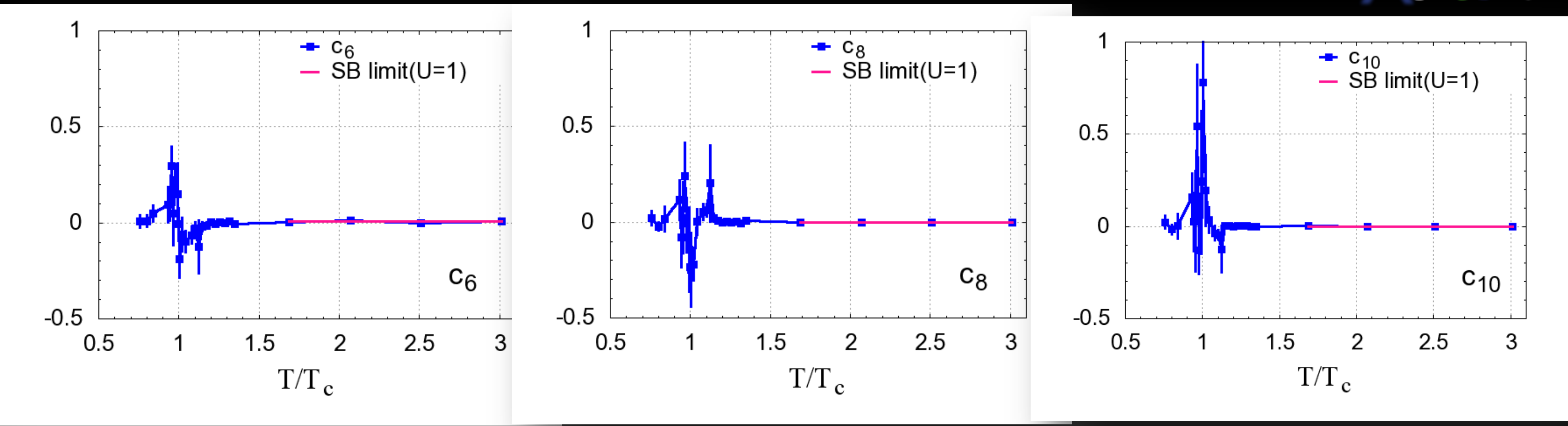
Taylor coefficients



- *At high T, almost converge up to O((mu/T)^4).*
- *Near and below Tc, slow convergence. Higher order terms are non-negligible.*

We are preparing for higher order coefficients and those at low T

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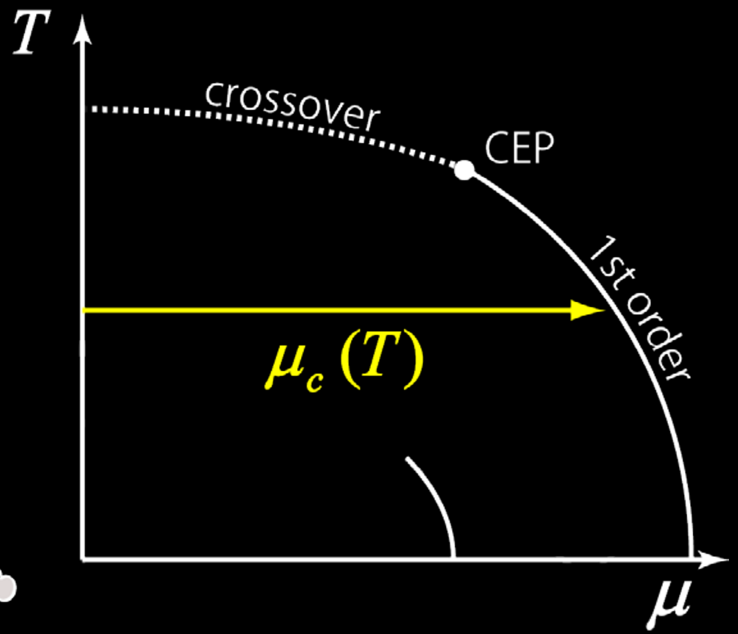
EoS & Phase boundary

Phase boundary can be obtained from convergence radius (in principle)

$$r(T) = \lim_{n\to\infty}\left|\frac{c_n}{c_{n+2}}\right|$$



$$\mu_c = T\sqrt{r(T)}$$

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Lee-Yang zeros

- *Zeros of partition function (Lee-Yang) implies phase transtions*

$$Z(\xi) \to 0$$

$$F = -T \ln Z(\xi) \qquad \frac{\partial}{\partial\mu} F = -T \frac{1}{Z} \frac{\partial Z}{\partial \mu}$$

$$\xi = \exp(-\mu/T)$$

- *LY zeros are distributed on the complex fugacity plane.*
- *LY zeros approach to Re[xi] AXIS as V -> 0.*

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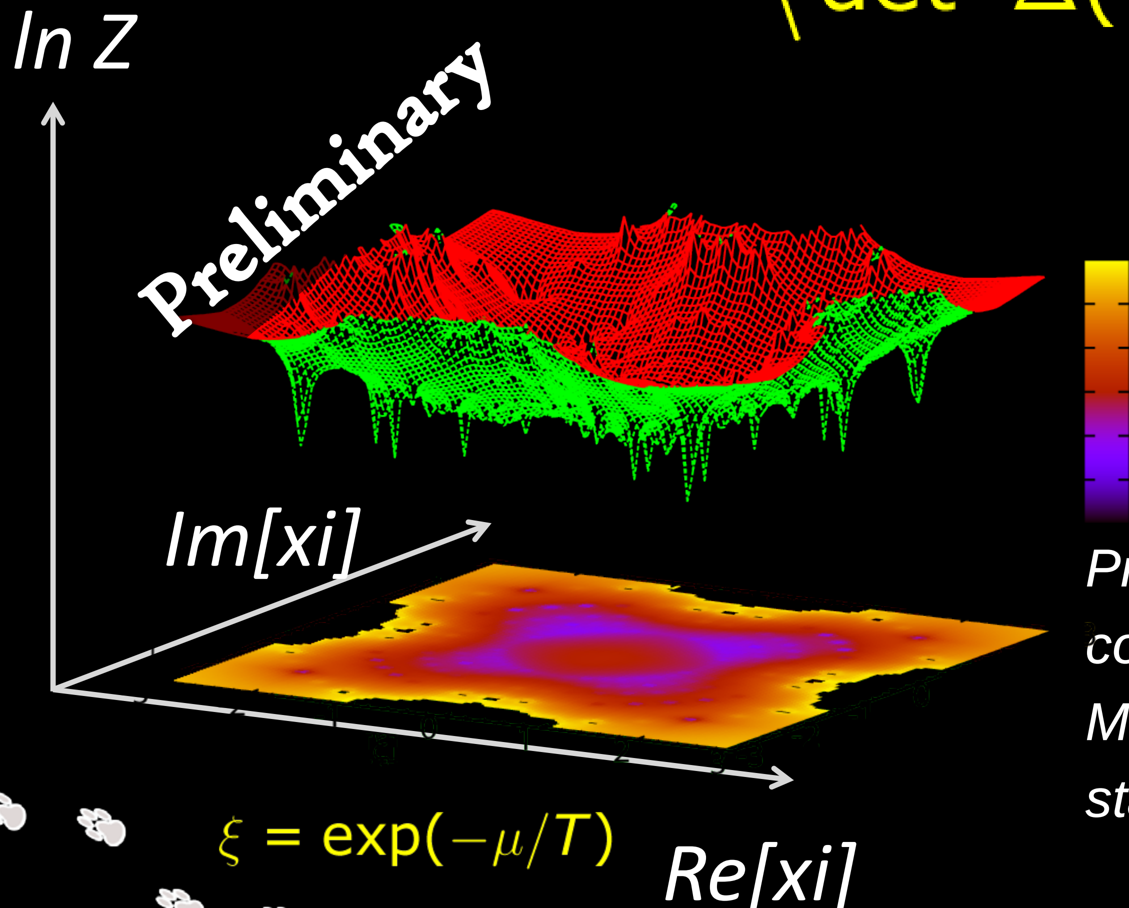
Lee-Yang zeros

$$Z(\xi) = \left\langle \frac{\det^2 \Delta(\mu)}{\det^2 \Delta(0)} \right\rangle_0$$

$$F = -T \ln Z(\xi)$$



$$\xi = \exp(-\mu/T)$$

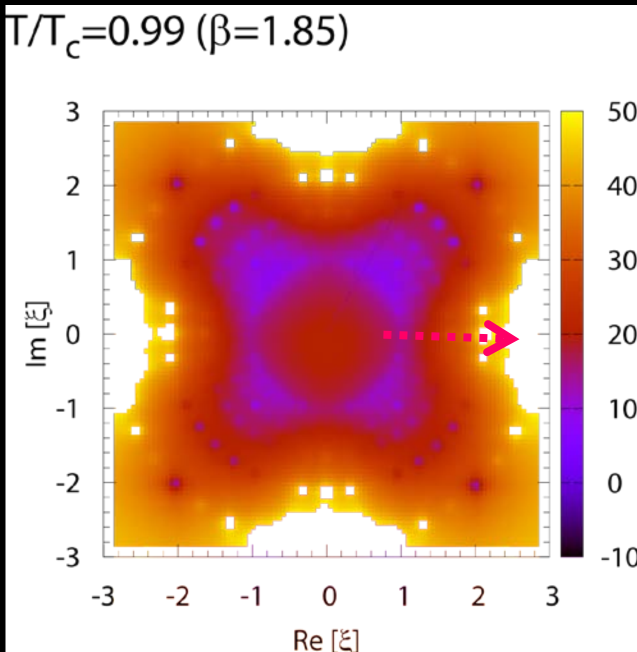
Preliminary data :

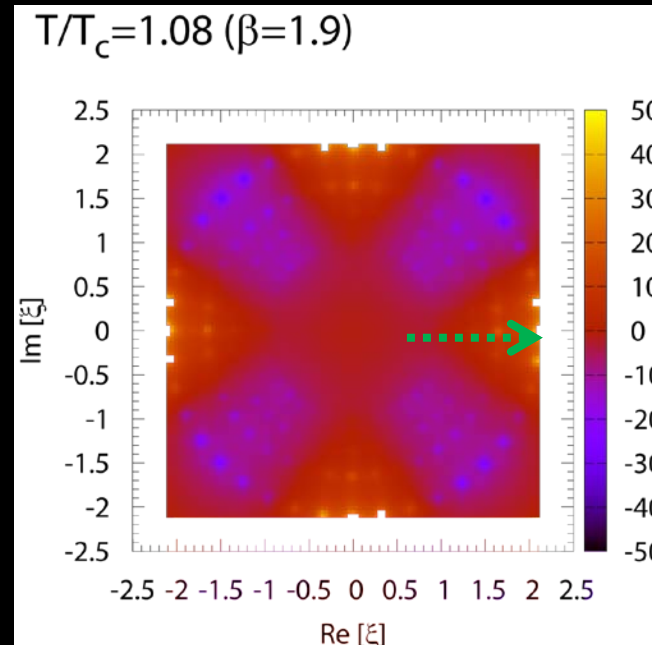
config : clover-Wilson

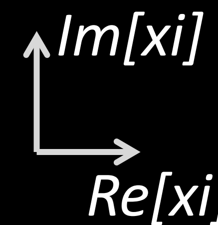
Measurement : w/o clover-term

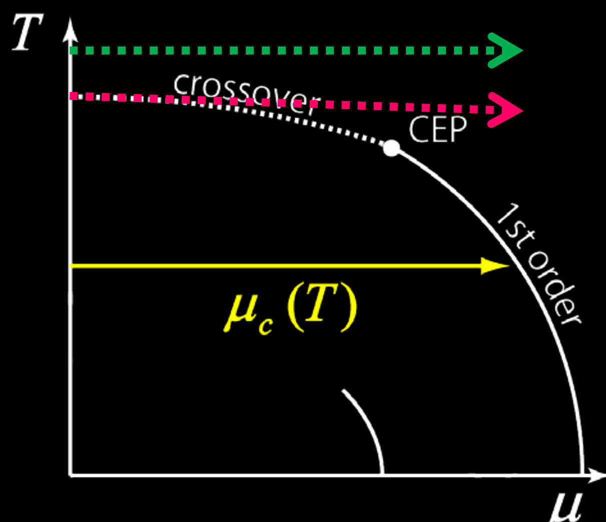
statistics 100

Lee-Yang zeros









quark : inside unit circle

anti-quark : outside unit circle

Imaginary chemical potential

$$Z(\mu) = \int \mathcal{D}U (\det \Delta(\mu))^{N_f} e^{-S_G}$$

- *Sign problem is absent for pure imag. mu*
  - *Generating configs. at imaginary mu*
  - *approach to real mu*
    - *analytic continuation*
    - *canonical approach*

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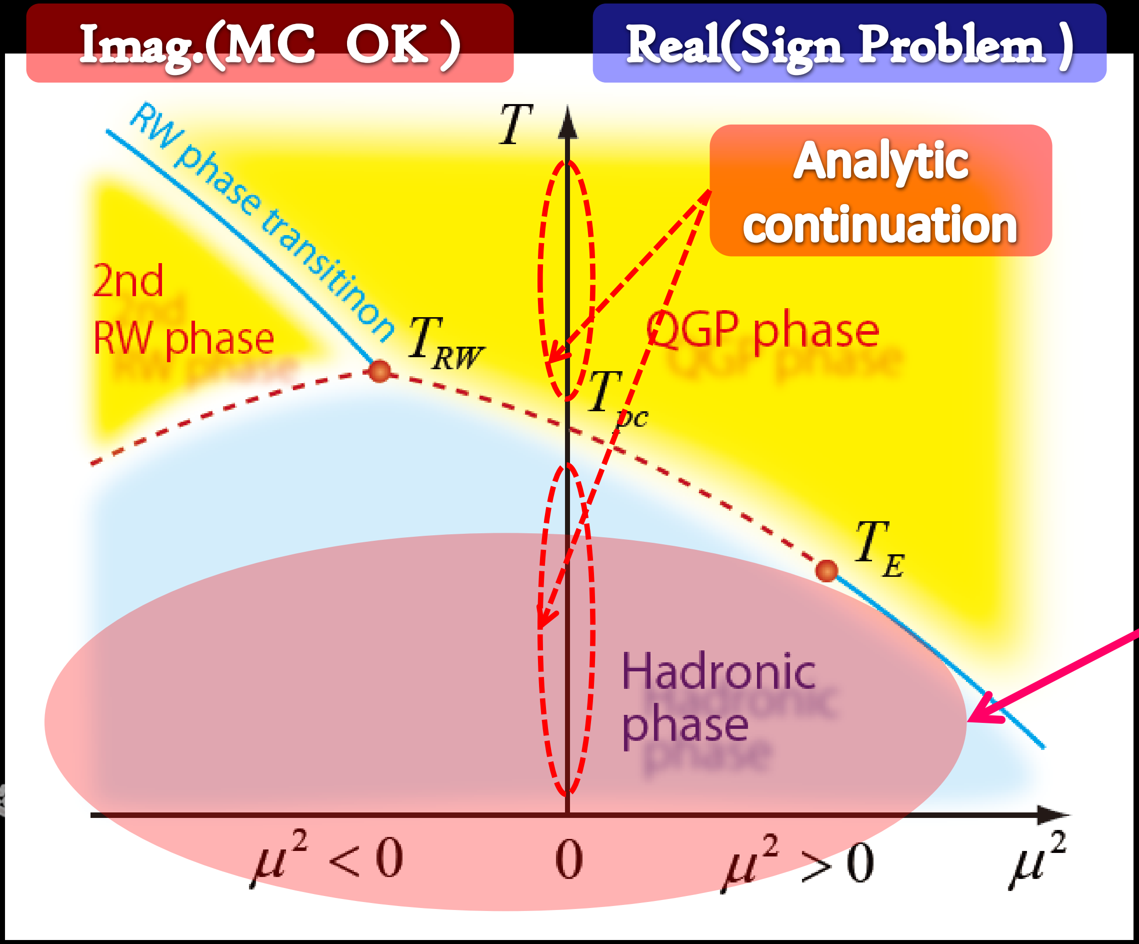
Imaginary chemical potential

- MC simulations in mu^2<0 region and analytic continuation
- This approach may be useful for the study of low-T region.



For properties of this phase diagram e.g. KN&AN, PRD83,114507 , arXiv:1104.2142.

- wide regions
- RW periodicity

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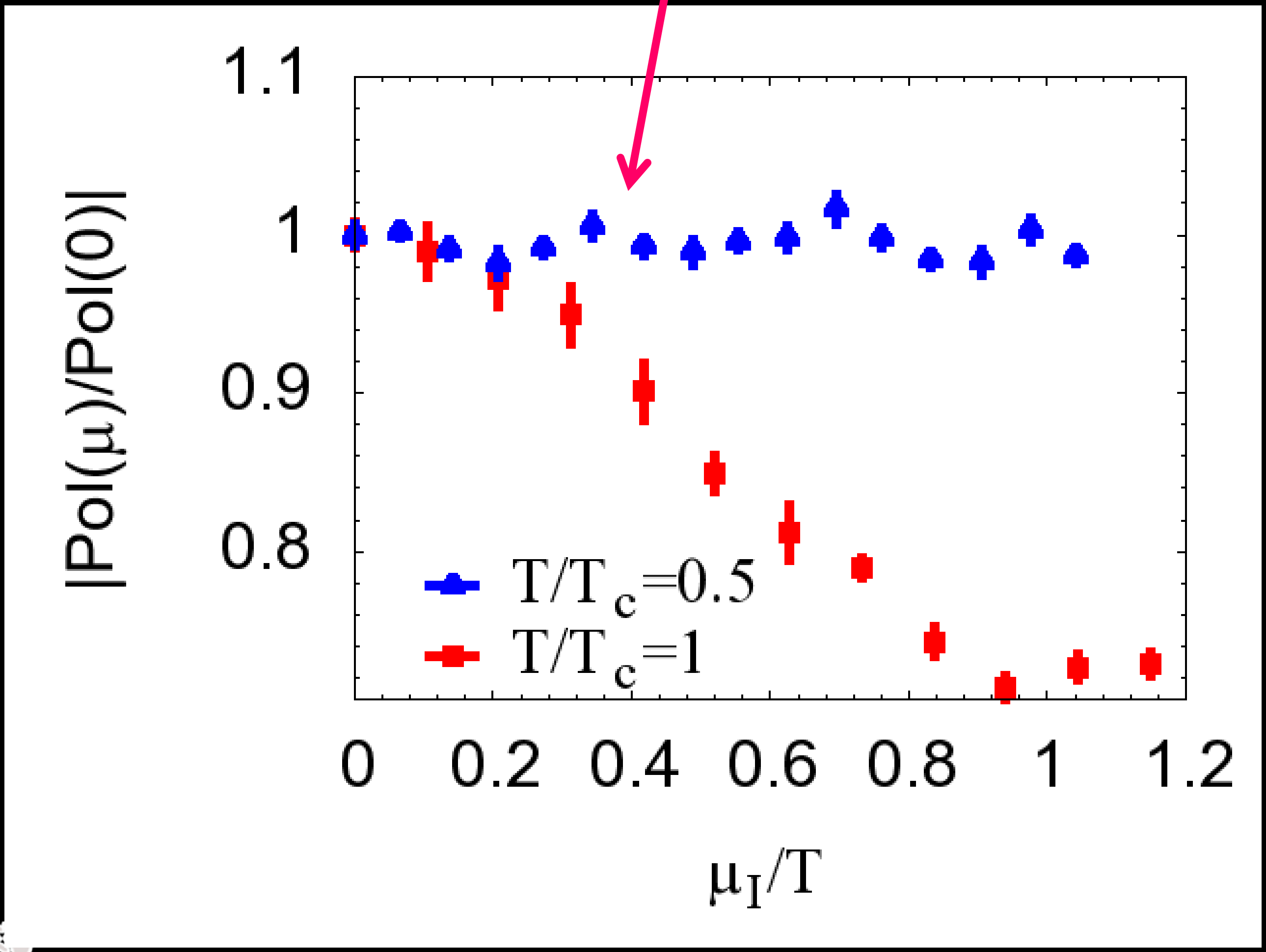
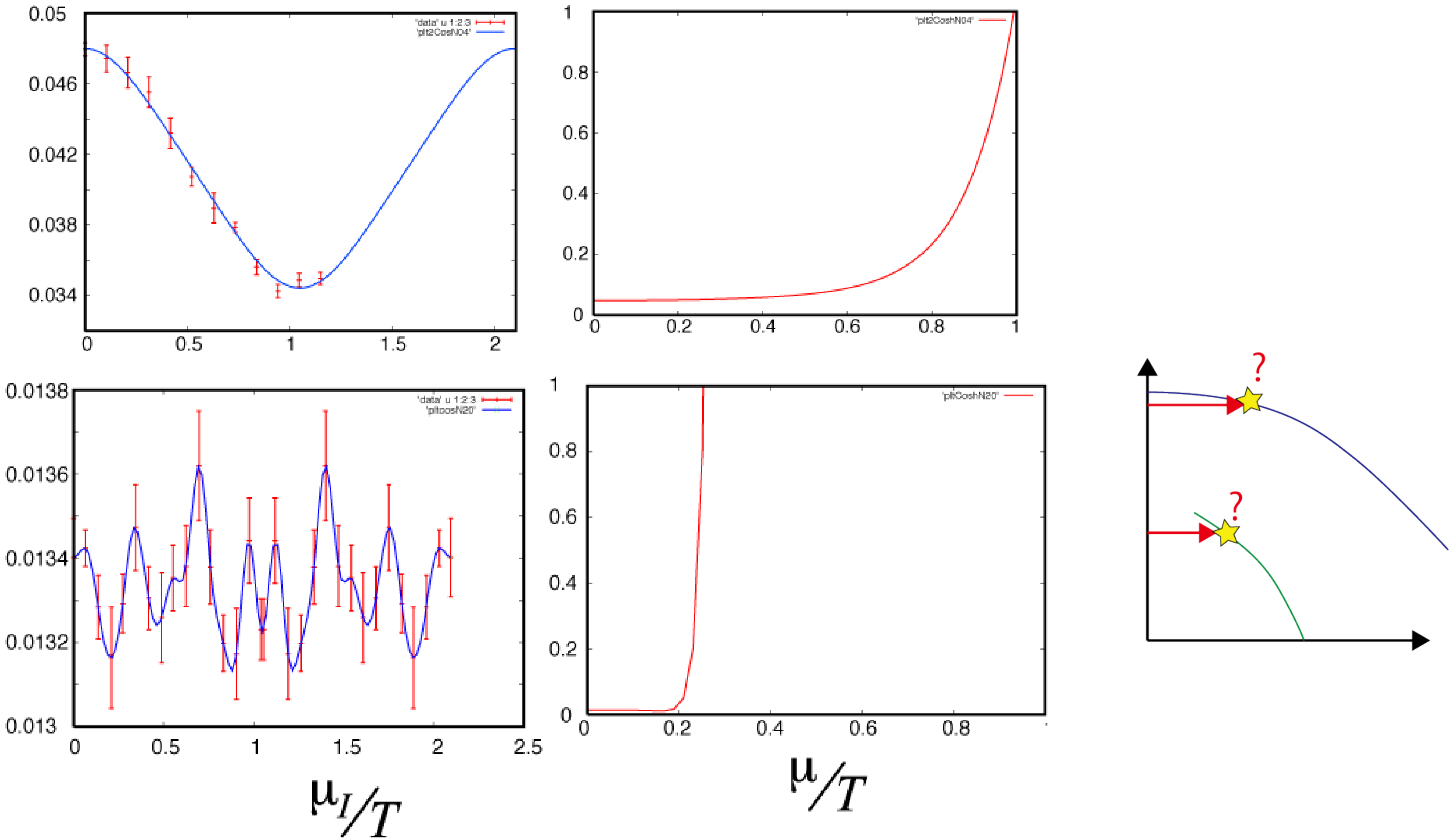
Polyakov loop at low T

We found the effect of chemical potential at T/Tc=0.5.



Fit & analytic continuation

RW periodicity -> Fourier series (cos) -> cosh



Consistency

- *Each approach has its own difficulty, consistency check is helpful*

$$Z(\mu) = \int \mathcal{D}U (\det \Delta(\mu))^{N_f} e^{-S_G}$$

- *Taylor : truncation error*
- *MPR : overlap & sign problems*
- *Imaginary : uncertainty of functional form*

- *They would be consistent if calculations are done in enough precision.*

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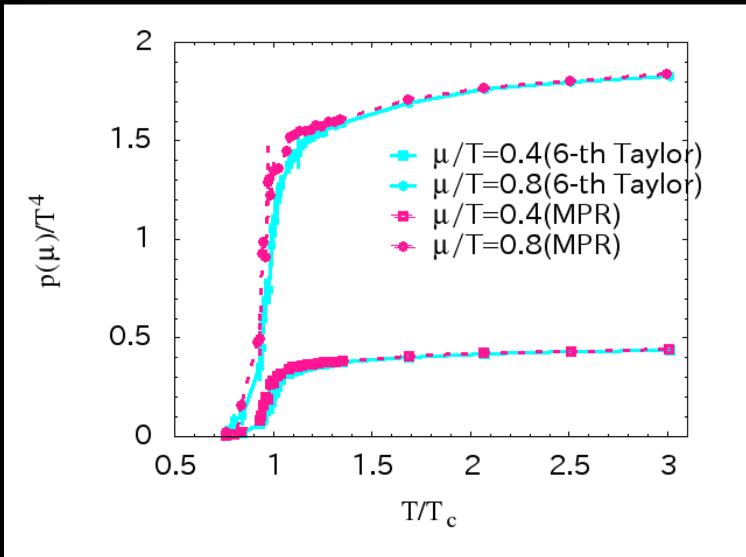
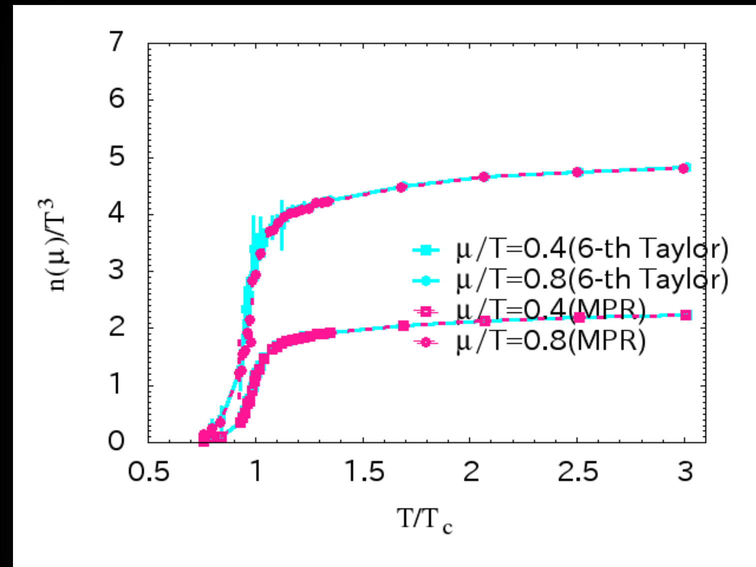
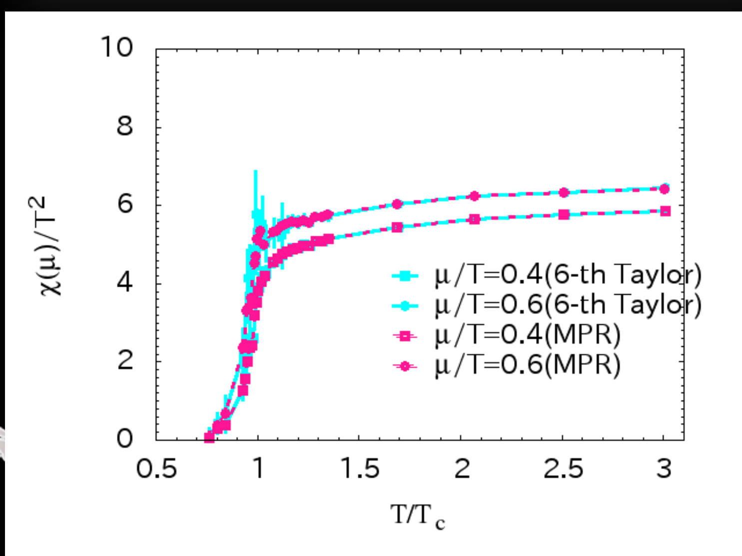
Consistency







- *Taylor and MPR are consistent*
  - *consistent up to mu/T = 0.6*
  - *Taylor : truncation error*
  - *MPR : overlap & sign problem*

Summary

- *We are studying several ideas towards low T & high mu*
  - *Taylor, Lee-Yang, Imaginary etc*
  - *phase boundary, EoS ...*

- *Further investigations are in progress*
  - *larger volume, increasing statistics , low T (T=0.5Tc-Tc)*

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